A PROPOSED COURSE OF STUDY IN HIGH SCHOOL ALGEBRA

by

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CHAPTER I

INTRODUCTION

Algebra, as it is taught in the American secondary school, appears to be both difficult to learn and difficult to teach. Evidence of this is found in numerous studies revealing the large percentage of failures in first year courses, and in the results of investigations as to errors and difficulties experienced by students. Thorndike, after an exhaustive study of the latter type, sums up his findings in the following words: "It does not seem an exaggeration to say that on the whole these students of algebra had mastery of nothing what-so-ever."[1] To this may be added the persistent complaints of teachers of advanced courses in mathematics, and in subjects other than mathematics in which algebra is used, that many of their students are unable to use their algebra correctly. A recent survey, 1943, made in connection with the government training program for aviators seems to confirm the truth of both of these statements.[2] This survey was made in order to

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1. Thorndike, Edward L. "The Strength of Mental Connections Formed in Algebra." Mathematics Teacher, 15:318-31.
2. National Committee on Mathematical Requirements. Reorganization of Mathematics in Secondary Education. U.S. Bureau of Education, Monogram 17.

determine whether or not students who have finished their formal schooling can be relied on to do accurate computation. Two thousand persons ranging in grade level from the ninth grade through the first year in college were included in the study. The test used in this survey consisted of one hundred items covering computation in the fundamentals of both arithmetic and algebra. It was given in forty different high schools and colleges throughout the United States. The average score made on this test was only forty-six. The seriousness of the situation revealed by this survey lies in the fact that deficiencies in computational skills impose serious handicaps on the persons concerned. They will hinder the individual in his mental reactions whenever quantitative elements are involved, whether these elements are encountered in school studies or in the practical affairs of life. It is significant that certain war industries, military establishments, and teacher-training institutions have found it necessary to institute remedial programs in order to overcome the computational deficiencies of individuals who have been victims of the low standards of achievements found in many of our public schools.

This condition is due, primarily, to the fact that the major purpose for instruction in algebra in the past has been to meet requirements set up by college entrance examination boards, and many of the courses of study and textbooks

which we have today have been arranged with that purpose in view. The work of the pupil has been of the blind imitation and mechanical manipulation type. It seems probable that the aim, content, and method of the course have occasioned the high percentage of failures in the subject. By failures is meant not only those who failed to make a passing mark in the course, but also those who "got credit only."

For some time the writer has felt the need for a more comprehensive course of study in algebra--one that would make the course more meaningful to the pupil, and one that would be based on insight and appreciation rather than on imitation and mechanical manipulation. Therefore, this study has been undertaken for the purpose of furnishing teachers of algebra with a complete handbook containing teaching units arranged in logical sequence and in unit form; and suggestions as to methods of presentation, drill, testing, and evaluation.

The Problem

The purpose of this study is: first, to determine what should be the aims of instruction in algebra; second, what materials should be used in making a course of study in algebra in order to attain these aims, and in what order should these materials be arranged; third, how should these materials be presented to attain the accepted goals for algebra instruction.

Method of Treatment

All available current educational literature on the subject of the teaching of mathematics in general and algebra in particular was examined in an attempt to determine what the commonly accepted aims and objectives for the teaching of algebra are. The findings of this study are given in detail in the following chapter on aims and objectives.

The selection and organization of the subject matter and teaching procedures which appear to offer the greatest value toward the satisfaction of these accepted aims were made from the sources in the preceding paragraphs. The two sources of information of greatest value used in this study were the 1923 report of the National Committee on the Reorganization of Mathematics in Secondary Schools,[3] and a survey report published in 1932 by the United States Bureau of Education.[4] The latter report was especially valuable because of the large number of courses of study included in the survey.

To set up general aims and objectives in any course is not sufficient. These are generally broad, rather inclusive statements, or at least are so regarded by many teachers

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3. National Committee on Mathematical Requirements. op. cit.
4. National Survey of Secondary Education. Instruction in Mathematics. U.S. Bureau of Education Bulletin 17, Monograph 23.

who are prone to lose sight of the fact that each single unit of material contributes to the final structure of achievement. Just as a class period is ineffective and wasteful of time and effort when specific aims and objectives are lacking, so is a course of study which does not take account of the aims to be accomplished in each unit. The aims of a course shape its character. Therefore, an attempt has been made to justify each unit of subject matter included in this course of study in terms of the accepted aims set up.

Related Studies

Practically all committees charged with the problem of reconstructing the mathematical curriculum have come to a common conclusion. The Committee of Ten in 1893 recommended instruction in algebraic symbols and in simple equations.[5] It advocated that a child's geometrical training should begin as early as possible; that systematic instruction in concrete and experimental geometry should begin at the age of ten years, and that it should occupy one hour per week for at least three years. These recommendations followed closely the prevailing practice in Europe. It was not presumed that algebra and geometry should be taught in the early grades as separate subjects, but that they be taught in

5. Monroe, Walter S. and Weber, Oscar F. The High School, p. 295.

connection with arithmetic.

Thirty years later, in 1923, the National Committee on Mathematical Requirements said in its report:

> In recent years there has developed among progressive teachers a very significant movement away from the older rigid division in "subjects" such as arithmetic and algebra, each of which is "completed" before the other is begun, and toward a rational breaking down of the barriers separating these subjects in the interest of an organization of subject matter that will offer a psychologically and pedagogically more effective approach to the study of mathematics. ... The advocates of this new method of organization base their claims on the obvious and important relations between arithmetic and algebra, which the student may grasp before he can gain any real insight into mathematical methods, and which are inevitably obscured by a strict adherence to the conception of separate "subjects." ... The newer method enables the pupil to gain a broad view of the whole field of elementary mathematics early in his high school course. In view of the large number of pupils who drop out of school at the end of the eighth or ninth school year, or who for other reasons cease their study of mathematics, this fact offers a weighty advantage over the older type of organization under which pupils studied algebra alone during the ninth school year to the complete exclusion of all other types of mathematics.[6]

One of the most valuable series of studies in the field of mathematics was made by Dr. Breslich of the University of Chicago. This series of investigations was begun in 1928 and completed in 1931.[7] The study was concerned with problems that relate to effective methods and procedures in

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6. National Committee on Mathematical Requirements. op. cit., p. 10.
7. Breslich, Earnest R. Problems in Teaching Secondary School Mathematics, pp. 10-15.

teaching mathematics, and to the evaluation of the results of teaching. This study was based on the recommendation made in 1927 by the National Committee. The first of the series was devoted to the study of problems arising from the teaching activities which arise during the teaching of a unit to a class, making assignments, supervision of pupil study, and conducting recitations. The second was concerned with the psychological considerations such as motivating the study of mathematics, maintaining pupil interest, developing effective habits of study, providing for individual differences, supplying sufficient practice materials, doing corrective teaching, and eliminating typical errors and difficulties. The third of the group deals with methods of presenting subject matter such as the individual method, the lecture method, the question-answer, the laboratory method, the directed-study method, and the project method. The fourth and last of the group pertain to the evaluation of the results of teaching, the construction and administration of objective tests to determine the pupil's apperceptive mass, his mathematical ability, and his habits of study, tests for research purposes, and diagnostic tests whose results might be used to improve teaching. The findings of these studies have been a valuable source of information for curricular revision and improvement.

CHAPTER II

AIMS AND OBJECTIVES FOR INSTRUCTION IN MATHEMATICS

The Development of Aims and Objectives in Mathematics Since 1893

The disciplinary function of mathematics began to be emphasized about 1825, and by the middle of the century it represented the dominant value. Mental arithmetic, frequently called "intellectual arithmetic," was considered to possess a very high disciplinary value. The following is a statement from the preface of a textbook published in 1873:

> The science of arithmetic, until somewhat recently, was much less useful as an educational agency than it should have been. Consisting mainly of rules and methods of operations, without representing the reasons for them, it failed to give that high degree of mental discipline which, when properly taught, it is so well calculated to afford. But a great change has been wrought in this respect; a new era has dawned upon the science of numbers; a "royal road" to mathematics has been discovered, so graded and strewn with the flowers of reason and philosophy that the youthful learner can follow it with reason and pleasure; and one of the most influential agents in this work has been the system of mental arithmetic.
>
> The importance of this change can hardly be over estimated. The study of mental arithmetic, introduced by Warren Colburn, to whom teachers and pupils owe a debt of gratitude which can never be paid, affords the finest mental discipline of any study in the public schools. When properly taught, it gives quickness of perception, keenness of insight, toughness of mental fiber, and an intellectual power and grasp that can be acquired by no

> other elementary branch. An old writer on arithmetic quaintly called his work "The Whetstone of Wit." Mental arithmetic is, in my opinion, truly a whetstone of wit. It is a mental grindstone; it sharpens the mind and gives it the power of concentration and penetration. To omit a thorough course of mental arithmetic in the common school is to deprive the pupil of one of the principal sources of mental power.[1]

In the period from 1875 to 1890, some writers began to emphasize what was generally designated as the cultural value of mathematics. This attitude is expressed clearly in the following statement quoted from a paper read by W.H. H. Hudson before the Educational Society, London, in 1886:

> I maintain, therefore, that algebra is not to be taught on account of its utility, not to be learnt on account of any benefit which may be supposed to be got from it; but because it is a part of mathematical truth, and no one ought to be wholly alien from that important department of human knowledge.[2]

The work of the Committee of Ten, which reported in 1893, marked the beginning of the present era of secondary school expansion and development. In its report this Committee insisted that the disciplinary function of mathematics be minimized, and that the aims and objectives, the content, and the methods of instruction be modified so that the practical phases would be emphasized. The Committee

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1. Monroe, W.S. Development of Mathematics as a School Subject. U.S. Bureau of Education Bulletin No. 10, pp. 91-92.
2. Smith, D.E. The Teaching of Mathematics, p. 167.

also recommended that the different courses in mathematics be

> abridged and enriched; abridged by omitting entirely all subject-matter which perplex and exhaust the pupil without affording any really valuable mental discipline, and enriched by a greater number of exercises in simple calculation and the solution of concrete problems.[3]

The Committee recommended further the use of "oral exercises" in algebra, and suggested that the proof of the binomial theorem and other difficult propositions be omitted.

The 1913 report of the Committee on Mathematics of the Commission on the Reorganization of Secondary Education gave three chief aims for instruction in mathematics in the secondary school. They were:

1. To inspire and facilitate the acquisition of knowledge in an important field of human thought; mathematics is necessary to the comprehension and mastery of nature.
2. To develop the ability to apply this knowledge to practical and theoretical investigations.
3. To develop and strengthen the ability to preceive exact relations and to make inferences correctly. The teacher's constant aim should be to train the pupil to think and to formulate clearly the results of his thinking.[4]

In addition to these aims the Committee recommended a list of attainments which might reasonably be expected of students completing a high school course and presenting

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3. Report of the Committee of Ten on Secondary School Studies, p. 105.
4. McLean, George Edwin. Present Standards of Higher Education in the United States. U.S. Bureau of Education, Bulletin No. 4, p. 171.

themselves for admission to college. They are:

1. A fair degree of accuracy and rapidity in calculations, and a fair knowledge of the application of numbers to the solution of the common problems of life.
2. A fair degree of skill in making algebraic transformations.
3. The ability to use the equation as an instrument in the solution of problems.
4. The ability to interpret algebraic results.
5. A fair comprehension of what constitutes a proof in mathematics.
6. A good knowledge of the facts of elementary algebra, and plane and solid geometry.[5]

In the recommendation concerning the organization of materials and methods of instruction the Committee stated:

> We recognize the fact that the students of the last high school year, because of their greater maturity, have much more ability to grasp the abstract thinking of mathematics than do students of the first year. The material offered in the courses and the methods of instruction should be determined with this in view. At first the simpler and more concrete ideas of the subject should be dealt with. In later stages, more complicated mechanical work and formal theory should be introduced gradually.[6]

In 1916 the Mathematical Association of America organized a committee to be known as The National Committee on Mathematical Requirements. It was the purpose of this Committee to give impetus to the movement which had been developing in the direction of reform in the teaching of mathematics. The 1920 preliminary report of this Committee illustrates the views of the Committee on the reorganization

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5. McLean, George Edwin. op. cit., p. 172.
6. Ibid.

of mathematics in secondary schools. The following is a summary of the recommendations of this report:

> The committee believes that introductory mathematics, ordinarily conceived as separate courses in algebra, geometry, and trigonometry, should be given in connection with the solving of problems and the executing of projects in fields where students already have both knowledge and interest. This would make the study of mathematics more nearly approximate a laboratory course, in which individual differences could be considered and effective devices of supervised study be utilized. The minimum of the course might well in this way be cared for in the recitation period, reserving the outside work for allied projects and activities for the more advanced pupils.
>
> A content thus instrumentally selected will be free of the old formal puzzles, the complex instances, the verbal problems which in the past have wasted so much time and destroyed so much potential interest; and will, on the other hand, extend across the divisions heretofore separating algebra, geometry, and trigonometry.[7]

The Committee also suggested that courses be planned for four groups of pupils:

> (1) The "general readers" whose needs lie largely in the "interpretative" functions of mathematics; (2) those who, expecting to enter trades, would have a small but still definite need for "practical" mathematics; (3) those who, as prospective engineers, would need a considerable body of content determined by the demands of engineering study and practice; (4) those specializing in mathematics who would wish a content determined by the satisfactions inherent in the activity and by the demands of further study.[8]

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7. Problems of Mathematics in Secondary Education. U.S. Bureau of Education, Bulletin No. 1, p. 24.
8. Ibid.

In the 1920 report there are mentioned for the first time interests of students, individual differences, and integration of the different mathematical subjects. This, within itself, is proof that we had come a long way toward the reorganization of mathematics for the general benefit of the masses rather than for the favored few.

The 1923 report of the Committee on Mathematical Requirements has become widely recognized as authoritative. Accepting the customary classification of aims into practical and utilitarian, disciplinary, and cultural, the Committee gave a more satisfactory statement of them than had been available previously. The following is an excerpt of the report:

> The primary purpose of the teaching of mathematics should be to develop those powers of understanding and analyzing relations of quantity and of space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and action which will make these powers effective in the life of the individual.[9]

Under the practical and utilitarian aims were given: (1) accuracy and facility in numerical computation; (2) an understanding of the language of algebra and ability to use it readily and intelligently; (3) the ability to

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9. National Committee on Mathematical Requirements. The Reorganization of Mathematics in Secondary Education. U.S. Bureau of Education, Bulletin 23, p. 10.

interpret correctly graphic representation of various kinds; (4) a familiarity with geometric forms common in nature, in industry, and in life; and (5) the acquisition of ideas and concepts in terms of which the quantitative thinking of the world is done.

Under the disciplinary aim the following were stated: (1) the ability to use those ideas and concepts in terms of which the quantitative thinking of the world is done in the practical situations of life; (2) the mental habits and attitudes needed to make the above training effective in the everyday life of the individual; (3) training in functional thinking, as this term is used in mathematics.

Under the culture aims the Committee emphasized: (1) appreciation of beauty in the geometrical forms of nature, art, and industry; (2) the inculcation of ideals of perfection as to logical reasoning, discrimination between the true and the false, et cetera; (3) an appreciation of the extent of the influence of mathematics in the development of our civilization, the progress of science, the growth of industry, and the establishment of systems of philosophy.

In the conclusions of this report the Committee gave the following additional recommendations that were highly significant of the direction in which mathematical reforms were moving:

> (1) It is very desirable to bring mathematical writing into closer touch with good usage in English writing in general. This is desirable because mathematics should be a genuine help toward the acquisition of good habits in the speaking and writing of English. (2) The committee recommends that teachers of mathematics and writers of textbooks eliminate, so far as possible, obsolete technical terms and symbols. It seems perfectly natural that a printed discussion should strike the pupil as an expression of reasonable ideas in terms of reasonable English forms. The fewer technical terms introduced, the less likely is the subject to give the impression of being difficult and a mere juggling of words and symbols. (3) The committee suggests that examining bodies, contributors to mathematical journals, and authors of textbooks endeavor to follow the general principles formulated in this report.[10]

The report of this Committee has had an important influence on the point of view of teachers. Authors of textbooks almost invariably have claimed that their books conform to its recommendations. Teachers in training institutions have studied this report diligently, and writers on methods of teaching continue to devote many of their pages to a discussion of its views.

In 1929 the North Central Association of Colleges and Secondary Schools recommended that an extensive study of mathematics, "as it exists at the present time in the schools of the United States," be made by the National Office of Education. This survey was made under the

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10. National Committee on Mathematical Requirements. op. cit., p. 84.

direction of Dr. Leonard V. Koos of the University of Chicago, and the committee report was published in 1932. The investigation was made, primarily, to determine the similarities and the differences in the aims and objectives, content, and methods of teaching and the results accomplished in the field of mathematics throughout the public schools of the United States. Seventy-nine schools throughout the country were included in this study. These schools were selected because they published their own courses of study, and their work in mathematics was considered outstanding.

Table I shows the regional distribution of the cities included in this survey.[11]

The first phase of this survey (page 4) was devoted to a study of the methods of treating the major phases of instruction in mathematics. Fifty-seven courses of study were used. The findings of the study are shown in Table II.

The results of this study are significant. They show conclusively that, although we had come a long way since 1893, we were not at the journey's end by any means; that, while the organization of course of study outlines had been improved since 1920, they were far from being the important

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11. National Committee on Mathematical Requirements. Instructions in Mathematics. U.S. Bureau of Education, Monograph 23, p. 2.

TABLE I

DISTRIBUTION OF CITIES INCLUDED IN THE KOOS STUDY
BY REGIONS AND BY POPULATION GROUPS

Region	Number of Cities by Population Groups				
	Less than 10,000	10,000 30,000	30,000 100,000	100,000 500,000	More than 500,000
New England	0	0	2	4	1
Mid-Atlantic	1	2	9	3	4
Southern	1	1	2	3	0
Mid-West	4	3	12	7	5
West	3	1	3	6	2
TOTAL	9	7	28	23	12

TABLE II

METHODS OF ORGANIZATION OF COURSES OF STUDY IN MATHEMATICS AS SHOWN BY THE KOOS STUDY

Phase	Informal Statement	Bare List	Organized Outline	Parallel Column	Units	Para-graph	No State-ment
Objectives	5	32	4	6	0	1	9
Content	5	16	20	10	5	0	1
Procedure	16	7	5	10	0	7	12
Evaluation	5	10	7	0	1	0	34
Individual differences	7	2	1	4	0	1	42
Miscellaneous	15	4	1	0	0	0	39
References	0	21	2	2	0	0	32

teaching aid they were intended to be. As a result many teachers, especially those with less teaching experience, were "teaching" textbooks rather than children. "The forest was still hidden by the trees."

General Objectives for Instruction in Secondary School Mathematics

The second phase of this investigation by the National Committee on Mathematical Requirements of 1932 was the analysis of the objectives contained in fifty-seven courses in use at the time the survey was made. The objectives were listed under seven headings, and the number of courses of study containing each was tabulated. In Table III the findings of this analysis are shown.[12]

This table indicates that disciplinary values still were given considerable prominence by the schools represented. It is also evident that the practical and utilitarian aims were considered of most importance.

In summarizing its findings with reference to objectives, the Committee states:

> The objectives of mathematics as formulated by the National Committee of 1923 seemed to have influenced markedly those appearing in the outlines examined. The following characteristics

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12. National Committee on Mathematical Requirements. Instructions in Mathematics. U.S. Bureau of Education, Monograph 23, p. 19.

TABLE III

OBJECTIVES FOR INSTRUCTION IN SECONDARY SCHOOL MATHEMATICS AS SHOWN IN THE SURVEY MADE BY L.V. KOOS

Objectives	Number Listing
1. Accuracy in fundamental processes	35
2. Knowledge and power to apply mathematical concepts	22
3. Specific knowledge useful in life	28
4. Disciplinary values	28
5. Cultural values	22
6. Specific future needs	8
7. Exploration and guidance	15

seem to prevail:

1. Aims which relate to the practical objectives of instruction seem to predominate.
2. They include the application of mathematics to the social and economic environment of the child.
3. Informational as well as computational mathematics is emphasized.
4. The introduction and correlation of mathematics from higher courses is stressed.
5. Disciplinary and cultural aims appear in the majority of outlines, but less often than the practical.

Although the aims reflect a leaning toward the practical side of mathematics, the outlines do not indicate a decided connection of such aims with teaching materials and procedures. There seems to be need of more careful breaking up of objectives

> so that they will not only connect with actual materials of instruction, but also the degree to which they do connect may be measured.[13]

The Committee did not make any direct or specific recommendations. By implication it did suggest: (1) a psychological rather than a logical organization of materials of instruction; (2) a closer connection between aims, materials, and procedures; (3) better trained teachers.

The statements of aims and objectives in present-day courses of study do not differ materially from those found by the Committee in 1932. This is shown by the following aims and objectives as quoted from one of the recent publications on the subject of teaching mathematics in high schools.

I. Practical aims
   1. To give each pupil a mastery of those phases of mathematics which are necessary for living in his social and economic environment.
   2. To give each individual those phases of mathematics which are necessary for his vocation.
   3. To enable individuals to recognize mathematical situations so that they may refer them to specialists if they are unable to solve them.
   4. To enable individuals to discover new and more efficient methods of procedure.

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13. National Committee on Mathematical Requirements. Instructions in Mathematics. U.S. Bureau of Education, Monograph 23, pp. 65-72.

II. Preparatory aims
   1. The acquisition of a sufficient knowledge of the elements of mathematics which will serve as a basis for further study if his present knowledge is not sufficient to meet the requirements of his vocation.
   2. The mastery of the elements of mathematics and mathematical methods preparatory to further study in that field.

III. Cultural aims
   1. To develop the appreciation of the values of mathematics as a tool for solving the problems of our environment.
   2. To give the student sufficient knowledge of mathematics to enable him to make the application implied above.
   3. To develop the vocabulary through the study of mathematical terms.
   4. To develop an aesthetic value through the study of mathematical forms.

IV. Disciplinary aims
   1. To develop an understanding of the mathematical method in solving problems.[14]

In all the articles read by the writer, coming either directly from the military authorities or the United States Bureau of Education working in cooperation with them, no criticisms were found of the aims and objectives for instruction in mathematics. The criticisms which have been offered deal with the fact that we have been satisfied with minimum attainments, and that the procedure used for measuring attainments has been inadequate. The following excerpt

14. Minnick, J.H. Teaching Mathematics in Secondary Schools, pp. 38-48.

is indicative of the trend of all these criticisms:

> The committee making this report feels that adjustments must be made, in the field of mathematics, but no extensive reorganization of the mathematics courses should be undertaken. The reason is fairly obvious. The modern program has been emphasizing the concepts of mathematics that are of permanent worth in our industrial civilization. These basic and permanent ideas, such as ratio, proportion, the language of algebra, geometric concepts, and above all arithmetic fundamentals, are the concepts so desperately needed in our war effort. There is no need to recognize the materials of the mathematics program in order to meet the needs of the armed forces. WHAT WE NEED TO DO IS TO TEACH MORE THOROUGHLY THE BASIC MATERIALS ALREADY IN THE PROGRAM AND PRESENT IDEAS OF PERMANENT WORTH RATHER THAN SPECIALIZED TECHNIQUE.[15]

On the basis of the findings of these investigations, and committee reports, the writer offers below what seem to be the more commonly accepted aims and objectives for instruction in mathematics in secondary schools:

1. To increase skills in computation.
2. To increase understanding of basic concepts and principles of mathematics.
3. To provide a wider range of application of mathematics to other fields and to life needs.
4. To achieve more power in mathematical reasoning and problem solving.

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15. Mallory, Virgil S. "Essential Mathematics for Minimum Army Needs." The Mathematics Teacher, March, 1943, pp. 243-47.

5. To secure a thorough understanding of measurement.
6. To become familiar with the properties of the common geometric forms.
7. To be able to express and use mathematical relationships.
8. To learn to use and appreciate symbolism.
9. To be able to recognize the inter-relationships of the various mathematical fields.
10. To understand how mathematics is related to practically all other branches of learning.
11. To enjoy mathematics for its own sake.

Objectives for Instruction in Algebra

In the teaching of algebra, as in any other kind of work we set out to do, we can not expect to realize our aims unless they are specific and are defined clearly at the outset. The first step, therefore, in planning a course of study in algebra is to prepare a list of desirable objectives which we hope to attain. Such a list should be the result of much study, thought, and discussion on the part of those who will actually use it, who know the work of the classroom, and who themselves have some concept of what can be expected of the pupils.

There are two kinds of objectives to be kept in mind: the great central ones, and those that more specifically concern themselves with algebraic concepts. The former have to do with the realization of aims and objectives that

are not peculiar to the field of algebra alone, but which are to be sought in all fields of knowledge and particularly in the field of mathematics. For example, the most important major objective in teaching of any junior high school mathematics is to develop well-educated, useful citizens. This purpose is much the same in literature, history, or art. However, major objectives alone are not sufficient. All algebra and each algebraic activity should have a specific objective, and that objective should be carefully geared to one or more of the more general aims. If this is not true, it is very easy for teachers to "lose sight of the forest because of the trees." A great deal of time and effort can be wasted when specific aims and objectives are lacking. They should not be elaborate and "lovely" statements of things hoped for, but simple and clear-cut statements of objectives to be achieved. The more elaborate the statement of objectives, the less likely they will be put into practice. The aims of a course should shape its character.

The aims and objectives of this course of study were selected after a careful examination of all available current literature on the subject of instruction in algebra. The examination was made in order to try to determine what are the commonly accepted aims for the teaching of algebra. Three important criteria were used: (1) the intrinsic

worth of algebra itself; (2) the needs of students in general, and especially those of the Tucson schools; and (3) the interests of the students who are to study the subject.

These aims are:

1. To develop an understanding of the formula, the graph, and the equation as useful tools in solving problems.
2. To teach the student to know the meaning of positive and negative numbers, and to be able to interpret them in his reading.
3. To give the student a more accurate and ready working knowledge of the four fundamental operations: addition, subtraction, multiplication, and division.
4. To teach the value and use of literal representation.
5. To drill in algebraic technique for the purpose of increasing speed and accuracy in the use of fundamentals.
6. To give pupils who may desire to specialize in the fields of science or mathematics sufficient acquaintance with algebraic concepts and skill in their use to enable them to pursue such work successfully.

CHAPTER III

CONTENT OF THE COURSE OF STUDY FOR INSTRUCTION IN HIGH SCHOOL ALGEBRA

Subject content for instruction in algebra has changed more than any other branch of mathematics. In the ninth century, algebra changed from a subject dealing with puzzle problems solved by mere rule to being quite largely a subject in which the validity of rules was established. In the sixteenth century, it changed from a subject concerned with rational numbers and the quadratic equation to one which dealt with irrationals and with higher equations. In the seventeenth century, it changed from a subject with awkward symbols into one with a convenient symbolism and concerned with purely abstract formalism--work with polynomials which were rarely used in life situations, and the solution of equations which had but little practical value. This type of content continued to constitute the main course in algebra until about 1850. The period 1850 to 1893 was another period of significant change in the materials used for instruction in algebra. This change was governed principally by changes in the college entrance requirements. However, the course continued to have little practical value except to a very small group of individuals who were particularly

interested in mechanical and civil engineering, or in the field of physical science.

The 1893 report of the Committee of Ten made few recommendations regarding the content of mathematics on the high school level. In algebra the use of "oral exercises" was advised, and the suggestion was made that the proof of the binomial theorem and other difficult propositions be omitted.[1]

The 1913 Committee on Mathematical Requirements was the first group to make any extensive study of the trends in algebra, and also the first to make any definite recommendations concerning the content of a course of study in algebra. These recommendations were made, however, for college entrance requirements only, and the Committee stated that no significance was to be attached to the order in which they were listed. They were:

1. The fundamental laws of algebra.
2. The general view of algebraic numbers.
3. The four fundamental operations as applied to integral, fractional, and irrational expressions.
4. Factoring.
5. Binomial theorem for positive integral exponents.
6. Solution of equations in one variable, including simple quadratics, fractional, and irrational equations.

1. Report of the Committee of Ten on Secondary School Studies, p. 105.

7. Solution of systems of equations in two variables, including linear system, linear quadratic system, and a few of the quadratic systems that occur more frequently in practice.
8. Ratio and proportion.
9. The statement and solution of problems.[2]

This Committee also recommended that arithmetic, algebra, geometry, and trigonometry should be regarded and treated as difficult phases of the same great subject--mathematics--and that algebra be divided into two portions; i.e., elementary algebra, to be given in the first year of high school, and a more advanced course to follow plane geometry in the third or fourth year of high school.

The 1920 preliminary report of the Committee on Mathematical Requirements concerning the content of the course in algebra was based on the 1913 report. However, some recommendations were made concerning arrangement of materials in both courses. These recommendations were:

I. Minor Requirements
   1. Simple formulas
   2. Positive and negative numbers
   3. Development of variables
   4. Four fundamental operations
   5. Factoring
   6. Fractions
   7. Graphs and simple graphic representations
   8. Ratio
   9. Linear equations in two unknowns
   10. Exponents and simple radicals

2. McLean, George Edwin. Present Standards of Higher Education in the United States. U.S. Bureau of Education, Bulletin No. 4, p. 172.

II. Major Requirements
    1. Quadratic equations
    2. Exponents and radicals of higher order
    3. Binomial theorem
    4. Graphs and graphic representations of the second degree.[3]

The recommendations made by the 1923 Committee were practically the same as the 1913 and 1920 reports except they were given in more detail. They were:

I. Formula
    1. As a concise language
    2. As a short-hand rule for computation
    3. As a general solution
    4. As an expression of the dependence of one variable upon another.
II. Graphs and graphic representation in general
    1. Representing facts, statistics, etc.
    2. Representing dependence
    3. Solving problems
III. Positive and negative numbers, their meaning and use
IV. The equation--its use in solving problems
    1. Linear equations in one unknown
    2. Simple cases of quadratic equations
    3. Equations in two unknowns
    4. Simple application of ratio and proportion
V. Algebraic technique
    1. The fundamental operations. Their connection with the rules of arithmetic should be brought out and made to illuminate numerical processes.
    2. Factoring. Common factors of the terms of a polynomial. The differences of two squares. Trinomials of the second degree that can be easily factored by trial.

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3. National Committee on Mathematical Requirements, Preliminary Report. The Reorganization of the First Courses in Secondary School Mathematics. U.S. Secondary School Circular, No. 5.

3. Fractions. The intimate connection with the corresponding processes of arithmetic to be made clear. The four fundamental operations should be considered only in connection with simple cases.
4. Exponents and radicals. Confined to simplest material required for the treatment of formulas.
5. Stress checking solutions.[4]

In 1923 a careful examination of twelve algebra textbooks ranging in date of publication from 1881 to 1920 revealed a number of significant changes in content. The recommendations of the National Committee on Mathematical Requirements were used as a basis of comparison. Of the topics that this Committee recommended for inclusion in the course in algebra, only 71 per cent were found in the text of 1881. All of them were included in the 1920 text. On the other hand, all the topics recommended for omission were included in the text of 1881, and 47 per cent of them still were included in the 1920 text.[5]

Although algebra texts still retain much of the content that characterized the texts of twenty-five to forty years ago, it is apparent that there have been significant changes in the topics included, as well as in arrangement and the nature of treatment. These changes may be listed

4. National Committee on Mathematical Requirements. Reorganization of Mathematics in Secondary Schools. U.S. Bureau of Education, Bulletin 17, p. 32.
5. Flagg, Elinor Bertha. "The Trends of the Content of First Year Algebra Texts." Unpublished Master's thesis, University of Illinois, 1923.

under three headings: (1) the elimination of materials no longer considered worth while; (2) the removal to a higher level of topics for which there is no immediate need but which have deferred values in more advanced courses, in subjects other than algebra, and in life situations; (3) bringing down to a lower level topics which traditionally have been taught at higher levels.

The 1932 Committee made a topical analysis of twenty-eight courses of study in algebra. These data, showing the percentage of schools offering the various topics, the order in which the various topics were presented, and the average number of weeks allotted to each, are shown in Table IV.[6]

The foregoing analysis reveals a wide variation in the order in which the different topics were introduced. For example, only 89 per cent placed the study of the fundamental operations first; 79 per cent followed the study of fundamentals with the beginning study of equations; and 57 per cent placed the study of the formula as item number five, following the study of factoring. This situation probably is due, in part, to the fact that no recommendations have been made concerning the best order of arrangement of

6. National Committee on Mathematical Requirements. The Reorganization of Mathematics in Secondary Schools. U.S. Bureau of Education, Bulletin 17, Monograph 23, p. 34.

TABLE IV

CONTENT OF TWENTY-EIGHT COURSES OF STUDY AS FOUND BY THE KOOS COMMITTEE IN 1932

Topic	Average % Counting	Average Order Presented	Average Time in Weeks
1. Fundamental operations in algebra	89	1	4.9
2. Factoring	86	4	4.4
3. Fractions	86	6	5.5
4. Exponents and radicals	86	9	3.4
5. Equations	79	2	3.8
6. Simple quadratics	71	10	2.7
7. Graphs	71	7	1.9
8. Positive and negative numbers	68	3	1.7
9. Linear equations	64	8	2.8
10. Formula meaning and use	57	5	1.0
11. Ratio and proportion	39	11	2.0
12. Equations in two unknowns	30	12	3.0

materials for instruction in algebra, with the exception of those made in the 1920 Committee report; and those recommendations were not stressed to any extent, but were offered as a suggestion for material arrangement.

In all the course of study outlines for algebra examined by the writer and in other current literature pertaining to content materials very little deviation from the various committee recommendations was found. Textbooks conform pretty closely to the 1923 recommendations and course of study outlines, more often than not, are made to fit some particular text. The following outline is indicative.

Division of Secondary Education

Tucson Public Schools

OUTLINE OF COURSE OF STUDY IN FIRST YEAR ALGEBRA

Text: Algebra and Its Uses; Silverstein, Newell, Harper

GENERAL OBJECTIVES:

A. PRACTICAL

ALGEBRA SHOULD DEVELOP

1. The ability to solve more successfully quantitative problems concerning the home, industry, and the government.

2. An understanding of

   a. Common mathematical symbols, and

   b. Some common business practices

3. The foundation for technical training

B. LOGICAL

A STUDY OF ALGEBRA SHOULD DEVELOP

1. The ability to acquire useful ideas for quantitative thinking

2. The ability to think independently

3. Better habits of concentration and persistence

4. An appreciation of precision, clearness, accuracy, neatness, and thoroughness, and a distaste for vagueness and incompleteness.

C. CULTURAL

A STUDY OF ALGEBRA SHOULD DEVELOP

1. An appreciation of the great contributions of mathematics to our present civilization, e.g. in astronomy, engineering, and the various other sciences

2. The power of concentration and persistence and the encouragement of systematic and orderly habits of work

3. An increased respect for truthfulness and honesty

4. An appreciation of the language of algebra as the principal tool of all mathematics above the level of arithmetic.

5. An appreciation of the contribution of mathematics to the war effort

D. PROFICIENCY TO BE EXPECTED

1. The minimum course in Algebra should include the following:

   a. The fundamental operations involving monomials, binomials, and trinomials

   b. Graphs (simple)

   c. Formulas

   d. Simple and quadratic

   e. Simultaneous linear equations in two unknowns, and

   f. Powers and radicals

FIRST SEMESTER

I. First Six Weeks: (pp. 1-119 new text)

1. Study terms and symbols
2. Simple formulas
3. Graphs
4. Sign numbers and evaluating formulas
5. Simple equations

II. Second Six Weeks: (pp. 158-198)

1. Simple equations continued
2. Fundamental operations involving binomials and polynomials
3. Reviews on pp. 1-119
4. Reviews of arithmetic recommended
5. It is recommended that pp. 120-158 be postponed until the second semester as honor work

III. Third Six Weeks: (pp. 198-225)

1. Special products and factoring
2. Reviews
3. Review of arithmetic. Fundamental processes with whole numbers, decimals, and common fractions.

SECOND SEMESTER

IV. Fourth Six Weeks: (pp. 226-263)

1. Solving quadratics by factoring (2 or 3 lessons)
2. Fractions

V. Fifth Six Weeks: (pp. 264-328)

1. Fractional equations
2. Simultaneous linear equations in two unknowns

VI. Sixth Six Weeks: (pp. 329-384)

1. Square roots and radicals
2. Quadratic equations
3. Selected reviews in algebra and arithmetic

REFERENCE: Mathematics for the Emergency by Lapp, Knight, Rietz. (Scott, Foresman and Company). Contains good reviews in arithmetic and algebra.

It is to be noted that the Tucson Course of Study in First Year Algebra follows exactly the textbook arrangement of materials given in Algebra and Its Uses by Silverstein, Newell, Harper, with one exception--it is recommended that the material presented on pages 120 to 157 inclusive be postponed until the second semester and then be offered as "honor work."

In the more recent committee reports referred to in the preceding chapter, no changes in the materials were recommended for the courses in algebra. Here again the same criticisms were offered to algebra as were offered for the other phases of mathematics: that there should be a closer

tie-up between aims and objectives and the materials of instruction; that the study of the mechanics of algebra should not be an end in itself, as it so often has been in the past, but a means toward some purposeful activity. In the various training programs connected with the war effort, both civilian and military, workers have had to "relearn" their mathematics. Especially is that true of those in the technical branches of the service and in industry. And when we remember that these last-mentioned individuals are high school graduates, we can not but wonder if our school system is not guilty as charged.

In the following units of instruction the writer has endeavored to combine aims and objectives, materials, and teaching procedures in such a manner that the study of algebra will not be a formal memorizing of formulas or facts, or mechanical manipulations based on rote memory or blind imitation. If algebra is to function in the everyday life of the student, he not only must understand those functions, but also must be able to apply them in problem situations.

Summary

If algebra is to be made a living subject, it must be approached gradually and the mind gradually led into the field of abstract thought. In the grades, vivid mental images are very often established, thus materially aiding

the process of recall. History and geography books are illustrated profusely, the arithmetic and the general science texts are filled with projects, but algebra is not so motivated. The result is that it becomes vastly different from anything the student has ever had before. Thus, the big problem confronting algebra teachers is how the subject can be presented most successfully. At the very outset many students who enter algebra classes do not understand the significance of the subject, its relation either to practical problems or to the study of higher mathematics or to science. The first few lessons then, in most cases, will determine the success or the failure of the student.

Some teachers of mathematics tend to over-emphasize speed, sometimes at the sacrifice of accuracy. Particularly is this true in algebra. Schultze, in The Teaching of Mathematics in Secondary Schools, says that too much emphasis has been placed on formal examinations and "spectacular effects." Too much is attempted in the time allotted, with insufficient assimilation of the materials studied. Pupils are not taught how to study algebra; they are drilled in abstract formulas. The result is that neither memory nor reasoning is developed properly. This fact alone accounts for many needless failures in the subject and, often times, a development of distaste for higher mathematics in general.

The content of a course of study in algebra should give evidence of being more than a mere logical outline of the basic textual materials. In the first place, the selection should relate specifically to the broad and to the detailed aims or objectives which have been adopted, and should reflect pupils' present-day as well as their future life needs. In the second place, the organization should be psychological in that it indicates serious effort to organize the material so that it will be within the grasp of the pupil and, as far as possible, appeal to his interests. In the opinion of the writer, the best type of such an organization is some form of the "unit" plan. In the third place, there should be included materials for re-teaching and relearning, which are too infrequent in many of our present-day courses of study.

The provision which is made for individual differences will reflect, more than any other item, the carefulness of attention given to the construction of the course of study. There should be specific materials included for this purpose, as well as procedures suggested for individual progress, such as the use of the directed study or the laboratory plan of instruction. A variety of activities should be suggested which will aid in the attainment of specific types of learning, such as those for obtaining knowledge, habits, and attitudes and skills.

The complete course of study outline should give attention to general teaching procedures, and to procedures connected with specific materials or specific types of learning. They should contain also suggestions relative to additional subject matter. In the accompanying course of study outline, each unit outline contains suggested teaching activities and procedure in the presentation of the subject matter, and the use of corrective and practice materials.

The lack of standards and of suggestions for testing is a weakness of many of our courses of study in high school algebra. Outlines should contain general standards and, in so far as they may be presented with definiteness, more detailed statements of expected outcomes. These standards should be kept definitely in mind in making suggestions concerning tests that will objectively measure pupil attainment.

The latest efforts in schools in which revision has progressed over a period of years give evidence of more careful attention to the mechanical make-up of course of study outlines as a means of making them more attractive and usable. Such outlines should be made more attractive, as well as forceful, through the use of mechanical aids. In a program of continuous revision, the outline should provide space in which teachers' suggestions and criticisms may be recorded.

CHAPTER IV

A SUGGESTED COURSE OF STUDY IN HIGH SCHOOL ALGEBRA

In the preceding discussion the writer has traced the development of aims and objectives for instruction in the field of mathematics generally, and of algebra in particular. Changes in materials for instruction considered appropriate for the attainment of these objectives also have been indicated. Findings of the various national committees and the recommendations of recognized authorities relative to "what is" and "what ought to be" in the field of mathematics have been cited. From these recommendations the writer has set up what seem to be the most commonly accepted aims and objectives for instruction in algebra.

The following course of study consists of materials which seem to have the greatest possibilities toward the attainment of these objectives. Each subject matter unit is preceded by both primary and immediate objectives in keeping with the general objectives set up and proposed for the entire course of study for high school algebra. Each unit is followed by suggested teaching procedures which, in the opinion of the writer, will be most effective in making algebra a vitalized subject in the high school curriculum.

UNIT I. USE OF SYMBOLS

Suggested Time: 2 to 3 Weeks

I. Primary Objective

A. To assist pupils to understand and appreciate the use of symbols to express quantitative relationships

II. Immediate Objectives

A. To develop ability of the pupils to interpret the meaning of symbolic expressions

B. To enable pupils to express and comprehend relationships in symbols

C. To develop pupil skills in solving simple literal expressions

D. To teach pupils the purpose and value of various algebraic expressions

III. Materials

A. Introduction. This unit is intended as an introduction to the symbolic mathematical language of algebra. It should acquaint the pupil with the concept of literal numbers, and familiarize him with the symbols and the methods of algebra. Since most pupils will have studied literal numbers and symbolism in the eighth grade, this unit probably will require only a brief review treatment.

B. Subject content

1. Use and meaning of literal numbers

2. Simple algebraic expressions and formulas from oral and written statements

3. Evaluation of simple formulas

4. Finding values from tables; making tables

5. Adding and subtracting similar terms

6. Use of the signs of operation

   a. Meaning and use of the signs of aggregation

   b. Omission of signs

7. Becoming familiar with new expressions

   a. Literal number

   b. Formula

   c. Similar terms

   d. Dissimilar terms

   e. Factor

   f. Coefficient

   g. Product, sum, difference, quotient

   h. Monomial, binomial, trinomial, polynomial

IV. Suggested Teaching Procedure

A. Pre-test. Since it is supposed that all pupils have had some simple algebra in the eighth grade, it would be well to give a simple test in order to estimate the present ability of the class with reference to:

1. Ability to interpret simple algebraic expressions

2. Ability to express oral and written statements in symbols

3. Ability to use fundamental processes in combining literal numbers

B. Introducing the unit

1. The use of letters to represent numbers can be explained by dots and dashes of the Morse Code, letters to identify assigned frequencies of radio stations, KVOA, KTUC, and initial letters in simple familiar formulas as: $A = LW$, $P = 21 + 2W$, et cetera

2. Attention should be called to the fact that pupils have been using symbols throughout their study of mathematics in the use of such signs as +, -, x, ÷, et cetera

3. A comparison of literal numbers with denominate numbers may be effective in establishing the concept of symbol. A simple problem from arithmetic such as: "If a walk is 59 feet long and 4 feet wide, how many square feet of concrete will be required to lay it?" will lead to similar problems in which letters can be used instead of numbers. This can be illustrated further by the familiar simple interest formula $I = PRT$.

4. Emphasis should be placed upon orderliness and neatness in solving problems in sequential steps. The following example illustrates:

   Remove parentheses and collect terms

   $(X + Y) - (X^2 - Y^2) + (-X + Y)$

   Solution

   $(X + Y) - (X^2 - Y^2) + (-X + Y)$

   $X + Y - X^2 + Y^2 + X + Y$

   $-X^2 + 2Y + Y^2$

5. The use of symbols and literal numbers can be illustrated clearly by the formula for finding the area of a rectangle. A rectangle may be drawn on the board and divided into squares. These squares can be counted and it may then be developed that the area can be

found by using the formula, area equals length times width. Letters may then be substituted and the formula A = LW be developed. Show the effect of substituting different values for A, L, and W.

6. A brief historical sketch on the development of algebra usually is effective in arousing interest in the subject.

C. Drill

1. Pupils should be given drill materials in terms of their needs. Drill work can be done effectively through dictated exercises, and mimeographed materials for individual practice.

2. Each new concept should have sufficient drill to establish understanding, and a reasonable degree of accuracy and efficiency.

D. Demonstrations

1. The teacher should give explanations and demonstrations orally, on the blackboard, and individually to illustrate each new concept. These demonstrations SHOULD NOT BE TOO BRIEF OR HURRIED. Remember that algebra is a cumulative study, and that "a chain is as strong as its weakest link."

2. After the class seems to understand the new processes involved, demonstration work by all the pupils should follow, the teacher giving help where needed. After all, we learn mainly by doing.

V. Evaluation

A. Teacher's check list

1. Do the pupils understand how and why letters are used to represent general numbers?

2. Do the pupils know the meaning of the new mathematical terms studied?

3. Can the pupils make and apply simple formulas?

4. Can the pupils collect similar terms?

5. Can the pupils evaluate simple formulas and algebraic expressions?

6. Do the pupils appreciate the idea of the dependence of an algebraic quantity upon the values of its various terms and factors?

7. Can the pupils express changing or related values?

B. Tests. The teacher should give short tests from time to time to ascertain the achievement of the class and as a basis for remedial work. For example:

1. Combine the similar terms in the following:

   a. $6A + A - 2A + 4A$

   b. $4X - 2X + X - 3X + 5X$

2. Find the sum:

   a.
   $$\begin{array}{r} 3Y + 2 \\ 5Y - 3 \\ \hline 8Y + 4 \end{array}$$

   b.
   $$\begin{array}{r} 4X - 8 \\ 6X + 7 \\ \hline 3X - 5 \end{array}$$

3. Find the product:

   a. $12 \times 9W$

   b. $16X \times 4$

4. Find the quotient:

   a. $\frac{12X}{3}$

   b. $\frac{210Y}{5}$

5. Five-ninths of a certain number is 55. What is the number?

6. If $P = A + 3B + 4C$
   Find P if $A = 2$, $B = 5$, $C = 3$.

7. Selective exercises

   a. The algebraic expression $3X + 4$ is a monomial, term, binomial, quotient.

   b. In the formula $A = LW$, L is a term, factor, sum, product.

8. True-False

   a. In the formula $A = LW$, the length and width are to be added together.

   b. In the expression $4Y - 1$, 4 is called a coefficient.

To the Teacher: Please summarize the results of your experience in teaching this unit, and offer suggestions for its improvement.

I. Primary Objective

II. Immediate Objectives

III. Materials

IV. Suggested Teaching Procedures

V. Evaluation

UNIT II. POSITIVE AND NEGATIVE NUMBERS

Suggested Time: 1 to 2 Weeks

I. Primary Objective

A. To acquaint pupils with the concept of signed numbers and to acquire skill in their use.

II. Immediate Objectives

A. To teach the meaning and importance of positive and negative numbers

B. To teach pupils to express temperature reading, altitudes, debts, et cetera by the use of signed numbers

C. To familiarize the pupils with the principles of making graphs and diagrams using signed numbers

D. To develop pupil skill in the fundamental processes involving signed numbers

III. Materials

A. Introduction. This unit introduces a new system of numbers, negative numbers. They should receive enough attention to make pupils feel at ease in using them to express quantities, and in using the laws governing their operations.

B. Subject content

1. Expressing temperatures, latitude, longitude, assets, debts, et cetera with positive and negative numbers

2. Plotting points on coordinate axes

3. Construction and reading of graphs

4. Addition, subtraction, multiplication, and division of literal numbers

5. Addition and subtraction of polynomials with positive and negative signs

6. The rules of signs

7. Study of new words and terms

   a. Positive

   b. Negative

   c. Signed number

   d. Directed number

   e. Absolute value

   f. Sign of equality

   g. Sign of operation

   h. Abscissa

   i. Ordinate

   j. Axis, et cetera

IV. Suggested Teaching Procedure

A. Pre-test. Informal questions to determine the pupils' knowledge of signed numbers may precede the formal introduction of the unit.

B. Introducing the unit

1. Point out that plus and minus signs have two meanings:

   a. To indicate addition and subtraction

   b. To indicate kind of numbers

2. Temperature readings are an application of signed numbers within the experience of most pupils. They are a good medium for introducing the unit.

3. After familiar zero points have been reviewed, other examples should follow readily.

4. Illustrate the plus and minus concept with statistics on a football game showing "yards gained," "yards lost," et cetera

C. Drill

1. Extensive oral and written drill in expressing signed quantities should be given. Drill on addition, subtraction, multiplication, and division of signed numbers. Check with frequent short objective tests and do remedial work with individual pupils on the basis of test errors.

2. Several exercises involving the "laws of signs" in the fundamental processes should be solved by each of the pupils to develop skill in the use of fundamental operations.

3. Oral and written exercises dictated by the teacher, mimeographed sheets, or exercises from a standard textbook also provide excellent drill materials.

D. Demonstrations

1. Show that subtraction is the reverse of addition, and that a number may be subtracted by adding its opposite. Subtract signed numbers on a number scale by counting the distance between two numbers on the scale.

2. Number scales and diagrams can be used effectively to demonstrate addition, subtraction, and multiplication. Most textbook writers explain division as the opposite of multiplication.

3. Use the time line in subtraction to show difference in time between dates.

V. Evaluation

A. Teacher's check list

1. Does each pupil understand the meaning and importance of positive and negative numbers?

2. Can each pupil express numbers with the correct sign?

3. Can each pupil state orally the "laws of signs?"

4. Can each pupil add, subtract, multiply, and divide signed numbers accurately?

B. Testing

1. Achievement tests should be given as each new topic is completed to ascertain further needs of pupils.

2. A test at the end of the unit, including exercises of each type studied, should be given to determine present attainments. For example:

a. Express the following assigned numbers:

1) An altitude of 250 feet above sea level

2) 15 below zero

3) A gain of $5.00

4) 8 pounds overweight

5) 300 years B.C.

b. Find the sum, difference, and product:

1) $\begin{array}{r} -12 \\ +\ 5 \\ \hline \end{array}$ 2) $\begin{array}{r} -10 \\ -\ 3 \\ \hline \end{array}$ 3) $\begin{array}{r} +6 \\ +7 \\ \hline \end{array}$ 4) $\begin{array}{r} +9 \\ -4 \\ \hline \end{array}$

c. Find the quotient:

1) $\frac{-12}{+6}$ 2) $\frac{+18}{+3}$ 3) $\frac{-25}{-5}$ 4) $\frac{+45}{-9}$

d. An elevator started from the ground floor and made the following trips: Up 5 floors, down 2, up 3, down 6, up 3. Using signed numbers, express its final position.

e. A thermometer registered 8° below zero at midnight. It rose 20° by six o'clock the next morning. Express the morning reading with the correct sign.

To the Teacher: Please summarize the results of your experience in teaching this unit and offer suggestions for its improvement.

I. Primary Objective

II. Immediate Objectives

III. Materials

IV. Suggested Teaching Procedures

V. Evaluation

UNIT III. LINEAR EQUATIONS

Suggested Time: 1 to 2 Weeks

I. Primary Objective

A. To assist pupils to understand and develop skill in the use of linear equations

II. Immediate Objectives

A. To familiarize the pupil with common terms used in working with equations

B. To teach simple functional relationships as expressed by equations and graphs

C. To develop pupil skill in solving simple equations by the use of axioms

D. To help pupils acquire skill in analyzing verbal problems, selecting significant data and translating them into equations.

III. Materials

A. Introduction. The equation may be introduced more effectively by the use of materials from geometry and the percentage relations of general mathematics. Throughout this entire unit, the importance of developing the ability to recognize examples of quantitative relationships and of functional dependence should be kept in mind. Frequent opportunities should be given for the development of generalizations which may result from verbal problems.

B. Subject content

1. Application of the formula as an equation

2. Development of the concept of simple equations

3. Study of common terms in working with equations, such as:

a. Coefficient

b. Equation

c. Root

d. Variable

e. Axiom

f. Transposition

g. Coordinate

h. Conditional equation

i. Identity

j. Equivalent equation

4. Meaning and use of transposition

5. Solving simple literal equations

6. Checking solutions of equations; values of the check

7. Principles of addition, subtraction, multiplication, and division in solving equations

8. Making equations from verbal statements

9. Expressing one quantity in terms of another

10. Use of simple functional relations, variables, constants

11. Construction and interpretation of graphs

IV. Suggested Teaching Procedure

A. Pre-test. Formulate a brief test to measure the present ability of the pupils with reference to the following:

1. Ability to select pertinent data from a stated problem and to formulate an equation

2. Knowledge of terms used in working with equations

3. Recognition of dependence in tables and graphs

4. Ability to solve linear equations

B. Introducing the unit

1. For demonstration work, select problems that are within the everyday experience of most of the class. Illustrate effectiveness of equations and formulas in contrast to arithmetical methods.

2. Emphasize orderly arrangement and sequential steps in solving equations. Work out pattern with the class.

3. Problem solving is to be emphasized. Make certain that pupils can read problems understandingly. Begin with simple problems containing familiar terms. Use simple number problems, then proceed to real problems.

4. The comparison of an equation to a balance is effective in the demonstration of the axioms.

5. The following skills and concepts should be employed in presenting the unit:

   a. Understanding of terms used

   b. Thinking problem through carefully. What is given? What is to be found?

   c. Dependence of one quantity upon another

6. The following steps may be given pupils as aids to problem solving:

   a. Read problem carefully two or three times

b. Determine what is to be found

c. Determine how to find it; i.e., what operations to use

d. Write equation

e. Solve for the unknown quantity

f. Check

C. Drill

1. Drill on expressing numbers and relationships in problems and use of algebraic symbols, and use practice sheets containing both oral and written problems.

D. Demonstrations

1. See directions in Unit I. Demonstration work by the teacher never should be too brief or cover too much material. Be sure pupils understand. Review each phase.

V. Evaluation

A. Teacher's check list

1. Are pupils familiar with the common terms used in working with equations?

2. Can pupils translate written and verbal statements into equation form?

3. Do pupils appreciate the value of the equation in working with mathematical problems?

4. Can pupils solve equations with reasonable speed and accuracy?

5. Have pupils mastered such skills as the use of addition, subtraction, multiplication, division, et cetera in working with equations?

B. Tests. The instructor should formulate achievement tests emphasizing the materials covered in the unit. The sampling should not be too brief. The following will serve to illustrate:

1. Solve the following in sequential steps and check:

   a. $X + 2 = 3$

   b. $X - 1 = 2$

   c. $2X + 4 = 8$

   d. $3X - 6 = 18$

   e. $2X + 3 = x + 4$

   f. $3X - 6 = 2X - 2$

   g. $4X - 7 + 5X - 9 = 61 + 5X - 3X$

   h. $17Y + 16Y - 5Y - 4 = 5Y + 8 - 3Y + 2 + 16Y$

2. Solve the following. State the axiom you use in each step. Check your solution.

   a. $\frac{N}{3} = 12$

   b. $\frac{X}{2} - \frac{X}{5} = \frac{9}{10}$

   c. $\frac{3W}{5} - \frac{4W}{3} = \frac{4}{9} - W$

3. Remove the parentheses and solve. Check your work.

   a. $6X + (7X - 11) = 15X - (4X + 50)$

   b. $(3Y + 5) - (2Y - 7) = (4Y + 9) - (2Y - 11)$

   c. $(6X - 8) - (9X + 4) = (9 - 13Y) - 11Y$

4. Solve the following and prove your answer.

a. What number increased by 45 equals 63?

b. Three times a certain number equals 15 diminished by the number. What is the number?

c. If 9 be added to 7 times a certain number, the result equals 16 diminished by the number. What is the number?

d. The sum of two numbers is 45. If four times the smaller be decreased by twice the larger, the result is 12. What are the numbers?

To the Teacher: Please summarize the results of your experience in teaching this unit and offer suggestions for its improvement.

I. Primary Objective

II. Immediate Objectives

III. Materials

IV. Suggested Teaching Procedures

V. Evaluation

UNIT IV. FUNDAMENTAL ALGEBRAIC OPERATIONS, AND OPERATIONS WITH EXPONENTS

Suggested Time: 3 to 4 Weeks

I. Primary Objective

A. To teach fundamental operations with signed literal quantities, including operations with exponents

II. Immediate Objectives

A. To teach pupils the meaning and use of exponents

B. To teach the application of the laws of exponents in multiplication and division of monomials and polynomials

C. To teach pupils to evaluate algebraic expressions and formulas containing exponents

D. To teach pupils how to solve equations by performing indicated operations

E. To teach pupils how to apply these principles in solving problems

III. Materials

A. Introduction. This unit is intended to prepare the pupil to understand and perform all of the fundamental operations of algebra preceding special factoring methods.

B. Subject content

1. Learning and using rule for order of operations

2. Literal expressions

3. Meaning of exponents

4. Powers higher than the first

5. Laws of exponents in multiplication

6. Laws of exponents in division

7. Multiplication of monomials and polynomials

8. Division of monomials and polynomials

9. Parentheses preceded by positive and negative signs

10. Equations involving parentheses

11. Simple equations involving simple fractions

12. Application of principles in verbal statements

IV. Suggested Teaching Procedure

A. Pre-test. (See Unit I)

B. Introducing the unit

1. Begin the study of exponents by a review of formulas from seventh and eighth grade arithmetic, such as:
$A = \pi r^2$, $V = e^3$, $A = S^2$, et cetera

2. Stress law of exponents for multiplication. Show that the exponent indicates a short form of multiplication, just as multiplication indicates addition.

C. Drill

1. See Unit I. Sufficient drill should be done on each new topic to establish it as an algebraic habit.

V. Evaluation

A. Teacher's check list

1. Do pupils understand the meaning of the new terms introduced in this unit?

2. Can pupils raise a number to an indicated power?

3. Can pupils multiply and divide monomials correctly?

4. Are operations with polynomials correct?

5. Can pupils remove parentheses correctly?

6. Can pupils check their own work?

B. Tests. The teacher should formulate tests to check on each topic introduced. Tests should not be extensive, but should cover each item studied. For example:

1. Find the sum, the difference, and the product in the following:

a. $-8X$ / $6X$ b. $-21XY$ / $-5XY$ c. $25A^2$ / $15A^2$

2. Find the quotient

a. $\frac{25X^2}{5X}$ b. $\frac{-30X^4Y}{6X}$

c. $(36X^4Y^3 + 6X^2Y^2)$

3. Evaluate the following if $\pi = 3.14$, $R = 10$ in.

a. $A = \pi R^2$

b. $C = 2R\pi$

c. $V = \frac{4}{3R^3\pi}$

4. Multiply

a. $4X^2 + 6X - 5$ by $X + 3$

5. Divide

a. $3Y^2 - 22Y - 24$ by $Y - 3$

6. Simplify the following:

a. $16 - (4X + 8) + 16X$

b. $12Y - 2\left[X - 5\,(X + 2)\right] + 4\,(5X + 2)$

To the Teacher: Please summarize the results of your experience in teaching this unit and offer suggestions for its improvement.

I. Primary Objective

II. Immediate Objectives

III. Materials

IV. Suggested Teaching Procedures

V. Evaluation

UNIT V. SPECIAL PRODUCTS AND FACTORING

Suggested Time: 3 to 4 Weeks

I. Primary Objective

A. To familiarize the student with the special products and to develop skill in factoring.

II. Immediate Objectives

A. To familiarize the student with the different types of special products

B. To teach the value of factoring in the solution of quadratic equations

C. To familiarize the pupil with the use of special products as short cuts in mathematics.

III. Materials

A. Introduction. The mastery of this unit will be of value to the student throughout all higher mathematics. To master type forms is difficult, but the student who does so has gone a long way toward a foundation for real learning in mathematics. If special products and factoring are developed together, the student will more easily grasp the meaning of form.

B. Subject Content

1. Terms used in special products and factoring

a. Binomial

b. Trinomial

c. Square

d. Cube

2. The square of a binomial (sum and difference)

3. Product of sum and difference of two terms

4. Product of two binomials having a common term

5. Product of two binomials having similar terms

6. Finding prime factors of algebraic expressions

7. Monomial factors

8. Factoring perfect square trinomials

9. Factoring products of each of the above-mentioned types

10. Factoring by grouping

IV. Suggested Teaching Procedure

A. Pre-test. Determine present ability or understanding of pupils with reference to the following:

1. Factoring, using simple arithmetical expressions

2. Similarity between factoring and division

3. Knowledge of terms used in unit

B. Introducing the unit

1. Illustrate the use of the special product method in arithmetical multiplication

2. Show that factoring is the reverse of multiplication, using simple arithmetic

3. Increase problem difficulties gradually

4. Use cross-product method in teaching how to find the product of two binomials

C. Drill

1. Drill on methods of selecting both factors, stating the principle used. A thorough knowledge of these principles is a "must" because they form a major part of the foundation for future work in algebra.

2. Review frequently

D. Demonstrations. (See Unit I)

V. Evaluation

A. Teacher's check list

1. Do pupils understand and appreciate the value and use of special products and factoring in everyday mathematical problems?

2. Have pupils become familiar with the terms used in this study?

3. Are pupils able to recognize type forms?

4. Can pupils use these type forms with reasonable speed and accuracy?

B. Tests. The teacher should test frequently, emphasizing the materials covered. For example:

1. Solve the following:

a. $(2X + 3)(X + 5)$

b. $N^2 - 7N + 12$

c. $(4X - 6Y)(4X + 6Y)$

d. $(AX + AY + BX + BY)$

To the Teacher: Please summarize the results of your experience in teaching this unit and offer suggestions for its improvement.

I. Primary Objective

II. Immediate Objectives

III. Materials

IV. Suggested Teaching Procedures

V. Evaluation

UNIT VI. FUNDAMENTAL OPERATIONS WITH FRACTIONS AND FRACTIONAL EQUATIONS

Suggested Time: 3 Weeks

I. Primary Objective

A. To develop proficiency of pupils in the use of algebraic fractions and their practical application in the solution of fractional equations

II. Immediate Objectives

A. To give pupils mastery over principles governing the use of fractions in the fundamental operations

B. To teach the law of signs as applied to fractions

C. To assist the pupil in becoming skillful in the use of fractions

D. To teach principles governing the finding of the L.C.M. and L.C.D. of fractions

E. To teach pupils how to reduce fractions to equivalent fractions

III. Materials

A. Introduction. The methods dealing with fractions in algebra are the same as those the pupils have already studied in general mathematics. Only the technique is different. The usual algebraic definition for the sum of two fractions is usually given in this form:

$$\frac{X}{Y} + \frac{A}{B} = \frac{BX + AY}{BY}$$

This is too difficult and mechanical. Emphasize simpler manipulations with fractions within the experience of pupils.

B. Subject content

1. Reduction of fractions to lowest terms

2. Addition and subtraction of simple fractions

3. Multiplication and division of simple fractions

4. Solving simple fractional equations

5. Solving problems involving fractions

6. Complex fractions; the four fundamental principles

IV. Suggested Teaching Procedure

A. Pre-test. The teacher should test the ability of the pupils with reference to:

1. Reduction of fractions

2. Knowledge of terms; i.e., numerator, denominator, equivalent, et cetera

3. Finding L.C.D.

4. Solving simple fractional equations

B. Introducing the unit

1. Review previous study of arithmetical fractions and proceed gradually to quantities

2. Stress meaning of terms

3. Drill on methods of changing signs of fractions

4. Eliminate the term "cancel"; use divide

5. Study multiplication and division first

6. Review and emphasize factoring

C. Drill. (See Unit I)

V. Evaluation

A. Teacher's check list

1. Are pupils familiar with common terms?

2. Are they reasonably competent and accurate in their addition, subtraction, multiplication, and division?

3. Do they understand finding L.C.D.?

4. Do they understand the "why" for the changing of signs?

5. Can they reduce fractions to lowest terms?

6. Do they understand the "why" for inversion in division?

B. Tests. The teacher should formulate achievement tests emphasizing the materials covered. The following illustrate possibilities:

1. True-False

a. $\frac{X + 4(X-Y)}{3(X-Y)} = \frac{X + 4}{3}$

b. $\frac{A}{B} + \frac{D}{C} = \frac{AC + BD}{BC}$

2. Perform the indicated operations

a. $\left(\frac{X^2 + 2X + Y}{Y}\right)\left(\frac{X^2}{X + Y}\right)$

3. Write the equation for the following and solve:

a. Separate 64 into two parts such that 2/3 of the smaller part exceeds 1/2 the larger part by 3.

To the Teacher: Please summarize the results of your experience in teaching this unit and offer suggestions for its improvement.

I. Primary Objective

II. Immediate Objectives

III. Materials

IV. Suggested Teaching Procedures

V. Evaluation

UNIT VII. SOLVING SYSTEMS OF LINEAR EQUATIONS

Suggested Time: 2 Weeks

I. Primary Objective

A. To familiarize the pupil with the solutions and uses of simultaneous equations in two unknowns

II. Immediate Objectives

A. To familiarize the pupil with common terms used in this study

B. To teach functional relationships as expressed by equations and graphs in two unknowns

C. To develop pupil skill and accuracy in solving equations by addition and subtraction, substitution, and the graph

III. Materials

A. Introduction. Algebra again appears as a tool subject in this study. It should be kept in mind that the ability to recognize quantitative relationships and functional dependency is essential. Generalizations should be developed from word problems.

B. Subject content

1. Definition of linear equation

2. Addition-subtraction method

3. Substitution method

4. Graphical method

5. Formulating verbal statements into equations

6. Solving word problems

7. Solving number-relation problems

8. Checking solutions

9. Checking use of terms: variables, roots, elimination, substitution, inconsistent, et cetera

10. Solving systems of equations using three or four unknowns

IV. Suggested Teaching Procedure

A. Pre-test. A brief test should be given to determine pupils' present ability with reference to:

1. Terms used in the unit

2. Solving equations with two unknowns

3. Graphs and use of tables

4. Selection of data, and formulation of equations

B. Introducing the unit

1. In introducing this unit, use problems within the everyday experiences of the pupils

2. Review simple equations and the idea of balance

3. The following skills and concepts should be kept in mind in presenting the unit:

a. Understanding terms used

b. Equation solving by:

1) Elimination method

2) Substitution method

3) Graphical method

c. Review the following:

1) Axioms, parentheses, negative signs, and L.C.D.

d. Emphasize importance of graphs

e. Not much time should be spent on fractional linear equations

C. Drill. (See Unit I)

V. Evaluation

A. Teacher's check list

1. Do pupils understand the three solutions?

2. Can they use these methods with reasonable speed and accuracy?

3. Are they familiar with the common terms used?

4. Do they understand and appreciate the value of two variables in the solution of problems?

B. Tests. The teacher should formulate achievement tests emphasizing the material covered in this unit. The following illustrate:

1. Solve by elimination:

a. $2A + 3Y = 1$
$3A - 2Y = 21$

b. $X - 2 = Y$
$3X - 5 = 2Y$

2. Graph the following:

a. $3X - Y = 5$
$2X + Y = -2$

3. Write the equations necessary and solve:

a. The sum of two numbers is 100. Twice the first minus three times the second is equal to 10. What are the numbers?

To the Teacher: Please summarize the results of your experience in teaching this unit and offer suggestions for its improvement.

I. Primary Objective

II. Immediate Objectives

III. Materials

IV. Suggested Teaching Procedures

V. Evaluation

UNIT VIII. SQUARES AND SQUARE ROOTS OF ARITHMETICAL NUMBERS AND LITERAL NUMBERS

Suggested Time: 2 Weeks

I. Primary Objective

A. To develop skill in finding squares and square roots of monomials and polynomials, and the application of these skills to practical problems

II. Immediate Objectives

A. To familiarize the pupil with the terms used in working with squares and square roots; for example, factor, root, exponent

B. To develop pupil speed and accuracy in finding squares and square roots

C. To assist the pupil in acquiring the knowledge necessary to apply the principles of squares and square roots to practical problems

III. Materials

A. Introduction. This unit is general and should cover the operations involved in finding squares and square roots of numerical and algebraic expressions, using the table of squares and square roots, and the application of the rule of Pythagoras.

B. Subject content

1. Squares and square roots of arithmetical numbers

2. Using table of squares and square roots

3. Squares and square roots of monomials

4. Squares and square roots of polynomials

5. Finding square roots by computation

6. Application of the rule of Pythagoras

IV. Suggested Teaching Procedures

A. Pre-test. The teacher should formulate a brief test to measure the knowledge which pupils already have with reference to the following:

1. The ability to find the squares and square roots of numbers from 1 to 144

2. Recognition of powers which are perfect squares

3. Recognition of perfect square trinomials

4. Understanding the meaning of square and square root

B. Introducing the unit

1. The teacher should call attention to practical uses of squares and square roots--problems of indirect measurement solved by the use of the rule of Pythagoras. Need for this study as a foundation for further study in mathematics and science should be understood by the pupils.

2. The following concepts and skills should be emphasized:

a. Meaning and use of terms

b. Analysis of problems and selection of pertinent data

c. Determining approximate square roots

C. Drill. (See Unit I)

V. Evaluation

A. Teacher's check list

1. Do pupils understand the meaning of terms and concepts used?

2. Can they find squares and square roots with reasonable speed and accuracy?

3. Can pupils apply the principles and concepts studied in solving equations and word problems?

B. Tests. Achievement tests emphasizing the materials studied in this unit should be given. These tests help the teacher and pupils locate points of weakness which should be reviewed and drilled on. For example:

1. Find the value of:

a. 7^2

b. $\overline{(3)}^{1^2}$

c. $\sqrt{81}$

d. $(.16)^2$

e. $\sqrt{\frac{25}{169}}$

f. $\sqrt{1.44}$

g. $\sqrt{29929}$

h. $\sqrt{X^2 - 4XY + 18X + 4Y^2 - 36Y + 81}$

2. Complete:

a. The square root of a quantity is called one of the two _____ of the number.

b. A number has two square roots. One is _____, the other _____.

c. In the expression X^6, the 6 is called _____.

d. The relation between the sides of a right triangle is expressed by _____.

3. Solve:

a. An 18-foot ladder reaches a window 15 feet from the ground. How far from the wall of the building is the foot of the ladder?

To the Teacher: Please summarize the results of your experience in teaching this unit and offer suggestions for its improvement.

I. Primary Objective

II. Immediate Objectives

III. Materials

IV. Suggested Teaching Procedures

V. Evaluation

UNIT IX. EXPONENTS AND RADICALS

Suggested Time: 3 Weeks

I. Primary Objective

A. To teach the meaning and practical uses of the various kinds of exponents, and to develop accuracy in performing the fundamental operations with radicals.

II. Immediate Objectives

A. To help pupils attain a knowledge of simple radicals and exponents

B. To teach the meaning and use of the common terms used in this study; for example, index, power, radical, exponent, et cetera

C. To help pupils attain accuracy and skill in performing operations peculiar to exponents and radicals

III. Materials

A. Introduction. The first part of this unit summarizes and extends the laws used in working with powers having positive integral exponents. These are extended to include negative numbers, fractions, and zero exponents. The study of equations also will be extended to include the solution of those in which the unknown is under the radical sign.

B. Subject content

1. Fundamental laws of exponents

a. Addition and subtraction

b. Multiplication

c. Division

d. Power of a product

e. Power of a power

2. Meaning and use of X^1

3. Meaning and use of X^0

4. Meaning and use of X^{-n}

5. Problems involving the use of exponents

6. Similar radicals

7. Simplification of radicals

8. Addition, subtraction, multiplication, and division of radicals

9. Equations containing radicals

IV. Suggested Teaching Procedures

A. Pre-test. Pupils should be given a brief test to determine their present ability to:

1. Deal with the fundamental operations involving radicals and exponents

2. Change quantities from radical to exponent form

3. Simplify radicals

4. Understand and use terms such as power base, index, radical, fractional exponent, zero exponent, negative exponent, rational and irrational numbers.

B. Introducing the unit

1. Show how the concepts of radicals and exponents are related to those of squares and square roots, also how time and effort can be saved by using exponents and radicals.

2. Develop a thorough understanding of the term "radical expression."

3. Explain thoroughly rules of signs in involution and evolution with numbers.

V. Evaluation

A. Teacher's check list

1. Are the pupils familiar with the meaning and the use of the terms in this unit?

2. Can they use the fundamental laws of exponents with ease and understanding?

3. Do they understand the meaning of a negative exponent? the "0" exponent?

4. Can they change from fractional exponents to radical forms, or vice versa?

5. Are they reasonably accurate and skillful in solving equations containing radicals?

B. Tests. The teacher should give achievement tests emphasizing the materials studied in this unit. For example:

1. Write in radical form:

a. $X^{1/3}$ c. $4\,X^{1/2}Y^{2/3}$

b. $Y^{3/4}$ d. $(X^2 + 2Y^2)^2$

2. Rewrite, using fractional exponents:

a. Y c. X^3Y^2

b. Z^2 d. $N^2 - M^3$

3. Multiple choice:

a. All expressions with zero exponents are equal to

1) 0

2) 1

3) the number itself

b. Y^4Y^8 is equal to

1) Y^4

2) 4

3) Y^{32}

4) Y^{12}

To the Teacher: Please summarize the results of your experience in teaching this unit and offer suggestions for its improvement.

I. Primary Objective

II. Immediate Objectives

III. Materials

IV. Suggested Teaching Procedures

V. Evaluation

UNIT X. QUADRATIC EQUATIONS

Suggested Time: 3 to 4 Weeks

I. Primary Objective

A. To teach the principles governing the solution of second degree equations, and to develop skill in their application

II. Immediate Objectives

A. To familiarize pupils with the different kinds of quadratic equations

B. To develop pupil ability and skill in using the various methods for solving quadratics

III. Materials

A. Introduction. The principles involved in this unit are necessary to a mastery of algebra. This unit is a comprehensive study of all the methods for solving second degree equations; that is, factoring, completing the square, the formula, and graphing.

B. Subject content

1. Definition of quadratic equations

2. Complete and incomplete quadratics

3. Solution by factoring (review)

4. Solution by completing the square

5. Development of the quadratic formula

6. Solving problems using the three methods

7. Solution by graphing

8. Literal quadratic equations

IV. Suggested Teaching Procedures

A. Pre-test. The teacher should give a brief test to determine how much the pupils already know concerning:

1. Recognition of the different second degree equations

2. Terms used in the study of this unit

3. Solving simple second degree equations

4. Recognition of perfect squares

B. Introducing the unit

1. Introduce the study by a comparison with linear equations

2. Review the use of familiar formulas

3. Demonstrate application of principles to word problems

4. Demonstrate and drill on the derivation of the formula from the standard quadratic equation
$AX - BX - C - 0$

5. Review the construction of simple graphs preparatory to the study of graphs of second degree equations

6. Emphasize terms used in this unit

V. Evaluation

A. Teacher's check list

1. Are pupils familiar with the different kinds of quadratic equations?

2. Can they use the four methods for solving quadratics with equal skill?

3. Are the relationships between roots and coefficients understood?

4. Do pupils understand the development of the quadratic formula from the quadratic equation?

5. Do pupils understand the meaning and use of complex and imaginary roots?

B. Tests. The teacher should formulate achievement tests emphasizing the materials covered in this unit. The following suggestions illustrate:

1. True-False:

   a. A quadratic equation may or may not have two roots.

   b. The graph of a quadratic equation may be a straight line.

2. Solve the following quadratics by the formula:

   a. $Y^2 + 5Y + 4 = 0$

   b. $25X^2 - 26X + 1 = 0$

3. The sum of two numbers is 8 and the sum of their squares is 41. What are the numbers?

4. If the roots of an equation are 4 and 7, what is the equation?

To the Teacher: Please summarize the results of your experience in teaching this unit and offer suggestions for its improvement.

I. Primary Objective

II. Immediate Objectives

III. Materials

IV. Suggested Teaching Procedures

V. Evaluation

UNIT XI. RATIO AND PROPORTION

Suggested Time: 2 to 3 Weeks

I. Primary Objective

A. To develop a knowledge of the principles of ratio and proportion and the application of these principles to problems of everyday life

II. Immediate Objectives

A. To familiarize the pupils with the different types of everyday problems solved by using ratio and proportion

B. To give pupils an understanding of ratio used as a fraction, or as indicated division

C. To develop skill in the solution of problems involving ratio and proportion

III. Materials

A. Introduction. The practical phase of this unit should be stressed; the use of ratio and proportion in the solution of such problems as costs, measurement, mixing concrete and feeds should be emphasized. This study is one of the most important phases of practical life mathematics.

B. Subject content

1. Definition of ratio, and proportion relationship

2. Writing simple ratios as fractions

3. Different methods of expressing a proportion

4. Equations in proportion form

5. Solving word problems

6. Variation as proportion (direct and inverse variation)

7. Study and use of common terms, for example; ratio, proportion, variation, inverse variation, constant, variable, direct variation, function, mean proportion

8. Formation and solution of problems involving inverse variation

IV. Suggested Teaching Procedures

A. Pre-test. The teacher should formulate a brief test covering the more elementary phases of the unit to determine the present ability of the pupils with reference to the following:

1. Writing ratios in fractional form

2. Solving problems by proportion

3. Using formulas as expressions of variation

4. Terms used in working with ratio and proportion

B. Introducing the unit

1. Review the study of ratio and proportion in general mathematics

2. Begin with the common fraction and indicated division

3. Emphasize the common unit and rules of means and extremes

4. From the formula $Wl = wL$, develop the proportion $\frac{W}{w} = \frac{L}{l}$

C. Drill. (See Unit I) The teacher should give sufficient drill to establish the fundamental concepts of the unit.

V. Evaluation

A. Teacher's check list

1. Do pupils understand that ratio is a simple fraction?

2. Can pupils solve problems of proportion and variation with speed and accuracy?

3. Do they understand the relationship of one variable with another?

4. Do they understand checking ratio to determine proportion?

5. Are they familiar with the meaning and use of the study of ratio, proportion, and variation?

B. Tests. The teacher should use achievement tests to emphasize the materials studied in this unit. For example:

1. In the proportion $\frac{X}{Y} = \frac{A}{B}$

   Find Y if X = 2, A = 4, and B = 6

   Find B if X = 3, Y = 7, and A = 12

2. Find three numbers whose sum is 110 and whose ratio is 1:4:6

3. True-False:

   a. All ratios are equations

   b. Four equal ratios form a proportion

4. Solve the following for X:

$$\frac{16 - X}{4} = \frac{12}{6}$$

To the Teacher: Please summarize the results of your experience in teaching this unit and offer suggestions for its improvement.

I. Primary Objective

II. Immediate Objectives

III. Materials

IV. Suggested Teaching Procedures

V. Evaluation

UNIT XII. SYSTEMS OF EQUATIONS

Suggested Time: 3 Weeks

I. Primary Objective

A. To further clarify in the minds of the pupils the meaning of the equation by graphing, and to teach the pupils how to find values that are common to two or three equations in a system

II. Immediate Objectives

A. To teach pupils how to recognize a circle, ellipse, parabola, hyperbola, and equilateral hyperbola by their equations

B. To teach pupils how to solve systems of equations such as one quadratic and one linear equation, two quadratics, and three linear equations

III. Materials

A. Introduction. The teacher should review the material taught relative to systems of linear equations as a basis for the introduction of the more advanced materials contained in this unit.

B. Subject content

1. Review systems of linear equations, introducing three equations with three unknowns

2. Solution of linear equations by using determinants

3. Graphing second degree equations

a. Circle

b. Ellipse

c. Equilateral hyperbola

d. Parabola

e. Hyperbola

4. Solving mixed systems of equations by substitution

5. Solving two quadratics with two variables by addition, subtraction, substitution, and graphing

6. Solving written problems involving quadratic systems, using the following terms:

a. Indeterminate equation

b. Inconsistent equation

c. Simultaneous equations

d. Linear systems

e. Quadratic systems

IV. Suggested Teaching Procedures

A. Pre-test. The teacher should formulate a brief test covering materials studied in Units VII and XI. The review work will be based on the deficiencies revealed by this test.

B. Introducing the unit

1. This unit is an extension of materials covered in Unit VII. That material should be reviewed thoroughly before introducing problems involving simultaneous quadratic equations. Demonstrations by the teacher should clarify each step in the procedures used in the solution of the different types of problems.

C. Drill. (See Unit I)

V. Evaluation

A. Teacher's check list

1. Can pupils recognize the graph of a circle, an ellipse, a straight line, a parabola, and a hyperbola by seeing only the equation?

2. Can pupils graph the second degree functions of X and Y?

3. Do pupils understand how to solve quadratic systems and pair the values correctly?

B. Test. The tests covering the materials of this unit should be adequate for measuring the attainments of the pupils. At least two should be given. The following illustrate the type of question that might be used:

1. Solve by graphing, addition or subtraction, and substitution methods:

a. $X + Y = 10$
$2X - 3Y = 0$

2. Solve by graphing and substitution:

a. $2X + Y = 7$
$X^2 + XY = 12$

3. Solve by graphing, addition or subtraction, and substitution:

a. $X^2 + Y^2 = 50$
$3X^2 + 2Y^2 = -95$

4. Solve by any one of the above methods:

a. $AB + CD = K$
$AC - YD = L$

b. $X^2 - Y^2 = 16$
$X - Y = 2$

c. $X + Y + 2 = 11$
$3X - 2Y = -3$
$3X + 2 = 10$

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5. Solve by determinants:

a. $2X + Y - 32 = 5$
$X + 2Y + 2 = 11$
$3X - Y + 22 = 4$

To the Teacher: Please summarize the results of your experience in teaching this unit and offer suggestions for its improvement.

I. Primary Objective

II. Immediate Objectives

III. Materials

IV. Suggested Teaching Procedures

V. Evaluation

CHAPTER V

RECOMMENDATIONS

It is suggested by the writer that this course in algebra be offered to tenth-grade pupils who have completed a course in general mathematics in the ninth grade. The guidance service offered by the high school should be such as to insure, within reasonable limits, that the pupils who enroll for courses in algebra have vocational plans that require more than an elementary understanding of mathematics, or that they wish to continue their study of mathematics because of the pleasure or cultural value they derive from it.

The materials included in this course of study have been selected in accordance with the recommendations of recognized authorities on curriculum construction. These materials are organized into twelve units. The last three units probably will be used only in classes where the pupils have had more than the average amount of experience with algebra in their ninth-grade course in general mathematics.

The writer has made an attempt to provide for individual differences on the basis of the pre-test of the materials to be included in the unit, and upon previous records. The greatest amount of achievement probably can be obtained

by using the class period as a laboratory period, except when general demonstrations and explanations are needed. This will enable the teacher to give more individual assistance to each pupil.

Pupils should be given drill materials in terms of their needs. Drill work can be done effectively either at the board as dictated exercises, or by mimeographed work sheets for each pupil. Each unit should have sufficient drill to establish a reasonable degree of speed and accuracy.

The teacher should demonstrate each new concept and skill as needed in terms of problems within the experiences of the pupils. Pupils may also be used to give demonstrations as the unit is developed. This has been found to be a good method of motivation with certain members of a class.

Contrary to the accepted custom, no text references are listed in this study outline. It is intended for use with any good text. However, no claim for completeness is made. Provisions have been made for teachers to add materials which may be helpful in local situations, and for criticisms which might be valuable in future revisions of the course.

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