Optimal Operation of Water-Supply Systems

by
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PREFACE

This report constitutes the doctoral dissertation of the same title completed by the author in September 1969.

ACKNOWLEDGMENTS

ABSTRACT

The traditional metropolitan water-supply planning problem is characterized by two main steps:

(a) project future water requirements based on present rates of economic growth, and

(b) schedule water development projects to be introduced into the system on time to meet these predicted requirements.

The City of Tucson plans its water supply essentially in this manner. The prime objective of this phase of our research was to formally review the above problem and to formulate it in terms of concepts of management science. Implied commitments to accept Colorado River water and gradual changes in quality of Tucson's groundwater force serious consideration of the economic tradeoffs between alternative sources and uses of water. These alternatives lead to a need for a restatement of water-supply planning objectives in more precise forms than have heretofore been put forth. The doctoral dissertation by G. Clausen addresses itself to the above restatement with actual data on the Tucson basin.

The various water-supply planning objective functions including the traditional one are all expressions which maximize the difference between gains and losses involved with water development. They can be expressed mathematically and differentiated on the basis of how these gains and losses are defined. In the traditional sense, gains derived from meeting projected requirements are assumed to be infinite, and losses are taken to be actual project costs and not social costs associated with undesirable economic growth. Therefore, maximization of net gains is accomplished by minimizing project costs, and gains do not even have to be expressed. Consideration of alternatives, however, requires that gains be expressed quantitatively as benefits to individuals, communities, or regions, i.e., primary, secondary, or tertiary benefits. The same logic holds for the expression of total costs.

An objective function, used to express the water-supply problem in the Tucson Basin, considers gains as cash revenue to a hypothetical central water-control agency which sells water to the users within the basin. Losses are considered as marginal costs to the agency for producing, treating, and distributing water. The concept of economic demand is used to estimate the amount of water that municipal, industrial, and agricultural users will purchase at different prices. Linear demand functions are postulated. The possible sources of supply considered are groundwater from within the basin, groundwater from the neighboring Avra Valley Basin, reclaimed waste water, and Central Arizona Project water from the Colorado River. Constraints are formulated to allow for limits on water availability, for social limits on water prices, and for minimal requirements of each user over a specified time period; these permit a determination of optimal allocations of water under different conditions to answer "what if" questions, given the assumptions of the model. The resulting static model is termed a pricing model and is optimized by first decomposing the objective function into component parts with each part representing terms involving only one source

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of water. In instances involving inequality constraints, quadratic programming is used. In other instances where equality constraints or unconstrained conditions exist, Lagrange multipliers and calculus methods are used. These latter conditions arise when it is determined at which point certain constraints become inactive. In the completely general case, this type of decomposition is not possible, but it appears that in many specific uses objective functions of this nature can be profitably decomposed and optima determined much more conveniently than otherwise possible. The model clearly identifies the opportunity costs associated with the required use of Colorado River water in lieu of the cheaper Tucson groundwater.

CHAPTER 1

INTRODUCTION

It is becoming increasingly apparent that the aims of large-scale water development are changing. Water is no longer thought of as a resource to be exploited by anyone as he sees fit for his own economic gain or need, but it is thought of as a resource belonging to all the people to be exploited as they determine for the most benefit of all concerned. This means in general to use water to help improve the quality of the total environment, i.e., the entire region or the entire country. These new aims require sound planning procedures capable of considering all the alternatives possible for water use in the total environment. As stated by the National Academy of Science (1968), "the best cure for a threatening water shortage is not necessarily more water; savings in water use, or transfer of water use to less consumptive, higher yield applications, or discovery of new techniques of water management may offer better solutions."

This investigation is an effort directed toward clarifying the objectives of water-supply problems and introducing new planning techniques. It should be emphasized here that it is the planning process and not the plan itself that we are most interested in. The purpose is to develop processes, attitudes and perspectives which make sound planning possible rather than to simply have a water-supply plan. The actual plan is likely to become obsolete shortly after it is developed due to reappraisal and reevaluation of both objectives and data input. The process, which created the plan if thoughtfully conceived, nurtured, and controlled, is not.

In operations research we attempt to formalize the general planning process and then proceed to use the process to achieve the best plan possible under the circumstances. First, the physical-economic system with which we are dealing must be defined and the interaction of the system variables must be quantified. Next, we need a measure of system effectiveness expressible in terms of the system variables. This measure of effectiveness is referred to as an objective function. Finally, we must be able to identify which values of the system variables yield optimum effectiveness. Just as all but the simplest decisions are made pursuant to the consideration of certain requirements, there also exist in most cases certain constraints concerning the variables which must be considered. The constraints are in the form of physical, economic, or social limitations.

Most prior applications of operations research to water-supply problems have been concerned with formulating a model to fit a particular programming technique. Usually only one objective function and one programming technique are presented. The objective functions are not clearly interpreted in terms of welfare economics. The above are major criticisms often leveled at operations research studies. Therefore, in chapter 2, all the various water-supply planning objective functions are formalized in general mathematical terms. Certain objective functions

imply certain constraining relationships among the variables, and these are also set forth in general terms. The economic theory underlying the various objective functions is explained in the latter part of the chapter.

In chapter 3, the water-supply problem in the Tucson Basin, Arizona, is used as an illustrative example. It is used to demonstrate the application of one of the objective functions set forth in chapter 2. The objective function used is referred to as the pricing model and involves the consideration of water as a commodity rather than a free good. In the model, the sale of the commodity, water, is accomplished by a supposed central water-control agency, but this agency, being a public entity, is guided in its actions by the people.

The water-supply problems of the Tucson Basin are typical of the problems encountered in other arid and semiarid regions, that is, an economy with certain trends has been established on a groundwater supply which is being used at a faster rate than the average rate of recharge. This condition leads to a decline in the groundwater level to a point where the public realizes that in order to maintain the prevailing economic trend other water sources should be acquired. The question then is what is the most efficient plan for acquiring and allocating this water to the municipal, agricultural, and industrial users. Along with this, a basic premise of the study is that the alternative of altering the economic trend should also be considered. Specifically, this means using water in less consumptive, higher yield applications, however these may be defined.

The pricing model presented required input data concerning water availabilities, water costs, and demand functions relating the price charged for water to the amount expected to be used at that price. These data are obtained for the Tucson Basin model which essentially involve the determination of the optimal quantities of water from each source to be used by the municipal, agricultural, and industrial users in the basin. The sources of water considered are the Tucson Basin groundwater reservoir, groundwater from the neighboring Avra Valley Basin, reclaimed waste water, and Central Arizona Project water. Additional data describing the hydrologic-economic system are also obtained. The model is solved using a decomposition technique based on cost aggregation which is described in the text and a quadratic programming algorithm.

The distinctions of and relationships between static and dynamic models should also be made clear. A static model is one which simulates a situation which does not change with time or represents only one time period. In a dynamic model, the conditions change. A static model can be converted into a dynamic model, however, by redefining the coefficients in the objective function or changing the constraining conditions for each of a consecutive series of time periods. In this study, the model developed is assumed to represent conditions during one time period, but it could be expanded into a dynamic model as mentioned.

With studies of this type, professionals are admittedly encroaching on policy issues beyond their range of expertise. This point is also brought to the fore by Davis (1968) in his section on "The Task of Political Responsibility." It seems as if this overlapping of responsibility between the engineer-planner and the politician is not only inevitable, but, if pursued diligently, desirable because it can lead to inducing the politician and administrator to play more responsible roles in the planning process.

WATER ALLOCATION METHODS

Traditionally, the water-supply problem for a community or group of people has been handled in the following manner. The citizens first realize that a problem does exist, that is, they arrive at the belief that their existing supplies are not adequate to meet the projected requirements which they have decided will be necessary to fulfill the desires of future inhabitants of their community or area. Then a water-resource engineer is officially engaged to undertake a search for additional supplies, plan projects which will allow for the development of these supplies, and schedule the construction of these projects to come on line in time to meet the projected requirements. In many instances, the engineer is permanently engaged in the person, for instance, of a metropolitan water department to help define community water problems as well as solve them.

All engineers, however, are not satisfied with this type of approach and have argued that a more efficient use of the water resource should be made. This argument until recently has been overshadowed by the fact that surplus water had been available. But, as readily available sources dwindle and surplus water becomes more and more expensive to develop, it is beginning to be recognized that existing supplies are going to have to be utilized in a more efficient manner. This goes along with the general theme of improving the quality of our total environment rather than allowing the rather haphazard growth of resource use to continue unabated.

The proponents of this theme in the field of water resources point out two concepts which for the most part have been sorely lacking in the traditional approach. The first is that all the alternative and somewhat more imaginative schemes have not been taken fully into account and as long as they are not considered we cannot be assured of having devised the type of water-supply plan which will best meet our objectives. The most prominent among these alternatives are reuse of waste water, artificial recharge, and transfer of water between uses. The second concept is that water has not been considered as a commodity for which a market does exist. This means that economic principles governing the allocation of supplies between competing uses have not been employed. It must be recognized that the social and legal atmosphere surrounding any given situation may make the practical application of either or both of these two concepts very difficult, but these facts do not relieve the engineer of the responsibility to develop theoretical plans which encompass these concepts.

In this regard there are two general approaches that he can take: (1) meet projected requirements at minimum cost or (2) maximize net benefits accruing from water use. The approach taken depends on the above-mentioned social and legal constraints within which he must ultimately operate. In general, he must be satisfied with the first approach when working with private firms and certain low-level planning agencies such as small municipalities or irrigation districts which do not consider water as a limiting resource and are willing to pay high rates in order to maintain their present per capita use standards. In these cases, the idea of

considering alternative uses can be introduced. The second approach can
sometimes be taken when dealing with high-level planning agencies such as
state or regional planning boards which are in position allowing them to
judge at which place or in what activity a certain water resource can be
used most efficiently. In these cases, both the alternative uses and
commodity aspect of the water resource can be considered.

In this chapter general deterministic models, based on the above-
mentioned two concepts from which specific water-resource allocation prob-
lems can be deduced will be discussed. These concepts will first be
discussed in terms of general, deterministic, static models, that is,
models which do not change with time. Later, the time dimension will be
added to the models and discretization in space will be discussed. The
static models are presented first only for simplification. Also a need is
foreseen to explain the basis for the models in terms of economic systems
which have reached states of dynamic equilibrium (where the forces of
supply and demand are in balance) and can therefore be described as static.
Next, refinement possibilities for cases involving stochastic inputs will
be pointed out. A discussion of both the traditional optimization tools
and the relatively new mathematical programming techniques for optimizing
the models will follow. Finally, we will review and compare several
recent efforts by other investigators in this field.

Allocation Models

Classification of Problem Types
by Constraints

Whether or not we consider water as a limiting resource we must
consider capital as one. Therefore, in both of the above approaches we
cannot plan in a haphazard fashion but must plan so that we get the most
out of the money available. In this endeavor, formalized optimiation
techniques can be and have been used to find plans which either minimize
costs or maximize benefits according to certain objectives and subject to
various constraints.

These constraints fall generally into two categories: those which
require the total amount of water supplied by sources to be greater than
or equal to a specific requirement and those which require the total amount
of water used by uses from any one source to be less than or equal to the
amount available from that source. The former or requirements constraint
can be expressed as

$$\sum_{i=1}^{n} q_{ij} \geq Q_j ; \qquad j = 1, 2, \ldots, m. \tag{2-1}$$

where q_{ij} is the amount of water transferred from source i to use j and Q_j
is the total amount of water required by use j. The second or availability
constraint can be expressed as

$$\sum_{j=1}^{m} q_{ij} \leq Q_i ; \qquad i = 1, 2, \ldots, n \tag{2-2}$$

where Q_i is the total amount of water available from source i.

The two approaches mentioned earlier can be distinguished by the class of constraints present. The requirements approach must have both classes of constraints, but the benefit-maximization approach needs only the availability constraints. In other words, if we are to attempt to maximize benefits for any set of alternatives other than those specified by meeting projected requirements we must have the freedom to do so.

Requirements Approach

If projected requirements must be met and the objective Z is to minimize total cost, the function to be minimized has the following general form:

$$Z = \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij} q_{ij} \tag{2-3}$$

where c_{ij} is the cost of transferring a unit of water from source i to use j. Unit costs do not consider economics of scale and are implicitly linear. This is a basic assumption and implies the idea of independence among variables. We assume that c_{ij} is totally independent of not only its associated q_{ij} but also all other q_{ij}'s, i.e., c_{ij} is the same regardless of the level of its associated q_{ij}. Furthermore, it is not affected by the presence or level of other q_{ij}'s in the model. In this static model the q_{ij}'s are not independent of one another, as witnessed in the constraining relationships, and no assumption as to their independence has been made.

This type of problem is termed a "transportation or distribution problem" in operations research, and it was originally used to determine the most efficient shipping patterns for supply missions during World War II. It is helpful to be able to recognize the correspondence between industrial or military type problems and water-resources problems; there-fore, we will elucidate this point in the context of transportation problems. Suppose we have a number of manufacturing plants each producing a known quantity of the same product (these are analogous to sources of water supply). We also have a number of destinations, each of which has a particular requirement for this product (these are analogous to the various types of water users and their requirements). There is a shipping cost for shipping one unit of product from each manufacturing plant to each destination (these are analogous to costs of transferring water). The problem, then, is to determine how to distribute goods from the plants to the destinations in order to minimize total shipping cost and meet supply and requirement restrictions, i.e., how to meet the water-supply require-ment at least cost. The objective function is that given by (2-3). Simple algorithms are available for solving these type problems.

If it is feasible to assume that costs are linear functions of their associated quantities of water, the more realistic nonlinear relationship between c_{ij} and its associated q_{ij} could be substituted into the objective

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function as it stands in (2-3). The solution, however, would require a nonlinear optimization technique which would not be nearly as easy to apply in general as the well-known transportation algorithm.

It is interesting to note that in requirements-type models it is inherently assumed that the greatest benefits, both economic and social, direct and indirect, will arise when a predetermined amount of water is supplied to each use. Therefore, in this rather constrained situation, the least-cost solution is considered by some to also be the solution which maximizes net benefits.

Net-Benefits Approach

If the requirement constraints can be relazed from those given in (2-1), where a projected requirement for each use is specified, to the following

$$\sum_{j=1}^{m} \sum_{i=1}^{n} q_{ij} \geq Q \qquad (2-4)$$

where Q is the total amount of water required by the region as a whole and no specification is made as to individual use requirements, then the idea of net-benefit maximization can be introduced, that is, as soon as the requirements of the type (2-1) are relaxed, some freedom of choice is allowed and this is all that is needed to allow the net-benefit maximization approach to be used.

An objective function using constant unit net benefits b_{ij} can be maximized with linear programming techniques in the same manner as equation (2-3) and takes the form

$$Z = \sum_{j=1}^{m} \sum_{i=1}^{n} b_{ij} q_{ij} \qquad (2-5)$$

where b_{ij} is the benefit accruing from a unit of water being transferred from source i to use j. With linear benefit and cost functions, (2-5) is a simpler representation of

$$Z = \sum_{j=1}^{m} B_j - \sum_{j=1}^{m} \sum_{i=1}^{n} C_{ij} \qquad (2-6)$$

where B_j is the gross benefit arising from transferring some quantity of water to use j and C_{ij} is the total cost of transferring this water from source i to use j. In order to maximize the function, of course, both the benefits and costs would be expressed as functions of their associated quantities of water and could be nonlinear as well as linear.

If the requirement constraint can be relaxed altogether, the net benefit maximization approach can operate with complete freedom subject only to the availability constraints. The approach inherently assumes an overall controlling agency with the power to specify how much water each

type of user will be allowed. We can also visualize the existence of tiers of controlling agencies below this overall agency with each receiving an allotted amount of water from its superior and in turn allocating this to its indigenous sub-uses. For instance, an irrigation district could receive water from a basin-wide planning agency and in turn allocate this water to types of farming activities within its jurisdiction with an effort to maximize benefits to the irrigation district. In our society, however, we prefer to look at the lowest member of this tier first, that is, the individual. For instance, how much water should the individual farmer or industry purchase in order to maximize his or its individual profit? Whether we view this tier from the top or bottom, the supply-demand relationships have the same fundamental economic bases. These will be discussed in the next section, but before doing that let us analyze the term "benefit."

Definition of Benefits.

First of all, it seems that benefits resulting from the use of water are most quantifiably assessed in cases where the water is used in agriculture. The amount of water needed to grow a certain crop can be determined. So, at least in this case, a good estimate of primary or direct benefits, that is, benefits to the user of the water resource, can be obtained. Even these primary benefits are very difficult to estimate for all other types of uses. The benefit picture is not complete without the inclusion of indirect benefits, however. In most cases the total benefit picture is very difficult to describe because use of water by one sector of the economy creates reverberations throughout the whole economy which give rise to a myriad of indirect benefits and costs for which account should be made. Input-output models which attempt to show how the output from each sector of the economy is distributed among the other sectors and, likewise, how each sector obtains from the others its needed inputs are efforts directed toward putting monetary values on these indirect benefits. A detailed discussion of the application of these models is given by Martin and Carter (1962). Efforts such as these are prodigious works because of the immense amount of data required and are prepared on a large-area basis, for instance, nationwide or statewide. In the end, numerous assumptions still have to be made, but studies such as these are important as realistic methods of quantifying total benefits. In order to use these studies for sub-areas within the study area, the sub-areas must be considered as microcosms of the study area.

In the case of a water-supply system, the profits accruing from the sale of water can be considered as a measure of system effectiveness just as can the above-described benefits. In fact, profits are the measures of effectiveness of any business firm and as such can be thought of as benefits to the firm. If for some reason it would be desirable to operate a water-supply system as a business firm, the profits to the agency operating the system would be the total revenue paid by the water users less the total costs of obtaining, treating, and distributing this water. The objective here is to maximize the following function

$$Z = \sum_{j=1}^{m} R_j - \sum_{j=1}^{m} \sum_{i=1}^{n} C_{ij} \qquad (2-7)$$

where R_j is the total revenue collected by the agency for water from use j and C_{ij} is the total cost to the agency of transferring this water from source i to use j.

The fact is that water-supply systems are normally publicly owned and not operated to maximize profits per se. But this does not mean that in the interest of efficiency they cannot work toward maximizing profits within the publicly set constraints which they must operate. In fact, it will be shown later how a central water-control agency could be operated so as to maximize its internal profits for the sake of efficiency.

The introduction of price requires recognition of the concept of economic demand, the relation between water use and price. This concept recognizes that price has an effect on the amount of water used and that given the opportunity water users will adjust their use in relation to cost. The relative change in use with a change in price is known as the elasticity which is expressed mathematically as

$$\frac{dQ}{Q} = e \frac{dp}{p} \tag{2-8}$$

where p is the price per unit and Q is the quantity at which e, the elasticity is measured. Thus, if a doubling of price would decrease use by 20 percent, the elasticity is said to be -0.20. If the demand is elastic, use is dependent on price

There is also the possibility of cross elasticities being of importance in water supply, that is, the amount of water used by one sector might be affected by the amount used by another sector or, conversely, the price charged another sector. Cross elasticities can be expressed mathematically as

$$\frac{dQ_1}{Q_1} = e_1 \frac{dp_1}{p_1} + e_{1,2} \frac{dp_2}{p_2} \tag{2-9}$$

where p_1 is the price per unit charged the first sector, Q_1 is the quantity used by the first sector and e_1 is the elasticity between Q_1 and p_1. The price p_2 charged the second sector affects Q_1 through the cross elasticity value $e_{1,2}$. A considerable amount of elasticity does exist for water sales and, hence, demand considerations are important in planning.

We are now at the point of interacting with the basic ideas of public pricing policy and the theory of the firm. These ideas have been well expressed for quite some time in the economic literature, but they will be developed here purely in the context of water resources and for the purpose of superimposing this field of economics on the systems approach to water allocation problems as described above. The economic fundamentals used in the next section were gathered from Chiang (1967), Bonbright (1961), Maass et al. (1962), Hirshleifer, De Haven, and Milliman (1960), and Samuelson (1964).

Economic Principles

In very general terms, every organization's problem, whether this organization be a nation or an individual, is to maximize the difference between its gains and its losses (if we use the terms gain and loss in a very broad sense). The degree to which they do this can be thought of as the degree of economic efficiency which they attain. We will endeavor to show here in economic terms how the maximizing of economic efficiency within the framework of pure competition or monopolistic competition is the same as optimizing the objective functions set forth in (2-6) and (2-7) respectively.

Pure Competition

One of the basic economic axioms is that an optimum condition, from the point of view of overall economic efficiency, exists in the presence of a freely competitive market. Of course, in the water-supply field we do not have anything approaching a competitive situation, but if the forces operating in this situation are understood, the water-control agency can operate so as to simulate the competitive model. Thus, by focusing attention on the nature of a competitive market, the economic nature of a publicly owned water utility can be brought into perspective. Actually, it is common practice among economists to design public utilities to function in such a way as to emulate a competitive market.

This market is characterized by tne existence of enough consumers and producers to force the supply of and demand for a given commodity, in this case water, to an equilibrium condition. The equilibrium condition, as shown in Fig. 1, is found at the intersection of the supply and demand curves and prescribes a price per unit volume of water and a total level of water production that would occur in a purely competitive situation. In reality, this equilibrium condition is rarely attained because the market framework is continually shifting in time, and it takes time for the market conditions to catch up with each new shift. The reason that total production would be expanded out to the equilibrium point is that this is the only point at which individual consumers would be maximizing their net satisfaction while paying the exactly same price that in turn maximizes the net revenue of all the individual suppliers at this level of production. Everyone all this time acts only in his own self-interest. To see how these individual maximizations take place, it is necessary to look in more detail at the situation from both the consumer's and producer's viewpoint.

For each different unit price of water offered to the consumer, he will purchase a different quantity. Assuming that at each price, he can purchase whatever quantity he desires, he will purchase the amount which maximizes his net satisfaction (however he personally may define this) for the use of this water at that price.

In the competitive market situation, he is only offered water at a single price (the competitive price), but he can buy as much as he wants at this price. Therefore, in his own self-interest he will buy the amount of water indicated by his demand curve, and this will be the amount that maximizes his net satisfaction.
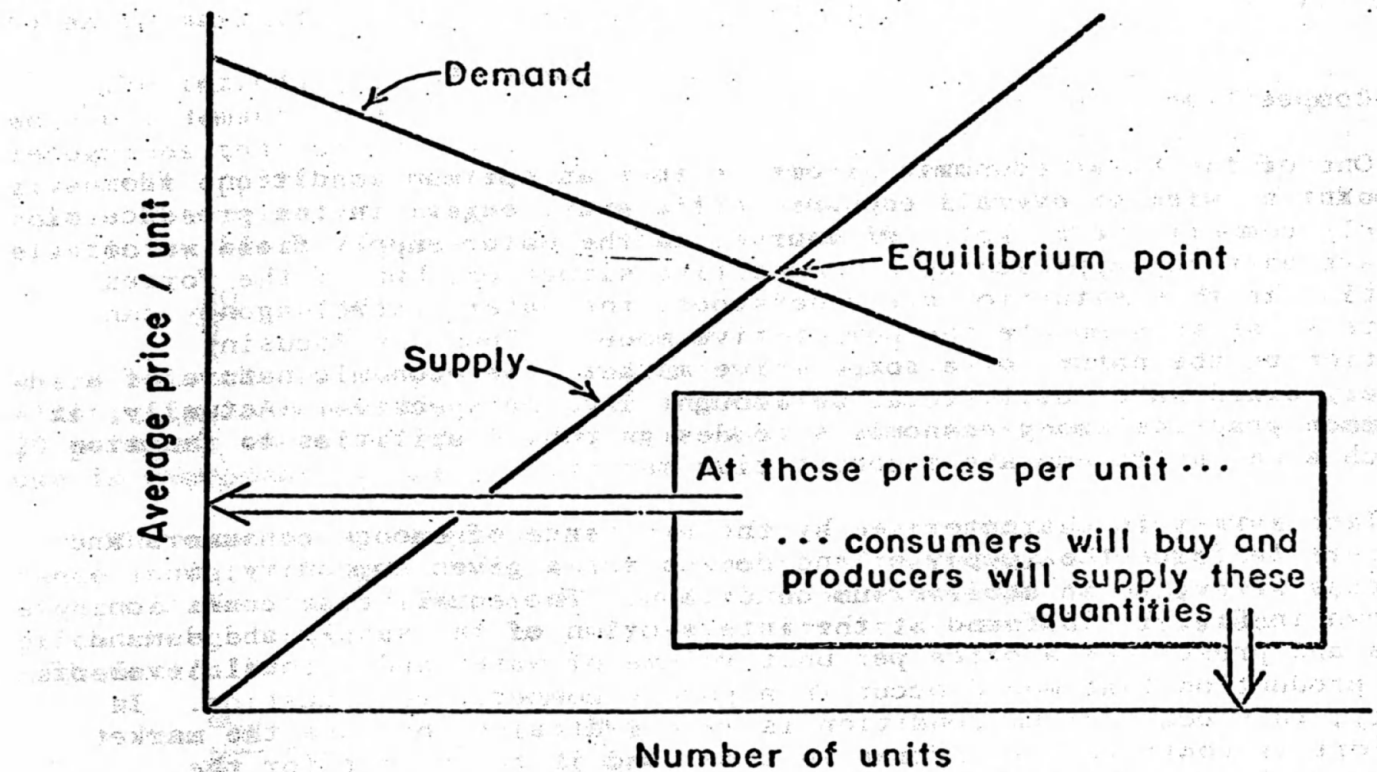
Figure 1.   Equalizing the Demand and Supply of a Commodity.

Fig. 2 shows an entire schedule relating the unit prices charged to a user of water to the quantity which he will use at that price. This is called a demand curve. Demand curves for water are thought of as having the general shape as shown in Fig. 2; however, they vary from user to user. Specific discussion of different types of demand curves is contained in chapter 3. The price paid per unit delivered declines as quantity increases, since the first units of water becoming available will naturally be applied to the most urgent needs and the user will pay a high price for these units, the next will be applied to somewhat less intense needs, etc. Eventually, prices in any use may become zero, reflecting a situation of a saturated demand for water. In this state, no more will be desired by this user even at a zero price. This, however, is not the type of demand curve faced by the supplier in a competitive market. The reason for this will be given shortly.

Now let us look in some detail at the water supplier's situation. The price which the user is willing to pay at the competitive equilibrium point for a quantity of water becomes the supplier's revenue. This price, as far as the individual supplier is concerned, is exogenous, that is, it is determined by market forces external to the individual supplier and he has no control over it. His total revenue function, then, will be

$$R = pQ \tag{2-10}$$

where R is the total revenue to be gained by the supplier upon producing an amount of water Q, and p is the unit price which the consumer pays. Similarly, his total cost function will be

$$C = c_A Q \tag{2-11}$$

where C is the total cost of supplying Q units of water, and c_A (also a function of Q) is the average unit cost. In his own self-interest, the supplier will sell the amount of water which will maximize his net returns (R - C). This amount will be indicated by the point where his marginal revenue equals his marginal cost. Marginal revenues and marginal costs at any level of production are obtained by taking the derivatives of (2-10) and (2-11), respectively, at that level of production, i.e., dR/dQ and dC/dQ. In a competitive market, however, the competitive price offered does not change as far as the supplier is concerned. Therefore, the price is both his average and marginal revenue. Since this is the case, the supplier should theoretically operate at the level indicated by the point where his marginal cost equals the competitive market price as shown in Fig. 3. The competitive market price necessarily appears as a horizontal line to the supplier. Fig. 4 shows this same operating level indicated by the point where there is the greatest difference between the supplier's total revenue curve R (always linear when he is entering a competitive market) and his total cost curve C. It should be kept in mind that in this present discussion, we are concerning ourselves with single suppliers and single users and their individual actions when faced with a competitive market.

At any production level Q, total cost C and average cost c_A are related to marginal cost c_M in the following manner
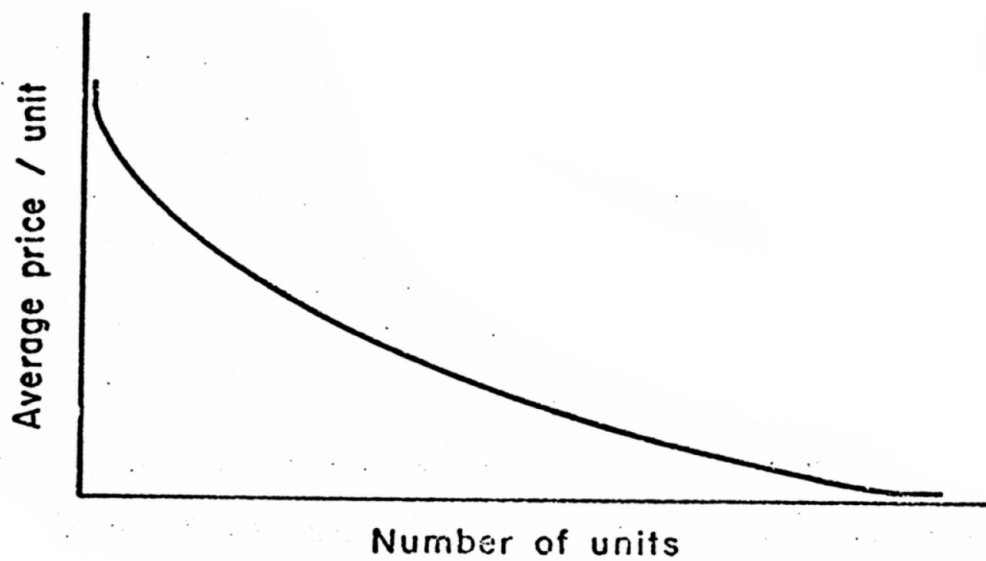
Figure 2. Hypothetical Demand Curve.
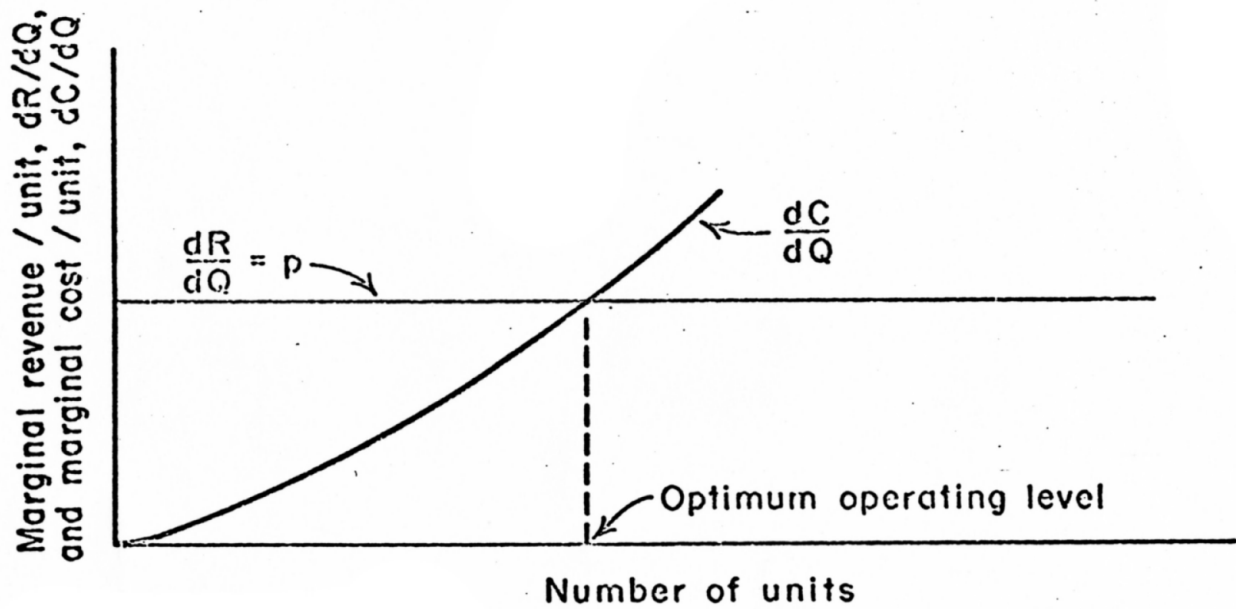
Figure 3. Supplier's Optimum Operating Level Indicated by Marginal Revenue and Marginal Cost Curves.
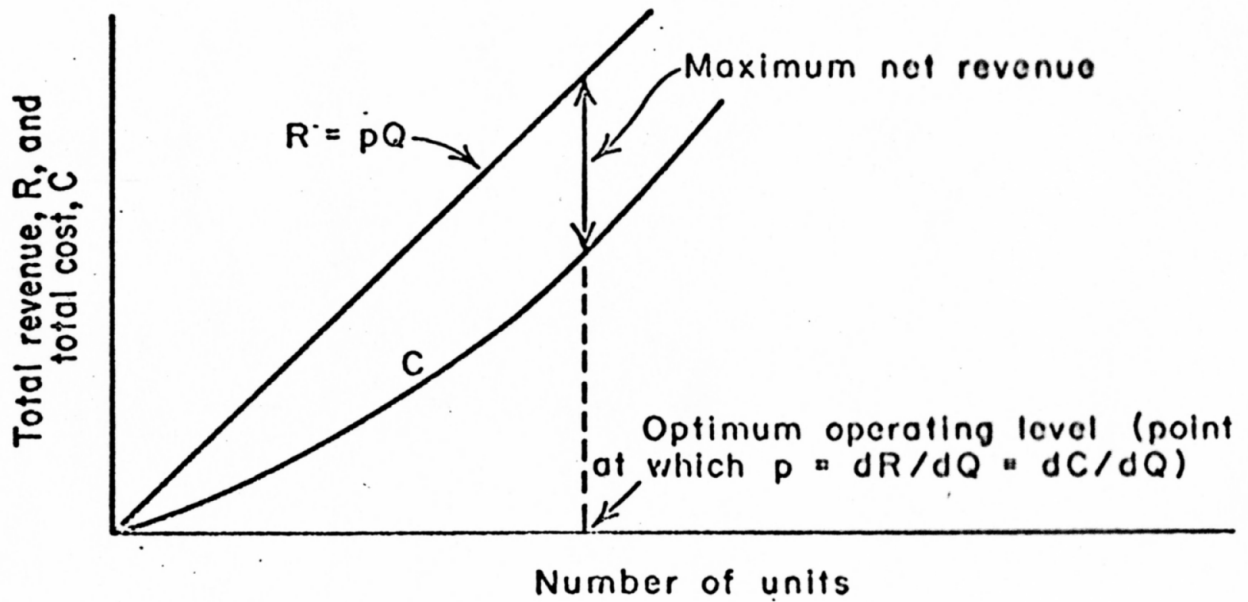
Figure 4. Supplier's Optimum Operating Level with Increasing Unit Costs.

$$c_M = \frac{dC}{dQ} = \frac{d(c_A Q)}{dQ} \quad . \tag{2-12}$$

In this discussion so far, we have pictured average costs as increasing whereas in reality average costs usually decrease throughout initial production levels due to economics of scale and only start to increase when the raw material resource (water in our case) starts to become scarce. Therefore, as long as the supplier's average costs are decreasing, he has every reason to expand his production further and further, since each additional increment in production brings him the same extra revenue but lower extra costs. This circumstance is shown in Fig. 5. If for some reason these types of producers never reached a point where average costs started increasing, they would continue to expand their output to a point where supply exceeds demand, perfect competition would cease to exist, and the market would have to readjust itself about a new equilibrium position. Therefore, in order for competitive market theory to hold true, each supplier's cost curve must be characterized by rising average costs beyond some point.

With this background, we are now in a position to extract a premise from the theory of competition which can be used to operate the central water-control agency in a manner which emulates this competitive condition of all-around net satisfaction. The premise is that maximum net satisfaction from the use of water is not achieved so long as consumers are willing to pay more for additional units of water than the additional cost incurred in producing these additional units. In the next section we will se how this premise is applied.

Monopolistic Competition

As alluded to above, the municipal water utility does not function in a competitive market. It must operate (as in reality do almost all types of economic enterprises) in an atmosphere of monopolistic competition which means, in this case, that the central water-control agency does not face a horizontal, competitive market demand curve, but instead must face the actual demand curves of its customers. But in so doing and according to the above-stated premise, if we operate at a point where marginal cost of producing, treating, and distributing water equals the price charged for the water, we will be maximizing net satisfaction from the use of this water.

Fig. 6 shows the water consumer's demand curve, the central water-control agency's marginal and average cost curves, and the point at which the marginal cost and the demand price are equal. This point, then, simulates the competitive market equilibrium point as discussed earlier.

Fig. 6 also shows "total consumer satisfaction" $S(Q)$ which is defined mathematically as

$$S(Q) = \int_0^Q p \, dQ \tag{2-13}$$

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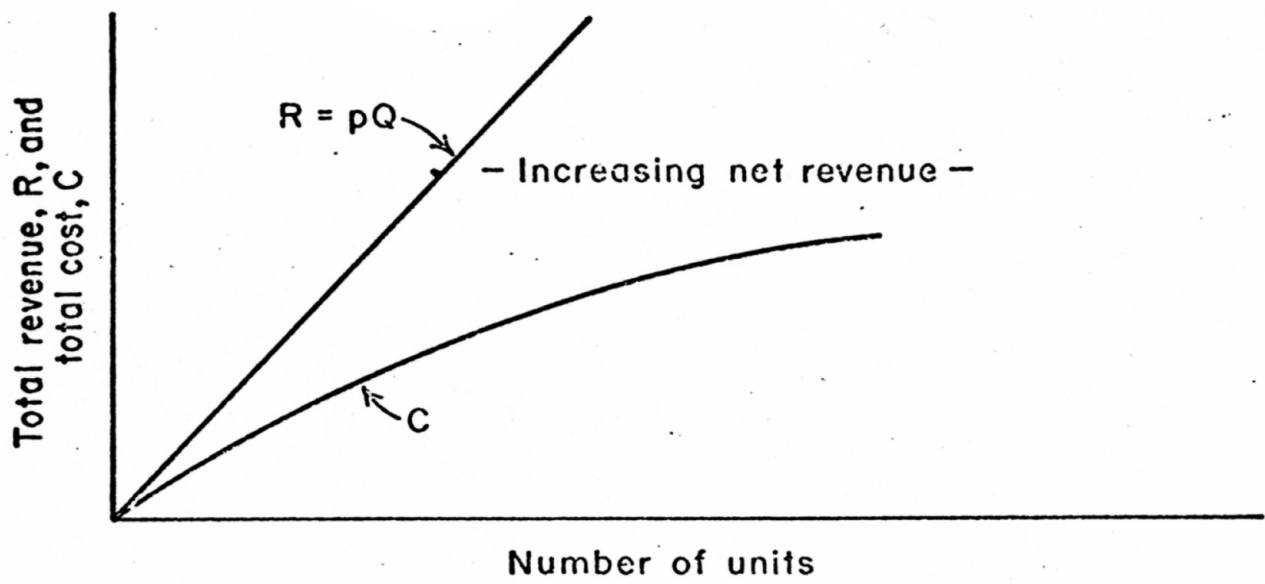
Figure 5. Supplier's Condition with Decreasing Unit Costs.
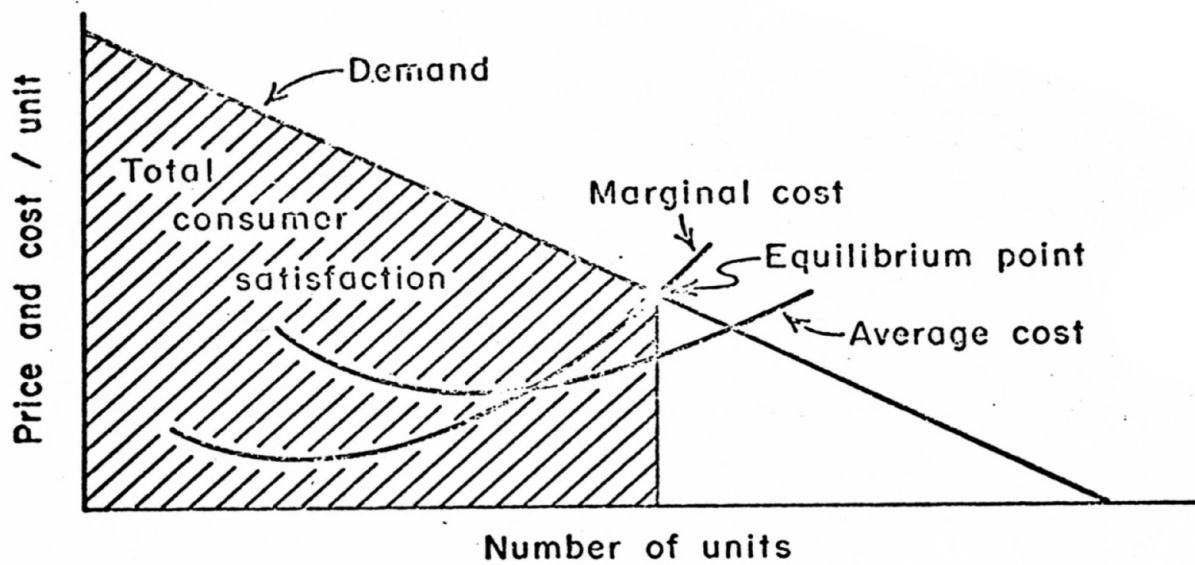
Figure 6. Equilibrium Condition Simulated at Point where Marginal Cost Equals Price.

or the area under the demand curve out to Q units of water. This total consumer satisfaction is also referred to in the economic literature as total "value-in-use" and in this the demand curve $p(Q)$ can be thought of as the marginal value-in-use curve, i.e., it is the slope of a total value-in-use function $S(Q)$. The following relationship then holds

$$p(Q) = \frac{d}{dQ} S(Q) .$$ (2-14)

Therefore, finding the point where marginal cost equals marginal value-in-use is the same as maximizing the following objective function

$$Z = V - C$$ (2-15)

where V is total value-in-use and C is total cost. With the above ideas in mind, (2-15) could also be written as

$$Z = \int_0^Q p(Q) \, dQ - \int_0^Q c_M(Q) \, dQ$$ (2-16)

where the first integral is the area under the demand curve out to Q, or the total value-in-use, and the second integral is the area under the marginal cost curve out to Q, or the total cost.

It is important to note here the similarity between (2-15) and (2-6) assuming that both are unconstrained. This is because total value-in-use V is a measure of the total benefits used in equation (2-6) and, therefore, the only difference between the two equations is that (2-6) is dealing with multiple uses and sources while (2-15) is only concerned with a single use and a single source. Thus the economic basis of (2-6) is established.

Just as in the case of the individual producer, however, problems arise when the average costs are decreasing relative to Q and this is often the case for water producers because of the normal aspects of economies of scale. Again, this is because the typical costs of production, treatment, and distribution all decrease on a per unit basis as the total amount of water involved increases. This increase, however, would theoretically be only up to a point where production would reach such a level that water would become scarce enough to cause the costs to start rising for each additional unit processed. Whenever average costs are increasing, they are exceeded by marginal costs. Therefore, as is the case with the demand curve shown in Fig. 7, the average cost exceeds the price when the equilibrium point falls in this range. This means that an actual monetary operational loss would be incurred by the water utility by operating at the equilibrium point under these conditions. This loss is shown graphically in Fig. 7. Proponents of marginal cost pricing for water utilities point out that this fact should not deter them from its use because the operational loss incurred can be made up by various means and the use of any other price would cause non-optimal use of the water resource. The most often suggested means for making up the operational loss is some form of tax-financed subsidy. But this in effect only disguises the full costs to the users and in actuality they are paying more than the marginal cost of supplying them with water. To protect themselves against this type of operational loss, therefore, most water utilities tend toward an average-cost-pricing system, that is, they set prices equal to average costs (which in this case include a normal rate of return on invested capital) in order to assure the financial solvency of the utility.
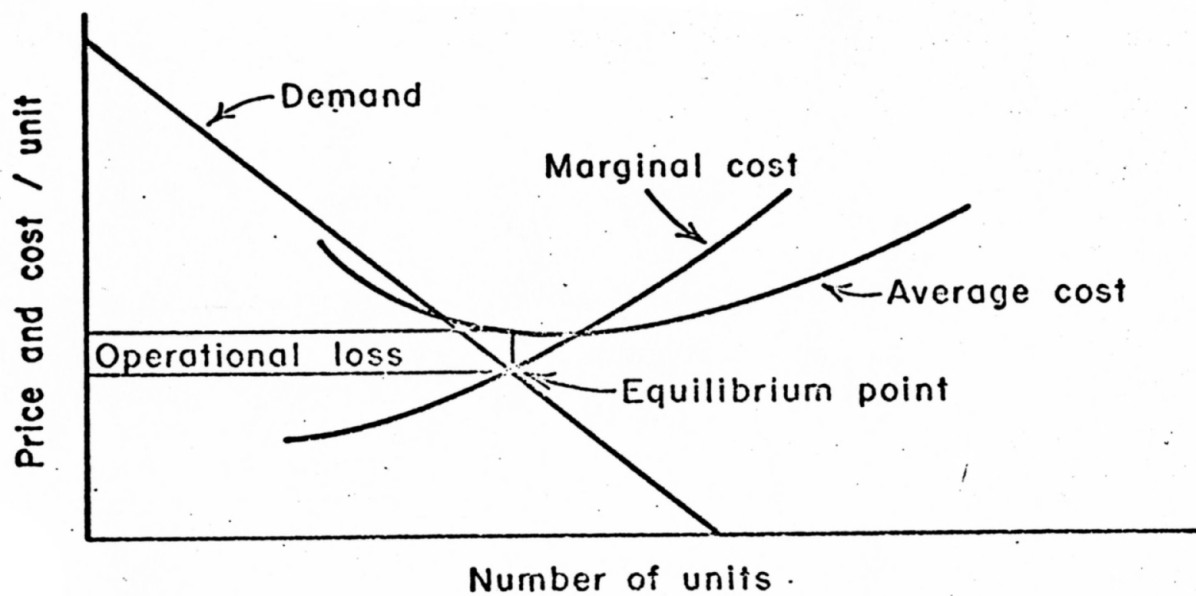
Figure 7. Equilibrium Condition where Average Costs are Decreasing with
Respect to Marginal Costs.

In the case where the demand curve intersects a rising average cost curve as shown in Fig. 8, the equilibrium-point production level will produce an operational gain to the utility. This is because the average costs are less than the price. Here, if the average-cost pricing system were used, for each of the additional units produced beyond the equilibrium point, the marginal cost (the additional cost of supplying the unit considered) is greater than the amount that anyone is willing to pay for the extra unit supplied. This violates the basic premise stated earlier and again leads to non-optimal use of the water resource.

Therefore, the marginal cost pricing system can be effectively used by a central water agency to simulate an ideal, competitive market as long as average costs are increasing. But, in the range of decreasing average costs, practical application of the theory raises the problem of how to recoup the operational loss.

The central water-control agency as postulated here is functioning as a publicly owned monopoly, but as seen in the previous section, it can be operated as if competitive conditions existed in order to maximize overall economic efficiency. Moreover, when average costs are increasing, the competitive-like operation not only maximizes overall economic efficiency, but also affords an operational gain in dollars to the water-control agency. This gain can either be retained by the public agency and accumulated as a future building fund or it can be redistributed to the consumer's as a dividend.

Let us now turn full attention to operational gain. It can be increased if we are willing to deviate from the perfectly competitive market structure and forego some amount of consumer satisfaction. This increase would be brought about by decreasing the production level. The reason that a water-control agency may be interested in this idea is twofold. In the face of impending expansion requirements, the operational gain would be accumulated as a building fund, and this would also accomplish the dual purpose of conserving water supply. This concept is shown in Fig. 9. Thus, if it were decided to operate at the sub-optimal level F instead of the competitive supply level I, the operational gain would increase from an amount equal to the area of rectangle BCKJ to an amount equal to the area of rectangle ADHG. The total consumer satisfaction would decrease from an amount equal to the area OEKI to an amount equal to area OEHF. The total amount of water used would decrease from amount I to amount F.

In the economic sense, a monopolistic condition exists if there is only one supplier and he is allowed to be concerned only with maximizing the net revenue to his operation.

The above-mentioned course of action is exactly the direction in which the monopolist would move as he endeavored to maximize his profits. Therefore, from his viewpoint the consumer demand curve becomes an average revenue curve, and he considers himself able to operate at whatever production level he chooses. This relationship is

$$r_A = \frac{R}{Q} = \frac{pQ}{Q} = p \qquad\qquad (2-17)$$

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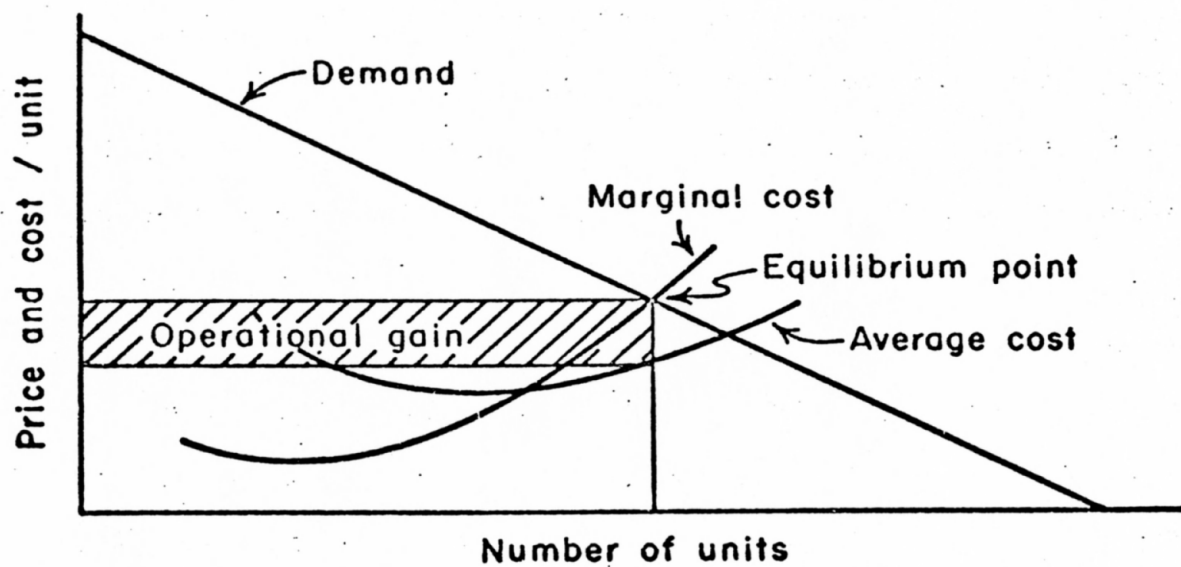
Figure 8. Equilibrium Condition where Average Costs are Increasing.
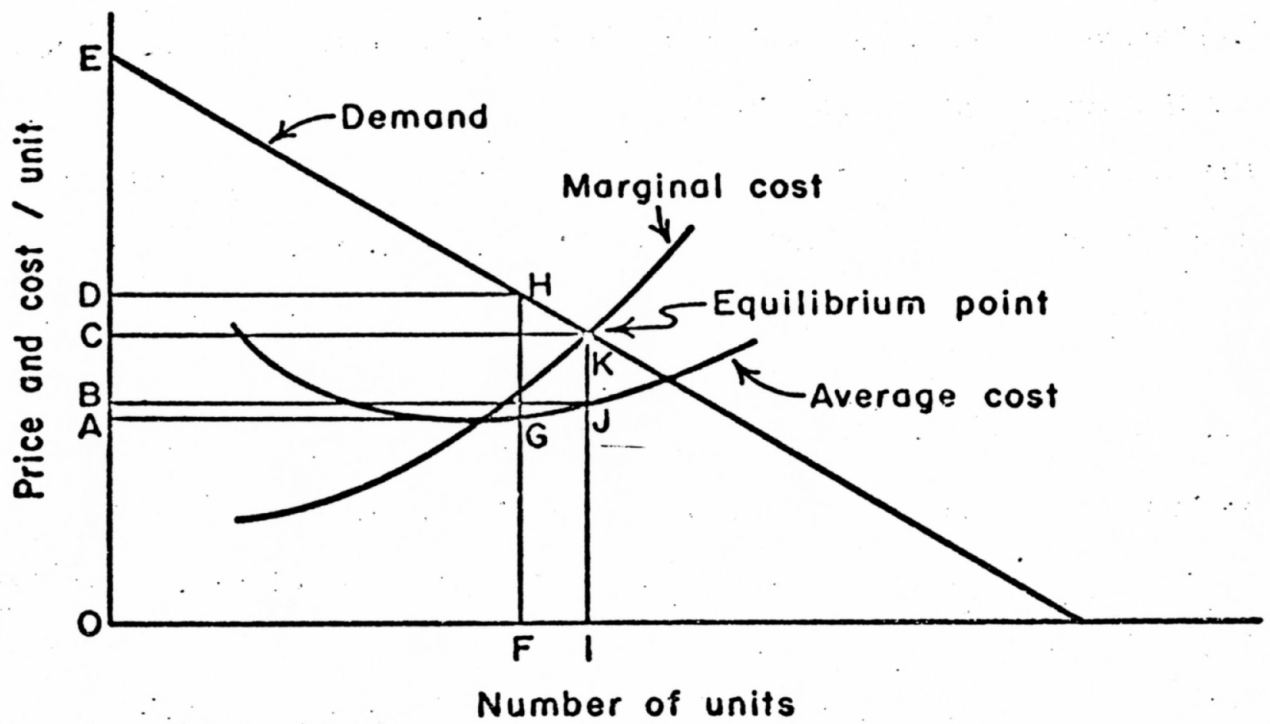
Figure 9. Effects of Operating at Non-Equilibrium Point.

where r_A is the average unit revenue at any level of production Q, R is total revenue, and p is the unit price taken from the demand curve at production level Q.

It is true, of course, that a municipal utility would never want to operate as a monopolist with the sole goal of maximizing profit; a utility, after all, only exists to serve the general public. The point is that the utility does face real, sloping demand curves and therefore is in a market characterized by monopolistic competition. We must understand this type of market as well as a purely competitive one.

Since the monopolist's only motive is to maximize his net revenue, he would continue to cut back his production to the point where his marginal revenue equaled his marginal cost. From the consumer's demand curve, the monopolist can derive his total revenue curve R as in (2-10). He can also derive his marginal revenue curve r_m from

$$r_m = \frac{dR}{dQ} = \frac{d(r_A Q)}{dQ} .$$
(2-18)

The monopolist's optimal production level and his corresponding maximum net revenue is shown in Fig. 10.

If the central water-control agency for the dual-purpose reason mentioned above desires to operate toward the monopolistic side of competition and maximize its net revenue, it can be seen from Fig. 9 that this is the same as maximizing the following objective function

$$Z = pQ - c_A Q$$
(2-19)

where p is price per unit, c_A is average cost per unit, and Q is any level of production. At this point, it is important to note the similarity between equations (2-19) and (2-7). Here, pQ is equivalent to total revenue R and $c_A Q$ is equivalent to total cost C. Therefore, the only difference between the two equations is that (2-7) is dealing with multiple uses and sources while (2-19) is only concerned with a single use and a single source. This, then, establishes the economic basis of (2-7).

Aggregation and Deaggregation of
Demands and Costs

The foregoing economic interpretations of equations (2-6) and (2-7) have been in the context of single demand curves and single cost curves. The curves can either be thought of as representing single sources and uses of water or in the more general sense as representing the aggregate demands and costs of multiple uses and sources. In most water-supply problems, it will behoove us to see how the aggregate curves can be broken down into their individual components and, conversely, how the individual curves can be aggregated. In this discussion it will be assumed that there is complete independence between the individual demands as well as between the individual costs.

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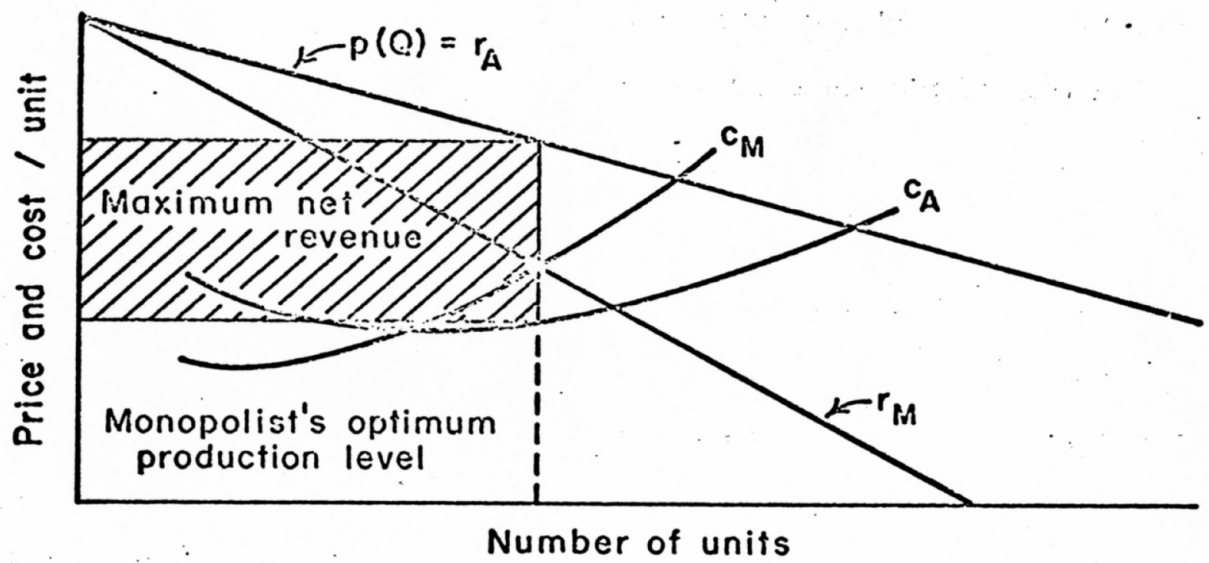
Figure 10. Optimum Monopolistic Production Level.

This aggregation and deaggregation of demands and costs will be looked at first within the context of a competitive market. Where complementary uses are involved, the individual demand curves can be added together to form an aggregate demand curve which will represent the total amount of water which will be taken by all consumers in the market at all possible prices. The aggregate demand curve is obtained by adding, for each ordinate (price), the abscissa of the individual curves of all the buyers in the market, i.e., the quantity demanded by each buyer at that price. The intersection of the supplier's marginal cost curve and this aggregate demand curve will then indicate the price to be paid by all users and the total amount of water that the supplier will furnish the market. This concept is shown in Fig. 11. Here in supplying demands D_A and D_B, the supplier would furnish a total amount Q_T. Consumer A would receive Q_A units, consumer B would receive Q_B units, and each would pay the same unit price p for the water. It can be seen that the amount of water transferred to A would be different if B were not present in the market; likewise, the amount of water transferred to B would be different if A were not present. Complementary sources of water can also be analyzed within the competitive market framework. In this case, the consumer considers water from any of the sources as the same commodity, but in obtaining and distributing this water the supplier incurs different costs depending on the source and quality.

Fig. 12 shows a demand curve p(Q), two average-cost curves (representing costs to the supplier to furnish water of the same quality but from different sources to the user), a total average-cost curve C_{AT} (horizontal summation of the individual average costs), and a total marginal-cost curve c_{MT} (derived from c_{AT} as in 2-12). Here, the supplier will operate at point (p, Q_T) and supply the demand with quantity Q_1 from source 1 and quantity Q_2 from source 2.

Finally, if both complementary sources and uses are involved as shown in Fig. 13, the water transfer will take place at the point (p, Q_T) with quantity Q_1 being supplied from source 1 and quantity Q_2 from source 2. Further, it is also shown that quantity Q_A will go to demand A and Q_B to demand B, each user paying a unit price p for the water. In this case

$$Q_T = Q_A + Q_B = Q_1 + Q_2 \qquad (2\text{-}20)$$

and water can be transferred from either source to either use while maintaining maximum economic efficiency. Operating the central water-control agency in such a way as to simulate the competitive market as depicted in Fig. 13 is the same as maximizing the following objective function

$$Z = \int_0^{Q_T} D_T(Q)\,dQ - pQ_T \qquad (2\text{-}21)$$

where Z represents net consumer satisfaction, the integral is total consumer satisfaction as shown in Fig. 6, and pQ_T is the total amount paid for water by the consumers.
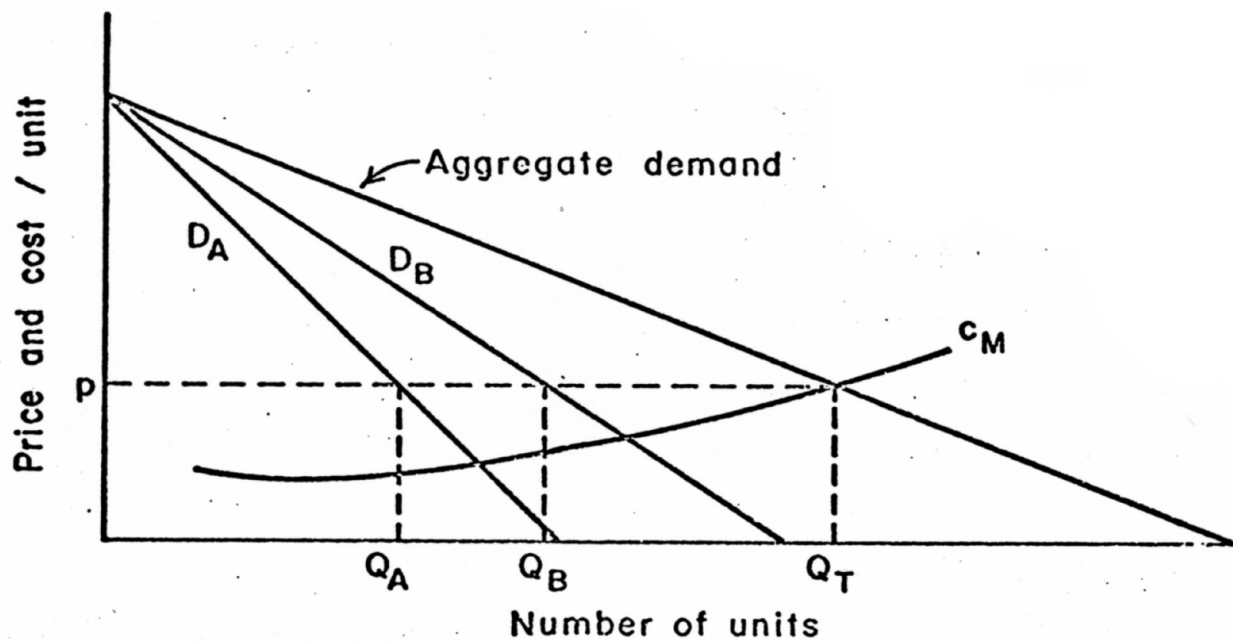
Figure 11. Complementary Uses and Aggregate Demand, Pure Competition.
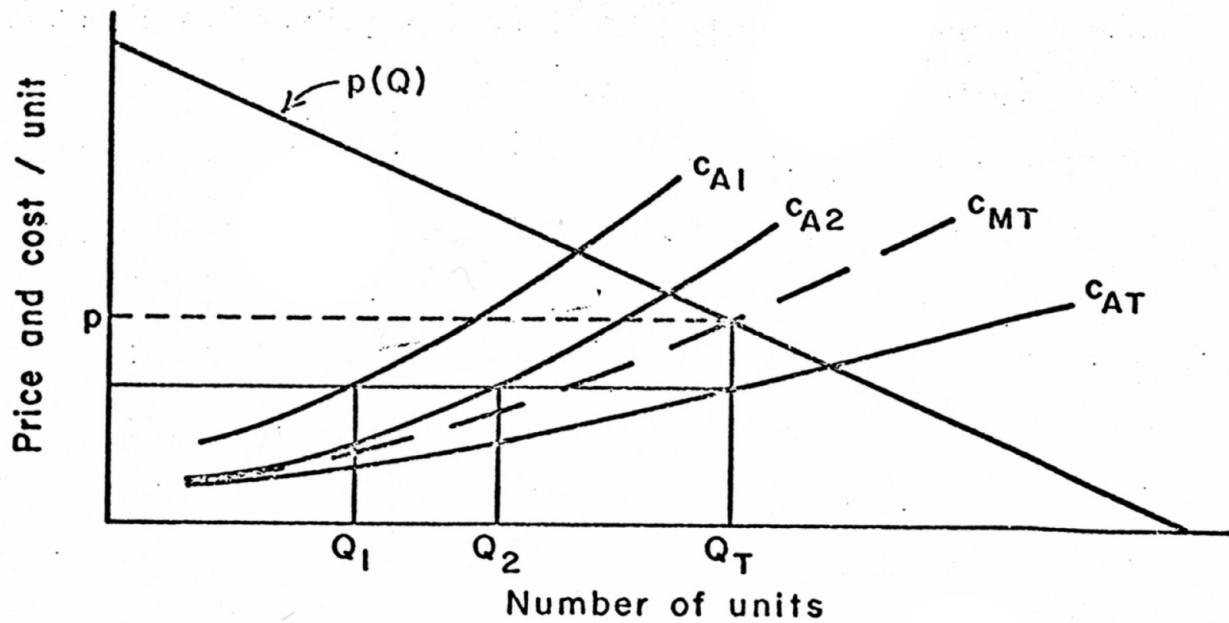
Figure 12.   Complementary Sources and Aggregate Cost, Pure Competition.
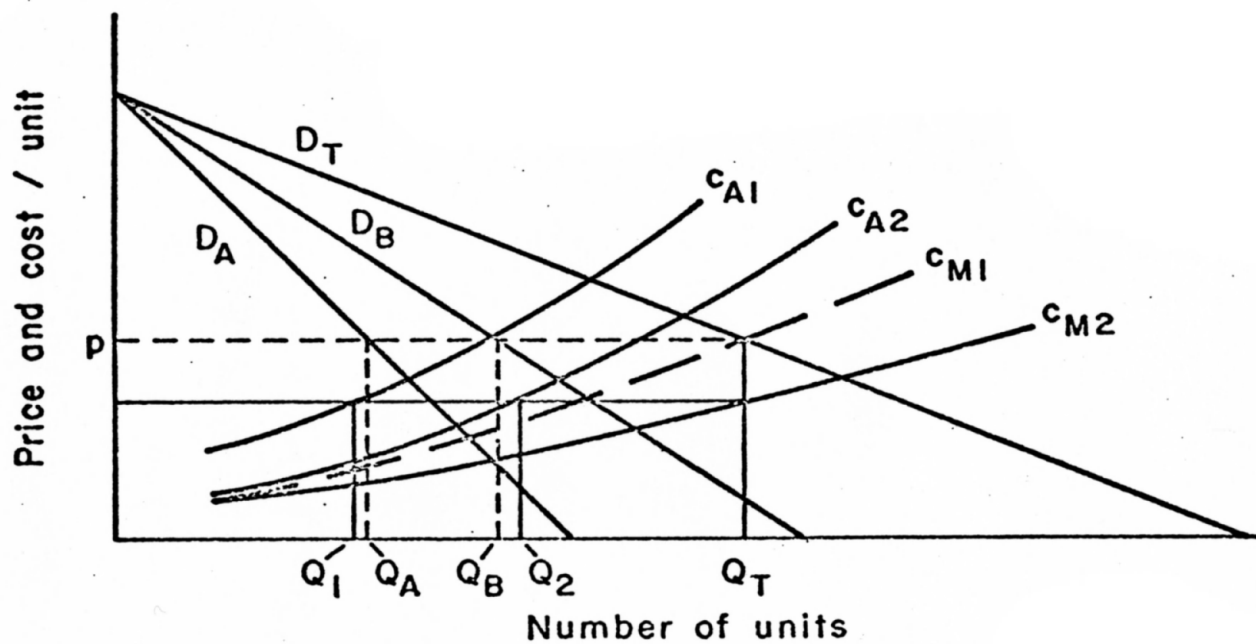
Figure 13. Complementary Uses and Sources with Aggregate Demands and Costs, Pure Competition.

The treatment of multiple uses and sources will now be reviewed in the framework of a monopolistic market. The central water-control agency will be cast as the monopolist who faces several different demands with each representing a different type of user who has at his disposal several sources of water with which to supply these demands. The user will be involved with a different cost curve for each source-use combination, i.e., if there are three sources and three uses there will be nine characteristic cost curves. This is because different costs of obtaining, treating, and distributing water are incurred for each source-use combination. The monopolist's objective is to determine the quantities of water which when transferred from each source to each use will maximize his net revenue. A situation involving three sources and two uses is depicted graphically in Fig. 14. From demand D_A, the supplier's marginal revenue function r_{MA} arising from this use can be obtained. The total average cost of supply use A, c_{ATA} is found by horizontally summing the individual average costs of supply use A from supplies 1, 2, and 3. These curves are labeled c_{A1A}, c_{A2A}, and c_{A3A}. The total marginal cost of supplying use A, c_{MTA} is obtained from c_{ATA}. The point of maximum net revenue is then indicated by the level Q_{TA} at which marginal revenue equals marginal cost, i.e., the intersection of r_{MTA} and c_{MTA}. The price to be charged user A, p, is also indicated by this point. When the total average cost is broken down into its component parts as shown in Fig. 14, the desired quantities of water Q_{1A}, Q_{2A}, and Q_{3A} to be transferred from sources 1, 2, and 3 to use A are indicated as well as the average unit cost of supplying use A, namely c_{AA}. The same analysis can be followed for use B to discover the quantities Q_{1B}, Q_{2B}, and Q_{3B} to be transferred from sources 1, 2, and 3 to use B.

The monopolist's objective in the situation depicted in Fig. 14 can be expressed in equation form as

$$Z = p_A Q_{TA} - c_{AA} Q_{1A} - c_{AA} Q_{2A} - c_{AA} Q_{3A} + p_B Q_{TB} - c_{AB} Q_{1B}$$
$$- c_{AB} Q_{2B} - c_{AB} Q_{3B} . \qquad (2-22)$$

Equation (2-22) has the same form as equation (2-19) and is related to equation (2-7) in the same way. It can be expressed in general form as

$$Z = \sum_{j=1}^{m} p_j Q_j - \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij} q_{ij} \qquad (2-23)$$

where p_j is the unit price to be charged user j, Q_j is the total amount of water used by j, c_{ij} is the cost of transferring a unit of water from source i to use j, and q_{ij} is the amount of water transferred from source i to use j.

Thus far in this section, objective functions which can be used in water-allocation problems have been formulated and their economic rationale
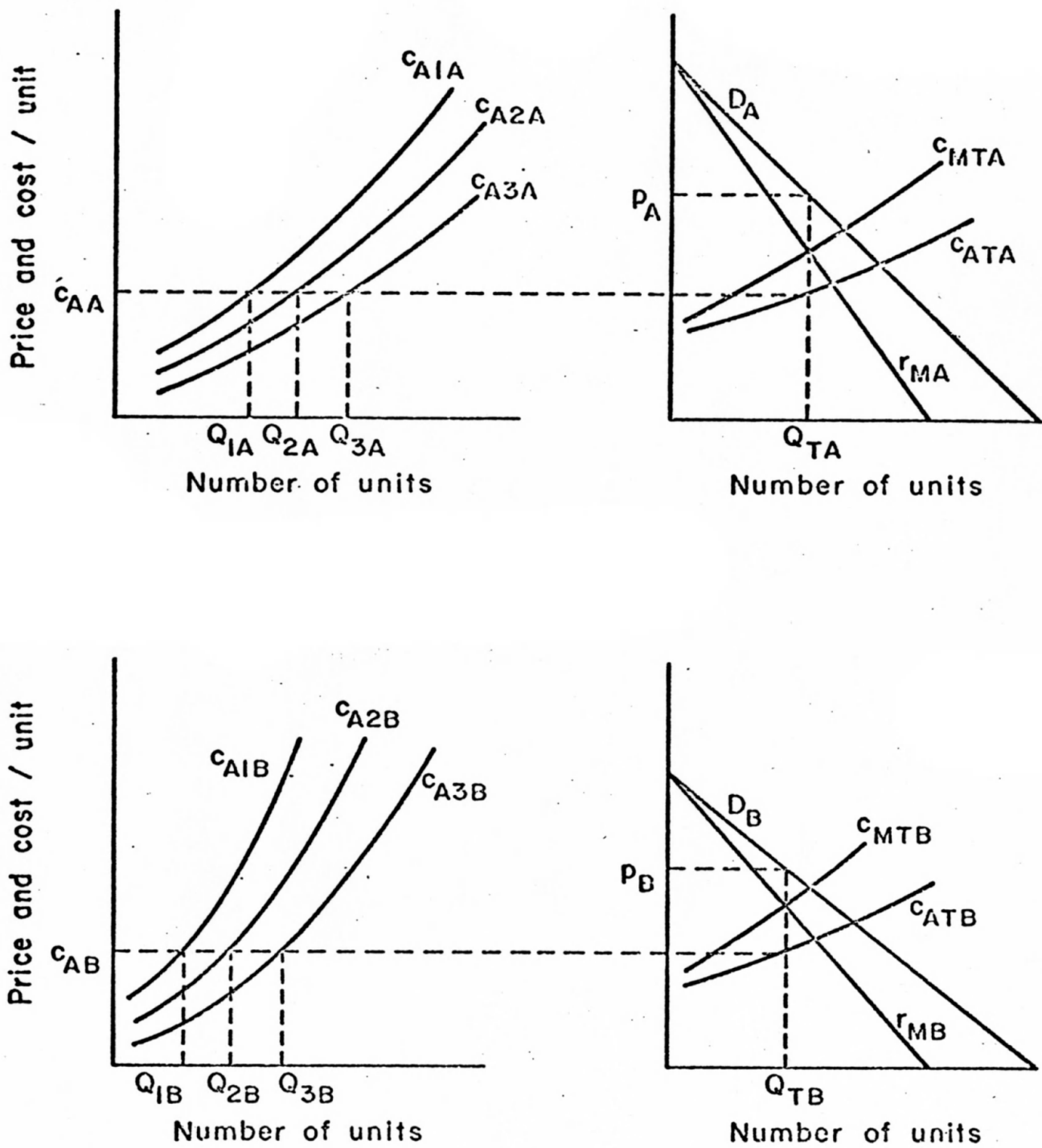
Figure 14. Complementary Uses and Sources with Aggregate Costs, Monopoly.

has been pointed out. It should be emphasized that constraining relations will usually always act to keep the system from reaching a theoretical equilibrium point or point of maximum profits. This simply means that the system will have to be operated as close to an unconstrained optimum as the constraints will allow.

Extensions into Space and Time

All of the equations presented thus far in this chapter have been presented in a single time frame and all of the sources and uses have been considered as independent with respect to their associated costs and water availabilities. We have really been thinking in terms of lumped systems, that is, lumped to the point where we can be assured of the independence criterion. For example, if one of the sources of water is a groundwater aquifer, our presentation so far requires that we consider the aquifer as a single source of water. We cannot theoretically consider separate well-fields or individual wells as single sources of water in these models because they are intimately related, i.e., the amount of water withdrawn from one well affects the amount of water which can be withdrawn from another well and the independence criterion does not hold. The same loss of hydrologic independence occurs when we consider surface sources and groundwater sources which are hydraulically connected or if the surface source recharges the groundwater source intermittently, i.e., the hydraulic connection is not always present. Aquifers, however, have been modeled as distributed systems, and currently the trend of research in this field is toward combining the management models discussed here and the aquifer analogs. This area of research will be discussed in a later section. Suffice it to say now that the independence problem must be overcome first.

The same independence criterion must yield an affirmative answer when applied to dynamic or multistage water-supply planning problems. For instance, in allocating water from various sources to several uses over a certain planning horizon, we have to make the allocation during each time period (or stage) within the horizon. The question then is this. Are the states of the variables describing the system at each stage independent of their states in other stages or are they dependent? If they are independent, the optimal allocations can be made at each stage, and the overall problem is optimized by merely considering the entire series of single stage decisions. If they are dependent, however, the optimal answer is only obtained by considering the entire stream of stages in toto. In other words, decomposition is impossible. In either case, any of the objective functions presented thus far can be expanded to encompass the time dimension by adding a "t" subscript to each of the variables. For example, (2-23) can be expanded to

$$ Z = \sum_{j=1}^{m} p_{jt} Q_{jt} - \sum_{j=1}^{m} \sum_{j=1}^{n} c_{ijt} q_{ijt} \tag{2-24} $$

where p_{jt} is now the unit price charged user j during time period t, Q_{jt} is the total amount of water used by j during the period t, c_{ijt} is the cost of transferring a unit of water from source i to use j during time period t, and q_{ijt} is the amount of water transferred from source i to use j during

time period t. Therefore, if independence between variables prevails, (2-24) can be decomposed into a series of individual optimization problems each containing only variables with the same "t" subscripts. If dependence prevails the overall objective function must be optimized as one large problem.

To make this point perfectly clear, a simple problem involving obvious dependencies in the form of "carry over" costs will be briefly stated at this point. The problem was posed by Duckstein and Kisiel (1968).

They considered the problem of a temporal capital investment policy for wells over two stages (two years). Given the initial production level X_0 = 50 acre-feet/year and requirements at each stage as D_1 = 30 acre-feet and D_2 = 35 acre-feet, let the cost of change in production level at each stage be R dollars/acre-feet,

$$R = 0.5(X_i - X_{i-1})^2 , \tag{2-25}$$

and the cost of surplus storage be C = $5/acre-feet. Assume no shortages. Find production schedule that minimizes costs. If we couple the two one-stage models, in period one (i=1) the solution

$$\min R_1 = 0.5(X_1 - 50)^2 + 5(X_1 - 30) \tag{2-26}$$

subject to

$$X = X_1 \geq 30 \tag{2-27}$$

is X_1^* = 45 and R_1^* = \$87.50, and in period two (i=2) the solution to

$$\min R_2 = 0.5(X_2 - 45)^2 + 5(X_2 - 35) \tag{2-28}$$

subject to

$$X_2 \geq 35 \tag{2-29}$$

is X_2^* = 40 and R_2^* = \$37.50.

The minimum cost of the production schedule is $R^* = R_1^* + R_2^*$ = \$125 at $\underline{X}^*$ = (45,40). On the other hand, if we solve it directly as a two-stage problem, the solution to

$$\min[0.5(X_1 - 50)^2 + (X_1 - 30) + 0.5(X_2 - X_1)^2 + 5(X_2 - 35)] \tag{2-30}$$

subject to

$$X_1 \geq 30 \text{ and } X_2 \geq 35 \tag{2-31}$$

is, by the calculus, X^* = (40,35) and R^* = \$112.50 in comparison to \$125.00.

But if the model were such that no year-to-year storage were necessary, no costs were involved in changing the production level, and requirements were specified at each stage, the same optimal allocation would take place regardless of the method used.

If the independence problem can be overcome, the next deterrent to the practical use of distributed-system models in water management is the availability of physical and economic data pertaining to each sub-source and each sub-use. As will be seen in chapter 3, economic data are particularly hard to estimate with much assurance of accuracy, and the task become more difficult as small units of area are analyzed.

Model Refinement under Uncertainty

In this section, we will discuss how the models described thus far might be handled in cases where the model parameters are not deterministic. Davis (1968) in dealing with the dimensions of uncertainty in water-planning models points out that the degrees of information surrounding particular model parameters may range from complete knowledge of the probability distribution to lack of knowledge of even the range of possibilities. Different degrees of model refinement are possible depending on the degree of uncertainty surrounding the parameters. In our context, the demand and cost functions in the objective function are estimates of the economic system and the amounts of water available in the constraints are estimates of the physical system; thus, some or all of the model parameters may in reality be random variables. The most obvious source of physical randomness in the water-allocation models is the case where a source of water is streamflow or, in a groundwater situation, where natural recharge is considered. On the other hand, when groundwater is being mined as a source of water there really is very little probability that a planned amount of withdrawal would not be available. However, future water quality is a very real uncertainty in both groundwater and surface water situations.

Sensitivity Analysis

If the systems being modeled really are non-deterministic but we have a deterministic model to plan their operation, we are in essence substituting mean or expected values of the parameters in question. This is a completely valid approach if no statistical evidence concerning the parameters in question can be obtained. For instance, in the case of economic data such as demand functions, any type of variability index would be a pure guess. In cases such as these, some type of sensitivity analysis can be useful, that is, the parameters in question can be varied over a certain range and the individual or combined effects of these perturbations on optimal values of the objective function can be noted. These type analyses afford us a way to present results embodying our admitted lack of knowledge about certain parameters and also point out directions in which further effort toward identification of parameter input should be expended. Data refinement is most worthwhile (with respect to the model) where it is concerned with the input parameters which cause the largest variations in output upon being perturbed the least.

If any of the random input parameters can be described by some types of variability indexes, these indexes can sometimes be incorporated into the operating model. Two such incorporations will be mentioned here.

Known Probabilities for Certain Parameter Values

An approach referred to as stochastic programming by Hillier and Lieberman (1968) offers a way of reformulating a mathematical model in order to include certain information we may have concerning the randomness of any of the model parameters. The type of information incorporated here concerns a series of values for any given parameter and the probabilities of occurrence of each of these values (the series of probabilities summing to one). The deterministic objective function is then replaced by an "expected value" function E(Z). For example, (2-23) would become

$$E(Z) = \sum_{j=1}^{m} E(p_j)Q_j - \sum_{j=1}^{m} \sum_{i=1}^{n} E(c_{ij})q_{ij} \; . \tag{2-32}$$

The most logical place for randomness to be considered in the pricing model of (2-23) is in the right-hand side constants of the availability constraint of the type shown in (2-2). Let us show here how the problem would be reformulated if one of the Q_i's in the set of (2-2) constraints would be expressed as a series of values of the following type

$$x_k Q_{nk} \; ; \qquad k = 1, 2, \ldots, \ell \tag{2-33}$$

where x_k is the probability associated with the quantity Q_{nk} and the x_k sum to one. Equation (2-32) would then become

$$E(Z) = \sum_{k=1}^{\ell} \sum_{j=1}^{m} x_k p_i Q_j + \sum_{k=1}^{\ell} \sum_{j=1}^{m} \sum_{i=1}^{n-1} x_k c_{ij} q_{ij} \tag{2-34}$$

and would be subject to the constraints

$$\sum_{j=1}^{m} q_{ij} \leq Q_i \; ; \qquad i = 1, 2, \ldots, n-1 \tag{2-35}$$

and the random nth constraint

$$\sum_{j=1}^{m} q_{nj} \leq x_k Q_{nk} \; ; \qquad k = 1, 2, \ldots, \ell \; . \tag{2-36}$$

Therefore, in this context, the objective function would be expanded by ℓ terms and the number of constraints would be increased as shown in (2-36). If more than one model parameter were considered random, we would become involved in joint probabilities and the equations would have to be expressed in matrix form.

Known Probability Distributions for Certain Parameters

Charnes and Cooper (1959) have developed an approach called chance-constrained programming, which can be used to modify models so that feasible solutions are allowed to have a certain probability, less than one, of violating a constraint. Explicitly, this formulation would allow us to

replace the availability constraints of (2-2) with a probability function P_r as follows

$$P_r\{ \sum_{j=1}^{m} q_{ij} = Q_i \} \leq x_i ; \qquad i = 1, 2, \ldots, n \qquad (2-37)$$

where the x_i are specified probabilities between zero and one. Within this framework we will again maximize the expected value function of (2-32), but the system of constraints will have to be modified differently than in the previous example. Hillier and Lieberman (1968) show how the constraints of (2-37) can be converted into legitimate inequality constraints if the Q_i are assumed to have normal distributions. Without presenting the details, our availability constraints could be reduced to

$$\sum_{j=1}^{m} q_{ij} = E(Q_i) + K_{x_i} \sigma_{Q_i} ; \qquad i = 1, 2, \ldots, n \qquad (2-38)$$

where $E(Q_i)$ and σ_{Q_i} are the mean and standard deviation of Q_i, respectively, and K_{x_i} is given as $[Q_i - E(Q_i)]/\sigma_{Q_i}$. As Hillier and Lieberman (1968) explain, the objective of this type of programming is to select the "best" non-negative solution that "probably" will turn out to satisfy each of the original constraints (assuming the x_i probabilities are reasonably close to one) when the random variables take on their values.

Application of Optimization Techniques

Up to this point in the discussion we have employed implicit, generalized functions to model the water-supply systems. It is desirable in a presentation such as this to use these implicit functions so that we do not become overburdened with detailed formulations and also so that the presentation will encompass many possible individual system peculiarities. This type of modeling has enabled us to present the essence of several different concepts in an easily understood manner. The real substance of a systems study, however, lies in determining the real physical, economic, or social relationships between the variables, i.e., it requires explicit formulation. This is because, in the end, the water resource analyst must produce numerical results which are the ultimate purpose of mathematical models in the first place. To this end, then, in this section we will consider some of the various mathematical methods which can be used to produce these results. The emphasis will be on associating the methods with general problem types, not on amplification of any particular method since a vast literature on the latter is currently available. We will attempt to show how this whole gambit of tools can be applied individually and in consort to optimize our water-resource models.

The first thing we want to know is whether or not the problem is decomposable, that is, can it be broken down into a series of independent subproblems which can in turn be individually optimized and then rejoined

to yield an overall optimum. It seems as if many problems may be of this type, but the basis for decomposition can be for a varied number of reasons and actually depends on the specifics of the problem. We will see an example of this in the next chapter.

Whether we are considering the whole or independent subparts, the next thing to look at is the constraint set. If no constraints exist, solution by the calculus is possible. If only equality constraints are present, Lagrangian multipliers provide an easy means of optimization. If the constraint set contains inequality constraints, we are usually forced into using some type of mathematical programming; but, if the problem is simple, some sort of informal search technique can sometimes be devised. The latter would mean, for example, optimizing the objective function alone and first of all noting which constraints are effective and which are not. Then set the effective constraints as equalities and reoptimize with the help of Lagrangians; continue the process until the objective function is optimized and the constraints are satisfied. Informal search cannot really be explained in general terms because it depends entirely on the problem structure.

Formal search techniques have been developed, however, which are coded and help us immensely in our optimization efforts. Linear, quadratic, and convex programmings are actually search techniques, and their development along with a certain decomposition principle, namely, dynamic programming, has afforded a great impetus to the entire field of operations research, let alone the study of water-resources systems. The formal search techniques are algorithms which have one thing in common -- they guarantee that each iteration is closer to the optimum than its predecessor. Furthermore, both linear and quadratic programming problems have closed-form solutions, and the algorithms indicate when this closed-form solution has been reached. Convex programming algorithms do not have closed-form solutions, but since with each iteration the solution is bettered, solutions very close to optimum can be reached.

Objective functions which are linear, such as (2-3) and (2-5), and constrained by linear inequalities can be solved with linear programming. Piecewise linearization of convex functions also yields itself to linear programming, however, and since linear programming codes are so well understood and capable of handling such a large number of variables, they can be used in this way to solve some nonlinear programming problems. All of the other objective functions besides (2-3) and (2-5) can be nonlinear and, therefore, the possibility exists of solving them all with linear programming codes.

Quadratic programming algorithms are variations of the basic simplex method of linear programming and were designed to handle quadratic objective functions with linear constraints. This condition leads to objective functions containing terms with integer powers only of the first and second order, i.e., the function is quadratic. This type of programming is employed in the example problem given in the next chapter. If the demand and cost functions could not be assumed linear, then either convex programming (of which there are several algorithms) or piecewise linearization would have to be employed.

Dynamic programming is actually a decomposition principle used in multistage problems which arise when outputs from one stage or decision are also inputs to other stages or decisions. In other words, a series of decisions are to be made in time or space, but because the individual decisions (but not the properties of each stage) are in some way dependent upon each other, the optimal overall policy will not necessarily be found by considering each decision separately. Therefore, these problems must be handled by trying to find optimal values for all decisions simultaneously. Dynamic programming is a method which while considering the string of interacting decisions in toto, also splits the problem into subproblems, each involving only one variable or a portion of the variables which can be considered one at a time. This type of decomposition is best applied in water-supply problems when we are considering allocation to a large number of uses from only one source or to a single use over a number of time periods from a single source. Once the model is established, a functional relationship is derived which distributes any total allocation to all of the component subproblems.

After one has developed a "feel" for a given water-supply model it is often possible to decompose and use several different optimization techniques on the different components ranging from the calculus to some type of mathematical programming. Actually mathematical programming techniques find their fullest utilization only when the problem to be optimized is very large. In typical water-supply problems, however, often the problem is not very large because there simply are not large numbers of sources and uses to be considered. In these cases, it is felt that it is best to use "home-made" search techniques and in so doing come to a better realization of the intricacies of the problem at hand. Furthermore, the premise that, with a minimum period of programming instruction, the computer can assume the entire burden of calculation is simply not valid even though it is expressed openly in many texts. Oftentimes, bitter experience with the computer combined with programming problems makes an experienced analyst very cautious in examining his reams of numerical printouts. Thus, in the end he is finally forced to the old-fashioned numerical techniques to verify the calculations for at least a sample case.

Recent Investigations

Recent applications of operations research methods in the water-supply field have not been mentioned up to this point because the intent was to describe the various objective functions and their interrelations in general terms and to avoid any specifics. Now that the groundwater has been laid, however, we can take a look at the pertinent literature to see where each work fits into this framework of objectives.

The requirements approach using an objective function as given in (2-3) was used by Dracup (1966) to model the water-supply system in the San Gabriel Valley in southern California. He compiled data on the quantities of water available from five sources and the requirements by four uses over a thirty-year planning period. A network of unit costs associated with the possible source-use transfers was also estimated. These three types of data constitute the needed data input for this type of model. Dracup used a general linear programming code to find the least-cost combination of

meeting the given requirements. This type of model also fits quite nicely into the framework of the transportation method of linear programming which allows a much simpler means of solution if one is concerned about such things. It is definitely conceptualized easier if set up as a simple transportation network. There has been some debate as to whether Dracup's model need be optimized in toto, over the entire time period, or whether it can be decomposed on annual basis and solved year by year. Since there are no carry-over costs or other parameter dependencies, it would seem that decomposition would be possible if so desired. These comments should not detract from the beauty of Dracup's work however, which lies in his method of presentation -- it is easily read and easily understood. In order for our work with operations research to permeate the practical realm, more lucid presentations such as Dracup's are definitely needed in the water-supply field.

Objective functions using constant unit net benefits as shown in (2-5) were used by McLaughlin (1967) to model parts of water-supply systems in several South American river basins and by Heaney (1968) to model part of the Colorado River Basin water-supply system. Both used linear programming codes to find the values of the variables (amounts of water used by subregions) which maximized net benefits. In addition to the availability constraints, Heaney also bound the upper limits of water transfers by using the projected water requirements as upper limits. These types of models are usually optimized without requirements constraints, however, and the data then needed consists of availabilities and constant unit net benefits. As explained earlier, the latter dictates an inelastic demand for water which we have implied is not the case. Both of these models are capable of incorporating the elasticity concept, however, but then they would have to be solved using piecewise linearization of the total benefit function if linear programming solutions were to be retained. One reason for using objective functions of the form (2-5) is a lack of price elasticity data. When (2-5) is used, an assumption of complete price inelasticity is implicitly made.

Moving on to the nonlinear case as encompassed in objective functions of the type (2-6), Buras (1963), Bear and Levin (1964), and Burt (1964) all used a modified form of this to maximize the present value of net benefits over time accruing from groundwater use and the conjunctive use of a groundwater and surface reservoir. Buras and Bear used dynamic programming exclusively whereas Burt used a combination of search techniques (trial and error) to optimize their models. Essentially, input consisted of demand curves or total benefit approximations, costs, and a planning period. Then using the total number of years in the planning period, a year-by-year pumping policy was obtained which maximized present value of net benefits. The pumping schedule is then referred to as the "optimal yield" of the aquifer rather than the "safe yield."

Domenico (1967) approached the "optimal yield" problem in a slightly different manner. Input in Domenico's model consists of initial depths of water, a constant annual pumping rate, and an interest rate. Then, output is the number of years water should be mined at the given rate to yield maximum present net worth. Using a modification of (2-6) Domenico solves his mathematical model with the calculus.

Flinn (1969) considers the allocation of the annual safe yield from a
servoir using an objective function of the type (2-5) and terms the total
nefits "net social payoff" after Samuelson (1952). His models are based
ideas presented by Takayama and Judge (1964). The use of linear demand
hedules by the various users results in a quadratic objective function.
e use of quadratic programming is advocated to solve for the optimum
atial allocation of water. Flinn's work is much along the lines of the
terial presented in the next chapter of this dissertation in that
sically the same input (costs, demands, and water availabilities) is
quired. Flinn, however, chooses to operate his system as close to the
mpetitive market equilibrium point as possible, whereas, the application
the next chapter operates as a constrained monopoly.

Brown and McGuire (1967) were concerned with developing a pricing
licy for the Kern County Water Agency, California. They also adopted an
jective function of the type (2-15) and suggest implementing the price
anges through the means of a pump tax.

The groundwater portions of all the above-mentioned models have been
mped systems, i.e., basin-wide, average water levels have been used,
d this implies that static conditions are reached immediately after
mping disturbances take place (like extracting water from a bucket). The
y to correct for this physical discrepancy is to tie the management model
to an analog model of the groundwater basin so that the true distributed-
rameter system can be represented. Martin, Burdak, and Young (1969) have
ne just this for an agricultural problem in Pinal County, Arizona where
aquifer is being mined for irrigation needs. Objective function (2-5)
s been used allowing for linear programming to be employed in the
nagement portion of the model. After the management model is optimized
r one time period, the analog is operated to determine the new water
vels according to the pumping pattern determined by the management model.

We have shown in this discussion of the literature that operations
search methods can play an active part in the water-supply planning
oblem. Optimization techniques are available to handle almost any
oblem -- linear or nonlinear, lumped or distributed, spatial or time
riant -- if the necessary data are available.

CHAPTER 3

APPLICATION OF THE PRICING MODEL:

TUCSON BASIN, ARIZONA

In this chapter a very realistic problem, which is eminently suited to illustrate the use of the pricing model discussed in chapter 2, will be described and worked out. As the problem evolves, it also affords a good example of work within an area of operations research referred to as "solution strategy" (Geoffrion, 1968), that is, reduction of a large-scale optimization problem to a sequence of simpler derived optimization problems. We will be concerned with the problem of how to allocate present and future sources of water to various classes of water users in the Tucson Basin, Arizona. There is not enough water in the sense that all users cannot continue to enjoy their present rates of use at the present prices charged for water for any extended period of time. This situation exists because the water supply is pumped entirely from the groundwater reservoir within the basin at a rate exceeding the average annual recharge and alternative sources are all more expensive to obtain. Hence, costs will be continually increasing as activity within the basin grows with time. We will be concerned here with the optimal possible short-run policy, given the framework of existing market conditions.

After describing the hydrologic and physical setting along with the water-supply system as it exists now and as it is postulated to exist in the problem, we will discuss the various sources and costs of water and the availability constraints. We will next introduce the demand functions, formulate an objective function, discuss decomposition, and introduce a policy constraint. Finally we will give and discuss numerical results.

General Hydrologic Description

The Tucson Basin in southern Arizona is an intermontane trough within the Basin and Range province of the southwestern United States. It is about 1,000 square miles in area and is bounded on the east and west sides by mountains. A protrusion of the Santa Catalina Mountains from the east forms a four mile wide narrows at the northwest end of the basin out of which flow both occasional surface runoff and groundwater. The basin is drained to the northwest by the influent Santa Cruz River and a tributary, Rillito Creek. The basin is filled with semiconsolidated fluvial deposits eroded from the adjacent mountains. These deposits have been encountered at depths up to 2,000 feet in the center of the basin. Flood plains within the basin are as much as a mile wide and underlain by unconsolidated gravel and sand. The basin deposits and the flood plains are hydraulically connected and form the groundwater reservoir. Depth to water is 100 to 200 feet throughout most of the basin but ranges between 5 feet near the flood plains to 500 feet or more elsewhere.

Average annual precipitation varies from 11 inches on the basin floor to 30 inches on some of the surrounding mountain areas. The summer

thunderstorm season accounts for most of the surface runoff, but this runoff is less than one percent of the total rainfall.

The Water-Supply System

At the present time, as stated earlier, the water supply for the basin is pumped entirely from the groundwater reservoir within the basin. The total amount now used is about 150,000 acre-feet per year. None of the runoff is used in the basin because the streams are dry for long periods and there are no surface storage reservoirs. The metropolitan area of Tucson is located in the northern part of the basin. The city's Department of Water and Sewers in supplying most of the municipal and industrial needs of the area pumps about 40 percent of the total amount used in the basin. The rest of the water is pumped by a few small water companies which supply some outlying residential areas; individual industries, mainly mines in the southern part of the basin; farmers and other private users with their own wells. The largest of these private users in and near Tucson are the University of Arizona, Tucson School District No. One, and Davis-Monthan Air Force Base. The agricultural water is used to irrigate lands mostly lying along the Santa Cruz River northwest and south of Tucson.

The Tucson Department of Water and Sewers has hopes of obtaining overall control of the water resources within the basin and if successful in so doing could be considered as a monopolist with complete price-setting power for water users. In this problem it will be assumed that this type of control is an accomplished fact.

It will also be assumed that the "central water-control agency" which we have alluded to will have the following limited sources from which to draw:

(a) groundwater from within the basin,

(b) groundwater from Avra Valley (a neighboring basin),

(c) reclaimed waste water, and

(d) Central Arizona Project water.

The uses will be divided into the following categories:

(a) domestic,

(b) industrial, and

(c) agricultural.

After some deductions, the question is how should water from these sources be dispersed among domestic, industrial, and agricultural uses, and what prices (the policy instruments) should then be charged to maximize profits to the central water-control agency. Since this process of maximizing profits may lead to very high, socially unacceptable prices, a policy constraint will be added which attaches a certain disutility to upward changes from the existing situation.

Before proceeding, it will be helpful to refer to Fig. 15 which is a schematic of the possible transfers which can take place within the postulated model. The agricultural products capable of being produced in the Tucson Basin are identical to those which can be produced in Avra Valley, and it seems illogical to expect a water transfer between these two areas for the purpose of irrigation. Therefore, this particular type of transfer will not be considered in the model.

Water Availabilities

Complete management of the water resource, as envisioned in this problem, must include a deliberate and significant search for new technology. As Smith (1967) points out "the management problem is dynamic and the arsenal of tools required to cope with it must be equally dynamic." In this regard there are several sources of water supply available to the Tucson Basin in addition to the four deemed most pertinent in this example problem -- some of these options have been explored and some have not. They are listed in Table 1 along with references to pertinent investigations and cost figures if available. All of these options must be kept in mind; when and if the political-technological picture clears, they should all receive careful consideration. The four sources of water chosen to be studied here are the only ones which are either currently being used or for which plans to implement their use are currently being carried out. Therefore, they are the only sources which can logically be considered in a short-run policy study such as this and the only ones which will be discussed in terms of amounts of water available and costs of obtaining this water.

Groundwater, Tucson Basin

There is a very large amount of groundwater available in the Tucson Basin; however, it is not all extractable, it becomes more costly to obtain as the depth to water increases, and it is generally thought to be impaired in chemical quality and temperature as depth increases. Smoor (1967) studied the horizontal variation in chemical quality of the basin's groundwater and found it to be significant, yet regional distribution patterns did emerge. Knowledge of the groundwater quality is vitally important in the planning process because it is the biggest, single, possible deterrent to the accessibility of an enormous supply. The extent of this supply in just the upper 500 feet of the basin-fill material was estimated from data obtained from Matlock, Schwalen, and Shaw (1965) to be 18 million acre-feet. This is not an estimate of recoverable water but simply the volume in place using a value for specific yield of 0.15 and a present average depth to water of 250 feet. To put the immensity of this figure in perspective, the current rate of withdrawal could be sustained for some 90 years before this volume of water would be used. This is assuming an average annual rate of natural recharge of about 56,000 acre-feet per year which is an estimate associated with the above-mentioned specific yield of 0.15.

This supply is not certain, however, and the general philosophy of the regional water authorities has been to consider the Tucson Basin groundwater reservoir as a base supply, but also to be continually seeking outside supplies which when implemented will help "conserve" this base supply. The

Figure 15. Schematic of Possible Water Transfers Within Model
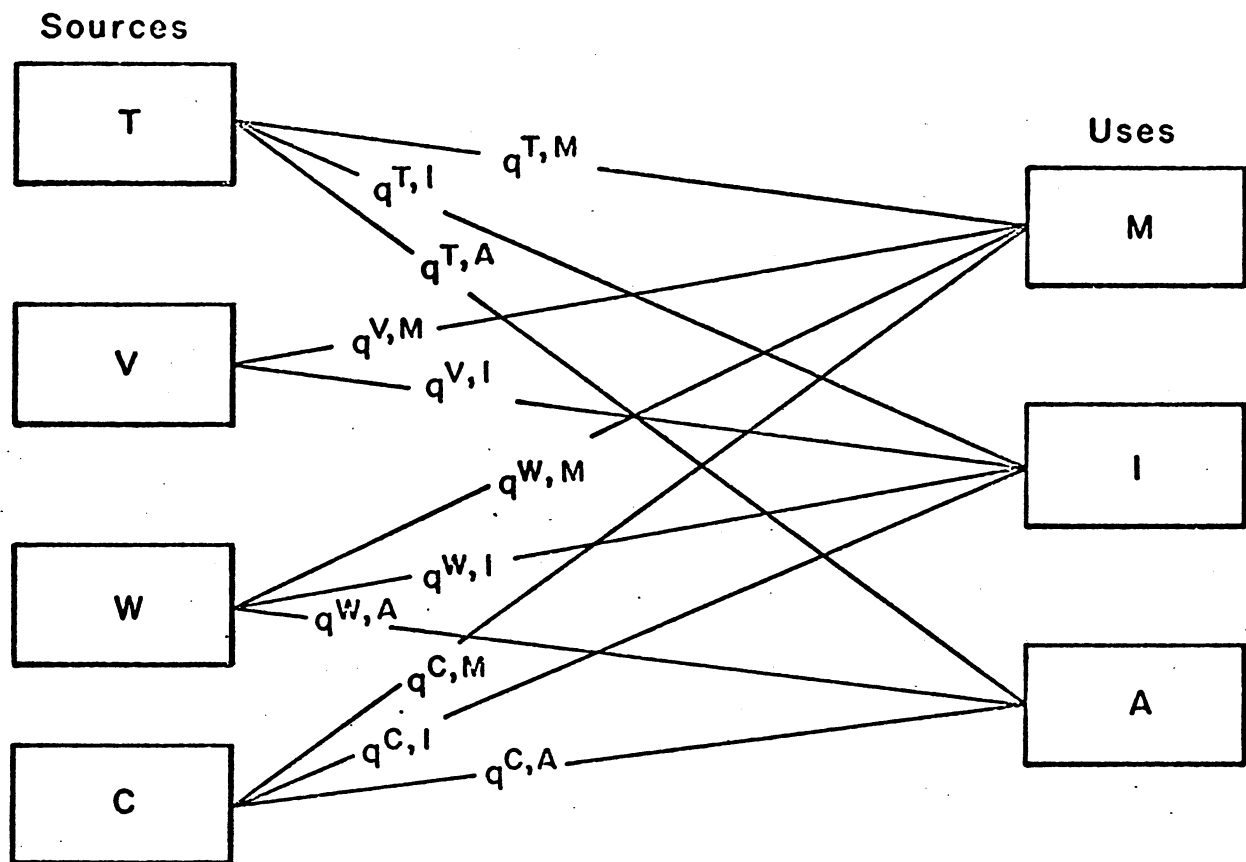
T = Groundwater, Tucson basin

V = Groundwater, Avra Valley

W = Reclaimed waste water

C = Central Arizona Project water

M = Municipal

I = Industrial

A = Agricultural

TABLE 1.--Optional sources of water, pertinent references, and relative costs

Source	Reference	Cost Estimate ($/acre-foot)
Runoff Induced by Land Treatment[a]	University of Arizona, Water Resources Research Center (1965)	23 - 95[b]
Desalination, Gulf of California	Seale and Post (1965)	145[c]
Transfer from Low-value Uses	Kelso and Jacobs (1967)	31 - 68[d]
Surface Runoff	Rillito Creek Hydrologic Research Commission (1959)[e]	
Groundwater, San Pedro River Basin		
Urban Runoff		
Cloudseeding		
Evaporation Suppression		
Artificial Recharge		
Seepage Control		
Increased Irrigation Efficiency		

a. Includes covering land surface with plastic and gravel and treating land surface with salt.

b. Treatment costs only for first 20 years of operation

c. Total production and transportation costs to Tucson

d. Total costs of diverting water from agricultural to urban use.

e. Compilation of surface runoff data from Tillito Creek and tributaries.

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economic consequences of such implementation will be one of the items
which this example problem will help to estimate. The next three of these
additional supplies which are currently virtually assured of being
implemented will now be discussed.

Groundwater, Avra Valley

The City of Tucson has purchased land in the southern part of Avra
Valley, drilled large wells, and presently has a 42-inch, 15-mile long
pipeline constructed to convey water to the Tucson area. The amount
involved will be about 11,000 acre-feet per year increasing to about 22,000
acre-feet per year in the near future. As in the Tucson Basin, there is
a large amount of water potentially available in Avra Valley. Again, using
a value for specific yield of 0.15 and a present average depth to water, in
this case, of 320 feet, the volume in the upper 500 feet of basin-fill
material in Avra Valley was estimated at about 6 million acre-feet. Data
used in making this estimate was obtained from White, Matlock, and
Schwalen (1966). The current annual withdrawal for agricultural purposes
was about 115,000 acre-feet per year in 1966.

Presently citizens in Avra Valley are bringing suit against the City
of Tucson in an effort to enjoin the city from obtaining Avra Valley water
until just compensation has been made by the city. The basis for this
action is that Arizona water law does not favor the use of groundwater
outside the basin from which it is pumped. This suit will probably
culminate in the form of additional costs to the city in obtaining Avra
Valley water.

Reclaimed Waste Water

In fiscal year 1967-68, the Tucson Department of Water and Sewers
treated 26,800 acre-feet of waste water, and the Pima County Sanitary
District No. One treated about 7,200 acre-feet for a total of 34,000 acre-
feet. This is equal to about 55 percent of the amount used by the total
sewered area. In addition to these main sources of waste water, there are
several industries in the basin producing sizable amounts of waste water
which do not enter the sewer system. Tucson Gas and Electric Company
released about 500 acre-feet in 1968 and Hughes Aircraft Company released
about 350 acre-feet. There are several other minor sources such as the
Mineral Hill underground copper mine located 15 miles south of Tucson which
pumps about 35 acre-feet per year from their main shaft into a nearby stream
bed. This gives a total of about 35,000 acre-feet of waste water originating
in the Tucson Basin annually. Only 12,000 acre-feet or 34 percent of the
total available waste water is presently being reused. This water is reused
to irrigate land in the northern part of the basin. In this problem, we will
assume that the present contract which the Tucson Department of Water and
Sewers has concerning this 12,000 acre-feet of waste water can be renegotiate
This will allow waste water to be used for other purposes if deemed desirable
Discounting the mine effluent, there is currently about 35,000 acre-feet of
waste water, most of which is already being reclaimed, available for reuse in
the Tucson metropolitan area.

Central Arizona Project Water

The City of Tucson is scheduled to receive some 112,000 acre-feet of water from the Central Arizona Project, which was recently authorized by Congress. In addition to this, irrigation districts may form within the basin and contract for additional water. The first water is scheduled to reach Tucson around 1975 and will come from the yet to be constructed Charleston Dam on the San Pedro River. This water is expected to amount to about 14,000 acre-feet per year. The remaining imported water is to come from the Colorado River supposedly around 1980.

The preceding data result in the following four constraints on the availability of water from the four sources concerned. Referring to Fig. 15 for notation and using quantities in thousands of acre-feet, we have:

$$q^{T,M} + q^{T,I} + q^{T,A} \leq G^{MAX}$$

$$q^{V,M} + q^{V,I} \leq 11$$

$$q^{W,M} + q^{W,I} + q^{W,A} \leq 35$$

$$q^{C,M} + q^{C,I} + q^{C,A} \leq 112 \ . \tag{3-1}$$

The term, G^{MAX}, will be the annual withdrawal of groundwater from the Tucson Basin with the exact amount to be decided by a central water-control agency. It is assumed that the current City of Tucson contract to sell 12,000 acre-feet of reclaimed waste water for irrigation purposes can be renegotiated. This leaves 35,000 acre-feet of waste water available for reuse. Even though the bulk of Central Arizona Project water will not arrive for at least 10 years, for purposes of illustration in this problem, we will assume that it is available now. Since no negative quantities can be transferred, we also have the constraints that all the quantities should be non-negative.

Water Costs

A defensible water-rate schedule should be based on accurate, reliable cost analyses. In this regard, quite reliable data can be obtained concerning costs involved in the existing water-supply system in the Tucson Basin (City of Tucson Department of Water and Sewers, 1967 and 1968; Nelson and Busch, 1967; Gilkey and Beckman, 1963). It is interesting to note at this point that a study of Afifi (1967) showed that only about 45 percent of the water utilities in Illinois keep cost information on a regular basis. It cannot be emphasized too strongly how important this type of data is to the formulation of a rational water-rate schedule. The real difficulty in a problem such as this is to predict costs for the various source-use relationships which do not yet exist. In this example problem only operating costs will be used, and they will be assumed to be linear, that is, the same unit costs will hold in each case for any quantity of water transferred.

In order to facilitate good estimation of the various operating costs, each one was broken down into three components -- production costs, treatment costs, and distribution costs. For each one of the water transfers shown in Fig. 15, there is an associated cost. A summary of the component costs and the total individual costs is contained in Table 2. The superscripts used in Table 1 identify the costs with their respective water-transfer routes as shown in Fig. 15. The costs will now be discussed in terms of the three components.

Production Costs

The factors involved in determining the unit operating costs of producing groundwater, that is, pumping it to the land surface, are the depth to water, pump efficiency, fuel costs, and repair and maintenance of pump and well. The figure used for $c^{T,M}$ also includes the average annual cost of drilling new wells while $c^{T,I}$ and $c^{T,A}$ do not. If the central water-control agency did exist there would be few new wells drilled for irrigation purposes. This is because the total amount of water allocated to agriculture would decline annually. This idea is expressed in a table of projected water requirements constructed by the Tucson Department of Water and Sewers (Rauscher, 1968). The average annual cost of drilling new wells is not included in $c^{T,I}$ because this sector will probably be expanding at the expense of agriculture and therefore be able to take over existing irrigation wells when needed. The costs of producing Avra Valley groundwater were taken to be the same as those for producing it in the Tucson Basin. These costs were taken from the City of Tucson Department of Water and Sewers, Annual Report 1967-68 (1969). The value of $c^{T,A}$ was obtained from the agricultural demand curve, described later in the text, knowing the present amount of agricultural use. The costs of Central Arizona Project water -- $c^{C,M}$, $c^{C,I}$, and $c^{C,A}$ -- were taken from estimates currently being made by officials of the U. S. Bureau of Reclamation.

Treatment Costs

The groundwater pumped from the Tucson Basin and Avra Valley requires no treatment other than small amounts of chlorine at some locations. The present cost of reclaiming waste water by the Tucson Department of Water and Sewers was taken as the cost to treat it for reuse by agriculture. Costs of reclaiming waste water for reuse by the municipality and industry were estimated from Watt (1968) who gives the cost of tertiary treatment as four times the cost of primary treatment. No attempt was made to estimate costs of reusing the industrial effluents mentioned earlier. The Central Arizona Project water will have to be treated, if so desired, by the purchaser, and this cost was also estimated from current statements made by officials of the U. S. Bureau of Reclamation.

TABLE 2.--Component and total costs of water transfers in $/acre-feet

Cost	Production	Treatment	Distribution	Total
$c^{T,M}$	23.20	0	39.30	62.50
$c^{T,I}$	18.00	0	2.00	20.00
$c^{T,A}$	5.00	0	3.00	8.00
$c^{V,M}$	23.20	0	59.30	82.50
$c^{V,I}$	18.00	0	22.00	40.00
$c^{W,M}$	0	44.00	39.30	83.30
$c^{W,I}$	0	44.00	39.30	83.30
$c^{W,A}$	0	11.00	4.00	15.00
$c^{C,M}$	55.00	20.00	39.30	114.30
$c^{C,I}$	55.00	20.00	39.30	114.30
$c^{C,A}$	10.00	0	6.00	16.00

Distribution Costs

If a user is connected to a distribution system, the distribution costs include administration, engineering, metering, and maintaining the system. The figures used for this portion of the costs were taken from the annual report of the City of Tucson Department of Water and Sewers (1967 and 1968) and also from estimates currently being made by local water authorities. These costs also include depreciation of plant and equipment.

Summary of Costs

Table 2 is a list of the estimates of component and total costs we will use in this example problem.

Water Demand Functions

Let us first state that in actuality each farming operation, each industry, each type of municipal use (households, lawn sprinkling, city parks, etc.), and even each individual has his or its own particular demand curve for water and, furthermore, these change over time. This necessitates, therefore, a certain amount of aggregation before we can even start to estimate them. In this problem we have three aggregations: agricultural uses, municipal uses, and industrial uses.

The determination of demand functions for water, despite the growing number of empirical investigations in recent years, remains a matter of approximation. The approximation usually concerns the form of the particular mathematical function chosen, as well as the numerical specification of the parameters to go into the chosen function. Let us first look at some of the rationale behind the choice of functions.

The total demand curve can generally be thought of as occurring in three merging portions. They are, in order of increasing elasticity, obligated demand, intermediate demand, and potential demand. These are depicted in Fig. 16 and labeled 1, 2, and 3 respectively. The obligated demand is made up of uses such as drinking and washing -- type of uses which we are prepared to pay almost any amount to retain. The intermediate demand can be pictured as made up of uses which we desire to retain, but we will definitely make an effort to cut back on the amount of water involved when the price goes up. The potential demand consists of uses which we do not really want or need, but if given the water at a low price we will "invent" them just to be using the water. In the municipal sector, these latter uses may be excessive lawn sprinkling or hosing down patios and driveways instead of sweeping them. In the agricultural sector, these uses are growing very low-value crops. In naming these uses, of course, one is always biased, and argument as to which type of use is or not a potential use is always present.

Often times municipal and industrial demand curves are expressed functionally as rectangular hyperbolas which are mathematically simple representations of the general demand curve shown in Fig. 16. Demand,
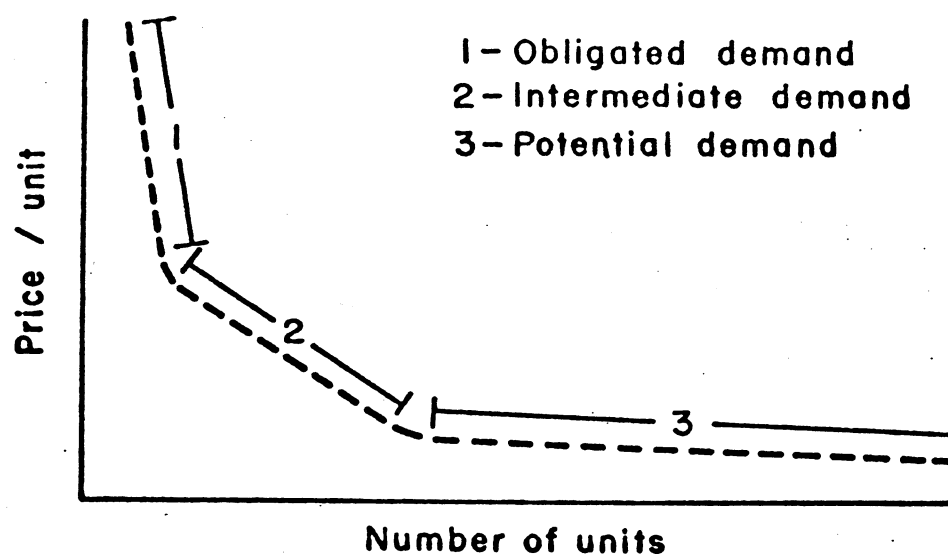
Figure 16. Characteristic Portions of Demand Curves

when approximated as a rectangular hyperbola (Fig. 17), has the form

$$Q = (\frac{B}{p})^k \qquad (3-2)$$

where Q is quantity, p is price per unit, and B and k are constants. In this case, the demand curve has constant elasticity and its value is the negative of k. This can be shown using equation (2-8) where

$$e = \frac{dQ/dp}{Q/p} . \qquad (3-3)$$

Rewriting equation (3-2) as

$$Q = B^k p^{-k} \qquad (3-4)$$

and substituting it and its derivative with respect to p

$$\frac{dQ}{dp} = -kB^k p^{-k-1} \qquad (3-5)$$

into (3-3) gives

$$e = \frac{-kB^k p^{-k-1}}{\frac{B^k p^{-k}}{p}} = \frac{-kB^k p^{-k-1}}{B^k p^{-k-1}} = -k . \qquad (3-6)$$

On the other hand, agricultural demand curves are most often represented as straight lines because both the obligated and potential portions are thought of as being truncated producing a curve like the one shown in Fig. 17. This truncation is said to occur on the upper end because there is a somewhat definite point beyond which prices could not be raised if the farming operation were to make a profit. On the lower end, the farmer actually could deteriorate his land with too much water and therefore would not buy anymore water no matter how low the price is. The slope of a straight line should not be confused with elasticity -- they are different. Actually, a straight-line demand curve is elastic near the price axis, unitary elastic at the halfway point, and inelastic near the quantity axis. The equation for the demand function, when approximated as a straight line can be derived from the following, more specific form of equation (2-8)

$$\frac{Q - \overline{Q}}{\overline{Q}} = e \frac{p - \overline{p}}{\overline{p}} . \qquad (3-7)$$

Thus, if we have data for the existing use rate $\overline{Q}$, the existing price per unit $\overline{p}$, and an estimate of elasticity e; we can obtain the parameters for a straight-line demand function. Likewise, a set of these same three types of data when used in equation (3-2) will give us the parameters for a hyperbolic demand function.

If, however, experimental data in the form of a series of price-quantity values are available, curves can be fitted to this data and equation parameters calculated from the fitted curves. This is how the municipal and agricultural demand curves will be derived in this example problem. The industrial demand curve will be derived by estimating the parameters first as mentioned above. All three of the demand curves will be assumed to be linear; a nonlinear function, as explained in chapter 2,
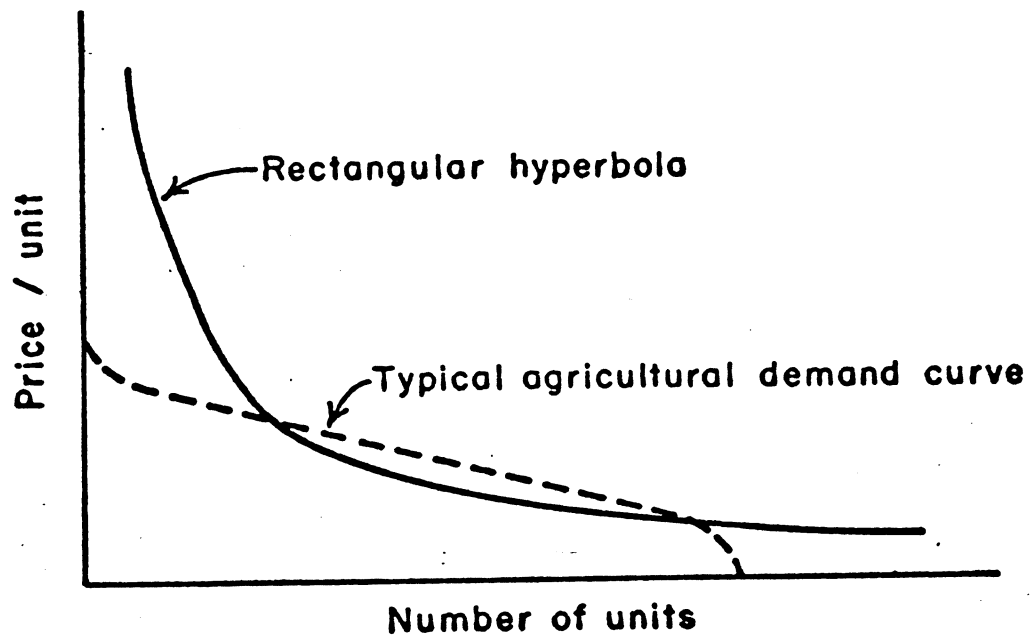
Figure 17. Commonly Used Shapes of Demand Curves

might increase the complexity of the problem solution immensely. Also, we will be dealing only with limited portions of the total demand curve, and in these cases the linear representations are judged to be just as valid as any other representation considering the paucity of the data. We will mainly be concerned with rather small deviations from the existing price-quantity values, which to some extent justifies the use of a linear demand function.

Before going on to a detailed discussion of the demand curves as derived for the Tucson Basin, it should be kept in mind that these, of course, are only estimates; later in the chapter we will discuss the consequences of "wrong" estimates.

Municipal Demand

A series of price-quantity values for municipal use was obtained using empirical relationships suggested by Howe (1968) and based on the well-known "Hopkins Study" (Linaweaver, Geyer, and Wolff; 1968). This study done at Johns Hopkins University served to gather and interpret residential water-use data from 11 metropolitan areas throughout the country. The two study areas most hydrologically similar to and also closest to Tucson were San Diego, California and Denver, Colorado. It was determined that municipal water use could best be studied when broken down into household use and sprinkling use. The factors most influential in determining household use were the market value of the dwelling unit and the price charged for the water. The factors most influential in determining sprinkling use were the irrigable area surrounding the dwelling unit, the average potential evapotranspiration during the sprinkling season, the average precipitation during the sprinkling season, and the price charged for the water.

Using these factors, Howe (1968) developed the following expressions by which to estimate municipal demand curves

$$q_{a,d} = 206 + 3.47v - 1.30p_w \qquad (3\text{-}8)$$

where $q_{a,d}$ is household use in gallons per day per dwelling unit, v is the market value of the dwelling unit in thousands of dollars, and p_w is the price charged for water,

$$q_{s,s} = 3657r_s^{0.309} p_s^{-0.930} \qquad (3\text{-}9)$$

where $q_{s,s}$ is the summer sprinkling use in gallons per day per dwelling unit, p_s is the price charged for water, and r_s is defined as

$$r_s = b(w_s - 0.6\rho_s) \qquad (3\text{-}10)$$

where b is the irrigable area in acres surrounding the dwelling unit, w_s is the average summer potential evapotranspiration (calculated by the Thornthwait method) for the area in inches, and ρ_s is the average summer precipitation for the area in inches.

An aggregate municipal demand curve representing both household and sprinkling use was estimated for the Tucson Basin using this method. The variables needed for this locale are listed in Table 3.

The median market value of homes was taken from the U. S. Bureau of Census (1960) and represents the median market value of owner-occupied residences as of April, 1960. At that time there was an average of 3.3 persons per dwelling unit, and this figure is used later to estimate the number of dwelling units from population data. The "summer" sprinkling season in Tucson was taken to include the entire year as many residents having lawns usually plant and water rye grass during the winter months.

To estimate the total amount of water used for municipal purposes, equations (3-8) and (3-9) were combined as follows

$$Q_M = \{(q_{a,d})365 + (q_{s,s})365\}N_{du} \tag{3-11}$$

where Q_M is the total amount of water in gallons per year and N_{du} is the number of dwelling units. Further expansion of equation (3-11) yields

$$Q_M = (206 + 3.47(11.6) - 1.30p_w)\ 365N_{du} + \{3657\ (0.25(87$$

$$- 0.6 \times 11))^{0.309}\ p^{-0.930}\}\ 365N_{du}\ . \tag{3-12}$$

Using a present population of 315,000 in the metropolitan area with the corresponding number of dwelling units equal to 95,300 and changing the units of p_w to dollars per acre-foot and Q_M to acre-feet per year, gives

$$Q_M = \frac{613 - p_w}{2.33 \times 10^{-2}} + \frac{3.01 \times 10^6}{p_w^{0.930}}\ . \tag{3-13}$$

In 1967-68 the price paid for water by residents served by the Tucson Department of Water and Sewers ranged from 20 cents per thousand gallons to over $1.00 per thousand gallons in some hard to serve outlying areas. The average price paid by these users was 36 cents per thousand gallons or $110 per acre-foot. When this price is used in equation (3-13) the resulting quantity is 59,700 acre-feet. This number matches quite nicely with the often-quoted estimate of 60,000 acre-feet as the current annual rate of municipal use in the Tucson area. Data taken from equation (3-13) are plotted in Fig. 18. Also seen in Fig. 18 is a linear representation (by the method of least squares) of the portion of these data between p_M equal to $110 per acre-foot and $210 per acre-foot. The latter, then, is the municipal demand curve which will be used for calculations in this problem. The equation for this linear demand curve is

$$Q_M = 88.10 - 0.258p_M \tag{3-14}$$

where Q_M is total municipal demand in thousands of acre-feet and p_M is price in dollars per acre-foot. Point elasticities along this line can be calculated using equations (2-8) and (3-14) in the following manner

TABLE 3.--Data used for estimating demand for municipal water, Tucson Basin

Variables	Values
Median market value of dwelling unit, v	$11,600
Average irrigable area surrounding dwelling unit, b	0.25 acre
Average "summer" potential evapotranspiration, w_s	87 inches
Average "summer" precipitation	11 inches

$$Q_M = 88.1 - 0.258 p_M$$

Quantity (thousands of acre feet)

Price (dollars / acre foot)
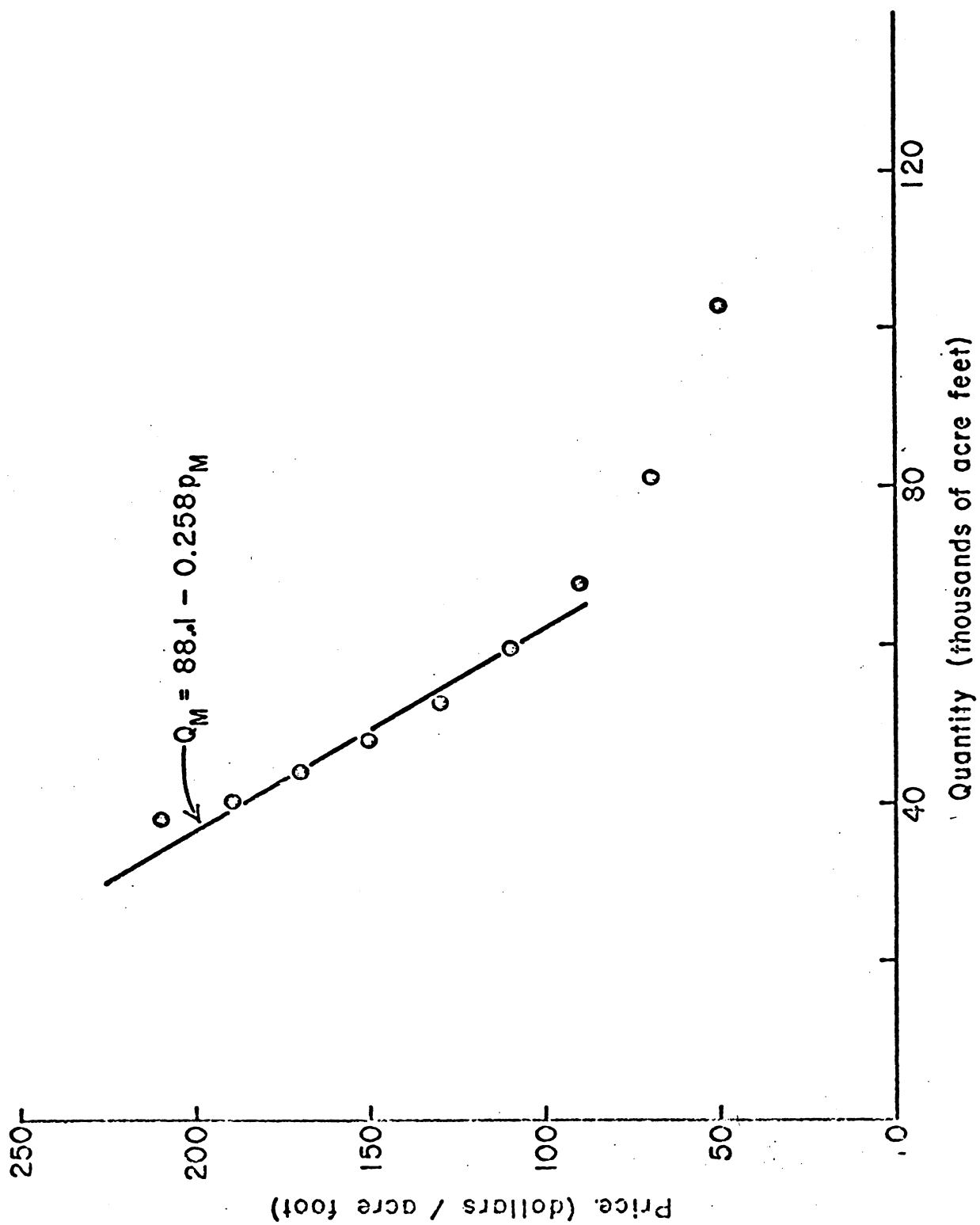
Figure 18. Municipal Demand Curve, Tucson

$$\frac{dQ_M}{dp_M} = -0.258 \tag{3-15}$$

$$\frac{Q_M}{p_M} = \frac{88.10 - 0.258p_M}{p_M} \tag{3-16}$$

and

$$e_M = \frac{dQ_M/dp_M}{Q_M/p_M} = \frac{-0.258p_M}{88.10 - 0.258p_M} \;. \tag{3-17}$$

The elasticities range from -0.48 where p_M equals \$110 per acre-foot to -1.59 where p_M equals \$210 per acre-foot. This means that from the present price of \$110 per acre-foot a 10 percent increase in price will lead to a 4.8 percent decrease in municipal water use. Conley (1967) in reviewing several studies on municipal price elasticities concludes that values ranging from -0.30 to -0.35 are most likely for the western United States. The existing municipal elasticity in Tucson is higher than these latter figures probably because the people realize that they are living in a desert environment and are privately and publicly concerned about the conservation and proper use of their water resource.

Agricultural Demand

The agricultural sector's response to water prices can be studied through a "net-revenue coefficient" approach. This is done by calculating all of the variable costs (those costs that change with changes in output), except the cost of water, which are involved in producing an acre of a given crop. These variable costs are then subtracted from the farmer's gross revenue per acre generated by the harvesting and marketing of this crop; the residual is the net revenue per acre. Since all variable costs except water are considered, this net revenue, presumably, is the maximum amount the farmer could afford to pay for water and still make a normal profit (normal profit is included in the variable costs). When the net revenue is divided by the total water requirement per acre for this crop, a net-revenue coefficient is obtained which is the maximum amount the farmer should be willing to pay for an acre-foot of water. Then, if these net-revenue coefficients in dollars per acre-foot for each crop are plotted against the amounts of water used in the area for each crop (on an accumulated basis starting with the highest coefficient and ending with the lowest), as shown in Fig. 19, a stepped function is obtained which is an estimate of the area's demand curve.

This type of study was made for the purposes of this problem using crop surveys and net-benefit coefficients for Pima County, Arizona (the Tucson Basin lies wholly within Pima County) obtained from the Departments of Agricultural Engineering and Agricultural Economics, The University of Arizona. The variable costs used in calculating the net-revenue coefficients included such items as power and material expenses, machinery repair, and labor costs. The data obtained is shown in Table 4. When the fourth row of this table is plotted against the fifth row, a stepped
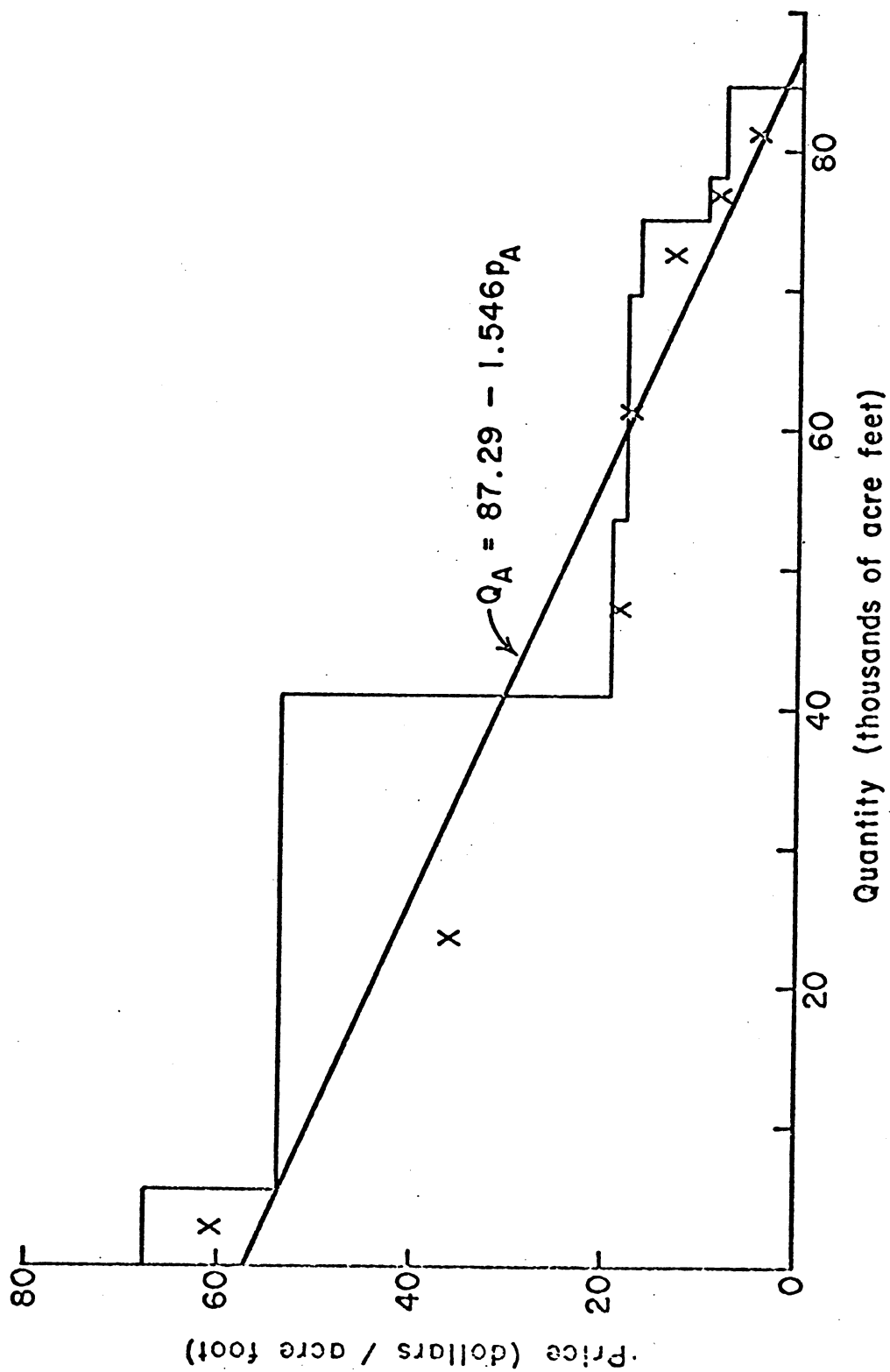
Figure 19. Aggregate Demand for Irrigation Water, Tucson Basin, 1967

The figure axes read: Price (dollars / acre foot) on the vertical axis and Quantity (thousands of acre feet) on the horizontal axis. The fitted line is labeled $Q_A = 87.29 - 1.546 p_A$.

TABLE 4.--Data used for estimating demand for irrigation water, Tucson Basin

	Crops							
	Pecans	Cotton	Barley	Sorghum	Safflower	Alfalfa	Pasture
(1) Acreage	1610	7203	4520	6144	1479	539	1691
(2) Acre-feet/acre	3.5	5.0	2.5	2.75	5.0	3.5	4.0
(3) Acre-feet	5635	36017	11299	16895	5178	2696	6765
(4) Accumulation of (3)	5635	41652	52951	69846	75024	77720	84485
(5) Net-revenue coefficient ($/acre-foot)	67.80	53.61	19.11	18.27	17.00*	10.51	8.00*

*Estimated by author after talking with members of the Agricultural Economics Department, The University of Arizona.

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function (Fig. 19) is obtained which represents an estimate of the agricultural demand as it existed in 1967, the time during which the surveys were made. To smooth out the steps in the demand schedule for the purposes of this problem, a linear regression equation was fitted to these data by the method of least squares. The equation for this linearized demand curve is

$$Q_A = 87.29 - 1.546p_A \qquad (3-18)$$

where Q_A is total agricultural demand in thousands of acre-feet at a price in dollars per acre-foot of p_A. It was assumed that the midpoints, marked X, of the vertical portions of the steps were most stable with respect to price changes; these points, therefore, were used as observations for fitting the estimating equation.

The irrigation water requirements used were averages while studies have shown that for a given crop some farmers use twice the amount of water that others use on comparable land. This, coupled with the fact that there is a wide range in net-benefit coefficients, points out the tremendous incentives farmers have for saving water. In other words, when prices are raised, they have the opportunity to save water by adopting more efficient irrigation practices or substituting higher valued crops and using less acreage.

The amount of water used for irrigation in the basin has been dropping a little each year due to increased pumping lifts in some areas and the phasing out of some low-valued crops. Whereas Table 4 shows a total agricultural use of 84,485 acre-feet in 1967, the current estimates are that about 75,000 acre-feet will be used in 1969. At this latter rate of annual use, the demand equation (3-18) gives a current price that the farmer is paying for water of $8.00 per acre-foot. In this case, the price can be also interpreted as the cost of supplying irrigation water from the Tucson Basin groundwater reservoir since the existing distribution system would continue to be used if the central water-control agency was in control. Using data in Nelson and Busch (1967), it was estimated that the cost of pumping the water to land surface is currently $5.00 per acre-foot; therefore, it appears that the cost of maintaining an irrigation distribution system in the basin is $3.00 per acre-foot. These latter stated costs appear in Table 2 as costs of production and distribution.

Elasticities on the agricultural demand curve range from -0.20 at a price of $8 per acre-foot to -1.13 at a price of $30 per acre-foot. At prices higher than about $30 per acre-foot, it is considered that crop substitution and the effect of water costs on final product prices would serve to invalidate the demand curve. For example, according to Table 4, alfalfa would be the first crop to cease to be planted if water prices increased, and yet theory tells us that as the production of alfalfa drops, its market value would increase, thus increasing its net-revenue coefficient. Also with a water-price increase, higher value crops will be substituted for crops of lower value and again increase the net-revenue coefficients.

In order to incorporate these discrepancies some economists (Hartman and Whittelsey, 1960; Moore, 1962) have developed general equilibrium models (solved using linear programming techniques) with which to determine the

crop program which would maximize net farm income. This crop program is then used along with the net-revenue coefficients to develop a demand curve. For comparison with the above-mentioned agricultural elasticities, Moore (1962) calculated elasticities ranging from -0.14 at $5 per acre-foot to -1.58 at $25 per acre-foot for crop land in southern California.

Industrial Demand

There has been little objective study of true industrial demand for water. Much of the reason for this dearth of information is based on the fact that the "value added per acre-foot of water" in most industries is very high. For instance, Tijoriwala, Martin, and Bower (1968) indicate that the value added per acre-foot of water intake in the industrial sectors of the Arizona economy ranges from $1,684.79 for primary metals to $140,331.18 for fabricated metals and machinery. While, for comparsion, the figures in the agricultural crop sectors range from $13.52 for food and feed grains to $126.14 for vegetables. "Value added" is the total cost of production paid out as earned incomes to the owners of factors of production (wages, interes proprietor's profits, water costs, etc.). If "value added per acre-foot of water" is low, indication is that a large portion of the total production costs are accounted for by water; likewise, high values added per acre-foot of water indicate that only a small portion of the total production costs are attributable to water. Thus, if the price of water were to increase assuming this increase could not be passed on to buyers of the product, firms where value added per acre-foot of water is quite small would face a "profit squeeze" and be forced to change their water-using habits. On the other hand, high values added per acre-foot of water are often times interpreted as meaning that these associated firms are able to pay a very high price for water and still maintain high profits and, further, that they would not be forced to lower their usage if the price of water were to rise. This condition, if it were true, would produce an inelastic demand curve within any range of reasonable prices and lead to a demand curve described simply as a vertical line through a point indicated by the existing price-quantity values.

The point to be made here is that even though it can be shown that industry is able to pay more for water, they are certainly not always willing to do so if the price is raised and are perfectly capable of changing their internal water-use patterns and reducing their water intake if they can save money by so doing. This means more than a reduction in the amount of intake water due to in-plant treating and subsequent recirculation which would essentially maintain the existing status of the gross amount of water applied per unit of output, but also means the capacity to reduce the gross water applied through actual changes in the manufacturing process. The latter notion is exemplified by data (Sewell et al., 1968) which show the gross water applied in petroleum refining to vary from less than 200 to more than 4,000 gallons per barrel of crude oil processed for 159 refineries surveyed.

This leads one to believe that it would be possible to quantify industrial demand curves for water by analyzing the alternative water-use patterns in terms of their associated costs and the amount of water used with each alternative. Then, it is only logical that as the price of water

rises to a point where a new use pattern becomes economical, the firm would switch over to this use pattern requiring less water and so on. These data, then, would constitute a stepwise demand curve. Needless to say, these types of data are not easily obtainable due to the understandable unwillingness of plant officials to divulge the required information. But, nonetheless, this does not rule out the possibility of estimating such data if one is familiar with the manufacturing processes.

There is an industrial demand study worthy of note, however, not only because it is an attempt to quantify industrial demand, but also because it introduces the more general subject of how uncertainty in water supply may affect the demand. Rather than study the demand curve for each type of industry Turnovsky (1968) devised an "index of per capita industrial production" which involved summing the industrial payrolls in a given community and converting this sum to a per capita term. The index was then used to help estimate the industrial water demand in each of 19 Massachusetts towns as

$$X = \beta_0 + \beta_1\sigma^2 + \beta_2 p + \beta_3 IP \qquad (3-19)$$

where X is the per capita industrial water demand, the β's are regression coefficients, σ^2 is the variance of supply, p is the price, and IP is the above-mentioned industrial production index. Of the three variables, IP turned out to be the least significant in predicting per capita industrial demand, but price and variance were highly significant.

In the Tucson Basin, there are three types of uses which make up the preponderance of the industrial water-use component. The largest of these is a group of four copper mines located in the southern end of the basin about 20 miles south of Tucson. Three of these mines are open pit and currently each of them obtains water from its own wells. Data taken from Gilkey and Beckman (1963) and Larson and Henkes (1968) indicate that their total new water intake is about 12,000 acre-feet per year at present, and all practice recirculation within the concentrators. The fourth operation is an underground mine using about 600 acre-feet per year which it obtains from within its main shaft. The mines use 200 to 250 gallons of water per ton of ore processed.

The amount of water used by mines will change drastically in the period 1970-1975 as both the Duval Corporation and the Anaconda Company bring large, new, open pit properties into operation. These two operations are expected to increase the water use by mines to about 42,000 acre-feet per year by 1975.

The second largest industrial user in the basin is Tucson Gas and Electric Company which withdrew 4,400 acre-feet of water from its own wells in 1968. This operation practices cooling-tower recirculation to the extent that their total in-plant use is about 22,000 acre-feet per year.

The other large industrial use is by a large manufacturing plant operated by the Hughes Aircraft Company which used 415 acre-feet of water in 1968. This water was again obtained from private wells.

This gives a total industrial use within the basin at present of about 17,500 acre-feet annually. The cost of obtaining water by the mines in 1962 was about $20 per acre-foot (Gilkey and Beckman, 1963), and since this type of use makes up the largest portion of total industrial use, this figure was used as an average current price of water to industrial users in this problem.

Since all the industries in the basin are already practicing recirculation, the price elasticity for this demand is probably rather low. For instance, Cootner and Lof (1965) have indicated that for electric steam generation plants with cooling towers already installed (as is the case with Tucson Gas and Electric Company), the price elasticity for water would be about -0.15. Price elasticity is still present because of their ability to increase the condenser size (at increased cost, of course) and thus increase the efficiency of each unit of water in disposing of heat.

It appears, according to Kaufman (1967), that there are more opportunities for conserving water in the mining operations even though, in this case, recirculation within the ore concentrators has already been adopted. These opportunities include recirculation at other stages in the milling process.

With the above-stated facts in mind, it was decided to estimate the industrial price elasticity at the present price-quantity values as a -0.20. These data then give us one point on the demand curve and the elasticity at this point. With these we can derive a linear expression for industrial demand using equation (3-7) as follows:

$$\frac{Q_I - 17.5}{175} = -e_I \frac{p_I - 20}{20} \tag{3-20}$$

or

$$Q_I = 21.0 - 0.175p_I$$

where Q_I is total industrial demand in thousands of acre-feet and p_I is in dollars per acre-foot.

In this context, costs are the unit, variable costs of delivering water from the groundwater reservoir within the basin (the only current source) to each of the three sectors. Since at present irrigators and industrialists are self-supplied, their costs are the same as the prices they are now paying for water in that they are not charging themselves any more for water than it costs them to obtain it. The municipal users, served by the Department of Water and Sewers, however, are not self-supplied, and the prices which they pay for water exceed the variable costs of delivery. If our hypothetical central water-control agency existed now and was operating the existing water supply system under the conditions as specified in Table 5, the municipal sector, by equation (2-11), would be the only sector from which a net revenue would be generated. This net revenue could be calculated simply as follows:

$$R_N = 59,700(110.00 - 62.50) + 75,000(8.00 - 8.00)$$
$$+ 17,500(20.00 - 20.00) = \$2,835,750 \tag{3-22}$$

TABLE 5.--Present uses, prices, costs, elasticities, and demand functions

Sector	Use (acre-feet/year)	Price ($/acre-foot)	Cost ($/acre-foot)	Elasticity	Demand Function
Municipal	59,700	110.00	62.50	-0.48	$Q_M = 88.10 - 0.258p_M$
Agriculture	75,000	8.00	8.00	-0.17	$Q_A = 87.29 - 1.546p_A$
Industry	17,500	20.00	20.00	-0.20	$Q_I = 21.00 - 0.175p_I$

During 1967-68, the Tucson Department of Water and Sewers had a net operatin revenue of $2,202,924 upon delivering 44,500 acre-feet of water for municip; use.

The Objective Function

As stated before, there is presently only one source of water being use in the Tucson Basin -- the Tucson Basin groundwater reservoir. All of the data presented herein concerning this source are current, i.e., they represe the situation as it is believed to exist in the spring of 1969. Likewise, each of the three types of uses currently exist and their demand curves are presented as an estimate of the situation only as it exists now. The other sources of water, however, are not actually being used in the system yet, an therefore the data presented concerning them represent conditions as they ai expected to arise when these sources enter the system.

The pricing model, then, can be used to allocate water within the watei supply system as it exists now and can also be used to allocate water within the system as it is expected to exist when the new supply sources become availabe. Actually, ·in this problem we will first allocate water on an annual basis within the existing system and then look at how this allocatior. would be affected if each of the new supply sources were to be currently brought on line.

If the four sources of water which have been described were all presently available, the problem would be as follows: Allocate the water from these sources to the municipal, agricultural, and industrial users in such a way as to maximize profits to the central water-control agency. This stated objective can be formulated as a nonlinear mathematical programming problem whose general form is as shown in equation (2-15). The objective function to be maximized is

$$Z = p_M(q^{T,M} + q^{V,M} + q^{W,M} + q^{C,M}) - c^{T,M}q^{T,M} - c^{V,M}q^{V,M}$$

$$- c^{W,M}q^{W,M} - c^{C,M}q^{C,M} + p_A(q^{T,A} + q^{W,A} + q^{C,A}) - c^{T,A}q^{T,A}$$

$$- c^{W,A}q^{W,A} - c^{C,A}q^{C,A} + p_I(q^{T,I} + q^{V,I} + q^{W,I} + q^{C,I})$$

$$- c^{T,I}q^{T,I} - c^{V,I}q^{V,I} - c^{W,I}q^{W,I} - c^{C,I}q^{C,I} \qquad (3\text{-}23)$$

subject to the following availability constraints:

$$q^{T,M} + q^{T,A} + q^{T,I} \leq Q_T$$

$$q^{V,M} + q^{V,I} \leq Q_V$$

$$q^{W,M} + q^{W,A} + q^{W,I} \leq Q_W$$

$$q^{C,M} + q^{C,A} + q^{C,I} \leq Q_C \qquad (3\text{-}24)$$

and nonnegativity constraints

$$q_{ij} \geq 0 \ . \tag{3-25}$$

The general form of these constraints is shown in equation (2-2). The known cost coefficients (c_{ij} in equation (2-15) expressed in dollars per acre-foot are

$c^{T,M}$ = cost of transferring water from Tucson Basin groundwater reservoir to municipal uses,

$c^{V,M}$ = cost of transferring water from Avra Valley to municipal uses,

$c^{W,M}$ = cost of transferring waste water to municipal uses,

$c^{C,M}$ = cost of transferring Central Arizona Project water to municipal uses,

$c^{T,A}$ = cost of transferring water from Tucson Basin groundwater reservoir to agricultural uses,

$c^{W,A}$ = cost of transferring waste water to agricultural uses,

$c^{C,A}$ = cost of transferring Central Arizona Project water to agricultural uses,

$c^{T,I}$ = cost of transferring water from Tucson Basin groundwater reservoir to industrial uses,

$c^{V,I}$ = cost of transferring water from Avra Valley to industrial uses,

$c^{W,I}$ = cost of transferring waste water to industrial uses, and

$c^{C,I}$ = cost of transferring Central Arizona Project water to industrial uses.

"Cost of transferring" refers to the total cost of production, treatment, and distribution as given in Table 2. The price variables (p_j in equation 2-15), expressed in dollars per acre-foot, are

p_M = price paid for water by municipal users,

p_A = price paid for water by agricultural users, and

p_I = price paid for water by industrial users.

The quantity variables (q_{ij} in equations 2-2 and 2-15), expressed in acre-feet, are

$q^{T,M}$ = quantity of water transferred from Tucson Basin groundwater reservoir to municipal uses,

$q^{V,M}$ = quantity of water transferred from Avra Valley to municipal uses,

$q^{W,M}$ = quantity of waste water transferred to municipal uses,

$q^{C,M}$ = quantity of Central Arizona Project water transferred to municipal uses,

$q^{T,A}$ = quantity of water transferred from Tucson Basin groundwater reservoir to agricultural uses,

$q^{W,A}$ = quantity of waste water transferred to agricultural uses,

$q^{C,A}$ = quantity of Central Arizona Project water transferred to agricultural uses,

$q^{T,I}$ = quantity of water transferred from Tucson Basin groundwater reservoir to industrial uses,

$q^{V,I}$ = quantity of water transferred from Avra Valley to industrial uses,

$q^{W,I}$ = quantity of waste water transferred to industrial uses, and

$q^{C,I}$ = quantity of Central Arizona Project water transferred to industrial uses.

The availability constants (Q_i in equation 2-2) on the right-hand side of the constraints, expressed in acre-feet, are

Q_T = total amount of water available from Tucson Basin groundwater reservoir (depends on political decision rule to be discussed later),

Q_V = total amount of water available from Avra Valley (11,000 acre-feet),

Q_W = total amount of waste water available (35,000 acre-feet), and

Q_C = total amount of Central Arizona Project water available (112,000 acre-feet).

The demand functions given in Table 5 can be restated as

$$p_M = 341.47 - 3.876Q_M$$

$$p_A = 56.46 - 0.647Q_A \qquad (3\text{-}26)$$

$$p_I = 120.00 - 5.714Q_I$$

but

$$Q_M = q^{T,M} + q^{V,M} + q^{W,M} + q^{C,M}$$

$$Q_A = q^{T,A} + q^{W,A} + q^{C,A} \qquad (3\text{-}27)$$

$$Q_I = q^{T,I} + q^{V,I} + q^{W,I} + q^{C,I}$$

therefore

$$p_M = 341.47 - 3.876(q^{T,M} + q^{V,M} + q^{W,M} + q^{C,M})$$

$$p_A = 56.46 - 0.647(q^{T,A} + q^{W,A} + q^{C,A}) \tag{3-28}$$

$$p_I = 120.00 - 5.714(q^{T,I} + q^{V,I} + q^{W,I} + q^{C,I}) \ .$$

These equations (3-27) along with the cost coefficients given in Table 2 can be used in equation (3-23) to give the following more specific objective function which is a quadratic function in q_{ij}.

$$\begin{aligned}
Z = \ & 341.47(q^{T,M} + q^{V,M} + q^{W,M} + q^{C,M}) \\
& - 3.876(q^{T,M} + q^{V,M} + q^{W,M} + q^{C,M})^2 - 62.50q^{T,M} - 82.50q^{V,M} \\
& - 83.30q^{W,M} - 114.30q^{C,M} + 56.46(q^{T,A} + q^{W,A} + q^{C,A})^2 \\
& - 8.00q^{T,A} - 15.00q^{W,A} - 16.00q^{C,A} \\
& + 120(q^{T,I} + q^{V,I} + q^{W,I} + q^{C,I})^2 - 20.00q^{T,I} - 40.00q^{V,I} \\
& - 83.30q^{W,I} - 114.30q^{C,I} \ .
\end{aligned} \tag{3-29}$$

When the availability constraints are used in equations (3-24), the constraints become

$$\begin{aligned}
q^{T,M} + q^{T,A} + q^{T,I} &\le Q_T \\
q^{V,M} + q^{V,I} &\le 11 \\
q^{W,M} + q^{W,A} + q^{W,I} &\le 35 \\
q^{C,M} + q^{C,A} + q^{C,I} &\le 112
\end{aligned} \tag{3-30}$$

and

$$q_{ij} \ge 0 \ . \tag{3-31}$$

Equation (3-39) can be maximized subject to the constraints, equations (3-30), using quadratic programming techniques, all eleven q_{ij} variables, and all four inequality constraints. But because of the particular economic structure of the problem (specifically, the manner in which the cost coefficients are arrayed), it can be decomposed into four independent sub-problems thereby greatly reducing the computational effort required for its solution. The basis for and reasoning behind this decomposition will now be explained.

Decomposition of the Objective Function

There are innumerable ways by which large-structured mathematical programming problems can be manipulated in order to facilitate their solutions. The effort that has been devoted to this subject is quite varied because each worker has been involved with particular problems, each

of which has its own peculiarities and each of which must be attacked some-
what differently so that these individual peculiarities can be exploited.
The impetus behind the manipulations and strategies is usually to overcome
the mathematical problem of computer capacity. But there is another
important reason why these manipulations and strategies should not be
overlooked. It is possible to generate problems which appear to require
programming techniques for their solutions, but which actually can be
solved somewhat simply. If detailed programming techniques are used in
these situations, they cloud the general understanding of the problem and
sometimes imply that certain insights are missing. These manipulations and
strategies, however, when applied can often prove quite fruitful in
revealing some of these insights. We will show in this problem how this
type of "information feedback" occurred when attempts were made to use a
solution strategy.

One of these solution-strategy concepts is referred to as the principle
of decomposition. That is, some large problems can be broken into parts --
variables and constraints can be grouped so that each constraint or set of
constraints involves only one set of variables while other constraints
involve sets of different variables. When the matching groups (subproblems)
are solved, their solutions can be reassembled to give an overall solution.
In searching for some sort of decomposition possibility in this problem, it
was found that certain economic principles could be used as guidelines
along which decomposition could take place. These principles will first be
explained, and then the actual decomposition of this particular problem will
be described.

Keeping this explanation in terms of water resources, let us first
consider a case where only one user is being supplied with water. The user
confronts us with a single linear demand curve, and we have the option of
supplying him from either of two sources. We are faced with a different
unit cost of supply from each source. This situation is pictured in Fig. 20
From the supplier's point of view, the demand curve becomes the average
revenue curve. For each supply, the average and marginal costs are pictured
as single horizontal lines because, in this case, they are constant unit
costs, and average costs equal marginal costs. Area ABCD is the profit to
be gained by supplying the user with water from source A, and the area EBCF
is the profit to be gained by supplying the user with water from source B.
The area identified as "Δ profit" in Fig. 20 is the increase in profit to
be obtained by supplying the user with the least-cost source (in this case
source A). Therefore, it is obvious that we will obtain the most profit by
supplying the user from source A. It is also obvious that the "Δ profit"
area will always be present no matter what supply level is chosen; therefore
it will always be best to supply the user from source A. We can generalize
this statement by saying that whenever we are faced with a linear demand
curve and constant unit costs of supply, at any level of supply we will
obtain the most profit at this level by supplying from the least-cost
source. For sake of clarification, this concept is also shown in Fig. 21,
but this time using a total revenue curve and total cost curves. Again, it
can be seen that at any supply level we will obtain the most profits by
supplying from the least-cost source. Actually this principle is
intuitively realized by most water-source planners in their obvious desire
to use the least expensive sources of water early in an area's development
stage and then start seeking out more expensive sources of supply to meet
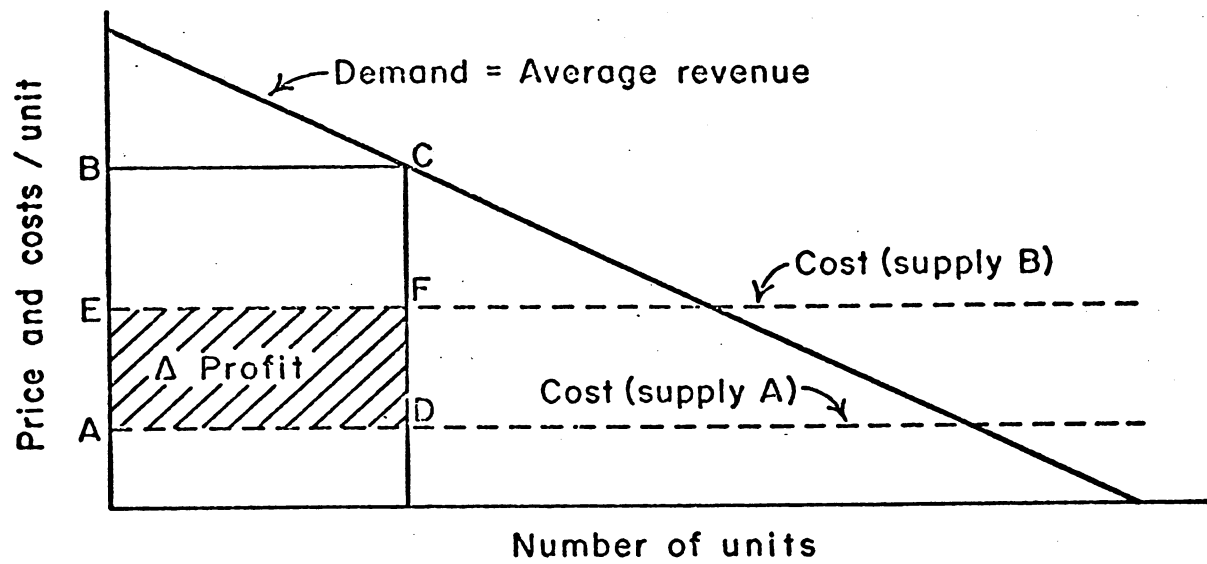
Figure 20. Single User, Two Possible Supply Sources, Unit Prices, and Costs
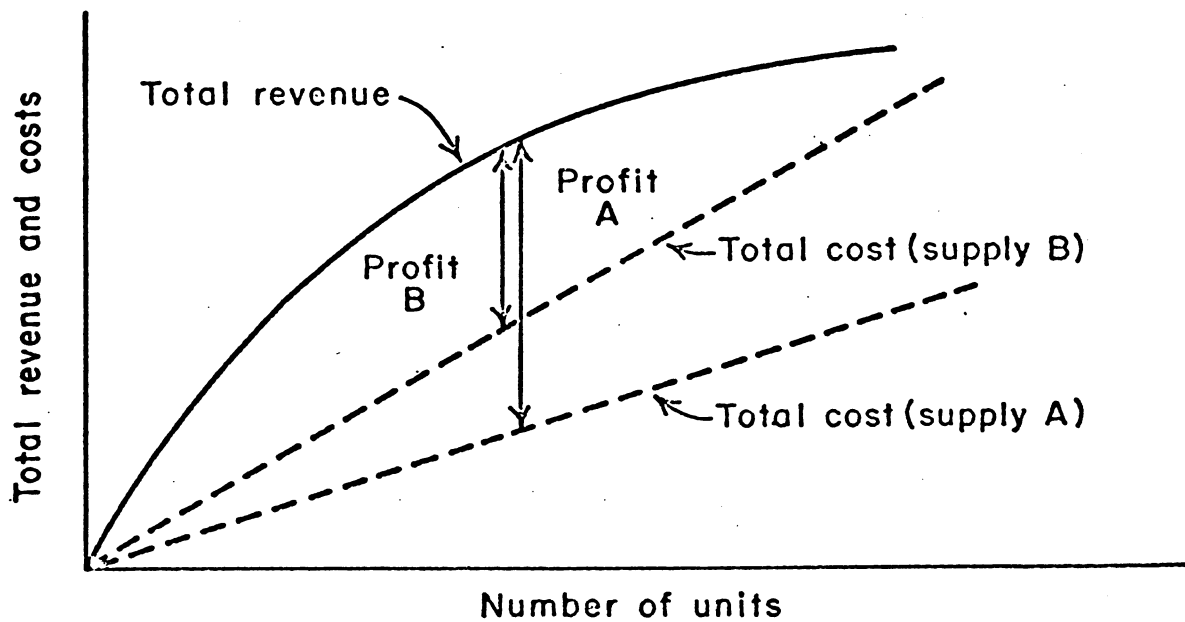
Figure 21. Single User, Two Possible Supply Sources, Total Revenue, and Total Costs

expected requirements once the area's economy is relatively established. This idea is vital to the decomposition of this problem.

We can expand this idea by stating that if we are faced with supplying several users each with his own characteristic demand curve, by the same above reasoning we would be in the best profit position if we supplied each user with his particular least-cost source. Now, in this specific problem, the total cost structure is such that the least-cost source (Tucson Basin groundwater reservoir) to municipal users is also the least-cost source to the agricultural and industrial users. Further, the next least expensive supply source (Avra Valley) to municipal users is also the next least expensive source to the other users. The same thing holds true for the third least expensive source (waste water) and the most expensive source (Central Arizona Project water). This categorized array of costs can be seen in the "total cost" column of Table 2.

These circumstances, then, afford us a guideline along which the overal problem as expressed in equations (3-23), (3-24), and (3-25) can be decomposed, that is, first allocate in an optimal manner only water from the Tucson Basin groundwater reservoir to all three users. This subproblem is formulated in the same way as the overall problem except that only the applicable quantity variables ($q^{T,M}$, $q^{T,I}$, and $q^{T,A}$) need be considered. The objective of this subproblem is to maximize

$$Z_T = p_M q^{T,M} - c^{T,M} q^{T,M} + p_A q^{T,A} - c^{T,A} q^{T,A}$$
$$+ p_I q^{T,I} - c^{T,I} q^{T,I} \tag{3-32}$$

subject to

$$q^{T,M} + q^{T,A} + q^{T,I} \leq Q_T \tag{3-33}$$

and

$$q^{T,M}, \ q^{T,A}, \ q^{T,I} \geq 0 \tag{3-34}$$

where Z_T refers to the profit obtained from allocating only water from the Tucson Basin groundwater reservoir. This is a quadratic programming problem in only three variables and can be solved using Wolfe's (1959) modified simplex technique by hand very easily. Since we are using an inequality type availability constraint, none of the individual allocations will go beyond the level indicated by the point where marginal revenues equal marginal costs, but any of them may be less than this level because of the availability constraint, that is, an optimization with no availability constraint would call for supplying all sources at the level indicated by the point at which marginal revenues equal marginal costs; beyond this the solution would be nonoptimal. But because of the availability constraint, all or some of the uses might not be supplied up to this level by the least-cost source. If for any use the level of allocation from this least-cost source is the same level as that indicated by the point where marginal revenues equal marginal costs, no further allocation need be made to this use. This would be the case for use A in an example involving three uses and four sources depicted in Fig. 22. If for any use where the allocation
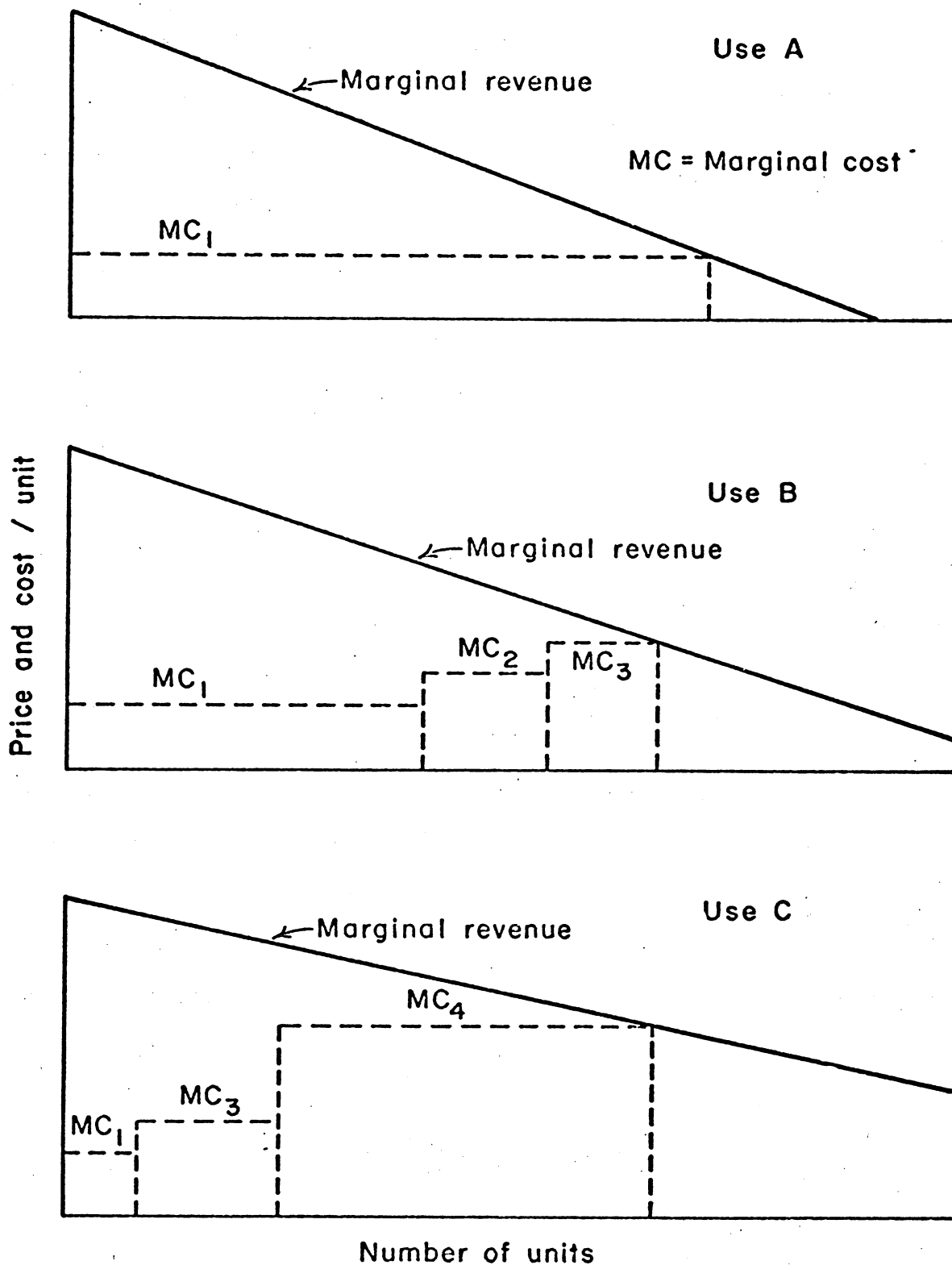
Figure 22. Demonstration of Decomposition Principles

from the least-cost source is less than that indicated by the point where marginal revenues equal marginal costs, further allocation must be made from the next least-cost source. Let us say that this was the case in our example Tucson Basin problem. If use A were municipal use, our next step would be to allocate water from Avra Valley, but we know that Avra Valley water cannot go to agricultural use, therefore, we allocate all of it to industrial use (use B in Fig. 22). The third subproblem would be again formulated in the same way as the overall problem except that only the applicable quantity variables ($q^{W,A}$ and $q^{W,I}$) need be considered. Remember, municipal use has already been fully allocated and the third least-expensive source is waste water. The objective of this third subproblem would be to maximize

$$Z_W = p_A q^{W,A} - c^{W,A} q^{W,A} + p_I q^{W,I} - c^{W,I} q^{W,I} \qquad (3\text{-}35)$$

subject to

$$q^{W,A} + q^{W,I} \leq Q_W \qquad (3\text{-}36)$$

and

$$q^{W,A}, \; q^{W,I} \geq 0 \qquad (3\text{-}37)$$

where Z_W refers to the profit obtained from allocating just waste water. This amounts to another simple quadratic programming problem, this time with only two variables. In this third subproblem it is possible that the allocation to use B, if added on to the preceding allocations, would exceed the level indicated by the point where the marginal cost of this source equals the marginal revenue. In this case, the allocation to this use would be made only to the point where marginal revenue equals marginal cost, and the remainder of this allocation would be added on to use C. Lastly, in Fig. 22, the most expensive source would be allocated to use C out to the point where the marginal cost of this source equals the marginal revenue of use C.

What we have actually done in this decomposition procedure is break the overall problem down into a series of very simple quadratic programming problems (in our example, these can be solved very simply with hand calculations) and an elementary bookkeeping procedure. The main benefit, however, is the much deeper insight we gained into the problem at hand.

Unconstrained Optimization

The previous section explained why, in maximizing our problem (equations 3-29, 3-30, and 3-31), we would first maximize the profit to be obtained from allocating only water from the Tucson Basin groundwater reservoir. Now, if we do not limit the amount of water available from this source, or, for instances, set Q_T equal to the amount of water now used in the basin (150,000 acre-feet annually), the first subproblem is formulated as

$$Z_T = 341.47q^{T,M} - 3.876(q^{T,M})^2 - 62.50q^{T,M} + 56.46q^{T,A}$$
$$- 0.647(q^{T,A})^2 - 8.00q^{T,A} + 120.00q^{T,I} - 5.714(q^{T,I})^2$$
$$- 20.00q^{T,I} \qquad\qquad (3\text{-}38)$$

subject to

$$q^{T,M} + q^{T,A} + q^{T,I} \leq 150 \qquad\qquad (3\text{-}39)$$

and

$$q^{T,M}, \; q^{T,A}, \; q^{T,I} \geq 0 \; . \qquad\qquad (3\text{-}40)$$

Its maximum is obtained with the following set of quantities and corresponding prices

$$q^{T,M} = 35{,}990 \text{ acre-feet}, \; p_M = \$201.99$$

$$q^{T,A} = 27{,}460 \text{ acre-feet}, \; p_A = \$ \; 32.23$$

$$q^{T,I} = 8{,}750 \text{ acre-feet}, \; p_I = \$ \; 70.00 \; . \qquad\qquad (3\text{-}41)$$

The total allocation to all uses is 82,200 acre-feet. This is less than 150,000 acre-feet and, therefore, this means that we have allocated out to the point where marginal revenues equal marginal costs for all three uses and any further allocation from this source or any other source would be suboptimal. This set of prices and quantities, therefore, also maximizes the overall problem. The total profits in this case is $6,365,400. This is an increase in profit of $3,529,600 over what the central water control agency would be generating if presently in operation.

There are, however, important reasons for not attaching too much significance to this result. The first is that the linear demand functions which we used are hardly acceptable for such violent deviations from the initial values. The second is that the result implying an 83 percent rise in the price of municipal water, a 300 percent rise in the price of agricultural water, and a 250 percent rise in the price of industrial water -- would not likely be acceptable. The third is that the result also calls for a 46 percent reduction in the total annual use in the basin; this also is too drastic to be acceptable. For these reasons a policy constraint is introduced into our example problem and will be explained in the following section.

The Policy Constraint

To accomodate for the fact that large price rises are unsatisfactory from the social point of view, a constraint will be added to our problem of the following form

$$K = n_M \frac{p_M - 110}{110} + n_M \frac{p_A - 8}{8} + n_I \frac{p_I - 20}{20} \; . \qquad\qquad (3\text{-}42)$$

The n_I coefficients are weights attached to the relative deviations of a projected price from the existing water prices. This type of constraint was used by Louwes, Boot, and Wage (1963) in an interesting problem involving the optimal use of milk in the Netherlands. The weights n_I will be determined in such a way that we have

$$n_M : n_A : n_I = \overline{P}_M \overline{Q}_M : \overline{P}_A \overline{Q}_A : \overline{P}_I \overline{Q}_I \qquad (3\text{-}43)$$

where the $\overline{p}_I$ and $\overline{Q}$ values refer to existing prices and quantities. Hence the ratio of the n_I's is equal to the ratio of the total revenues received by the central water control agency from each of the three types of uses. The n_I's have been scaled so as to add to 10, which gives

$$n_M = 8.7, \; n_A = 0.8, \; n_I = 0.5 . \qquad (3\text{-}44)$$

This implies that if all prices increase by 10 percent, K = 1.0. The value of K is zero in the existing situation. As we allow K to increase from zero, we know from our unconstrained optimum study that prices will tend to rise; therefore, protests from farmers, city dwellers, and industrial interests will become stronger. This is why we shall refer to equation (3-42) as the social constraint.

Substituting the n_I values of (3-44) into (3-42), we can specify the constraint numerically as

$$K = 8.7 \, \frac{p_M - 110}{110} + 0.8 \, \frac{p_A - 8}{8} + 0.5 \, \frac{p_I - 20}{20} \qquad (3\text{-}45)$$

or

$$K = 0.079 p_M + 0.100 p_A + 0.025 p_I - 10 . \qquad (3\text{-}46)$$

Our problem can now be stated as follows: maximize equation (3-29) subject to (3-30), (3-31), and (3-46) for various values of K, by determining the quantities to be allocated and the corresponding prices.

In formulating this constraint a certain value judgment has been made. Since the municipal users presently pay the most for water on a per acre-foot basis and because of their numbers also contribute by far the greatest revenue to the water agency, the judgment was made that their water prices should be perturbed the least. In the constraint, a perturbation in municipal water prices is weighted heavily; whereas, the other uses are not so heavily weighted.

The Numerical Results

As explained earlier, in this problem as described thus far, it will always be best to allocate water from the Tucson Basin groundwater reservoir first. We also know from previous discussion that as we relax K, the policy constraint, optimal solutions will tend toward higher prices and lower quantities of water delivered. To state this another way, increasing K will result in optimal solutions using less total water annually than the

152,000 acre-feet now being pumped in the Tucson Basin. If we consider this 152,000 acre-feet per year as an arbitrary maximum amount which we will allow to be pumped, rather than some lesser amount, we can explore the option of using only water from the Tucson Basin groundwater reservoir and be able to drop the availability constraint. For this reason, and also because of the fact that groundwater within the basin is the only active source at present, we will first look at the results obtained using only this source of water. Our problem at this point, then, has been reduced to the following: maximize equation (3-38) subject to equations (3-39), (3-40), and (3-46) for various values of K.

Results for K = 0

We will first consider the problem for K = 0 and answer the question, "What is the maximum profit obtainable for socially neutral situations?" The results of this solution are given in the second column of Table 6. It can be seen that a readjustment in prices was called for with the municipal price decreasing and the agricultural and industrial prices increasing. This in itself is an indication that the existing price to municipal users is closer to the optimum than the other prices because at this point a greater increase in profit per acre-foot can be obtained by increasing the price to agricultural and industrial users. Our K = 0 constraint requires, however, that in order to increase these latter two prices, the municipal price must decrease, and this is what happened. The Lagrangian λ_4 pertaining to the policy constraint turned out to be positive, another indication that profits can be increased by increasing prices. The value of the Lagrangian multiplier is commonly referred to as the shadow price in economics. This is because these values are measures of the cost of the constraint in the sense that they give the increase or decrease in revenue obtainable when the constraint is relaxed by one marginal unit. Thus, in this case, if we let K = 0.1 instead of K = 0, the Lagrangian λ_4 gives the increase in the value of the objective function for infinitesimal increases in K. A closer approximation is obtained by taking the average λ_4 value of K = 0 and K = 0.1.

Since our main concern in this entire problem is how to increase profits and we know that in so doing we must ultimately raise all prices, it does not seem realistic to consider lowering the price to municipal users. Therefore, at this point we will introduce three more constraints into our problem which simply do not allow the prices to fall below their present values. They are

$$p_M \geq 110.00$$

$$p_A \geq 8.00 \tag{3-47}$$

$$p_I \geq 20.00 \ .$$

Since we are using quantities of water as our decision variables, we need to reformulate these constraints in terms of the q_{ij} variables rather than the prices. This can be accomplished by the appropriate substitutions of equations (3-28) into the above inequality constraints. This gives

TABLE 6.--The numerical solutions using only water from the Tucson Basin groundwater reservoir for various values of K

	Existing Situation (1)	K = 0 (2)	K = 0.1 (3)	K = 0.2 (4)	K = 0.3 (5)
p_M	110.00	104.14	110.00	110.00	110.00
p_A	8.00	11.56	9.10	9.90	10.54
p_I	20.00	24.35	20.00	20.80	22.23
Q_M	59,700	61,230	59,700	59,700	59,700
Q_A	75,000	69,320	73,220	71,980	71,000
Q_I	17,500	16,740	17,500	17,360	17,110
$\sum_{i=1}^{m} (p_i - c)Q_i$	2,835,570	2,870,183	2,916,292	2,986,400	3,054,245
$\sum_{i=1}^{m} Q_i$	152,200	147,390	150,420	149,040	147,810
Active Constraints*		4	1,3,4	1,4	1,4
$\lambda 4$		63,912	71,521	69,047	67,068

*These numbers refer to the following constraints:

1. $q^{T,M} \geq 59.7$

3. $q^{T,I} \geq 17.5$

4. $K = 0.079 p_M + 0.100 p_A + 0.025 p_I - 10$, for various values of K.

TABLE 6.--Continued

	K = 0.4 (6)	K = 0.5 (7)	K = 0.6 (8)	K = 0.7 (9)	K = 0.8 (10)
p_M	110.00	110.00	110.00	110.40	111.30
p_A	11.19	11.83	12.47	12.88	13.07
p_I	23.66	25.09	26.51	27.27	27.69
Q_M	59,700	59,700	59,700	59,620	59,380
Q_A	69,990	69,000	68,010	67,380	67,080
Q_I	16,860	16,610	16,360	16,230	16,150
$\sum_{i=1}^{m} (p_i - 1) Q_i$	3,120,725	3,184,564	3,246,257	3,302,604	3,362,032
$\sum_{i=1}^{m} Q_i$	146,550	145,310	144,070	143,230	142,020
Active Constraints*	1,4	1,4	1,4	4	4
λ_4	65,059	63,080	61,101	59,819	59,234

*These numbers refer to the following constraints:

1. $q^{T,M} \geq 59.7$

3. $q^{T,I} \geq 17.5$

4. $K = 0.079 p_M + 0.100 p_A + 0.025 p_I - 10$, for various values of K.

TABLE 6.-- <u>Continued</u>

	K = 0.9 (11)	K = 1.0 (12)	K = 2.0 (13)	K = 3.0 (14)	K = 4.0 (15)
p_M	112.19	113.09	122.04	131.00	139.95
p_A	13.26	13.45	15.34	17.23	19.13
p_I	28.11	28.53	32.70	36.88	41.06
Q_M	59,150	58,920	56.610	54,300	51,990
Q_A	66,790	66,500	63,570	60,650	57,720
Q_I	16,080	16,010	15,280	14,550	13,810
$\sum_{i=1}^{m}(p_i-c)Q_i$	3,420,886	3,479,752	4,031,218	4,564,953	4,959,886
$\sum_{i=1}^{m}Q_i$	142,020	141,430	135,460	129,500	123,520
Active Constraints*	4	4	4	4	4
λ_4	58,649	58,064	52,216	46,368	40,520

*These numbers refer to the following constraints:

1. $q^{T,M} \geq 59.7$

3. $q^{T,I} \geq 17.5$

4. $K = 0.079p_M + 0.100p_A + 0.025p_I - 10$, for various values of K.

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TABLE 6.--<u>Continued</u>

	K = 5.0 (16)	K = 6.0 (17)	K = 7.0 (18)	K = 8.0 (19)	K = 9.0 (20)
p_M	148.90	157.86	166.81	175.76	184.72
p_A	21.02	22.91	24.80	26.70	28.58
p_I	45.23	49.41	53.59	57.77	61.94
Q_M	49,680	47,370	45,060	42,750	40,440
Q_A	54,670	51,870	48,950	46,010	43,110
Q_I	13,080	12,350	11,620	10,890	10,160
$\sum_{i=1}^{m} (p_i - c)Q_i$	5,334,163	5,653,797	5,913,083	6,113,567	6,255,880
$\sum_{i=1}^{m} Q_i$	117,430	111,590	105,630	99,650	93,710
Active Constraints*	4	4	4	4	4
$\lambda 4$	34,672	28,824	22,976	17,128	11,280

*These numbers refer to the following constraints:

1. $q^{T,M} \geq 59.7$

3. $q^{T,I} \geq 17.5$

4. $K = 0.079p_M + 0.100p_A + 0.025p_I - 10$, for various values of K.

TABLE 6.--<u>Continued</u>

	K = 10.0 (21)	K = 10.94 (22)	K = 11.0 (23)
p_M	193.67	201.99	202.62
p_A	30.47	32.23	32.36
p_I	66.12	70.00	70.29
Q_M	38,130	35,990	35,820
Q_A	40,180	37,460	37,250
Q_I	9,430	8,750	8,700
$\sum_{i=1}^{m} (p_i - c)Q_i$	6,339,267	6,365,400	6,363,848
$\sum_{i=1}^{m} Q_i$	87,740	82,200	81,770
Active Constraints*	4	4	4
λ_4	+5,432	0	-415

*These numbers refer to the following constraints:

1. $q^{T,M} \geqq 59.7$

3. $q^{T,I} \geqq 17.5$

4. $K = 0.079 p_M + 0.100 p_A + 0.025 p_I - 10$, for various values of K.

$$q^{T,M} + q^{V,M} + q^{W,M} + q^{C,M} \leq 59.7$$

$$q^{T,A} + q^{W,A} + q^{C,A} \leq 75.0 \tag{3-48}$$

$$q^{T,I} + q^{V,I} + q^{W,I} + q^{C,I} \leq 17.5$$

With these constraints (3-48) in the problem, the solution for K = 0 will be the same as the existing conditions since in order for any one of the prices to move up in value at least one of the others would have to move down, but this cannot happen because of the constraints (3-48). As soon as the policy constraint is relaxed, however, the prices will start to move up. In the next section, we will discuss the results obtained for alternative values of K. The problem is now stated as follows: maximize equation (3-38) subject to (3-39), (3-40), (3-46), and (3-48) for various values of K.

Results for Alternative Values of K

Table 6 gives the solution values of prices and quantities, the profit obtained, and the Lagrangians associated with the exactly satisfied constraints for alternative values of K. Apart from the policy constraint only the first and third constraints of (3-48) are effective for the lower values of K. The absolute maximum, already discussed in an earlier section, is obtained where K = 10.94. Up to this point it is seen that the Lagrangian λ_4 values are positive but decreasing, implying that the incremental increases in profit are getting less as we approach the absolute maximum. At this point λ_4 is zero meaning that any further increase in prices will lower profit. As K is raised beyond 10.94, λ_4 becomes negative and would get larger if K were raised further.

Now let us see how the information contained in Table 6 could be used by the central water-control agency for the twofold purpose of increasing profits and decreasing the total amount of water used in the basin. The last major increase in water rates for municipal users in the City of Tucson was instigated in October, 1964, and amounted to about 30 percent. Let us assume that another 30 percent increase is now warranted and use Table 6 to see how, according to our model, it can be optimally accomplished, that is, how can we obtain the most profit out of a 30 percent price increase? This question is answered in column 14 of Table 6. Sell municipal water for $131.00 per acre-foot, agricultural water for $17.23 per acre-foot, and industrial water for $36.88 per acre-foot. This amounts to a 19 percent rise in municipal rates, a 115 percent rise in agricultural rates, and a 84 percent rise in industrial rates. These increases reflect the structure of our policy constraint which inhibits municipal rate changes in comparision with the others on the grounds that municipal rates are already comparatively high and municipal users contribute a preponderance of the total revenue. The total profits would be increased by 61 percent or by about $1,729,000, and the total amount of water used in the basin would be decreased by 15 percent or by 22,700 acre-feet.

Let us repeat that these results are based on demand function which are at best approximations and the further we deviate from the existing situation

the more liable are our results. The consequences of these uncertainties will be discussed later.

Results Involving All Sources

The use of the pricing model will now be expanded to obtain results involving all the sources heretofore mentioned as future possibilities of supply. We know from our discussion thus far that these sources will not enter the solution unless forced to do so. We will force their entrance by restating the inequality-type availability constraints as equalities. First let us set up four operating rules which concern the Tucson Basin ground- water reservoir and the concept of "safe yield." (This latter term is truly one of the most ambiguous terms in hydrology -- to the author it simply means some sort of planned utilization of the groundwater reservoir based on the wants and needs of the community.) Three of these rules will dictate three different annual rates of withdrawal based on three different rates of estimated average annual natural recharge. These reflect a policy whose purpose is to maintain or build up the present average groundwater level within the basin. A fourth rule is also considered which calls for importir all water used in the basin in a concerted effort to build up the average groundwater level. In these cases we will assume that we are required to supply the present annual rate of use, 152,200 acre-feet, and therefore we will use equality constraints starting with the amount of water to be used from the least expensive source. To this amount we will successively add on, in the form of equality constraints, the amounts of water to be used from the next least expensive sources until the specified requirement is met. For instance, the first operating rule assumes an average natural recharge rate of 79,000 acre-feet per year and limits the pumping within the basin to this amount. Therefore, we need to add to this 11,000 acre-feet from Avra Valley, 35,000 acre-feet of waste water, and 27,200 acre-feet of Central Arizona Project water to make the total requirement of 152,200 acre-feet. Our problem will then be the following: maximize (3-29) subject to (3-31), (3-38), and

$$q^{T,M} + q^{T,A} + q^{T,I} = 79$$
$$q^{V,M} + q^{V,I} = 11$$
$$q^{W,M} + q^{W,A} + {-}^{W,I} = 35 \tag{3-49}$$
$$q^{C,M} + q^{C,A} + q^{C,I} = 27.2$$

The second, third, and fourth operating rules will be along the same lines as the first, but with annual pumpages within the basin limited to 56,000 acre-feet, 33,000 acre-feet, and zero acre-feet. Again, the total requirement is 152,200 acre-feet in all cases. Before discussing the result let us make it perfectly clear how they were obtained.

Operating rule number one will be used as an explanatory example, and Fig. 23 will be used for illustration. The solution procedure is as follows

1. Allocate 79,000 acre-feet of water from the Tucson Basin ground- water reservoir to all uses by maximizing equation (3-38) subject to (3-40), and
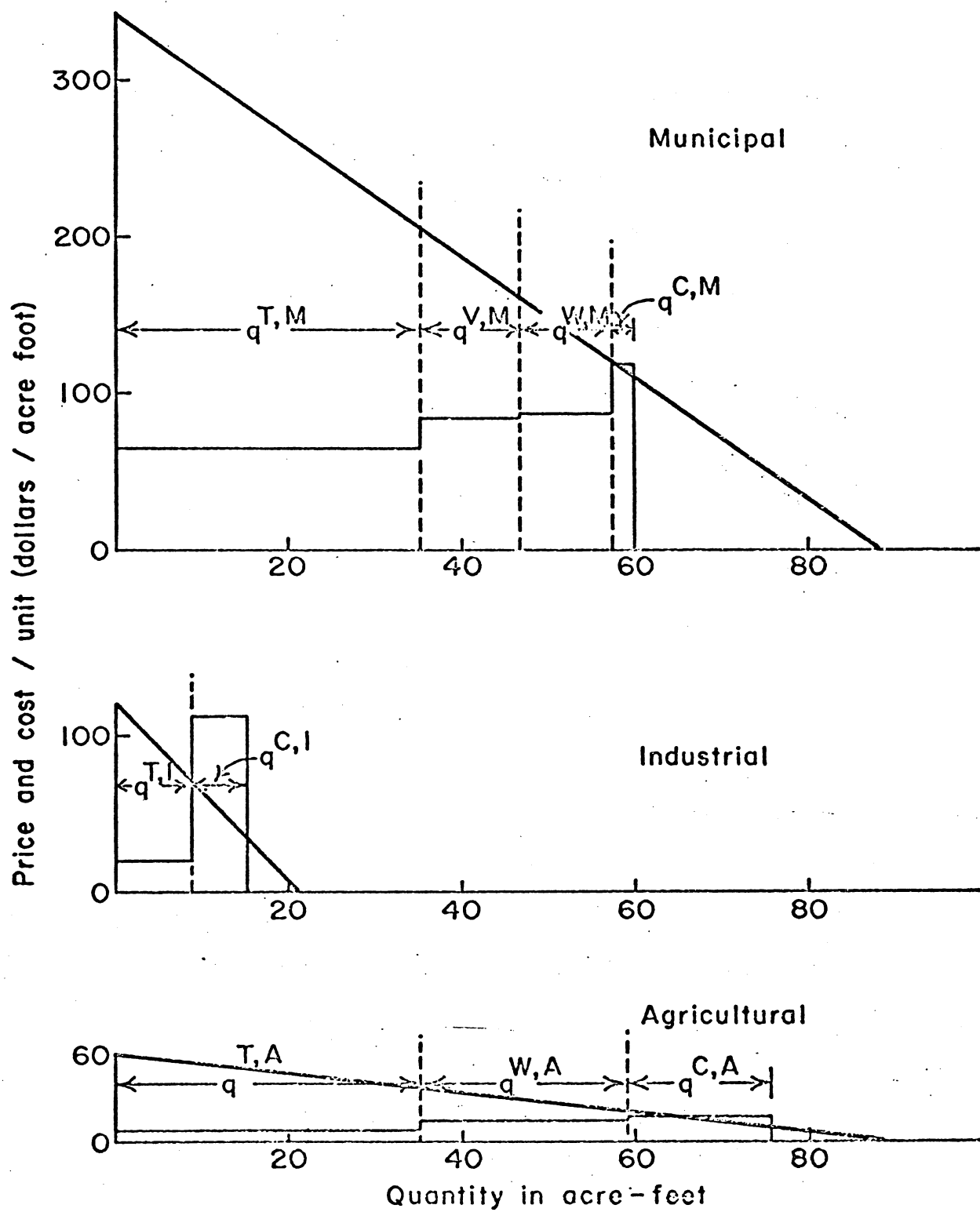
Figure 23. Allocation under Operating Rule No. 1

$$q^{T,M} \leq 59.7$$

$$q^{T,A} \leq 75.0 \qquad \text{(3-50)}$$

$$q^{T,I} \leq 17.5$$

and

$$q^{T,M} + q^{T,A} + q^{T,I} = 79.0 \ . \qquad \text{(3-51)}$$

2. Allocate 11,000 acre-feet of water from Avra Valley to municipal and industrial uses by maximizing

$$Z_2 = 203.59q^{V,M} - 3.876(q^{V,M})^2 - 82.50q^{V,M}$$
$$+ 71.61q^{V,I} - 5.714(q^{V,I})^2 - 40.00q^{V,I} \qquad \text{(3-52)}$$

subject to

$$q^{V,M} \leq 24.13$$
$$q^{V,I} \leq 9.03 \qquad \text{(3-52)}$$

and

$$q^{V,M} + q^{V,I} = 11.0 \ . \qquad \text{(3-54)}$$

To obtain the new demand functions used in (3-52) we simply shift the vertical axes out to the point of the most recent allocation as shown by the dashed lines in Fig. 23. The slope of the demand functions remain the same, but their ordinal intercepts change as shown. The right-hand side constants in (3-53) are simply the right-hand side constants in (3-50) deleted by the amount of the previous allocations.

3. Allocate 35,000 acre-feet of waste water to all uses by maximizing

$$Z_3 = 160.95q^{W,M} - 3.876(q^{W,M})^2 - 83.30q^{W,M} + 33.84q^{W,A}$$
$$- 0.647(q^{W,A})^2 - 15.00q^{W,A} + 71.61q^{W,I} - 5.714(q^{W,I})^2$$
$$- 83.30q^{W,I} \qquad \text{(3-55)}$$

subject to

$$q^{W,M} \leq 13.13$$
$$q^{W,A} \leq 41.04 \qquad \text{(3-56)}$$
$$q^{W,I} \leq 9.03$$

and

$$q^{W,M} + q^{W,A} + q^{W,I} = 35.0 \ . \qquad \text{(3-57)}$$

4. Allocate 27,200 acre-feet of Central Arizona Project water (the amount needed in addition to water from the other three less

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expensive sources to meet the total requirement of 152,200 acre-feet) by maximizing

$$Z_4 = 116.37q^{C,M} - 3.876(q^{C,M})^2 - 114.30q^{C,M} + 18.63q^{C,A}$$
$$- 0.647(q^{C,A})^2 - 16.00q^{C,A} + 71.61q^{C,I} - 5.714(q^{C,I})^2$$
$$- 114.30q^{C,I} \qquad \qquad (3\text{-}58)$$

subject to

$$q^{C,M} \leq 1.63$$

$$q^{C,A} \leq 16.54 \qquad \qquad (3\text{-}59)$$

$$q^{C,I} \leq 9.0$$

and

$$q^{C,M} + q^{C,A} + q^{C,I} = 27.2 \ . \qquad \qquad (3\text{-}60)$$

At this point, the sum of the right-hand side constants in (3-59) necessarily equal the right-hand constant in (3-60). Therefore, the final allocation can only be

$$q^{C,M} = 1.63$$

$$q^{C,A} = 16.54 \qquad \qquad (3\text{-}61)$$

$$q^{C,I} = 9.03 \ .$$

The allocations resulting from the application of all four different operating rules are shown in Table 7. In general, the allocations remain fairly stable except for the agricultural sector. As the decisions are made to reduce pumping from within the basin, agricultural use from this source should decline and be supplanted by Central Arizona Project water. The reasonableness of this course of action will be discussed later.

From the allocations in Table 7, a table of costs (Table 8) was calculated. Because of the specified requirement of using 152,200 acre-feet of water for all uses, the total revenue under each operating rule remained the same. As more expensive sources of water were used to replace the groundwater supply from within the basin, the costs necessarily increased. What our model actually shows us in this case is the least-cost method of supplying the required amount of water under the conditions specified. For all four operating rules the total revenue was $7,527,000; therefore, profits for the four rules were

Rule #1	$1,111,200
Rule #2	$ 617,900
Rule #3	$ 92,900
Rule #4	-$1,103.800

The profits rendered under each of these operating rules are all less than the profits now accruing with the water-supply system operating in its present state. This profit, it will be remembered, is $2,835,570. The difference in the present profits and each of the other possible profits

TABLE 7.--Optimal allocation considering present requirement and all sources, in acre-feet

	Tucson Basin Groundwater	Avra Valley	Waste Water	CAP	Total
Rule #1 (Q_T = 79,000 acre-feet)					
Municipal	35,570	11,000	11,500	1,630	59,700
Agricultural	34,960		23,500	16,540	75,000
Industrial	8,470			9,030	17,500
Rule #2 (Q_T = 56,000 acre-feet)					
Municipal	32,570	11,000	11,420	4,710	59,700
Agricultural	17,000		23,580	34,420	75,000
Industrial	6,430			11,070	17,500
Rule #3 (Q_T = 33,000 acre-feet)					
Municipal	28,780	11,000	11,910	8,010	59,700
Agricultural			23,090	59,910	75,000
Industrial	4,220			13,280	17,500
Rule #4 (Q_T = zero)					
Municipal		11,000	35,000	13,340	59,700
Agricultural				75,000	75,000
Industrial				17,500	17,500

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TABLE 8.--Costs of water supply under optimal allocation procedures, in dollars

	Tucson Basin Groundwater	Avra Valley	Waste Water	CAP	Total Cost
Rule #1 (Q_T = 79,000 acre-feet)					
Municipal	2,223,250	948,750	957,950	186,540	
Agricultural	279,730		352,500	264,540	
Industrial	169,380			1,033,160	6,415,800
Rule #2 (Q_T = 56,000 acre-feet)					
Municipal	2,035,880	948,750	951,450	538,700	
Agricultural	136,020		353,670	550,740	
Industrial	128,700			1,265,190	6,909,100
Rule #3 (Q_T = 33,000 acre-feet)					
Municipal	1,798,312	948,750	992,020	916,340	
Agricultural	84,440		346,370	830,540	
Industrial				1,517,330	7,434,100
Rule #4 (Q_T = zero)					
Municipal		948,750	2,915,500	1,566,370	
Agricultural				1,200,000	
Industrial				2,000,360	8,630,800

then gives us a measure of the opportunity cost of implementing each of the
rules. They are

     Rule #1 $1,724,370
     Rule #2 $2,217,670
     Rule #3 $2,742,670
     Rule #4 $3,939,370

Results Using Estimated Data for 1975

The purpose of this section is to demonstrate how the pricing model
can be used in planning over a time horizon. In the case of the Tucson
Basin, the static model can be expanded generally as shown in equation
(2-24). Any logical amount of water can be bought or pumped each year
regardless of how much was bought or pumped the previous year; therefore,
we will assume that hydrologic dependencies from year to year do not
affect the model. Likewise, economic dependencies can be assumed to have
no effect. The problem facing us then is to predict water requirements,
water demands, water costs, and water availabilities in the future as
input data at each stage of the model.

Needless to say, this type of data cannot be predicted very
accurately. However, intelligent attempts can be made and must be made if
we wish to plan for the future at all. As mentioned earlier, in the Tucson
Basin the use of water by mines is expected to increase sharply in the near
future. It is estimated that by 1975 the mines would like to be using about
41,600 acre-feet annually, but to stay at or near this new level for
awhile. This fact gives us a good reason to try to predict the overall
water-supply picture in 1975. By so doing we can suggest plans for
coping with this rather substantial perturbation in the water-supply system.

A summary of the predicted requirements and demands for each sector in
1975 is shown in Table 9. The estimated requirements were taken from
Rauscher (1969). The estimates 1975 population in the Tucson Basin is
388,000.

The municipal and industrial demand curves for 1975 were estimated by
simply shifting the present demand curves horizontally out to the point
indicated by the new price-use relationship assuming present-day prices.
In other words, it is assumed that municipal users would use 73,100 acre-
feet at a price of $110 per acre-foot and industrial users would use
47,500 acre-feet at a price of $20.00. The slopes of the functions, there-
fore, remain the same and their equations can be derived based on this
information. Actually, by doing this we have assumed that at every price
the total use in 1975 will be a constant amount more than it would be
presently. This constant amount is the difference between future and
present uses at present prices.

The agricultural demand curve was estimated by assuming the same
elasticity at the future use rate of 58,700 acre-feet at $8.00 per acre-
foot as at the present use rate of 75,000 acre-feet at $8.00 per acre-foot.
This tends to keep ordinal intercepts of the present and future demand
curves very nearly the same, but increases the slope of the function as

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TABLE 9.--Requirements and demands predicted for 1975

Sector	Requirement (acre-feet per year)	Demand Function
Municipal	73,100	$Q_M = 101.5 - 0.258p_M$
Agriculture	58,700	$Q_A = 68.6 - 1.236p_A$
Industry	47,500	$Q_I = 51.0 - 0.175p_I$

total agricultural use decreases. Thus, as total agricultural use decreases the low-value crops will be eliminated and the high-value crops retained. T agricultural demand curve for 1975 then was estimated using a price-use point of 58,700 acre-feet and $8.00 and an elasticity of -0.17.

All of the water-transfer costs were assumed to remain the same as given in Table, and the 1975 availabilities were estimated as follows:

Tucson Basin groundwater	56,000 acre-feet
Avra Valley groundwater	22,000 acre-feet
Waste water	40,300 acre-feet
Central Arizona Project	112,000 acre-feet

The figure for Tucson Basin groundwater is one of the natural recharge estimates. It is assumed that the full amount of Avra Valley water will be available, that is, all that the present system can handle. The waste water is a straight percentage of the municipal use, 55 percent as indicated earlier. The Central Arizona Project water is again assumed to be available so that we can compare these results with those earlier.

The optimization of this 1975 model was accomplished in exactly the same way as indicated in the preceding section using the 1975 parameters and assuming that the predicted requirements had to be met. The resulting allocations are given in Table 10. These results can be compared with the results for operating Rule #2 in Table 7, since they both assume an average annual natural recharge rate of 56,000 acre-feet.

After 1975 it is estimated that the municipal requirement will continue to expand with the population at its present rate of increase (about 2,000 acre-feet per year), while industrial requirements level off and agricultural requirements decrease by about 1,400 acre-feet per year (Rauscher, 1968). If this is true, the trend of optimal allocations will be to supply the agricultural and industrial users with more and more of their water from the CAP. This is assuming a constant relative price range.

Interpretation of Results

The most obvious conclusion to be drawn here is that a central water control agency could profit by using only groundwater from the Tucson Basin and less of it than is now being used. Undoubtedly there is a tendency to dismiss this conclusion as plain "horse sense." It is true that perhaps any good water-resources consultant could have given the same broad conclusion just from his experience. But those years of experience were not available in this case, and the answer was still obtained. The operations research framework and the rigor it demands helped make this possible.

In the particular case of the Tucson Basin it is quite possible that a water-rate increase may be considered as a way of financing the introduction of outside sources. If such action is considered, the results in Table 6 will help in planning this rate increase. This is assuming that there is basic agreement on the form of   policy constraint which favors larger increases in agricultural and industrial rates than municipal rates.

TABLE 10.--Optimal allocation considering 1975 requirements and all
sources, in acre-feet

	Tucson Basin Groundwater	Avra Valley	Waste Water	CAP	Total
Municipal	36,420	14,580	13,890	8,210	73,100
Agricultural			22,110	36,590	58,700
Industrial	19,580	7,420	4,300	16,200	47,500

In the results involving all four sources of water, the objective is constrained by forcing the system to meet the present requirements, and because of this the present price structure remains intact. A form of internal efficiency has been accomplished, however, by allocating the water in such a way as to maximize profits to a central water-control agency. In the case of the Tucson Basin, by retaining the price structure we are operating the system as a point close to maximum net satisfaction or maximum net social payoff -- that is, maximum under the assumption used in developing the demand curves. It will be remembered that the demand curves were assumed to pass through the present price-use relationships and that the present marginal costs of both agricultural and industrial water were considered as their present prices. Therefore, in these latter two cases we are operating at the point where marginal cost equals price and this is where net satisfaction is greatest for these two sectors. In the case of municipal water, the price of $110.00 is greater than the marginal cost of $62,50, when delivering 59,700 acre-feet. Therefore, we are theoretically operating at a point which yields less net satisfaction than possible.

This leads to the rather envious results of operating as close to the point of maximum net social payoff as the constraints will allow and at the same time operating a central water-control agency so as to maximize its profits under these conditions.

Effect of Changes in Elasticities on Problem Solution

In this section we will perform a sensitivity analysis to determine the effect on the optimal solution of the pricing model if the demand elasticities take on other possible values. In particular, we will vary the point elasticities by plus and minus 10 percent at the present price-use coordinates to see what changes in prices this will bring about at the optimal solution for $K = 13.0$ in the case where the only source considered is groundwater. It will be recalled that setting $K = 13.0$ allows for a 30 percent overall price increase.

The influences of changes in the elasticities on the demand functions can be determined from (3-7). From (3-7) we derive

$$Q = \frac{\overline{Q}}{p} ep - \overline{Q}(e - 1) . \tag{3-62}$$

If e is changed by an amount Δe, we obtain

$$Q = \frac{\overline{Q}}{p} (e + \Delta e)p - \overline{Q}(e + \Delta e - 1) . \tag{3-63}$$

Equation (3-63) can be used to derive a new linear demand curve for each perturbation in elasticity. Then, the adjusted demand curves can be entered into the pricing model and the changes in output over the original unperturbed output can be noted.

A 10 percent change in the point elasticities given in Table 4 yields the following Δe values:

$$\Delta e_M = .048$$

$$\Delta e_A = .017 \tag{3-64}$$

$$\Delta e_I = .020 \ .$$

The subscripts M, A, and I again designate municipal, agricultural, and industrial sectors. The adjusted demand functions can now be calculated for a positive 10 percent change in elasticity using (3-63). They are:

$$Q_M = 85.49 - .234p_M$$

$$Q_A = 86.48 - 1.434p_A \tag{3-65}$$

$$Q_I = 20.65 - .158p_I \ .$$

For a negative 10 percent change in elasticity, the adjusted demand functions are:

$$Q_M = 91.22 - .287p_M$$

$$Q_A = 89.02 - 1.753p_A \tag{3-66}$$

$$Q_I = 21.35 - .192p_I \ .$$

Table 11 contains the numerical results of changing each of the point elasticities individually and in unison on the optimal prices at K = 13.0. It can be observed that individual changes in any elasticity always result in optimal price changes of the same sign for the corresponding use, and price changes of the opposite sign for the other two uses. In other words, a positive change in any single elasticity makes it profitable to transfer less water to the corresponding use and more water to the other two uses. In effect, the demand curve whose elasticity has been positively perturbed becomes steeper. This allows the same profit to be made by transferring less water to this use, i.e., by increasing the price to this use. At the 30 percent overall price increase level, the largest change in price occurs when the municipal demand elasticity is decreased. This causes a negative change in price to municipal users of $1.76 per acre-foot. As K increases, however, the effect of changes in elasticity also increases. If all the elasticities are either overestimated or underestimated the results will partially offset each other as can be seen in the last two rows of Table 11.

TABLE 11.--Effects of 10% elasticity changes on water prices ($/acre-foot), at K = 13.0

	Resulting water-price changes and % changes					
	p_M	%	p_A	%	p_I	%
$+e_M$	+1.09	+0.83	-0.68	-3.94	-1.51	-4.10
$+e_A$	-0.47	-0.36	+0.65	+3.77	-0.22	-0.58
$+e_I$	-0.37	-0.38	-0.08	-0.45	+1.59	+4.31
$-e_M$	-1.76	-1.34	+0.68	+3.94	+1.50	+4.07
$-e_A$	+0.90	+0.69	-0.90	-5.21	+0.42	+1.14
$-e_I$	+0.14	+0.11	+0.04	+0.23	-1.41	-3.82
$+e_M, e_A, e_I$	+0.34	+0.26	-0.13	-0.75	-0.23	-0.62
$-e_M, e_A, e_I$	-0.61	-0.47	-0.23	-1.33	+0.44	+1.19

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

In this final chapter, we will link the entire decision-making process which has been presented thus far, anticipate and answer various critical comments, and point out directions in which the use of operations research in water resources seems to be moving.

The principal conclusions to be drawn from this analysis are:

1. The choice of both water-supply planning objectives and source-use relationships should be from a broader base of alternatives than traditional practice allows.

2. Relationships between water use and price (the concept of economic demand) can be quantified and should be incorporated in water-supply planning models.

3. The pricing model presented is a valid water-supply planning objective function.

4. The pricing model can be used, as it was for the Tucson Basin, to indicate a water-supply plan which maximizes both net social payoff to the community and actual monetary profits to the central water-control agency.

5. Optimization with detailed programming techniques can cloud the general understanding of the problem and sometimes implies that certain insights are missing. Decomposition can often prove quite fruitful in revealing some of these insights and also leads to simpler solutions of nonlinear functions.

It appears that continued research into the planning of conjunctive use water-supply systems should be directed toward combining the management models with aquifer analogs. This would involve representing the management model as a distributed parameter system and thus a more detailed data collection program than was attempted in this analysis would be required. Data on costs and demands would be required from each subsector of the basin.

The purpose of this effort has been to present the water-supply planning problem in all of its ramifications from choosing the objectives and describing the system to optimizing the model and noting its plausibility. The framework was built in chapter 2 where we discussed possible objective functions, showed their economic basis, mentioned the traditional and more recent solution methods, and saw what work in this field has gone before. Now let us reflect for a moment on that part of the presentation.

In general terms, every organization's planning objective is to maximize the difference between its gains and losses, however these may be expressed. These gains and losses are functions of certain variables (some

controlled by the organization, some uncontrolled), and hence we can express the organization's objective in mathematical form. This is precisely what we did in the first part of chapter 2 for a series of water-supply planning objectives. But, how does the water-resources engineer with operations research capabilities choose a specific objective function for his particular project? The answer is that in practice he does not choose. At the university level he is free to speculate and theorize, but in practice the objective function is determined by the public administrator and the type of data available. If the public administrator is reflecting the consensus of community values, then indeed he will be working with the correct objective, but the fact is that he is often a victim of power politics and reflects individual interest or interest-group demands. When the individual interests do not coincide with the public consensus, then diseconomies will likely appear and detract from the representativeness of the objective in maximizing net community gains from the use of its water resource. Let us not be naive, however, and think that this consensus of community values is easy to obtain, for it is not. The reason interest groups are so strong is that they are easy to listen to -- they interject themselves, whereas the general public does not. The American democratic system is so styled as to answer to the interest groups, and only when their views conflict mightily with the public are the people aroused to the point where they react. But the beautiful thing about the system is that this reaction can take place and on occasion it does take place. In fact, it is taking place to a minor degree in this effort, through the presentation of water-supply planning objectives which are alternatives to the standard requirements approach.

The models were presented and compared in a static framework because this is how their basic natures can be seen most clearly. These same models are the building blocks of a planning model which extends over time; therefore, it was not thought necessary to clutter the explanations with additional time subscripts. To extend the models over a time horizon, it is necessary to make predictions as to what the input data will be in each time period and also to state quantitatively any economic or hydrologic interdependencies which may be present between stages. It was seen that if the interdependencies do exist they must be entered into the model, and then the model has to be solved as a whole, i.e., simple decomposition by time periods cannot take place. A current criticism of some of the multi-stage planning models is that all of these interdependencies have not been taken into account and if the models are used assuming independence they are used incorrectly. These types of mistakes are very noticeable in the water-supply models when optimal solutions call for drastic reallocations from stage to stage. We know from an engineering viewpoint alone that this would be impractical to implement and, therefore, some sort of dependency function is required to eliminate these possibilities from the model. The other big question when working with planning models over time is whether or not to use the present value concept and, if it is used, what interest rate should be applied. In this effort, we have concentrated on variable costs which are not incurred until the water transfer takes place, and the attitude has been taken that benefits accruing from the transfers are most important to the future recipients. For these reasons, the present-value concept in the 1975 model was not used, and thus, the problem of deciding on an interest rate was evaded. The whole question of interest rate is presently a point of much controversy at all levels of planning, and it is not out purpose here to discuss it further.

As concerns model building and optimization in general we have implied throughout the presentation that the problem should not be approached with any particular optimization technique in mind. In this way we will not be biased to formulate a model which conforms to any particular technique. The model at best will always be an approximation to reality, but the differences with reality should be kept as confined as possible, and this is best done by worrying about solution techniques after the model has been formulted. As was mentioned, the engineer now has quite an array of optimization tools available to him, and only after the model has been formulated should he consider modifying it for the sake of optimization.

A case study of a portion of the water-supply planning problem in the Tucson Basin was presented in chapter 3. It is only a portion of the total problem because political and legal realities have been overlooked and also because the basin has been considered as a lumped system. But, nevertheless, this endeavor has merit to the extent that it provides a conceptual guide-line for sound economic development of the water resource. The extent to which realities require deviation from a plan such as this can be likened to the extent to which social goals deviate from economic goals.

The establishment of some sort of central water-control agency in the Tucson Basin seems likely at present. An organization known as the Tucson Urban Area Regional Reviewing Committee and made up of representatives from Tucson, South Tucson, and Pima County already exists and is considering basin-wide water problems, but they have authority only to suggest various courses of action, not to implement them. However, elsewhere where ground-water basin management is also a vital public concern central water authorities have been established and can claim some success. A good example is the Orange County Water District in California (Weschler, 1968). It has responsibility for and authority to implement all aspects of management and control of the groundwater basin in the county. A pump tax levied on units of groundwater pumped by individuals provides a means for regulation of the extraction rate.

The other part of the problem not accounted for in this analysis is the design of an actual distribution network within the basin. This would require definition of the system in terms of spatially distributed parameters, and economic data not normally considered worthy of the required effort would have to be gathered. Detailed hydrologic data are much more readily available. The models as they were presented are capable of handling a spatially distributed system if the appropriate data are available and, thus, a distribution network could be designed. The same type of data required in the problem in chapter 3 would be needed for each subarea within the basin. The dependency problem of well interference would be accounted for by operating the models in conjunction with an analog of the groundwater system as was done by Martin, Burdak and Young (1969).

The pricing of public water supplies is not publicly acceptable as a device for limiting demand at the present time. It does seem, however, in arid and semiarid areas where people are unusually conscious of possible water shortages that the idea may gain a certain amount of acceptance. This process is in fact a naturally occurring one even now. This is witnessed by the dissolution of marginally valued irrigation projects as the cost of pumping and delivering water increases with increased depths to water.

The basic pricing model was used in all the applications in chapter 3 as a means of efficiently operating a central water-control agency. The reduction in demand would presumably occur if not otherwise constrained in this model. However, the policy constraint and requirement constraints can be used to keep the total water use at any level desired.

We have concentrated here on questions of allocative efficiency, but as pointed out by Weisbrod (1968) the effects of income distribution are just as important in water-resource project evaluation. This is evidence on a national scale by the authorization of projects from particular geographic regions in spite of their relatively low measures of economic efficiency. In one of the applications in chapter 3, an implicit decision concerning distributional equity was made in the formulation of the policy constraint. The weights developed for the different types of uses favored the municipal users over the agricultural and industrial users. Up to the point where K = 0.6, profits would be made from the industrial and agricultural sectors only. Above this value of K, municipal users would provide an increasing percentage of the profits but, on an individual basis, the irrigators and industrialists would contribute the most. This would have the effect of redistributing income from these irrigators and industrialists to the municipality.

In general, we may conclude that the pricing model has potential as a means of regulating groundwater withdrawal and allocating water in a conjunctive use situation. The dual purpose of economic efficiency and distributional equity can be quantitatively explored in this framework, but implementation of results requires the establishment of a central water-control agency. Before this can be accomplished, more politicians are needed who recognize that alternative choices in water-supply management do exist and and who are willing to review the alternatives without bias and take responsibility for the plan of their choice.

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