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# EVALUATION OF FLOOD FORECASTING-RESPONSE SYSTEMS 

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## Technical Reports on Natural Resource Systems

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The value of a forecast system in preventing urban property damage depends on the accuracy of the forecasts, the time at which they are received, the response by the floodplain dweller and the efficacy of that response. A systems model of the overall flood forecast-response system is developed. Evaluation of the system is accomplished by a decision theoretic methodology. A case study is done for Milton, Pennsylvania, which evaluates the present system and potential changes to it. It is concluded that the sequential nature of the forecast sequence must be considered in modeling the flood forecast-response system if a meaningful evaluation of the economic value of the system is to be obtained. Methodology for obtaining the parameterization of the model from the available data is given. Computer programs have been written to handle a good portion of the calculations. While more work is needed on obtaining accurate parameterization of certain parts of the model, such as the actual response to forecasts; use of the procedures and programs as they now stand produces reasonable evaluations.

## EXTENDED ABSTRACT

A stochastic model of a Flood Forecast-Response process (abbreviated henceforth FFR process) has been developed. The purpose of this model is to provide a means for quantitative evaluation of effectiveness of FFR systems in reducing flood damage.

General description of FFR process. The forecasting service collects data which are used to provide flood forecasts (i.e. forecasts of the flood crest). These forecasts are communicated to various public and private organizations which disseminate them to potential users threatened by the oncoming flood. The floodplain dweller is the decision maker who must then make a decision about an action aimed at reducing his potential loss due to flooding.

The FFR process has been conceptualized in the form of a system shown in Figure 1-1. This system is composed of two subsystems: (1) the forecasting system which includes the hydrometric system, forecasting model and dissemination system, and (2) the response system which includes the decision-making process followed by protective actions taken by the floodplain dweller. The efficiency of the FFR system is determined by a number of interrelated factors such as: structure and reliability of the hydrometric network, performance of the forecasting model (i.e. timeliness and accuracy of the generated forecasts), speed and reliability of the dissemination process, decision behavior of the floodplain dweller, and stochastic nature of the actual flood process.

Admittedly, the complexity of the factors involved is enormous. This research effort has been aimed at developing a model which would include the essential aspects of the $F F R$ process and yet be computationally tractable.

ModeI of FFR process. The basic model of the forecast-response process is formulated for a single decision maker. The sequence of forecasts of the flood crest and the actual river stages are described by means of a Markov process. The decision maker's response to the sequence of forecasts is formulated as a random duration, multistage decision process. At each decision time, $k$, the state of the process is a four-tuple ( $\alpha, i, h, w$ ) where
$\alpha$ - the degree of response already achieved (due to the decisions already made),
$i$ - the current flood level,
h - the forecasted flood crest,
w - binary randon variable indicating whether or not more forecasts will be issued.

The decision to be made, $d$, is the degree of response (measured on an interval scale [no response, full response]). The law of motion for the process is a two-branch Markov chain of order one defined on the two-tuple ( $i, h$ ) with branches determined by the binary variate $w$. The loss function for the process represents the cost of implementing a given degree of response and the damage caused by the flood crest when it eventually arrives. Minimization of the expected value of the loss throughout the whole flood process (with the aid of a dynamic programing algorithm) yields an optimal strategy, $S^{*}$. This strategy relates the decision maker's optimal degree of response $d(k)$ at the time $k$ to the degree of response already achieved $a(k)$ and to the
information contained in the forecast message, namely $i(k)$ and $h(k)$. For a given strategy the expected annual loss is computed. In actuality, the decision maker may not behave optimally. His actual response is, therefore, described by an actual strategy, $S^{a}$.

Measures of effectiveness. Evaluation of the FFR system with respect to an individual flood plain dweller is measured by a performance vector and an efficiency vector. The components of the performance vector are in terms of the annual expected value which is determined by the expected reduction in flood loss due to the use of the system under three different conditions:
(1) The potential value is obtained by assuming perfect forecasting system (no errors in the forecasts and infinite forecast lead time) and an optimal response strategy $\mathrm{S}^{*}$.
(2) The optimal value is obtained for the actual forecast accuracy and the optimal response strategy $\mathrm{S}^{*}$.
(3) The actual value is obtained for the actual forecast accuracy and the actual response strategy $s^{\text {a }}$.

From these values the overall efficiency as well as the efficiencies of the forecast subsystem and response subsystem can be obtained. Together these six values give a thorough evaluation of the forecast-response system.

Case study. The methodology developed in this study has been applied to evaluate the $\operatorname{FFR}$ system at Milton, Pennsylvania. The results of this evaluation are presented to demonstrate the potential of the model. In addition, sensitivity analyses are performed to illustrate that the model can provide answers to a variety of problems that are paramount to efficient design as well as operation of flood forecast-response systems.

Considerable amounts of detailed theory and notation are required for the evaluation methodology developed in this report. This report presents the theory and methodology in full detail. For the reader desiring to obtain an understanding of the work without becoming completely immersed in the theory, the first chapter contains the introduction, conclusions and a summary of most of the theory and methodology. Chapters 2 and 3 give the details of the theory. Chapter 4 gives a general description of the procedures used for developing case studies. Details of the Milton, Pennsylvania case study are given in Chapter 5. The work to date on the Victoria, Texas case study is in Chapter 6. A flow chart and manual for the associated computer package is contained in Chapter 7. The listing of the computer package is given in a separate yolume.

A list of principal symbols is given prior to Chapter 1 ; a detailed listing of the notation used in the theoretical chapters of the report is at the end of Chapters 2.

In a few instances the notation and definitions in this report do not strictly conform to NWS usage. It is believed these deviations were necessary for the theoretical model that was developed.

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Flood Forecast-Response System

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## LIST OF PRINCIPAL SYMBOLS

$\alpha$
degree of response
unit damage function
unit cost function
forecasted flood crest
actual flood crest
flood level
decision time
loss function
cost function
stage-damage-response function
location step
maximum possible damage
unit reduction function
probability
structural category
strategy
forecast indicator

## Chapter 1

## Introduction, Conclusions and Summary

## 1. INTRODUCTION

The effectiveness of flood forecasts in reducing urban property damage is dependent on the accuracy and timing of the forecasts, the manner in which they are disseminated to the public and the public's response to the forecast information received. In this report a comprehensive systems model of the whole flood forecast response system is developed which enables the quantitative evaluation of such a system. This model represents a significant advance in evaluation methodology due to $i$ ts explicit recognition of the sequential nature of flood forecasts and the responses to them, as well as simultaneously considering the performance of the forecasting model, the speed of the dissemination process, the decision behavior of the flood plain dweller, the nature and location of the structures in the flood plain and the stochastic nature of the actual flood process in the evaluation procedure.

Flood forecasts are provided by the National Weather Service so that the flood plain dweller may take actions to reduce or eliminate the losses caused by flooding. This forecast service may be considered to be imbedded in an overall flood forecast-response system which is shown schematically in Figure 1-1. The National Weather Service collects data which are used to provide forecasts. These forecasts are communicated to various public and private organizations which disseminate them to the potential user who is threatened by a flood event. The flood plain dweller is the decision maker who must then decide upon what action to take to reduce his potential losses due to flooding. The effectiveness of the action taken by the flood plain dweller in reducing his losses is the measure used in this report to evaluate the worth of the overall flood forecasting response system and its components.

A good forecast is valueless if it is not received by those living in the flood plain in time to take protective action. It is also valueless if those endangered flood plain dwellers who do receive the forecast in time, do not understand that they must take action. Conversely, an alert community with an effective civil defense organization is severely handicapped if it is provided with poor flood forecasts.

Previous studies (Sniedovich et al., 1975; Sniedovich and Davis, 1977) have developed a decision theoretic systems framework for the analysis of flood forecast-response systems. The overall system as shown in Figure 1-1 is composed of two subsystems: 1) the forecasting system which includes all parts of the overall system from the collection of the field data, through the development and dissemination of the forecast, and 2) the response system which includes the decision making and actions taken by the flood plain dweller. A decision theoretic approach was taken so that the uncertainties in the forecasts and in the flood plain dweller's perception of the situation could be explicitly incorporated into the analysis. Evaluation of the flood forecast-response system, with respect to an individual flood plain dweller, was measured by a performance vector and an efficiency vector. The components of the performance vector are in terms of the value of the forecast-response system under three different conditions. The value of a forecast-response system is the reduction in flood loss to be expected by use of the system. The potential value is obtained by considering perfect forecasts and perfect responses; the optimal value considers the actual forecast accuracy and the optimal response to these forecasts; the actual value considers the actual response to the actual forecasts. From these values the overall efficiency


Figure 1-1. Flood Forecast-Response System
as well as the efficiency of the forecast subsystem and response subsystem can be obtained. Together these six values give a thorough evaluation of the forecast-response system.

In the previous work cited above, the methodology was used to study the tradeoff between forecast accuracy and lead time as well as the effect on the value of the system of misperception by the flood plain dweller of his location in the flood plain. No actual flood forecast-response systems were evaluated in the previous work. The methodology was developed on the basis of a single forecast and required detailed information about many system components that was not generally available. This project was developed with the purpose of remedying these deficiencies.

### 1.1 Objective

The objective of this project was to develop a decision theoretic systems methodology which would enable the practical evaluation of the National Weather Service's river forecast system and to apply the developed methodology to a case study. Development of such a methodology would require:

1) Construction of a model of the flood forecast-response system which considers the sequential nature of the forecasting process and the resultant decisions by the flood plain dweller.
2) Development of an evaluation methodology based on the sequential model.
3) Determination of a precise statement of the information needed for quantifying the flood forecast-response model and for its evaluation.
4) Development of algorithms to convert the information known about the various components of the actual flood forecast-response system to the form needed for the computer model of the system.
5) Development of further knowledge of the human factors involved in the response of the flood plain dweller and the development of models to indicate the level of response by flood plain dwellers to flood warnings.
6) Expanding the evaluation model from the consideration of an individual flood plain dweller to evaluation on a regional and national basis.
2. CONCLUSIONS
1) A systems model of the sequential flood forecast-response system has been developed which enables the quantitative evaluation of such systems for their effectiveness in reducing urban property damage.
2) A sequential model is necessary for the proper evaluation of a flood forecast-response system because of the interaction between the sequence of forecasts on the rising limb and the limitations in the rate of response by the flood plain dweller.
3) It is necessary to consider the interaction among all parts of the system when evaluating the whole system or a segment of the system.
4) The data base necessary for the evaluation needs improvement, especially in the area of response to flood warnings.
5) Data is sufficient for making analyses of flood forecasting-response systems in some locales. The case study of Milton, Pennsylvania demonstrates the potential of the model and the evaluation methodology.
6) Evaluation of the flood forecast-response systems was conducted for Milton, Pennsylvania, and Victoria, Texas. These systems have different characteristics and their evaluation vectors differed markedly.

## 3. SUMMARY

### 3.1 System Model

An actual flood forecast-response system is extremely complicated. The goal of the modeling effort was to produce a model which would include the essential aspect of the forecast-response system and yet be computationally traceable. The resultant model is a compromise between these goals. The following section is a description of the mathematical model of the flood forecast-response system. A rigorous definition of the model is given in Chapter 2.

In considering losses due to flooding, it is believed that the quality of the NWS forecasts provided for riverine flood hazards is such that loss of life can be prevented and the necessary response on the part of the flood plain dweller to prevent such losses is relatively clear. However, the actions to be taken to reduce the economic loss from possible flooding is not always clear to the flood plain dweller. Such actions have an economic and possibly a psychic cost. Only economic losses and the factors which determine them are considered in the model. The benefits of action taken to reduce flood damage may be an uncertain quantity due to forecast errors, actual and imagined, and due to the lack of perception and understanding of the potential hazard by the flood plain dweller. On the basis of expected benefits and costs or very possibly some other criteria, the flood plain dweller decides what action to take after receipt of a flood forecast.

The amount of flood damage that can be prevented by action on the part of the flood plain dweller is limited; there is a maximum reduction in flood damage that can be accomplished. In the mathematical model, the flood plain dweller's action, termed the degree of response and denoted by the Greek
letter $a$ is represented by the fraction of the maximum possible damage reduction that is accomplished by the action chosen. The cost of such action rises monotonically with the degree of response.

The sequential nature of the flood forecasting process is recognized in the system model by indexing the forecast times, i.e., the first forecast, the second forecast, etc. The decision maker's response is indexed by the time of the forecast on which it is based even though the actual time of decision is later than the forecast time. The state of the system at any forecast time, $k$, is given by the current flood level $i(k)$, the forecasted flood crest $h(h)$ and the degree of response already achieved $\alpha(k)$. At this point the flood plain dweller makes the decision, $d(k)$, about the desired level of protection for the next time period, $\alpha(k+1)$ subject to constraints on the allowable change in the degree of response which depend on the time available to implement the action. The decision $d(k)$ at time $k$ is conditioned by the value of the state of the system $(\delta(k), i(k), h(k))$. The function, $S$, which gives the flood plain dweller's decision as a function of time of the forecast and the state of the system is called the response strategy.

The state of the system at the next forecast time is described by the same variables, if there is indeed another forecast. If there are no forecasts to come, the terminal state of the system is the actual flood crest hh. Whether or not the next forecast will be the last is indicated by an indicator function $w(k)$ which has the value of 0 if there are no more forecasts or the value of 1 if there are to be more forecasts. The current value of $w(k)$, however, is not known to the flood plain dweller. The flood plain dweller's action ends whenever: 1) he has been flooded, 2) the flood receeds without reaching his location.

A law of motion describes the progression of states of the system as the forecast series is issued and the flood crest arrives. Since the state of the system is a random variable, the law of motion is a probability distribution for the state variables representing the flood level, the forecasted crest, the actual crest and the next forecast indicator:
$P[i(k+1), h(k+1) \mid i(k), h(k), w(k)=1], P:[h h(k) \mid i(k), h(k), w(k)=0]$. The initial Condition of the states of the system, i.e., their value at the forecast time $k=1$, is also defined by a probability distribution.

The sequence of decisions made by a specific flood plain dweller for a particular flood warning sequence is termed a policy. The factors which lead the flood plain dweller to choose a specific policy may vary from flood warning to flood warning and are only partially understood. They have been discussed in the previous report (Sniedovich et al., 1975) and are examined in relation to the objectives of this project in a later chapter.

The choice of policy for a particular flood event determines a sequence of states of the system; such a sequence of states is termed a trajectory. Since this trajectory sequentially describes the flood levels and degree of response, it can be used to determine the loss inflicted on the flood plain dweller by the flood occurrence.

The loss to the flood plain dweller are the costs of responding to the forecasts and the damage caused by the flood. Because costs are incurred in responding the flood plain dweller can suffer flood losses even if he is not flooded. These losses are described by a loss function: L. This function is postulated to be additively separable for each forecast time. Costs of responding are fully charged at each decision time while the effectiveness of the decision depends on the time available for implementation of the decided action.

Because there are constraints on the rate at which the response can be increased, it is possible that the actual response achieved will be lower than planned due to the arrival of the flood waters. The time available for the flood plain dweller to implement his decision is termed consumer time. After the last forecast this time is effectively unlimited if the flood plain dweller is not flooded. If he is flooded, the consumer time is dependent on the flood plain dweller's location in the flood plain, the lead time of the forecast, the processing time of the forecast and the dissemination time. The forecasting process starts with the collection of the data on which the forecast is based. The time extending from the data collection to the actual crest of the flood is defined as the lead time. The time required to produce the forecast is the processing time. The time between issuance of the forecast and its receipt by the flood plain dweller is the dissemination time. If the processing time and dissemination time are zero, then the lead time is equal to the consumer time. The lead time is the maximum possible consumer time. By its definition lead time is a purely hydrologic variable.

This mathematical model of the flood forecast-response system provides the information necessary to evaluate the system.

### 3.2 Model Structure

The choice of structure for the mathematic model presented in the previous section is a tradeoff between data requirements and availability, computational accuracy and cost, and between flexibility and depth of analysis.

The evaluation procedures ultilize the mathematical model to calculate the required information. The optimal strategy is calculated by stochastic dynamic programming. For a system described by four states, Bellman's "curse of dimensionality" is in full ascendency and can very easily overpower even a large computer. Therefore, the first consideration in determining the model structure was to have reasonable computation times.

The second consideration in determining the model structure was the project objective of obtaining a model that could be used to evaluate a flood forecastresponse system for large regions anywhere in the United States. Satisfaction of this objective requires flexibility as the charactertstics of the nation's rivers and flood plains are quite variable in different areas. Further, the availability and form of data describing the characteristics of rivers and flood plains is also variable.

The third requirement for the model structure was the ability to produce reasonably accurate, credible and useful evaluations. A useful evaluation is one that can be obtained from the evaluation procedure by the expenditure of reasonable preparatory effort and computational time, and which provides information of practical use to those analyzing flood forecast-response systems.

The compromise model structure involves 1) discretization of variables, 2) deterministic lead times, 3) a limited number of categories encompassing all structural types, 4) unit damage and cost functions for these categories and 5) some restriction and simplifications on the law of motion. The resultant structure is outlined below and given in detailed form in chapter 4.

The variables $i$ and $h$ describing flood level and forecasted flood crest have been discretized. Effectively the flood plain has been terraced into "steps" for a finite distance above flood stage. When used to describe the flood plain dweller's location in the flood plain, the variable is denoted by $m$.

For the last forecast, $(w(k)=0)$, the lead time is described deterministically as a function of the forecast time $k$. This lead time is then used to compute the consumer time, the time available for the flood plain dweller's response. All structures in the flood plain have been classified into a limited number of categories, indexed by $r$. For the case study presented later in the report there were seven categories: one story residences, two story residences, trailers, two different types of commercial establishments and two different types of industrial establishments. All structures in a category have similar stage-damage curves. This similarity has been exploited by the unit function concept.

It is assumed that there is a maximum possible damage, $M D$, to a structure resulting from floodwaters of an arbitrarily high stage. The unit damage function, $\delta(h h)$ is that fraction of the MD that would be caused by flooding to level hh. The unit reduction function, $M R(h h)$, is that fraction of the maximum damage that can be reduced by taking full protective action, $\alpha=1.0$, prior to the flood. The stage-damage-response function, specifying the damage caused to a structure by a flood crest hh when the flood plain dweller has taken a protective response of level $\alpha$, is calculated as follows:

$$
L D(\alpha, h h)=\operatorname{MD}[1-\alpha M R(h h)] \delta(h h) .
$$

The cost of response at the level $\alpha$ is defined by the cost function, $L C(\alpha)$, in terms of the maximum damage and a unit cost function $\gamma(\alpha)$ as follows:

$$
L C(\alpha)=M D_{\gamma}(\alpha) .
$$

Each structural category, $r$, is assumed to have unique unit functions $\delta_{r}, \gamma_{r}$ and $M R_{r}$. Therefore each structure on the flood plain may be specified by the vector ESTABLISHMENT $=(m, r, M D)$ where $m$ is the step of the flood plain in which the first floor of the structure is located, $r$ is the category, and MD is the maximum damage. If the maximum damage for each establishment in the flood plain is indexed within categories, $M D_{j r}$, then a complete inventory of information about structures on the flood plain may be stored in a very efficient manner.

The unit performance of a flood forecast-response system for establishment ( $m, r, M D_{j r}$ ) can be calculated in terms of the unit potential value, the unit optimal value and the unit actual value. The system efficiency can be calculated directly from these numbers. Obtaining actual values is done by multiplying unit values by the maximum damage. Any other flood forecastresponse system with the same characteristics except for a different value of maximum damage can be evaluated by use of the calculated unit values and a multiplication by the particular maximum damage.

In the law of motion, it is assumed that the probabilities describing the next values of the flood level, forecasted flood crest or the actual flood crest are stationary. That is, they are
independent of how many forecasts have been issued. The nonstationarity in the flood forecast sequence is recognized by conditioning the probability distribution of the forecast indicator, $w(k)$, on the number of forecasts issued; thus the law of motion itself is not stationary.

Further simplification is obtained by expressing the law of motion as the product of conditional distributions. The high correlation between current flood level and forecasted crest motivated removal of the current river levels from the conditioning of these distributions. Further simplification is obtained by assigning zero probability to all future actual flood stages and forecasted crests, that are below the current stage. The net result is a structure for the law of motion that is easier to use and that is easier to fit to the available data.

The potential value of a flood forecast-response system is independent of the characteristics of the forecasting subsystem. The law of motion is not used in this calculation: a stage-probability relation is all that is needed. For the evaluation model, this relationship is derived from the law of motion. This ensures compatibility between the values in the performance vector. Comparison of the derived stage-probability relationship with one obtained directly from the data by standard methods provides a check on the law of motion.

## Evaluation of a Reach of River

The flood forecast-response system for a reach of river is composed of many individual systems each of which has the same forecasting and disseminating subsystem components, but differ in characteristics of the response system. The performance vector of the flood forecast-response
system for a reach is the sum of the performance vectors for each individual system. Use of the unit function concept together with the assumption that the number of response strategies is limited, allows the calculation of the performance vector for a reach to be made in a reasonable amount of computing time.

The detailed structure for the evaluation of the flood forecastingresponse system for a reach of river is given in Chapter 2.

### 3.3 Evaluation of Performance

The evaluation of a flood forecast-response system is intended to be a measure of its future performance with respect to an individual flood plain dweller or a group of flood plain dwellers. This evaluation is expressed by three measures of effectiveness, called the performance vector, and three efficiencies called the efficiency vector.

Future performance is expressed as an expectation since the future can only be described in terms of probability. The measures of performance and efficiency depend on the characteristics of the situation being studied. The mathematical model imbeds the characteristics of the forecasting part of the system in the law of motion, processing time, dissemination time and lead time. The nature of the losses are determined by the loss function and the constraints on flood plain dweller's response. The resultant of the flood plain dweller's decision process is characterized by the strategy.

A perfect forecasting system and the best possible response by the flood plain dweller would given the maximum possible reduction in flood damage that could be obtained by a flood-forecasting response system. Adjustment is made for flood frequency to give a yearly expected reduction in flood damage for an ideal system. This is the potential value of the system.

Perfect forecasts are not available to the flood plain dweller. The forecast river stages may be in error and the time available for the flood plain dweller to make his response is limited, due to a limited lead time. Time is also required for processing and disseminating the forecast. The expected annual reduction of flood damage that may be accomplished by a strategy of optimal response to the actual characteristics of the forecast system is termed the optimal value of the forecasting response system.

The flood plain dweller lacks the information and direction to make an optimal response. The actual value of the flood forecast response system is the annual expected reduction in flood damage for the actual response strategy applied to the actual forecast system.

The efficiency of the forecast system is a measure of the effectiveness of the actual forecast system with perfect response; its value is the quotient of the optimal value divided by the potential value. The efficiency of the response system is a measure of the effectiveness of the actual response compared with the optimal response. Its value is obtained by dividing the actual value by the optimal value. The efficiency of the overall system is obtained by dividing the actual value by the potential value.

The costs involved in implementing a poor response may be greater than the reduction in flood damage accomplished. In this case the actual value, response efficiency and actual efficiency will be negative. A better response strategy in these cases would be to take no action.

The values and efficiencies defined above are only applicable to the overall flood forecast-response system being considered. The measures of value and efficiency for the two main subsystems are dependent on the characteristics of both subsystems, e.g., the efficiency and value of the forecast system is heavily dependent on the characteristics of the response system. Consider two overall systems which differ only in the rate at which response can be accomplished; the forecast efficiency and optimal value of the identical forecast systems will be greater for one in which the overall system contains a response system capable of responding at a rapid rate. Conversely, the slowly responding response system will show a higher value when connected to a forecast system having a longer lead time.

The actual strategy used by the flood plain dweller is a necessary part of the model. The next section summarizes the human factors model developed to study the factors determining this strategy.

### 3.4 Human Factors Summary

The human factors mathematical model for response to warnings assumes that the decision maker (DM) begins to respond when he is sufficiently sure that a flood will reach his property. His degree of certainty that this witl happen is represented by a subjective probability, the value of which depends on his past experience with floods and losses and on the warnings he receives. When his subjective probability exceeds a threshold, he takes a characteristic course of action that will result in savings should he be flooded. The amount of savings he can accomplish is limited by the time available to him, and he stops his protective action if the flood reaches his property, or if the crest occurs below it. Following a flood incident, the decision maker learns from that experience. The specific features of this are described below.

It is assumed that when the decision maker arrives on the flood plain, his subjective probability of a flood $p(F)$ and of a loss given a flood $p(L \mid F)$ are both essentially zero. The DM revises his subjective probability of a flood $p(F)$ toward the historical value for his area, to an extent dependent on his willingness to learn, whenever a flood occurs. Between floods the probability decays exponentially toward zero. The tendency to be concerned about flooding right after floods and for that concern to diminish in time has been widely reported in the literature.

Similarly the subjective probability of a loss given a flood, $p(L \mid F)$ is revised toward the experienced frequency with each loss, again to a degree dependent on the willingness to learn. And it too decays between losses.

Figure 1-2 shows the model's output for the time course of the subjective probabilities of a flood and of a loss given a flood for a DM who began residing on the Milton, Pennsylvania flood plain (at level m=4) in 1940 in time for that year's flood. The initial zero probabilities are quickly modified toward their historical values, but a long period without loss, such as that before Agnes, produces a very low prior probability of loss.

The DM's subjective probability of a loss at any time a warning has not been given is assumed to be $p(F) p(L \mid F)=p(F, L)$. This would be indicative of the DM's willingness to take precautions prior to a flood, seek insurance or abatement projects, or learn how better to protect his property.

When a warning is issued, it is assumed that the DM revises his prior probability, the current value of $p(L \mid F)$, to obtain a posterior value. The model for revision is the prescriptive Bayesian model in which the posterior odds are obtained by multiplying the prior odds by the likelihood ratio for the data (i.e., for the set of warnings received) with the modification that a subjective likelihood ratio is used, one which is closer to unity than the correct "historical" value. This means that the DM's revision is "conservative," i.e., he changes his opinion less than the warnings actually warrant.

Figure 1-3 shows the revisions of the prior from Figure 1-2 that the model indicates would have occurred during the warning sequences of the floods of 1972 (Agnes) and 1975. The likelihood ratios were calculated from the entire historical record for Milton from 1940 as set out in Chapter 5. It is particularly interesting to note that the early predictions, being for low crests, result in downward revision of the prior--i.e., the $D M$ is led to believe that he is less likely to suffer a loss than he previously

Figure 1-2. Subjective uncertainty for floods and flood losses for Milton,


thought. Since early forecasts for small floods are also for low crests and since most floods don't cause a loss for the DM on level 4, this is correct behavior on his part.

When the revised prior probability of a loss exceeds a threshold value, characteristic of the individual $D M$, he then takes protective action. The model does not define the nature of the protective action, except that it assumes a fixed sequence such that the proportion of possible protection achieved is a function of the time spent working at it. Certain major protective efforts, such as complete evacuation of goods, will not be undertaken unless there is even greater subjective certainty that a loss will occur, and the model assumes successively higher thresholds for actions such as these.

Following a flood incident and its outcome of loss or no loss, the DM revises his probabilities of flood and of loss. In addition, he could be expected to modify either his threshold for action or his degree of belief in the warnings he receives, or both if there had been discrepancies among his revised probability, the warnings given and the actual outcume. If he suffered a loss but had a low revised probability of loss in spite of warnings he would be more inclined to believe warnings next time, and to act sooner. If there were no loss, but he had taken protective action on the basis of a high probability of loss he would be less likely to believe the warnings. The model in its present form does not prescribe exactly how this revision should be made, but Figure $4-2$ in the theory chapter gives the conditions for the changes in model parameters. If the DM learns, as a result of the flood or at any time, how better to protect his property, the change in his knowledge
is reflected in a suitable change in the function describing the amount of protection he can achieve with time.

The mathematical model described above does not take into account many of the aspects of how people react to flood warnings. It does not consider the warning source, the social context, the beliefs of others, and the possibility that learning may occur during the warning sequence. In order to better understand these aspects of the problem, a simulation model is proposed and has been developed in preliminary form. It is based on current psychological theory about decision making (Janis and Mann, 1977)* and adaptation to flood hazard (Kates, 1970) and upon a variety of sociological computer simulations of opinion revision and social interaction.

The structure of the simulation is detailed in the theory chapter (see Figures 3-5 and 3-6 of that chapter). Basically it assumes that warnings result in interaction with others which tends to confirm or discount them and that if it appears too risky to ignore them there is a search for an acceptable response, which also may involve interaction with others and learning from them.

The further development of such a model would permit a rather comprehensive exploration of the interaction of warning system, response knowledge and local conditions.
*References for the human factors model are listed at the end of Chapter 3.

### 3.5 Field Survey

The decision model has not been verified by a field survey. As part of the project a reconnaissance survey of Victoria, Texas was made to determine the feasibility of obtaining information that would be helpful in modeling the decision making process in a flood event. There were 26 interviews of flood plain residents.

Sixteen of the interviews were with residents in the Guadalupe River flood plain. These people are flooded regularly. They are aware of the neighborhood flood problem but most do not believe that they will be flooded. In a flood situation most of these people do not take much action to protect their property. They are aware of the flood danger but do not respond to forecasts.

Ten of the interviews were conducted with residents in a new development in the Lone Tree Creek 100 -year flood plain. These people have experienced flooding of their streets but not of their houses and do not believe they are in danger of being flooded.

This survey was too small and too limited geographically to be able to draw firm conclusions. It neither verifies nor disproves the human factors models developed. One could say that the residents of the Guadalupe flood plain need a threshold probability of one (water on the porch) in order to take the small amount of action they do to preserve their property. Essentially their response is zero. Those in the new subdivision have a prior probability of zero for a flood; they therefore have not considered taking any action.

The survey did indicate that it would be difficult to obtain the estimates of probabilities that would be needed to calibrate a model. A
revised questionnaire was developed which should enable future surveys to obtain the pertinent information with shorter interviews. The proposed questionnaire is in Chapter 6.

Further work on human factors should emphasize interaction between theoretical work, laboratory experimentation and feedback from field surveys.

### 3.6 Information Requirements

Programs have been developed to implement the evaluation methodology. To use these programs information describing a particular flood forecastresponse system must be obtained and transformed into the parameters and functions needed for these programs.

The forecast subsystem requires information on the law of motion, initial conditions at the start of the forecasting sequence, the expected number of floods per year, processing time for the forecast and dissemination time for the forecast. This information is specific for presently operating systems and is obtained from historic records such as river forecast verification reports.

The initial conditions are described by the parameters of a multinomial distribution. The law of motion, in its conditional form is described by binomial distributions.

The expected number of floods per year can be estimated directly from the data. The problem is what data to use as there is usually a longer record of flood peaks than there are of forecasts. If a longer record, reconstructed to existing conditions is available, it should be used.

Some information for the response subsystem, such as location, structure type and maximum flood damage are specific and is obtained from local data. Other information such as constraints on response, the damage and cost functions may also be obtained locally, from data. However, the unit concept may be used for these functions based on data obtained from the literature. This is more convenient and the programming is based on unit functions. The optimal strategy is calculated by the program in the course of the evaluation procedure. The actual strategy may be obtained from surveys of flood plain inhabitants or by developing models for human response based on general human factors knowledge.

Evaluating the effect of changes in any part of the system requires the parameters describing the new system. These can often be obtained by analyzing the effects of such change on the parameters of the present system.

The unit functions and constraint functions needed for the evaluation have been developed in general and are in the programs. The specific information needed to describe a particular flood forecast-response system must be developed for each case. Some of the needed parameters may be taken directly from the data, others are so involved as to require subsidiary.programs for their calculation.

The information requirements are listed in Table 2, Chapter 2 and in the detailed description of the computer programs.

### 3.7 Programs

Program SONIA evaluates the flood forecast-response system for a single decision maker, program ROSALIE evaluates the forecast-response system for a river reach.

To obtain the specification of the law of motion two subsidiary programs were developed. Program FORCAST analyzes the dáta about actual and forecast flood levels which is obtained from forecast verification forms. Statistical summaries are prepared and sets of conditional empirical distributions are produced. The mean values of these distributions are smoothed by hand and are input to program PARAMT. This program calculates the parameters of the binomial and multinomial distributions describing the law of motion and the initial conditions. Finally program LAWMO computes the law of motion using the parameters supplied by PARAMT.

Programs SONIA and ROSALIE compute the law of motion in the same manner as LAWMO by the use of internal functions which when called give the information needed for an evaluation. LAWMO on the other hand prints out the entire law of motion. This enables comparison with the historic data and the analysis of this data as obtained from FORCAST.

Program DWELLER is designed to take field inventory of structures as prepared by the Corps of Engineers and condense the information contained therein to the parameters required for SONIA and ROSALIE. It is expected only small modifications would be needed to handle other types of inventories such as those done by Day and Lee (1976).

The actual response is determined by a function which is called for in SONIA and ROSALIE. Printouts of the program are in the appended computer package.

### 3.8 Case Studies

The case study that motivated the specific form of much of the model structure was Milton, Pa. It is believed however, that the techniques used for Milton are generally applicable to other flood prone communities.

Milton, Pa. has a population of about 8,000 and is located on the West Branch of the Susquehanna River in northeastern Pennsylvania. Flood data was provided by the River Forecast Center at Harrisburg. An inventory of structures in Milton was obtained from Baltimore district of the Corps of Engineers. Information regarding the dissemination of forecasts in Milton was given by the Susquehanna River Basin Commission. No information was available on the actual strategy used by those effected by floods. Two actual strategies were used: 1) the pure strategy which consists of taking the maximum possible action when the forecast indicates flooding and no action otherwise, and 2) a strategy based on the human factors models.

Flood stage at Milton is 18.2 feet. Since 1889 there have been 17 flood events, stages in excess of 34 feet being reached in 1936 and 1972. The law of motion was based on river forecast verification reports extending from 1959 to 1975. These reports are shown in Tables 1-1a and 1-1b.

As the data on the verification report was not as complete as desired, additional data was used from williamsport after suitable transformation. Williamsport and Milton have similar flood and forecast characteristics. The composite data and the resultant law of motion are shown later in the printout of programs FORCAST and LAWMO. Comparisons of the actual flood frequencies during the calibration period with those obtained from the law of motion are shown in Figure 1-4. The complete flood record extends from 1889 while the flood verification record starts in 1959. In recent years there have been a higher proportion of flood flows, thus Figure l-4 is not representative of

| $\qquad$ | Piver forecast center <br> Harrisburg, PA |  |
| :---: | :---: | :---: |
|  | RIVEA DISTRICT OFFICE |  |
|  | Herrishurg, P4 |  |
|  | REPORT FOR: |  |
|  | MONTM | YEAR |



 .9: act: $:=$ :een: (9) ice aition (10) other.


Table l-la. River Forecast Verification Report for Milton, Pa.


 ad.41s: ant (3) bee action (10) other.

| ver ano.station | $0$ |  | forecast |  |  |  |  | observeo |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | When issued |  | $\begin{aligned} & \text { CREST } \\ & \text { on } \\ & \text { STAGE } \end{aligned}$ | date | time | $\begin{aligned} & \hline \text { CREST } \\ & \text { STAGE } \end{aligned}$ | date | time |
|  |  |  | date | time |  |  |  |  |  |  |
| quehanna liver |  |  |  |  |  |  |  |  |  |  |
| edar Run | 12 | $\begin{gathered} 6.90 \\ 25 / 1300 \\ 13.30 \\ 26 / 0700 \end{gathered}$ | 25 | 1400 | 8-9 | 26 | 0100 | $13.40$ | 26 | 1100 |
|  |  |  | 26 | 1115 | 13-14 | 26 | 1300 |  |  |  |
| arsey Shoro | 26 | unknown | 25 | 1400 | 19 | 26 | 1900 |  |  |  |
|  |  | 13.07 | 25 | 2330 | 19.8 | 26 | 0300 |  |  |  |
|  |  | 25/2230 |  |  |  |  |  |  |  |  |
|  |  | $\begin{aligned} & 18.76 \\ & 26 / 0700 \end{aligned}$ | 26 | 1115 | 31-32 | 27 | 0400 | 28.52 | 26 | 2000 |
| illiamsport | 20 | 7.17 | 25 | 1400 | 17:20 | 26 | 2200 |  |  |  |
|  |  | 25/1300 |  |  |  |  |  |  |  |  |
|  |  | 12.90 | 25 | 2330 | 18 | 26 | 1700 |  |  |  |
|  |  | 25/1900 19.75 | 26 | 0530 | 24 | 26 | 2300 |  |  |  |
|  |  | 26/0400 | 26 |  |  | 26 |  |  |  |  |
|  |  | 18.97 | 26 | 1115 | 31-32 | 27 | 0800 |  |  |  |
|  |  | 26/0700 |  |  |  |  |  |  |  |  |
|  |  | $\begin{aligned} & 23.18 \\ & 26 / 1300 \end{aligned}$ | 26 | 1615 | 28 | 26 | 2400 | 27.2 | 26 | 2130 |
| iuncy | $\square$ | Unknowa | 25 | 2330 | 22 | 26 | 2000 |  |  |  |
|  |  |  | 26 | 1615 | 31 | 27 | 0500 | 37.19 | 26 | 2100 |
|  |  | Enlenown | 27 | 0230 | 31 | 27 | 0500 |  |  |  |
| iliton | 19 | 13.25 | 25 | 2330 | 19. | 26 | 2100 |  |  |  |
|  |  | 25/1900 |  |  |  |  |  |  |  |  |
|  |  | Unimown | 26 | 0530 | 26.0 | 27 | 0400 |  |  |  |
|  |  | 19.75 | 26 | 1115 | 30-31 | 27 | 1200 |  |  |  |
|  |  | 26/0700 |  |  |  |  |  |  |  |  |
|  |  | Unknow | $\begin{aligned} & 26 \\ & 27 \end{aligned}$ | $\left\lvert\, \begin{aligned} & 1615 \\ & 0230 \end{aligned}\right.$ | $\begin{aligned} & 30 \\ & 30 \end{aligned}$ | $\begin{aligned} & 27 \\ & 27 \end{aligned}$ | 0500 0500 | 29.5 | 27 | 0400 |
|  |  |  |  | -230 |  |  |  |  |  |  |
| Lewisbure | 18 | 7.10 | 25 | 1400 | 161 ${ }^{2}-17$ | 27 | 0500 |  |  |  |
|  |  | 25/1300 |  |  |  |  |  |  |  |  |
|  |  | 11.59 25/1900 | 25 | 2330 | 18 | 26 | 2200 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

Table 1-1b, River Forecast Verification Report for Milton, Pa.
EVALUATICN GIF THE FLOOD FURECAST-RESPOVSE SYSTEM FIR A REACH









the whole record．The Corps of Engineers has developed a probability discharge curve for Milton based on existing river conditions．Flood crest frequencies， as calculated from the law of motion and adjusted for the long record，are compared with the probability curve calculated by the Corps of Engineers for existing conditions．（Figure 1－5）．

Consumer time，the time available for taking action in response to a forecast was based on a forecast processing time of about three hours，a lead time of five hours for the first forecast and 12 hours for the last forecast and a dissemination time of one hour．These numbers represent an averaging and a smoothing of data from the verification report．Calculations show that the maximum consumer time available to the decision maker who will be flooded after the last forecast range from one half hour to eight hours with an average of four hours．

Basic evaluations are done for three situations．Individual evaluations were done for an industrial structure，the ACF plant，located six feet above floodstage in the 17 －year flood plain and a large residence containing two stories，a basement and high quality furnishings located 12 feet above flood stage in the 50 －year flood plain．Finally an evaluation for the whole town was done．

Table 1－2a and 1－2b show the actual parameters used to describe the forecast system．Further evaluations were done based on changes in descriptions of the system．

The unit damage functions for Milton were developed from the inventory data supplied by the Corps．The other unit functions were developed from more general types of data．Some typical functions will be given in this chapter， for more detail see Chapter 5 on applications．

Figure 1-6 shows the unit damage functions for two story houses. Figure 1-7 shows the unit cost function for residential. Figure 1-8 shows the unit reduction function and Figure 1-9 shows the constraint function. Knowledge of the shape of these curves is important in interpreting the evaluations for the forecast system.

The numerical procedures involyed in the calculations are approximations. Sensitivity analyses were run to determine the best tradeoff between accuracy and computer time. These are discussed in the chapter on the case study. The properly "tuned" numerical procedures were fixed and the desired evaluations run.

The evaluations for the three cases discussed are given in Table 1-3. These values differ from those in the actual computer printout. The computer programs are based on the data from 1959-1975 while the values presented in the tables are adjusted to reflect the record from 1889-1975.

Notice that the potential value of the forecasts is considerably higher than the optimal value. The potential value assumes an infinite lead time whereas in Milton the lead times are limited and therefore the action that can be taken is limited. If the actual response is the pure strategy, that is if the flood plain dweller only responds when the forecast indicates he will be flooded, the expected annual losses are very close to those that would occur if there were no response, sometimes more. The actual response as determined by the human factors model, (Chapter 2) which indicates that the flood plain dweller may respond in advance of a forecast indicating he will be flooded, gives lower flood losses than for the case of the pure strategy.

An important point here is that the efficiency of the forecast subsystem is held down by characteristics of the response system as well as by inaccuracies in forecasting the rising limb of the flood. It takes 24 hours to achieve a


Figure 1-6. Unit Damage Function for Two Story House

Commercial
Figure 1-7. Unit cost function for residential and commercial structures


Figure 1-8. Unit Reduction Function for a Residence

Figure 1-9. Constraint Function for Residential and Commercial

| Structure (s) | Residence | ACF plant | all of Milton |
| :---: | :---: | :---: | :---: |
| Elevation above flood stage, ft. | 12 | 6 |  |
| Maximum possible damage,\$ | 46,900 | 3,500,000 | 48,599,580 |
| Expected annual loss, \$ perfect forecast and response | 336 | 95,910 | 883,363 |
| no response | 472 | 176,842 | 1,541,249 |
| optimal strategy | 424 | 160,415 | 1,404,766 |
| actual strategy pure | 510 | 207,846 | 1,788,478 |
| human factors | 471 | 170,699 | 1,510,198 |
| Performance, \$ |  |  |  |
| potential value | 136 | 80,932 | 657,886 |
| optimal value | 48 | 16,427 | 136,483 |
| actual value pure | -38 | -31,003 | -247,229 |
| human factors | 1.0 | 6,143 | 31,051 |
| Efficiency |  |  |  |
| forecasting system | . 35 | . 20 | . 20 |
| response pure | -. 79 | -1.89 | -1.81 |
| human factors | . 021 | . 37 | . 23 |
| overall pure | -. 28 | -. 38 | -. 37 |
| human factors | . 007 | . 076 | . 047 |

Table 1-3. Evaluation of Milton, Pa.,
Flood Forecast-Response System


Table 1-4. The Effect of Eliminating the Cost of Response and the Constraints on the Rate of Response

| Structure (s) and <br> elevation above flood stage | Lead Time Change | Processing Time Change | Residence 12 | ACF plant <br> 6 | Milton |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Effici | encies |  |
| forecasting system | 0 | 0 | . 35 | . 20 | . 20 |
| response <br> (human factors) |  |  | . 015 | . 37 | . 34 |
| overall |  |  | . 005 | . 076 | . 070 |
| forecasting system | +6 | 0 | . 41 | . 22 | . 22 |
| response <br> (human factors) |  |  | . 018 | . 44 | . 38 |
| overall |  |  | . 007 | . 095 | . 084 |
| forecasting system | 0 | -2 | . 44 | . 24 | . 20 |
| response (human factors) |  |  | . 019 | . 56 | . 56 |
| Overall |  |  | . 008 | . 13 | . 11 |

Table 1-5. The Effect of Changes in Consumer Time and Processing Time

Structure
elevation above
flood stage

ACF Plant

Threshold Probabilities
Efficiencies
forecasting system . 18 . 18
response system . 32 . 26
Overal1 . 058 . 046

Table 1-6. The Effect of a Change in Threshold Probabilities in the Human Factors Model on the Efficiency of the Flood Forecast-Response System
evaluation of the flcco forecast-response system for a reach


|  |  |  |  |
| :---: | :---: | :---: | :---: |
| STRUCTURAL CATEGORY |  |  | CATEGORY OF DMs |
| $r$ |  | q |  |
| 1 | One story house |  |  |
| 2 | Two story house | 1 | Residential |
| 3 | Trailer |  |  |
| 4 | Commercial- arage type |  |  |
| 5 | Commercial-store | 2 | Commerical |
| 6 | Industrial - group 1 |  |  |
| 7 | Industrial - group 2 | 3 | Industrial |

Table 1-7a. Notation for Tables, Showing Partition of Maximum Damage, Potential Value, Optimal Value and Actual Value for Milton, Pa.
$* * * * * \# \# \# \# \# \# \# \# \# \# \# \# \#$
$\#$
$\#$ DtCISION MAKERS
$\#$
$\# \# \# \# \# \# \# \# \# \# \# \# \# \# \# \# \# \# \#$


$$
E(M, R)=
$$

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | $E(M)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $4 \frac{1}{2}$ | $\bigcirc \cdot{ }^{\circ} \mathrm{CO}$ | C．000 | $\begin{aligned} & 0.000 \\ & 0.0000 \end{aligned}$ | C． CcC | $\text { C. } 000$ |  |  |  |
| $\frac{1}{2}$ | C．CCO | O．000 | $\begin{aligned} & 0.000 \\ & 0.000 \end{aligned}$ | C．OCC C．OCC | $\begin{aligned} & 0 . C O O \\ & 0.01 c \end{aligned}$ | 0.000 0.000 | 0．000 | $\begin{array}{r} 0.000 \\ .191 \end{array}$ |
| 4 | C．CCO | C．cic | 0.600 | c．ccl | $\bigcirc .000$ | 0．0c0 | －198 | － 226 |
| 5 | C．CCO | －16C | 0.050 | c．cco | ．02E | ． 057 | 0.000 | ． 245 |
| $t$ | c．cco | －Cés | $0 . \mathrm{Ci}) \mathrm{C}$ | C．CCl | － C 2 C | ．065 | 0.000 | －169 |
| 7 | －CII | －C0゙4 | 0.050 | O．CCC | c．00c | 0.000 | 0.000 | ． 095 |
| \％ | －Ci3 | ．042 | C． 000 | c．ccc | c． COO | 0.000 | 0.000 | ． 055 |
| ¢ | $0.6 C O$ | －620 | c．cio | C．CCC | C．COC | 0.000 | 0.000 | ． 020 |
| $E(R)$ | ．C23 | ． 417 | 0.000 | C．CCC | ． 058 | .122 | ． 330 |  |

$P V=1$ 19上と075．
$N P(M, R)=$

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | NP（M） |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | C．ClO | 0.100 | 0．000 |  |  |  |  |  |
| 2 | O．OCO | O．CCO | c． 000 | 0.000 | C．OCC | c．000 $0 . c 00$ | $\begin{aligned} & 0.000 \\ & 0.000 \end{aligned}$ | $0.000$ |
| 4 | 0.060 | O．CCO | C．OCO | O．0口C | －CE1 | C．CCC | ． 357 | ． 411 |
| 5 | －．cco | －C13 | C． $\mathrm{CCO}_{0}$ | O．CCO | $C . C C C$ | －．coc | ． 254 | ． 272 |
| $\stackrel{0}{0}$ | U．CCC | －cer | C．CCO | －0．00 | － 06 | － 0 as | 0.000 | － 215 |
| 7 | －CCl | － 007 | C．OCO | 0.000 | C．OCL | c．051 | 0.000 0.000 | ． 072 |
| 8 | －．4Cl | －COI | C． 000 | 0.000 | c．OCC | C．OOC | 0.000 | ． 001 |
| 9 | 0.060 | －CCu | c．0CJ | －． 030 | C．CCC | $O .00 C$ | 0.000 | .000 |
| NP（R） | －CC2 | ． 163 | C．OCO | C．0．00 | ． 144 | ． 137 | .614 |  |

Table 1－7b．Partition of Maximum Damage and Potential Value by Flood Plain Step and Structural Category

```
OV = & 40c412.
```

$N O(M, R)=$

|  | 1 | 2 | 3 | 4 | 5 | $\theta$ | 7 | $N O(M)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| M 1 | C.000 | 0.100 | 0.000 | 0.000 | C. 000 | c.coc | 0.000 | 0.000 |
| 2 | 0.060 | O.CCO | U.000 | 0.000 | C.CCC | C.COC | 0.000 | 0.000 |
| 3 | C. Cuc | 0.60 | C. ccc | 0.00 C | - 040 | C.coc | . 312 | . 353 |
| 4 | C. Oco | - Cく2 | c. 000 | 0.600 | C.OCC | c. COC | . 222 | . 247 |
| 5 | $C .060$ | -C98 | C.OCC | C.COC | - 051 | - 692 | 0.000 | -281 |
| 7 | 0.0 CO | - 028 |  | 0.000 | .031 | -041 | 0.000 | . 101 |
| 7 | . 063 | - $C 12$ | C. 000 | -.COO | COCO | 0.00 C | 0.000 | . 015 |
|  | . CCl | -CCl | $0 . c 0 c$ | 0.000 | C.OCC | C.000 | 0.000 | . 002 |
| 9 | 0.060 | - cco | c. 000 | 0.000 | c.0cc | 0.00 C | 0.000 | . 000 |
| NO(R) | . 004 | 162 | 0.000 | 0.000 | . 162 | .133 | 538 |  |

```
AV = 1 -7CCCB5.
```

$\operatorname{NA}(M, K)=$


Table 1-7c. Partition of Optimal Value and Actual Value by Flood Plain Step and Structural Category

The effect on efficiency of a two hour reduction in
process time in Milton, Pa., as a function of elevation
in the flood plain.
state of maximum response to a flood warning. From the last forecast the average time available for response (CT-consumer time) is under four hours. Therefore, the optimal response requires anticipating actions whose value depends on the accuracy of the information contained in the rising limb of the forecast. By way of comparison Table 1-4 shows the value of the forecasting system assuming no constraints on the rate of response. Also shown is the case where the costs of response are zero. The efficiency of the system markedly improves in both cases. With no constraints on the rate of response the efficiencies of the system are very high.

The effect of reducing the processing time and of increasing the lead time of the system is shown in Table 1-5. No other factors were changed. Performance of the system improved marginally with the change. These changes would be necessary to simulate radar and QPF. Such simulations would also require changes in the law of motion. At present the law of motion is not flexible enough to allow such changes to be conveniently made.

A reduction in processing time increases the consumer time for all flood plain dwellers equally. In Milton, Pa., this increase is proportionally more valuable to flood plain dwellers on the lower steps of the flood plain than to those on higher levels. Figure 1-10 shows how a two hour reduction in processing time changes the efficiency of the forecasting system, response system, and the overall system across the steps of the flood plain.

In the human factors model of response, discussed in summary previously and in detail in Chapter 3, the response depends on the flood plain dweller's subjective probability assessment of the probability of being flooded. Action is taken when this probability exceeds a threshold value. The calculations in Table 1-6 show that lowering these threshold values improves the performance of the flood forecast response system. Also shown in this table is an evaluation
for the town of Milton assuming a threshold of 0.5 for residences， 0.4 for commercial structures and 0.3 for industries．

Who in Milton needs the flood forecast－response system and who benefits？ Table l－7b shows the distribution of the maximum damage possible in Milton by structural category and step of the flood plain．Tables $1-7 b$ and $1-7 \mathrm{c}$ show the distribution of the potential，optimal and actual（based on the pure response）values of the forecast－response system．While $45 \%$ of the maximum potential flood damage in Milton can occur to residences，only $11 \%$ of the potential value of the flood forecast response system accrues to residential structures．If all decision makers used the optimal strategy $17 \%$ of the optimal value would accrue to the residential structures．If the actual strategy used by the flood plain decision makers is the pure strategy，i．e．， no response until the forecast crest is at or above flooding level，then respond as fast as possible，then less than $9 \%$ of the actual value of the forecast－response system accrues to the residential structures．

This difference has two main explanations：1）industrial and commerical structures are more concentrated in the lower parts of the flood plain，and 2）industrial and commerical decision makers are able to make better responses to the flood forecasts．

### 3.9 National and Regional Evaluation

In this section a tentative examination is made of the guidelines necessary to use the forecast-response models, MFS(DM) and MFS(REACH), (described earlier), for the evaluation of the forecasting-response systems from a national point of view. The assumption is made here that the flood prone areas of the U.S. can be divided into several flood prone regions according to some criteria. Each region, not necessarily contiguous, consists of "similar" flood reaches and the flood forecasting-response system can be evaluated only once for a region with some possible slight modifications for each individual reach. The mathematical model for evaluating a region is called MFS(REGION).

The following are some of the more important concerns that must be considered developing the MFS(REGION) model:
i) Identifying the data available that directly or indirectly provides the information needed for the evaluation procedure.
ii) Developing relations and approximations between the data available and the information needed.
iii) Developing the methodology for the determination of "similar" reaches. The basis for this determination may range from a listing of similar economic and hydrologic characteristics to statistical procedures such as cluster analysis.

## 1. Approach Framework

The general framework of approaching the national model will be summarized below. Difficulties associated with the approach from the real data view point are discussed in the next section.

The framework of the MFS(REGION) model, although not absolutely specified, will be developed following these general steps:
i) Studying Milton and other case studies and comparing the data necessary for this study to data available on other reaches.
ii) Deciding on some characteristics of reaches (e,g. geographical location, major river, economic background, etc.) to claim that two or more reaches are similar. These would be evaluated by one run of MFS(REACH) plus a possible simple linear transformation.
iii) Then, through the many clustering techniques available, organization of reaches into a unit called region can be accomplished. Thus, if we index all reaches in a region by a number in index Set $J$, the expected annual loss for a region is simply the sum of expected annual losses of all the reaches in that region, i.e.

$$
E L(\text { REGION })=\sum_{i \in I} E L(\operatorname{REACH}(i))
$$

2. Some Examples of Details of the MFS (REGION) Using Milton Case Study

Perhaps the best place to start seeking needed information for the MFS (REGION) is the already worked out Milton case study. Through step by step investigation of input data to Milton and other case studies we can throw some light on possible shortcuts, approximations and extrapolations for obtaining data on other flood reaches.
2.1. Difficulties - There exist many sources of information containing many kinds of hydrological and socioeconomical data for reaches. One difficulty, however, lies in the fact that each source would have to be individually consulted for the type of information they offer. Further, much of this data is not available in libraries but has to be obtained from district offices of particular government agencies such as the Corps, SCS, River Forecast Center, etc. Another difficulty rises from the large number of reaches to be evaluated. There are about 1000 different forecast points in this country for which
some sort of data is available or at least obtainable. These problems force one to seek generalizations to approximate some reaches based on a known particular reach. Hydrological data from the U.S.G.S. are easily obtainable but the data needed to derive the law of motion must be obtained from the River Forecast Centers. In many cases the record may not be long enough. An approach that may be fruitful would be to investigate the possibility of extrapolating the law of motion, obtained from river forecast verification records, to other forecast points in a similar region with suitable adjustment. Such adjustment would be based on data that is more easily available such as area of the watershed, basin slope, characteristics of the rain gage network and channel stability.

In their work for the NWS on potential flood damage reduction in the Connecticut River basin, Day and Lee (1976) made a regional evaluation based on extrapolation from a small number of communities actually surveyed. As a part of their current research for the NWS, they are determining the applicability of this type of extrapolation to other areas of the country. Their results will provide information for the construction of regional evaluation models.

The use of unit functions for the economic factors is an example of the aggregate data in the present methodology. Once structural categories have been established and the unit functions developed, the remaining economic information needed to evaluate a new reach is the information called for in the vector ESTABLISHMENT. If maximum damage can be related to assessed valuation then all that is necessary to know is the distribution of the assessed value of each structural category among the various "steps" of the flood plain.

This type of analysis must be done separately and jointly for each bit of information required for the evaluation methodology. With such an
analysis accomplished, competing models and methodologies for the region can be created and evaluated.

In summary, the work on MFS(REACH) has already started and sources of information are being studied. Also, theoretical possibilities for conducting the approximations, extrapolations, and clustering techniques will be continued in the future. The work for MFS(REGION) starts by looking for distinctive properties for reaches, then the clustering of reaches into REGIONS so as to simplify the national evaluation. This evaluation of national model reduces to evaluation of few regions.

## Discussion

A model of the flood forecast-response system has been developed which takes into account the sequential nature of the forecasting sequence and the nature of the response to that sequence. Further, a methodology has been developed which enables a quantitative evaluation to be made of flood fore-cast-response systems in terms of expected reduction in flood damage. This methodology may be used to evaluate systems presently in use or to calculate the benefits to be expected from changes that might be made to present systems.

Comparison of this sequential model with the single prediction model previously developed (Sniedovich et al., 1975) shows that a sequential model is necessary for the investigation of the performance of a flood forecastingresponse system where the forecasts are sequential in nature. The responses necessary to reduce flood damage take time to accomplish, therefore, the time available for response must be accounted for in the model. Further, the accuracy of the forecasts on the rising limb of the flood as well as the final forecast of the crest are considered in the sequential model.

The systems model of the flood forecasting response system developed during this project includes more facets of the actual system in a quantitative manner, than any previous model or evaluation procedure. Yet, the model is still very simplistic when compared with the actual system. On the other hand, the information requirements for this simple model are such, that for many particulars, data are either insufficient or not available.

The most serious simplification is in regard to lead time. Lead time is considered in the modeling and evaluation procedure, but in a manner that is fixed, abeit in a probabilistic manner, for all possible flood events. The forecast lead time is not one of the state variables and its effect on the flood plain dweller's actual choice of response is not considered. Thus,
the benefits to be expected from better accuracy in forecasting the lead time of a flood event may not be evaluated. However, the effect of changing the average lead time of a forecasting system can be analyzed and was considered as part of the case study.

The data required to implement the evaluation procedure are sometimes difficult to obtain. An inventory of structures and the amount of damage they would sustain, subject to floods of various depths, is possible to obtain for a community, though it may be costly. The other information required to ascertain flood losses such as the amount of damage that can be prevented by proper action, the cost of such action and the constraints on the rate at which such actions may be undertaken was obtained by generalization of other studies and may be in error. Information for the commercial and industrial situations is particularly sparse.

The use of unit damage functions to represent all members of a class of structures is a technique that saves much computer time. The deviation of individual residences from the unit damage function of the residential category is not great; these deviations may reasonably be expected to average out over a large number of structures. Deviations for commercial and industrial categories are greater and as there are a smaller number of these types of structures, effective averaging may not take place. The case study showed that there is a significant difference between evaluations using the individual damage functions and those using the category damage function for the industrial structures.

It should be noted that the commercial categories include governmental type buildings such as schools and fire houses as well as typical commercial structures such as stores and gas stations.

Information about the forecasting subsystem may be obtained from forecast verification reports. However, most records are not long enough to allow an unambiguous construction of the bivariate conditional distribution representing the "law of motion" of the sequence of forecast crests and actual river stages. The use of other hydrologic data and verification records from other forecast points proved necessary for the case study done in this report.

The present formulation and method of calculation for the law of motion does not lend itself easily to sensitivity analysis. There is no simple way of unambiguously simulating a more accurate or less accurate forecast sequence.

No hard data are available on the way in which a flood plain dweller determines his response to a flood forecast. The flood plain dweller's actual response strategy was determined in two ways for the evaluations in this report:

1) the pure strategy, i.e., no response is made until the forecast indicates flooding, then a maximum response is made and 2) the strategy considering human factors, obtained from a quantitative model of human behavior considering the previous flood history of the location and the sequence of forecasts for the impending flood event. Both strategies are reasonable. Other researchers have used the pure strategy for evaluating flood forecasts; its use assumes the flood plain dweller follows forecast "instructions". The strategy considering human factors is a quantitative model based on qualitative descriptions from the literature of people's reactions to floods and other disasters. The form of the model reflects the large body of knowledge concerning human decision making which has been obtained from laboratory experimentation in the human decision making process. The small amount of lab work done during this project corroborates the chosen form of the model.

However, there has been no field verification of this model nor has field data been used in choosing the parameters. The pilot survey of flood
plain habitants in Victoria, Texas, done as part of this project, gave some indication that the form of our model may be correct. It also indicated the difficulty of ascertaining values for the parameters.

Computational needs have led to the replacement of continuous variables with discretized variables, the most obvious being the division of the flood plain into steps. Where discretization in numerical procedures is used, sensitivity analyses have been made and a level of discretization has been chosen which does not cause significant error. The choice of steps with a width of three feet for the Milton, Pa. case study is considered to add minimal error to the evaluation methodology: however this has not been checked because the law of motion would have to be completely reworked for each change in step width.

To summarize, a mathematical model of the sequential flood forecastresponse system has been built which enables a quantitative evaluation of the system to be made. This evaluation is dependent on information which in some cases is not available, difficult to obtain or is deliberately approximated, in order to reduce computation requirements.

It is believed that a large portion of the information needed for an accurate system evaluation, with the possible exception of the actual response strategy, can be obtained and constructively used if the user is willing to bear the costs involved. It is also believed that by the use of generalized data, approximating techniques and models of human decision making, the effect of changes in parts of the flood forecasting-response system can be effectively studied.

The case study of Milton, Pa. provides an illustration of what can be done. The efficiency of the present system was obtained and shown to be low. The efficiency of both the forecast and response subsystems were also low. In
the sequential model the performance of these two subsystems are much more dependent than in the single forecast model. When the constraints on the rate at which response could be accomplished were removed all efficiences improved markedly. Efficiencies also improved when consumer time and lead time were lengthened.

Although the model and evaluation methodology were not designed to evaluate the performance of the overall flood forecast-response system in its role of eliminating the loss of life during flood events, it is fruitful to attempt to obtain some estimate of the efficiency of the system in its lifesaving role. Perhaps the biggest difference is in the response system, actions to protect life can be taken much faster than actions to protect property. This is certainly the case in Milton. Therefore it is reasonable to believe that the economic evaluation made without constraints on the rate of response gives a reasonable approximation to the efficiency of the system for saving lives.

An evaluation of the Milton, Pa., flood forecastina-response system made without constraints showed a higher efficiency than the evaluation made with contraints.

Analysis indicates a better performance may be obtained for property protection if the flood plain dweller starts his response earlier and if the time available for him to respond is lengthened. The fact that the pure response is sometimes worse than making no response indicates that improvement of the forecast accuracy on the rising limb would also be beneficial.

### 3.11 Suggested Research

Needed work on modeling and evaluation of flood forecast-response systems may be divided into four areas: 1) refinement of the model and computational procedures, 2) development of better methods for obtaining the necessary information, 3) extention of the methodology to evaluating regions and the nation, and 4) obtaining a better understanding of the decision making process which determines the flood plain dweller's response to flood warning. At present the model and computational procedures are believed to be in better shape than the data base. The use of the law of motion should, however, be made more flexible.

First it is recommended that further work stress the human factors involved in determining the response to flood warning by the joint use of theoretical studies and laboratory experiments, backed up by field studies. The Victoria survey is a start and the questionnaire developed as a result of that survey should enable the collection of additional data in a more rapid and efficient manner. Secondly, methods for obtaining good approximations of the information needed for the evaluation procedures based on data that are available with reasonable expenditures of time and effort, are needed. If these two recommended research objectives are accomplished extention of the evaluation procedures to a regional and national scale should be relatively routine.

## 4. REFERENCES

Day, J. and Lee, K., Flood Damage Reduction Potential of River Forecast Services in the Connecticut River Basin, NOAA Memorandum NWS Hydro-28, National Weather Service, Office of Hydrology, Silver Spring, Md, 1976.

Sniedovich, M., et al., "The Evaluation of Flood Forecasting-Response Systems: A Decision Theoretic Approach", Report, Office of Hydrology, Contract 3-35108, National Weather Serice, Silver Spring, Md, 1975.

Sniedovich, M. and Davis, D., "Evaluation of Flood Forecasting-Response Systems", Journal of the Water Resources Planning and Management Division, ASCE, Vo1. 102, WR1, April 1976, pp. 77-87.

## Chapter 2

# Evaluation of Flood Forecast-Response Systems-Theory 

## 1. INTRODUCTION

This study is aimed toward providing a methodology for measuring the effectiveness of flood forecast-response systems in reducing flood damage. The research reported herein is an extension of the previous studies accomplished at the Department of Hydrology and Water Resources at the University of Arizona (Kisie1 and Duckstein, 1973; Sniedovich et al., 1974; Sniedovich et al., 1975; Sniedovich and Davis, 1977). The proposed methodology has two distinguished features. First, it results from a systems theoretic analysis of the entire flood forecast response process. Second, it is furnished with a mathematical model of a forecast-response process; this model is believed to be at present the most elaborated and the most comprehensive mathematical formulation of the flood forecast-response processes.

THE SYSTEM
Two components of the Flood Forecast-Response System (abbreviated henceforth FFR) are: (1) the forecasting system (which includes: the hydrometric system, the forecasting model, and the dissemination system) and (2) the response system (which includes the decision model and the protective system) (Figure 2-1).

The hydrometric system: The field data are collected and transformed into hydrologic data.

The forecasting model: The hydrologic data provided by the hydrometric system are used as input to the forecasting procedures which, in return,
provide flood forecasts.
Dissemination system: The forecasts issued by the forecasting model are delivered to the flood plain dweller or the Decision Maker (abbreviated henceforth $D M$ ) by the dissemination system such as radio, TV, telephone, etc.

Decision model: The decision behavior of the DM is described by a decision model. The information (forecast) that comes into the decision model is used to plan the response i.e. allocation of resources to the flood protective activities.

Protective system: After making the decision, the DM takes certain protective actions.

The forecasting system is viewed as an information system and as such its performance is measured through the potential and actual use of the information provided by it. The response system is viewed as a decision system and as such its performance is measured through the optimal and actual response strategies. The relationship between these subsystems is modeled so that the contribution of each component to the overall performance of the entire system can be determined.

### 1.2 SCOPE OF THE METHODOLOGY

The informational structure of the complete evaluation methodology envisioned by the authors is given in Figure 2-2. The key element of this methodology is the Flood Forecast-Response Model (FFR) which is constructed in three modes:

1. $\operatorname{FFR}(D M):$ Flood Forecast-Response Model for a single $D M$,
2. FFR(REACH): Flood Forecast-Response model for a REACH (a collection of all DMs responding to the flood forecasts issued for the same forecast point),
3. FFR(NATION): Flood Forecast-Response Model for a NATION (a collection of all REACHES in the NATION).

Three types of inputs to each of those models are:

1. hydrologic input,
2. economic input,
3. human factors input.

The output is an evaluation of the effectiveness of:

1. a FFR system for a single $D M$,
2. a FFR system for a REACH,
3. all FFR systems in the NATION.

This report presents $\operatorname{FFR}(D M)$ and $\operatorname{FFR}(R E A C H)$. $\operatorname{FFR}($ NATION $)$ has not yet been developed and it is suggested here only as a natural extension of the research accomplished thus far.

The heart of the entire evaluation methodology is the Flood ForecastResponse Model for a single Decision Maker. It is a mathematical description of the physical forecast-response process which takes place during floodings. The forecasting part of the system supplies a sequence of flood forecasts for a given point on the river. The inaccuracy of these forecasts and the random nature of the river stages are described by means of a Markov process. The response part of the system is a single decision maker. His decision behavior in responding to flood forecasts is modeled under several psychological postulates.

### 1.3 OUTLINE OF THIS CHAPTER

Section 1 describes $\operatorname{FFR}(D M)$. We begin with a general mathematical formulation of the model (Section 1.1). The DM's response to the sequence of flood forecasts is formulated as a multistage decision process. The measures of effectiveness of the FFR system are defined. In Section 1.2, particular
model components are analyzed in depth and described with detailedness which allows for computer implementation.

Section 2 describes $\operatorname{FFR}($ REACH $)$. First, a general formulation of the model is given (Section 2.1) and extreme computational requirements, which preclude implementability of the model, are disclosed. In Section 2.2 we try to overcome this problem. It is shown that under certain mild'postulates, a very efficient model can be obtained in which the computational complexity can be reduced by the order of magnitude.

The final section (Section 3) summarizes the elements of both models.


Figure 2-1. Flood Forecast-Response System

FIGURE 2-2. Informational structure of the evaluation methodology
2. FORECAST-RESPONSE MODEL FOR A SINGLE DECISION MAKER

### 2.1 GENERAL MATHEMATICAL FORMULATION

### 2.1.1 MULTISTAGE DECISION PROCESS

Definition 1. The set of decision times, $K$, is an initial segment of the set of positive integers.

Specifically,

$$
K=\{k: k=1,2,3, \ldots, K N\}
$$

It is assumed that the sequence of decisions matches the sequence of forecasts. Hence, $K N$ is the maximum number of forecasts expected with a positive probability, or

$$
K N=\min \{j: P[(\# \text { of forecasts })>j]=0, j=1,2, \ldots\} .
$$

The real time interval between any two decision times, $\Delta t$, is not necessarily constant, but it is known at $k=1$.

Definition 2. The state space, $\Omega$, is a cartesian product $A \times I \times H \times W$ where

$$
\begin{aligned}
& A=\{\alpha: \alpha \varepsilon[0,1]\}, \\
& I=\{i: i=1,2, \ldots, I N\}, \\
& H=\{h: h=1,2, \ldots, I N\}, \\
& W=\{w: w=0,1\} .
\end{aligned}
$$

Hence

$$
\Omega=\{x: x=(\alpha, i, h, w)\}
$$

where:
$\alpha$ - the degree of response already achieved (due to the decision already made),

## i - the current flood level,

$h$ - the forecasted flood crest, or the actual flood crest following the last forecast (for clarity, the actual flood crest will be denoted by hh; theoretically, $h$ and hh are assumed to be the same state variable),
$w$ - forecast indicator $=\left\{\begin{array}{l}0, \text { no more forecasts will be issued, } \\ 1, \text { at least one more forecast will be issued. }\end{array}\right.$ Conceptually, the flood plain is discretize into IN steps. Both i EI and $h \varepsilon H$ correspond to the steps of the flood plain. The degree of response $\alpha$ is a cardinal measure of the DM's response defined arbitrarily on the closed interval of real numbers $[0,1]$. We shall speak of:

| no response if | $\alpha=0$, |
| :--- | :--- |
| partial response if | $0<\alpha<1$, |
| full response if | $\alpha=1$. |

Later in the development, $\alpha$ will be related to physically meaningful and measurable parameters.

An example of the state of the system follows. $x(3)=(0,5,2,5,1)$ is a state associated with the 3 rd decision time ( $k=3$ ), when the degree of response already achieved is 0.5 ( $\alpha=0.5$ ); the current flood level is $2(i=2)$, the forecast of the flood crest is $5(h=5)$, and at least one more forecast will be issued ( $w=1$ ).

Definition 3. The decision set, $D$, is a set-valued mapping defined on $\Omega \times K$, the set of state-time pairs:
$D=\{D(x, k): x \in \Omega, k \varepsilon K\}$
where $D(x, k)$ is the set of admissible decisions available to the $D M$ at decision time $k \varepsilon K$ when the state of the system is $x \varepsilon \Omega$. An element $d$ of $D$ is a degree of response. Hence, $D(x, k) \subseteq A$ for every $x \in \Omega, k \varepsilon K$.

A stochastic description of the forecast-response process is expressed by transition probabilities.

Definition 4. A law of motion, $\Phi$, is a conditional probability distribution on $\Omega$ of the following form:

$$
\Phi=\left\{P\left[x^{\prime} \mid x, d, k\right]: x^{\prime}, x \varepsilon \Omega, d \varepsilon D(x, k), k \varepsilon K\right\}
$$

where $P\left[x^{\prime} \mid x, d, k\right]$ is the conditional probability of the state of the system $x^{\prime}$ at decision time $k+1$ given that at decision time $k$ the state of the system is $x$ and the decision $d$ is made. In the present model, however, we restrict our attention to the following particular form of the law of motion:

$$
P[\cdot \mid i, h, w, k]= \begin{cases}P[i(k+1), h(k+1) \mid i(k), h(k), k] & \text { for } w(k)=1 \\ P[h h(k) \mid i(k), h(k), k] . & \text { for } w(k)=0\end{cases}
$$

where the value of $w(k)$ is determined according to the probability

$$
P[w(k) \mid k], \quad \text { for all } k \varepsilon K \text {. }
$$

The law of motion reads:
a) If at least one more forecast beyond the decision time $k$ will be issued $(w(k)=1)$, then $P[\cdot \mid i, h, w, k]$ is the probability that at decision time $k+1$ the actual flood level will be $i(k+1)$ and the forecast issued will indicate cresth(k+1) given that at decision time $k$ the current flood level is $i(k)$ and the forecasted flood crest is $h(k)$.
b) If no more forecasts beyond decision time $k$ will be issued (w $(k)=0)$ then $P[\cdot \mid i, h, w, k]$ is the probability that the actual flood crest will be $h(k)$ given that at the decision time $k$ the current flood level is $i(k)$ and the forecasted flood crest is $h(k)$.
c) $P[w(k) \mid k]$ is the probability of the forecast indicator $w(k)$ being one or zero at the decision time $k$.

At this point we have to make clear that the law of motion described above bears the following two assumptions:
a) the forecast indicator variable, $w$, is independent of the remaining coordinates of the state vector $x$,
b) the sequence $\{w(k)\}$ forms a Markov chain of order zero. Definition 5. A trajectory, $\bar{x}$, is a sequence of states indexed by $k \varepsilon\{1,2, \ldots$, $K N, K N+1\}$. The following notation will be used:
$\bar{x}_{k}$ - the sequence of states for decision times not less than $k$,
$x(k)$ - the state of the system at the $k$-th decision time.
Definition 6. A policy, $\bar{J}$, is a sequence of decisions indexed by kek. The following notation will be used:
$\bar{d}_{k}$ - the sequence of decisions for decision times not less than $k$, $d(k)$ - the decision made at the $k$-th decision time.

Since at each decision time the DM may choose a decision from a set of decisions available, depending on the state of the system at that time, his response may be expressed as a function defined on the state and decision times sets with values in the decision set.

Definition 7. A strategy (response strategy), $S$, is a function defined on $\Omega X K$ with values in $D . S(x, k)$ is the decision made at decision time $k$ when the state of the system is $x . S_{k}(x, j), j \leq k \leq K N$ is a strategy for times not less than $k$ if $(x, j)$ is the initial state-time. A set of feasible strategies is $\sigma=\{S: S(x, k) \varepsilon D(x, k), x \in \Omega, k \varepsilon K\}$.

With each realization of the process, described by ( $\bar{x}, \bar{d}$ ), one can associate certain values representing the total loss caused to the DM.

Definition 8. A loss function, $L$, is a real valued function defined on the triple $\left(\bar{x}_{k}, \bar{d}_{k}, k\right)$, where $k \in K$, and $\bar{x}_{k}$ and $\bar{d}_{k}$ are, respectively, a trajectory and a policy whose domains are restricted to times not less than $k$. For brevity, notation $L(\bar{x}, \bar{d}, k)$ will be used. It is assumed that the loss function is separable, viz., that it admits the representation:

$$
L(\bar{x}, \bar{d}, k)=\sum_{n=k}^{K N} L(x(n), x(n+1), d(n), n)
$$

Specifically, the following structure of the loss function is postulated:
$L(x(k), x(k+1), d(k), k)=\left\{\begin{array}{l}L_{1}(\alpha(k), d(k), k) \text { for } w(k)=1, \quad k=1, \ldots, K N-1 . \\ L_{1} \text { represents the cost of implementing } d(k) \\ \text { given the degree of response already } \\ \text { achieved } \alpha(k) \text {. Note the implicit assumption } \\ \text { that the decision } d(k) \text { is implemented in } \\ \text { the time interval }\left[t_{k}, t_{k+1}\right] .\end{array} \quad \begin{array}{l}L_{0}(\alpha(k), h h(k), \alpha(k+1), d(k), k) \text { for } w(k)=0, k \varepsilon K . \\ L_{0} \text { represents: } \\ \text { a) the cost of implementing } d(k) \text { given the } \\ \text { degree of response already achieved } \alpha(k), p l u s \\ b) \text { the damage caused by the flood crest hh(k) } \\ \text { given the final degree of response } \alpha(k+1) .\end{array}\right.$

It is assumed that on $A$, the cost of response is monotonic increasing function, and the flood damage is monotonic decreasing function.

Definition 9. A flood forecast-response process for a single DM (abbreviated $\operatorname{FFR}(D M)$ ) is a quintuple ( $\Omega, K, D, \Phi, L$ ).

Theorem 1. In $F F R(D M)$ the expected Zoss $E[L(x, S, k)]$ associated with a strategy $S \varepsilon \sigma$ and an initial state-time ( $x, k$ ) is a uniquely determined quantity and may be obtained from the following algorithom:
(a) For $\mathrm{K}=\mathrm{KN}$ set $\mathrm{d}=\mathrm{S}(\mathrm{x}, \mathrm{KN})$ and compute

$$
V(x, K N)=E\left[L\left(x, x^{\prime}, d, K N\right)\right] .
$$

(b) For $\mathrm{k}<\mathrm{KN}$ set $\mathrm{d}=\mathrm{S}(\mathrm{x}, \mathrm{k})$ and compute

$$
V(x, k)=E\left[L\left(x, x^{\prime}, d, k\right)+V\left(x^{\prime}, k+1\right)\right] .
$$

Finally, set

$$
E[L(x, S, k)]=V(x, k)
$$

Proof: See Yakowitz (1969, p. 28-29, 33)
The decision behavior of the flood plain dweller may be characterized by the response strategy. Three types of strategies are identified: Definition 10. An optimal strategy is a strategy $S^{*} \varepsilon \sigma$ such that

$$
E\left[L\left(x, S^{*}, k\right)\right]=\min _{S_{\varepsilon \sigma}} E[L(x, S, k)] \text { all } x \in \Omega, k \varepsilon K .
$$

The set of all optimal strategies will be denoted by $\sigma^{*}$.
Definition 11. An actual strategy is a strategy $S^{\text {a }} \varepsilon \sigma$ used actually by the $D M$. By definition

$$
E\left[L\left(x, S^{*}, k\right)\right] \leq E\left[L\left(x, S^{a}, k\right)\right] .
$$

The set of all actual strategies will be denoted by $\sigma^{a}$.
Definition 12. A pure strategy of the $D M$ located on the step $m$ is a strategy $S_{\varepsilon \sigma}$, satisfying for all $x \varepsilon \Omega$ and $k \varepsilon K$ the following conditions:

$$
S(x, k) \quad\left\{\begin{array}{l}
=0 \quad \text { for } k<\min \{t: h(t) \geq m, t \in K\} \\
=\max \{d: d \varepsilon D(x, k)\} \text { for } k \geq \min \{t: h(t) \geq m, t \varepsilon K\}
\end{array}\right.
$$

Most of the works in flood forecasting evaluation consider solely pure response, assuming $K=\{1\}$, and $\max \{d: d \varepsilon D(x, 1)\}=1$.
$S^{P}$ is completely specified by the definition. $S^{a}$ is expected to be
generated by a human response model described elsewhere. Construction of $S^{*}$ is shown below.
Theorem 2. In $\operatorname{FFR}(D M)$, an optimal strategy $S^{*}$ may be constructed by the dynamic progromming algorithon as follows:
(a) $S^{*}(x, K N)=d^{*}$ where for any state $x, d^{*}$ is found through the solution to $V(x, K N)=\min _{d \varepsilon D(x, K N)} E\left[L\left(x, x^{\prime}, d, K N\right)\right]$,
(b) $S^{*}(x, k)=d^{*}, k<K N$, where for any state-time $(x, k), d^{*}$ is found through the solution to the recursive equation

$$
V(x, k)=\min _{d \varepsilon D(x, k)} E\left[L\left(x, x^{\prime}, d, k\right)+V\left(x^{\prime}, k+1\right)\right] .
$$

The expected loss associated with the initial state-time ( $x, k$ ) and the strategy $S^{*}$ is

$$
E\left[L\left(x, S^{*}, k\right)\right]=V(x, k)
$$

Proof. The $F F R(D M)$ is formulated as an adaptive control process for which the dynamic programming algorithm was proven to provide an optimal feasible solution (Yakowitz, 1969)

We show specific details of the above algorithm.
(a) $S^{*}(\alpha, i, h, K N)=d^{*}$, where for any state $(\alpha, i, h), d^{*}$ is a solution to $V(\alpha, i, h, K N)=\min _{\operatorname{deD}(x, K N)} \sum_{h h \in H} L_{o}\left(\alpha, h h, \alpha^{\prime}, d, K N\right) \cdot P[h h \mid i, h, K N]$,
(b) $s^{*}(\alpha, i, h, k)=d^{*}, k<K N$, where for any state-time $(\alpha, i, h, k), d^{*}$ is a solution to the recursive equation
$V(\alpha, i, h, k)=$

$$
\begin{aligned}
& \min _{\operatorname{d\varepsilon D}(x, k)} \underset{i^{\prime}}{\{ } \sum_{h^{\prime} \varepsilon I}\left(L_{j}(\alpha, d, k)+V\left(\alpha^{\prime}, i^{\prime}, h^{\prime}, k+1\right)\right) \cdot P\left[i^{\prime}, h^{\prime} \mid i, h, k\right] \cdot P[w=1 \mid k]+ \\
&\left.\sum_{h h \varepsilon H}^{\Sigma} L_{0}\left(\alpha, h h, a^{\prime}, d, k\right) \cdot P[h h \mid i, h, k] \cdot P[w=0 \mid k]\right\} .
\end{aligned}
$$

As has been shown above, the optimal strategy $S^{*}$ is a function defined on the variable ( $\alpha, i, h, k$ ). Clearly, at the decision time $k$, an optimal decision $d^{*}$ is chosen according to the degree of response already achieved $\alpha$, the actual flood level $i$, and the forecast of the flood crest $h$.

Definition 13. An initial condition for the $F F R(D M)$ is defined by a probability distribution $\Phi_{0}=\left\{P\left[x\left(k_{0}\right)\right]: x\left(k_{\alpha}\right) \varepsilon \Omega\right\}$ of the initial state $x\left(k_{0}\right) \varepsilon \Omega$ at a specified initial decision time $k_{0} \varepsilon K$. For short-hand we shall denote $x\left(k_{0}\right)=x_{0}$. The expected loss associated with the strategy $S$ is defined as

$$
E[L(S)]=E\left[V\left(x_{0}, k_{0}\right)\right]
$$

The value $E[L(S)]$ represents the expected loss per one flood
event. Very often economic analysis is conducted in terms of annual losses. Definition 14. The expected annual loss, $E L$, associated with a strategy $S$ is defined as

$$
E L=E\left[V\left(x_{0}, k_{0}\right)\right] \cdot E[N]
$$

where $E[N]$ is the expected number of flood events per year. In order to determine $\Phi_{0}$ and $E[N]$, a precise definition of a flood event is needed. Definition 15. A flood, $F$, is an occasion on which at least one forecast of the flood crest would be issued by a given forecasting system. It is assumed that the forecasting system has a well defined set of rules which determine initiation of the forecasting process from hydrometeorological conditions on each occasion in a consistent manner.

### 2.1.2 MEASURES OF EFFECTIVENESS

Measures of effectiveness are to relate the system performance to accomplishment of goals. It seems desirable to distinguish between the performance of the forecast-response system as a whole and the performance of its major components, namely forecasting system and response system. In this way, the relative effectiveness of various improvements in the one component may be compared with the alternatives of improving the other component. The measures of effectiveness for each of the two subsystems and for the overall system will now be defined.

Although part of the flood damage may be reduced by implementation of a response strategy, even with a perfect forecasting system (no errors in the forecasts and large lead-time) and an optimal response strategy ( $S^{*} \varepsilon \sigma^{*}$ ) some damage will still occur. There is an upper bound to the preventable damage. Definition 16. The potential value, PV.

Assume: (1) a perfect forecasting system which at the decision time $k_{0}=1$ predicts the actual value of the flood crest with an "infinite" lead time,
(2) an optimal response of the $D M$ who at $k_{0}=1$ chooses an optimal strategy $\mathrm{S}^{* *} \varepsilon \sigma$.

Then the potential value of the forecast-response system is defined as $P V=E L^{0}-E L^{* *}$
where EL ${ }^{\circ}$ denotes the expected annual loss with "no response" from the DM, and $E L^{* *}$ is the expected annual loss under strategy $S^{* *}$. For the specific type of the loss function introduced in Definition 8, we have

$$
P V=\underset{h h}{E\left[L_{0}(0, h h, 0,0,1)\right] \cdot E[N] \quad-E\left[\min _{h h} d \varepsilon D^{\prime} . L_{0}(0, h h, d, d, T)\right] \cdot E[N]}
$$

PV by definition is the expected loss with no response minus the expected loss with an optimal response to a perfect forecast. It is an upper bound of the damage reduction one may expect in a given flood forecast-response system. Obviously, for real systems PV is rarely realized.

Forecasts are seldom perfect. Sequential forecasting is employed to reduce the uncertainty of long lead-time forecasts. Remaining uncertainty and sequential inflow of information must be accounted for by an optimal response strategy.

Definition 17. The optimal value, OV.
Assume: (1) a forecasting system having law of motion $\Phi$,
(2) an optimal response $S^{*} E \sigma^{*}$ of the $D M$ to the sequence of forecasts generated by $\boldsymbol{\phi}$.

Then the optimal value of the forecast-response system is given by

$$
O V=E L^{O}-E L^{*}
$$

where $E L$ * is the expected annual loss under strategy $S^{*}$. The difference between PV and OV is that OV accounts for the uncertainty in the forecasts (quantified in terms of $\Phi$ ). Still, for both PV and OV, the optimal response strategy is assumed to be used by the DM. Thus, OV represents the optimal value of the information provided by the given forecasting system.

Since often (if not always) the actual response strategy is not optimal, the actual value of the information provided by the forecasting system is less than OV.

Definition 18. The actual value, AV.
Assume: (1) a forecasting system having law of motion $\Phi$,
(2) an actual response strategy $S^{a} \varepsilon \sigma$ is used by the DM.

Then the actual value at the forecast-response system is defined as

$$
A V=E L^{\circ}-E L^{a}
$$

where $E L^{\text {a }}$ denotes the expected annual loss incurred by the $D M$ under strategy $S^{a}$. AV may be viewed as a measure of the performance of the overall forecastresponse system since it is computed with the actual law of motion, $\Phi$, and the actual response strategy, $s^{a}$.

In order to present the effectiveness of both the forecasting and the response systems, as well as the effectiveness of the overall forecast-response system, the following measures are defined.

Definition 19. The performance of the forecast-response system, PE, is defined by the vector:

$$
P E=\{P V, O V, A V\}
$$

Definition 20. The efficiency, EC, of the forecast-response system is defined by the vector:
$E C=\{E F, E R, E O\}$,
with
$E F=O V / P V ; E R=A V / O V ; E O=A V / P V$,
where
$E F$ is the efficiency of the forecasting system,
$E R$ is the efficiency of the response system,
EO is the overall efficiency.
The following relations hold:
(1) $A V \leq O V \leq P V$;
(2) $E O \leq E F ; E O \leq E R ; 0 \leq E F, E R, E O \leq 1$,
(3) $E O=E F \cdot E R$.

While PE is designed for evaluating alternative forecast-response systems, EC should be used for evaluating the components of a given forecast-response system (forecasting system vs response system). Together, PE and EC provide a basis for making decisions concerning allocation of resources to activities involved in a forecast-response system.

### 2.2 DETAILED MODELING

### 2.2.1 LEAD TIME

The $\operatorname{FFR}(D M)$ model developed in the previous section does not consider the timing of the flood crest. The authors felt that the conditioning of the strategy on three dimensional state space already presents such high computational complexity that inclusion of one variable more, without detailed analysis of its relevancy, could seriously affect the economics of the computational solvability of the problem. The lead time (to be defined rigorously soon) is the variable that was chosen to be the "safety-valve" in analysis of the trade-off between theoretical precision and computational feasibility of the model. In the sections to follow, we give a rigorous treatment of this problem. First, the elements affecting the lead time are analyzed in depth. Next, three possibilities of including the lead time in the model are discussed; these are:

1. state space approach,
2. limited state space approach,
3. parametric approach.

Analysis of Elements Affecting the Lead Time
The variable of concern is the flood crest defined by a two-tuple ( $\mathrm{h}, \xi$ ) on the product space $H \times T$, where $h \varepsilon H$ is the crest magnitude, and $\xi \varepsilon T$ is the time of occurrence. Let $t_{k}$ be the time of origin of the forecast ( $=$ time of making the observations upon which the forecast is based). Suppose that a forecast originating at the decision time $k \varepsilon K$ (real time $t_{k} \varepsilon T$ ) is ( $\left.h(k), \xi_{k}, k\right)$ (Figure 3a).
Definition 21. The lead time, $\lambda \varepsilon \Lambda$, of the forecast originated at $k$ is defined by the relation:

$$
\lambda(k)=\xi_{k}-t_{k}, \quad \text { for } \xi_{k} \geq t_{k}, k \in K .
$$

If $w(k)=1$, then the forecast ( $h, \xi, k$ ) will be followed by at least one more forecast. If $w(k)=0$, then the forecast $(h, \xi, k)$ is the last one, and it may be verified by the actual flood crest ( $h h, \xi^{\prime}, k$ ) with the actual lead time (Figure 3-3b).
$\lambda^{\prime}(k)=\xi_{k}^{\prime}-t_{k}, \quad$ for $\xi_{k}^{\prime} \geq t_{k}, \quad k \in K$.
Let $\left\{t_{k}\right\}$ be such that $t_{k+1}-t_{k}=\Delta t$ for all $k \varepsilon k$. Suppose that the total observed rainfall input for the flood event is defined on $\left[t_{B}, t_{E}\right.$, for $t_{B}, t_{E} \varepsilon T$. For a given watershed, the lead time $\lambda(k) \varepsilon \Lambda(k \varepsilon K)$ is a function of two independent vectors: FORECASTER and EVENT. Thus, we have $\lambda$ (FORECASTER, EVENT,k). The coordinates of FORECASTER are;

1. DELAY of the set of decision times $\left\{t_{j}, \ldots, t_{K N}\right\}$ in relation to the time interval $\left[t_{B}, t_{E}\right.$ ], defined as DELAY $=t_{T}-t_{B}$.
2. PORTION ( $k$ ), of the total rainfall input used for the forecast of the crest originating at $k$. For some $t\left(t_{B} \leq t \leq t_{E}\right), ~ P O R T I O N(k)=\left[t_{B}, t\right]$. The coordinates of EVENT are:
3. INITIAL conditions of the watershed.
4. SHAPE of the actual rainfall input function (For a linear rainfallrunoff relation it may be compressed to the location of the centroid).

Vector EVENT is beyond the hydrologist's control; therefore it is considered as a random vector being a source of the natural uncertainty for the lead time. FORECASTER, however, characterizes the hydrologist's ability to forecast the flood event before its actual occurrence. In the viewpoint taken here, FORECASTER has a deterministic nature in the sense that its value provides a fixed characteristic of a given forecasting system.

For example, FORECASTER who needs $2 \cdot \Delta t$ hours for initiation of his

FLOOD LEVEL $\uparrow$

a. FORECAST $(h, \xi, k)$ FOR $w(k)=1$

b. FORECAST $(h, \xi, k)$ FOR $w(k)=0$

Figure 2-3. Definition of the Lead time
functioning since the rainfall beginning, and who predicts the runoff solely on the basis of the precipitation observed up to the time of forecast preparation will have the coordinates (see Figure 2-4a):

DELAY $=2 \cdot \Delta t$,
PORTION $(k)=\left[t_{B}, t_{k}\right], k \in K$.
FORECASTER who does not have any delay in the initiation of his functioning, and who uses QPF for 2. $\Delta$ t hours forward will have the coordinates (see Figure 2-4b):

```
DELAY = -\Deltat ,
PORTION (k) = [t }\mp@subsup{\mp@code{B}}{B}{\prime}\mp@subsup{t}{k+2}{}],k\inK
```


## . State. Space Approach

To formally include the lead time the following changes are necessary: Definition 2'. The state space, $\Omega$, is a cartesian product $A \times I \times H \times \wedge \times W$, with $\Omega=\{x: x=(\alpha, i, h, \lambda, w)\}$.

Definition 4'. A law of motion, $\Phi$, is a conditional probability distribution on $\Omega$ of the following form:

$$
\Phi=\left\{P\left[x^{\prime} \mid x, d, k\right]: x^{\prime}, x \in \Omega, d \in D(x, k), k \in K\right\}
$$

with $P$ specified as

$$
P[\cdot \mid i, h, \lambda, w, k]= \begin{cases}P[i(k+1), h(k+1), \lambda(k+1) \mid i(k), h(k), \lambda(k), k] & \text { for } w(k)=1, \\ P\left[h h(k), \lambda^{\prime}(k) \mid i(k), h(k), \lambda(k), k\right] & \text { for } w(k)=0 .\end{cases}
$$

Definition 8' A loss function, $L$, is a real valued function satisfying separability condition, and having the form
$L(x(k), x(k+1) d(k), k)= \begin{cases}L_{1}(\alpha(k), d(k), k) & \text { for } w(k)=1, k=1, \ldots, k N-1, \\ L_{0}\left(\alpha(k), h h(k), \lambda^{\prime}(k), \alpha(k+1), d(k), k\right) \text { for } w(k)=0, k \in K .\end{cases}$
As one can see, there is no theoretical difficulty in including the lead time as an explicit state variable in the $\operatorname{FFR}(D M)$ model. However, two practical factors have to be given a careful consideration. First, the computational burden in the dynamic programming algorithm increases now to four

(a) DELAY $=2 \Delta t, \quad$ PORTION $(k)=\left[t_{B}, t_{k}\right]$

(b) DELAY $=-\Delta t \quad$ PORTION $(k)=\left[\dagger_{B},{ }^{\dagger} k+2\right]$

FIGURE 2-4. Definition of the FORECASTER
dimensions, wrapping thus our problem in the "curse of dimensionality". Secondly, successful estimation of the law of motion from the data available seems to be rather illusory, even though certain simplifications due to the stochastic independence of some of the state variables could be made.

In conclusion, we are forced unequivocally to incorporate the lead time in a different way than through the state vector. Undoubtediy, any alternative approach will require additional assumptions simplifying the actual forecast-response process. But at this point it is felt that by including the lead time, even simplistically, the $F F R(D M)$ model is still gaining some informational value. Next sections present the alternative approach

Limited State Space Approach
The following assumptions concerning the actual lead time $\lambda^{\prime} \varepsilon \Lambda$ are made: Assumption 1. $\lambda^{\prime}$ is independent of $i$ and $h$, and hh is independent of $\lambda^{\prime}$. Note that this assumption is in accordance with behavior of linear systems. Inasmuch as rainfall-runoff relations do not deviate far from linearity, the assumption is not unreasonable.

Assumption 2. $\left\{\lambda^{\prime}(k): k \varepsilon K\right\}$, defined for $w(k)=0$, form a sequence of independent variables in K.

Required modifications of the $\operatorname{FFR}(D M)$ model include definitions $2^{\prime \prime}$ and $8^{\prime}$, given in the previous section, and a new form of the law of motion. Definition 4". Under the circumstances given in definition 4, the law of motion is modified as follows:

$$
P[\cdot \mid i, h, \lambda, w, k]= \begin{cases}P[i(k+1), h(k+1) \mid i(k), h(k), k] & \text { for } w(k)=1, \\ P[h h(k) \mid i(k), h(k), k] \cdot P\left[\lambda^{\prime}(k) \mid k\right] & \text { for } w(k)=0 .\end{cases}
$$

The major advantage of this approach is that although the state vector and the loss function contain the lead time, the strategy $S$ remains defined
on only $A \times I \times H \times K$, as in the original $F F R(D M)$ model. The computational complexity increases by one integration at each stage of the dynamic programming algorithm, but there is no increase in the dimensionality of the problem.

The optimal strategy, $S^{*}$, may now be viewed as a strategy of an optimizing DM who extracts from the flood forecast only the values of $i$ and $h$. Yet, he accounts for the timing of $h$ by taking an expectation over actual lead time, $\lambda^{\prime}(k) \varepsilon \Lambda(k \varepsilon K)$, being now an independent random variable. The distribution $P\left[\lambda^{\prime}(k) \mid k\right]$ may be either the historical one or the DM's own prior, representing his belief concerning the actual lead time at $k$ if $w(k)=0, k \varepsilon K$.

Parometric Approach
Under Assumptions 1 and 2 we define the following:
Definition 22. The average actual lead time, $L T(k)$, of the forecast $h(k)$, for $w(k)=0$, $k \varepsilon K$, is given by the equation
$L T(k)=E\left[\lambda^{\prime}(k)\right] \quad$ for all $k \varepsilon K$.
The use of $L T(k)$ in the model will be shown in the definition of the consumer time. (Section 1.2.2.).

Simulation AZgorithm For Obtaining $P\left[\lambda^{\prime}(k) \mid k\right]$ or $L T(k)$.
0 . For a given basin, describe the forecasting system in terms of FORECASTER $=$ (DELAY, (PORTION $(k)\})$.

1. Generate $\underline{E V E N T}_{j}=$ (INITIAL, SHAPE $_{j}$ on the closed interval PORTION(k).
2. Transform EVENT $j$ into forecast $(h, \xi, k)$ for $w(k)=0$.
3. Get $\lambda^{\prime}(k)_{j}$.
4. Repeat 1-3 for all $j$.
5. Repeat 1-4 for all k.
6. From $\left\{\lambda^{\prime}(k)_{j}\right\}{ }_{j>0}$, obtain $P\left[\lambda^{\prime}(k) \mid k\right]$ or $L T(k)$ for all $k \varepsilon K$.

### 2.2.2 TRANSMISSION TIMES

Definition 23. Processing time, PT, is the length of a closed time interval defined on the set of decision times $K$ by the relation

$$
\operatorname{PT}(k)=t_{k}^{\prime}-t_{k}, \quad k \varepsilon K
$$

where:
$t_{k}$ - time of origin of the forecast (= time of making the observations upon which the forecast is based),
$t_{k}^{\prime}$ - time of issuing the forecast by the forecaster.
Physically, the processing time incorporates the time needed for data acquisition and the time needed for forecast preparation.
Definition 24. Dissemination time, $D T$, is the length of a closed time interval defined on the set of decision times $K$ by the relation

$$
D T(k)=t_{k}^{\prime \prime}-t_{k}^{\prime}, \quad k \varepsilon K
$$

where:
$t_{k}^{\prime}$ - time of issuing the forecast by the forecaster,
$t_{k}^{\prime \prime}$ - time of receiving the forecast by the DM.
In the above development, $\{P T(k)\}$ and $\{D T(k)\}$ are fixed characteristics of the forecasting system and the dissemination system, respectively. From the DM's viewpoint, there is a need for defining one more element which we shall call consumer time. It is the actual net time available to the DM located on $m$ for implementing the decision $d(k)$, when the states of the system at $k$ and $k+1$ are $x(k)$ and $x(k+1)$ respectively. Specific definition of the consumer time depends on the definition of the lead time. Subsequent development assumes Definition 22 (parametric approach) and gives only an approximation to the exact consumer time. The proposed approach should be viewed as a compromise between theoretical exactness and computational simplicity.

Definition 25. Consumer time, CT , is the length of a closed time interval defined on $\Omega X K$ for a DM located on mel such that

$$
C T(x(k), x(k+1), k, m)=\max \{0, \text { "value" }\}
$$

with "value" specified as follows:

| state |  | "value" |
| :--- | :---: | :---: |
| $w(k)=1$ | $i(k+1)<m$ | $\Delta t(k)$ |
|  | $i(k+1) \geq m$ | $\beta \Delta t(k)-P T(k)-D T(k)$ |
|  | $h h(k)<m$ | $\infty$ |
|  | $h h(k) \geq m$ | $\beta L T(k)-P T(k)-D T(k)$ |

$\beta$ is a real valued function defined on $\Omega$ for location $m$ with values in $[0,1]$. It accounts for timing of the event $\{i \geq m\} .{ }^{1 /}$ Inasmuch as at $k, h h(k), w(k)$, and $\mathrm{i}(\mathrm{k}+1)$ are random variables, the consumer time, CT , is also a random variable.

1
For example, under assumption of linear interpolation between states $x(k)$ and $x(k+1)$ on the time interval $\left[t_{k}, t_{k+1}\right]$, we have

$$
B(x(k), x(k+1), m)= \begin{cases}\beta(i(k), i(k+1), m)=\frac{m-i(k)}{i(k+1)-i(k)} & \text { for } w(k)=1 \\ \beta(i(k), h h(k), m)=\frac{m-i(k)}{h h(k)-i(k)} & \text { for } w(k)=0\end{cases}
$$

### 2.2.3 DECISION CONSTRAINTS

Particularizing the decision set, $D$, it is assumed that for any statetime $(\alpha, k), D(\alpha, k)$ represents a lower and an upper bound on the decision $d(k)$. The upper constraint may be due to limited availability of physical resources (e.g., means of transportation for evacuation), limited availability of manpower, etc.

Hence, it is postulated that there exists a unique, real-valued mapping dd from $[0, \infty)$ into $A$ such that for any $t \varepsilon[0, \infty), \operatorname{dd}(t)$ is the maximum degree of response which can be achieved in the time interval [ $0, t$ ]. Accordingly, if at decision time $k$, the degree of response already achieved is $\alpha(k)$, and the consumer time is $C T(k)$ then the maximum degree of response that $c a n$ be achieved at $k+1, \alpha(k+1)$, is constrained by

$$
\alpha(k) \leq \alpha(k+1) \leq d d\left(d d^{-1}(\alpha(k))+C T(k)\right) .
$$

dd will be termed the decision constraint function.

### 2.2.4 INCORPORATION OF THE CONSUMER TIME

The following decision mechanism, incorporating the consumer time, $C T$, is assumed. At the decision time $k$, the $D M$ chooses $d(k)$, and the cost of response is computed from $\alpha(k)$ and $d(k)$. However, at $k+1$ not necessarily $\alpha(k+1)=d(k)$ since the actual net time available for implementation of $d(k)$ (i.e., the consumer time, $C T(k)$ ) is a random variable.

Referring to the particular form of the decision set, specified in Section 1.2.3, the degree of response $\alpha(k+1)$ actually achieved at time $k+1$ is determined by the relation
$\alpha(k+1)=\min \left\{d(k), d d\left(d d^{-1}(\alpha(k))+C T(k)\right)\right\}$.
Let us summarize the concepts introduced in Sections 1.2.1 through 1.2.4.

1. For all k\&K,
a) the forecasting system is characterized by the average actual lead time, $\operatorname{LT}(k)$, and the processing time, $\mathrm{PT}(k)$,
b) the dissemination system is characterized by the dissemination time, $D T(k)$.
2. For all $k \in K$, the consumer time, $C T(k)$, is a random variable defined by the random variates $h(k), w(k), i(k+1)$ for the given values of $L T(k), P T(k), D T(k)$, and $f(k)$.
3. For all $k \varepsilon K, \alpha(k+1)$ is a random variable induced by the random variate $C T(k)$, for the given values of $\alpha(k)$ and $d(k)$.
4. $\alpha(k+1)$ enters the loss function for $w(k)=0$.

### 2.2.5 LOSS FUNCTION

Postulated loss function is given in Definition 8. Assuming further stationarity in $k$, we may write

$$
L(x(k), x(k+1), d(k))= \begin{cases}L_{1}(\alpha(k), d(k)) & \text { for } w(k)=1 \\ L_{0}(\alpha(k), h h(k), \alpha(k+1), d(k)) & \text { for } w(k)=0\end{cases}
$$

for all keK. Next, two real-valued functions are assumed to exist:
a) Cost function, $L C(\alpha)$, specifying the cost of response of degree $\alpha$,
b) Stage-damage-response function, $L D(\alpha, h h, m)$, specifying the damage caused to the establishment located on step $m$ by the actual flood crest hh, given the final degree of response $\alpha$.

Now the loss function, $L$, can be established in terms of the LC and LD functions as follows:

$$
\begin{array}{ll}
L_{1}(\alpha, d)=L C(d)-L C(\alpha) & \text { for } w=1 \\
L_{0}\left(\alpha, h h, \alpha^{\prime}, d\right)=L C(d)-L C(\alpha)+L D\left(\alpha^{\prime}, h h, m\right) & \text { for } w=0
\end{array}
$$

where $\alpha^{\prime}=\alpha(k+1)$ is the final degree of response, as defined in Section 1.2.4.
Proposed form of LD and LC will now be developed. Let $\{y(m): m=1, \ldots$, IN $\}$ be a set of step elevations above an arbitrary level and $z$ denote the depth of flooding measured from the first floor level. For an establishment located on m , the following relation holds:

$$
\begin{equation*}
z(h h, m)=y(h h)-y(m), \quad h h \geq m, h h \varepsilon H . \tag{1}
\end{equation*}
$$

Define
MD - maximum possible damage to the establishment due to flood of any magnitude with no response, $\alpha=0.0$,
$M R(z)$ - unit reduction function expressing the reducible fraction of $M D$ induced by full response, $\alpha=1.0$, when the depth of flooding is $z$, $\delta(z)$ - unit damage function (Bhavnagri and Bugliarello, 1965, 1966),
$\gamma(\alpha)$－unit cost function，$\alpha \in A$ ．
The fundamental assumption is now made that，for a given establishment，the stage－damage function，LD，can be specified by a linear equation

$$
\begin{equation*}
\operatorname{LD}(z)=\operatorname{MD\delta }(z) \tag{2}
\end{equation*}
$$

Furthermore，if a linear relation between the degree of response，$\alpha$ ，and the reduced damage is accepted，then in view of（2），the stage－damage－response function assumes the form

$$
\begin{equation*}
\operatorname{LD}(\alpha, z)=\operatorname{MD}[1-\alpha \operatorname{MR}(z)] \delta(z) . \tag{3}
\end{equation*}
$$

The basic expression for LD follows now from（1）and（3）

$$
\begin{equation*}
\operatorname{LD}(\alpha, h h, m)=\operatorname{MD}[1-\alpha M R(h h, m)] \delta(h h, m) \tag{4}
\end{equation*}
$$

In a similar fashion，the cost function，LC，can be specified by a linear equation
$L C(\alpha)=M D_{\gamma}(\alpha)$.
Now，it is a simple matter to verify
$L_{p}(\alpha, d)=\operatorname{MD}[\gamma(d)-\gamma(\alpha)]$,
$L_{o}\left(\alpha, h h, \alpha^{\prime}, d\right)=\operatorname{MD}\left\{\gamma(d)-\gamma(\alpha)+\left[1-\alpha^{\prime} M R(h h, m)\right] \delta(h h, m)\right\}$
Somewhat more must be said about the rationale of using unit functions concept．Suppose that a flood forecast－response system has to be evaluated for a finite number of establishments．With the unit functions concept a procedure for developing LD and LC may now be outlined as follows：

1．Partition all establishments in the flood plain into a finite number－ of structural categories，say set $R=\{r: r=1,2, \ldots, R N\}$ ．

2．For each category $r \& R$ ，find $\delta_{r}, \gamma_{r}$ ，and $M R_{r}$ ．
3．Describe each establishment（ $\equiv$ decision maker）in the flood plain by specifying vector ESTABLISHMENT $=(m, r, M D)$ ．

4．With $J_{r}$ the set indexing the establishments within category $r \in R$ ，for an establishment $j r, j \varepsilon J_{r}, r \in R$ ，

$$
L_{1}(\alpha, d)=M D_{j r}\left[\gamma_{r}(d)-\gamma_{r}(\alpha)\right]
$$

and

$$
L_{o}\left(\alpha, h h, \alpha^{\prime}, d\right)=M D_{j r}\left\{\gamma_{r}(d)-\gamma_{r}(\alpha)+\left[1-\alpha^{\prime} M R_{r}(h h, m)\right] \delta(h h, m)\right\}
$$

Twofold advantage of the above approach is apparent. First, each individual establishment is characterized by relatively simple information (only three numbers!). Second, linearity of the loss function with respect to MD and the fact that the multiplicator of MD does not depend on the particular $O M$ $j \varepsilon J_{r}$ but only on the structural category $r \varepsilon R$ and the location step $m$, are the utmost important properties which lead to an efficient computationally FFR(REACH) model. This will be clearly demonstrated in the further development of the model.

### 2.2.6 PURE STRATEGY

Pure strategy has been specified in Definition 12. Here, an operational equation for computing $S^{p}$ is given. For the $D M$ located on the step $m$,

$$
S^{P}(\alpha, i, h, k)=\left\{\begin{array}{l}
\alpha(k) \text { if } \quad i \geq m \\
\alpha(k) \text { if } \quad h<m \\
d d\left(d d^{-1}(\alpha(k))+L T(k)\right) \quad \text { otherwise }
\end{array}\right.
$$

### 2.2.7 INITIAL CONDITION

It is assumed that in the initial state-time $\left(x_{0}, k_{0}\right)=\left(\alpha_{0}, i_{0}, h_{0}, k_{0}\right)$, $\alpha_{0}=0$ and $k_{0}=1$. Hence, $P\left[x_{0}\right]=P\left[i_{0}, h_{0}\right]$ for $a l l i_{0} \varepsilon I, h_{0} \varepsilon H$, and the expected annual loss, EL, associated with a strategy $S$, can be computed from the equation:
$E L={\underset{\substack{i_{0} \\ h_{0} \varepsilon I}}{\Sigma} V\left(\alpha_{0}, i_{0}, h_{0}, k_{0}\right) \cdot P\left[i_{0}, h_{0}\right] \cdot E[N] . . ~}_{\text {. }}$

### 2.2.8 DYNAMIC PROGRAMMING ALGORITHM

A general dynamic programming algorithm for $\operatorname{FFR}(D M)$ has been given in Section 1.1.1. The algorithm presented herein uses parametric approach to lead time and employs specific form of the loss function as developed in Section 1.2.5.

For a DM having ESTABLISHMENT $=(m, r, M D)$ the algorithm proceeds as follows:

| (a) $\mathrm{S}^{*}(\alpha, \mathfrak{i}, \mathrm{~h}, \mathrm{KN})=\mathrm{d}^{*}$, where for any state ( $\alpha, \mathrm{i}, \mathrm{h}$ ), $\mathrm{d}^{*}$ is a solution to |  |
| :---: | :---: |
|  |  |
| (b) | $S^{*}(\alpha, i, h, k)=d^{*}, k<K N$, where for any state-time ( $\alpha, i, h, k$ ) , $d^{*}$ is a solution to the recursive |
|  | $V(\alpha, i, h, k)=\min _{d \in D(\alpha, k)}\left\{\gamma_{r}(d)-\gamma_{r}(\alpha)+\sum_{i, \varepsilon I} V\left(\alpha^{\prime}, i^{\prime}, h^{\prime}, k+1\right) \cdot P\left[i^{\prime}, h^{\prime} \mid i, h, k\right] \cdot P[w=1 \mid k]+\right.$ |
|  | $h^{\prime}{ }^{\prime} \varepsilon H$ |

$\left.\underset{h h \in H}{\Sigma}\left[1-\alpha ' M R_{r}(h h, m)\right] \cdot \delta_{r}(h h, m) \cdot P[h h \mid i, h, k] \cdot P[w=0 \mid k]\right\}$
$(\alpha, i, h, k)$ will be called unit expected loss associated with the initial state-time ( $\alpha, i, h, k$ ) and the
strategy $S^{*}$. For a given initial condition $\alpha_{0}, k_{0}$ and $\left\{P\left[i_{0}, h_{0}\right]\right\}$, the unit expected annual loss,
associated with the strategy $\mathrm{S}^{*}$, is specified by the relation
$V A^{*}=\sum_{i} V\left(\alpha_{0}, i_{0}, h_{0}, k_{0}\right) \cdot P\left[i_{0}, h_{0}\right] \cdot E[N]$
and the expected annual loss, $E L{ }^{*}$, associated with the strategy $S^{*}$, is
$E L{ }^{*}=M D \cdot V A^{*}$

### 2.2.9 MEASURES OF EFFECTIVENESS

The concept of unit damage functions brings about some computational advantage also for the measures of effectiveness. For a DM having ESTABLISHMENT $=(m, r, M D)$, the performance of the forecast-response system may be redefined as follows. Definition $16^{1}$. The potential value, PV.

By definition

$$
P V=E L^{0}-E L^{* *} .
$$

With VA the unit expected annual loss,

$$
P V=M D\left(V A^{0}-V A^{* *}\right),
$$

and defining the unit potential value as

$$
P V U=V A^{\circ}-V A^{* *},
$$

we obtain

$$
P V=M D \cdot P V U .
$$

The unit expected annual loss associated with the strategy $s^{\circ}(=$ no response) is given by the equation

$$
V A^{O}=\sum_{h h \varepsilon H} \delta_{r}(h h, m) \cdot P[h h] \cdot E[N]
$$

The unit expected annual loss associated with an optimal strategy $s^{* *}$ in response to a perfect forecast is specified by the equation

$$
V A^{* *}=\sum_{h h \in H d_{\varepsilon D}(0,1)} \min _{r}\left\{\gamma_{r}(d)+\left[1-d M R_{r}(h h, m)\right] \delta_{r}(h h, m)\right\} \cdot P[h h] \cdot E[N]
$$

By analogous reasoning the optimal and actual values are redefined as follows. Definition 17'. The optimal value, OV.
$O V=M D \cdot O V U$
where the unit optimal value is given by
$O V U=V A^{0}-V A^{*}$.
Definition 18'. The actual value, AV.

$$
A V=M D \cdot A V U
$$

where the unit actual value is given by $A V U=V A^{0}-V A^{a}$.
3. FORECAST-RESPONSE MODEL FOR A REACH

### 3.1 GENERAL MATHEMATICAL FORMULATION

The $F F R(R E A C H)$ model has been developed under the following assumptions:
(i) The forecasting and dissemination subsystems of the FFR(REACH) model are the same as in the $\operatorname{FFR}(D M)$ model; however, the response subsystem is now defined as a collection of all DMs in the reach.
(ii) On each hypothetical step, $m$, there are $G(m)$ DMs. Each $D M$ is indexed by mg ; thus $(\dot{D M})_{m g}$ is the gth $D M$ on the mth step of the reach, where $m=1,2, \ldots, I N ; g=1,2, \ldots, G(m)$.
(iii) Each element of the response subsystem becomes now a vector having IN $\sum_{m=1} G(m)$ components.
(iv) A general case is considered in which the measures of effectiveness for the reach are obtained by mere "summation" over all DMs in the reach.
(v) The definitions of the elements of the $\operatorname{FFR}($ REACH $)$ model which are identical
with those of the $\operatorname{FFR}(D M)$ model are not repeated here. Basically, then, the mathematical formulation of the model is limited to indexing of the appropriate model elements by $\mathrm{mg}, \mathrm{m}=1, \ldots, \mathrm{IN}$; $g=1, \ldots, G(m)$.

Definition IR. The set of decision times, $K$, does not change.
Definition 2R. The state space for (DM) ${ }_{m g}$ is
$(\Omega)_{m g}=\left\{(x)_{m g}:(x)_{m g}=\left((\alpha)_{m g}, i, h, w\right)\right\}$
where

$$
(x)_{\mathrm{mg}}=\text { state of the system for }(\mathrm{DM})_{\mathrm{mg}}
$$

and
$(\alpha)_{\mathrm{mg}}=$ the degree of response already achieved by (DM) mg .
Definition 3R. (D) $\mathrm{mg}_{\mathrm{mg}}$ is the decision set for (DM) mg .
Definition 4R. The law of motion, $\Phi$, remains unchanged since
it depends only on the forecasting system and not on the DM.
Definition 5R. $(\bar{x})_{m g}$ is a trajectory for (DM) ${ }_{m g}$.
Definition 6R. ( $\bar{d})_{m g}$ is a policy for (DM) mg .
Definition 7R. $\quad(S)_{m g}$ is a strategy, and $(\sigma)_{m g}$ is the set of feasible
strategies for (DM) mg .
Definition 8R. $(L(\bar{x}, \bar{d}, k))_{m g}$ is a loss function for (DM) ${ }_{m g}$.
Definition 9R. A flood forecast-response process for a reach (abbreviated
$\operatorname{FFR}(R E A C H)$ ) is a tuple of the form
$\left(\left\{(\Omega)_{\mathrm{mg}},(\mathrm{D})_{\mathrm{mg}},(L)_{\mathrm{mg}}\right\}, K, \Phi\right)$.
Definition 10R. $\left(S^{*}\right)_{m g}$ is an optimal strategy for $(D M)_{m g}$.
Definition llR. $\left(S^{\mathrm{a}}\right)_{\mathrm{mg}}$ is an actual strategy for (DM) $\mathrm{mg}^{\text {• }}$
Definition 12R. $\quad\left(S^{P}\right)_{m g}$ is a pure strategy for (DM) mg.
Definition 13R. The initial condition, $\Phi_{0}$, is assumed to be independent of the $D M$ Definition 14R. (EL) $)_{\mathrm{mg}}$ is the expected annual loss for (DM) ${ }_{\mathrm{mg}}$. The expected annual loss for the reach is defined by the equation

$$
E L(R E A C H)=\sum_{m=1}^{I N} \underset{\sum_{g=1}}{G(m)}(E L)_{m g} .
$$

Definition 15R. Definition of the flood, F, does not change.
Definition l6R. The potential value of a forecast-response system for the reach, PVR, is defined as follows:
where $(P V)_{m g}$ is the potential value for $(D M)_{m g}$.
Definition 17R. The optimal value for the reach, OVR, is defined as follows:

$$
O V R=\sum_{\mathrm{m}=1}^{\mathrm{IN}} \sum_{\mathrm{g}=1}^{\mathrm{G}(\mathrm{~m})}(\mathrm{OV})_{\mathrm{mg}}=E L^{\circ}(\text { REACH })-\dot{E L}^{*}(\text { REACH })
$$

where $(O V)_{m g}$ is the optimal value for $(D M)_{m g}$.
Definition 18R. The actual value for the reach, AVR, is defined as follows:

$$
A V R=\sum_{\sum_{m=1}^{I N} \underset{\mathrm{~g}=1}{\mathrm{G}(\mathrm{~m})}}^{(A V)_{\mathrm{mg}}=E L^{0}(\text { REACH })-E L^{\mathrm{a}}(\text { REACH }) .}
$$

where $(A V)_{m g}$ is the actual value for (DM) $)_{m g}$.
Definition 19R. The performance of the forecast-response system for the reach
is defined by the vector:

$$
\operatorname{PE}(\text { REACH })=\{P V R, O V R, A V R\} .
$$

Definition 20R. The efficiency of the forecast-response system for the reach
is defined by the vector:

$$
E C(R E A C H)=\{E F R, E R R, E O R\}
$$

where

$$
\begin{aligned}
& E F R=O V R / P V R, \\
& E R R=A V R / O V R, \\
& E O R=A V R / P V R .
\end{aligned}
$$

The following relations hold:

1) $A V R \leq O V R \leq P V R$,
2) $E O R \leq E F R ; E O R \leq E R R ; 0 \leq E F R, E R R, E O R \leq 1$.

Definition 21R. Definition of the lead time, $\lambda$, does not change. Definition 22R. Definition of the average actual lead time, LT, does not change.

Definition 23R. Processing time is the same for all DMs since it is a characteristic of the forecasting system.

Definition 24R. Dissemination time, $(D T(k))_{m g}$, is defined for (DM) ${ }_{m g}$ as

$$
(D T(k))_{m g}=\left(t_{k}\right)_{m g}-t_{k}^{\prime}, \quad k \varepsilon K
$$

where $\left(t_{k}^{\prime \prime}\right)_{m g}$ is time of receiving the forecast by $(D M)_{m g}$, and $t_{k}^{\prime}$ is time of issuing the forecast by the forecaster. Since $t_{k}^{\prime}$ depends only on the forecasting system, it is not indexed by mg.

Definition 25R. Consumer time, ( $C T)_{m g}$, is indexed by mg since by definition it depends on $m$ and (DT) mg .
Decision constraints. $(d d(t))_{m g}$ is the decision constraint function for ( $\left.D M\right)_{m g}$. Loss function. If each $D M$ is assumed to have different loss function, then the loss function postulated in Section 1.2 .5 can be written for ( $D M)_{m g}$ as follows:

$$
(L(x(k), x(k+1), d(k)))_{m g}= \begin{cases}\left(L_{1}(\alpha(k), d(k))\right)_{m g} & \text { for } w(k)=1, \\ \left(L_{0}(\alpha(k), h h(k), \alpha(k+1), d(k))\right)_{m g} & \text { for } w(k)=0\end{cases}
$$

Hence, using the proposed form for (LC) $)_{\mathrm{mg}}$ and $(L D)_{\mathrm{mg}}$, we can write the following:

$$
\left(L_{p}(\alpha, d)\right)_{m g}=(M D)_{m g}\left[\gamma_{r}(d)_{m g}-\gamma_{r}(\alpha)_{m g}\right],
$$

and

$$
\left(L_{0}\left(\alpha, h h, \alpha^{\prime}, d\right)\right)_{m g}=(M D)_{m g}\left\{r_{r}(d)_{m g} g^{-r_{r}}(\alpha)_{m g}+\left[1-\left(\alpha^{\prime}\right)_{m g} M_{r}(h h, m)\right] \cdot \delta_{r}(h h, m)\right\} .
$$

The important property of the above loss function is its multiplicative form, where the random variable hh appears only in the unit damage function, $\delta$, and unit reduction function, MR, which are independent of the index $g$. This fact substantially simplifies the dynamic programming computations.

### 3.2 FORMULATION OF AN EFFICIENT MODEL

General formulation of the FFR (REACH) model requires that the FFR system be evaluated for each $D M$ on the REACH. Since in each evaluation run the dynamic programming algorithm has to be executed twice (once to find EL* and once, without optimization, to find $E L{ }^{\text {a }}$ ), computational burden precludes any realistic application of the model. ${ }^{1 /}$ It is, therefore, of utmost importance to find a way of reducing the computational complexity. The subsequent sections explore such possibilities. It is shown that under relatively mild assumptions, a very expedient and computationally efficient model for evaluating $\operatorname{FFR}($ REACH $)$ can be obtained.

1/
For the Milton case study, evaluation of FFR(DM) takes on the average 80 sec CP time on CDC 6400. With 900 establishments in Milton, evaluation of FFR(REACH) would take $72,000 \mathrm{sec}$ or 20 hours CP time!

### 3.2.1 MAIN THEOREM

Theorem 3. Consider a FFR(DM) model in which an optimal strategy $S^{*}$ is generated according to the algorithm described in Section 2.2.8. If for any two $D M s$ a and $b$, located on the some step $m$, the following relations hold:
(i) $(d d(t))_{a}=(d d(t))_{b}$ for $a t z t \geq 0$,
(ii) $(D T(k))_{a}=(D T(k))_{b} \quad$ for all $k \varepsilon K$,
(iii) $(r)_{a}=(r)_{b}$,
then $\left(S^{*}\right)_{a}=\left(S^{*}\right)_{b}=S^{*} \quad$ for all $S^{*} \varepsilon \sigma^{*}$.
Furthermore, if VA* is the unit expected annual loss associated with on optimal stragety $S^{*} \varepsilon \sigma^{*}$ then the expected annual losses for $D M s a$ and $b$ are given respectively by

$$
(E L *)_{a}=(M D)_{a} \cdot V A^{*}
$$

and

$$
(E L *)_{b}=(M D)_{b} \cdot V A^{*}
$$

Proof. The proof of this follows directly from inspection of the dynamic programming algorithm given in Section 1.2.8m
3.2.2 COMPUTATION OF EXPECTED ANNUAL LOSS, ASSUMPTION 2.

To simplify further the computations involved in obtaining EL(REACH), in the following sections two assumptions will be introduced, and the consequence upon the computational schemes will be derived.

Assumption 1. ALI DMs on a specific step, m, of the flood plain choose strategies from a set of strategies having $U(m)$ elements, where

$$
U(m) \leq G(m), m=1,2, \ldots, I N .
$$

The set of all strategies for the reach is now

$$
\{S(\mathrm{mu}): m=1, \ldots, I N ; u=1, \ldots, U(m)\}
$$

Let $G(m u)=\{g\}$ be the finite set indexing the $D M s$ on step $m$ who use the strategy $S(\mathrm{mu})$. The expected annual loss for all DMs in $G(m u)$ is

$$
E L(m u)=\sum_{g \varepsilon G(m u)}^{\Sigma}(E L)_{\mathrm{mg}}
$$

Summation over all $U(m)$ strategies gives the expected annual loss for the step m

$$
E L(m)=\sum_{u=1}^{U(m)} E L(m u) .
$$

Finally, summation over all steps gives the expected annual loss for the reach

$$
E L(R E A C H)=\sum_{m=1}^{I N} E L(m),
$$

or

$$
E L(R E A C H)=\sum_{m=1}^{I N} \sum_{u=1} \quad U(m) \quad \sum_{g(m u)} \quad(E L)_{m g}
$$

If, in addition to Assumption 1 , all DMs in $G(m u)$ satisfy the conditions of Theorem 3, then we can write

$$
E L(R E A C H)=\sum_{m=1}^{I N} \sum_{u=1}^{U(m)} \sum_{\varepsilon G(m u)}^{\Sigma}(M D)_{m g} \quad V A(m u)
$$

where $V A(m u)$ denotes the unit expected annual loss associated with the strategy $S(m u)$. Now denoting

$$
\begin{equation*}
M D(\mathrm{mu})=\sum_{\mathrm{g} \mathrm{\varepsilon G}(\mathrm{mu})}^{\Sigma}(M D)_{\mathrm{mg}}, \tag{1}
\end{equation*}
$$

the following equation is obtained

$$
E L(\text { REACH })=\sum_{m=1}^{I N} \sum_{u=1}^{U(m)} M D(m u) \cdot V A(m u) .
$$

Thus the expected annual loss for the reach has been expressed in terms of $M D(m u)$ - the maximum possible damage to the group of establishments associated with the strategy $S(m u)$ and $V A(m u)$ - the unit expected annual loss associated with the strategy $S(\mathrm{mu})$.

### 3.2.3 COMPUTATION OF EXPECTED ANNUAL LOSS, ASSUMPTION 2.

Denote

$$
M D(R E A C H)=\sum_{m=1}^{I N} \sum_{g=1}^{G(m)}(M D)_{\mathrm{mg}} .
$$

Notice that $M D(R E A C H)=\sup$ (stage-damage function for the reach with no response from any $D M$ ).

Assumption 2. There exists a distribution function $\{n(m u)\}$ on the set of integer numbers $\{m u: m=1, \ldots, I N ; u=1, \ldots, U(m)\}$, such that

$$
\begin{aligned}
& \begin{array}{cc}
\text { IN } & U(m) \\
m=1 & \sum_{u=1} \\
n(m u)=1,
\end{array} \\
& \text { (ii) } n(m u)=n(u \mid m) \cdot n(m) \text {, } \\
& \text { (iii) } M D(m u)=n(m u) \cdot M D(\text { REACH })
\end{aligned}
$$

In other words, the distribution $\{\eta(m u)\}$ partitions the maximum possible damage for the reach, $M D(R E A C H)$, among the strategies used by the $D M s$ in the given reach, or, more exactly, among the groups of establishments associated with particular strategies from $\{S(\mathrm{mu})\}$.

Now Equation (2) can be written as follows

$$
E L(\text { REACH })=\sum_{m=1}^{I N} \sum_{u=1}^{U(m)} \eta(m u) \cdot M D(R E A C H) \cdot V A(m u),
$$

and denoting

$$
\begin{equation*}
V A(R E A C H)=\sum_{m=1}^{I N} \underset{\sum_{u=1}}{ } \quad \mathrm{~V}(\mathrm{~m}) \tag{3}
\end{equation*}
$$

the final result is established

$$
\begin{equation*}
E L(R E A C H)=M D(R E A C H) \cdot V A(R E A C H) . \tag{4}
\end{equation*}
$$

Clearly then, the expected annual loss for the reach has been obtained as a product
of two values. The first one, $\operatorname{MD}(\mathrm{REACH})$, is the maximum possible damage for the reach; the second one, VA(REACH), is the unit expected annual loss associated with the set of strategies $\{S(m u)\}$ and the distribution $\{n(m u)\}$.

### 3.2.4 DISCUSSION

In the previous sections two methods of computing EL(REACH) have been established.

1. In the first method, the computation of EL (REACH) is performed according to Equations (1) and (2). Theorem 3, in conjunction with Assumption 1, allows grouping the DMs according to the similar input characteristics and strategies. This fact increases significantly the computational efficiency since now the dynamic programming algorithm is executed only once for each group of the DMs as apposed to one execution for each individual DM, as a straightforward approach would require. Still, each DM contributes to the EL(REACH) individually in the sense that the complete input information for each DM is required and is used directly in the computations.
2. The second method employs Equations (3) and (4). In addition to the conditions of Theorem 3 and Assumption 1, Assumption 2 is introduced. Under this circumstance an expedient Equation (4) has been obtained where EL(REACH) is computed in terms of the maximum possible damage for the reach, which can be obtained from an "ordinary" stage-damage function, and in terms of the unit expected annual loss.

Although in the second method the DMs are not considered individually, this method, theoretically, is equivalent to the first approach. The very appealing, from both the computational $/$ and implementational viewpoints, second method reveals the efficiency of our approach combining stochastic dynamic programming with the concept of unit loss functions.

[^0]There is a practical problem as to how one can identify the $U(m)$ different strategies for every meI. From condition (iii) of Theorem 3 it is known that to have the same strategy the DMs must have the same value for $r$. In one view of discussion from Section 2.2.5., it seems reasonable and advantageous to assume that $U(m)=R N$ for every meI and that $u=r(r=1, \ldots, R N)$.

### 3.2.5 DISTRIBUTION OF THE PERFORMANCE

Let $P E(m u)=\{P V(m u), O V(m u), A V(m u)\}$ be the performance of the FFR system for a group of DMs associated with the strategy $S(m u)$ ( $m=1, \ldots, I N$; $u=1, \ldots, U(m))$.

Definition 26R. Distribution of the performance is a triplet of the distributions $v=\left\{\nu_{p}(m u), \nu_{0}(m u), v_{a}(m u): m=1, \ldots, I N ; u=1, \ldots, U(m)\right\}$ such that
(i) $P V(m u)=v_{p}(m u) \cdot P V R$,
(ii) $\mathrm{OV}(\mathrm{mu})=v_{0}(\mathrm{mu}) \cdot \mathrm{OVR}$,
(iii) $\quad A V(m u)=v_{a}(m u) \cdot A V R$.

Computation of $v$ can be accomplished as follows:

$$
\nu_{p}(m u)=\frac{P V(m u)}{P V R}=\frac{\eta(m u) M D(R E A C H) \cdot\left[V A^{0}(m u)-V A^{* *}(m u)\right]}{M D(R E A C H) \cdot P V U}
$$

hence

$$
v_{p}(m u)=\eta(m u) \cdot \frac{\left[V A^{0}(m u)-V A^{* *}(m u)\right]}{P V U},
$$

and in a similar manner

$$
\begin{aligned}
& v_{o}(m u)=\eta(m u) \cdot \frac{\left[V A^{0}(m u)-V A^{\star}(m u)\right]}{O V U}, \\
& v_{a}(m u)=\eta(m u) \cdot \frac{\left[V A^{0}(m u)-V A^{a}(m u)\right]}{A V U}
\end{aligned}
$$

### 3.2.5 DISTRIBUTION n

A further discussion on the interpretation and derivation of the distribution $\eta$ is in order.

Marginaz $n(y)$
Instead of discrete steps $m(m=1, \ldots, I N)$, consider a continuous variable y measuring elevation above certain level. Let the stage-damage function . for the reach, $S D$, be defined on a closed interval $\left[y_{0}, y_{m}\right]$ such that $\operatorname{SD}\left(y_{0}\right)=0$ and $\operatorname{SD}\left(y_{m}\right)=M D(\operatorname{REACH})$.

For any $y \varepsilon\left[y_{0}, y_{m}\right], S D(y)$ expresses the cummulative damage on the elevation interval $\left[y_{0}, y\right]$ caused by flood of the magnitude $y$. The maximum possible damage for a particular level $y \varepsilon\left[y_{0}, y_{m}\right]$ is defined as

$$
\begin{equation*}
M D(y)=\frac{d S D(y)}{d y} . \tag{1}
\end{equation*}
$$

By integrating Equation (iii) in Assumption 2, Section 2.2.3., over $u$, we obtain

$$
\begin{equation*}
M D(y)=\eta(y) \cdot M D(R E A C H) \tag{2}
\end{equation*}
$$

Now from (1) and (2), it follows that

$$
n(y)=\frac{1}{M D(R E A C H)} \frac{d S D(y)}{d y},
$$

or

$$
\begin{equation*}
n(y)=\frac{1}{\operatorname{SD}\left(y_{m}\right)} \frac{\operatorname{dSD}(y)}{d y} . \tag{3}
\end{equation*}
$$

Precisely, the marginal distribution $n(y)$ has been expressed in terms of the stage-damage function. $n(y)$ shows how the maximum possible damage for the REACH is distributed along the elevation.

Conditional $n(r \mid y)$
Suppose that the number of strategies $U(y)$ at any level $y$ is equal to the
number of structural categories (which holds if the assumptions of Theorem 3 are satisfied) so that in notation of Assumption $2 u=r, r \in R$. Distribution $\eta(r \mid y)$ tells then how the maximum possible damage, $M D(y)$, at the given elevation $y$, is distributed among structural categories reR.

If the exact distribution $n(r \mid y)$ is not available, there are many ways of approximating it. Basically, derivation of $n(r \mid y)$ requires an answer to a question: what is the spatial distribution of the various establishments on the flood plain. Inasmuch as this type of information is fundamental for any type of economic analyses of flood plains, assessment of $n(r \mid y)$ does not seem to introduce any additional difficulty beyond that encountered in the traditional flood studies.
4. SUMMARY OF THE MODEL ELEMENTS

All elements defining the general $\operatorname{FFR}(D M)$ and $F F R(R E A C H)$ models are specified in Table 2-1. They are grouped according to the input, output, and internal elements. Those elements which are exactly the same in both models are not repeated for $\operatorname{FFR}(R E A C H)$.

On the basis of the specific structures for particular model elements, which were proposed in Section 1.2., a general statement of the input information for $\operatorname{FFR}(D M)$ and $F F R(R E A C H)$ models has been prepared (Table 2-2).

Table 2-1. Summary of the elements of the general models

| Element | FFR(DM) | FFR (REACH) |
| :---: | :---: | :---: |
| Input Elements |  |  |
| 1. Set of decision times | $X=\{k: k=1, \ldots, k N\}$ |  |
| 2. Discretization of the | $I=\{i: j=1, \ldots, i K\}$ |  |
| flood plain |  |  |
| 3. Decision set | $D=\{0(x, k)\}$ | $\left\{(0)_{\text {mg }}\right\}$ |
| 4. Law of motion | - $=\left\{P\left[x^{\prime} \mid x, d, k\right]\right\}$ |  |
| 5. Loss function | $L=(\bar{x}, \bar{d}, k)$ | $\left\{(L(\sqrt{x}, \mathrm{~J}, \mathrm{k}))_{\mathrm{mg}}\right\}$ |
| 6. Actual response strategy | $s^{\text {a }}=\left\{S^{\text {a }}(x, k)\right\}$ | $\left.\left.\mathfrak{f ( ~} s^{\text {a }}\right)_{\text {mg }}\right\}$ |
| 7. Inftial condition | $*_{0}=P\left[x_{0}\right]$ |  |
| Internal Elements |  |  |
| 8. State space | $\Omega=A X I X H X H$ | $\left\{(\Omega)_{\text {mg }}=(A)_{\text {mg }} \times I \times H \times W\right\}$ |
|  | $\Omega=\{x: x=(\alpha, f, h, w)\}$ |  |
|  | $A=\{a: a c[0,1]\}$ |  |
|  | $1=\{i: j=1, \ldots, I N\}$ |  |
|  | $H=\{h: h=1, \ldots, I N\}$ |  |
|  | $H=\{w: w=0,1\}$ |  |
| 9. Trajectory | $\overline{\mathrm{x}}=\left\{\mathrm{x}\langle\mathrm{k}): \mathrm{x}=1, \ldots, x^{(1)+1\}}\right.$ | \{ $\left.(\bar{x})_{\text {mg }}\right\}$ |
| - 10. Policy | J= $\{d(k): k \varepsilon K\}$ | $\left.\boldsymbol{\{}(\mathrm{d})_{\mathrm{mg}}\right\}$ |
| 11. Strategy | $S=\{S(x, k)\}$ | \{ $\left.(S)_{\text {mg }}\right\}$ |
| 12. Expected loss | $E[L(x, S, k)]$ | $\left\{E\left[(L(x, S, k)]_{\text {mg }}\right]\right\}$ |
| Output Elements |  |  |
| 13. Optimal strategy | $S^{*}=\left\{S^{*}(x, k)\right\}$ | ( $\left.\left(S^{*}\right)_{\text {mig }}\right)$ |
| 14. Expected annual losses |  | EL ${ }^{0}$ (REACH), EL ${ }^{\text {a }}$ (REACH) |
|  |  | EL* (REACH), EL** (REACH) |
| 15. Performance | $P E=\{P Y, O Y, A Y\}$ | PE(REACH $)=$ \{PYR, OVR, AVR $\}$ |
| 16. Efficiency | $E C=\{E F, E R, E D\}$ | $E C(R E A C H)=\{E F R, E R R, E O R\}$ |

Table 2-2. Statement of the input information

| Subsystem | Input Element | FFR(DM) | FFR(REACH) |
| :---: | :---: | :---: | :---: |
| FS | 1. Set of decision times <br> 1.1 Max number of forecasts <br> 1.2 Time interval between decision times | $\begin{aligned} & \mathrm{KN} \\ & \{\Delta t(k)\} \end{aligned}$ |  |
| RS | 2. Discretization of the flood plain <br> 2.1 Number of steps <br> 2.2 Elevation of a step | IN $\{y(i)\}$ |  |
| RS | 3. Decision constraint function | ${d d_{r}}^{(t)}$ | \{ $\mathrm{dd}_{r}(\mathrm{t})$ \} |
| FS | 4. Law of motion <br> 4.1 Transition distribution for $w(k)=1$ <br> 4.2 Transition distribution for $w(k)=0$ <br> 4.3 Distribution of the forecast indicator <br> 4.4 Distribution of the actual crest | $\begin{aligned} & \{P[i(k+1), h(k+1) \mid i(k), h(k), k]\} \\ & \{P[h h(k) \mid i(k), h(k), k]\} \\ & \{P[w(k) \mid k]\} \\ & \{P[h h]\} \end{aligned}$ |  |
| RS | 5. Loss function <br> 5.1 Set of structural categories <br> 5.2 Unit damage function <br> 5.3 Unit cost function <br> 5.4 Unit reduction function <br> 5.5 Description of the DM <br> 5.6 Description of the REACH | $\begin{aligned} & R=\{r: r=1, \ldots, R N\} \\ & \delta_{r}(z) \\ & Y_{r}(a) \\ & M R_{r}(z) \\ & (m, r, M D) \end{aligned}$ | $\begin{aligned} & \left\{\delta_{r}(z)\right\} \\ & \left\{r_{r}(\alpha)\right\} \\ & \left\{\operatorname{MR}_{r}(z)\right\} \\ & \{n(m, r)\}, M D(\text { REACH }) \end{aligned}$ |
| RS | 6. Actual response strategy | $s^{\text {a }}$ | $\left\{S_{m r}^{\text {a }}\right\}$ |
| FS | 7. Initial condition | $a_{0}, k_{0}, P\left[i_{0}, h_{0}\right]$ |  |
| FS | 8. Additional information <br> 8.1 Processing time <br> 8.2 Dissemination time <br> 8.3 Average actual lead time <br> 8.4 Expected number of floods per year | $\begin{aligned} & \{P T(k)\} \\ & \{D T(k)\} \\ & \{\operatorname{LL}(k)\} \\ & E[N] \end{aligned}$ |  |

Abbreviations: FS - Forecasting subsystem
RS - Response subsystem

| A | set of degrees of response |
| :---: | :---: |
| $\alpha$ | degree of response |
| AV | actual value |
| AVR | actual value for the reach |
| AVU | unit actual value |
| $\beta$ | function in the definition of the consumer time |
| CT | consumer time |
| d | decision |
| $\bar{d}$ | policy |
| dd | decision constraint function |
| D | decision set |
| DM | Decision Maker |
| DT | dissemination time |
| $\delta$ | unit damage function |
| $\Delta t$ | time interval between decision times |
| E | expectation |
| EC | efficiency |
| EC(REACH) | efficiency for the reach |
| EF | efficiency of the forecasting system |
| EFR | efficiency of the forecasting system for the reach |
| EL | expected annual loss |
| $E L^{\text {a }}$ | expected annual loss associated with $s^{\text {a }}$ |
| $E L^{\circ}$ | expected annual loss with "no response" |
| EL* | expected annual loss associated with S* |
| EL** | expected annual loss associated with S** |
| EL (REACH) | expected annual loss for the reach |
| E0 | overall efficiency |
| EOR | overall efficiency for the reach |
| ER | efficiency of the response system |
| ERR | efficiency of the response system for the reach |
| $\eta$ | distribution partitioning MD(REACH) |
| F | flood |
| FFR | Flood Forecast-Response System (or Model) |
| FFR(DM) | Flood Forecast-Response Model for a Single Decision Maker |
| FFR(REACH) | Flood Forecast-Response Model for a Reach |


| g | index labeling DMs on a given step |
| :---: | :---: |
| $G(m)$ | number of DMs on step m |
| Y | unit cost function |
| h | forecasted flood crest |
| hh | actual flood crest |
| H | set of flood crests |
| i | flood level |
| I | set of flood levels |
| IN | number of steps in the flood plain |
| j | an integer |
| J | set of integer numbers |
| k | decision time |
| $k_{0}$ | initial decision time |
| K | set of decision times |
| KN | max number of forecasts |
| L | loss function |
| $L_{1}$ | loss function for $w=1$ |
| $L_{0}$ | loss function for $w=0$ |
| LC | cost function |
| LD | stage-damage-response function |
| LT | average actual lead time |
| $\lambda$ | lead time |
| $\Lambda$ | set of lead times |
| m | location step |
| MD | maximum possible damage |
| MD (REACH) | maximum possible damage for the reach |
| MR | unit reduction function |
| $N$ | number of floods per year |
| $v_{\text {a }}$ | distribution partitioning AVR |
| $\nu_{0}$ | distribution partitioning OVR |
| $\nu_{p}$ | distribution partitioning PVR |
| OV | optimal value |
| OVR | optimal value for the reach |
| OVU | unit optimal value |
| $p$ | probability |
| PE | performance |


| PE(REACH) | performance for the reach |
| :---: | :---: |
| PT | processing time |
| PV | potential value |
| PVR | potential value for the reach |
| PVU | unit potential value |
| $\Phi$ | law of motion |
| ${ }^{\circ}$ | probability distribution on the initial state |
| $r$ | structural category |
| R | set of structural categories |
| RN | number of structural categories |
| S | strategy |
| $s^{\text {a }}$ | actual strategy |
| $s^{P}$ | pure strategy |
| S* | optimal strategy |
| S** | optimal strategy under perfect forecast |
| SD | stage-damage function |
| $\sigma$ | set of feasible strategies |
| $\sigma^{*}$ | set of optimal strategies |
| $\sigma^{\text {a }}$ | set of actual strategies |
| $t$ | time |
| $t_{B}$ | time of the rainfall beginning |
| $t_{E}$ | time of the rainfall end |
| T | time space |
| $u$ | index labeling strategies on a given step |
| $U(\mathrm{~m})$ | number of different strategies used on step m |
| $v$ | auxiliary variable also unit expected loss |
| VA | unit expected annual loss |
| $V A^{\text {a }}$ | unit expected loss associated with $S^{\text {a }}$ |
| $V A^{0}$ | unit expected loss with "no response" |
| VA* | unit expected loss associated with S* |
| VA** | unit expected loss associated with S** |
| VA (REACH) | unit expected annual loss for the reach |
| w | forecast indicator |
| W | set of forecast indicators |
| $x$ | state vector |
| $\bar{x}$ | trajectory |

initial state
time of the occurrence of the crest
elevation of a step
depth of flooding from first floor
state space

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## 1. MATHEMATICAL MODEL

The mathematical model of human response to flood warnings consists of 4 interconnected parts: 1) uncertainty about flooding and loss prior to a flood, 2) sequential inference based on warnings, 3) protective action, and 4) learning.

### 1.1 Prior uncertainty

The decision maker's uncertainty about the occurrence of a flood incident and his uncertainty about suffering a loss when a flood incident occurs are represented by subjective probabilities.

Define:
$P_{n}(F \mid t)=$ the decision maker's subjective probabilities of a flood incident in the next time interval $\Delta t$, at time $t$ following the occurrence at $t_{n}$ of the nth flood incident he has experienced. $t_{n} \leq t \leq t_{n+1}$
$\tau_{F} \quad=a$ forgetting time constant which governs the rate at which the expectation of a flood incident diminishes as time passes without a flood incident.

The subjective probability of a flood incident decays according to

$$
\begin{equation*}
P_{n}(F \mid t)=P_{n}\left(F \mid t_{n}\right) \exp \left[-\left(t-t_{n}\right) / \tau_{F}\right] \tag{1}
\end{equation*}
$$

The next occurrence of a flood incident increases the probability toward the true value to an extent dependent upon the willingness of the decision maker (DM) to learn. The true value is approached since there are objective sources of information besides the DM's own experience, e.g. the weather
service, from which he may learn the true probability of a flood incident. Define:
$\pi=$ the actual (or historical) value of the probability of a flood incident in the next $\Delta t$
$\beta=a$ learning parameter, dependent on the individual. When it is close to 1 , learning is more rapid. $0<\beta<1$.

The occurrence of a flood incident at time $t_{n}$ modifies the subjective probability of a flood incident that was held just prior to the incident, $P_{n-1}\left(F \mid t_{n}\right)$, to give a new value $P_{n}\left(F \mid t_{n}\right)$ according to

$$
\begin{equation*}
P_{n}\left(F \mid t_{n}\right)=\beta \pi+(1-\beta) P_{n-1}\left(F \mid t_{n}\right) \tag{2}
\end{equation*}
$$

By (1) this may be written in terms of the probability immediately following the ( $n-1$ ) st flood as

$$
\begin{equation*}
P_{n}\left(F \mid t_{n}\right)=\beta \pi+(1-\beta) P_{n-1}\left(F \mid t_{n-1}\right) \exp \left[-\left(t_{n}-t_{n-1}\right) / \tau_{F}\right] \tag{3}
\end{equation*}
$$

This representation of the probability of a flood incident is designed to mimic the observed behaviors: (1) long periods without a flood reduce peoples' belief that they will occur, (2) there are sources of information used to obtain a knowledge of flood probability other than the proportion of times a flood incident was experienced, (3) expectation of floods and expectation of loss appear to be uncoupled, and (4) the probability of a flood incident is usually underestimated.

The form of the model for $P_{n}(F \mid t)$ is a mathematical learning model of a sort often found to apply to sequential changes in response tendency with intermittent reinforcement (Bush and Mosteller, 1955).

It is assumed that when the DM arrives on the flood plain, the values of $p_{0}(F \mid t)$ and $p_{0}(L \mid F, t)$ are both zero. This is consistent with the facts for rarely flooded areas, but only approximate for areas with more frequent flooding. In Atlanta, for example, the study by James, et al (1971) found that $57 \%$ of the respondants did not know about flooding when they arrived (i.e., $P_{0}(F \mid t)=0$ and $p_{0}(L \mid F, t)=0$ ) and of the rest, $48 \%$ said it didn't bother them at all (i.e., $\left.p_{0}(L \mid F, t)=0\right)$.

The subjective probability of a flood incident $P_{n}(F, t)$ is relevant in that the joint probability $P(L, F \mid t)=P_{n}(F \mid t) P_{m}(L \mid F, t)$ would determine the floodplain dweller's preparation for protective measures that must be accomplished before an incident. But once a warning has been given, the relevant subjective probability is the probability of a loss given a flood incident $P_{n}(L \mid F, t)$.

We define:
$P_{m}(L \mid F, t)=$ the $D M^{\prime}$ 's conditional subjective probability of a loss given a flood incident at time $t$ when he has previously experienced $m$ flood losses, the mth at $t_{m} . t_{m} \leq t \leq t_{m+1}$.
$\tau_{L} \quad=a$ forgetting time constant which governs the rate at which the expectation of loss diminishes with time.

The conditional subjective probability decays according to

$$
\begin{equation*}
P_{m}(L \mid F, t)=P_{m}\left(L \mid F, t_{m}\right) \exp \left[-\left(t-t_{m}\right) / \tau_{L}\right] \tag{4}
\end{equation*}
$$

The next occurrence of a flood loss increases the probability toward the value that would be estimated from the proportion of flood incidents in which a loss occurred, $m /(n+1)$ to an extent dependent on the $D M^{\prime} s$
willingness to learn. This value rather than the true value is approached because information about the true value is not widely available and the DM can be expected to keep some mental accounting of his losses.

Define:
$\delta=\mathrm{a}$ learning parameter, dependent on the individual. $(0 \leq \delta \leq 1$ generally, although it may take on values $>1$ if that does not cause a probability value to exceed 1)

The occurrence of a flood loss at time $t_{m}$ modifies the subjective probability of a loss given a flood that was held just prior to the incident $P_{m-1}\left(L \mid F, t_{m}\right)$ to give a new value $P_{m}\left(L \mid F, t_{m}\right)$ according to

$$
\begin{equation*}
P_{m}\left(L \mid F, t_{m}\right)=\delta\left(\frac{m}{n+1}\right)+(1-\delta) P_{m-1}\left(L \mid F, t_{m}\right) \tag{5}
\end{equation*}
$$

Thus the subjective probability of a loss is revised only when losses occur and diminishes with time between losses. Its revision reflects the actual loss history as field investigations suggest it should (Roder, W., 1961).

This form of learning model was found accurately to represent probabilities assigned to flood loss in a laboratory simulation experiment undertaken in connection with the present research. An example of one subject's sequential estimates and the model fitted to them is shown in Figure 3-1.

### 1.2 Sequential inference based on warnings

When a flood incident occurs at time $t$, the prior probability of loss given a flood at that time $P_{m}(L \mid F, t)$ is revised as successive warnings are given.

Define:
$W_{k} \quad=$ the sequence of warnings through the kth warning, which gives such information as the current river level $\mathfrak{i}$ and the forecast crest $h$.


$$
\begin{aligned}
P\left(W_{k} \mid F, t, L\right)= & \text { probability that the sequence of } k \text { warnings } W_{k} \text { is given } \\
& \text { if a loss occurs. } \\
= & \text { the non-occurrence of a loss }
\end{aligned}
$$

The optimal revision of the prior probability of a loss given a flood is the Bayesian revision, which can be written

$$
\begin{equation*}
\frac{P\left(L \mid F, t, W_{k}\right)}{1-P\left(L \mid F, t, W_{k}\right)}=L_{0}\left(W_{k}\right) \frac{P(L \mid F, t)}{1-P(L \mid F, t)} \tag{6}
\end{equation*}
$$

where the likelihood ratio $L_{0}\left(W_{k}\right)$ is

$$
L_{0}\left(W_{k}\right)=\frac{P\left(W_{k} \mid F, t, L\right)}{P\left(W_{k} \mid F, t, \bar{L}\right)}
$$

The model assumes that the $D M$ revises his subjective prior $P(L \mid F, t)$ in the optimal Bayesian fashion but that underestimates the impact of the evidence and uses a subjective likelihood ratio $L_{s}$ that differs from the objective likelihood ratio $L_{0}$.

$$
\begin{equation*}
L_{s}\left(W_{k}\right)=L_{0}\left(W_{k}\right)^{c} \quad 0<c<1 \tag{7}
\end{equation*}
$$

There is substantial empirical evidence from laboratory experiments indicating that people revise their prior probabilities on receipt of new data in a manner roughly consistent with this model where c may depend upon the diagnosticity of the data, upon the value of $L_{0}$ (Phillips \& Edwards, 1966). It is assumed that c is a characteristic of the individual DM and reflects his willingness to believe the warning. A low value of $c$ results in the warning having little effect on the DM's prior.

Substituting $L_{s}$ into (6) and solving for the revised subjective probability gives

$$
\begin{equation*}
P\left(L \mid F, t, W_{k}\right)=\frac{L_{0}\left(W_{k}\right)^{c}\left(\frac{P(L \mid F)}{1-P(L \mid F)}\right)}{1+L_{0}\left(W_{k}\right)^{c}\left(\frac{P(L \mid F}{1-P(L \mid F)}\right)} \tag{9}
\end{equation*}
$$

### 1.3 Protective action

When the DM is sufficiently sure that he shall suffer a loss, he will act.

Define:

$$
T_{1}=a \text { threshold level of probability }
$$

The DM will begin to take action to protect his property on the kth warning such that

$$
\begin{equation*}
P\left(L \mid F, t, W_{k-1}\right)<T_{1} \tag{10}
\end{equation*}
$$

$$
\text { and } P\left(L \mid F, t, W_{k}\right) \geq T_{1}
$$

It is assumed that a particular $D M$ has a set of protective actions which he will carry out in a fixed order if he is sufficiently sure that he will suffer a loss.

Define:

$$
\alpha(\tau)=\frac{\text { Damage the } D M \text { can prevent in } \tau}{\text { Maximum damage the } D M \text { can prevent }}
$$

Thus $\alpha$ is the proportion of the total possible protection achieved by time $\tau$, and the function is characteristic of the particular DM. With the total preventable damage, it completely specifies protection.

The protective action $\alpha(t)$ is divided into $s$ segments

$$
\begin{array}{lr}
0-\alpha\left(\tau_{1}\right) & \text { segment } 1 \\
\alpha\left(\tau_{1}\right)-\alpha\left(\tau_{2}\right) & \text { segment } 2 \\
\cdot \cdot \cdot \cdot \cdot & \cdot \\
\alpha\left(\tau_{s-1}\right)-1.0 & \text { segment } s
\end{array}
$$

To be undertaken，each segment requires that the subjective probability of loss exceed its threshold value，i．e．the protective action represented by the nth segment will not begin until the threshold $T_{n}$ is reached and will cease if the probability is revised to a value below $T_{n}$ ．

It is assumed that once the DM begins work on segment $n$ ，he will continue until he completes it or until $P\left(L \mid F, t, W_{k}\right)$ is revised to a value less than $T_{n}$ ．Once segment $n$ is completed the $D M$ will continue with segment $n+1$ provided $P\left(L \mid F, t, W_{k}\right) \geq T_{n+1}$ ．

Protective action may thus be interrupted before $\alpha$－ 1 if the subjective probability of a loss either is revised downward enough or does not rise fast enough．

The multiple threshold model is adopted in recognition of the fact that the costs of more extreme actions such as complete evacuation of property， and their consequences，such as lost production time if no flood loss is sustained，may be such that a higher level of certainty is necessary to initiate them．The threshold levels $T_{n}$ ，like the function $\alpha(\tau)$ ，are character－ istic of the individual DM．

1－4 Learning
Following a flood event，the $D M$ is assumed to evaluate and perhaps revise：

1）his prior uncertainty
2）his belief in the warning system
3 ）his plan of protective action and thresholds for it．


Figure 3-2:: Changes in model parameters following a flood event in which the decision maker had either no loss or a loss.

The revision of the subjective probabilities of a flood event and of loss given such an event were described in the section on prior uncertainty.

The DM's confidence in the warning system is represented by the quantity c in (8) which determines the impact of the warning on his belief. His reaction to the warning system is embodied in the threshold value $T_{1}$. Warning sequences which produce behavior appropriate to the actual event (loss or no loss) do not result in change in $c$ or the $T$ values, but ones which produce inappropriate behavior do cause changes. The warning sequence can be characterized by revision of the DM's prior with the objective likelihood ratio $L_{0}\left(W_{k}\right)$ (i.e. $c=1$ ) and this is compared with the DM's actual revision with $L_{s}=L_{o}\left(W_{k}\right)^{c}$. In the case of loss or no loss the appropriate changes in c or $\mathrm{T}_{1}$ made, as shown in Figure 3-2. If the warning sequence does not move the $D M$ to protective action and there is no loss, he has acted appropriately and does not change his parameters. But if there is a loss, he increases $c$ and may decrease his threshold for action if it would not have been reached even had $c$ been equal to 1.0 . If the $D M$ is moved to protective action, and there is a loss, he has acted appropriately and does not change. But if there is no loss he decreases c and may increase his threshold for action.

There is no theoretical basis for adopting a particular model of the way in which $c$ and $T_{1}$ are updated. It is tentatively assumed that these parameters will increase in proportion to the amount by which they are less than 1 and decrease in proportion to their value. Thus if the conditions in Figure 3-2 indicate an increase, the new values will be

$$
\begin{align*}
& C+C+K_{C}(1-C)=K_{C}+C\left(1-K_{C}\right) \\
& T_{1}+T_{1}+K_{T}\left(T_{1}\right)=K_{T}+T_{1}\left(1-K_{T}\right) \tag{11}
\end{align*}
$$

If the plan of protection, knowledge of protective actions, or skill at carrying them out has changed, then this change is represented by an appropriate change in $\alpha(\tau)$ and in the total preventable damage. But the model does not include a mechanism for this.
2. SIMULATION MODEL (OUTLINE)

The mathematical model described in Section 1 has been designed for evaluation purposes. Since it does not explicitly involve the characteristics of the warning dissemination and response system that can be influenced by legislation and policy, it cannot readily be used as a basis for system design. It is the purpose of the simulation model to attempt to relate those characteristics (e.g. warning source, neighborhood awareness of flood hazard, whether the warning gives response information, etc.) to the level of response and to damage reduction.

The data requirement for such a model is much greater than for the more abstract mathematical evaluation mode1. In addition, verification of the components of the model becomes more difficult for the very reason that it is more specific and explicit. The advantages of a simulation model in relating specific system characteristics to behavior are paid for in the uncertainty that must be allowed for in the results--however it may still be revealing of system interrelationships even the inputs are inaccurate.

The theoretical viewpoint taken is a combination of the views of Janis and Mann (1977) on human decisions in difficult choice situations and of Kates (1970) on human adjustment to flood hazard. Because of the central importance of sequences of revised and timely flood warnings in the present research, the simulation model has been developed in a dynamic form suitable to a sequence of inputs and decisions.

Only the outline of the model has been completed and can be presented here. It provides no more than a basis for further research and implementation. But even in its present form it appears to be helpful in conceptualizing the $D M^{\prime} s$ problem and the principal features of the situation that affect his response.

The Janis-Mann model is diagramed in Figure 3-3 (from Janis and Mann, 1976). Its form is based on extensive data taken in a variety of contexts and purports to be of broad theoretical significance. It consists of three sequential parts: 1) evaluation of the risk of not responding, 2) evaluation of the risk of responding, and 3) selection of a response. If the risk of not responding is acceptable, the DM does not act but awaits developments-this corresponds to the initial threshold in the mathematical model. If the risk of not responding is unacceptable and that of responding is acceptable, the DM responds. This assumes he has a response in mind. If he does not and there is time, he searches for a better response.

The Kates model of hazard adjustment parallels this formulation (and predates it). It is diagramed in Figure 3-4. Kates introduces it by saying (Kates, 1970, p. 16)
"The presence of a natural hazard encourages human action to minimize its threat and mitigate its effects. For any individual managerial unit the decision process is a complex but interesting one, and it has been a focus of hazard research for many years.

A model of decision-making applicable both to the choice of resource and natural hazard adjustment has been developed. This model by White (1961) is heavily influenced by the work of Simon (1957) particularly in the notions of "bounded rationality" and "satisficing." The work also parallels the complex model of resource use developed by Firey (1960).

Over the years, variants of this approach have been tested in different hazard and resource use situations. Two emphases can be found in this work: to develop a sharper, more predictive decision-making model and to incorporate individual personality characteristics into it.

The sub-model presented in Figure $3-4$, then, is really the current state of our decision making theory strung together in an operative sequence."


FIGURE 3-3. Janis and Mann's Model


Figure 3-4. Kates' Model

It should be noted that neither Kates nor the Janis-Mann model explicitly allow for the decision processes to be iterated in a period prior to the event. Indeed, the Kates model is not specifically linked to single events but describes a process of adjustment that takes place over a number of events.

A synthesis of these two models that takes account of the sequential nature of flood warnings is shown in Figure 3-5. It explicitly incorporates the fact that the DM obtains information from his peers and either revises or confirms his opinions in discussion with them. This feature of exposure of one's opinions to the opinions of others and revision on the basis of the extent of agreement has been important in sociological simulations for some years (Gullahorn and Gullahorn, 1963), and is in agreement with field studies which show that people's perceptions of hazard warning depend on their immediate social context or reference group (McLuckie, 1973).

The general model of Figure 3-5 has been expanded in the light of sociological simulation studies with a view toward developing an actual computer simulation. A review of related sociological simulations and the expanded model are presented in Section 3, following.

## 3 SIMULATION MODEL (Preliminary Development)

Additional research has been conducted into the development of a simulation model to represent the individual's response to actual flood warnings. This model includes such things as receiving the warning, interacting with peers, attempting to confirm warning and then responding to the warning. This type of simulation could be used to test various types of warning systems in an attempt to improve the present system. A more detailed discussion of the present model that has been developed will be


Figure 3-5. General Simulation Model
presented later, but first a short discussion of previous studies of human interaction studies which provide the background for the present work will be given.

In previous studies of the simulation of social systems (and human interactions in flood situations are part of a social system), two major breakdowns have developed. Those breakdowns include: (a) simulations of total system processes and (b) simulations of microbehavioral (or individual) processes. Since we will be concerned with microbehavioral processes, studies of total system processes will not be discussed.

In considering previous studies on individual information-processing mechanisms and the social systems in which they are embedded, one finds models of varying degrees of complexity and subject material. Studies on such things as adult socialization to an urban environment, political socialization, individual processing of social communications and social exchange in decision-making all represent studies of importance to the processes involved in flood situations.

In the simulation of socialization in an urban setting, Hanson et al. (1967) consider the socialization of new migrants into an urban situation and how they become familiar with the social structure of the community through communication with existing members. The parallel to the flood situation is obvious since new members entering a flood plain need to gather information from present members on the threat of floods and previous responses.

The model developed by McPhee (1963) concerning individual voters and how interpersonal interactions influence voter preference provides a basis for part of the model that we currently have under consideration, In

McPhee's model, a voting population is input along with initial dispositions. The population then interacts in an attempt to find a final preference (as a flood plain population would attempt to find an appropriate response). The final decision of the voter is then remembered for the next election (or in our case, flood threat).

The third major type of microbehavioral simulation deals with individual processing of social communications. A good example of this type of simulation was developed by Abelson and Bernstein (1963) in which each simulated citizen was exposed to several sources of information from the media such as radio, television, etc. The developers of the model were concerned with the influence of each source on the individual's opinion and how many sources were required to formulate that opinion.

The fourth area of previous study is that social exchange in decision making. Gullahorn and Gullahorn (1963) have developed a computer model of a social man in which face-to-face interaction causes an updating of the simulated man's knowledge during a decision making process. The contact between people causes reinforcement (both positive and negative) based on differences of opinion.

In addition to these simulation types, a behavioral model developed by Janis and Mann (1977) is also being implemented into the present simulation model. In their model, the individual is first asked to weigh the risks involved in making the decision. If the risks are too great, the individual checks to see if a better response is possible. If it is not, the decision maker avoids the situation. If there is a better solution possible but no time to find it, the individual panics and if there is time, more information is sought.

The structure of a model that attempts to accommodate the previous work cited above and to reflect actual sequences of behavior is presented in Figure 3-6b. Table 3-1 lists the actions.

Basically, there are four processes, that of discussion, decision making, physical response and learning. The first process, that of discussion follows the individual's interpretation of the warning. After the flood plain dweller becomes aware of the flood situation, he or she will attempt to reaffirm their interpretation of the hazard (called "warning confirmation") by discussing it with family members, neighbors, etc. and by seeking other information sources such as TV or radio. In addition to reaffirming the situation, discussion will also consider the various possible response strategies available to the discussion group. This discussion thereby provides the individual with information that will aid in the next process of decision making.

In the decision making process, the potential victim is required to make a decision concerning what response he or she feels is warranted and what level of response is possible in light of constraints such as protective materials on hand such as sandbags, physical strength, money available, etc., in order to reduce potential damage. Several considerations are involved in this decision making process such as seriousness of the risks involved in the response, hope for better solutions and time involved in making the decision (Janis and Mann, 1977).

Once the decision has been made about what response to attempt, the actual physical response is made with respect to constraints on time and material. This response is then followed by a learning process in which the individual is able to ascertain the effect of the response on reducing flood damage and thereby increase his or her knowledge with respect to further floods.


Figure 3-6a. Flow diagram of the preliminary simulation model. (The event may be either a warning or the cresting of the flood). The sequence is described in Table 3-1.


Figure 3-6b. Flow diagram of the preliminary simulation model, left hand side.

The development of a detailed model to represent human response in a flood situation has not been previously attempted and is therefore a potentially important contribution to understanding the action of people in a flood hazard. The mathematical model of the previous section is an evaluative model to be used in conjunction with the dynamic programming model mentioned elsewhere in this report. It provides a means of mimicking the actual strategy of an individual in the flood plain in response.

The second model, that of the simulation of actual behavior that is outlined above, could also make an important contribution if completed, since it will permit a detailed explanation of the interaction of the warning and the response systems.

## TABLE 3-1

## Flood Response Sequence

I. Preliminary Events
A. Flood danger - observed by NWS through use of rain gages
B. Initial warning issued

1. Contains weather information
2. Time of flood crest and height of flood crest
II. Stimulation and Observation
A. Person hears warning
3. Makes initial interpretation of the message depending upon his predisposition towards floods; including $P(F), P(L \mid F)$, $P(L \mid F W)$ threshold, personality and experience
4. Person considers source of the information and may try to confirm information from another source
B. Person observes external factors
5. Checks the weather conditions to determine if rainy, etc.
6. Checks local rivers, streams to see if they are rising, etc.
7. Checks general reactions of neighbors and other community members to ascertain what is being done
8. Checks what flood material (sandbags, etc.) is on hand
III. Disćussion
A. Initial discussion
9. Person discusses situation with family members and local neighbors individually in an attempt to confirm the situation
10. If discussion confirms flood threat, the person attempts to find larger groups of family and neighbors to discuss what response should be taken
11. If the threat of flood is not confirmed, the person develops a "wait and see" attitude and checks for new information and next flood warning
B. Response discussion
12. Response is discussed in larger groups and possible actions are considered
13. Materials on hand are ascertained and cost of response is discussed
14. Response may be determined to be unwarranted and then additional information and warnings are sought
IV. Decision Process (Response desired)
A. The individual considers the response suggested by the group and weighs it in consideration with his own risk factor
15. Risk reasonable - the person makes the response in light of constraints on time, experience, etc.
16. Risks unreasonable - the person asks himself if some other response might be better in light if individual risks (if not, he ignores the situation or gives a feeble response)
B. In consideration of other possible solutions the individual asks if there is sufficient time to find a better solution
17. No time remaining (risk unreasonable) - panic response
18. Time remaining - search out new information that will assist in finding a more satisfactory response
V. Learning Process
A. Short-run

Sees what effect the present response has on the current situation. Waits for flood or next updated warning which may change the level of response. Considers accuracy of forecasts previously given.
B. Long-run

Remembers the losses that occurred in relation to the actions taken, remembers how precise the warnings given were, etc. [In other words, he updates initial dispositions of $P(F), P(L \mid F)$, thresholds]. He then repairs damages and waits for the next flood.

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## 1. MODELING THE LAW OF MOTION AND INITIAL CONDITION

### 1.1. INTRODUCTION

Modeling the law of motion, $\Phi$, by means of parametric distributions presents a research problem in itself. Virtually, nothing has been done in this area of hydrology. The only reference known to the authors is the study by Grayman and Eagleson (1971, p. 265) who found the ratio of the forecasted flood crest to the actual flood crest to be log-normally distributed. Theoretical difficulty arrises because of the complexity of $\Phi$ which is multivariate, conditional, and dynamic. Besides the lack of theoretical studies, there is also a shortage of real data from which $\Phi$ could be estimated; primarily this is because the proposed herein concept for modeling flood forecast-response processes is the first one which fosters the need for describing sequential flood forecasts and actual flood stages in probabilistic terms.

The above remarks are intended to explain or to justify the approach followed by the authors. In short, $\Phi$ is described by a conceptual model derived from a set of seemingly reasonable assumptions concerning the flood forecasting process. Within the model, a parametric family of distributions is employed. For the present case study, multinomial family has been chosen. By no means do the authors try to motivate or to defend their choice on the physical or statistical grounds. The multinomial family has been selected mainly for the following reasons:

1. its discrete form allows for using it directly in the dynamic programming algorithm,
2. it has well developed theory and relatively simple form for joint distributions,
3. the estimators for parameters are relatively easy to get.
1.2. CONCEPTUAL MODEL OF $\Phi$ AND $\Phi_{0}$

### 1.2.1. Assumptions

Recall the definition of the law of motion:

$$
\Phi= \begin{cases}\{P[i(k+1), h(k+1) \mid i(k), h(k), k]\} & \text { for } w(k)=1 \\ \{P[h h(k) \mid i(k), h(k), k]\} & \text { for } w(k)=0 \\ \{P[w(k) \mid k]\} & \end{cases}
$$

and of the initial condition:

$$
\Phi_{0}=\left\{P\left[i\left(k_{0}\right), h\left(k_{0}\right)\right]\right\} \text { for some } k_{0} \varepsilon K .
$$

The following assumptions concerning the structure of the Markovian process described by $\Phi$ and $\Phi_{0}$ are made. Let $i \varepsilon I$, heH, and hheh. For every keK:
(i) $\quad h(k) \geq i(k)$,
(ii) $i(k+1) \geq i(k)$,
(iii) $h h(k) \geq i(k)$,
(iv) probability distributions involving i, $h$, and hh are stationary. Assumptions (i) - (iii) bound the domain of the definition of $\Phi$ (Figure 4-1). Conceptually, they derive from the nature of the flood-forecasting process. Assumption (i), requiring the forecasts of the crest to lead the actual flood level, is a prerequisite of any useful forecasting. Assumption (ii), requiring the flood levels to form a monotone-increasing sequence, although not always met in practice, seems to be sufficiently accurate for the purpose of economic evaluation. Assumption (iii) defines the flood crest. Finally, Assumption (iv) is dictated primarily by the scarcity of the historical data.

$I, H=(1,2, \ldots, I N) \quad i(k) \in I, h(k) \geqslant i(k), \quad i(k+1) z i(k), h(k+1) \geqslant i(k+1)$
Figure 4-1. Domain of the Transition Probabilities for $w(k)=1$

### 1.2.2 Transition Probability for $w(k)=1$

For $w(k)=1$, the stationary transition probability can be written as

$$
P[i(k+1), h(k+1) \mid i(k), h(k)]=P[h(k+1) \mid i(k+1), i(k), h(k)] \cdot P[i(k+1) \mid i(k), h(k)]
$$

The right-hand side of this expression is now simplified by eliminating the conditioning on $\mathrm{h}(\mathrm{k})$ in the first term and weakening the conditioning on $\mathrm{i}(\mathrm{k})$ through the use of Assumption (ii):

$$
\simeq P[h(k+1) \mid i(k+1) ; i(k+1) \geq i(k)] \cdot P[i(k+1) \mid h(k), i(k+1) \geq i(k)]
$$

Informally, this step can be explained by high correlation between $\mathfrak{i}(k)$ and $h(k)$ ( 0.7965 for Milton, Pa ). The second term on the right-hand side can be obtained by truncating and normalizing $\mathrm{P}[\mathrm{i}(\mathrm{k}+1) \mid \mathrm{h}(\mathrm{k})]$, so that

$$
P[i(k+1), h(k+1) \mid i(k), h(k)]=P[h(k+1) \mid i(k+1)] \cdot P[i(k+1) \mid h(k)] / A[i(k), h(k)]
$$

where

$$
\begin{aligned}
& P[h(k+1) \mid i(k+1)] \text { is defined for } h(k+1) \geq i(k+1) \\
& P[i(k+1) \mid h(k)] \text { is defined for all } i(k+1) \varepsilon I
\end{aligned}
$$

and

$$
\begin{equation*}
A[i(k), h(k)]=\sum_{i(k+1) \geq i(k)} P[i(k+1) \mid h(k)] \tag{2}
\end{equation*}
$$

### 1.2.3 Transition Probability for $w(k)=0$

For $w(k)=0$, the stationary transition probability is approximated by
$P[h h(k) \mid i(k), h(k)] \simeq P[h h(k) \mid h(k), h h(k) \geq i(k)]$
Again, conditioning on $i(k)$ has been weakened by the use of Assumption (iii) which is enforced by truncating and normalizing $\mathrm{P}[\mathrm{hh}(\mathrm{k}) \mid \mathrm{h}(\mathrm{k})]$ so that

$$
\begin{equation*}
P[h h(k) \mid i(k), h(k)] \cong P[h h(k) \mid h(k)] / B[i(k), h(k)] \tag{3}
\end{equation*}
$$

where
$\mathrm{P}[\mathrm{hh}(\mathrm{k}) \mid \mathrm{h}(\mathrm{k})]$ is defined for all $\mathrm{hh}(\mathrm{k}) \varepsilon \mathrm{H}$
and

$$
\begin{equation*}
B[i(k), h(k)]=\sum_{h h(k) \geq i(k)} P[h h(k) \mid h(k)] \tag{4}
\end{equation*}
$$

### 1.2.4 Distribution of $w(k)$

Distribution $\{P[w(k) \mid k]\}$ can be obtained directly from an analysis of historieal sequences of flood forecasts or from an analysis of the storm durations. The first approach is straightforward; the second one is similar to that of Sittner (1976). To illustrate, let $T$ be storm duration, and let $\Delta t$ be the real time interval between decision times ( $\Delta t$ being constant for all keK). Then

$$
\begin{equation*}
P[w(k)=1 \mid k]=P[T>k \Delta t \mid T>(k-1) \Delta t], \quad k \varepsilon K . \tag{5}
\end{equation*}
$$

In words, right side of (5) is the probability of having storm longer than $k \Delta t$ given that it lasts already longer than $(k-1) \Delta t$. However, storm longer than $k \Delta t$ implies that at least one more forecast beyond decision time $k$ will be issued. This is exactly what $P[w(k)=1 \mid k]$ means. For $w(k)=0$, being the complement of the event $w(k)=1$, we have

$$
\begin{equation*}
P[w(k)=0 \mid k]=1-P[w(k)=1 \mid k], \quad k \varepsilon K . \tag{6}
\end{equation*}
$$

For some real data, Sittner (1976) shows $\{P[w(k) \mid k]\}$.

### 1.2.5 Parametric Distributions

Developed in the previous section, the model of $\Phi$ and $\Phi_{0}$ requires the following distributions:

1. $\{P[h(k+1) \mid i(k+1)]\}$
2. $\{P[i(k+1) \mid h(k)]\}$
3. $\{P[h h(k) \mid h(k)]\}$
4. $\{P[w(k) \mid k]\}$
5. $\left\{P\left[i_{0}, h_{0}\right]\right\}$

Distributions 1-3 and 5 are assumed to be members of the multinomial family (the elementary properties of the multinomial family are summarized in Appendix) Specific distributions employed for $\Phi$ and $\Phi_{0}$ and their parameters are given in Table 4-1. Distributions 1-3 are positive binomial. Due to their similarity, only one of them is described at length.

For every heH, $\{P[h h \mid h]\}$ is positive binomial with parameters IN and $\mathrm{QHH}(\mathrm{h})$. The maximum likelinood estimator (Equation $\mathrm{A}-12$ ) of $\mathrm{OHH}(\mathrm{h})$ is:

$$
\hat{Q} H H(h)=\hat{E}[h h \mid h]\left[1-(1-\hat{Q} H H(h))^{I N}\right] / I N
$$

Since IN iṣ known, only $\hat{E}[h h \mid h]$ has to be determined for every heH. For this purpose, the historical values of $E[h h \mid h]$ are plotted versus $h$. Then a free hand interpolating curve is fitted, and in this way $\hat{E}[h h \mid h]$ is determined for every $h \in H$.

Distribution $\left\{P\left[i_{0}, h_{0}\right]\right\}$ is positive trinomial with parameters (2IN,QIO,QHO); 2IN is known whereas QIO and QHO are estimated from Equation A-12 with inputs ( $2 I N, \hat{E}\left[t_{0}\right]$ ) and (2IN, $\hat{E}\left[h_{0}\right]$ ), respectively.

For $\{[w(k) \mid k]\}$, historical frequences, after being smoothed along the $k$ axis, are used.

| Element | $\underset{X}{\text { Variable }}$ | Distribution of $X$ | Parameters in the Likelihood Equation |  | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\hat{E}(X)$ | $N$ |  |
| $P[h(k+1) \mid i(k+1)]$ | $h-i+1$ | $\mathrm{BP}(\mathrm{IN}-\mathrm{i}+1, \mathrm{QH}(\mathrm{i})$ ) | $\hat{E}[h \mid i]-i+1$ | $\mathrm{IN}-\mathrm{i}+1$ | $h \geq i$ |
| $P[i(k+1) \mid h(k)]$ | i | $B P(I N, Q I(h))$ | $\hat{E}[\mathrm{i} \mid \mathrm{h}]$ | IN |  |
| $P[h h(k) \mid h(k)]$ | hh | BP(IN, QHH (h) | $\hat{E}[\mathrm{hh} \mid \mathrm{h}]$ | IN |  |
| $P\left[i_{0}, h_{0}\right]$ | $\left(i_{0}, h_{0}\right)$ | MP(2IN,QIO,QHO) | $\hat{E}\left[\mathrm{t}_{0}\right], \hat{E}\left[h_{0}\right]$ | 2IN,2IN | $h_{0} \geq i_{0}$ |
| $\mathrm{P}[\mathrm{hh}]^{* /}$ | hh | $B P(I N, Q H)$ | $\hat{E}[\mathrm{hh}]$ | IN |  |

*/ \{P[hh]\} used in FFR model is computed as a marginal distribution of $\Phi$.
TABLE 4-1. ELEMENTS OF THE LAW OF MOTION AND INITIAL CONDITION:
DISTRIBUTIONS AND PARAMETER ESTIMATORS

### 1.3 DISTRIBUTION OF THE ACTUAL FLOOD CREST

Distribution $\{P[h h]\}$ of the actual flood crest hheH ought to be obtained from the law of motion, $\Phi$, rather than directly from the historical record, in order to maintain the consistency of the overall computations. The importance of such an approach derives from the fact that the computation of $\Phi$ involves certain approximations as well as fitting parametric distributions (positive binomial in the present case). Thus, if any bias is introduced to $\Phi$ it should also be reflected in $\{P[h h]\}$, otherwise, computed expected annaul losses cannot properly be compared. Note that $\{P[h h]\}$ is the distribution of the terminal state of a discrete time, vector valued Markov chain $\{i(k), h(k)\}$ whose law of motion is $\Phi$ and initial condition is $\left\{P\left[i_{0}, h_{0}\right]\right\}$ where $i_{0}=i\left(k_{0}\right), h_{0}=h\left(k_{0}\right)$ for specified $k_{0} \varepsilon K$.

For the given terminal state hheh, let $v(i, h, k)$ denote the probability of an eventual transition from the state $(i, h) \varepsilon I X H$ at the decision time k $k$ to the terminal state hheH. The exact timing of this transition is not known; however, the transtion must occur not later than at the KNth decision time.

From an elementary Markov property, P[hh] (hheH) can be computed according to the following recursive algorithm:
(a) for $k=K N$

$$
v(i, h, K N)=P[h h \mid i, h, K N]
$$

(b) for every $k<K N$

$$
v(i, h, k)=\sum_{\substack{i^{\prime} \varepsilon I \\ h^{\prime} \varepsilon H}} v\left(i^{\prime}, h^{\prime}, k+1\right) \cdot P\left[i^{\prime}, h^{\prime} \mid i, h, k\right] \cdot P[w=1 \mid k]+P[h h \mid i, h, k] \cdot P[w=0 \mid k]
$$

(c) and for a given initial condition at the decision time $k_{0} \varepsilon K$,

$$
P[h h]=\sum_{\substack{i_{0} \varepsilon I \\ h_{0} \varepsilon H}} v\left(i_{0}, h_{0}, k_{0}\right) \cdot P\left[i_{0}, h_{0}\right]
$$

### 1.4 VERIFICATION OF THE LAW OF MOTION

Verification of the law of motion, $\Phi$, presents a substantial problem both theoretically, because of multidimensionality, and practically, because of scarcity of the historical data. For these reasons, the verification problem has been approached, again, from an engineering standpoint: regardless of the difficulties, the engineer wants somehow to gain the confidence that $\Phi$ properly describes the system being modeled. In the verification method advocated herein, first, the multidimensionality of $\Phi$ is reduced using a dynamic programming algorithm; next, a one dimensional goodness-of-fit test is applied, and graphical comparisons are made.

Let $P[h h \mid F]$ be the distribution of the actual flood crest computed from $\Phi$ as given in Section 1.3. Here, the conditioning of hh on flood, $F$, is shown explicity. (In fact, the whole law of motion is conditioned by the occurrence of flood F.) With FS the flood stage, event "F" is assumed to be equivalent to the event "hh $\geq$ FS" (hh, FSeH). Accordingly,

$$
P[h h, F]=P[h h \mid F] \cdot P[F] \quad \text { hheH }
$$

can be written as

$$
P[h h, h h>F S]=P[h h \mid h \geq F S] \cdot P[h h>F S] \quad h h, F S \in H .
$$

and so

$$
P[h h]=P[h h \mid F] \cdot P[F] \quad \text { for every } h h \geq F S
$$

Now $\{P[h h]\}$ can be compared with historical frequency distribution $\{\hat{\mathrm{P}}[\mathrm{hh}]\}$ for $h h \geq$ FS. More formally, we can test the hypothesis $H_{0}$ : distribution of the flood crest hh for $h$ h $\geq$ FS is governed by $\Phi$. With $n$ the number of floods in the record, the statistic

$$
D_{n}=\sum_{h h=F S}^{I N} \frac{n(\hat{P}[h h]-P[h h])^{2}}{P[h h]}
$$

is approximately $x^{2}$ distributed with $k=I N-F S$ degrees of freedom. $H_{0}$ is not rejected at the significant level $\alpha$ if $D_{n} \leq \tau_{\alpha, k}$. In addition, by plotting exceedence probabilities corresponding to $\{P[h h]\}$ and $\{\hat{P}[h h]\}$, a visual judgment on the goodness of $\Phi$ can be made.

## 2. CONCEPT OF CATEGORY-UNIT LOSS FUNCTIONS

Flood damage depends on the type of structure, its contents, depth of flooding, water velocity, sediment load, etc., etc. If only the depth of flooding is known, prediction of flood damage will be uncertain because of lack of detailed knowledge of the structure and its contents and of the characteristics of the flood other than depth. Homan and Waybur (1960) were among the first to use a statistical approach to flood damages. Wilson et al., (1974) have used the concept of damage probability matricies to analyze flood plain regulation in Pennsylvania.

Stage-damage curves for every structure in a community are usually not available. Even when they are available, the evaluation of a flood forecastresponse system on a house by house basis would be infeasible because of prohibitive amounts of computational time. It has been noted by Grigg and Helweg (1975, p. 385), among others, "that houses of one type had similar depth-damage curves regardless of actual value." This similarity allows the use of unit stage-damage functions (Bhavnagri and Bugliarello, 1965, 1966) to describe the fraction of total value of a house that is lost by flooding to a particular level. Figures $4-2,4-3$ and $4-4$ show unit damage functions derivec from many sources for one story houses, two story houses and commercial establishments. Although there are some discrepancies between particular functions, it seems that, in general, the unit damage function concept can be accepted for evaluation of communities.

For the evaluation methodology developed in this project, a community is divided into a small number of structural categories and a unit damage function is developed for each. While the validity of the categorization of urban structures is not accepted by some (James, 1965), our studies show a reasonable
similarity between damage curves of structures in a category. There is more similarity between structures in one of the residential categories than between structures in an industrial category.

The arguments for using the category-unit damage function approach may be summarized as follows:
(1) The variability in flood damage estimates based on category and flood stage "averages out" for the evaluation procedures which consider many different possible floods and all the structures in a community.
(2) The amount of data needed is considerably reduced. Such a reduction enables considerable savings in data aquisition effort, in data storage requirements and in computation time.

The category-unit function concept was extended to the cost of response and the reduction in damage obtained by a response. Constraints on response have also been categorized. In the following sections, covering the detailed application of these concepts, it is shown how a large quantity of information is condensed by using these concepts. As a result, each structure is described by a three element vector consisting of the location step, the category, and the maximum damage that the structure can sustain in a flood.


FIGURE 4-2. Comparison of Unit Damage Functions for One Story Houses

## Figure 4-2 Key

| Curve | Source | Original Source |
| :---: | :---: | :---: |
| 1 | Grigg and Helweg, 1975, Fig. $\text { 3, p. } 386$ | U.S. Dept. Agriculture, Soil Conservation Serivce, 1974, stage-damage curve used in Lisle, Illinois (letter from R. D. Murphy', |
| 2 | Grigg and Helweg, 1975, Fig. $\text { 3, p. } 386$ | U.S. Federal Insurance Administration, 1970, flood hazard factors, depth-damage curves, standard rate tables. |
| 3 | Grigg and Helweg, 1975, Fig. $\text { 3, p. } 386$ | U.S. Army Corps of Engineers, 1970, guidlines for flood insurance studies. |
| 4 | Bhavnagri and Bugliarello, 1965, Fig. 8, p. 170 | Sheaffer, 1960. |
| 5 | Kunreuter and Sheaffer, 1970, Fig. 1, p. 664 | U.S. Army Corps of Engineers, unpublished data |
| *6 | Homan and Waybur, 1960, Fig. $2, \text { p. } 34$ | Stanford Research Institute, 1958, reside: tial schedules of flood damage data. |
| *7 | Schaake and Fiering, 1967, <br> Table 5, p. 923 | U.S. Army Corps of Engineers, unpublished data. |
| *8 | Lee, et al., 1976, Fig. 2-13, curve B, p. 70. | Grigg (FIA), modified. |



FIGURE 4-3. Comparison of Unit Damage Functions for Two Story Houses

Figure 4-3 Key

Curve

5 Kunreuter and Sheaffer, 1970, Fig. 1, p. 664

6 Lee, et al., 1976, Fig. 2-13, curve D, p. 70

7 Friedman, 1975, Table V-2, p. 120

8 James, L.D., 1972, Table 4, p. 12.

Original Source
U.S. Dept. Agriculture, Soil Conservation Service, 1974, stage-damage curve used in Lisle, Illinois (letter from R. D. Murphy).
U.S. Federal Insurance Administration, 1970, flood hazard factors, depth-damage curves, elevation frequency curves, standard rate tables.

Grigg and Helweg, 1975, Fig. U.S. Army Corps of Engineers, 1970, guidelines for flood insurance studies.
U.S. Army Corps of Engineers, Baltimore Dìstrict, unpublished flood-damage survey data from Susquehanna River Basin.
U.S. Army Corps of Engineers, unpublished data.

Grigg (FIA), modified.

HUD unpublished collation of U.S. Army Corps of Engineers, TVA, and USGS data.


FIGURE 4-4. Comparison of Unit Damage Functions for Cormercial Estabiishments

| Curve | Source | Original Source |
| :---: | :---: | :---: |
| 1 | Day, 1969, Fig. 4, p. 941 | Not Specified |
| 2-5 | Bhavnagri and Bugliarello, | Scheaffer, 1960 |
|  | 1965, Fig. 8, p. 170 | Curve 2 - Drugstore |
|  |  | 3 - Coveralls manufacturer |
|  |  | 4 - Auto repair shop |
|  |  | 5 - Shoe distributor |

6 Kates, 1962, Fig. 11, p. 99
7 Aitken, 1976, Table 1
3. SET OF STRUCTURAL CATEGORIES, $R$

The set of structural categories $R=\{r\}$ and the set of categories of DMs $Q=\{q\}$ have been defined on the basis of an analysis of the available damage and cost data as follows:

| STRUCTURAL CATEGORY |  | CATEGORY OF DMs |  |
| :---: | :--- | :---: | :---: |
| $r$ |  | .$q$ |  |
| 1 | One story house |  |  |
| 2 | Two story house | 1 | Residential |
| 3 | Trailer |  |  |
| 4 | Commercial-garage type | 2 | Commerical |
| 5 | Commercial-store type |  |  |
| 6 | Industrial - group 1 | 3 | Industrial |
| 7 | Industrial - group 2 |  |  |

## 4. DECISION CONSTRAINT FUNCTION, dd

Decision constraint functions, dd, are specified in Table 2. Below some comments on the data sources are provided.

1. RESIDENTIAL DMs

For residential DMs, dd was obtained through interpretation of the data in Day (1973, pp. 10-12, Fig. 9). One has to bear in mind, therefore, that dd is subject to all assumptions made by Day. In particular, the given equation is a synthetic relation derived from a family of synthetic stagedamage functions for the conditions of no warning ( $t=0$ ), limited warning time ( $t \varepsilon[12,24]$ ), and maximum practical evacuation ( $t>24$ ). Owing to Day's assumption of "100 percent response to the warning", the damage reduced by a protective action under the given warning condition is directly a function of the warning time available. By converting then the reduced damage to the degree of response, $d d(t)$, the points in Figure $4-5$ were obtained, and a function was fitted.
2. COMMERCIAL DMS

For commercial DMs, dd was obtained from a supermarket stage-damage curve (Day, et a1., 1969, p. 941, Fig 4) by the same type of interpretation as was used for residential DMs. Since the data for a supermarket are the only ones available at the time, the developed dd (Figure 4-6) has been applied to all other types of. commercial DMs.

## 3. INDUSTRIAL DMS

For industrial DMs, dd was developed from the data published in Bock and Hendrick (1966, p. 42, Table 4-4C). The given equation should be viewed as a generalized relation in the sense that it does not apply to any specific DM, but it is a result of a statistical analysis of 100 industrial DMs who gave their estimates of the "advanced warning time needed ... to minimize any losses due to river conditions." The points plotted in Figure 4-7 represent the cumulative distribution of the warning time needed for DMs using either flood warnings or forecasts of the river stage or forecasts of the river flow (first three rows in Table 4-4C in Bock and Hendrick, 1966). The survey was conducted in the Connecticut River Basin.

TABLE 4-2. DECISION CONSTRAINT FUNCTIONS

| CATEGORY OF DMs | EQUATION | PARAMETERS |
| :---: | :---: | :---: |
| 1. RESIDENTIAL | $d d(t)= \begin{cases}a t^{b} & 0 \leq t \leq 24 \\ 1 & 24<t\end{cases}$ | $\begin{aligned} & a=.01048 \\ & b=1.434 \end{aligned}$ |
| 2. COMMERCIAL | $d d(t)= \begin{cases}a t^{b} & 0 \leq t \leq 24 \\ 1 & 24<t\end{cases}$ | $\begin{aligned} & a=.01611 \\ & b=1.299 \end{aligned}$ |
| 3. INDUSTRIAL | $d d(t)=a\left(1-e^{-(t / g)}\right)+b \quad 0 \leq t$ | $\begin{aligned} & a=.89 \\ & b=.11 \\ & g=26.0 \end{aligned}$ |




5. UNIT DAMAGE FUNCTION, $\delta$

The unit damage functions $\delta_{r}(r=1,2,3,4,5)$ were derived from the generalized stage-damage curves provided by the Corps of Engineers (1977). Given stage-damage functions were classified into one of the five structural categories in a manner shown in Table 4-3. For industrial damages in Milton, PA., the Corps data contain stage-damage functions for 15 individual industries. These were classified into two groups ( $r=6,7$ ) on the basis of the similarity of the stage-damage relations (Figure 4-14).

For each structural category $r(r=1, \ldots, 7)$ a single unit damage function $\delta_{r}$ was developed. $\delta_{r}$ was assumed to be adequately described by a polynomial. Polynomial coefficients (Table 4-4 and 4-5) were obtained by the method of least squares. Figure $4-8$ to $4-13$ present the points obtained from the data and the fitted polynomials.

TABLE 4-3. CLASSIFICATION OF THE STAGE-DAMAGE FUNCTIONS OF THE CORPS OF ENGINEERS INTO STRUCTURAL CATEGORIES FOR DEVELOPING UNIT DAMAGE FUNCTIONS

| STRUCTURAL CATEGORY | INCLUDED STAGE-DAMAGE FUNCTIONS <br> FROM THE CORPS DATA <br> (SYMBOLS AS USED BY THE CORPS, 1977) |
| :---: | :---: |
| 1. One story house | $\begin{aligned} & \text { AIYAA, AINAA, AIYAA (seepage), } \\ & \text { BIYAA, BINAA, BIYAA (seepage), } \\ & \text { CIYAA, CINAA, CIYAA (seepage) } \end{aligned}$ |
| 2. Two story house | $\begin{aligned} & \text { A2YAA, A2NAA, A2YAA (seepage), } \\ & \text { B2YAA, B2NAA, B2YAA (seepage), } \\ & \text { C2YAA, C2NAA, C2YAA (seepage) } \end{aligned}$ |
| 3. Trailer | TL, TA, TS |
| 4. Commercial-garage type | Garage in good and poor condition (2 curves) |
| 5. Commercial-store type | Store in good and poor condition, with and without basement (4 curves) |
| 6. Industrial-group 1 | Industries number: $2,4,6,8,9,12,13$ |
| 7. Industrial-group 2 | Industries number: $\begin{aligned} & 1,3,5,7,10,11, ~ \\ & 14,15\end{aligned}$ |

TABLE 4-4. UNIT DAMAGE FUNCTIONS

$$
\delta(z)=c(0)+c(1) z+\ldots+c(n) z^{n} \quad(z \text { in feet })
$$

| STRUCTURAL CATEGORY | n | i | c ${ }^{(1)}$ | DOMAIN <br> z (feet) |
| :---: | :---: | :---: | :---: | :---: |
| 1. One story house | 8 | 0 1 2 3 4 5 6 7 8 | $\begin{array}{r} .148905 \\ .852040 \times 10^{-1} \\ .820734 \times 10^{-2} \\ -.235219 \times 10^{-2} \\ -.133283 \times 10^{-3} \\ .393609 \times 10^{-4} \\ .366432 \times 10^{-7} \\ -.225617 \times 10^{-6} \\ .805889 \times 10^{-8} \end{array}$ | 0-15 |
| 2. Two story house | 4 | 0 1 2 3 4 | $\begin{aligned} & .110007 \\ & .271166 \times 10^{-1} \\ & .137889 \times 10^{-2} \\ & -.399962 \times 10^{-4} \\ & -.326650 \times 10^{-7} \end{aligned}$ | 0-24 |
| 3. Trailer | 4 | 0 1 2 3 4 | $\begin{aligned} & .163062 \\ & .136326 \\ & .133073 \times 10^{-1} \\ & -.415330 \times 10^{-2} \\ & .239271 \times 10^{-3} \end{aligned}$ | 0-9 |

TABLE 4-4. Continued

| STRUCTUREAL CATEGORY | n | i | $c(i)$ | $\begin{gathered} \text { DOMAIN } \\ z \text { (feet) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 4. Commercial-garage type | 9 |  | $\begin{array}{r} .439931 \times 10^{-1} \\ .707324 \times 10^{-1} \\ .157361 \times 10^{-1} \\ -.302723 \times 10^{-2} \\ -.576608 \times 10^{-3} \\ .978475 \times 10^{-4} \\ .887390 \times 10^{-5} \\ -.143767 \times 10^{-5} \\ -.476253 \times 10^{-7} \\ .741636 \times 10^{-8} \end{array}$ | 0-11 |
| 5. Commercial-store type | 6 | 0 1 2 3 4 5 6 | $\begin{aligned} & .402845 \\ & .138426 \\ & .899010 \times 10^{-3} \\ & -.220052 \times 10^{-2} \\ & .506582 \times 10^{-4} \\ & .143909 \times 10^{-4} \\ & -.648618 \times 10^{-6} \end{aligned}$ | 0-11 |

TABLE 4-4. Continued

| STRUCTURAL CATEGORY | $n$ | i | $c(i)$ | $\begin{aligned} & \text { DOMAIN } \\ & z(\text { feet }) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| 6. Industrial group 1 | 4 | $\begin{aligned} & 0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \end{aligned}$ | $\begin{aligned} & -.151948 \times 10^{-1} \\ & .229184 \\ & .296703 \times 10^{-2} \\ & -.374736 \times 10^{-2} \\ & .218115 \times 10^{-3} \end{aligned}$ | 0-10 |
| 7. Industrial group 2 | 4 | $\begin{aligned} & 0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \end{aligned}$ | $\begin{array}{r} -.122059 \times 10^{-1} \\ .511755 \times 10^{-1} \\ .129084 \times 10^{-1} \\ -.125563 \times 10^{-2} \\ .307060 \times 10^{-4} \end{array}$ | 0-18 |

TABLE 4-5. UNIT DAMAGE FUNCTIONS
FOR INDIVIDUAL INDUSTRIES

$$
\delta(z)=c(0)+c(1) z+\ldots+c(n) z^{n} \quad \text { (z in feet) }
$$

| INDUSTRY NUMBER | n | i | $c(i)$ | $\begin{gathered} \text { DOMAIN } \\ z(\text { feet }) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 4 | $\begin{aligned} & 0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \end{aligned}$ | $\begin{array}{r} .113609 \times 10^{-1} \\ .508476 \times 10^{-1} \\ .234403 \times 10^{-1} \\ -.296726 \times 10^{-2} \\ .983617 \times 10^{-4} \end{array}$ | 0-15 |
| 2 | 5 | $\begin{aligned} & 0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \end{aligned}$ | $\begin{aligned} & .343074 \times 10^{-2} \\ & .119280 \\ & -.212670 \\ & .164148 \\ & -.374811 \times 10^{-1} \\ & .270833 \times 10^{-2} \end{aligned}$ | 0-6 |
| 3 | 9 | $\begin{aligned} & 0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & 6 \\ & 7 \\ & 8 \\ & 9 \end{aligned}$ | $\begin{array}{r} -.358809 \times 10^{-3} \\ -.182655 \times 10^{-1} \\ .503030 \times 10^{-1} \\ -.387605 \times 10^{-1} \\ .127559 \times 10^{-1} \\ -.203970 \times 10^{-2} \\ .178442 \times 10^{-3} \\ -.879646 \times 10^{-5} \\ .230246 \times 10^{-6} \\ -.249547 \times 10^{-8} \end{array}$ | 0-19 |
| 4 | 5 | 0 1 2 3 4 5 | $\begin{aligned} & .769231 \times 10^{-3} \\ & .250868 \\ & .397582 \times 10^{-1} \\ & -.207838 \times 10^{-1} \\ & .249126 \times 10^{-2} \\ & .961538 \times 10^{-4} \end{aligned}$ | 0-10 |

TABLE 4-5. Continued

| INDUSTRY NUMBER | n | i | $\dot{c}(\mathrm{i})$ | DOMAIN <br> z (feet) |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 8 | $\begin{aligned} & 0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & 6 \\ & 7 \\ & 8 \end{aligned}$ | $\begin{array}{r} .526621 \times 10^{-3} \\ .296391 \\ -.580215 \\ .388595 \\ -.113273 \\ .173689 \times 10^{-1} \\ -.146879 \times 10^{-2} \\ .649504 \times 10^{-4} \\ -.117474 \times 10^{-5} \end{array}$ | 0-13 |
| 6 | 6 | $\begin{aligned} & 0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & 6 \end{aligned}$ | $\begin{aligned} & -.608727 \times 10^{-9} \\ & -.267000 \\ & .162078 \times 10^{1} \\ & -.111875 \times 10^{1} \\ & .326944 \\ & -.442500 \times 10^{-1} \\ & .227778 \times 10^{-2} \end{aligned}$ | 0-6 |
| 7 | 5 | $\begin{aligned} & 0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \end{aligned}$ | $\begin{array}{r} -.147308 \times 10^{-1} \\ .779934 \times 10^{-1} \\ -.400704 \times 10^{-1} \\ .103248 \times 10^{-1} \\ -.875578 \times 10^{-3} \\ .241699 \times 10^{-4} \end{array}$ | 0-15 |
| 8 | 5 | 0 1 2 3 4 5 | $\begin{aligned} & -.321096 \times 10^{-2} \\ & .169330 \\ & .320523 \\ & -.103711 \\ & .139642 \times 10^{-1} \\ & -.682692 \times 10^{-3} \end{aligned}$ | 0-7 |

TABLE 4-5. Continued

| INDUSTRY NUMBER | n | i | c(i) | $\begin{gathered} \text { DOMAIN } \\ z(\text { feet }) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 9 | 5 | $\begin{aligned} & 0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \end{aligned}$ | $\begin{array}{r} .769231 \times 10^{-3} \\ .250868 \\ .397582 \times 10^{-1} \\ -.207838 \times 10^{-1} \\ .249126 \times 10^{-2} \\ -.961538 \times 10^{-4} \end{array}$ | 0-10 |
| 10 | 8 | $\begin{aligned} & 0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & 6 \\ & 7 \\ & 8 \end{aligned}$ | $\begin{array}{r} .118287 \times 10^{-3} \\ -.155758 \\ .317152 \\ -.141854 \\ .335083 \times 10^{-1} \\ -.455018 \times 10^{-2} \\ .353576 \times 10^{-3} \\ -.145761 \times 10^{-4} \\ .246966 \times 10^{-6} \end{array}$ | 0-14 |
| 11 | 2 | 0 1 2 | $\begin{aligned} & .658741 \times 10^{-2} \\ & .123845 \\ & -.243357 \times 10^{-2} \end{aligned}$ | 0-10 |
| 12 | 4 | $\begin{aligned} & 0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \end{aligned}$ | $\begin{array}{r} -.577922 \times 10^{-2} \\ -.525758 \times 10^{-1} \\ .171364 \\ -.415152 \times 10^{-1} \\ .318182 \times 10^{-2} \end{array}$ | 0-6 |

TABLE 4-5. Continued

| INDUSTRY NUMBER | n | i | $c(i)$ | $\begin{aligned} & \text { DOMAIN } \\ & z \text { (feet) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| 13 | 6 | $\begin{aligned} & 0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & 6 \end{aligned}$ | $\begin{aligned} & .350272 \times 10^{-2} \\ & .129447 \\ & -.323180 \\ & .281859 \\ & -.837375 \times 10^{-1} \\ & .103746 \times 10^{-1} \\ & -.460185 \times 10^{-3} \end{aligned}$ | 0-8 |
| 14 | 5 | $\begin{aligned} & 0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \end{aligned}$ | $\begin{array}{r} .719814 \times 10^{-2} \\ -.216535 \times 10^{-1} \\ .254931 \times 10^{-1} \\ -.634810 \times 10^{-3} \\ -.212429 \times 10^{-3} \\ .119076 \times 10^{-4} \end{array}$ | $0-13$ |
| 15 | 4 | 0 1 2 3 4 | $\begin{array}{r} .193116 \times 10^{-2} \\ .138736 \times 10^{-1} \\ .255778 \times 10^{-1} \\ -.297453 \times 10^{-2} \\ .110414 \times 10^{-3} \end{array}$ | 0-12 |



FIGURE 4-8. Unit Damage Function For One Story House ( $r=1$ )


FIGURE 4-9. Unit Damage Function For Two Story House ( $r=2$ )


FIGURE 4-10. Unit Damage Function For Trailer ( $r=3$ )


FIGURE 4-11.. Unit Damage Function for Commercial StructureGarage Type ( $r=4$ )


FIGURE 4-12. . Unit Damage Function For Commercial StructureStore Type ( $r=5$ )



FIGURE 4-13. Unit Damage Functions For Industry In Milton, Pa.



FIGURE 4-14. Unit Damage Functions For Individual Industries in Milton, Pa.

## 6. UNIT COST FUNCTION, $\gamma$

Unit cost functions for residential and commercial DMs were developed by the authors. Due to lack of data, it was not feasible to develop $\gamma$ for industrial DMs; therefore, for the purpose of the case study, $\gamma$ for commercial DMs was assumed to apply also to industrial DMs. Equations for $\gamma$ are specified in Table 4-6 and shown in Figures 4-15 to 4-16.

TABLE 4-6. UNIT COST FUNCTIONS

| CATEGORY OF DMs | EQUATION | PARAMETERS |
| :---: | :---: | :---: |
| 1. Residential | $r(\alpha)=c \cdot{ }^{\text {b }}$ | $\begin{aligned} & c=.0169 \\ & b=2.1 \end{aligned}$ |
| 2. Commercial | $\gamma(\alpha)=c \cdot \alpha{ }^{\text {b }}$ | $\begin{aligned} & c=.0670 \\ & b=1.3 \end{aligned}$ |
| 3. Industrial | $\gamma(\alpha)=c \cdot \alpha^{b}$ | $\begin{aligned} & c=.0670 \\ & b=1.3 \end{aligned}$ |


7. UNIT REDUCTION FUNCTION, MR

Unit reduction functions, MR, are specified in Table 4-7 and shown in Figures 4-17 through 4-19. The following are some comments on the data sources. 1. RESIDENTIAL DMS

For residential DMs, MR was obtained from the stage-damage functions given by Day (1973), p. 11, Fig. 9) and interpreted herein as follows:
no warning condition $\quad \Rightarrow \quad \alpha=0$,
maximum practical evacuation $\quad \Rightarrow \quad \alpha=1.0$.
2. COMMERCIAL DMs

For commercial DMs, MR was extracted from supermarket stage-damage curves (Day et al., 1969, p. 941, Fig. 4). The following interpretation was assumed:
no warning condition $\quad \Rightarrow \quad \alpha=0$,
24 hour warning $\quad \Rightarrow \quad \alpha=1.0$.
3. INDUSTRIAL DMs

For industrial DMs, MR was developed from the average stage-damage function for the establishments $E_{0}$ and $E_{d}$ in Kates (1965, p. 64, Table 11). The following interpretation was used:
loss bearing $\quad \Rightarrow \quad \alpha=0$,
emergency action $\quad \Longrightarrow \quad \alpha=.3$.
The reason for having $\alpha=.3$ is that Kates' emergency action assumes warning time $t=6$ hours; from decision constraint function (Figure 4-2) we get $\operatorname{dd}(6)=.3$

TABLE 4-7. UNIT REDUCTION FUNCTIONS

$$
\operatorname{MR}(z)=c(0)+c(1) z+c(2) z^{2}+\ldots+c(n) z^{n} \quad \text { (z in feet) }
$$

| CATEGORY OF DMs | n | 1 | $c(i)$ | $\begin{aligned} & \text { DOMAIN } \\ & z \text { (feet) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1. Residential | 4 | 0 | $\begin{array}{r} .247612 \\ .990691 \cdot 10^{-1} \\ -.153879 \cdot 10^{-1} \\ .777969 \cdot 10^{-3} \\ -.129309 \cdot 10^{-4} \end{array}$ | 0-24 |
| 2. Commercial | 6 | 0 1 2 3 4 5 6 | $\begin{aligned} & .170094 \cdot 10^{-2} \\ & .118480 \cdot 10 \\ & -.651081 \\ & .181678 \\ & -.268257 \cdot 10^{-1} \\ & .198784 \cdot 10^{-2} \\ & -.580882 \cdot 10^{-4} \end{aligned}$ | 0-11 |
| 3. Industrial | 3 | 0 | $\begin{aligned} & .400455 \\ & .136726 \\ & -.200947 \cdot 10^{-1} \\ & .738636 \cdot 10^{-3} \end{aligned}$ | 0-15 |



FIGURE 4-17. Unit Reduction Function For Residential DMs (Data Source: Day, 1973)


FIGURE 4-18. Unit Reduction Function For Commercial DMs (Data Source: Day et al., 1969)


FIGURE 4-19. Unit Reduction Function For Industrial DMs
(Data Source: Kates, 1965)

## 8. VECTOR ESTABLISHMENT $=(m, r, M D)$

Vector ( $m, r, M D$ ) has been determined for each establishment on the basis of the methodology and field inventory data provided by the Corps of Engineers (1977). Location step, m, can be obtained directly from the field data. Structural category, $r$, and maximum possible damage, MD, can be determined as follows. For industrial establishments $(r=6,7), r$ and MD are specified directly in the field inventory data for each individual industry. For residential and commercial establishments ( $r=1,2,3,4,5$ ), $r$ and MD have to be obtained from a set of descriptors collected for each establishment during field inventory. For completeness of the presentation we describe briefly the procedure.

1. A set of descriptors (Table 4-8) is used to characterize the establishments in the flood plain.
2. Each establishment is described in accordance with the schedule given in Table 4-9.
3. $r$ is determined according to Table 4-10.
4. $M D$ is computed as follows:

For $C L=(A, B, C, T, N)$
$M D=M D 1 \cdot M D 2$
and for $C L=(G, S)$
$M D=M D T \cdot A R$
where MD1 and MD2 are defined by the values of the descriptors, as given in Table 4-11 and 4-12.

## REMARKS

1. Computed MD represents maximum possible direct total damage, that is the damage to the structure and to the contents.
2. For $C L=(G, S)$ the Corps data contain only structural damage. Damage to the contents was assumed after Homan and Waybur (1960, p. 7) to be 1.28 times the structural damage.
3. Values for MD1 are at 1977 price level and have been determined from the values given by the Corps at 1963 price level. Price ratios used were 1.50-1.90, depending on the class of structure.

TABLE 4-8. SET OF DESCRIPTORS FOR ESTABLISHMENTS IN THE FLOOD PLAIN

| DESCRIPTOR |  | SET OF VALUES |  |
| :---: | :---: | :---: | :---: |
| TY | Type | $R$ $C$ | Residential <br> Commercial |
| CL | Class | $\begin{aligned} & A \\ & B \\ & C \\ & T \\ & \text { N } \\ & \text { G } \\ & \text { S } \end{aligned}$ |  |
| ST | Number of stories | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ |  |
| BA | Basement | $Y$ | $\begin{aligned} & \text { Yes } \\ & \text { No } \end{aligned}$ |
| SI | Size | $\begin{aligned} & \text { L } \\ & \text { A } \\ & \text { S } \end{aligned}$ | Large <br> Average <br> Small |
| FU | Furnishings | H A L | High <br> Average <br> Low |
| CO | Condition | $\begin{aligned} & G \\ & P \end{aligned}$ | Good <br> Poor |
| AR | Floor plan area |  | [Square feet] |

Source: Corps of Engineers, 1977.

TABLE 4-9. DESCRIPTORS REQUIRED TO CHARACTERIZE AN ESTABLISHMENT

| If |  | then there must be data for: |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TY | CL | ST | BA | SI | FU | CO | AR |
| R,C | $A, B, C$, | $\checkmark$ | $V$ | $\checkmark$ | $\checkmark$ |  |  |
|  | T, N |  |  | $\checkmark$ | $\checkmark$ |  |  |
| C | G |  |  |  |  | $\checkmark$ | $\checkmark$ |
|  | S |  | $\checkmark$ |  |  | $V$ | $\checkmark$ |

TABLE 4-10. DETERMINATION OF $r$

| CL | ST | $r$ |
| :---: | :---: | :---: |
| A | 1 |  |
| B | 1 | 1 |
| C | 1 |  |
| N |  | 2 |
| A | 2 | 3 |
| B | 2 | 2 |
| C |  | 4 |
| G |  | 5 |
| S |  |  |

TABLE 4-11. COEFFICIENT MD1

| CL | ST | BA | CO | MD1 |
| :---: | :---: | :---: | :---: | :---: |
| A |  |  |  | 58000 \$ |
| B |  |  |  | 35000 \$ |
| C | 1 |  |  | 12000 \$\$ |
|  | 2 |  |  | 16000 \$ |
| T |  |  |  | 6500 \$ |
| N |  |  |  | 5000 \$ |
| G |  |  | G | 9.1 \$/sq. ft. |
|  |  |  | P | 5.0 |
| S |  | $Y$ | G | 7.1 " |
|  |  |  | P | 3.9 |
|  |  | $N$ | G | 5.7 |
|  |  |  | P | 3.2 " |

Source: Data of the Corps of Engineers (1977) Updated by the authors to 1977 price level.

TABLE 12. COEFFICIENT MD2

| SI | FU | CL |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A | B | C | T | N |
| L | H | 1.30 | 1.34 | 1.54 | 1.57 | 2.18 |
|  | A | 1.11 | 1.20 | 1.21 | 1.23 | 1.37 |
|  | L | . 95 | 1.06 | . 88 | . 89 | . 59 |
| A | H | 1.17 | 1.14 | 1.35 | 1.35 | 1.69 |
|  | A | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
|  | L | . 84 | . 86 | . 70 | . 70 | . 43 |
| S | H | 1.06 | . 91 | 1.10 | . 64 | 1.00 |
|  | A | . 90 | . 78 | . 79 | . 46 | . 76 |
|  | L | . 74 | . 67 | . 57 | . 33 | . 27 |

Source: Corps of Engineerș (1977). Column for T was computed by the authors; correction coefficient for furnishings was assumed to be the same as in Column C.

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## APPENDIX

## ELEMENTARY PROPERTIES OF THE MULTINOMIAL FAMILY*/

## 1. MULTINOMIAL DISTRIBUTION

### 1.1 Joint

Let $X=\left(X_{1}, \ldots, x_{k}\right)$ be a random vector each of whose components $X_{j}$ has for the sample space finite set of nonnegative integers $[0,1,2, \ldots, N]$ and each of whose samples $\left(x_{1}, \cdots, x_{k}\right)$ satisfies $\sum_{j=1}^{k} x_{j} \leq N$. Random vector $x$ has k-variate multinomial distribution with parameters $N$ and ( $p_{1}, \ldots, p_{k}$ ), which will be referred to as $M\left(N ; p_{1}, \ldots, p_{n}\right)$, if the probability function is specified by equation

$$
\begin{equation*}
P\left(x_{1}, \ldots, x_{k}\right)=N!\prod_{j=1}^{k+1}\left(p_{j}^{x_{j}} / x_{j}!\right) \tag{A-1}
\end{equation*}
$$

where it is understood that

$$
x_{k+1}=N-\sum_{j=1}^{k} x_{j} \text { and } p_{k+1}=1-\sum_{j=1}^{k} p_{j}, \quad 0<p_{j}<1 \quad(j=1, \ldots, k) \text {. The joint }
$$

moments of $X$ are:

$$
\begin{align*}
E\left(X_{j}\right) & =N p_{j} \\
\operatorname{Var}\left(X_{j}\right) & =N p_{j}\left(1-p_{j}\right)  \tag{A-2}\\
\operatorname{Cov}\left(X_{i} x_{j}\right) & =-N p_{i} p_{j} \quad i, j=1, \ldots, k ; \quad i \neq j .
\end{align*}
$$

[^1]
### 1.2 Joint of a Subset of $x$

Since the probability that $\sum_{j=1}^{k+1} x_{j}=N$ is 1, the joint distribution of any subset $x_{a_{1}}, \ldots, x_{a_{s}}$ of $x$ is also multinomial $M\left(N ; p_{a_{1}}, \ldots, p_{a_{s}}\right.$ Lith $(s+1)$ th variable equal to $N-\sum_{j=1}^{s} x_{a_{j}}$ and $(s+1)$ th parameter $p$ equal to $1 \cdot \sum_{j=1}^{s} p_{a_{j}}$.

### 1.3 Marginal

The marginal distribution of any $X_{j}$ is binomial with parameters $N$ and $p_{j}$, which will be referred to as $B\left(N, P_{j}\right)$. The probability function is

$$
\begin{equation*}
P\left(x_{j}\right)=N!\left[x_{j}!\left(N-x_{j}\right)!\right]^{-1} p_{j}^{x_{j} q_{j}^{n-x_{j}}} \tag{A-3}
\end{equation*}
$$

where $q_{j}=1-p_{j}$.
2. POSITIVE MULTINOMIAL

### 2.1 Joint

If the sample space of each $X_{j}(j=1, \ldots, k)$ is reduced to the finite set of positive integers $[1,2, \ldots, N]$ then the distribution ( $A-7$ ) is called positive multinomial and will be referred to as $M P\left(N ; P_{1}, \ldots, P_{k}\right)$. The distribution function is specified by the equation

$$
\begin{equation*}
P\left(x_{1}, \ldots, x_{k}\right)=N!\prod_{j=1}^{k+1}\left(p_{j} x_{j} / x_{j}!\right) \cdot A^{-1} \tag{A-4}
\end{equation*}
$$

where

$$
\begin{equation*}
A=1-\left[\sum_{j=1}^{k}\left(1-p_{j}\right)^{N}-(k-1)\left(1-p_{k+1}\right)^{N}\right] \tag{A-5}
\end{equation*}
$$

### 2.2 Marginal

Marginal distribution of any $X_{j}$ is positive binomial, $B P\left(N, p_{j}\right)$ with the distribution function

$$
\begin{equation*}
P\left(x_{j}\right)=N!\left[\left[x_{j}!\left(N-x_{j}\right)!\right]^{-1} P_{j}^{x_{j}} q_{j}^{n-x_{j}} \cdot\left[1-q_{j}^{N}\right]^{-1}\right. \tag{A-6}
\end{equation*}
$$

The first two moments of $x_{j}$ are:

$$
\begin{align*}
& E\left(X_{j}\right)=N p_{j} /\left(1-q_{j}^{N}\right), \\
& \operatorname{Var}\left(X_{j}\right)=N p_{j} q_{j} /\left(1-q_{j}^{N}\right)-N^{2} p_{j}^{2} q_{j}^{N} /\left(1-q_{j}^{N}\right)^{2} \tag{A-7}
\end{align*}
$$

## 3. APPROXIMATION

The routine computation of multinomial probabilities from ( $A-1$ ) can be tedious. The following proved to be useful approximations to $M\left(N ; p_{j}, \ldots, p_{k}\right)$ :

$$
\begin{equation*}
P\left(x_{1}, \ldots, x_{k}\right) \equiv(2 \pi N)^{-\frac{k}{2}}\left(\prod_{j=1}^{k+1} p_{j}\right)^{-\frac{1}{2}} \exp \left[-\frac{1}{2} \sum_{j=1}^{k+1}\left(x_{j}-N p_{j}\right)^{2} / N p_{j}\right] \tag{A-8}
\end{equation*}
$$

An equivalent approximation to $B(N, P)$ is

$$
\begin{equation*}
P(x) \equiv(2 \pi N \cdot p q)^{-\frac{1}{2}} \exp \left[-\frac{1}{2}(x-N p)^{2} / N p q\right] . \tag{A-9}
\end{equation*}
$$

## 4. ESTIMATORS

With known $N$ and $k$, the maximum likelihood, unbiased estimator at $p_{j}$ ( $j=1, \ldots, k$ ) in ( $A-1$ ) is

$$
\begin{equation*}
\hat{p}_{j}=\hat{E}\left(X_{j}\right) / N \tag{A-10}
\end{equation*}
$$

where for the known sample frequency distribution $\left\{z_{( }\left(x_{j}\right): x_{j}=0,1, \ldots, N\right\}$ of $x_{j}$

$$
\begin{equation*}
\hat{E}\left(x_{j}\right)=\sum_{x_{j}=0}^{N} z\left(x_{j}\right) \cdot x_{j} . \tag{A-11}
\end{equation*}
$$

The estimator of $p_{j}(j=1, \ldots, k)$ for positive multinomial distribution given by ( $A-4$ ) is

$$
\begin{equation*}
\hat{p}_{j}=\hat{E}\left(X_{j}\right)\left(1-\left(1-\hat{p}_{j}\right)^{N}\right) / N \tag{A-12}
\end{equation*}
$$

With respect to $\hat{p}_{j},(A-12)$ has the form of the fixed point so that it can be solved by iteration. An alternative estimator is

$$
\begin{equation*}
\tilde{p}_{j}=\left(\hat{E}\left(X_{j}\right)-z(1)\right) /(N-z(1)) . \tag{A-13}
\end{equation*}
$$

$\tilde{p}_{j}$ can be used as an initial value in ( $A-12$ ) for the iteration although it can stand alone since usually the asymptotic efficiency $\underset{n \mid \infty}{\left[\operatorname{imm}_{n}\{\operatorname{var}(\hat{p}) / \operatorname{var}(\tilde{\mathrm{p}})\}\right]}$ of $\tilde{p}$ is quite high.

# Chapter 5 <br> Inputs for Milton Case Study 

## 1. INTRODUCTION

The case study for Milton, Pa., was described in Chapter 1. Three types of information were required: hydrologic, economic, and response strategy. Chapter 4 discusses the methodology of obtaining the specific parameters needed for the case study. Some of these results, such as the unit functions, are general and may be used in evaluations other than Milton. The hydrologic parameters are specific; this chapter covers their derivation for Milton, Pa. and the derivation of the human factors model of response based on previous flood history and the forecast sequence.

The main emphasis in this chapter is the details of the determination of the law of motion for Milton, Pa., based on the methodology presented in Chapter 4 and the data obtained from Flood Forecast Verification Reports. First, the net result of this work is shown in the format used to input the data to the computer.

The first step in obtaining these numbers involves a complete analysis of the historical forecast record. Using the results of this analysis, graphs are drawn to provide smoothed values of needed descriptors. From these values the parameters of the fitting distribution are obtained. Using these, the complete law of motion is obtained. Finally a synthetic historical record of peak flows is derived from the law of motion and compared with the actual record.

In the last section, the effect of the hydrologic variables and the forecast sequence on the response strategy as determined by the human factors model, for Milton, is shown.
2. SUMMARY OF THE INPUT DATA


TABLE 5－1．Continued
3. INPUT FOR THE FORECASTING SYSTEM
3.1 ANALYSIS OF HISTORICAL FORECASTS

|| || || || || || \| \| \| \| \| \| \| \| \| \| \| \|



ORNM=RNONOO



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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$$
\begin{array}{lllllllll} 
& 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$$
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0 & 0 & 0 & 0 & \hat{0} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0
\end{array}
$$

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& 0 & 0 & \text { N } & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0
\end{array}
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\begin{array}{llllllllll}
\text { N } & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\underset{\sim}{\sim} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\underset{y}{ \pm} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
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0 & 0 & 0 & 0
\end{array}
$$

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\begin{array}{lllllllll} 
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0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$$
\begin{array}{lllllllll} 
& 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$$
\begin{array}{lllllllll} 
& 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & n & 0 & 0 \\
0 & 0 & 0 & 0 & & 0 & 0 & 0
\end{array}
$$

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\begin{array}{lllllllll}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0
\end{array}
$$

$$
-N m=\text { in } 0 \sim \infty
$$

TABLE 3

$$
\underset{\sim}{\Xi}
$$

| table 4 |  | $\mathrm{P}(\mathrm{I}(\mathrm{K}+1) \mathrm{H}(\mathrm{H}+1) / \mathrm{H}(\mathrm{K}) \mathrm{)}$ |  |  | $\mathrm{H}(\mathrm{K})=1$ |  |  | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{H}(\mathrm{K}+1)$ |  |  |  |  |  |  |  |  |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 |  |  |  |
| $I(K+1)$ | 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 2 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 3 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 4 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 5 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 6 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 7 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 8 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 9 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |






o $0000^{\circ} 00000^{\circ} 0$ $0000^{\circ} 00000^{\circ} 0$ $0000 \cdot 00000 \cdot 0$ $0000^{\circ} 00000 \cdot 0$ $0000^{\circ} 00000^{\circ} 00000^{\circ} 00000^{\circ} 0$ 0.00000 .00000 .00000 .0000 0.00000 .00000 .00000 .0000 $0000^{\circ} 00000^{\circ} 00000^{\circ} 00000^{\circ} 0$ $0000^{\circ} 00000^{\circ} 00000^{\circ} 00000^{\circ} 0$

## n

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## 3

 $0000^{\circ} 0$(K)

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\begin{aligned}
& 0.0000 \\
& 0.0000
\end{aligned}
$$

| $(I(K+1)$ | $H(K+1) / H(K))$ | $H(K)=7$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | .3333 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

0
$-N m \geq n \infty \times \infty$
TABLE 4
$I(K+1)$
$\mathbf{P}(I(K+1), H(K+1) / H(K)) \quad H(K)=8$

|  | － | － | － | － | $\bigcirc$ | － | － | $\bigcirc$ | $\bigcirc$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\sigma$ | O | － | O | － | － | － | － | － | － |
|  | － | － | － | － | － | － | － | － | － |
|  | $\bigcirc$ | 0 | － | 0 | － | － | － | O | － |
|  | － |  | － | － | － | － |  | － | － |
|  | 0 | 0 | － | － | － | － | 0 | 0 | 0 |
|  | O | O | O | － | － | O | $0$ | $0$ | O |
|  | O | O | O | － | O | O | O | － | － |
| $\infty$ | $\bigcirc$ | 0 | － | － | 0 | 0 | 0 | － | 0 |
|  | － | － | － | $\stackrel{+}{+}$ | － | － | $\dot{\circ}$ | $\dot{\square}$ | $\dot{0}$ |
|  | － | － | － | － | － | － | － | － | － |
|  | － | O | O． | O | O | O | 0 | O | O |
| － | － | － | 0 | O | O | O | － | － | O |
|  | － | － | 0 | O | － | － | 0 | － | 0 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | O | O | O | － | O | O | － | ㅇ | O |
|  | － | － | O | O | ㅇ | O | O | ㅇ | 앙 |
| － | － | － | O | O | O | － | O | 0 | 0 |
|  | 0 | 0 | － | 0 | 0 | － | － | － | 0 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 |
|  | － | O | － | 0 | － | － | O | － | 0 |
|  | O | － | O | O | － | O | － | O | － |
| $\sim$ | － | － | － | － | － | － | － | O | O |
|  | － | － | － | － | － | － | － | － | 0 |
|  | － | － | － | － | － | － | $\dot{0}$ | － | － |
|  | 0 | － | － | － | － | 0 | 0 | 0 | 0 |
|  | － | － | － | － | O | O | O | － | O |
|  | 0 | O | － | 0 | O | O | O | O | O |
| $=$ | － | O | － | － | O | － | － | － | O |
|  | 0 | － | － | O | 0 | 0 | O | － | 0 |
|  | $0 \cdot$ | 0 | $\dot{0}$ |  | 0 | $0 \cdot$ | 0 | $\stackrel{\circ}{\circ}$ | 0 |
|  | － | － | － | － | － | － | － | － | － |
|  | O | O | O | O | O | O． | O | O | O |
| $m$ | O | O | O | O | O | O | O | O | O |
|  | － | － | － | 0 | O | － | － | 0 | O |
|  | $\stackrel{\circ}{-}$ | 0 | $\bigcirc$ | $\dot{\circ}$ | $\dot{-}$ | ${ }^{\circ}$ | － | $\dot{\circ}$ | 0 |
|  | － | － | － | － | － | － | － | － | O |
|  | O | O | O | O | O | O | － | O | O |
| $N$ | O | O | O | O | O | － | － | O | O |
|  | $\bigcirc$ | － | 0 | O | － | O | 0 | － | O |
|  | 0 | 0 | 0 | 0 | 0 | － | 0 | 0 | 0 |
|  | － | O |  | － |  |  |  |  |  |
|  |  |  | － | O | － | － | O | 0 | O |
| － | O | O | － | O | O | O | O | O | O |
|  | O | O | $0$ |  |  |  | $8$ | O | 8 |
|  | － | $0$ | $0$ | $0$ | O | O | O | 0 | O |
|  | － | － | － | 0 | － | 0 | － | 0 | 0 |

TABLE 4
$I(K+1)$
TABLE 4

$I(K+1)$

$$
\begin{aligned}
& \mathrm{E}(\mathrm{UH} / \mathrm{H}) \\
& 0.0000 \\
& 3.0000 \\
& 2.8000 \\
& 0.0000 \\
& 5.0000 \\
& 0.0000 \\
& 7.0000 \\
& 0.0000 \\
& 0.0000
\end{aligned}
$$

$$
\begin{array}{lllllllll}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$$
\begin{array}{lllllllll}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$$
\begin{array}{rllllllll}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & - & 0 & 0
\end{array}
$$

$$
\begin{array}{lllllllll} 
& 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\end{array} 0
$$

$$
\begin{array}{lllllllll}
\text { O } O \text { O } & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & - & 0 & 0 & 0 & 0
\end{array}
$$

$$
=\begin{array}{llllllll}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}
$$

$$
\begin{array}{rllllllll}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & - & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$$
-N m \nexists n \infty \quad \infty \quad \sigma
$$



$$
H(K)
$$



- 응
.$a$
.0000
$\infty$
.0000
- $\quad \stackrel{\circ}{\circ}$
- 응
^ 응
$=\quad \stackrel{\circ}{\circ}$



| - | の | $\pm$ |
| :---: | :---: | :---: |
| R | $\cdots$ | a |
| $\checkmark$ | $\cdots$ | N |
| $\bigcirc$ | n | $\Omega$ |
| - | - | - |
| 11 | 1 | 11 |
| $\bar{z}$ | $B$ | $\bar{z}$ |
| $\underline{2}$ | $\mathcal{L}$ | 己 |
| 0 | 0 | ¢ |


$\begin{array}{llll} & \text { の } & 0 & 0 \\ 0 & \text { a } & 0 \\ 0 & 0 \\ 0 & & 0\end{array}$
$\begin{array}{llll} & \infty & 0 & \\ 0 & \infty & 0 \\ & 0 & \infty & 0 \\ 0 & & 0 \\ & 0 & & 0\end{array}$
F F F F
$\begin{array}{lll} & 0 & \text { ت } \\ 0 & 0 & \text { ت } \\ & 0 & \\ 0 & & \end{array}$

$=\quad \begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0\end{aligned} \quad \stackrel{m}{m}$
$\begin{array}{llll} & & 0 \\ n & n & 0 \\ n & n \\ n & n & \infty \\ 0\end{array}$

픙 $\quad \begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0\end{aligned}$
$\infty$
table
TABL
HH
P(H)
P(HH.GE.M)

$$
\begin{aligned}
& P(I(K 0)) \\
& .7778 \\
& .2222 \\
& 0.0000 \\
& 0.0000 \\
& 0.0000 \\
& 0.0000 \\
& 0.0200 \\
& 0.0000 \\
& 0.0000
\end{aligned}
$$

$$
\left.\begin{array}{r}
\hline 0.0 \\
\mathrm{O} \\
\hline 0 \\
0 \\
0 \\
0
\end{array}\right)
$$

$$
\begin{gathered}
P(I(K 0), H \\
H(K 0) \\
1 \\
.1728 \\
0.0000 \\
0.0000 \\
0.0000 \\
0.0000 \\
0.0000 \\
0.0000 \\
0.0000 \\
0.0000 \\
.1728
\end{gathered}
$$

$\begin{array}{ccc}7 & 8 & 9 \\ 3.0000 & 1.5000 & 3.0000 \\ 0.0000 & 14.0000 & 14.0000\end{array}$
0 $\begin{array}{ll}0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \text { N } & 0\end{array}$

|  |  | 0 |
| :--- | :---: | :---: |
| 0 |  |  |
| 0 | 0 |  |
| 0 | 0 |  |
|  | 0 | 0 |

$\begin{array}{ccc} & 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ & - & 0 \\ & & N \\ & 0 & 0 \\ & 0 & 0 \\ 0 & 0 \\ & 0 & 0\end{array}$

| － |  | $n$ | 0 |
| :---: | :---: | :---: | :---: |
| $z$ |  | $m$ | － |
| $<$ | N | $\infty$ | $\bigcirc$ |
|  |  | ） | ก |
| $x$ |  | $\dot{m}$ | $\stackrel{\sim}{\circ}$ |
| $\checkmark$ |  |  |  |
| $\square$ |  |  |  |
| a |  | $\stackrel{\infty}{\sim}$ | $\bigcirc$ |
|  | $r$ | N | － |
|  |  | N | $\bigcirc$ |
|  |  | $\stackrel{\sim}{\sim}$ | 0 |

TABLE 10
$K$
PT（K）
LT（K）
3.2 GRAPHS OF THE EXPECTED VALUES, PROBABILITY P[w(k)], PROCESSING AND LEAD TIMES.


FIGURE 5-2. $\quad \hat{E}[i(k+1) / h(k)]$
Milton, Pa., West Branch Susquehanna


FIGURE 5-3. $\hat{E}[h h(k) \mid h(k)]$
Milton, Pa., West Branch Susquehanna


FIrURE 5-4. robability that at least one more forecast will be issued
Milton, Pa., West Branch Susquehanna


FIGURE 5-5. Processing time and average actual lead time Milton, Pa., West Branch Susquehanna

### 3.3 PARAMETERS OF MULTINOMIAL DISTRIBUTIONS

EVALUATION OP THE PLOOD PORECAST-RESPONSE SYSTEM parabeters of the moltinomial disteibutions

RIVER : WEST bRANCH SUSQUEHANNA
gORECAST PJINT : MILTON, PENNSTLVANIA

|  | parameter | $E$ | $N$ | $p$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | QH 1 | 2.400 | 9 | . 2458 |
| 2 | QH2 | 2.500 | 8 | . 2932 |
| 3 | QH3 | 2.400 | 7 | . 3200 |
| 4 | Q+4 | 2.300 | 6 | . 3561 |
| 5 | QH5 | 2.100 | 5 | . 3825 |
| 6 | QH6 | 1.800 | 4 | . 3867 |
| 7 | QH7 | 1.600 | 3 | . 4399 |
| 8 | QH8 | 1.300 | 2 | . 4638 |
| 9 | QH9 | 1.000 | 1 | 1.0000 |
| 10 | QI 1 | 1.000 | 9 | . 01.46 |
| 11 | QI2 | 1.400 | 9 | . 0885 |
| 12 | QI3 | 1.900 | 9 | . 1732 |
| 13 | QI4 | 2.600 | 9 | . 2725 |
| 14 | QI5 | 3.400 | 9 | . 3721 |
| 15 | QI6 | 4.200 | 9 | . 4650 |
| 16 | QIT | 5.000 | 9 | . 5552 |
| 17 | QI8 | 5.800 | 9 | . 6444 |
| 18 | QI9 | 6.600 | 9 | . 7333 |
| 19 | QHE1 | 1.000 | 9 | . 0146 |
| 20 | QHH2 | 2.000 | 9 | . 1887 |
| 21 | QHH3 | 3.000 | 9 | . 3235 |
| 22 | QHE4 | 4.000 | 9 | . 4421 |
| 23 | QHH5 | 5.000 | 9 | . 5552 |
| 24 | QHH6 | 6.000 | 9 | . 6666 |
| 25 | $24 \mathrm{H7}$ | 7.000 | 9 | . 7778 |
| 26 | QHH8 | 8.000 | 9 | . 8889 |
| 27 | Q4\% 9 | 9.000 | 9 | 1.0000 |
| 28 | QH | 3.778 | 9 | .4165 |
| 29 | QIO | 1.222 | 18 | . 0271 |
| 30 | QHO | 2. 542 | 18 | . 1298 |

3.4 PRINTOUT OF THE LAW OF MOTION AND INITIAL CONDITION

 LAW OF MOYION AND INITIAL CONOITION
COMPUTEC FKOM MULTINCMIAL DISTRIBUTIONS
$=$ FORECASTED FLOCD CREST
$=$ ACTUAL FLOGDCREST
=CURRENFLCOC LEVEL
= OECISOM TIME
$=$ INITAL IIME.
$=$ PROBABILITY



$$
\begin{aligned}
& \text { P(H(K+1)/I(K+1)) }
\end{aligned}
$$

$$
\begin{aligned}
& \text { table 2a: }
\end{aligned}
$$

$P(1(K+1) / H(K))$
$\sigma$
$\infty$


TABLE 4A:

| I(K+1) |  | H(k+1) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|  | 1 | . 2263 | . 3469 | . 2919 | . 1349 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 2 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 3 | 0.0000 | c. 0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 4 | 0.0000 | c.0c00 | 0.0000 | 0.0000 | c. 0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 5 | 0.0000 | c.0c00 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | $0.0000^{\circ}$ | 0.0000 |
|  | 6 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | c. 0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 7 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | c. 0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 8 | 0.0000 | 0.0000 | c. 0000 | 0.coco | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 9 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | c. 0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 |

$I(K)=1 \quad H(K)=2$


| $1(k+1)$ |  | $H(K+1)$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|  | 1 | . 08887 | .1360 | . 1144 | . 0529 | C. 0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 2 | 0.6000 | . 0831 | .1384 | . 1261 | . 0628 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 3 | 0.0000 | 0.0000 | . 0430 | . 0699 | . 0590 | . 0258 | 0.0000 | 0.0000 | 0.0000 |
|  | 4 | 0.0000 | 0.0000 | 0.0000 | 0.cocc | C. 0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 5 | 0.00000 | c.00c0 | 0.0000 | 0.0000 | c. 0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | . 6 | 0.0000 | 0.0000 | 0.0000 | 0.1000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 7 | 0.0000 | c. 0000 | 0.0000 | 0.0000 | c. 0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 8 | 0.0000 | 0.0000 | 0.0000 | 0.000C | C.000C | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 9 | 0.0000 | c. 0000 | c. 0000 | 0.0000 | C. 0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

TABLE $4 A: \quad P(I(K+1), H(K+1) / I(K), H(K)) \quad I(K)=1 \quad H(K)=4$

$I(K)=1 \quad H(K)=5$






- 6 $\begin{array}{lllllllll}0 & 0 & 0 & 0 & 0 & 0 & N & N & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & N & N & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{N} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0\end{array}$ 0.0000 0858
$\begin{array}{ll}\infty & 0 \\ 0 \\ 0 & 0 \\ 1 & 0\end{array}$





 0.000
TABLE 4A: $P(I(K+1), H(K+1) / I(K), H(K))$ 1


$I(K)=2 H(K)=3$

8

$0000^{\circ} 0 \quad 0000^{\circ} 0$
0.00000 .0000
$\stackrel{\sim}{n}$

(2)

TABLE 4A: P(I(K+1),H(K+1)/I(K),H(K)) I(K)=2 H(K)=5 $\begin{array}{lllllllll}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dot{0} & \dot{1} & 0\end{array}$
 0.00000 .00000 .0000
TABLE $4 A: P(I(K+1), H(K+I) / I(K), H(K)) \quad I(K)=2 \quad H(K)=6$
TABLE $4 A: P(I(K+1)$, H(K+II/I(K),H(K)) $I(K)=2 \quad H(K)=7$

$$
6
$$

$$
0.00000 .0000
$$

$$
0.00000 .0000
$$

$$
0.0000 \quad 0.0000
$$

$$
0.0000^{\circ}
$$

$$
0.0000
$$ .0480 .0097 0.00000 .0000 $0.0000 \quad 0.0000$ …….....................




$I(K)=3 \quad H(K)=4$

$$
o
$$ $0000 \cdot 0$ $0000 \cdot 0$ 0.0000 0.0000 $0000^{\circ} 00000^{\circ} 00000^{\circ} 00000^{\circ} 0$

 $0000^{\circ} 0$
 $00000^{\circ}$ $\circ$
8
0
0
0
 $0000^{\circ} 00000^{\circ} 00000^{\circ} 0$ $0000^{\circ} 00000^{\circ} 00000^{\circ} 0$
 - 000



TABLE \&A: $P(I(K+1), H(K+1) / I(K), H(K)) \quad I(K)=3 \quad H(K)=7$




$$
0000^{\circ} 00000^{\circ}
$$

$$
0000 \cdot 0
$$

$I(K)=4 \quad H(K)=4$
"TABLE 4A: P(I(K+1),H(K+1)/I(K),H(K))

$$
6
$$

0.00000 .0000

$L$
$0000^{\circ} 00000^{\circ} 0$
0.00000 .0000
0.00000 .0000
.2663 .1063
0.00000 .0000
응
9ZLE.
응
c. 00000
$c .0000$
3
$0000 \cdot$
(K)
TABLE 4A: P(I(K+1),H(K+1)/I(K),H(K)) $\quad I(K)=4 \quad H(K)=5$

$I(K)=4 \quad H(K)=6$
TABLE $4 A: \quad P(I(K+1), K(K+1) / I(K), H(K))$

$\cdots .0000$
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
8
.0000
0000
$0000 \cdot 0$
20.0000
.0909 .0237
.0324
.0000
0.0000
$\cdots$
0.0000




TABLE 4A: $P(I(K+1), H(K+1) / I(K), H(K)) \quad I(K)=5 \quad H(K)=5$

$$
6
$$




5
0.0000 c. 0000 C. 0000
0.0000 $0.0000 \quad 0.0000 \quad 0.0000$ 0.0000 $\square-2$ 3

$$
\begin{array}{r}
.2098 \\
0.0000
\end{array}
$$

$$
\begin{aligned}
& c .0000 \\
& c .0000
\end{aligned}
$$

$$
\begin{array}{r}
\text { C. } 0000 \\
0.0000
\end{array}
$$

$$
.1086 .1141
$$

$$
0.00000 .0000
$$

$$
0.0000
$$

$$
0000^{\circ} \mathrm{C}
$$ $0000^{\circ} 0$

6...0.0.000. C.0C00 0.0000 0.000C .0000 4-- -0.0000 50.0000


$$
.1809
$$

$$
0.0000 \quad 0.0000
$$

$$
0.0000
$$

$$
0.0000 \quad 0.0000
$$

$$
.04720 .0000
$$

$$
.0418 \quad 0.0000
$$

$$
0.0000 \quad 0.0000
$$

$$
0.0000 \quad 0.0000 \quad 0.0000 \quad 0.0000
$$ 0

0
0
0
0
$i$
0
0
0
0
0
0
0
0
0
0
0 $0000 \cdot 0 \quad 0000^{\circ} 0 \quad 0000^{\circ} 0$

## 

 $0000 \cdot 0 \quad 0000 \cdot 0 \quad 0000 \cdot 3$ $-\cdots--\quad-\quad-\quad-\quad-$ 


$I(K)=5 \quad H(K)=7$
$P(I(k+1), H(k+1) / I(k), H(k))$

|  |  |
| :---: | :---: |
| 0. | 9 |
| 0.0000 | 0.0000 |
| 0.0000 | 0.0000 |
| 0.0000 | 0.0000 |
| 0.0000 | 0.0000 |
| .0291 | 0.0000 |
| .0572 | 0.0000 |
| .0743 | .0150 |
| 0.0000 | 0.0000 |
| 0.0000 | 0.0000 |



1.
TABLE $4 A: \quad P(I(K+1), H(K+1) / I(K), H(K)) \quad I(K)=5 \quad H(K)=8$


$$
\begin{array}{r}
10 \\
\sigma \\
\sigma \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
i \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{array}
$$

$$
\begin{array}{ll}
0 & 0 \\
0.0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{array}
$$

$$
0000^{\circ} 00000 \cdot 0
$$

$$
0000 \cdot 0 \quad 0000 \cdot 0
$$

$$
\begin{array}{ll}
0000^{\circ} 0 & 00000^{\circ} \\
0000^{\circ} 0 & 6 \angle 55^{\circ}
\end{array}
$$

$$
\begin{array}{ll}
0000^{\circ} 0 & 6 L 5 \mathrm{I} \cdot \\
00000 & 0000 \cdot 0
\end{array}
$$

TABLE 4A: P(I(K+I),H(K+1)/I(K),H(K)): I(K)=6 H(K)=6

$$
\begin{gathered}
0000 \cdot 0 \\
0000 \cdot 0 \\
0000 \cdot 0 \\
0000 \cdot 0 \\
L
\end{gathered}
$$

$$
\begin{aligned}
& 9 \\
& =(x) H
\end{aligned}
$$

## .4106 .4314

 0.00000 .0000 0.00000 .0000 $0000^{\circ} 0$
$I(K)=6 \quad H(K)=8$
$P(I(K+1), H(K+1) / I(K), H(K))$
"TABLE 4A:

table 4a: P(I(K+1),H(K+1)/f(X),H(K)) I(K)=7 H(K)=7






TABLE 5A: $P(H H(K) / I(K), H(K)) \quad I(K)=1$

 | $\circ$ |
| :--- |
| 8 |
| 0 |
| $\vdots$ | 0.0000 0.0000

$0.0000 \quad 0.0000$ 0.0000
.0921


$1!$
$1:$
$I(K)=2$
TABLE SA: $P(H H(K) / I(K), H(K))$



TABLE 5A: $P(H H(K) / I(K), H(K)) \quad$ I(K) $=5$

1
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0 $0000^{\circ} \mathrm{O} \cdot 0000^{\circ} 0$
$0000^{\circ} 00000^{\circ} 0$ $0.0000 \quad 0.0000 \quad 0.0000$ $0.0000 \ldots 0.00000 .0000$ $.3623 \quad .1381$ .2663
1.0000 0.00001 .0000

$$
\begin{aligned}
& 0000^{\circ} 0 \\
& \varepsilon 992^{\circ} \\
& 9667^{\circ} \\
& 0000^{\circ} 0 \\
& 0000^{\circ} 0 \\
& 0000^{\circ} 0 \\
& 0000^{\circ} 0 \\
& 0000^{\circ} 0 \\
& 0000^{\circ} 0
\end{aligned}
$$

$$
L
$$



$$
\begin{aligned}
& \text { TABLE 5A: } P(H H(K) / I(K) \neq H(K)) \quad I(K)=7
\end{aligned}
$$



8 $0000 \cdot 0$

$$
0000^{\circ} 0
$$

TABLE 5A: P(Hh(K)/I(K),H(K))
$0000 \cdot 0$
$0000 \cdot 0$
$0000 \cdot 0$
$0000 \cdot 0$
6 .0000

$$
<
$$

9

$\zeta$

$$
\begin{array}{ll}
\circ & 0 \\
\hline & 0 \\
0 & 0 \\
\dot{0} & \dot{0} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{array}
$$




2
$H H(K)$
0.0000
0.0000
$0.00 C C$ 0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0

$$
H(K)^{1}
$$

 $0000 \cdot 0 \cdot 0000 \cdot 0$ $0000 \cdot 0 \quad 0000 \cdot 0$ 0.0000 $0000^{\circ} 1$ $0000^{\circ} 0$
$0000 \cdot 0$
0000.0
$0000^{\circ} 0$

$I(K)=9$

TABLE GA: $P(I(K O), H(K D))$

### 3.5. VERIFICATION OF THE LAW OF MOTION

# VERIFICATION OF THE LAW OF MOTION FOR MILTON 

Verification of the law of motion, $\Phi$, for Milton, Pa., has been done in a manner described in Chapter 4, Section 1.4. The results are summarized in Table 5-2. In view of the $D_{n}$ statistic, it is clear that $\Phi$ cannot be rejected as an appropriate exceedence model at the significance level $\alpha=.10$. Figure $5-6$ shows plots of historical and computed exceedence probabilities. Admittedly, these results prove the validity of the conceptual approach to modeling $\Phi$. . It should be kept in mind that this almost remarkable performance of the model has been achieved with a very small amount of data which served as a guide, rather than as real numbers to which multivariate distributions could be fitted.

$\begin{array}{ll}\text { Probability of Flood P[F] }=.353 \\ x^{2} \text { Statistic } & D_{9}=12.3 \\ \text { Threshold Value } & \tau .10,8=13.4\end{array}$

4. INPUT FOR THE RESPONSE SYSTEM

### 4.1 Discretization of the flood plain

The Milton flood plain has been discretized into nine steps. Figure 5-7 shows location of the steps in relation to historical floods and characteristic return periods.
4.2 Distribution $n^{* /}$

Distribution $\{\eta(m r)\}$ which partitions the maximum possible damage $\operatorname{MD}($ REACH $)$ among steps $\{\mathrm{m}: \mathrm{m}=1, \ldots, 9\}$ and structural categories $\{r: r=1, \ldots, 7\}$
is shown in Table 5-3 and in Figure 5-8. As one can notice, from 34 non-zero elements many are very small numbers. To save on the computation time, 34 non-zero elements in Table 5-3 have been clustered subjectively to give nine non-zero elements in Table 5-1, Section 1. The arrows in Table 5-3 show how the clustering procedure was accomplished.

[^2]840
48599.58
$E(M, R)$

OMN
$M D R$

NUMBER OF DECISION MAKERS
DMil $=840$

㝕
ELEV
$\mathrm{Y}(\mathrm{m})$



 | 1 ionials |  |
| :--- | :--- |
|  |  |

A. DISTRIBUTION OF MDR ACROSS THE STEPS.

### 4.3 Human Response Applied to the Milton Case Study

The mathematical model of human response to flood warnings has had to be modified, in some respects, for application. The specific adaptations are:

1. The effect of the subjective probability of a flood $p(F \mid t)$ has been ignnred, i.e., no account has been taken of whether the joint probability of a flood and a loss $p(F \mid t) p(L \mid F, t)$ is sufficiently high to induce preparations for a flood which would improve response. This aspect is presumed to be represented within the response function $\alpha(\tau)$.
2. The effects of previous floods on the threshold $T_{1}$ and likelihood ratio exponent $c$ have been ignored.
3. The likelihood ratio for a warning sequence has been assumed to depend only on the last warning of the sequence and to be a function only of the actual river level $i$ and the forecast crest level $h$. This has been necessary because of the limited amount of data on forecasts.

The likelihood ratio is defined as

$$
L_{0}(i, h \mid m, k)= \begin{cases}\infty & i \geq m \\ \frac{p\left(i, h \mid h^{\prime} \geq m, k\right)}{p\left(i, h \mid h^{2}<m, k\right)} & i<m\end{cases}
$$

where $h^{\prime}$ is the actual crest and $h^{\prime} \geq m$ is equivalent to $L$, a loss, $k$ is the forecast number, $i$ is the current level and $h$ is the forecast crest. It is assumed that $h \geq i$ and that there is no dependence on $k$ other than through the values of $h$ and $i$. Then

$$
L_{0}(i, h \mid m)=\frac{p\left(i, h \mid h^{\prime}>m\right)}{p\left(i, h \mid h^{\prime}<m\right)}
$$

It is further assumed that $i$ and $h$ are independent.

$$
\begin{aligned}
& L_{0}(i, h \mid m)=\frac{p\left(i \mid h^{\prime} \geq m\right) p\left(h \mid h^{\prime} \geq m\right)}{p\left(i \mid h^{\prime}<m\right) p\left(h \mid h^{\prime}<m\right)} \\
&= \frac{p\left(h^{\prime} \geq m \mid i\right) p(i)}{p\left(h^{\prime} \geq m\right)} \\
& \frac{\left[1-p\left(h^{\prime} \geq m \mid i\right)\right] p(i)}{1-p\left(h^{\prime} \geq m\right)} \cdot \frac{p\left(h^{\prime} \geq m \mid h\right) p(h)}{p\left(h^{\prime} \geq m\right)} \\
&= \frac{p\left(h^{\prime} \geq m \mid i\right) p\left(h^{\prime} \geq m \mid h\right)\left[1-p\left(h^{\prime} \geq m \mid h\right)\right] p(h)}{1-p\left(h^{\prime} \geq m\right)} \\
& {\left[1-p\left(h^{\prime} \geq m \mid i\right)\right]\left[1-p\left(h^{\prime} \geq m \mid h\right)\right]\left[p\left(h^{\prime} \geq m\right)\right]^{2} }
\end{aligned}
$$

where:

$$
\begin{aligned}
& p\left(h^{\prime} \geq m\right)=\sum_{h^{\prime} \geq m}^{\Sigma} p\left(h^{\prime}\right) \\
& p\left(h^{\prime} \geq m \mid i\right)=\sum_{h^{\prime} \geq m}^{\Sigma} p\left(h^{\prime} \mid i\right) \\
& p\left(h^{\prime} \geq m \mid h\right)=\sum_{h^{\prime} \geq m}^{\sum} p\left(h^{\prime} \mid h\right)
\end{aligned}
$$

These values were estimated from the data for Milton for values of $i, m, h$ and $h^{\prime} 1$ through 9 using the same definitions of level as in the optimal decision program.
4. The prior probability of loss given a flood was derived by assuming that the DM has been on the Milton flood plain for the past twenty
years, that he began with $p_{0}(L \mid F, t)=0$ and that equations (4.4) and (4.5) have been followed since with the arbitrarily chosen but plausible values of

$$
\begin{aligned}
\tau_{L} & =10 \text { years } \\
\delta & =.7 \text { for residential and cormercial } D M^{\prime} s \\
\delta & =1.0 \text { for industrial } D M^{\prime} s
\end{aligned}
$$

It is assumed that the industrial DMs use the best estimate of the frequency.
5. The threshold for response was chosen to be relatively high in view of widespread reports of unwillingness to respond to warnings.
$T_{1}=.5$ for residential $D M^{\prime} s$
$T_{1}=.4$ for commercial $D M ' s^{\prime}$
$T_{1}=.3$ for industrial DM's
The lower values for more economically oriented DM's is plausible in view of their responsibilities and the size of their operations.
6. Thresholds for action above $T_{1}$, i.e., $T_{2}, T_{3} \ldots T_{n}$, were assumed not to exist, since it was not clear how to estimate them.
7. The exponent $c$ to which the likelihood ratios are raised before revision of the prior is assumed to be

$$
\begin{aligned}
& c=.6 \text { for residential } D M^{\prime} s \\
& c=.7 \text { for commercial } D M^{\prime} s \\
& c=.8 \text { for industrial } D M^{\prime} s
\end{aligned}
$$

The value of .6 is typical in the experimental literature (e.g., Phillips and Edwards, 1966) and again it was assumed that the more economically motivated DM's would interpret the warnings more objectively.
8. The response functions $\alpha(\tau)$ were derived from the decision constraint functions used in the optimal decision program.
a) Residential $D M^{\prime} \mathrm{s}$

The time required for maximum protection was assumed to be 36 hours and the same shape of curve as that of the decision constraint function was assumed, giving

$$
a(\tau)=.0029 \quad 1.63 \quad 0<t \leq 36
$$

b) Commercial DM's

The same procedure as in (a) was used giving

$$
\alpha(\tau)=.0202 t^{1.09} \quad 0<t \leq 36
$$

c) Industrial DM's

The actual decision constraint function used in the optimal decision program was fitted by

$$
\alpha(\tau)=.13 t^{.5} \quad 0<t \leq 60
$$

The values given above enable a calculation of the response $d$ in terms of an increment in a for each type of $D M$ given values of $i, h, m$, and $\alpha$. The prior odds ratio $p(L \mid F) / p(\bar{L} \mid F)$ is determined from the learning equations (4) and (5) of Chapter 3, in connection with the flood history of Milton for the past 20 years. Then for any forecast $(i, h)$ the likelihood ratio $L_{0}(i, h)$ is raised to the power $c$ and if that value is greater than threshold $T_{1}$, the DM acts, and the amount of his action $d$ is the increase in $\alpha$, from the current value, that can be accomplished in 6 hours (the assumed time between forecasts).

The results are discussed in Chapter 1. However, the sensitivity of the model to some of the parameter choices can be seen in Figure 5-9 which shows the threshold forecasts needed for the residential DM's at levels 4 , 5 and 6 to start protecting their property as influenced by different values of time on the flood plain, threshold probability of a loss, and learning constant. (The standard case is the one described above). Lower thresholds,
shorter times on the flood plain and higher learning constants all make response start with less extreme forecasts. The inclusion of a second threshold gives a second forecast boundary needed to be reached before completion of protection is possible. For the most part, actual forecasts too closely follow the rise of the river to reach the threshold in sufficient time to allow protective action according to this model--even with the most favorable parameter values.

$\delta=.6$

$\delta=.8$


15 yrs. residence


$$
T_{1}=.5, T_{2}=.95
$$



Figure 5-9. Sensitivity of the warning threshold to variations in model parameters

## Chapter 6

## Case Study - Victoria, Texas

1. Reconnaissance Survey of Floodplain Residents, Victoria, Texas

During the week of May 27 th to June 4 th, twenty-six residents living in the one hundred year floodplain in Victoria, Texas were interviewed. Sixteen persons were interviewed in the Green Addition neighborhood along the Guadalupe River and ten in the Tanglewood subdivision in the Lone Tree Creek floodplain.

The two neighborhoods are of very different ages, racial mixtures and socioeconomic levels and were chosen for study because they represent the opposite ends of the socioeconomic spectrum of the floodplain dwellers.

The Green Addition is an older low income neighborhood with average house values of about $\$ 13,000$. All of the dwellings are single family frame units without basements. Many were seen to be set on blocks which raised their floors some twelve to eighteen inches above ground surface level. A small proportion of the people live in mobile homes. Racially, the subdivision contains a mixture of Mexican-Americans and Anglos. The education level of the residents is low. Only a few of the people interviewed had graduated from grade twelve and several had never gone to school at all.

A high proportion of the residents in the Green Addition are elderly, retired, or semi-retired people who have lived in the neighborhood for thirty or forty years. The subdivision does not have very many young married couples with children.

The Green Addition has flooded regularly in the past and experienced a flood in late April of this year. This flood was still very much on the minds of the people interviewed.

The Tanglewood subdivision in the Lone Tree Creek floodplain is a newer middle to high income residential development. At the present time, residential construction is proceeding very rapidly in the Lone Tree Creek floodplain and development plans call for several thousand new homes to be built in the next year or so.

The Tanglewood subdivision is the wealthiest of the neighborhoods along Lone Tree Creek. Houses are large--all have been built within approximately the last thirteen years and are valued at $\$ 50,000$ and upwards.

All of the people interviewed were well educated. Each respondent had as a minimum a grade twelve education and over one-third had university degrees. Most are employed in managerial or professional capacities. Most of the residents have lived in Tanglewood for ten or eleven years.

The neighborhood has been flooded in the past several years and although the streets have been inundated to a depth of six inches to twelve inches, none of the residents reported water entering their homes.

## 2. Survey Aims

The principal objective of this reconnaissance survey was to field test the questionnaire developed to determine the resident's attitudes, perceptions, and responses to the flood hazard. In addition, it was hoped that the survey might provide some very general insights into the resident's awareness and response patterns.

## 3. Summary of Results

It should be emphasized that only twenty-six persons were interviewed. This is certainly too small a number to obtain a representative sample of the floodplain dweller's opinions. Also, the respondents were selected from two very particular neighborhoods. It might very well be that the residents of
other neighborhoods would respond differently. These results should, therefore, be treated with caution and regarded as preliminary findings which will require a full-scale survey for verification.

All of the residents interviewed in Tanglewood believe that their neighborhood is not in danger of being flooded. Only one person reported having experienced a flood in the subdivision and she regarded it as a very minor event.

None of the residents have ever suffered a flood loss and. none have made any preparation for protecting their property. Also, none of the respondents have purchased flood insurance. When asked "Why not?" the standard reply was that they did not need it.

When asked what they would do in the event of a flood, most said they would get out and only two mentioned moving things to a higher location in the house.

Most of the residents were aware that Victoria had a flood warning system and over half mentioned having heard warnings issued over the radio. The general feeling, however, was that the warnings were for the people along the Guadalupe, and although of interest in an academic way, did not apply to their neighborhood.

In summary, the people interviewed in Tanglewood are not aware of the flood problem or do not admit to it and, consequently, warnings are ignored as the residents do not believe they apply to them.

As mentioned earlier, the residents of the Green Addition experienced a flood on April 28th, 1977 and a number of people were evacuated.

The residents are aware that their neighborhood is flood-prone -- fifteen out of sixteen stated that the neighborhood has a flood problem with one person answering maybe. However, when asked if their own house was in danger of
flooding, only half believed that their property would be affected by a flood. Those who did not expect to be flooded usually explained it in terms of their house being higher than the others around, being on blocks or that the floodwaters did not flow by their property. Many of the residents mentioned flooding as a disadvantage to living in the Green Addition.

Most of the respondents remembered other floods, particularly the 1936 flood and those caused by hurricanes Carla and Beulah. Only a few mentioned that they had suffered damage from flooding and only one person had obtained a small amount of compensation from the Red Cross for linoleum and furniture damage.

None of the respondents, in talking about floods, displayed any sign of fear. In fact, two people made the specific point that they were not afraid of floods. Floods do not appear to be regarded as particularly frightening by the people interviewed.

None of the interviewees expect there to be any loss of life from future floods. Most expect to merely be inconvenienced, while about one-third expect some property damage. Ten of the sixteen stated that they were aware of the flood problem before they moved to the neighborhood. However, they moved because housing was cheap, housing was available, or they had family or friends living in the Green Addition.

Few of the residents have made any preparations for a flood. One respondent was in the process of putting her house on eighteen inch blocks after the April flood while several mentioned having raised their houses following earlier floods. One family owns a canoe in case they have to evacuate. Only three of the sixteen respondents have purchased flood insurance. The most common reasons given for not purchasing it were either that they could not afford it, did not need it, it was too expensive, or that they did not know it was available.

The residents were favorable to the general idea of flood warnings. Thirteen thought them a good idea while eleven were aware that Victoria had a flood warning system and four did not.

In the event of a future flood, twelve of the sixteen stated that they would stay in their houses while four would evacuate in response to warnings. The general attitude of those who plan to remain in their homes is one of "wait and see." Two people stated that they would "stay at home as usual." Most said they would only get out when the floodwaters entered their homes and they could no longer remain there. Several mentioned that during the April 1977 flood they had driven their automobiles out to higher ground and then waded back through the rising floodwaters to remain in their houses.

Two of the respondents stated that they would pick up things in the yard in response to a warning while three said they would move a few things to higher locations in the house.

Several people mentioned that the April 1977 flood warning had been wrong and that the floodwaters arrived before they were expected. However, eleven people said that they believed the warnings as issued by the weather office, while two did not and three did not know. One man remarked that he believed the warnings now but "did not use to."

None of the residents knew how high they were above river flood stage. However, a few were aware of what the critical flood stage was for their neighborhood.
4. Summary

Based on a very limited sample of twenty-six persons living in the Tanglewood and Green Addition neighborhoods, the attitudes and responses of the residents to the flood hazard and flood warnings appear quite different.

The residents of Tanglewood are not aware of the flood problem, have made no preparations for a flood, and do not believe that the flood warnings as issued by the Weather Service apply to them, but rather are for the people living in the bottomlands of the Guadalupe. These people, therefore, probably will not respond to future flood warnings and in the event of a major flood, will be caught unawares.

The residents of the Green Addition are aware of the flood problem, although half do not believe their property will be affected and most knew of the hazard prior to moving to the neighborhood. The residents do not fear flood and expect merely to be inconvenienced or to suffer minor property damage in future floods. The majority have made no preparations in the event of a flood. Most are aware of the flood warnings as issued by the Weather Service and the majority believe them. However, their reaction to a flood warning would be to "wait and see what happens" and to remain in their homes. A few would pick up items in their yards and move some things upstairs.

In the event of a major flood, the residents cannot be expected to evacuate prior to the flood or to take any action which might substantially reduce their losses. Instead, most will have to be rescued from their homes. The behavior of the residents, therefore, may very well increase flood damage costs rather than reduce them.

## 5. Evaluation

Flood forecasts are provided for the Guadalupe River by the Fort Worth River Forecast Center. Flooding by the Guadalupe causes damage to the Green Addition. The other flood prone area is along the Lone Tree Creek and is subject to flash floods. Since the methodology developed in this report is not designed for the flash flood situation the evaluation of the flood fore-cast-response system for Victoria is essentially an evaluation of the Green Addition in Victoria.

The Green Addition is purely residential consisting of one and two story dwellings. The unit functions previously developed were used. An inventory of these structures was obtained from Day and Lee (1977). Most of the structures are over 11 feet above flood stage, above the 10 year flood plain. In recent years there have been about 3 flood events per year above flood stage.

Forecast data obtained from the Fort Worth RFC was used to determine the Law of Motion. A comparison of this data and the derived Law of Motion is given in Table 6-1. A chi-square test at the $1 \%$ level shows no significant difference. A graphical comparison is given in Figure 6-1.

Graphs of the processing time and lead time as a function of forecast time are shown in Figure 6-2. They may be compared with Figure 5-5 for Milton, Pennsylvania. Dissemination time is one hour. The response on the part of the floodplain dweller was assumed to be the pure strategy, the maximum possible response when the forecast indicates flooding and no action otherwise.

The results of the evaluation are given in Table 6-2 for a single residence and for all of the Green Addition.

| Step hh | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Expectation E[hh] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| From Historical Record |  |  |  |  |  |  |  |  |  |  |
| Probability P[hh] |  |  | . 2813 | . 1250 | . 2188 | . 1875 | . 1875 |  |  | 4.8750 |
| Exceedence fihh] | 1.0000 | 1.0000 | 1.0000 | . 7188 | . 5938 | . 3750 | . 1875 |  |  |  |
| $f[h h] \cdot P[F]$ | . 9167 | . 9167 | . 9167 | . 6589 | . 5443 | . 3438 | . 1719 |  |  |  |
| From the Law of Motion |  |  |  |  |  |  |  |  |  |  |
| Probability P[hh] | . 0037 | . 0277 | . 0883 | . 1714 | . 2271 | . 2233 | . 1628 | . 0728 | . 0229 | 5.413 |
| Exceedence $\mathrm{f}[\mathrm{hh}]$ | 1.0000 | . 9963 | . 9686 | . 8803 | . 7089 | . 4818 | . 2585 | . 0957 | . 0229 |  |
| $f[h h] \cdot P[F]$ | . 9167 | . 9133 | . 8879 | . 8070 | . 6498 | . 4417 | . 2370 | . 0877 | . 0210 |  |
| Probability of Flood P[F] $=.9167$ |  |  |  |  |  |  |  |  |  |  |
| Chi-square statistic $D_{32}=17.28$ |  |  |  |  |  |  |  |  |  |  |






FIGURE 6-2. Processing Time and Average Actual Lead Time Victoria, Texas, Guadalupe River

| Structure(s) | Residence | all of Victoria (Green Addition) |
| :---: | :---: | :---: |
| Elevation above flood stage, ft. | 11 |  |
| Maximum possible damage, \$ | 35,000 | 1,396,860 |
| Expected ánnual loss, \$ |  |  |
| perfect forecast and response | 1,346 | 50,085 |
| no response | 1,722 | 63,853 |
| optimal strategy | 1,540 | 57,201 |
| actual strategy pure | 1,697 | 62,918 |
| Performance, \$ |  |  |
| potential value | 377 | 13,768 |
| optimal value | 183 | 6,653 |
| actual, value pure | 25 | 935 |
| Efficiency |  |  |
| forecasting system | . 485 | . 483 |
| response system pure | . 139 | . 141 |
| overall |  |  |
| pure | . 068 | . 068 |

Table 6-2. Evaluation of Green Addition Victoria, Texas

## 6. Discussion

The evaluation of Victoria, Texas further demonstrates the applicability of the methodology presented in this report. The Law of Motion corresponds closely to the historical record. Given the inventory data for the structures in the flood plain and the historical forecast record, the evaluation of Victoria was "routine".

The Green Addition (Victoria, Texas) flood forecast-response system and the Milton, Pennsylvania flood forecast-response system differ in many ways. Most of the Victoria residences are located within a five foot elevation band which encompasses the 10-500 year return period floods. Milton has a much greater difference in elevation between structures and between the 10 and 500 year floods. There are only residences in the Green Addition whereas in Milton commercial and industrial structures as well as residences may be flooded.

Based on the reconnaissance survey of Victoria it may be assumed that the residents of the Green Addition do not make anticapatory responses to flood warnings. As sufficent data was not available to better define their response strategy the pure strategy was used in the evaluation as it is nonanticipatory.

The efficiency of the flood forecasting-response system for the Green Addition was higher than for Milton (Table 1-3). Of special note is the positive efficiency for the response system using the pure response in the Green Addition compared to a negative efficiency using the pure response in Milton. While many factors could explain these differences one factor stands out: the Victoria forecasts have a longer lead time which allows the flood plain dweller more time to complete his response.

The performance, in dollars, of the flood forecasting-response system for the Green Addition in Victoria, Texas is much greater, as a percentage of maximum possible damage, tha: n for Milton. This largely results from the higher efficiency of the system and the greater frequency of flooding.
7. Reference

Day J. and Lee K., Correspondence, University of Wisconsin, 1977.

QUESTIONNAIRE - FLOOD HAZARD STUDY ALONG GUADALUPE RIVER, VICTORIA, TEXAS

Hello, I'm from the University of Arizona. We are conducting a survey to find out how people feel about flooding in Victoria. I would be grateful if you would be kind enough to answer some questions.

1. How long have you been living in this neighborhood?
$\qquad$ years.
2. What are some of the things you like about living in this neighborhood?
3. Are there any disadvantages to living in this neighborhood?
4. There has been scme discussion recently about the threat of floods in Victoria. Do you think that your neighborhood is in danger of being flooded?

Yes $\qquad$
No
Maybe
Don't know $\qquad$
If yes, "did you know there was a flood problem before you moved here?"
Yes $\qquad$
No
Other $\qquad$
If yes, "why did you decide to move here?"
5. Have you ever experienced a llood?

Yes $\qquad$
No $\qquad$
(a) If yes, where?
(If no, go to 13.)
6. (b) If yes, when? (List years)
(1) $\qquad$
(2) $\qquad$
(3) $\qquad$
(c) If several floods - which flood was the worst?

Let me ask you a few questions about the last flood you mentioned.
7. How did you first find out about the last flood? (Probe)
8. Did you talk to your neighbors or friends about the possibility of a flood when you first learned of it?
9. What did you do when the last flood happened?

Note: If evacuation is mentioned - what finally persuaded you to leave?
10. Did you suffer any losses during the last flood?

Yes $\qquad$
No $\qquad$
Other $\qquad$
If yes, what were your losses?
21. Did you get any compensation for your losses?

Yes
No
$\qquad$
$\qquad$
If yes
(a) How much (approximately)?
(b) From whom?
12. Why did you decide to continue to live here after the last flood? (Probe)

Let me ask you something now about the chances of another flood occuring in this neighbornood.
13. How often do you think it will flood here. Will it flood:
(a) Every year
(b) Once every two years $\qquad$
(c) Once every five years
(d) Once every ten to twenty years
(e) Once every twenty to fifty years
$(f)$ Once in 100 years $\qquad$
(g) Never $\qquad$
(h) Other $\qquad$
74. When do you expect the next flood to happen? Do you think it will happen:
(a) This year
(b) In the next several years
(c) In the next 10 to 20 years $\qquad$
(d) Between 20 and 100 years
(e) Never $\qquad$
15. Do you think your house will be affected by the next flood?

Yes
No $\qquad$
Maybe
Don't know $\qquad$
16. If yes, in what way would you expect to be affected? (Check list only-do not show or read to respondent.) (Probe how much.)
(a) Loss of Iife.
(b) Damage to home and property.
(c) Some property damage.
(d) Inconvenience.
(e) No damage.
(f) Other (specify).
17. If a flood were to occur what would you do? (Check list only-do not show or read to respondent.) (Probe)
(a) Nothing.
(b) Get out.
(c) Remove some possessions and get out.
(d) Clear out house.
(e) Move things upstairs.
(f). Protect home.
(g) Stay here.
(h) Other.

If respondent mentions staying in the house, "Why would you stay here?"

Do you have any materials on hand to protect your property?
Yes $\qquad$ No $\qquad$
18. If you knew for sure that a flood was going to occur twelve to twentyfour hours from now, what would you do?

Let me ask you a few questions about flood warnings.
19. Do you think flood warnings are a good idea?

Yes $\qquad$
No
Other $\qquad$
If no, why not?
20. Does Victoria have a flood warning system?

Yes
Maybe
Other
No Don't know _

If yes, how are flood warnings given in Victoria? (Probe)
21. What do you think is the best way to warn people of a flood?
22. Do you believe the flood warnings?


Don't know $\qquad$
Other $\qquad$
If No, why not?
23. What do you usually do when you hear a flood warning?
24. Do you know how high the river stage will have to be for you to have water in your home?
Don't know $\qquad$
No $\qquad$
Other $\qquad$
25. If yes, how high? (feet)
26. Who do you think should be responsible for doing something about the flood problem? Do you think it is up to the:
(1) individual.
(2) federal government.
(3) county.
(4) city.
(5) community.
(6) other.
27. What do you think should be done to solve the flood problem?

Let me ask you a few quick questions about flood insurance.
28. Do you have flood insurance?

Yes $\qquad$
No
Other $\qquad$
If yes, what kind of insurance?

If no, why not?
(a) How old are you? years.
(b) What kind of work do you (usually) do?
(c) How many years of schooling did you complete? __ years.
(d) What is your husband's occupation?
wife's occupation?
THANK YOU for your time and cooperation. (You have been very helpful.)
30. To be completed after interview.
(a) Race
(b) Sex $\qquad$
(c) Was respondent very cooperative
(d) Was respondent somewhat cooperative
(e) Was respondent not cooperative
$(f)$ Was respondent very interested
somewhat interested
not interested
(g) Was respondent poorly informed informed very informed $\qquad$
31. Type of property:

Apartment $\qquad$
Single Family -

Trailer $\qquad$
Townhouse $\qquad$
Ranch $\qquad$
32. Estimation of socioeconomic level.

Very poor
Low middle $\qquad$ Middle $\qquad$ High $\qquad$
Poor $\qquad$
33. Address of property $\qquad$ -
34. Are there any obvious signs of flood adjustment measures, e.g., house on blocks or piles, trailer tied down, etc.?

Note what they are:

Computer Programs Developed for the Evaluation of Flood Forecast-Response Systems

1. INTRODUCTION

The seven programs needed for system evaluation are listed in this computer package. Each program is well commented. A manual explaining their use follows this introduction. Included in the manual is a diagram (Figure 7-1) tracing the flow of information from data, through the appropriate programming sequence, to evaluation.

The actual strategy function SA for SONIA is the human factors strategy while the actual strategy function for ROSALIE is the pure strategy. These strategies are interchangeable.

Program listings are in a separate volume.


FIGURE 7-1. FLOW DIAGRAM

## 2. MANUAL FOR FLOOD FORECAST-RESPONSE SYSTEM PROGRAMS

SONIA and ROSALIE are the main programs. The inputs to these programs are the outputs from the subsídiary programs (DWELLER, FORCAST, PARAMT, DATFIT, and LAWMO). The outputs from SONIA and ROSALIE are the actual evaluation of the flood forecast-response system (for a single decision maker, in the case of SONIA and for the reach, in the case of ROSALIE).

### 2.1 PROGRAM SONIA

SONIA evaluates the flood forecast-response system for a single decision maker in the reach.

## INPUT AND FORMAT

A. IDENTIFICATION

1. RIVER, name of the river (format ( $5 \mathrm{X}, 7 \mathrm{Al0}, \mathrm{A5}$ ))
2. POINT, name of the forecast point (format (5x, 7A10, A5))
3. STAGE, flood stage (format ( $5 \mathrm{X}, 7 \mathrm{~A} 10, \mathrm{~A} 5$ ))
B. FORECASTING SYSTEM
4. CONSTANTS AND ARRAYS
a. AN, number of discrete points in decision space
b. $I N^{1}$, number of steps in the flood plain
c. $K N^{2}$, maximum number of forecasts
d. RN, number of structural categories

AN, IN, KN, RN (format (5X, 4I 5))
e. DET (KN), time interval between decision times (format (5X, 15F5 •
f. $\operatorname{PT}(K N)$, processing time (format(5X; 15F5 •1))
g. $D T(K N)$, dissemination time (format( $5 \mathrm{X}, 15 \mathrm{~F} 5 \cdot 1$ )
h. $\mathrm{LT}(\mathrm{KN})$, average actual lead time (format( $5 \mathrm{X}, 15 \mathrm{~F} 5 \cdot 1)$ )
i. $\operatorname{PW}(2, K N)$, probability of $W(k)=1$ (format(5X, 15F5 .4))
j. PF, probability of flood occurrence (format(5X, 15F5 -4))
k. $Y(I N)$, elevation of a step (feet) (format(5X, 15F5 •1))

1. ENH(IN), expected number of floods per year given crest (format(5X, 15F5 •1))
m. PHH(IN), probability of actual crest (format(5X, 15F5 .4))
n. EN(IN, iN), expected number of floods per year given initial condition (format(5X, 15F5-3))
o. PO(IN, IN), initial condition (format(5X, 15F5.4))

## 2. FUNCTIONS

a. PIH(II, HH, I, H, K), law of motion for $W(K)=1$
b. $\mathrm{PH}(\mathrm{HH}, \mathrm{I}, \mathrm{H}, \mathrm{K})$, law of motion for $W(K)=0$

Notation:

$$
\begin{aligned}
& H H=\text { actual flood crest } \\
& I=\text { current flood stage } \\
& H=\text { forecasted crest } \\
& K=\text { decision time } \\
& I I=I(K+1)
\end{aligned}
$$

C. RESPONSE SYSTEM

Functions:
a. $D D(T, R)$, decision constraint function

NOTE: 1. Maximum $I N=$ 15. Arrays of size $I N(Y(I N), E N H(I N), \operatorname{PHH}(I N))$, must contain IN values.
2. Maximum $K N=15$ i.e. the program can handle up to 15 sequential forecasts. Arrays of size KN, (DET(KN), PT(KN), DT(KN), $\operatorname{LT}(\mathrm{KN})$, $\mathrm{PW}(2, \mathrm{KN})$ ), must contain KN values.
b. $\operatorname{DDIN}(D, R)$, inverse of $D D$
c. DELTA $(Z, R)$, unit damage function
d. $\operatorname{GAMA}(D, R)$, unit cost function
e. $M R(Z, R)$, unit reduction function
f. $S A(A I, I, H, K, M, R)$, actual strategy

Notation:
$T=$ time
$R=$ structural category
$D=$ point in decision space
$Z=$ depth of flooding from first floor
$A I=$ point in decision space
$M=$ location step
D. DECISION MAKER
a. IDENT, Identification (format(5X, 7A10, A5))
b. M, location step
c. R, structural category
d. MD, maximum possible damage in dollars

M, R, MD (format ( $5 \mathrm{X}, 2 \mathrm{I}, \mathrm{F} 10 \cdot 0$ ) )
E. OPTIONS
a. PRINT (1) $=0$, do not print optimal strategy
$=1$, print optimal strategy
b. PRINT (2) $=0$, do not print actual strategy
$=1$, print actual strategy
c. PRINT (3) $=0$, do not print optimal strategy under perfect forecast
= 1, print optimal strategy under perfect forecast.
PRINT (1), PRINT (2), PRINT (3) (format(5X, 4I5))
ORDER OF INPUT DATA - see input specification for SONIA (Tab;e 7-1)

Page 1: identification and notation
Page 2: data for forecasting system.
Page 3 and 4: data and notation for decision maker
Next 2(IN) pages: optimal strategies if PRINT (1) $=1$
Next 2(IN) pages: actual strategies if PRINT(2) $=1$
Next page: optimal strategy under perfect forecast if $\operatorname{PRINT}(3)=1$
Final page: summary of results - evaluation of the flood forecastresponse system for a single decision maker

SUBROUTINES
The subroutines called in SONIA are listed below.

1. SUBROUTINE DP ( $M, R, V$ ) - dynamic programming algorithm. DP returns to SONIA unit expected loss, $V(A N, I N, I N)$ for $K=1$. DP prints out the optimal strategy, $S(A N, I N, I N)$ for $K=K N, \ldots, 1$. DP calls the following functions:
a. $\operatorname{CT}(I, J, W, K, M)$, consumer time
b. $D D(T, R)$, decision time
c. $\operatorname{DDIN}(A, R)$, inverse of $D D$
d. $\operatorname{DELTA}(Z, R)$, unit damage function
e. $\operatorname{GAMA}(D, R)$, unit cost function
f. $\operatorname{MR}(Z, R)$, unit reduction function
g. $\operatorname{PIH}(I I, H H, I, H, K)$, law of motion for $W(K)=1$
h. $\mathrm{PH}(\mathrm{HH}, \mathrm{I}, \mathrm{H}, \mathrm{K})$, law of motion for $W(K)=0$

Notation:

$$
\begin{aligned}
W & =\text { law of motion } \\
J & =I(K+1) \text { if not last forecast } \\
& =H H \quad \text { if last forecast }
\end{aligned}
$$

2. SUBROUTINE GIV(M, R, V). For a given strategy GIV computes unit expected loss. GIV returns unit expected loss, $V(A N, I N, I N)$ for $K=1$. GIV prints out actual strategy, $S(A N, I N, I N)$ for $K=K N, \ldots, 1$. In addition to the functions called by DP, GIV calls

$$
S A(A I, I, H, K, M, R)=\text { actual strategy }
$$

3. SUBROUTINE EXPEC (V, VA). Computes unit expected annual loss. EXPEC returns unit expected annual loss, VA.
4. SUBROUTINE OSS(M, R, VAO, VASS). Gives unit expected loss with no response and with optimal strategy under perfect forecast. OSS returns unit expected annual loss with no response, VAO, and unit expected annual loss with optimal strategy under perfect forecast, VASS. OSS prints optimal strategy under perfect forecast, SSS(IN).

OSS calls the following functions:
a. $\operatorname{DELTA}(Z, R)$, unit damage function
b. GAMA $(D, R)$, unit cost function
c. $M R(Z, R)$, unit reduction function
5. SUBROUTINE $B B(M, I N, B)$. Computes elements of the consumer time, $B(I N, I N) . \quad B B$ returns $B(I N, I N)$.

Each subroutine and function subprogram are adequately commented making it easy to understand how each operates.

Program LAWMO provides the law of motion and initial condition. Program DWELLER provides M, R and MD. Program DATFIT provides decision constraint function, unit damage function, unit cost function and unit reduction function. Program FORCAST provides PT $(K), L T(K)$, and EN.

SONIA can handle up to DMN decision makers.

### 2.2 PROGRAM ROSALIE

ROSALIE evaluates the flood forecast response system for the reach.

## INPUT AND FORMAT

A. IDENTIFICATION

1. RIVER, name of the river (format( $5 \mathrm{X}, 7 \mathrm{~A} 10, \mathrm{~A} 5$ ))
2. POINT, name of the forecast point (format(5X, 7A10, A5))
3. STAGE, flood stage (format (5X, 7A10, A5))
B. FORECASTING SYSTEM

Same as SONIA
C. RESPONSE SYSTEM

Same as Sonia
D. DECISION MAKERS

1. IDENT, identification of the reach (format(5X, 7A10, A5))
2. DMN, number of decision makers in the reach
3. MDR, maximum possible damage for the reach in dollars DMN, MDR (format(5X, I5, F10.0))
4. $E(I N, R N)$, distribution partitioning MDR. Each row of table (format (5X, 15F5•4))
E. OPTIONS
5. PRINT (1) $=0$, do not print optimal strategies
= 1, print optimal strategies
6. PRINT (2) $=0$, do not print actual strategies
$=1$, print actual strategies
7. PRINT (3) $=0$; do not print optimal strategies under perfect forecast
$=1$, print optimal strategies under perfect response
PRINT (1), PRINT (2), PRINT (3) (format( $5 \mathrm{X}, 4 \mathrm{I} 5)$ )
ORDER OF INPUT DATA - see input specification for ROSALIE (Table 7-2)

### 2.3 PROGRAM DWELLER

DWELLER computes the input for the decision maker from the field inventory data of the Crop of Engineers for Milton, Pa.

1. For $\operatorname{FFR}(D M)$ (PROGRAM SONIA) this is $M, R, M D$.
2. For $\operatorname{FFR}($ REACH ) (PROGRAM ROSALIE) this is $D M N, M D R, E(M, R)$.

INPUT AND FORMAT
RIVER, name of the river (format( $5 X, 7 A 10, A 5$ ))
POINT, name of the forecast point (format(5X, 7A10, A5))
STAGE, flood stage (format (5X, 7A10, A5))
IN, number of steps in the flood plain (format (5X, 415))
YR, reference level, feet (format(5X, 15F5•1))
$(Y(I), I=1, I N)$, elevation of a step, feet (format (5X, 15F5.1))
NU, appraisal number
DYF, YF - YR where YF = first floor elevation
TY, type of property
CL, class
ST, stories
$B A$, basement
SF, size and furnishings
CO, condition
AR, floor plan area, square feet
R, structural category
MD, maximum possible damage in 1000 dollars
$N \mathrm{~N}, \mathrm{DYF}, \mathrm{TY}, \mathrm{CL}, \mathrm{ST}, \mathrm{BA}, \mathrm{SF}, \mathrm{CO}, \mathrm{AR}, \mathrm{R}, \mathrm{MD}$, (format(I5,5X,F3•0,3X,2A1, $\mathrm{F} 1 \cdot 0, \mathrm{~A} 1, \mathrm{~A} 3, \mathrm{~A} 1, \mathrm{~F} 6 \cdot 0, \mathrm{I} 2, \mathrm{~F} 8 \cdot 0) 1$.

Zeros appear in column 80 where adjustments in input data were made.

OUTPUT
Page 1: identification and notation
Page 2: data for forecasting system
Pages 3 and 4: data and notation for decision makers
Next 2(IN) pages: optimal strategies if PRINT (1) $=1$
Next 2(IN) pages: actual strategies if PRINT (2) = 1
Next page: optimal strategy under perfect forecast if PRINT (3) = 1
Final page: summary of results

## SUBROUTINES:

1. SUBROUTINE DP (M, R, V). Dynamic programming algorithm (Same as SONIA).
2. SUBROUTINE GIV(M, R, V). For the given strategy computes unit expected loss (Same as SONIA.).
3. SUBROUTINE EXPEC (V,VA). Unit expected annual loss. (Same as SONIA).
4. SUBROUTINE OSS(M, R, VAO, VASS). Unit expected annual loss with no response and with optimal strategy under perfect forecast. (Same as SONIA).
5. SUBROUTINE BB(M, IN, B). (Same as in SONIA).

Program LAWMO provides the law of motion and initial condition. DWELLER provides $\operatorname{DMN}$, MDR and $E(M, R)$. DATFIT provides unit damage function, unit cost function, unit reduction function and decision constraint function. Program FORCAST provides $\mathrm{PT}(\mathrm{K}), \mathrm{LT}(\mathrm{K})$ and EN .

M, location step
R, structural category
MD, maximum possible damage in 1000 dollars
DMN, number of decision makers in the reach
MDR, maximum possible damage for the reach in 1000 dollars
$E(M, R)$, distribution partitioning MDR
Page 1: identification, reach information and notation
Next pages (through DMN): data and input for single decision maker
Final page: input for the reach (DMN, MDR and $E(M, R)$
OPTIONS: None
SUBROUTINES:
SUBROUTINE ESTAB (M, R, MD). Computes vector ESTABLISHMENT (M, R, MD).
ESTAB returns $M, R$, and MD.
Program DWELLER requires no input from other programs. The input to DWELLER is the field inventory data of the Corp of Engineers for Milton, Pa. DWELLER can handle any number of decision makers.

### 2.4 PROGRAM FORECAST

FORCAST is a stochastic analysis of historical forecasts for developing the law of motion, initial condition and input to the human response model. INPUT AND FORMAT

RIVER, name of the river (format(5X, 7A10, A5))
POINT, name of the forecast point (format (5X, 7A10, A5))
STAGE, flood stage (format (5X, 7A10, A5))
IN, number of steps in the flood plain (format (5X, 15F5•1))

$N F$, number of forecasts in the file (format (5X, 1515))
IYR, year
K, decision time
I, current flood level (feet)
FLT, forecast lead time (hours)
PT, processing time
HH, actual flood crest (feet)
ALT, actual lead time (hours)
FUNCTIONS CALLED

1. $\operatorname{EXN}(P, 15)$, computes expectation
2. $\operatorname{NYEARS}(X, N)$, computes number of years in the file-

ORDER OF INPUT DATA - see input specification for FORCAST (Table 7-3) OUTPUT

Page 1: identification
Page 2: notation and table of contents
Page 3: input data and transformed data
Page 4: Table 1, correlation coefficients
Page 5: Table 2, $P(H(K+1) / I(K+1))$
Page 6: Table 3, $P(I(K+1) / H(K))$
Next 'IN' pages: Table 5, $\mathrm{P}(\mathrm{I}(\mathrm{K}+1), \mathrm{H}(\mathrm{K}+1) / \mathrm{H}(\mathrm{K}))$

Next page: Table 5, $P(H H(K) / H(K))$
Next page: Table $6, P(W(K)=1 / K)$ and $P(W(K)=0 / K)$
Next page: Table 7, $P(N), P(F)$ and $E(N)$
Next page: Table 8, $\mathrm{P}(\mathrm{HH})$ and $\mathrm{P}(\mathrm{HH} \mathrm{M})$
Next page: Table 9, $\mathrm{P}(\mathrm{I}(\mathrm{KO}), \mathrm{H}(\mathrm{KO}))$
Next page: Table 10, PT(K), LT(K)
SUBROUTINES

1. SUBROUTINE PR1 $(X, Y, P, M, N)$. Computes $P(Y=J / X=I)$. PRI returns $P(I, J)$ Notation:
$X=I(K)$ and $Y=H(K)$ if $P(H(K+1) / I(K+1))$ is required
$X=H(K)$ and $Y=I(K+1)$ if $P(I(K+1) / H(K))$ is required provided
$K(J+1)=K(J)+1(J=1, N F)$
$X=H(K)$ and $Y=H H(K)$ if $P(H H(K) / H(K))$ is required provided
$\mathrm{HH}(\mathrm{K})$ is not zero
$P=P I, P H$, or $P H H$
$M=$ size of array $X, Y$
$N=I N$
2. SUBROUTINE PR2 (PX, PY, PYX, N). PR2 computes PYX (I, J, K) $=P(Y=I$, $X=J / Z=K) . \quad$ PR2 returns $\operatorname{PYX}(I, J, K)$

Notation:
$P X=P H$
$P Y=P I$
PYX = PIH
$N=I N$
3. SUBROUIINE CORR (X, Y, N, RXY). Computes the correlation coefficient
between $X$ and $Y$. CORR returns $R(X, Y)$. CORR calls the following functions;
a. $\operatorname{PLUS}(X, N)$, sum of the elements of array $X$
b. $\operatorname{SQ}(X, N)$, sum of the squares of the elements of array $X$
c. $A B(X, Y, N)$, sum of the products $X(I) \times Y(I)$
d. SQRT, square root

Notation:
$X=$ any array of numbers
$Y=$ array of numbers (same size as $X$ )
$N=$ size of arrays $X, Y$
RXY = correlation coefficient
4. SUBROUTINE PR3(X, P,NF,N). Computes $\mathrm{P}(\mathrm{HH})$. PR3 returns $\mathrm{P}(\mathrm{HH})$ Notation:
$X=$ array containing the crests HH
$P=$ probability of the crest, $P(H H)$
$\mathrm{NF}=$ size of array X
$\mathrm{N}=\mathrm{IN}$
5. SUBROUTINE PR4(PH, P, PAB, N). Computes $\operatorname{PAB}(I, J)=P H(I, J) \times P(I)$ PR4 returns $\operatorname{PAB}(\mathrm{I}, \mathrm{J})$

Notation:
$P H=P(H(K+1) / I(K+1))$
$\mathrm{P}=\mathrm{P}(\mathrm{I}(\mathrm{KO}))$
PAB $=\mathrm{P}(\mathrm{I}(\mathrm{KO}), \mathrm{H}(\mathrm{KO}))$
6. SUBROUTINE $\operatorname{PN}(X, K, P, N F, N N)$. Computes $P(N)$, probability of $N$ floods per year ( $N=1,2, \ldots .$. .

Notation:
$X=Y R$
$K=$ decision times
$P=P(N)$
$N F=$ number of forecasts in the file
NN = number of years with floods
7. SUBROUTINE MIN(X, $N, X M I N)$. Computes XMIN, the minimum element in in array $X$. MIN returns XMIN.

Notation:
$X=$ any array of numbers (one dimensional)
$N=$ size of array $X$
XMIN $=$ minimum element in array $X$
8. SUBROUTINE MAX(X,N, XMAX). Computes XMAX, the maximum element in array $X$. MAX returns XMAX.

Notation:
$X=$ any one-dimensional array
$N=$ size of array $X$
XMAX = maximum element in array $X$
9. SUBROUTINE LT(X, $Y, R, M, N)$. Computes $L T(K)$, average actual lead time, and $\operatorname{PT}(K)$, average processing time. LT returns $R$, average actual lead time or average processing time.
Notation:
To get average processing time $X(J)=$ real $K(J)$ and $Y=\operatorname{PT}(J)$
( $J=1, N F$ )
For average actual lead time $X(J)=U(J)$, where $U(J)=$ real $K(J)$ 's
corresponding to non-zero $H H(J)^{\prime} s$, and $Y(J)=V(J)$, where $V(J)=$
non-zero HH(J)'s ( $J=1$, no. of floods)
$M=$ size of array $X$ and $Y$
$N=$ size of array $R$
10. SUBROUTINE $H M(P 1, P 2, N)$. Computes $P(H \geq M)$ where $M=1$, ..., IN.

HM returns $\mathrm{P} 2=\mathrm{P}(\mathrm{H} \geq \mathrm{m})$.
Notation:
Pl(J) = probability of $H$
$P 2(J)=$ probability of $H \geq M$ where $J=1$, IN.
$N=I N$
11. SUBROUTINE $K 2(X, Z, Y, M M, I K)$. Computes $P(W(K)=1 / K)$, and $P(W(K)=0 / K)$. $K 2$ returns $Z(J)$ and $Y(J)(J=1, I K)$.

Notation:
$X(J)=U(K)$ where $U(J)=$ real $K^{\prime}$ s corresponding to non-zero HH's, i.e., the $X(J)$ correspond to the last forecast.
$Z(J)=P(W(K)=1 / K)$
$Y(J)=P(W(K)=0 / K)$
$M M=$ number of floods in file
$I K=$ maximum $K$ in file.
12. SUBROUTINE $K O(X, A, N, K, J B), K O$ puts into array $A$ all years corresponding to all initial times, $K O=1$. KO returns $A(J B)$ and $J B$.

Notation:
$X=$ year
$A=$ years corresponding to $K=1$
$N=N F$
$K=$ decision time
$J B=$ size of array $A$

FORCAST can handle up to 200 forecasts as long as the maximum number of forecasts per flood, $K N$, does not exceed 15. If more than 200 forecasts are available the dimensions of the appropriate arrays must be increased.

### 2.5 PROGRAM LAWMO

LAWMO computes the law of motion and initial condition from multinomial distributions.

INPUT and FORMAT
RIVER, name of the river (format ( $5 \mathrm{X}, 7 \mathrm{~A} 10, \mathrm{~A} 5$ ))
POINT, name of the forecast point (format. $(5 X, 7 A 10, A 5)$ )
STAGE, flood stage (format( $5 \mathrm{X}, 7 \mathrm{~A} 10, \mathrm{~A} 5)$ )
IN, number of steps in the flood plain
KN , maximum number of forecasts
IN, KN, (format(5X,2I5))
DET, time interval between decision times (format( $5 \mathrm{X}, 15 \mathrm{~F} 5 \cdot 1$ )
PW $(2, K N)$, probability of $W(K)=1$ (format $(5 X, 15 F 5 \cdot 4)$ )
Y(IN), elevation of a step (format( $5 \mathrm{X}, 15 \mathrm{F5} \cdot \mathrm{l}$ ))
*PARAMETER OF THE POSITIVE MULTINOMIAL
FUNCTION PIH: IN, QH(IN), QI(IN)
FUNCTION PH: IN, QHH(IN)
FUNCTION PHH: IN, QH
FUNCTION PIHO: IN, QIO, QHO
ORDER OF INPUT: See input specification for LAWMO (Table 7-4). OUTPUT

Page 1: identification, input data, notation, and contents
Page 2: Table 2A, $P(H(K+1) / I(K+1))$
Page 3: Table $3 \mathrm{~A}, \mathrm{P}(\mathrm{I}(\mathrm{K}+1), \mathrm{H}(\mathrm{K}))$
Next 45 pages: Table $4 A, P(I(K+1), H(K+1) / I(K), H(K))$
Next 9 pages: Table $5 \mathrm{~A}, \mathrm{P}(\mathrm{HH}(\mathrm{K}) / \mathrm{I}(\mathrm{K}), \mathrm{H}(\mathrm{K}))$

[^3]Next page: Table $8 \mathrm{~A}, \mathrm{P}(\mathrm{HH})$ from the historical record Table $8 \mathrm{~B}, \mathrm{P}(\mathrm{HH})$ from law of motion

Next page: Table 9A, $P(I(K O), H(K O))$
FUNCTIONS CALLED

1. FUNCTION PIH(I2,H2,II,H1,K). Gives law of motion for $W(K)=1$. The parameters of the distribution are IN,QH(IN) and QI(IN), i.e., IN values for each. QH, QI are assigned values by data statements.

Notation:

$$
\begin{aligned}
12 & =I(K+1) \\
H 2 & =H H(K) \\
I 1 & =I(K) \\
H 1 & =H(K)
\end{aligned}
$$

$K=$ decision time
2. FUNCTION $\operatorname{PH}(H 2, I l, H 1, K)$. Gives law of motion for $W(K)=0$. The parameters of the distributions are IN, QHH(IN). QHH is assigned values by a data statement.

Notation:
$H 2=H H(K)$
$I I=I(K)$
$H 1=H(K)$
$K=$ decision time
3. FUNCTION PHH(H2). Gives probability of actual crest. The parameters of the distribution are IN and QH . QH is assigned a value by a data statement.

Notation:
$H 2=H H(K)$
4. FUNCTION PIHO (II, HI ). Gives initial condition. The parameters of the distribution are $I N, Q(1)-(Q I O)$, and $Q(2)-(Q H O) . Q(1)$ and $Q(2)$ are assigned values by a data statement.

Notation:

$$
\begin{aligned}
& I 1=I(K) \\
& H 1=H(K)
\end{aligned}
$$

The parameters $\mathrm{QH}(\mathrm{IN}), \mathrm{QI}(\mathrm{IN}), \mathrm{QHH}(\mathrm{IN}), \mathrm{QH}, \mathrm{QIO}$ and QHO are provided by program PARAMT.

### 2.6 PROGRAM PARAMT

Given $E$ and $N$, PARAMT computes the parameter, $P$, of the positive multinomial distribution.
$E=$ sample estimate of the expected value
$N=$ number of Bernoulli trials
$P=$ probability of positive multinomial
The maximum likelihood equation for $P$ is solved by iteration.
INPUT AND FORMAT:
RIVER, -name of the river
POINT, name of the forecast point
RIVER, POINT (format( $5 \mathrm{X}, 7 \mathrm{~A} 10, \mathrm{A5})$ )
PARA, name of the parameter
$E$, expected value
$N$, number of Bernoulli trials
PARA, $E, N(f) r m a t(A 5, F 5 \cdot 3, I 5))$

OUTPUT:
PARA, parameter
$E$, expected value
$N$, no. of Bernoulli trials
P, probability of positive multinomial
Program FORCAST provides program PARAMT with E and N.

### 2.7 PROGRAM DATFIT

This program fits polynomials to data. Any standard methodology may be used.
TABLE 7-2. INPUT SPECIFICATIDN FOR POSALIE
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$\begin{array}{r}-1 \\ - \\ -2 \\ -2 \\ -2 \\ -2 \\ \hline\end{array}$
$-1$
TABLE 7-3. INPUT SPECIFICATION FOR FORCAST



[^0]:    1
    To compare with the computational requirements of the general model, suppose that $I N=6$ and $U(m)=4$ for every $m \varepsilon I$, which gives 24 different strategies for the REACH. With an average computing time of $80 \mathrm{sec} C P$ time per strategy, the evaluation of $\operatorname{FFR}($ REACH $)$ requires 1920 sec or 0.53 hrs . CP time.

[^1]:    */Basic references are:
    Johnson, N.L. and Kotz, S., Discrete Distributions, Houghton Mifflin Company, Boston, 1969.
    Wilks, S.S., Mathematical Statistics, John Wiley \& Sons, Inc., New York, 1962.

[^2]:    */Notation in the program:
    $\eta=E$
    $m=M$
    $r=R$
    $M D($ REACH $)=M D R$

[^3]:    *The appropriate parameters are assigned in each function subprogram by the data statement.

