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LOW SURFACE BRIGHTNESS SPIRAL GALAXIES

The University of Arizona

PH.D.

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LOW SURFACE BRIGHTNESS
SPIRAL GALAXIES

by

William Romanishin

A Dissertation Submitted to the Faculty of the

DEPARTMENT OF ASTRONOMY

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

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THE UNIVERSITY OF ARIZONA
GRADUATE COLLEGE

As members of the Final Examination Committee, we certify that we have read the dissertation prepared by William Romanishin entitled Low Surface Brightness Spiral Galaxies

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ACKNOWLEDGMENTS

During the all-too-numerous years I have been a graduate student, many people have provided assistance of various kinds. Steve and Karen Strom, for the past 3-1/2 years, have provided scientific inspiration, and invaluable help in obtaining telescope time, fighting with bureaucrats in my behalf, and arranging for the part-time position at Kitt Peak which has kept me from starving. I am particularly grateful for the friendship and companionship of my colleague Susan Wilkerson, the most interesting, pleasant and beautiful person I have ever known. The residents of the infamous Park Avenue Apartment (Pete Worden, Eric Jensen, Steve Grandi, Gary Coleman, Todd Boroson and Brad Peterson) all helped to provide a friendly ambience during the years I lived there. I greatly enjoyed the company of numerous other people at Steward and Kitt Peak, among them Mark Adams, John Scott, Jim Liebert, John Stocke, Mark Giampapa, Paul Hintzen and John Kormendy. The many nights I spent eating Mexican food and/or drinking beer with some of these people were among the best times of the last 5-1/2 years.

During the last few months of writing this dissertation, when it seemed it would never end, many people went

out of their way to help me. The Stroms and Betty and Herb Schueler provided lodging. Worden, Grandi, Jensen and my parents provided monetary loans.

Many people at Kitt Peak helped in obtaining and reducing the data presented in this dissertation. On the Mountain, all the TA's and LTO's provided their usual high level of competent service, particularly Gus Maxey, who could always repair the telescopes and instruments I managed to put out of commission. Downtown I thank numerous programmers, computer operators, technicians and secretaries, particularly Betty Schueler, the only secretary I know of who works graduate student hours. I thank Dave Monet for helping me with the word processor at Kitt Peak, which I used for rough drafts of this work.

I would like to thank my scientific collaborators who have been patient with me this last year or so while I virtually ignored all the other projects I had started to agonize over this dissertation. These people include Roger Angel, Duane Carbon, Trinh Thuan and the Stroms.

I thank Arecibo Observatory and NRAO for providing telescope time for the 21-cm observations reported here. Nathan Krumm and G. R. Knapp provided invaluable help in obtaining this data, as I didn't know a circular feed from a doober.

Last, but not least, I thank my parents for all they have done for me. The best part of finishing this dissertation is that I can (at last) honestly tell them I won't be a student any longer.

TABLE OF CONTENTS

	Page
LIST OF TABLES	vii
LIST OF ILLUSTRATIONS	ix
ABSTRACT	x
1. INTRODUCTION	1
2. OBSERVATIONS	5
2.1 Selection of Galaxies	5
2.2 UBV _R Photometry	6
2.3 Surface Photometry	10
2.4 Infrared Photometry	43
2.5 HII Region Spectroscopy and Photometry	50
2.6 Long Slit Spectra Reduction	62
2.7 Neutral Hydrogen Radio Observations	65
3. ANALYSIS	75
3.1 Large-scale Photometric Properties	76
3.2 Stellar Metal Abundance from Broad-band Colors	87
3.3 HII Regions	99
3.4 Spiral Structure	114
3.5 Surface Mass Density	132
3.6 Neutral Hydrogen Content	135
3.7 Mass-to-light Ratio	138
4. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK	150
LIST OF REFERENCES	157

LIST OF TABLES

Table		Page
1.	Filter Combinations for Photoelectric Observations	7
2.	UBVR Photometry	11
3.	Luminosity and Color Profiles	25
4.	UVJHK Photometry	51
5.	Magnitudes of HII Regions	59
6.	IIDS HII Region Data	60
7.	HII Region Photographic Emission Line Ratios	61
8.	X, Y Positions of HII Regions on Prints . .	63
9.	Neutral Hydrogen Data	68
10.	Disk Parameters	83
11.	Morphological Types from K_1	86
12.	Old Disk Colors	95
13.	Average LSB HII Region Parameters	100
14.	Comparison of HII Regions in LSBs and M 33, M 101	102
15.	Model Stellar Clusters with $M_U = -14$	111
16.	Arm Strength Parameters	129
17.	Roberts, Roberts and Shu Parameters	131
18.	Surface Mass Density	134
19.	Hydrogen Mass to Luminosity Ratio	136

LIST OF TABLES--Continued

Table		Page
20.	Hydrogen Mass to Total Mass Ratio	139
21.	Annular M/L_B Measurements	144
22.	M/L_B from 21-cm Widths	147

LIST OF ILLUSTRATIONS

Figure		Page
1.	(U-B) vs. (U-R) Color-color Plot	44
2.	(B-V) vs. (U-R) Color-color Plot	45
3.	(V-R) vs. (U-R) Color-color Plot	46
4.	Size vs. Absolute Magnitude Plot	77
5.	R Profile for NGC 4517A	80
6.	Photographs of LSB Galaxies	116
7.	Rotation Curve for UGC 06983	141
8.	Rotation Curve for UGC 07557	142
9.	Rotation Curve for NGC 4517A	143

ABSTRACT

This dissertation presents an observational overview of a sample of low surface brightness (LSB) spiral galaxies. The sample galaxies were chosen to have low surface brightness disks and indications of spiral structure visible on the Palomar Sky Survey. They are of sufficient angular size, (diameter > 2.5 arcmin) to allow detailed surface photometry using Mayall 4-m prime focus plates. In addition to photographic surface photometry, we obtained optical and infrared aperture photometry, HII region spectrophotometry, long-slit spectra and neutral hydrogen 21-cm radio observations for a selection of the sample galaxies.

The major findings of this dissertation are:

1. The average disk central surface brightness of the LSB galaxies is 22.88 magnitude/arcsec² in the B passband. This is a factor of 3 lower in intensity than the value found for most spirals studied up to now. The average disk scale length is about 4 kpc, almost as large as for a sample of giant late-type systems.

2. From broad-band color measurements of the old stellar population, we infer a low average stellar metallicity, on the order of $1/5$ solar. The old population was

observationally isolated in wavelength, by means of infrared observations, and spatially, by measuring optical colors of points in the disk where there has been no recent star formation.

3. The spectra and optical colors of the HII regions in the LSB galaxies indicate a lack of hot ionizing stars compared to HII regions in other late-type galaxies. We believe this indicates an initial mass function depleted in massive stars.

4. The average surface mass density, measured within the radius containing half the total mass, is less than half that of a sample of normal late-type spirals.

5. The average LSB galaxy neutral hydrogen mass to blue luminosity ratio is about 0.6, significantly higher than in a sample of normal late-type galaxies.

6. We find no conclusive evidence of an abnormal mass-to-light ratio in the LSB galaxies. However, there is a suggestion that the mass-to-light ratio in these galaxies may be slightly larger than normal.

7. Some of the LSB galaxies exhibit well-developed density wave patterns. The amplitude of these wave patterns range up to the amplitude found for normal spirals with well-developed spiral structure. The spiral density wave theory predicts weak shocks in the LSB galaxies.

8. A very crude calculation shows the lower metallicity of the LSB galaxies compared with normal late-type spirals might be explained simply by the deficiency of massive stars in the LSB galaxies.

These galaxies may prove useful in studying the connection between the initial mass function of a galaxy and its metallicity.

CHAPTER 1

INTRODUCTION

The detailed study of spiral galaxies has progressed rapidly in the last decade. Because of the rich variety of processes we are able to study in spiral galaxies, many different observational tools are needed to obtain even a rudimentary picture of a galaxy. The necessary tools include, but are not limited to: surface photometry in several colors, photoelectric optical and infrared aperture photometry, emission-line spectroscopy of HII regions, and 21-cm radio studies of neutral hydrogen. Observers have naturally concentrated on the nearby bright galaxies. Examples of recent detailed studies of the surface brightness distribution in bright spirals is presented by Schweizer (1976) and Talbot, Jensen and Dufour (1979). Examples of studies of the emission-line spectra of HII regions in spirals is provided by Searle (1971), Smith (1975) and Shields and Searle (1978).

Many parameters influence the evolution of spiral galaxies. We can study samples of galaxies with some observational parameter which differs from the majority of galaxies to determine the cause and effect of this

parameter on galactic evolution. In this way we hope to isolate the effects of one parameter. We chose to compare spirals having particularly low surface brightness disks with normal spirals. The disk surface brightness provides a convenient parameter by which to select a sample of "different" galaxies. It is easy to pick out galaxies with low surface brightness disks by visual inspection. One question we wish to answer is: Why do some spiral galaxies have low optical surface brightness disks, even though they have total luminosities comparable to some galaxies of normal surface brightness? Is it due to difference in mass distribution, or a radically different mass-to-light ratio or star formation rate? Another question we would like to ask is: Is the process of star formation different in these galaxies? By studying these "different" galaxies, we hope to learn more about the general process of galactic evolution. This dissertation presents an overview of the properties of spirals of low surface brightness.

An extensive catalog of low surface brightness galaxies was compiled by van den Bergh (1966). This list is referred to as the David Dunlap Observatory (DDO) dwarf galaxy list. Van den Bergh defined galaxies with low surface brightness and little central concentration of light to be "dwarf galaxies", implying low luminosity. Until 1975, little was known about the intrinsic luminosities of

the vast majority of the DDO dwarf galaxies because of the difficulty of measuring redshifts for these objects. In 1975, Fisher and Tully (1975) published a study of the DDO dwarf galaxies using observations of their neutral hydrogen content using the 21-cm radio line. They were able to measure redshifts for almost 2/3 of the complete DDO dwarf galaxy catalog. Using eye-estimate magnitudes from the Palomar Sky Survey, Fisher and Tully found that many of these galaxies were not intrinsically faint. Over half of the detected galaxies were brighter than $M_{pg} = -18$ ($H_0 = 50$ km/sec/Mpc).

The galaxies studied in this work are intrinsically luminous low surface brightness galaxies. In order to study their basic properties, the following types of data were obtained:

1. Photographic surface photometry in one, two or three colors to obtain accurate total magnitudes and colors, and to study the large-scale structure of the galaxies. The digital surface brightness maps are used to study the parameters of the spiral pattern, the color of the old disk stars, and the luminosities of the HII regions.
2. Photoelectric spectrophotometry of HII regions to study the metal abundance of the ionized gas, and the mass function of the stars ionizing the gas.

3. Infrared photometry to study the old stellar population.

4. Radio observations of the neutral hydrogen 21-cm line to study the gas content.

5. Long-slit photographic spectra to study the mass-to-light ratio as a function of position, and also to obtain some additional information on line ratios in the emission line spectra of HII regions.

CHAPTER 2

OBSERVATIONS

This chapter first describes the selection of the low surface brightness galaxy sample. Then, the observational techniques used to study these galaxies are described. In addition, the basic observational data on the low surface brightness galaxies is listed in this chapter.

2.1 Selection of Galaxies

The low surface brightness (LSB) spiral galaxies observed were chosen by examining most of the Palomar Sky Survey plates with galactic latitude greater than about 30° . The following criteria were used to select LSB spiral galaxies:

1. diameter between about 2.5 and 5 arcmin;
2. low surface brightness disk;
3. clear indications of spiral structure on glass copies of the Palomar Sky Survey.

The lower diameter limit was imposed so the galaxies would be large enough for detailed surface photometry. The upper diameter cutoff was imposed to eliminate galaxies too large to be studied with the 4-m prime focus camera. Many dozens

of LSB spiral galaxies were found below the 2.5 arcmin cutoff. Only a few were found above the 5 arcmin diameter cutoff. The sample is not intended to be complete in any sense. We are interested only in the intrinsic properties of typical members of the LSB class of galaxies. Discussion of some extrinsic properties of LSB galaxies, such as their luminosity function and space distribution, can be found in Thuan and Seitzer (1979).

The final observing list contained about 40 galaxies. Because of the varied nature of the observations, it was possible to obtain complete data for only a few galaxies.

2.2 UBVR Photometry

Photoelectric UBVR photometry was obtained for 35 galaxies, including all but one of the galaxies for which we have some plate material. Almost all observations were obtained with the Kitt Peak National Observatory (KPNO) 1.3-m telescope and the MK II computer-controlled photometer. The filter combinations used to define the bandpasses are listed in Table 1. A few galaxies were observed with the same equipment using the KPNO 2.1-m telescope.

At the 1.3-m telescope, the galaxies were acquired by offsetting from nearby stars. The acquisition television system, which does not have an integrating capability, was not sensitive enough to detect most of the LSB

Table 1. Filter Combinations for Photoelectric Observations -- All observations are with an S20 photocathode.

Photometric Band	Filters
U	Corning 9863 CuSO ₄
B	GG-385 BG-18 Corning 5-57
V	BG-18 GG-495
R	RG-610 KG-3

galaxies. Positions of the galaxy center, and several nearby stars, were measured from glass copies of the Palomar Sky Survey, using the KPNO two-dimensional Grant machine and the ASTRO reduction program. These positions, referenced to SAO stars, are accurate to about 1 arcsec. The 1.3-m computer-controlled drive is able to make offsets over the distances between the galaxies and the nearby reference stars (typically 1-3 arcmin) to within 1 or 2 arcsec. The accuracy of the telescope offsetting was checked many times each night, particularly after moving the telescope from one part of the sky to another, by offsetting between pairs of reference stars. After setting up on the galaxy coordinates, an electronic cursor on the television was set on a star in the field for guiding. The field was not visible on the television during an integration. After a sequence of galaxy and sky observations in all filters, lasting a minute or two, the guiding was checked. We chose sky measurement positions that were free of stars on the Palomar Sky Survey prints. These areas were 200 or more arcsec from the galaxies. We estimate that this was sufficient distance from the galaxies so that any correction for galaxy light in the sky beam would be less than 0.01 magnitude.

At the time of the first photoelectric observing runs, in late 1977, there was no extensive list of reliable

faint standards available for the standard Johnson R pass-band. We used standard stars from the list of Sandage (1973), who lists a red magnitude (which we will denote R_S) which is not exactly the same as the Johnson R. The primary standard stars used in the photometry were those listed by Sandage in M 67. Stars from the equatorial UBV catalog of Landolt (1973) were also observed each run. The UBV transformations obtained for the Sandage and Landolt standards were essentially identical.

Recently, R and I magnitudes on the Johnson system have been measured for a large number of the Landolt equatorial standards (Moffett and Barnes 1979). During our last observing run, we observed a number of these stars along with the usual Sandage standards. Using these observations, we derived the following transformations relating the Johnson R to the Sandage R_S :

$$R = R_S - 0.03 \quad , \quad (V-R)_S \leq 0.6 \quad (1)$$

$$R = R_S + 0.19 (V-R)_S - 0.15 \quad (2)$$

$$0.6 < (V-R)_S < 1.4$$

The difference between the two R systems is well-defined and small (< 0.04 mag) over the color range of interest for the galaxy photometry. All R magnitudes listed in this work have been corrected to the Johnson R system.

All UBVR photometry is listed in Table 2. The first column gives the galaxy name, the second the aperture diameter for the photoelectric photometry, in arcsec; the remaining columns list the V magnitude and the (B-V), (U-B) and (V-R) colors. Integration times were adjusted so that photometric uncertainties, derived from photon statistics, were typically 0.03 magnitude in V and 0.04 magnitude in all colors. All measurements with errors between 0.08 and 0.11 magnitudes are followed by a colon in Table 2.

2.3 Surface Photometry

We obtained photographic plates of 17 LSB galaxies. The prime focus camera of the Mayall 4-m telescope was used to obtain U, B_3 or R plates. The following plate, filter and exposure time combinations were used, resulting in an average photographic sky density of 0.7:

1. U -- IIIa-J plates baked in nitrogen or hydrogen were used with a UG2 filter for a typical exposure time of 2 hours and 15 minutes.
2. B_3 -- Baked IIIa-J plates were used with a GG385 filter. The typical exposure time was 45 minutes.
3. R -- Baked IIIa-F plates were used, with a RG610 filter, with sky reached in about 50 minutes.

The scale of these plates (18.55 arcsec/mm) is adequate for detailed surface photometry. One problem with

Table 2. UBVR Photometry^a

	Aperture	V	B-V	U-B	V-R
UGC 00334	35.3	16.03	0.55	-0.23	0.67
	58.6	15.31	0.47	-0.08	0.71
	82.2	14.95	0.49	-0.24	0.72
UGC A0033	35.3	15.37	0.79:	-0.12:	0.51
	82.2	14.30	0.51	0.09	0.50
DDO 48	82.2	15.27	0.49	-0.25:	0.64
UGC 04841	17.7	16.18	0.67	-0.01	0.74
UGC 04922	35.3	15.14	0.69	0.04	0.66
	58.6	14.52	0.57	0.04	0.66
DDO 79	82.2	14.32	0.55	-0.11	0.58
UGC 06446	82.2	14.39	0.48	-0.29	0.36
UGC 06682	82.2	14.77	0.65:	-0.11:	0.61:
NGC 3913	58.6	13.57	0.61	-0.09	0.64
	82.2	13.26	0.60	-0.04	0.64
UGC 06922	82.2	14.09	0.61	-0.12	0.64
UGC 06955	82.2	13.75	0.56	-0.07	0.58
UGC 06983	82.2	13.53	0.54	-0.14	0.58
NGC 4411B	35.3	14.33	0.68	-0.04	0.68
	82.2	13.25	0.56	-0.12	0.61
UGC 07557	35.3	15.19	0.41	-0.04	0.61
	82.2	13.89	0.61	-0.26	0.49
DDO 128	82.2	14.65	0.42	-0.13	0.51
UGC 07685	35.3	14.76	0.65	-0.08	0.62
DDO 144	13.3	16.39	0.48	-0.10	0.63
DDO 142	35.3	14.57	0.69	0.16	0.74
	82.2	13.30	0.68	-0.07	0.58

Table 2.--Continued

	Aperture	V	B-V	U-B	V-R
MCG-2-33-15	82.2	13.21	0.45	-0.13	0.59
UGC 08253	35.3	16.43:	0.23:	-0.02:	0.56:
UGC A0333	35.3	15.37	0.54	-0.15	0.63
	82.2	14.16	0.51	-0.17	0.54
UGC 08409	35.3	15.35	0.49:	0.06:	0.58:
	82.2	14.62	0.44	-0.07	0.52:
UGC 09061	35.3	13.78	0.94	0.52	0.92
	58.6	13.33	1.01	0.45	0.86
NGC 5774	35.3	13.93	0.60	-0.05	0.61
	58.6	13.41	0.54	-0.07	0.64
UGC 09614	35.3	16.10	0.48	0.03	0.62
	58.6	15.24	0.47	-0.31	0.41:
	82.2	15.19	0.38	-0.14	0.53
UGC 09875	35.3	15.96	0.64	-0.17	0.64
	58.6	14.90	0.39	-0.01	0.57
UGC 09910	35.3	16.06	0.12:	-0.14:	0.52
	58.6	15.71	0.14:	-0.16:	0.73:
	82.2	15.30:	0.37:	-0.29:	0.43:
NGC 5964	82.2	13.01	0.70	0.11	0.71
UGC 09938	58.6	15.88:	0.33:	-0.31:	0.47:
DDO 203	82.2	13.84	0.52	-0.12	0.57
UGC 11868	35.3	15.21	0.74	-0.01	0.78
DDO 214	35.3	14.95	0.46:	0.07:	0.74:
	82.2	13.57	0.81	-0.25	0.51
UGC 12522	35.3	15.88	0.52:	0.12:	0.85:
	58.6	15.18	0.77:	-0.05:	0.71:

Table 2.--Continued

	Aperture	V	B-V	U-B	V-R
DDO 219	35.3	15.60	0.66	0.10:	0.61
	82.2	14.56	0.50	-0.11	0.62
UGC 12845	35.3	15.06	0.89	0.17	0.81

- a. The nominal uncertainties are ± 0.04 magnitude in V and ± 0.05 in all colors. Values followed by a colon have uncertainties approximately twice the nominal value.

the prime focus camera is the complex large-scale background variation resulting from scattered light and the radial scale change. The galaxies studied here, with maximum size of 5 arcmin, are large enough that the background variations cannot be ignored, particularly in the outer, faint parts of the galaxies. A fairly simple method, described in detail below, was devised to correct for the background variation.

The first step in the reduction process was to scan the plates on the KPNO PDS microdensitometer. All scanning was done in the 12 bit density mode, which records plate density from 0.00 to 5.115 in 4096 steps. The aperture was a square 100 microns (1.86 arcsec) on a side, with a step size of 50 microns. Generally, a 1024 x 1024 raster or matrix of points was scanned around each galaxy, yielding an image 15.8 arcmin on a side. Each prime focus plate has a 16 step intensity calibration, exposed simultaneously with the galaxy exposure. A small raster on each of the density levels was scanned, along with a portion of the plate chemical fog.

All subsequent picture processing operations made use of the Interactive Picture Processing System (IPPS) at KPNO (Wells 1975). The densities of the step calibration exposure were fit to the following characteristic curve function (Goad 1975):

$$\log I = P_0 + P_1 Y + P_2 Y^2 + P_3 Y^3 \quad (3)$$

where

$$Y = D + P_4 \log_{10} (1 - 10^{-D}); \quad (4)$$

I is the intensity; D the photographic density; and the P 's are constants determined by the fit. All galaxy density rasters were converted to intensity rasters using the parameters determined for each individual plate.

Next, the intensity rasters were corrected for the 4-m background variation. Experimentation showed that the variation could be modeled, at least locally (i.e., over the area covered by a 1024^2 raster) as a linear decrease in the sky intensity I as a function of radius, with the radius measured from some point near the plate center:

$$I = I_0 - sR \quad (5)$$

where R is the distance of a pixel from the imaginary center point, I_0 the extrapolated intensity at the center point and s is a constant derived as explained below. In reality, the background variation is not radially symmetric over the entire plate, so the center of intensity is not necessarily the center of the plate. This variation is, of course, not the same as a simple tilted plane, as the sky isophotes were portions of approximately circular arcs. The procedures used to correct the background are outlined here:

1. We estimated the apparent center of the sky isophote arcs in the coordinate system of the raster, using the shape of the arcs as seen on the COMTAL display and a print of the plate.

2. We took an intensity scan along the longest possible radius vector appearing on the raster, yielding a plot and listing of the values of $I(R)$.

3. We made a linear fit to the radial intensity scan, ignoring the portions of the scan containing the galaxy or bright stars. This fit gives the constant s .

4. We created a raster containing a model of the background. This was done by first creating a raster, using the R subroutine in the EDIT menu, where each pixel was assigned the value $-(s/I_0)R$. The arithmetic subroutines were used to add 1.000 to this raster, yielding a raster with the intensity fulfilling equation 5, normalized so that I_0 was 1.000.

5. We divided the galaxy image by this raster.

After applying this correction algorithm, the next steps in the reduction were to find the basic properties describing the galaxy image: coordinates of center, position angle (θ) of the major axis, and ellipticity or ratio of semiminor axis b to semimajor axis a , (b/a) . The center was usually found by setting the COMTAL image display

cursor on the center of the galaxy and reading the coordinates (in pixels) of the cursor. There is an automatic, digital image center finding procedure, but in general these galaxies did not have the small sharp, well-defined nuclei needed to use this feature. The values of b/a and θ were found using the B/A subroutine. In this routine, the galaxy image was displayed so that the galaxy isophotes were clearly visible. The COMTAL cursor was positioned at either ends of the major and minor axes of a particular isophote and an ellipse was fit to these four points, yielding b/a and θ of the major axis. This was usually done for several isophotes and the results averaged. We used fairly faint isophotes, beyond the extent of the spiral arms, to try to derive parameters for the underlying disk.

The transformation constant s relating the photographic instrumental magnitudes to the absolute magnitude scale were found by comparing photoelectric aperture photometry with simulated aperture photometry done with the IPPS routine PHOT. PHOT sums the intensity of all pixels within a specified radius of a given point, and subtracts a sky value found by averaging the intensity values in a number of small apertures spaced along a circle at a large distance from the center of the aperture. This sky value is corrected for the presence of stars or plate defects

which might affect individual sky aperture measurements using a point rejection algorithm. Each plate was calibrated separately, using our photoelectric aperture photometry for all galaxies except NGC 45, where the R magnitude was extrapolated from the UBV magnitudes given by de Vaucouleurs and de Vaucouleurs (1972). The most significant source of uncertainty in this calibration procedure is the uncertainty in the photoelectric magnitudes, typically 0.04 magnitude. A potential major source of error is the centering of the aperture: if the aperture used in PHOT is not placed relative to the galaxy as was the photoelectric aperture, the calibration constant will be in error. When observing the galaxies photoelectrically, the error in centering was estimated to be no larger than 2"-3". The location of this same center on the digital image is probably better than 2". Changing the center of the PHOT aperture by 3" from the measured central position seldom changed the transformation coefficient by more than 0.01 magnitude, except for the smaller apertures, diameters 13.3" or 17.7". Some galaxies were observed photoelectrically with more than one aperture size. In these cases, the transformation coefficient was the average of the values for the individual values, with half weight given to the small aperture measurements.

To study the large-scale structure of a galaxy, we derived $I(r)$, the azimuthally averaged surface brightness as a function of the geometric mean radius, $r = \sqrt{ab}$. The intensity values, with the sky subtracted, in small (usually 2" diameter) apertures placed along elliptical paths are averaged for various values of b . A point rejection algorithm rejects particularly high or low points along each scan, to remove stars or plate defects. The values of b/a and position angle found using B/A are used for all b . This procedure can smear out some of the features in the profile, such as bars near the nucleus, which can have both different ellipticities and position angles from those adopted for the galaxy as a whole.

The first step in the profile measurement is the determination of the local sky value at each point in the image. Although the flattening procedure discussed above removed most of the systematic sky variations, we used as a final algorithm the Fourier sky fit discussed by Strom and Strom (1978). The sky intensity is measured in a large number of small apertures along a circle, centered on the galaxy, of radius R_0 . The resulting distribution of intensity as a function of angle, $I(\phi)$, was fit with a low-order Fourier series of the form $a_0 + f(\phi)$, after removing those points affected by stars or plate defects. At any point at radius r from the galaxy center, the background intensity

is computed as $a_0 + (r/R_0)f(\phi)$. This implicitly assumes that the sky varies linearly along a ray drawn from the galaxy center to R_0, ϕ . Many of the galaxies did not require this type of fit, as the intensity was constant along the circle. Some required a first order Fourier fit, i.e., a tilted plane, while only a very few required a second order Fourier fit.

At the IPPS the surface brightness profile is displayed on an interactive terminal. The $I(r)$ distribution is computed for increasing values of b until the sky level is reached, at which point $I(r)$ remains constant or changes very slowly compared to its change within the galaxy image. The final sky value is set equal to the average of the values of $I(r)$ from the edge of the galaxy to the sky point at the greatest galactocentric distance measured. Alternatively, an estimated value can be specified if this average value is not judged appropriate, following visual inspection. For example, an estimated sky value is entered if there is still a slight gradient in $I(r)$ past the maximum extent of the galaxy image. All the values of $I(r)$ are corrected for this final sky value, and a table listing values of $a, b, r, M(r)$ and $M_{\text{tot}}(r)$ is produced. $M(r)$ is simply $2.5 \log I(r)$, and $M_{\text{tot}}(r)$ is the integrated magnitude within the ellipse with geometric mean radius r . This surface brightness profile routine is known as ISOPH.

One problem we discovered late in the reduction process is that the point rejection algorithm, designed to delete star images, was also deleting a few pixels in particularly high surface brightness knots within the galaxy image. We estimated the size of this effect by comparing values of the total intensity derived from PHOT, which sums all pixels within a given radius, to the integrated intensity measured with ISOPH, for several galaxies with only faint superposed stars. The effect was detectable on the U images, but not on the R. This is as expected, as the contrast between the bright knots of newly formed stars and the surrounding galaxy is much greater in U than in R. The comparison of PHOT and ISOPH magnitudes shows the effect to be less than 0.05 magnitude in total U magnitude for NGC 3913, one of the galaxies which from visual inspection would appear to be affected the most.

Unfortunately, as is well known, (Burnstein 1978), surface photometry is often susceptible to large systematic errors, which are often difficult to evaluate internally. The best way to discover such errors is to compare results obtained by independent observers. As one form of external check, we derived profiles for three LSB galaxies using plates taken with the Palomar 1.2-m Schmidt telescope. These profiles agree well with the profiles obtained from the 4-m plates. These data are discussed below. The basic

integrity of the observing and reduction system, such as the calibration of the 4-m spot sensitometer, the accuracy of the density to intensity routine, and the correct performance of the IPPS algorithms, has been documented (Strom and Strom 1978).

The profiles of three galaxies, NGC 4411A and B, and UGC 07557, were measured on U and R plates obtained with the Palomar 1.2-m Schmidt telescope. These plates were scanned with a 20 micron square (1.34 arcsec square) aperture, at 20 micron separations. The calibration was provided by a set of Palomar spots on the plate. The spot densities were fit to a characteristic curve of the form (Strom et al. 1977):

$$\log I = a_0 + a_1 D + a_2 \log (1 - 10^{-D}) + 10^{a_3 (D - a_4)} \quad (6)$$

where the a's are constants determined by the fit. The a's for these particular plates have been determined by Strom (1978, personal communication). This calibration has yielded accurate profiles for elliptical galaxies on the same Schmidt plates. The subsequent analysis of these galaxy profiles proceeded as for the 4-m plates, except that no flattening or Fourier fit sky was necessary because of the uniform sky background on the Schmidt plates.

For NGC 4411A and B, and UGC 07557, the profiles derived from the 4-m plates and the Schmidt plates agreed very well. We compared each pair of 4-m and Schmidt profiles at each value of r . For surface brightness levels brighter than $\mu = 24 \text{ mag/arcsec}^2$ in both U and R, the average difference for all points was 0.01 magnitude, with a scatter of 0.04 magnitude. For $24 < \mu < 26.5 \text{ mag/arcsec}^2$, the average difference was 0.01, with a scatter of 0.21 magnitude. In this lower surface brightness range, small errors in setting the sky value can result in sizable systematic errors in the profiles. The scatter in the values of $I(r)$ averaged in ISOPH to obtain the sky provides an estimate of the uncertainty in the sky level. Appropriate small adjustments in the sky levels of the Schmidt profiles, within the errors in setting the sky, were made to improve the agreement between the 4-m and Schmidt profiles. With these adjustments, the scatter in the difference for the lower surface brightness region is decreased by a factor of two. We feel the error in setting the sky is primarily in the Schmidt profiles, because the estimated error in the sky was much less for the 4-m plates, simply because of the greater amount of information contained in the higher-scale 4-m images.

The small average difference between the 4-m and Schmidt profiles at $\mu < 26.5 \text{ mag/arcsec}^2$ shows that the

intensity scales are consistent. The small scatter in the differences for individual values of r , especially after small adjustments in the Schmidt sky levels, indicates that the level of random errors in the profiles is probably less than 0.05 mag for $\mu < 24$ mag/arcsec² and less than 0.10 mag for $24 < \mu < 26.5$ mag/arcsec².

The luminosity profiles are listed in Table 3. The first two columns list r , in arcsec, and the R surface brightness, in mag/arcsec², at that r . The (U-R) and (B₃-R) colors are listed when available. The last column lists the B surface brightness. For galaxies with only an R plate, the B surface brightness was obtained from the R using a constant (B-R), equal to that observed in the largest photoelectric aperture for the galaxy (see Table 2). The B profile for galaxies with both a U and R plates was obtained from the R profile and the value of (B-R) calculated at each r from (U-R), using the color interpolation equations (equations 9 and 10). For galaxies with plates in U, B₃, and R the B profile was obtained from the B₃ profile by the equation

$$B = B_3 + 0.3 (B-V) \quad (7)$$

(Schweizer 1976), where the (B-V) value at each r was found from the (U-R) color and the color interpolation formula.

Table 3. Luminosity and Color Profiles

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
NGC 45	2.3	20.91			22.12
(b/a=0.76)	7.0	21.08			22.29
	11.7	21.17			22.38
	16.4	21.28			22.49
	21.1	21.40			22.61
	25.8	21.51			22.72
	30.4	21.64			22.85
	35.1	21.82			23.03
	44.5	21.96			23.17
	58.5	22.26			23.47
	72.6	22.53			23.74
	86.6	22.86			24.07
	100.7	23.03			24.24
	114.7	23.23			24.44
	128.8	23.47			24.68
	142.8	23.69			24.90
	156.8	23.80			25.01
	170.9	23.90			25.11
	184.9	24.10			25.31
	199.0	24.38			25.59
	213.0	24.63			25.84
	227.1	24.73			25.94
	241.1	25.12			26.33
	255.2	25.70			26.91
	269.2	25.86			27.07
	283.2	26.44			27.65
	297.3	26.73			27.94

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
UGC 04841	1.2	20.97	1.68		22.52
(b/a=0.76)	3.4	21.27	1.47		22.68
	5.7	21.44	1.39		22.80
	8.0	21.61	1.35		22.94
	10.3	21.73	1.41		23.10
	12.6	21.85	1.29		23.14
	14.9	21.94	1.14		23.14
	17.2	22.09	1.06		23.24
	19.5	22.28	1.06		23.43
	21.8	22.40	1.11		23.58
	24.1	22.49	1.10		23.66
	26.4	22.59	1.06		23.74
	28.7	22.62	1.07		23.78
	31.0	22.67	1.15		23.88
	33.3	22.75	0.99		23.85
	35.6	22.82	1.04		23.95
	37.9	22.86	1.04		23.99
	40.2	22.93	1.04		24.07
	42.4	23.01	1.03		24.14
	44.7	23.11	1.07		24.26
	47.0	23.17	1.07		24.33
	49.3	23.24	1.06		24.39
	51.6	23.33	1.05		24.47
	53.9	23.37	1.02		24.49
	56.2	23.43	1.03		24.56
	58.5	23.48	0.97		24.57
	60.8	23.55	0.99		24.65
	63.1	23.59	0.98		24.69
	65.4	23.65	1.01		24.77

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	70.0	23.81	0.91		24.86
	76.9	24.05	0.88		25.08
	83.7	24.39	0.89		25.43
	90.6	24.74	0.84		25.75
	97.5	25.01	0.79		25.99
	104.4	25.29	0.72		26.22
	111.3	25.62	0.67		26.52
	118.2	25.91	0.73		26.85
<u>UGC 04922</u> (b/a=0.47)	1.5	20.92			22.15
	2.9	21.09			22.32
	4.4	21.30			22.53
	5.8	21.49			22.72
	7.3	21.68			22.91
	8.8	21.78			23.01
	10.2	21.91			23.14
	13.1	22.07			23.30
	17.5	22.31			23.54
	21.9	22.57			23.80
	26.3	22.78			24.01
	30.6	22.94			24.17
	35.0	23.10			24.33
	39.4	23.27			24.50
	43.8	23.44			24.67
	48.1	23.67			24.90
	52.5	23.95			25.18
	56.9	24.04			25.27
	61.3	24.22			25.45
	65.6	24.60			25.83

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	70.0	24.99			26.22
	74.4	25.20			26.43
	78.8	25.84			27.07
	83.1	25.83			27.06
<u>NGC 3913</u> (b/a=1.00)	1.0	19.09	1.54		20.55
	3.0	19.56	1.42		20.94
	5.0	20.12	1.36		21.46
	7.0	20.53	1.38		21.88
	9.0	20.82	1.34		22.15
	11.0	21.03	1.26		22.31
	13.0	21.20	1.20		22.44
	15.0	21.37	1.14		22.57
	17.0	21.58	1.11		22.76
	19.0	21.77	1.14		22.97
	21.0	22.03	1.10		23.20
	23.0	22.18	1.08		23.34
	25.0	22.32	1.01		23.44
	27.0	22.40	1.02		23.52
	29.0	22.50	1.01		23.62
	31.0	22.62	1.00		23.73
	33.0	22.78	1.03		23.91
	35.0	22.91	1.11		24.09
	37.0	23.03	1.15		24.24
	39.0	23.12	1.10		24.29
	41.0	23.20	1.09		24.37
	43.0	23.28	1.13		24.47
	45.0	23.33	1.16		24.54
	49.0	23.50	1.04		24.64

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	55.0	23.76	0.93		24.83
	61.0	23.85	0.94		24.92
	67.0	24.01	0.93		25.08
	73.0	24.35	0.94		25.42
	79.0	24.67	0.95		25.75
	85.0	25.03	0.92		26.09
	91.0	25.40	0.91		26.45
	97.0	25.97	0.75		26.92
	103.0	26.58	0.66		27.47
<u>UGC 06922</u> (b/a=0.70)	1.2	20.03	1.52		21.47
	3.6	20.40	1.44		21.79
	6.0	20.85	1.40		22.22
	8.4	21.30	1.36		22.64
	10.8	21.65	1.32		22.96
	13.2	21.94	1.20		23.18
	15.5	22.24	1.15		23.45
	17.9	22.45	1.05		23.59
	20.3	22.72	1.07		23.87
	22.7	23.00	1.12		24.19
	25.1	23.20	1.09		24.37
	27.5	23.34	1.06		24.49
	29.9	23.48	1.01		24.60
	32.3	23.67	1.04		24.81
	34.7	23.87	0.93		24.94
	37.1	24.04	0.96		25.12
	39.4	24.18	0.96		25.26
	41.8	24.36	0.91		25.41
	44.2	24.46	0.88		25.49

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	49.0	24.67	0.95		25.75
	56.2	25.00	0.87		26.03
	63.4	25.53	0.68		26.44
	70.5	26.09	0.67		26.99
	77.7	26.67	0.58		27.51
	84.9	27.05	0.60		27.90
<u>DDO 102</u> (b/a=1.00)	1.0	22.06	1.07		23.21
	3.0	22.21	1.11		23.39
	5.0	22.42	1.20		23.66
	7.0	22.60	1.12		23.79
	9.0	22.85	1.09		24.02
	13.0	23.14	1.27		24.42
	19.0	23.71	1.20		24.95
	25.0	24.09	1.17		25.31
	31.0	24.37	1.02		25.49
	37.0	24.59	0.84		25.60
	43.0	24.88	0.92		25.94
	49.0	25.12	0.77		26.08
	55.0	25.48	0.58		26.32
	61.0	25.69	0.58		26.53
	67.0	25.92	0.78		26.89
	73.0	26.26	0.71		27.18
	79.0	26.49	0.61		27.35
<u>UGC 06983</u> (b/a=0.67)	1.2	20.24	1.02	1.02	21.42
	3.7	20.68	1.13	1.03	21.88
	6.1	21.17	1.34	1.22	22.59
	8.6	21.42	1.29	1.30	22.90

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	11.0	21.71	1.20	1.12	23.00
	13.4	21.92	1.22	1.02	23.12
	15.9	22.05	1.20	1.05	23.28
	18.3	22.16	1.15	1.05	23.38
	20.8	22.26	1.12	1.03	23.46
	23.2	22.33	1.12	1.00	23.50
	25.7	22.41	1.09	0.99	23.57
	28.1	22.56	1.03	0.96	23.68
	30.5	22.57	1.05	0.97	23.70
	35.4	22.77	0.94	0.91	23.83
	42.8	23.07	0.90	0.90	24.11
	50.1	23.37	0.88	0.88	24.39
	57.4	23.57	0.95	0.91	24.63
	64.8	23.64	0.87	0.84	24.62
	72.1	23.98	0.83	0.85	24.96
	79.4	24.38	1.14	0.96	25.51
	86.7	24.69	1.14	0.94	25.81
	94.1	25.08	1.09	0.82	26.07
	101.4	25.44	1.10	0.83	26.44
	108.7	25.89	1.17	0.76	26.82
NGC 4411A (b/a=1.00)	1.0	20.39	1.17		21.61
	3.0	20.72	1.21		21.96
	5.0	21.03	1.28		22.32
	7.0	21.13	1.26		22.41
	9.0	21.35	1.31		22.66
	11.0	21.50	1.19		22.73
	13.0	21.57	1.15		22.78
	15.0	21.68	1.15		22.89
	17.0	21.85	1.11		23.03

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	19.0	21.99	1.16		23.20
	21.0	22.05	1.10		23.22
	23.0	22.14	1.04		23.28
	25.0	22.22	1.05		23.36
	27.0	22.33	1.02		23.45
	29.0	22.44	0.93		23.51
	31.0	22.59	1.02		23.71
	35.0	22.78	1.00		23.89
	41.0	23.09	1.13		24.28
	47.0	23.32	1.16		24.53
	53.0	23.54	1.25		24.81
	59.0	23.84	1.29		25.14
	65.0	24.14	1.34		25.47
	71.0	24.45	1.38		25.80
	77.0	24.82	1.51		26.26
	83.0	25.14	1.59		26.63
	89.0	25.58	1.41		26.95
	95.0	25.94	1.82		27.57
<u>NGC 4411B</u> (b/a=1.00)	1.0	19.73	1.49		21.15
	3.0	20.40	1.49		21.82
	5.0	20.73	1.43		22.12
	7.0	20.93	1.33		22.25
	9.0	21.12	1.26		22.40
	11.0	21.30	1.25		22.57
	13.0	21.41	1.31		22.72
	15.0	21.46	1.17		22.68
	17.0	21.62	1.09		22.79
	19.0	21.73	1.18		22.96

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	21.0	21.80	1.14		23.00
	23.0	21.87	1.10		23.04
	25.0	21.93	1.06		23.08
	27.0	22.07	1.05		23.21
	29.0	22.19	1.11		23.37
	31.0	22.33	1.07		23.48
	35.0	22.48	1.06		23.63
	41.0	22.71	1.07		23.86
	47.0	22.94	1.11		24.12
	53.0	23.05	1.06		24.20
	59.0	23.18	1.12		24.37
	65.0	23.43	1.14		24.63
	71.0	23.61	1.12		24.80
	77.0	23.87	1.11		25.05
	83.0	24.17	1.27		25.45
	89.0	24.61	1.49		26.03
	95.0	25.06	1.69		26.61
	101.0	25.44	1.68		26.99
	107.0	25.88	1.70		27.44
UGC 07557 (b/a=0.80)	1.1	21.12	1.10		22.29
	3.4	21.36	1.13		22.55
	5.6	21.63	1.14		22.83
	7.8	21.90	1.10		23.07
	10.1	22.04	1.00		23.15
	12.3	22.23	0.96		23.31
	14.5	22.41	1.00		23.52
	16.8	22.51	0.85		23.52
	19.0	22.59	0.86		23.61

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	21.2	22.64	0.77		23.60
	23.5	22.78	0.85		23.79
	25.7	22.87	0.91		23.92
	28.0	22.99	0.87		24.02
	30.2	22.98	0.95		24.06
	34.7	23.18	0.81		24.17
	41.4	23.40	0.83		24.40
	48.1	23.54	0.76		24.50
	54.8	23.68	0.77		24.64
	61.5	23.98	0.72		24.91
	68.2	24.19	0.72		25.12
	74.9	24.50	0.65		25.39
	81.6	24.95	0.50		25.74
	88.3	25.25	0.54		26.07
	95.0	25.51	0.57		26.34
	101.7	25.79	0.68		26.70
	108.5	26.33	0.83		27.33
<u>NGC 4517A</u> (b/a=0.62)	1.3	20.80	1.27	1.12	22.11
	3.8	20.97	1.24	1.09	22.24
	6.4	21.31	1.21	1.01	22.50
	8.9	21.51	1.26	1.07	22.77
	11.4	21.68	1.17	1.05	22.91
	14.0	21.88	1.23	1.04	23.10
	16.5	22.04	1.20	1.02	23.24
	19.1	22.13	1.14	1.03	23.33
	21.6	22.23	1.20	0.99	23.40
	24.1	22.33	1.09	0.96	23.46
	26.7	22.46	1.11	0.95	23.58
	29.2	22.57	1.06	0.93	23.66

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	34.3	22.69	1.02	0.92	23.77
	41.9	22.86	0.96	0.86	23.87
	49.5	23.04	0.95	0.88	24.07
	57.2	23.29	0.86	0.80	24.23
	64.8	23.57	0.90	0.83	24.54
	72.4	23.90	0.91	0.80	24.85
	80.0	24.17	0.89	0.81	25.12
	87.6	24.39	0.90	0.81	25.35
	95.3	24.66	0.99	0.84	25.65
	102.9	25.12	1.11	0.85	26.14
	110.5	25.57	1.34	0.91	26.67
	118.1	26.05	1.42	0.99	27.24
<u>DDO 144</u> (b/a=0.80)	1.1	20.65	1.01		21.77
	3.4	21.02	1.02		22.14
	5.6	21.28	1.15		22.49
	7.8	21.45	0.88		22.48
	10.1	21.69	0.90		22.74
	12.3	21.98	0.94		23.05
	14.5	22.15	1.00		23.26
	16.8	22.23	0.93		23.30
	19.0	22.41	0.94		23.48
	21.2	22.53	1.02		23.65
	23.5	22.58	1.04		23.72
	25.7	22.62	0.96		23.70
	28.0	22.66	1.13		23.85
	30.2	22.77	1.11		23.95
	34.7	22.98	0.99		24.08
	41.4	23.19	0.91		24.24

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	48.1	23.38	0.90		24.43
	54.8	23.50	0.90		24.55
	61.5	23.69	0.82		24.68
	68.2	23.96	0.85		24.97
	74.9	24.50	1.02		25.62
	81.6	25.01	1.02		26.13
	88.3	25.53	0.97		26.62
	95.0	26.55	0.65		27.44
<u>DDO 142</u> (b/a=0.86)	1.1	20.29	1.51	1.12	21.62
	3.2	20.90	1.60	1.22	22.34
	5.4	21.11	1.66	1.24	22.58
	7.6	21.21	1.60	1.23	22.66
	9.7	21.30	1.54	1.19	22.71
	11.9	21.38	1.52	1.18	22.78
	14.0	21.49	1.48	1.15	22.85
	16.2	21.61	1.48	1.14	22.96
	18.3	21.75	1.38	1.12	23.07
	20.5	21.84	1.35	1.14	23.18
	22.6	21.93	1.32	1.12	23.24
	24.8	22.00	1.35	1.08	23.28
	27.0	22.09	1.25	1.04	23.31
	29.1	22.18	1.24	1.05	23.41
	31.3	22.27	1.21	1.05	23.50
	35.6	22.52	1.16	1.02	23.71
	42.1	22.80	1.12	1.00	23.97
	48.5	23.05	1.10	0.99	24.21
	55.0	23.24	1.11	0.98	24.39
	61.5	23.42	0.99	0.91	24.48

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	67.9	23.65	0.97	0.92	24.72
	74.4	23.85	0.96	0.92	24.92
	80.9	24.00	0.83	0.89	25.03
	87.3	24.13	0.80	0.86	25.12
	93.8	24.39	0.86	0.85	25.38
	100.3	24.64	0.80	0.84	25.61
	106.8	24.88	0.79	0.80	25.81
	113.2	25.08	0.82	0.80	26.02
	119.7	25.20	0.86	0.87	26.21
	126.2	25.69	0.82	0.81	26.64
	132.6	26.38	0.79	0.70	27.21
<u>DDO 146</u> (b/a=0.71)	1.2	21.27	1.70	1.38	22.89
	3.6	21.38	1.42	1.16	22.74
	5.9	21.37	1.42	1.20	22.77
	8.3	21.47	1.45	1.22	22.90
	10.7	21.60	1.43	1.18	22.99
	13.1	21.71	1.46	1.21	23.13
	15.4	21.80	1.48	1.23	23.24
	17.8	21.93	1.43	1.20	23.34
	20.2	22.03	1.43	1.18	23.42
	22.6	22.10	1.42	1.20	23.50
	24.9	22.18	1.42	1.19	23.57
	27.3	22.27	1.36	1.21	23.68
	29.7	22.36	1.35	1.11	23.67
	32.0	22.41	1.30	1.13	23.73
	36.8	22.53	1.21	1.10	23.81
	43.9	22.69	1.15	1.08	23.94
	51.0	22.94	1.21	1.11	24.23
	58.2	23.18	1.28	1.13	24.50

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	65.3	23.30	1.30	1.17	24.66
	72.4	23.48	1.30	1.14	24.81
	79.5	23.67	1.31	1.16	25.02
	86.6	23.96	1.28	1.21	25.36
	93.8	24.33	1.40	1.25	25.78
	100.9	24.72	1.41	1.28	26.20
	108.0	25.07	1.32	1.29	26.55
	115.1	25.50	1.21	1.26	26.94
	122.2	25.92	1.02	1.14	27.22
	129.4	26.31	1.16	1.10	27.58
	136.5	26.75	1.01	1.04	27.95
	143.6	27.13	1.00	0.90	28.19
UGC A0333 (b/a=1.00)	1.0	21.13			22.17
	3.0	21.30			22.34
	5.0	21.61			22.65
	7.0	21.89			22.93
	9.0	22.15			23.19
	11.0	22.35			23.39
	13.0	22.54			23.58
	15.0	22.69			23.73
	17.0	22.80			23.84
	19.0	22.92			23.96
	21.0	23.07			24.11
	25.0	23.30			24.34
	31.0	23.58			24.62
	37.0	23.67			24.71
	43.0	23.87			24.91
	49.0	24.12			25.16
	55.0	24.52			25.56

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	61.0	24.94			25.98
	67.0	25.34			26.38
	73.0	25.26			26.30
	79.0	26.08			27.12
<u>NGC 5774</u> (b/a=0.75)	1.2	19.79	1.13	1.16	21.12
	3.5	19.94	1.29	1.23	21.36
	5.8	20.44	1.14	1.02	21.63
	8.1	20.62	1.07	1.03	21.81
	10.4	20.76	1.23	1.15	22.09
	12.7	21.03	1.18	1.04	22.25
	15.0	21.32	1.10	0.92	22.41
	17.3	21.46	1.26	0.99	22.64
	19.6	21.60	1.15	1.00	22.77
	21.9	21.75	1.12	0.97	22.89
	24.3	21.87	1.08	0.98	23.02
	26.6	22.01	1.10	0.95	23.13
	28.9	22.12	1.00	0.94	23.22
	31.2	22.23	1.13	0.92	23.32
	33.5	22.37	1.23	0.92	23.47
	35.8	22.55	1.09	0.93	23.65
	38.1	22.68	1.15	0.94	23.79
	42.7	22.88	1.15	0.87	23.92
	49.7	23.03	1.15	0.90	24.10
	56.6	23.15	1.18	0.90	24.23
	63.5	23.37	1.08	0.87	24.41
	70.4	23.55	1.18	0.96	24.69
	77.4	23.83	1.31	1.01	25.03
	84.3	24.18	1.36	0.99	25.37

Table 3. Luminosity and Color Profiles--Continued

	\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
	91.2	24.49	1.47	1.07	25.77
	98.2	24.90	1.76	1.11	26.25
	105.1	25.17	2.00	1.31	26.75
	112.0	25.52	1.92	1.25	27.03
<u>UGC 11868</u> (b/a=0.78)	1.1	21.11			22.63
	3.4	21.41			22.93
	5.7	21.58			23.10
	7.9	21.74			23.26
	10.2	21.92			23.44
	12.5	22.01			23.53
	14.7	22.17			23.69
	17.0	22.23			23.75
	19.3	22.37			23.89
	21.5	22.46			23.98
	23.8	22.54			24.06
	26.0	22.65			24.17
	28.3	22.78			24.30
	30.6	22.83			24.35
	32.8	22.89			24.41
	35.1	22.99			24.51
	37.4	23.10			24.62
	39.6	23.19			24.71
	41.9	23.25			24.77
	44.2	23.35			24.87
	46.4	23.51			25.03
	48.7	23.58			25.10
	51.0	23.78			25.30
	53.2	23.92			25.44

Table 3. Luminosity and Color Profiles--Continued

\sqrt{ab}	μ_R	U-R	B_3-R	μ_B
55.5	24.04			25.56
57.8	24.17			25.69
60.0	24.17			25.69
62.3	24.36			25.88
64.5	24.54			26.06
66.8	24.77			26.29
69.1	24.81			26.33
71.3	24.93			26.45
75.9	25.20			26.72
82.7	25.61			27.13
89.5	26.00			27.52
96.2	26.30			27.82
103.0	26.49			28.01

Most of the surface photometry data reduction was carried out using the interactive IPPS routines mentioned. In addition, a number of non-interactive programs were written to analyze the digital data. The most useful of these found the intensity values in small square rasters around a specified pixel. The COMTAL cursor was first used to read the coordinates of the desired spot on the picture. The program is similar in purpose to PHOT, but is more flexible. The intensity values are written to a disk file for further use.

For most galaxies, we obtained U and R plates. These colors provide a relatively long baseline in wavelength. The U plate emphasizes the young stellar population and the R the old. In order to estimate other colors from (U-R), we plotted all the UBVR colors against (U-R) and derived least-squares fits to all colors versus (U-R). This is necessary as most published models and analyses concern only UBVR colors. This procedure yielded the following relations, which we will refer to as the color interpolation equations:

$$(U-B) = 0.38 (U-R) - 0.49 \quad (8)$$

$$(B-V) = 0.38 (U-R) + 0.14 \quad (9)$$

$$(V-R) = 0.24 (U-R) + 0.35. \quad (10)$$

Figures 1-3 show plots of the color-color data. The rms deviation of individual points from the mean line is about 0.08 magnitude for (U-B) and (B-V), 0.06 for (V-R). These errors are larger than the observational errors, but not by a large factor. Apparently (U-R) contains much the same information as the other broad-band colors, at least for this set of photometry restricted to LSB galaxies.

2.4 Infrared Photometry

We obtained infrared photometry of 14 LSB galaxies, using the KPNO 1.3-m telescope and InSb detector system. There were two motivations for these observations: (1) the infrared magnitudes provide long-baseline colors useful in studying the stellar populations in galaxies and (2) we wished to see where the LSB galaxies fell in the H magnitude-HI velocity width plane (Aaronson, Huchra and Mould 1979), since one might expect the H mag-HI width relation to be different for galaxies of low surface brightness. Discussion of the second topic will be deferred until a more complete calibration of the H magnitude-HI velocity width correlation is available.

The galaxies were acquired as for the UBVR photometry, by offsetting from nearby stars using a television acquisition system. We observed 8 to 10 standard stars each night from the list of Frogel et al. (1978). These stars are on a magnitude system in which α Lyr has a

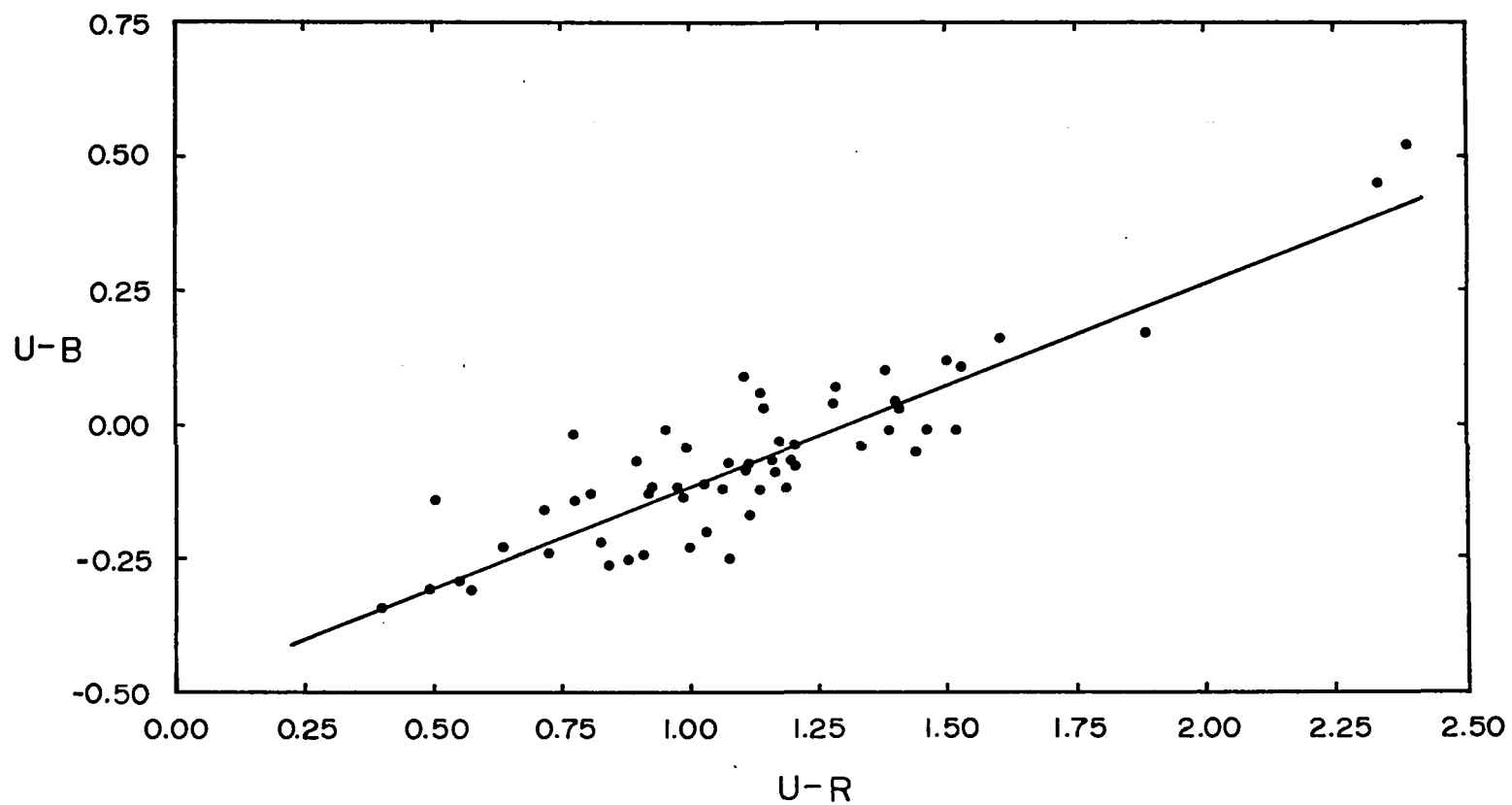


Figure 1. $(U-B)$ vs. $(U-R)$ Color-color Plot.

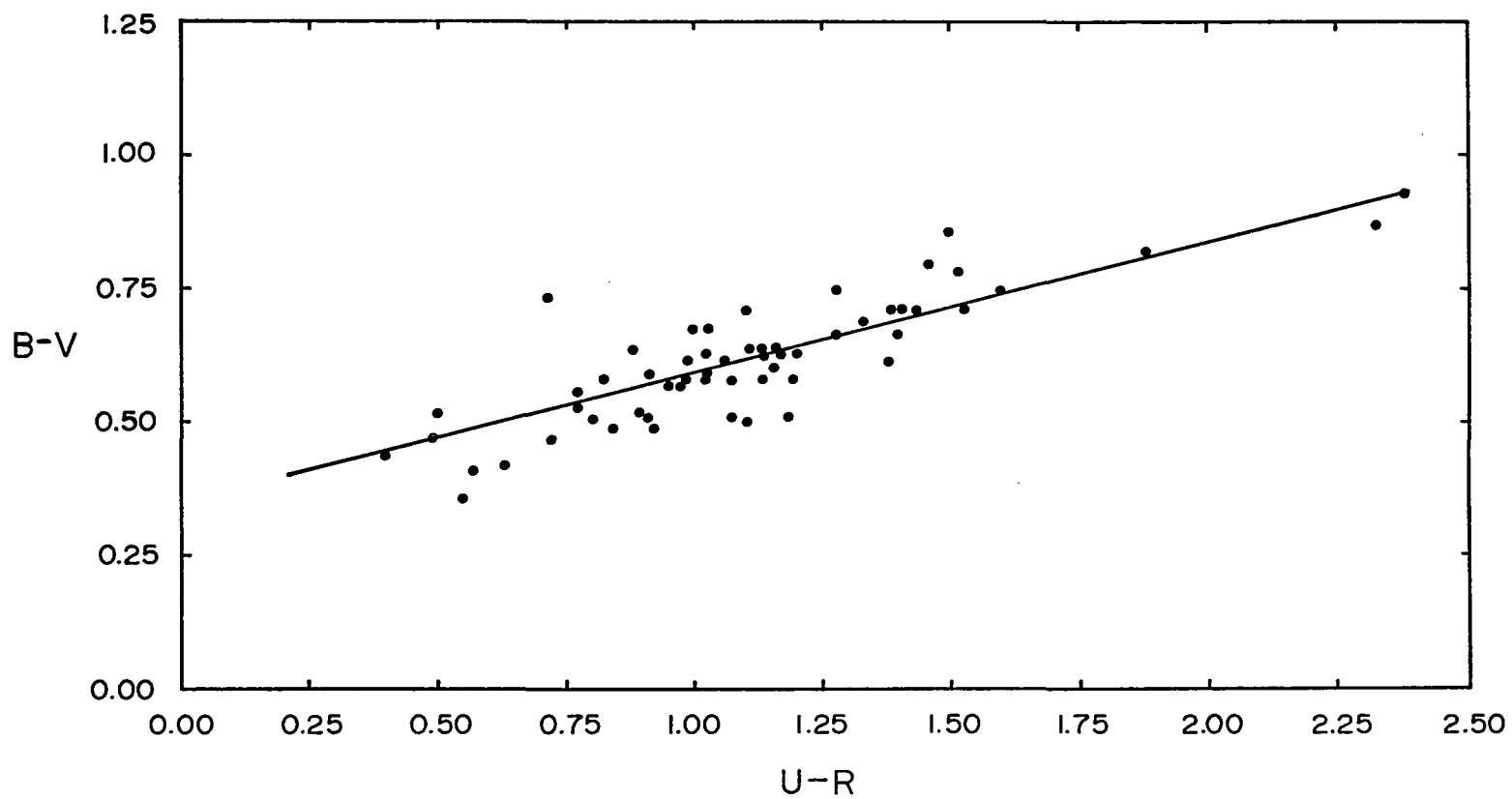


Figure 2. $(B-V)$ vs. $(U-R)$ Color-color Plot.

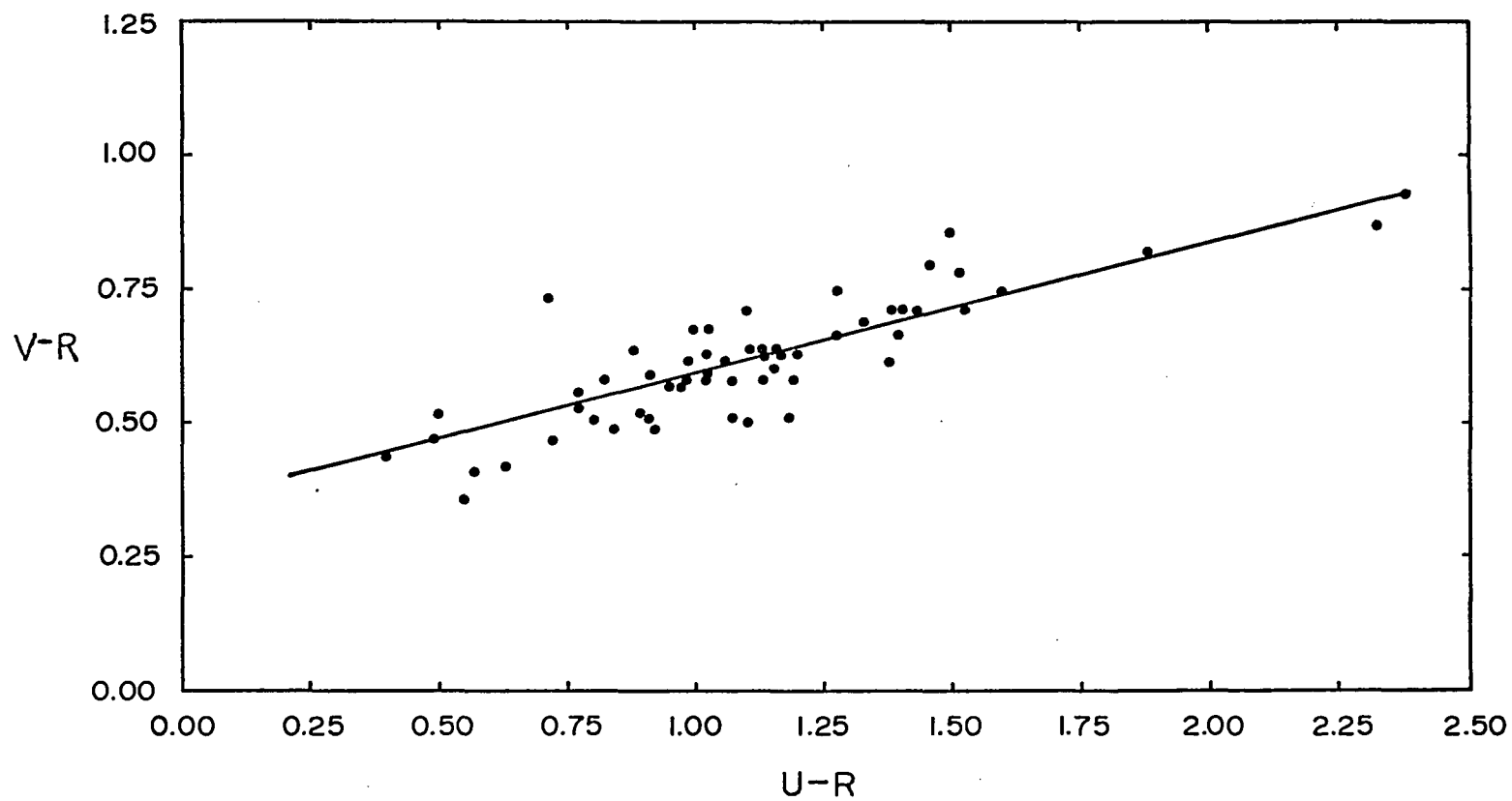


Figure 3. $(V-R)$ vs. $(U-R)$ Color-color Plot.

magnitude of 0.00 at all wavelengths. Most observations were made through an aperture of diameter 47 arcsec, with a very few made with a 33 arcsec diameter aperture. The 47 arcsec aperture provides a value of $\log A/D(0)$, the aperture size divided by the de Vaucouleurs diameter, near -0.6 for most of the galaxies in our sample. This aperture was chosen as this value of $\log A/D(0)$ is one of the standard aperture sizes used by Aaronson (1977). Internal errors were assigned to the measurements from the statistical fluctuations of repeated 10 or 20 second integrations. We checked these error estimates by observing a few galaxies on two or more nights to provide independent measurements. The assigned errors were found to be good estimates of the actual repeatability measured by the independent measurements. Because of the low surface brightness nature of these galaxies, they required long total integration times. First priority was given to obtaining H magnitudes for a number of galaxies, therefore J and K magnitudes were measured for only a fraction of the sample.

Sky positions were chosen to be free of all stars on the Palomar Sky Survey. These regions were typically 250 to 300 arcsec from the galaxy. At this beam size/throw ratio, and aperture/galaxy size ratio, the correction for flux in the reference beam is ≤ 0.01 magnitude (Aaronson 1977), so no correction was applied. A few galaxies were

observed with sky positions at 150 to 250 arcsec. We used the procedure of Aaronson (1977) to correct for the flux in the reference beam for these galaxies. This correction was always < 0.05 magnitude.

One problem affecting these observations is the variation of the response across the aperture caused by the optics necessary to image the signal on the small detector. We mapped the relative response across the aperture by driving a bright star across the aperture and recording the output of the PAR amplifier on a strip chart recorder. Using these cuts across the aperture, we made a model of the relative response across the beam. We then convolved this model beam with the digital image of the R photograph of each galaxy, using the IPPS system, and integrated the total flux in the convolved image. This flux was compared to the flux measured through a uniform aperture of the nominal diameter. The differences of these two measurements, in magnitudes, were used to correct the observed infrared magnitudes to magnitudes that would have been observed through a perfect aperture of the nominal size. This scheme was tested by observing the EO galaxy NGC 3379 which was observed by Frogel et al. (1978). We first corrected the Frogel et al. K magnitude to our aperture size (a very small extrapolation) by using their standard growth curve. The difference between this magnitude and our raw,

uncorrected K magnitude was 0.10 magnitude. A simple model of the light distribution of NGC 3379 was created, using the de Vaucouleurs $r^{\frac{1}{4}}$ law and the photometric parameters for NGC 3379 given by de Vaucouleurs (1953). Our procedure to measure the aperture correction yielded a correction of 0.10 magnitude using this artificial galaxy image, confirming the basic soundness of the procedure. The typical correction for the spiral galaxy magnitudes was 0.15 magnitudes. This value is larger than that computed for NGC 3379. This is expected, since the spirals do not have their light so strongly concentrated towards the center of the aperture where the relative response is greatest. We found no variation of the beam profile in the different infrared colors.

By far the largest error in the infrared magnitudes was simply the statistical uncertainty in the signal measurement. This produced a typical error of 0.05 magnitude. Observations of four bright galaxies observed by Frogel et al. (1978) showed very good agreement between our measured infrared colors and theirs, with a mean difference of 0.01 magnitude. This indicates that our measurements are on the same color system as theirs. We could not measure the accuracy of the beam correction procedure on any other galaxy than NGC 3379, as there was not sufficient information to create artificial galaxy images.

The results of the infrared photometry are listed in Table 4. The first column gives the measuring aperture diameter in arcsec, the second the H magnitude, and the remaining columns the (J-K), (H-K), (V-K) and (U-V) colors. The U and V magnitudes were derived from the plate material, using PHOT magnitudes for the appropriate aperture size, and the color interpolation formula. For a few galaxies, with only an R plate available, the U and V magnitudes were derived from the R PHOT magnitude and colors from the photoelectric photometry. All colors have been corrected for Galactic reddening and redshift following the precepts of Aaronson (1977). These corrections are very small, the largest being 0.03 magnitude.

For some of the galaxies in Table 4, an (H-K) color of 0.23 was assumed, in order to derive a K magnitude when only an H magnitude was measured.

2.5 HII Region Spectroscopy and Photometry

Measurements of the emission line ratios in the LSB spiral HII regions comes from two sources: (1) IIDS scans of individual HII regions obtained with the KPNO 2.1-m telescope and (2) the photographic spectra obtained for rotation curve study, described in detail in Section 2.6. The IIDS scans covered the spectral region from about 4500 Å to 7500 Å. The lines observed were [OIII] $\lambda\lambda$ 4959, 5007,

Table 4. UVJHK Photometry

	Aperture	H	J-H	H-K ^a	J-K	V-K	U-V
UGC 04841	33"0	12.70	0.44	0.36	0.80	2.68	0.69
UGC 04922	47.0	12.43	0.56	0.22	0.78	2.47	0.69
UGC 06922	47.0	12.02	0.47	0.21	0.68	2.68	0.57
NGC 4411B	47.0	11.56	0.47	0.21	0.68	2.56	0.57
NGC 4517A	47.0	12.13	0.48	0.24	0.72	2.46	0.52
NGC 5774	47.0	12.43	0.56	0.22	0.78	2.53	0.51
UGC 06983	47.0	11.96		(0.23)		2.54	0.50
UGC 07557	47.0	12.44		(0.23)		2.55	0.38
DDO 144	47.0	12.41		(0.23)		2.48	0.38
DDO 142	47.0	11.63		(0.23)		2.70	0.78
DDO 146	47.0	12.00		(0.23)		2.59	0.71
DDO 48	47.0	13.27					
UGC 05633	47.0	12.59					
UGC 06446	47.0	12.23					
UGC 06554	47.0	11.53					
DDO 128	47.0	13.00					
MCG-2-33-15	47.0	11.90					
DDO 203	47.0	12.16					

a. H-K values in parentheses are assumed, not measured.

H β , H α , [SII] $\lambda\lambda$ 6717, 6731 in first order and [OII] λ 3727 in second order. Because of the poor resolution of these scans (~ 15 Å full width at half maximum) the [NII] $\lambda\lambda$ 6548, 6584 lines are not cleanly split from the much stronger H α line, and little useful data can be obtained for the [NII] lines. The photographic spectra provide relative intensities for the [NII], H α , and [SII] lines.

At the time of our IIDS observations, problems with the IIDS reduction software prevented us from readily deriving an absolute calibration of our scans from observation of a standard star. We overcame this problem by observing a number of HII regions in M 101 that had been observed previously, and using the published line ratios as "standards". The published observations were taken from Searle (1971), Smith (1975) and Shields and Searle (1978). The following summarizes our procedure:

1. We first measured the height above the local continuum of the maximum of all the lines in all the HII regions. These measurements were in counts per channel. The lines were unresolved, so measurement of the height is equivalent to and much simpler than measuring the line integral.

2. We corrected intensities for each line for atmospheric extinction.

3. The published line ratios are presented corrected for reddening within the HII regions. We "uncorrected" these ratios for reddening by using the same reddening curve as was used by the original authors and by using the listed values of the reddening parameter for each individual region. This provided us with a list of line ratios as actually observed.

4. We took the logarithm of the ratios of the heights for various line pairs and compared these "instrumental" ratios to the actual observed logarithmic ratios. For each line ratio, the difference between the two logarithmic quantities should be constant. This difference provides a "calibration constant" to convert the instrumental ratios to real intensity ratios. This comparison was done between our measurements in M 101 and those of Smith (1975) and Shields and Searle (1978), separately.

One major potential problem in this calibration technique arises from the large spatial extent of the M 101 HII regions; a typical region has an angular size of order 20 arcsec. If our measurements were taken at a different position than the calibration observations, our ratios could be in error, as spatial inhomogeneities in the line ratios are not insignificant within a given HII region. The observations of Smith (1975) were taken with a 2 arcsec wide slit, centered on the highest surface brightness section of

the region. The Shields and Searle (1978) observations were made with a 7 arcsec diameter circular aperture, centered so as to maximize the $H\alpha$ count rate. Our data, obtained through an 8 arcsec aperture centered visually on the HII region, is thus very similar to the Shields and Searle data. Comparison of the calibration constants obtained from the two sets of published data enable a check on possible errors, such as any introduced by spatial inhomogeneities in the calibrating HII regions. The comparison is very encouraging -- the calibration constants differ by 0.04, 0.03 and 0.13 dex, for the $[OII]/H\beta$, $H\alpha/H\beta$, and $[SII]/H\alpha$ ratios, respectively. The difference in the $[SII]/H\alpha$ calibration is in the sense expected for spatial inhomogeneities, as the $[SII]$ lines form preferentially in the outer parts of the nebulae, and the Shields and Searle (1978) observations yield a higher $[SII]/H\alpha$ ratio than those of Smith (1975). Even though we had only three M 101 regions in common with Shields and Searle, as opposed to five in common with Smith, we adopt the calibration constants derived from the Shields and Searle (1978) observations for two reasons:

1. Their observations were obtained through an aperture almost identical to ours, and
2. The internal consistency of the calibration constants obtained for the separate regions is better, with a

standard deviation of 0.01, 0.02, and 0.06 for the 3 line ratios mentioned above, significantly better than those obtained from the Smith (1975) observations.

Considering the indirect nature of our calibration, it appears to have worked very well. The final errors in the intensity ratios measured for the HII regions in the LSB galaxies are certainly dominated by random errors, not systematic calibration errors.

The $[\text{OIII}]/\text{H}\beta$ ratio does not need to be calibrated in this way, as the lines are so close in wavelength that there should be no wavelength-dependent sensitivity change. Hence, our tabulated ratios are simply the ratio of the measured heights of the lines. Our $[\text{OIII}]/\text{H}\beta$ ratios for the M 101 regions were found to be in excellent agreement with those of Smith (1975) and Shields and Searle (1978), with a scatter in the difference between our measurements and the published ones of about 0.03 dex, with a negligible zero-point shift. Our measurements also yield the equivalent width of lines, in particular the equivalent width of $\text{H}\beta$, $W(\text{H}\beta)$. This measurement does not, of course, depend on the intensity calibration, but only on the linearity of the detector. Smith (1975) and Shields and Searle (1978) do not list equivalent widths, but Searle (1971) gives $W(\text{H}\beta)$ for three M 101 HII regions we observed. Our values of $W(\text{H}\beta)$ are larger, by a factor of 1.6 in the mean. As Searle

(1971) used a large aperture (14 or 20 arcsec diameter) for his measurements, we might expect our $W(H\beta)$ to be slightly larger, as his measurements must include more background continuum than our measurements, which would dilute the $H\beta$ line. One potential problem in the measurement of $W(H\beta)$ in the LSB regions is their relatively small angular size. If the continuum flux in the 8 arcsec aperture is dominated by background light because the HII region does not fill the aperture, we would measure an artificially small $W(H\beta)$. We tested for this effect by measuring the magnitude, on both the U and R plates, of an 8 arcsec diameter region centered on each HII region and in a nearby comparison region. The magnitudes of the region centered on the HII regions were in general a magnitude or more brighter than the comparison area, in both U and R. This implies that the majority of the continuum flux in the HII region comes from stars ionizing the region. We could derive an estimate of the correction to the $W(H\beta)$ for each individual region, but this was not done as equivalent data were not available for the published HII region data.

In Section 2.6, we describe the reduction of the photographic spectra and indicate how we can average rows of pixels to obtain the spectrum of a given region along the slit. To use the intensity ratio of $[SII]/H\alpha$ measured from these spectra, we first must see if there is a significant

change of sensitivity with wavelength in the system between 6560 and 6730 Å. There are two wavelength dependent factors in the variation of the response of this system with wavelength -- the blaze of the grating and the sensitivity of the image tube. The efficiency of the grating, blazed at 8000 Å, is rising with increasing wavelength in this spectral region, while the image tube response is falling (Kitt Peak National Observatory 1977). The changes are only a few percent in this spectral region, and to a good approximation they cancel, resulting in a uniform response with wavelength. Thus, the photographic line ratios were not corrected for variation of response with wavelength.

The visual absorption, A_v , was derived from the observed Balmer decrement, using the Purcell (1969) fit to the Whitford (1958) reddening curve. An intrinsic $I(H\alpha)/I(H\beta)$ ratio of 2.85 ($\log = 0.45$) was assumed. This is identical to the procedure used by Smith (1975). Because of the relatively small difference in extinction between $H\alpha$ and $H\beta$, small errors in the observed $H\alpha/H\beta$ ratio or in the assumed intrinsic intensity ratio can lead to large errors in A_v . For example, a change of only 10% (0.04 dex) in the observed $H\alpha/H\beta$ ratio leads to a change in A_v of about 0.25 magnitude. The only intensity ratio we measured significantly affected by reddening is the $[OII]/H\beta$ ratio. We corrected this ratio, using the computed A_v for each region

and the Purcell fit to the reddening curve. This correction was small, averaging 0.07 dex, and was made only to make our [OII]/H β ratio consistent with other published data.

The broad-band magnitudes of HII regions are useful in studying the properties of the associated star clusters. Table 5 lists the U magnitude and (U-R) color in an 8 arc-sec diameter circular aperture, for all HII regions observed with the IIDS. These magnitudes were obtained using the IPPS routine PHOT. We derived the equivalent (U-V) color, using the color interpolation formula, and M_U , the absolute U magnitude. Values of (U-V) and M_U are also listed in Table 5. These values have been corrected for internal reddening, assuming $A_V = 0.5$ magnitude for all regions. This A_V value is the average obtained from the H α /H β ratio for all LSB regions.

Table 6 lists the observed logarithmic line ratios for all HII regions observed with the IIDS. Also listed is the equivalent width of H β , $W(H\beta)$, in Ångstroms, and the visual extinction, A_V , derived from the H α /H β ratio for each region. The last column lists the distance of the HII region from the center of the galaxy, in units of the Holmberg radius, R_H . Table 7 lists the logarithmic line ratios observed photographically. The third column of Table 7 lists the equivalent [OIII]/H β , estimated from H α /[NII] and

Table 5. Magnitudes of HII Regions

	U	U-R	U-V ^a	M _u ^a
NGC 3913-2	17.64	-0.37	-0.9	-14.4
UGC 06922-4	18.57	0.77	0.0	-13.4
-5	19.04	0.50	-0.2	-13.0
UGC 06983-5	17.70	0.02	-0.6	-14.3
-7	17.28	0.92	0.1	-13.9
-8	18.09	0.30	-0.4	-14.7
NGC 4411A-17	18.40	0.46	-0.3	-13.9
-18	18.68	0.58	-0.2	-13.6
NGC 4411B-3	17.78	0.88	0.0	-14.5
-11	18.18	0.35	-0.3	-14.1
UGC 07557-3	18.42	0.45	-0.3	-13.8
-13	18.07	-0.09	-0.7	-14.2

a. Corrected for average internal reddening.

Table 6. IIDS HII Region Data

	[OII]/H β	[OIII]/H β	[SII]/H α	W(H β)	A _V	r/R _H
NGC 3913-2	0.73	-0.10	-0.40	117	1.3	0.3
UGC 06922-4	0.69	-0.08	-0.30	15	0.3	0.2
-5	0.65	0.85	----	19	1.3	0.5
UGC 06983-5	1.00	0.68	-0.36	25	1.1	0.8
-7	0.66	0.43	-0.49	49	0.6	0.6
-8	0.46	0.23	-0.42	12	0.6	0.1
NGC 4411A-17	0.46	0.35	-0.23	16	0.1	0.5
-18	0.55	0.13	-0.58	34	0.4	0.4
NGC 4411B-3	0.52	-0.23	-0.41	14	1.2	0.1
-11	0.43	0.42	-0.50	58	0.1	0.6
UGC 07557-3	0.39	0.43	-0.46	45	0.0	0.3
-13	0.49	0.52	-0.32	25	0.0	0.6

Table 7. HII Region Photographic Emission Line Ratios

	$[\text{SII}]/\text{H}\alpha$	$\text{H}\alpha/[\text{NII}]$	$[\text{OIII}]/\text{H}\beta$	r/R_{H}
UGC 06983-4	-0.96	0.99	0.71	0.7
-5	-0.71	0.88	0.67	0.8
-8	-0.47	0.47	0.24	0.1
-Bar	-0.54	0.50	0.36	0.0
-A	-0.80	0.93	0.69	0.8
UGC 07557-3	-0.59	0.59	0.56	0.5
-5	-0.44	0.81	0.65	0.8
-8	-0.46	0.75	0.62	0.4
-29	-0.29	0.57	0.56	0.3
-A	-0.03	0.42	0.05	0.0
NGC 4517A-1	-0.66	0.65	0.59	0.0
-16	-0.58	0.77	0.63	0.6
-20	-0.51	0.63	0.58	0.1
-30	-0.50	0.76	0.63	0.6

the relation between $[OIII]/H\beta$ and $H\alpha/[NII]$ given by Searle (1971).

Table 8 lists the coordinates of all HII regions on the galaxy prints, (Section 3.4). These coordinates are measured from the upper left hand corner of each set of four galaxy images, with x parallel to the short dimension of the page. The coordinates are in millimeters.

2.6 Long Slit Spectra Reduction

We obtained long slit spectra of three of the LSB galaxies, using the RC spectrograph on the Mayall 4-m telescope. These spectra provide the data which permit derivation of detailed rotation curves and also some emission line ratios. All spectra were taken approximately along the major axes of the galaxy. The exact position angle and slit center were chosen so the slit passed through as many condensations (HII regions) as possible. Because the exposures were long, 2 to 3 hours, we were only able to obtain one spectrum per galaxy. Along with each galaxy spectrum, we obtained a curvature plate with the telescope set at the hour angle corresponding to the midpoint of the galaxy spectrum and an intensity calibration wedge on a separate plate.

The 1200 line/mm grating was used, yielding a dispersion of about $28 \frac{\text{\AA}}{\text{mm}}$; the scale perpendicular to the dispersion was 25 arcsec/mm. The spectra were centered at 6600\AA and include the $H\alpha$, $[NII]$, and $[SII]$ emission lines.

Table 8. X, Y Positions of HII Regions on Prints -- Coordinates are measured from the upper left hand corner of each set of four galaxy images. The X axis is parallel to the short dimension of the page.

	X (mm)	Y (mm)
NGC 3913-2	34	47
UGC 06922-4	41	111
-5	48	109
UGC 06983-Bar	37	37
-4	10	40
-5	7	41
-7	42	50
-8	33	37
-11	63	35
NGC 4411A-17	23	33
-18	28	31
NGC 4411B-3	33	118
-11	29	131
UGC 07557-A	38	37
-3	47	43
-5	55	50
-8	24	25
-13	54	28
-29	42	39
NGC 4517A-1	37	40
-16	62	40
-20	31	37
-30	8	33

The galaxy spectra, along with the corresponding curvature plate, were digitized on the PDS microdensitometer. The resulting digital images were analyzed using the IPPS system, using the program RV written by A. Phillips and D. Tody. The entire operation is fairly complex. A brief outline of the process follows:

1. The spectra are converted from density to intensity using the same programs described earlier for the direct plates.

2. The curvature is mapped by tracing the lines on the curvature plate. The curvature parameters obtained from this mapping are used to create a new raster in which the effects of the curvature are removed from the galaxy spectra.

3. The vignetting perpendicular to the dispersion is mapped by measuring the intensity of a night-sky emission line across the spectra. The galaxy spectrum is divided by the inverse of the night-sky line intensity, yielding a spectrum with uniform response perpendicular to the dispersion.

4. The night-sky spectrum is obtained by averaging several regions which do not contain emission lines from the galaxy. This night-sky spectrum is subtracted from the entire raster, yielding a spectrum with only the galaxy emission lines left.

At this point we have a relative intensity raster containing the galaxy spectrum corrected for curvature, with the vignetting perpendicular to the dispersion removed, and with the night-sky emission spectra removed. We can now measure a rotation curve or emission line ratios. To measure the rotation curve, the program scans perpendicular to the dispersion in a small region flanking the specified emission line and measures the wavelength of the centroid of any emission regions. This produces a plot showing the velocity of each point on the spectra where the emission line signal was larger than some specified noise limit, specified in terms of the pixel-to-pixel variation in a blank region of the spectra near the emission line. To measure emission line ratios, we can obtain a plot of intensity versus wavelength for a given row of the matrix, or we can use the COMTAL cursor to specify a range of rows to be averaged together to obtain a spectrum of an entire emission region.

2.7 Neutral Hydrogen Radio Observations

Neutral hydrogen observations of LSB spiral galaxies were obtained with both the 91-m transit telescope of the National Radio Astronomy Observatory (NRAO) and the 305-m telescope of the National Astronomy and Ionosphere Center (NAIC) in Arecibo.

The NRAO 21-cm observations were obtained in December of 1977. We used the autocorrelator spectrometer with 192 channels, usually with a channel separation of 11 km/sec, although a few observations were made with 22 km/sec channel spacing. The signals from the two polarizations were averaged. Transformation of the data from antenna temperature to Jansky units was carried out through a standard curve of gain as a function of declination. This gain was checked several times during the run by observing continuum sources of known flux density. Comparison of our flux values with those of Fisher and Tully (1975) for 8 galaxies in common show the ratio of our measured flux to theirs to be 1.13 ± 0.11 (m.e.). One galaxy (DDO 146) had much larger than average discrepancies in both flux and line width. Without this galaxy, the flux ratio is 1.05 ± 0.11 (m.e.). Thus our flux scale is very close to that of Fisher and Tully, who went to great lengths to calibrate their scale.

The Arecibo 21-cm observations were obtained in August 1977. A 504 channel spectrometer was used, yielding spectra with a channel spacing of 4.1 km/sec. The system was calibrated by observing continuum sources to fix the temperature of the noise tubes. As the feed efficiency is dependent on frequency, separate calibrations were obtained for different velocity ranges.

Some of the galaxies observed at Arecibo were larger than the 3.2 arcmin FWHM size of the beam. Several galaxies were observed at a number of positions in an attempt to observe all the HI associated with a galaxy or to determine the separate HI content of galaxies in close pairs. These observations were made with beam centers of about 2 arcmin, using a 3 x 3 or 3 x 5 grid of points.

It is, of course, impossible to obtain a detailed HI map of an object comparable in size to the telescope beam. However, it is possible to use a grid of observations to derive a reasonable estimate of the total mass of HI. A simple model of the HI distribution in each galaxy was constructed, using the optical diameter of the galaxy and an HI versus radius curve similar to those obtained by Rogstad and Shostak (1972) for Scd galaxies. The model was convolved with the average profile of the Arecibo beam, including sidelobes. The model was iteratively altered to obtain better agreement between the actual observations and those predicted by the model. Once the differences between the model predictions and the actual observations were similar in size to the random errors in the observations, we integrated the best fit flux-position model to derive the total flux. This flux value is listed as F_{corr} in Column 6 of Table 9. Different initial models yielded final flux values within a few percent of one another, but with

Table 9. Neutral Hydrogen Data

Name (1)	V (2)	V _O (3)	ΔV (4)	F _H (5)	F _{corr} (6)	Telescope (7)
UGC 04841	1127	1288	167	5.79	6.48	NRAO
UGC 04922	1992	2024	247	3.44	3.75	NRAO
UGC 05633	1383	1257	177	3.68	3.86	NRAO
UGC 06446	646	729	139	8.04	8.85	NRAO
UGC 06682	1325	1436	83	2.96	3.08	NRAO
NGC 3913	953	1050	43	2.51	2.71	NRAO
UGC 06922	892	971	136	2.02	2.12	NRAO
DDO 105	901	922	155	6.66	7.79	NRAO
UGC 06983	1080	1168	189	7.83	8.85	NRAO
NGC 4411A	1280	1184	80	1.89	2.39	AO
NGC 4411B	1269	1174	77	2.33	3.61	AO
UGC 07557	937	835	144	3.50	6.16	AO
NGC 4517A	1530	1404	160	8.34	9.43	NRAO
NGC 4571	343	277	152	9.12	10.40	NRAO
DDO 144	1182	1064	112	3.10	3.32	NRAO
DDO 142	1433	1292	131	8.20	8.86	NRAO
DDO 146	1476	1334	168	4.86	5.30	NRAO
MCG-2-33-15	2390	2236	254	7.11	8.32	NRAO
UGC 08253	3317	3264	147	1.73	2.27	AO
UGC 08409	2804	2809	162	1.74	2.37	AO
UGC 09061	5442	5460	182	5.30	6.62	NRAO

Table 9.--Continued

Name (1)	V (2)	V _O (3)	ΔV (4)	F _H (5)	F _{corr} (6)	Telescope (7)
NGC 5774	1563	1561	142	6.66	13.58	AO
UGC 09614	3062	3086	130	1.25	1.61	AO
UGC 09875	1991	2094	86	1.15	1.55	AO
NGC 5964	1456	1502	180	5.08	10.77	AO
UGC 09938	1858	1989	101	0.92	1.20	AO
UGC 11868	1094	1349	118	2.11	2.25	NRAO
DDO 214	1693	1867	98	3.65	3.80	NRAO
DDO 219	2682	2844	130	1.90	2.02	NRAO

somewhat different distributions of HI. Another way to obtain the total HI content from a grid of observations is to treat the superposition of the beams as one large beam, and the sum of the antenna temperatures at each point in the grid as the flux of a point source in the large beam. The summed antenna temperature can then be converted to Jansky units, using the central gain of the artificial large beam. As expected, this method gave slightly smaller total flux values as the method described above, as each galaxy, although smaller than the large artificial beam, is large enough so it cannot be considered a point source relative to this beam.

The galaxy pair NGC 4411 A and B was observed with a 3 x 5 grid of points. The model fit was performed on the entire grid at once, and HI fluxes were ascribed to the individual galaxies by dividing the final model on a line halfway between the galaxies. The galaxy pair NGC 5774-5775 was also observed with a 3 x 5 grid of points. It was possible to crudely divide the flux from the points between the galaxies into contributions from the separate galaxies because of the different velocity profiles of the two galaxies. Then the model fit was performed on the separate 3 x 3 grids.

A direct comparison between our Arecibo and NRAO observations is possible for the galaxy UGC 07557. A single

NRAO observation gives a flux value, corrected for source to beam size, only 5% different from the flux obtained by the model fit to the Arecibo 3 x 3 point grid of observations. As the procedure applied to the Arecibo data is straightforward, with little chance of sizable error in deriving the total flux, this close agreement in final flux values suggests the flux density scales are similar.

By comparing the flux from the central beam to the total derived flux for the five galaxies for which total fluxes were obtained by the model fit, we derived the following equation to correct single-point Arecibo observations for source to beam size:

$$F_{\text{corr}} = F [1 + (0.65 a/\theta)^2]^{\frac{1}{2}} [1 + (0.65 b/\theta)^2]^{\frac{1}{2}} \quad (11)$$

In this equation, F is the single-beam flux, F_{corr} is the total flux, θ is 3.2 arcmin, the FWHM size of the Arecibo beam, and a and b are the Holmberg major and minor axes diameters, estimated from the Nilson diameter as in Fisher and Tully (1975). Use of the more accurate diameters available from the surface photometry change F_{corr} by only a few percent.

The observational data for the LSB spirals is listed in Table 9. The columns are as follows:

Column (1) -- The name of the galaxy.

Column (2) -- V , the heliocentric velocity (in km/sec). For most profiles, the velocity was determined in two independent ways. For straight sided profiles, we averaged the velocity of four points, the two where the profile reached zero intensity and the two at the high-intensity ends of the straight side portion of the scan. We also visually estimated the center of the profile and found the channel velocity corresponding to this point. The final velocity is the average of the two velocities obtained, which usually agreed to 5-10 km/sec. For gaussian or asymmetric profiles, we averaged the two points where the profile reached zero intensity and also estimated the centroid of the profile visually. These two velocities agreed to within 15 km/sec for most of these galaxies, and were averaged to obtain the final velocity.

Column (3) -- V_0 , the velocity of the galaxy corrected for galactic rotation. The form of the correction is from de Vaucouleurs, de Vaucouleurs and Corwin (1976) and assumes a galactic rotation velocity of 300 km/sec.

Column (4) -- ΔV , the full profile width at one-quarter of the peak intensity. We made a crude correction for the effects of instrumental resolution by subtracting 6 km/sec from the raw line widths obtained with the NRAO telescope and 3 km/sec from those obtained with the Arecibo telescope. Comparison of the line widths for seven objects observed by us and by Fisher and Tully (1975) (excluding DDO 146) shows

a mean difference of less than 1 km/sec and an rms difference of 12 km/sec. Comparison of three objects (not LSBs) observed by us at 11 km/sec channel spacing and by Dickel and Rood (1978) (NGC 2805, NGC 3319, NGC 4145) shows our line widths to be smaller by about 25 km/sec. As the Fisher and Tully line widths are measured at one-half peak intensity, we would expect our line widths to be slightly larger than theirs, but do not observe this. This, combined with the difference from the Dickel and Rood widths, suggests our line widths may be slightly underestimated. A comparison of 38 line widths for galaxies in common between Dickel and Rood (1978) and Shostak (1978) shows a rms difference of 22 km/sec. If we assume this as the size of the typical uncertainty in their line widths, the difference between our line widths and those of Dickel and Rood may simply be due to uncertainty in the measurements.

Column (5) -- The observed integrated hydrogen flux density F_H in units of $10^6 M_\odot/\text{Mpc}^2$. For the galaxies observed at more than one position with the Arecibo telescope, the flux in the beam centered on the object is listed in this column.

Column (6) -- The integrated hydrogen flux density corrected for source to beam size, F_{corr} , in units of $10^6 M_\odot/\text{Mpc}^2$. All NRAO observations were corrected as in Fisher and Tully (1975). The Arecibo observations were

corrected with equation 11, except for the galaxies observed at more than one position, for which the total flux determination was described previously.

The total hydrogen mass, M_H , can be derived from F_{corr} and the distance of the galaxy, D , in Mpc:

$$M_H = F_{\text{corr}} D^2 \quad (12)$$

Column (7) -- The telescope used to make the observations, where AO denotes the Arecibo telescope and NRAO the 91-m telescope.

CHAPTER 3

ANALYSIS

In this chapter, we use the observational material presented in Chapter 2 to study the physical properties of the LSB spiral galaxies. We also make extensive comparisons between the properties of the LSB and normal late-type spirals.

In the first section, the large-scale photometric properties of the LSB and normal galaxies are compared. We find that the LSB disk scale lengths are almost as large as those of Sb I and Sc I galaxies, while the average central surface brightness of the LSB disks is a factor of 3 lower than that characterizing the majority of spiral galaxies. In the second section, broad-band infrared and optical colors are used to show that the stellar metallicity in the LSB galaxies is significantly less than the solar value. Using the spectra and colors of the LSB HII regions, we show in the third section that the initial mass function for massive stars is significantly steeper in these galaxies than in other late-type spirals. The fourth section compares the properties of the spiral structure of normal and LSB spirals. This section includes a discussion of the

properties of the LSB spirals in terms of the spiral density-wave theory. In the fifth section, we compare the surface mass density of the LSB and normal galaxies. In the sixth section, we compare the neutral hydrogen content of normal and LSB spirals. We find that the LSBs have a high HI mass to luminosity ratio, and perhaps a high HI mass to total mass ratio. In the last section, we examine the mass-to-light ratio of the LSB galaxies. We find no conclusive evidence for an unusual mass-to-light ratio in these galaxies, although a slightly high value may be masked by the large uncertainties in our derived values.

3.1 Large-scale Photometric Properties

In this section, we use the luminosity profiles discussed in Section 2.3 to analyze two aspects of the large-scale structure of the LSB galaxies: (1) the position of the galaxies in the size versus absolute magnitude plane and (2) the decomposition of the luminosity profiles into disk and bulge components.

The positions of the LSB galaxies in the size-absolute B magnitude plane are shown in Figure 4. Total B magnitudes were determined from the photographic R magnitudes, using $(B-R)$ from the color interpolation equations (Section 2.3) for galaxies with U and R plates or from the photoelectric photometry for galaxies with no U plate. The

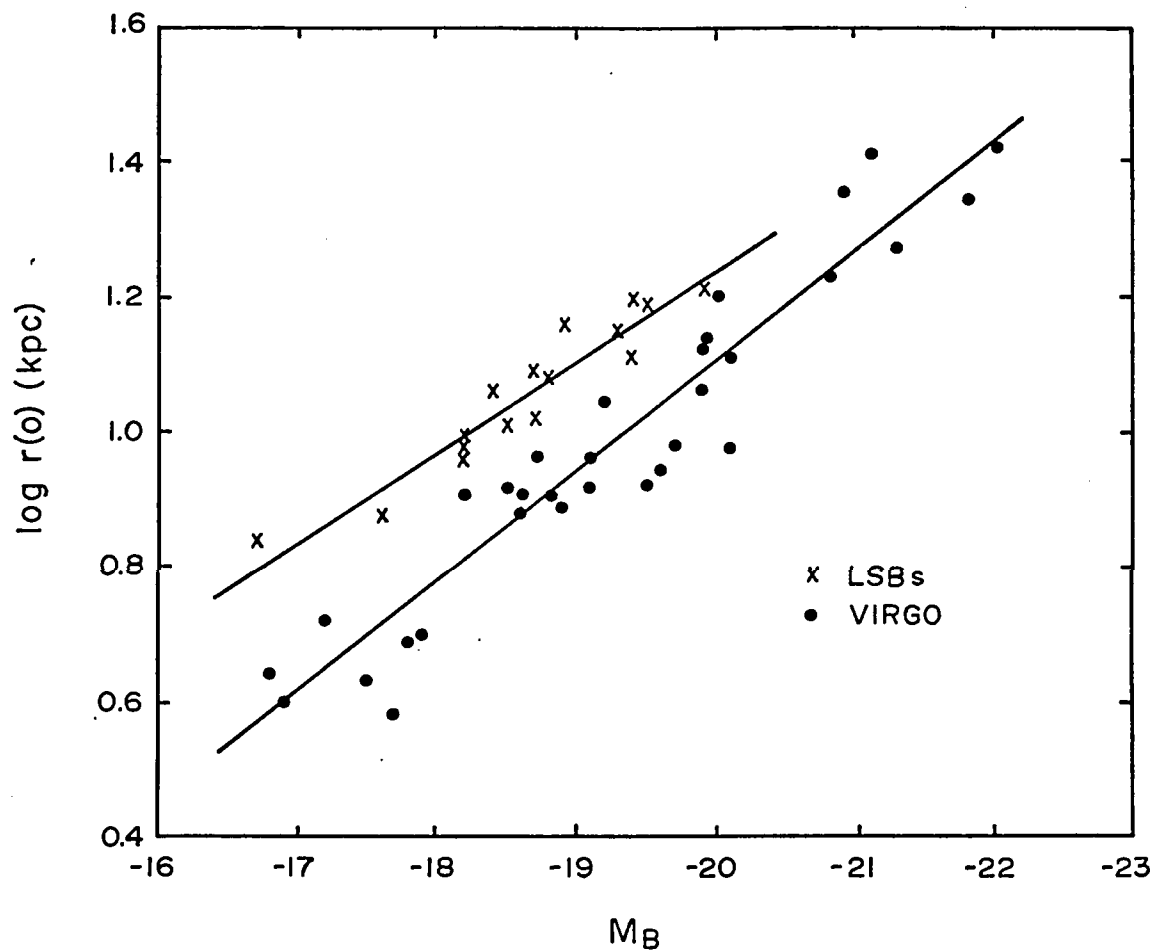


Figure 4. Size vs. Absolute Magnitude Plot.

vertical axis is the logarithm of the Holmberg radius, $r(0)$, corrected to face-on value. Also shown in Figure 4 are a number of normal spiral galaxies in the Virgo cluster studied by Peterson, Strom and Strom (1979). The LSB data was analyzed in a manner identical to the Virgo data. Galaxy distances were derived from the 21-cm velocities using $H_0 = 50$ km/sec/Mpc, or were taken as group distances consistent with this distance scale. The diameters were corrected to face-on values using the correction factor $(b/a)^{0.4}$. Peterson, Strom and Strom (1979) did not correct their magnitudes for internal reddening. Because the LSB galaxy sample is strongly biased towards face-on galaxies, while the normal Virgo sample is not, there will be a systematic error in the comparison if an internal absorption correction is not made. To crudely correct for this, we have corrected all galaxy magnitudes to face-on values, using the simple correction used by Tully and Fisher (1977):

$$A_B = 0.28 (a/b - 1) \quad (13)$$

Figure 4 reveals the expected result: the LSB galaxies are too faint at a given size or too large at a given absolute B magnitude. Least squares fits to the two sets of data yield the following relations between $\log r(0)$ and M_B :

$$\begin{aligned} \text{LSB: } \log r(0) = & 1.449 - 0.1344 M_B \\ & 0.225 \quad 0.0121 \end{aligned} \quad (14)$$

$$\text{Virgo: } \log r(0) = 2.167 - 0.1634 M_B \quad (15)$$

$$0.185 \quad 0.0096$$

Standard one-sigma errors are listed under each coefficient. These errors were calculated with the program LINFIT (Bevington 1969). At $M_B = -18.5$, the LSB galaxies have Holmberg radii a factor of over 1.5 times larger than the normal Virgo galaxies. At a fixed Holmberg radius of 10 kpc, the LSB galaxies are on average fainter than the normal Virgo galaxies by about 1.3 magnitudes.

A typical LSB galaxy luminosity profile, the R profile for NGC 4517A, is shown in Figure 5. The profile can be divided into three fairly distinct sections: (1) The dominant section of the profile is the exponential disk, extending from about $r = 30$ arcsec to $r = 100$ arcsec. (2) Near the center of the galaxy, the profile rises above the exponential disk extrapolated from further out. This section will be referred to as the central bulge. (3) Beyond 100 arcsec, the profile falls below the extrapolated disk.

Our analysis of the profiles was carried out using only the R profiles, as they are much less affected by the presence of spiral arms than the U or B_3 profiles. Disk parameters found using the R profiles were transformed to the B passband for galaxies with both U and R plates, using the color interpolation equations and the total (U-R) color of the galaxy.

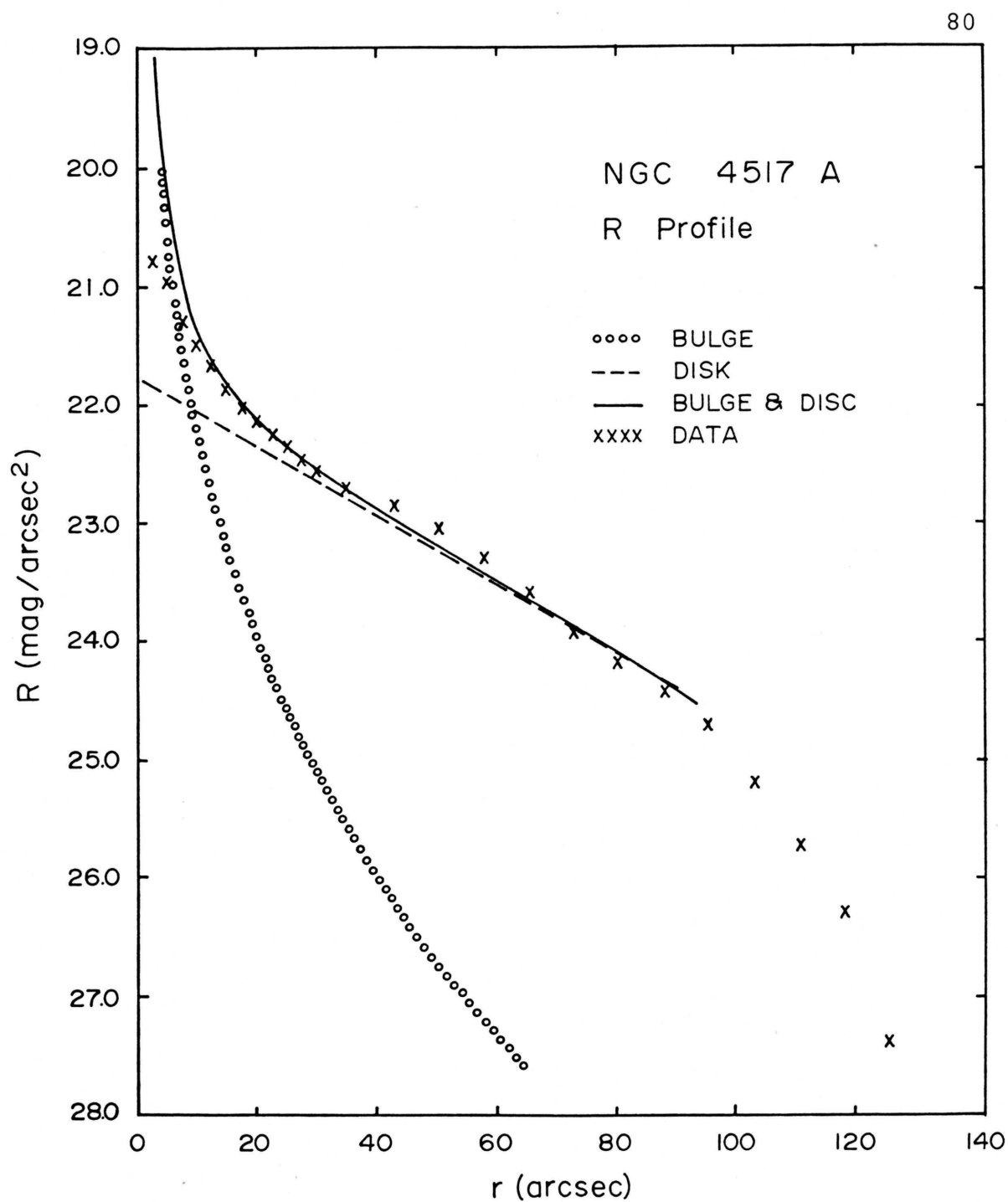


Figure 5. R Profile for NGC 4517A.

We decomposed the profiles into a contribution from the exponential disk, with intensity as a function of radius:

$$I(r) = I_0 \exp(-ar) \quad (16)$$

where α^{-1} is the scale length of the disk, and I_0 the central intensity, and a contribution from a spheroidal bulge component which follows the de Vaucouleurs law:

$$\log I(r) = I_{0v} 10^{-3.33 [(r/r_0)^{1/4} - 1]} \quad (17)$$

where r_0 is the radius containing half the total light, and I_{0v} is the intensity at r_0 . The de Vaucouleurs function is referred to as the " $r^{1/4}$ " law. The fits were made with an iterative least-squares fitting routine, similar to that used by Kormendy (1977a). We fit an exponential to the region where the disk was clearly dominant, subtracted this from the profile, and fit a $r^{1/4}$ profile to the difference. We then subtracted the $r^{1/4}$ fit from the profile, fit the difference with an exponential from $r = 0$ to the maximum extent of the disk as determined visually from the profile, and iterated the procedure several times. This procedure yielded well-determined parameters for the exponential disk. The parameters for the bulge were not as well determined, primarily because the bulge component in most of the LSB spirals does not seem to follow the $r^{1/4}$ law exactly. The

intensities at small radii were always too faint for the best fit de Vaucouleurs law component. Also, small changes in the limits of the disk fit changed the bulge parameters much more than the disk parameters.

The extrapolated disk central surface brightness values have been corrected for the geometric tilt effects and Galactic absorption as in Freeman (1970), to derive $B(0)_c$, the corrected central disk surface brightness. Values of $B(0)_c$ and α^{-1} , in kpc, are listed in Table 10. For 12 LSB galaxies with both U and R profiles, the average disk parameters are listed below, along with their standard deviations:

$$B(0)_c = 22.88 \text{ mag/arcsec}^2 \quad (\sigma = 0.39)$$

$$\alpha^{-1} = 4.04 \text{ kpc} \quad (\sigma = 1.24)$$

Schweizer (1976) found the following average disk parameters for six high-luminosity Sb I and Sc I spirals:

$$B(0)_c = 21.67 \text{ mag/arcsec}^2 \quad (\sigma = 0.35)$$

$$\alpha^{-1} = 4.95 \text{ kpc} \quad (\sigma = 1.24)$$

Disk scale lengths for the LSB galaxies were calculated with galaxy distances derived using $H_0 = 55 \text{ km/sec/Mpc}$ for consistency with the distances used by Schweizer. The value of $B(0)_c$ found by Schweizer is virtually identical to the value of 21.65 that Freeman (1970) found to characterize

Table 10. Disk Parameters

	α^{-1} (kpc)	$B(0)_c$ (mag/arcsec ²)
UGC 04841	4.97	22.76
NGC 3913	2.91	22.74
UGC 06922	2.22	23.82
UGC 06983	3.12	22.94
NGC 4411A	2.47	22.27
NGC 4411B	3.65	22.49
UGC 07557	3.62	23.04
NGC 4517A	5.49	23.24
DDO 144	3.95	22.95
DDO 142	4.80	22.60
DDO 146	5.30	22.90
NGC 5774	5.94	22.77

the majority of spiral galaxies for which he could find surface photometry. Freeman (1976) put forward the hypothesis that all non-dwarf disk galaxies have this same central surface brightness. Clearly, this is not the case. We use the normal z statistical test (Crow, Davis and Maxfield 1960) to determine whether or not there is a statistically significant difference between the means of samples of data from the LSB and normal galaxies. We use the experimental standard deviation, calculated from each sample, as an estimate of the true standard deviation. The normal test shows that the probability of there being no significant difference between $B(0)_c$ for the LSB and normal galaxies is less than 0.002%. The probability of there being no significant difference for α^{-1} is about 10%. The average LSB central surface intensity is a factor of 3 lower than Freeman's value. The LSB disk scale lengths are slightly smaller, on average, than the Sc I disk scale lengths, although the average values for the two samples are statistically only marginally different. Thus, the difference in $B(0)_c$ cannot be understood as an effect of the disk size.

The ratio of the bulge luminosity to total luminosity in a spiral galaxy is correlated with Hubble type (Yoshizawa and Wakamatsu 1975; de Vaucouleurs 1977). This ratio will be denoted K_1 , following de Vaucouleurs. We

determined the bulge luminosity following the procedure of Yoshizawa and Wakamatsu, by numerically integrating the luminosity above the exponential disk. Because the K_1 -type relation is defined for B magnitudes, we cannot directly use our K_1 , determined from the R profile. Since the bulge is redder than the disk, K_1 determined from the R profile will be too large. Therefore, we decreased K_1 by 0.7 magnitudes, the difference in B-R between the disk ($B-R \approx 1.1$) and the estimated bulge color ($B-R \approx 1.8$). The bulge color was estimated from the M_V -(U-V) plot of Aaronson (1977) and the color interpolation equations, using a typical bulge absolute magnitude ($M_V \sim -16$). Table 11 lists the values of K_1 , corrected to the B magnitude, and the type from the K_1 -type relation of de Vaucouleurs. Ten of the twelve galaxies listed in Table 11 have types in the Second Reference Catalog of Bright Galaxies (RC2) (de Vaucouleurs et al. 1976). In the RC2, all ten galaxies are classified as type Scd or later. The types listed in Table 11 derived from K_1 are, in general, slightly earlier than the RC2 types. Some of the disagreement is probably due to the intrinsic scatter in the K_1 -type relation. However, at least three galaxies (NGC 3913, UGC 06922 and NGC 5774) are definitely of early Hubble type as judged from their K_1 values.

Table 11. Morphological Types from K_1

	Log K_1	K_1 Type	RC2 Type
UGC 04841	-1.55	Sd	Sd
NGC 3913	-0.78	Sa	Sd
UGC 06922	-0.61	SO/a	---
UGC 06983	-1.62	Sdm	Scd
NGC 4411A	-1.44	Scd	Sdm
NGC 4411B	-1.26	Sc	Sdm
UGC 07557	-1.70	Sdm	---
NGC 4517A	-1.40	Scd	Sdm
DDO 144	-1.44	Scd	Sm
DDO 142	-1.33	Scd	Sm
DDO 146	-2.12	Im	Im
NGC 5774	-1.00	Sb	Sd

3.2 Stellar Metal Abundance from Broad-band Colors

In this section we discuss the interpretation of the optical and infrared colors of the LSB spiral galaxies. The primary factors affecting the intrinsic broad-band colors of a galaxy are: (1) the amount of light contributed by young stars, (2) the average stellar metallicity and (3) the initial mass function (IMF). It is often quite difficult to disentangle the effects of low metallicity from the effects of young stars on the basis of integrated colors, since both produce blue integrated colors. However, we are able to show that the blue colors observed for the LSB galaxies almost certainly result from the dominance of a metal-poor stellar population, with a characteristic metal abundance on the order of $1/10$ to $1/2$ solar.

We would like to compare the colors of the LSB galaxies with those of normal spirals of similar type. Of the six LSB spirals with measured (J-K) colors, four have types listed in the RC2. All four are listed as Sd or Sdm. The average colors for the four galaxies (Section 2.4), and standard deviation from the mean, are:

$$(U-V) = 0.57 \quad (\sigma = 0.08)$$

$$(V-K) = 2.56 \quad (\sigma = 0.09)$$

$$(J-K) = 0.75 \quad (\sigma = 0.06)$$

These colors are not corrected for internal reddening. As discussed earlier, these measurements are measured at an aperture corresponding to approximately $\log A/D(0) = -0.6$, one of the standard apertures used by Aaronson. We compare these colors with those of galaxies of type Scd and Sd observed by Aaronson (1977). The Scd and Sd spirals are the latest spirals observed by Aaronson. As there is a significant radial variation in the (U-V) and (V-K) colors, we use the (U-V) and (V-K) colors from his "restricted" class, which does not include galaxies measured only at very small values of $A/D(0)$. The (J-K) color is the average of all the Scd and Sd galaxies measured, as Aaronson finds no significant color gradient in (J-H) or (H-K), thus the measurement aperture size is not important. The average colors, uncorrected for internal reddening, are:

$$(U-V) = 0.62 \ (\sigma = 0.17)$$

$$(V-K) = 2.91 \ (\sigma = 0.10)$$

$$(J-K) = 0.90 \ (\sigma = 0.08).$$

The colors of the LSB galaxies are remarkably uniform. The average colors of the six galaxies with measured (J-K) colors, including the two without types listed in the RC2, are:

$$(U-V) = 0.59 \ (\sigma = 0.08)$$

$$(V-K) = 2.56 \ (\sigma = 0.10)$$

$$(J-K) = 0.74 \ (\sigma = 0.05)$$

The average colors of the 11 LSB galaxies with (V-K) colors, including those for which a (H-K) color was assumed (see Section 2.4), are:

$$(U-V) = 0.57 \ (\sigma = 0.13)$$

$$(V-K) = 2.57 \ (\sigma = 0.09).$$

The observed colors of spiral galaxies are affected not only by metallicity and the amount of light contributed by young stars, but also by reddening. We expect that a purely infrared color, like (J-K), will not be affected much either by reddening or by the light contributed by young stars. Thus (J-K) should be sensitive primarily to the metallicity and perhaps initial mass function of the old population. We computed several models of galaxies with constant star formation rates, using a galactic evolution program written by J. Huchra (see Huchra 1977 for details of this program). Huchra's models contain only stars of solar composition. These models reveal that (J-K) changes by only a small amount (0.02 magnitude at most) as (U-V) changes by up to one magnitude as the proportion of light from young stars changes in the course of the evolution of a model galaxy. Thus the effect on (J-K) of differences in the proportion of light from young stars should be negligible. We can estimate the maximum effect of reddening on (J-K), by making use of the ratio of the excess in (J-K), (E_{J-K}) , to the visual absorption (A_V) and

an estimate of A_V . The ratio E_{J-K}/A_V has a value of 0.16 from the standard reddening curve (Aaronson 1977). Using the individual values of $a/b-1$ for the Scd and Sd galaxies observed by Aaronson (1977), and the simple Tully and Fisher (1977) internal reddening law used in Section 3.1, we estimate that the average value of A_V for these galaxies is 0.2 magnitude or less. Thus, the correction to face-on colors makes the Scd and Sd galaxies 0.03 magnitude bluer in (J-K). The corresponding correction for the LSB galaxies makes them 0.01 magnitude bluer in (J-K). Thus the difference in (J-K) color between the LSB and Aaronson Scd and Sd galaxies should not be much affected by reddening differences. We would really like to compare intrinsic, rather than face-on, colors of the LSB and normal galaxies. As discussed below, we assume that any difference caused by a different face-on to intrinsic color transformation for the two samples of galaxies is negligible. Using the normal z test (Section 3.1) we find that the probability the face-on (J-K) values for the LSB and Scd, Sd spirals do not differ significantly is less than 0.2%.

The results from the galactic evolution models show that the light from young stars has negligible effect on (J-K). This implies that the blue and red supergiants do not affect (J-K) much. However, even if the supergiants do contribute a significant amount of light, the difference

between the (J-K) colors of the LSB and normal galaxies cannot be due to the difference in supergiant colors caused by differing metallicity between the two groups of galaxies. The ratio of blue to red supergiants (B/R) decreases with increasing radius in M 33 (Walker 1964). As the metal abundance also decreases with increasing radius, B/R decreases with decreasing metal abundance. Thus, the integrated supergiant colors become redder with decreasing metal abundance. We have two independent pieces of evidence that the average metal abundance in the LSB galaxies is less than that of normal galaxies of similar type: (1) the upper limit implied by the HII region line ratios (see Section 3.3), and (2) the optical colors of the old disk population (discussed later in this section). Because the LSB galaxies have lower metal abundance than the normal galaxies, proper inclusion of the supergiants in a model would tend to make the LSBs redder, not bluer, relative to normal spirals than if the difference in supergiant colors were not taken into account.

We investigated the effect of a change in the initial mass function (IMF) on (J-K) for a spiral galaxy, using the galactic evolution program mentioned above. Models were computed for a 15 billion year old galaxy with a constant star formation rate, and with a power-law IMF characterized by $x = 0.5, 1.35$ and 2.5 . The (J-K) color

for $x = 2.5$ was 0.04 magnitude redder than that for $x = 1.35$ and 0.07 magnitude redder than that for $x = 0.5$. Thus, changes in the IMF do not make a large change in the (J-K) color. In Section 3.3 we argue that the IMF in the LSB galaxies, at least for the massive stars, is steeper than that in a normal galaxy. If this holds true for less massive stars, we would expect this effect to tend to make the LSB galaxies redder in (J-K) than normal galaxies. However, we have no information on the low mass IMF in the LSB galaxies.

The LSB galaxies are significantly bluer in (J-K) than normal galaxies. We have argued that this difference cannot be due to the presence of young stars or reddening differences. Also, we have argued that a plausible difference in IMF between the LSBs and normal galaxies would make the LSB galaxies redder than normal in (J-K).

Aaronson et al. (1978) present evidence that, at least for globular clusters and elliptical galaxies, (J-K) is related directly to metallicity. This relation is defined primarily by globular cluster data, where the metal abundance of individual stars can be measured spectroscopically. The elliptical galaxies overlap with and extend the globular cluster color-metallicity relation to higher metallicity. All relations between color and line strength and metallicity found for globular clusters extrapolate

smoothly to elliptical galaxies (Faber 1977). Because of this smooth continuation, it seems reasonable to assume that the relations between color and metallicity found from globular clusters can be extrapolated to elliptical galaxies. We shall therefore assume $(J-K)$ provides a good measure of metallicity in old stellar populations. As $(J-K)$ is not much affected by the presence of young stars, reddening, or variations in the IMF, its use as a metallicity indicator appears justified for spiral as well as elliptical galaxies. This is a reasonable assumption, as $(J-K)$ essentially measures the effective temperature of the giant branch, which depends primarily on the chemical composition of the giants. From the globular cluster $(J-K)$ versus metallicity plots of Aaronson et al. (1978), we estimate that the face-on LSB $(J-K)$ of 0.74 corresponds to a metal abundance about $1/5$ solar. This estimate has a large uncertainty, which is difficult to precisely estimate, as there are few clusters near this $(J-K)$ to define the scatter in the $(J-K)$, metallicity diagram. Using an estimate of the scatter at lower metallicity as an estimate of scatter in $(J-K)$ versus metallicity at $(J-K)$ of 0.74, and an estimate of the error of the mean of the LSB $(J-K)$ value, we estimate this value of $(J-K)$ could correspond to metallicity between about $1/10$ and $1/2$ solar.

Another method of investigating the metal abundance of the older population is provided by the optical colors of the underlying disk stars. The color of the underlying old disk for seven LSB spirals was determined by measuring the colors in many small regions visually determined to be free of star formation, using the COMTAL display and the programs described in Section 2.3. These disk colors can be compared to the disk colors obtained by Schweizer (1976) for several giant spiral galaxies. The galaxies studied by Schweizer were brighter and more luminous than the LSB galaxies. Thus, Schweizer was able to make an estimate of the disk intensity level and color by finding the two minima corresponding to troughs between spiral arms, using azimuthal profiles in several bandpasses. We could not use this technique, as the signal-to-noise in azimuthal profiles of the LSB galaxies was not great enough. We measured (U-R) disk colors only for those galaxies for which there appeared to be patches of old disk stars not contaminated by obvious regions of active star formation. Schweizer measured (U-R) disk colors for four galaxies. One of these galaxies (M 81) is of early type (Sb) and will not be included in our average of the Schweizer disk colors. The other three galaxies are of type Sc or Scd and have similar disk colors. The average (U-R) disk color for these three galaxies is 1.71, with a standard deviation of

0.10, and the average (U-B) is 0.23 ($\sigma = 0.03$). These colors were measured at galactocentric distances from about 5 to 10 kpc. In M 101, one of these three galaxies, we know that the metal abundance in this region, at least in the gas, is approximately solar (Smith 1975). The (U-R) colors for the disks in the LSB galaxies are listed in Table 12.

Table 12. Old Disk Colors

	U-R	U-B
NGC 3913	1.21	
NGC 4411A	1.46	
NGC 4411B	1.41	
UGC 06983	1.14	0.04
UGC 07557	0.99	
NGC 4517A	1.19	0.04
DDO 144	1.05	

These colors are the average of 30 to 50 separate disk points. Only disk points within the de Vaucouleurs radius ($\mu_B = 25$) were used, as the dispersion for points at fainter levels increased significantly. The typical standard deviation in (U-R) of the points within one galaxy was

0.15 magnitude. The colors were computed for both 3×3 and 5×5 pixel boxes around each disk point. Strong contamination by young stars should show up as a difference between the two measurements, as the bigger box would pick up more of the region of star formation surrounding the disk points. The average colors of the small and large boxes agreed to within a few hundredths of a magnitude in all cases, with no systematic behavior as a function of box size. The average (U-R) disk color for the LSB spirals is 1.21 ($\sigma = 0.17$). The (U-B) color for the two galaxies for which B_3 plates are available is 0.04. The B magnitudes were calculated from the B_3 magnitudes using equation 7, with (B-V) estimated from the observed (U-R) and the color interpolation equations (Section 2.3). Thus the underlying disk colors in the LSB galaxies are bluer than the Schweizer spiral disks by about 0.5 magnitude in (U-R) and 0.19 magnitude in (U-B). Using the normal z test (Section 3.1), we find the probability that the (U-R) colors of the LSB and Schweizer spirals do not differ significantly is less than 0.02%. The (U-R) difference corresponds to a difference in both (U-B) and (B-V) of about 0.19 magnitude, using the color interpolation equations. Therefore, the old disk populations of the LSB spirals are about 0.2 magnitude bluer in both (U-B) and (B-V) than the disks of the galaxies studied by Schweizer.

The Schweizer spirals are all observed close to face-on ($b/a > 0.9$) as are the LSB galaxies, so there should be negligible difference in the optical colors caused by any correction to face-on colors. We would really like to compare the intrinsic disk colors, rather than the face-on colors. The correction from face-on to intrinsic color in a spiral galaxy is very complicated, as it depends on the distribution of the stars and dust, and the scattering and absorption properties of the dust. The small ($E_{B-V} \leq 0.02$ magnitude) Galactic polar reddening found by Sandage (1973) and Burnstein and Heiles (1978) is consistent with there being little or no correction from face-on to intrinsic color for the disks of spiral galaxies. Even if this correction is not negligible, we need only worry about the differential effect between normal and LSB galaxies. We, therefore, ignore any possible effect on our color comparisons caused by any correction from face-on to intrinsic colors.

The color difference between the LSB and normal disks can be attributed to a difference in metallicity between the old disk populations of the two classes of galaxies. Part of the color difference could be due to a difference in the residual star formation rate in the old disk regions measured. We assume this effect is negligible, although we cannot test the validity of this

assumption. Larson and Tinsley (1978) present a rough calibration of the change in UBV colors of a galaxy for a change in metal abundance. They give two vectors in the (U-B), (B-V) color plane for a change of metallicity of a factor of 4, one for red galaxies ($[U-B] = 0.5$, $[B-V] = 0.8$) the other for blue galaxies ($[U-B] = -0.2$, $[B-V] = 0.6$). If we start with the Schweizer disk colors and change the colors as appropriate to changing Z by a factor of 4, interpolation between the two vectors shows (U-B) would be 0.16 magnitude bluer at the lower metallicity and (B-V) 0.1 magnitude bluer. Larson and Tinsley state that the color vectors are uncertain by perhaps a factor of two. The difference in the optical colors of the old disk populations in the LSB and Schweizer galaxies can be plausibly understood as a metallicity difference, with the LSB disk stars a factor of four, or perhaps more, lower in metallicity than those in the Schweizer galaxies.

We have presented two methods of estimating the old stellar population metal abundance: (1) using (J-K) to measure the effective temperature of the giant branch and (2) using the difference in optical colors of the spatially isolated old disk population between the LSB and a sample of normal galaxies. Although both methods clearly have large uncertainties in calibration and observational data, they give similar results: the old stellar population in

the LSB spirals is metal-poor, with a metal abundance perhaps $1/4$ solar.

3.3 HII Regions

In this section we use the LSB HII region observations presented in Section 2.5 to study the metal abundance in the ionized gas and the mass function of the stars ionizing the HII regions. The LSB HII regions appear to have a softer ionizing radiation field than HII regions of similar metal abundance in normal spirals. We interpret this as a relative lack of massive stars in the clusters ionizing these HII regions. The optical colors of the HII regions also yield evidence for a lack of massive stars. The soft radiation field makes it difficult to use our measurements to directly measure the metal abundance in the ionized gas.

We averaged all the spectroscopic and photometric data presented in Section 2.5 to obtain values for a typical HII region in a LSB galaxy. Although there are of course differences among the HII regions in the LSB galaxies, as a group these regions are systematically different than HII regions in normal galaxies. The average values, along with their standard deviations from the scatter of the individual measurements are listed in Table 13.

Ideally, to study the ionizing radiation field, we would like to compare these measurements to measurements of HII regions of comparable metal abundance in normal

Table 13. Average LSB HII Region Parameters

Log [OIII]/H β	= 0.30 (σ = 0.33)	(IIDS only)
Log [OII]/H β	= 0.59 (σ = 0.17)	(IIDS only)
Log [SII]/H α	= -0.48 (σ = 0.19)	(IIDS and Photographic)
Log H α /[NII]	= 0.69 (σ = 0.18)	(Photographic only)
W(H β)	= 36 Å (σ = 30)	(IIDS only)
M _U	= -13.9 (σ = 0.48)	
A _V	= 0.6 Mag (σ = 0.5)	
(U-V)	= -0.32 (σ = 0.30)	

galaxies. Unfortunately, we cannot directly determine abundances from our data as we did not observe any lines sensitive to the electron temperature.

The $[\text{OIII}]/\text{H}\beta$ ratio is used as a crude metallicity indicator (see Jensen, Strom and Strom 1976 for a discussion). Therefore, we will compare the line ratios of the LSB HII regions with those of a similar range of $[\text{OIII}]/\text{H}\beta$ in galaxies of similar type. We took ratios for M 33 and M 101, both Scd galaxies, from Sarazin (1976). We isolated a range in $[\text{OIII}]/\text{H}\beta$ by taking only those HII regions observed with the IIDS with $\log [\text{OIII}]/\text{H}\beta$ between -0.03 and 0.63 , a range within one standard deviation of the LSB average. The average values for the line ratios and $W(\text{H}\beta)$ for HII regions with $[\text{OIII}]/\text{H}\beta$ in the restricted range in both the LSB galaxies and in M 33 and M 101 are listed in Table 14.

The line ratios for the LSB galaxy HII regions with $\log [\text{OIII}]/\text{H}\beta$ between -0.03 and 0.63 were compared with those of the normal galaxies in the same $[\text{OIII}]/\text{H}\beta$ range. We used the normal z test (Section 3.1) to compare the line ratio samples. The probability that there is no significant difference between both $W(\text{H}\beta)$ and $[\text{SII}]/\text{H}\beta$ in the LSB and normal spirals is less than 0.1% . The probability that $[\text{OII}]/\text{H}\beta$ does not differ significantly is less than 2% . The differences suggest that the radiation field in the LSB

Table 14. Comparison of HII Regions in
LSBs and M 33, M 101

<u>LSB with IIDS Observations</u>		
[OIII]/H β	-0.03	\rightarrow 0.63
[OII]/H β	0.49	(σ = 0.17)
[SII]/H β	0.02	(σ = 0.10)
W(H β)	34 \AA	(σ = 17)
<u>M 33, M 101</u>		
[OIII]/H β	-0.03	\rightarrow 0.63
[OII]/H β	0.33	(σ = 0.08)
[SII]/H β	-0.58	(σ = 0.28)
W(H β)	105 \AA	(σ = 38)

HII regions is softer than the in normal galaxies. The low-ionization lines [OII] and [SII] are stronger relative to $H\beta$ in the LSB galaxies. $W(H\beta)$, which decreases with decreasing hardness of the radiation field (Shields and Tinsley 1976), is smaller. The soft radiation field and consequent strong low-ionization lines could be caused by a large dust-to-gas ratio, or a relative dearth of the hottest, most massive stars, caused either by a steep mass function or a relatively low upper mass limit for stellar masses. These possibilities will be discussed in detail below.

Recall that the [OIII]/ $H\beta$ ratio increases with decreasing O/H abundance ratio, as the lower O abundance results in decreased cooling, a higher gas temperature and consequently a higher [OIII]/[OII] ratio. Taking the LSB average [OIII]/ $H\beta$ at face value, and a linear fit to the $\log [OIII]/H\beta - \log O$ abundance values derived by Shields and Searle (1978), we derive a metal abundance slightly less than solar. However, the apparently unusual nature of the ionizing radiation field in the LSB HII regions forces us to reconsider the relation between [OIII]/ $H\beta$ and metallicity. Ideally, we would like a grid of detailed HII region models such as those of Sarazin (1976) for various metal abundances and ionizing radiation fields. However, since these models do not exist, we do not know in detail

the behavior of $[OIII]/H\beta$ with differing metal abundance and T_{upp} , the effective temperature of the hottest star ionizing the HII region. Below, using $W(H\beta)$, we show that T_{upp} for the LSB regions is in the range 40,000 K to 43,000 K. A grid of simple dust-free HII regions models, for various values of abundances and T_{upp} have been published by Balick and Sneden (1976). From these models, we find that $[OIII]/H\beta$ drops as T_{upp} is lowered from 50,000 K, a typical value for an HII region in M 101 (Shields and Tinsley 1976), to 40,000 K. This implies that the metal abundance inferred from the $[OIII]/H\beta$ ratio for the LSB HII regions must be an upper limit. If there were more hot stars, producing a more normal radiation field, $[OIII]/H\beta$ would increase, resulting in our estimating a lower O abundance. Thus, because of the soft ionizing radiation field in the LSB HII regions, we cannot directly estimate the metal abundance from $[OIII]/H\beta$, but can only infer an upper limit. We need observations of some lines sensitive to the electron temperature in these HII regions to allow a more detailed chemical abundance analysis.

As Sarazin (1976, 1977) suggests, absorption of ionizing photons by dust can soften the radiation field in an HII region, if the dust absorption cross section varies with frequency in the Lyman continuum. He attributes the $W(H\beta)$ gradient seen in normal galaxies to a decrease in the

dust-to-gas ratio with increasing radius, and implicitly with decreasing metal abundance. High metal abundance (very low $[OIII]/H\beta$) HII regions can have $W(H\beta)$ values as low as those for the average LSB HII region. The prime argument against dust softening the radiation field in the LSB HII regions is the moderate to low metal abundance inferred above, and the assumption of a correlation of dust-to-gas ratio with metal abundance. We might hope to directly estimate the amount of dust in the HII regions by examining the reddening derived from the $H\alpha/H\beta$ ratio. The average value of A_V for the LSB HII regions is 0.5 magnitude ($\sigma = 0.5$). For comparison, we used the values of A_V listed by Smith (1975) for the HII regions in M 101. Dividing the M 101 regions into two groups of equal number on the basis of metallicity, we find an average A_V of 0.5 magnitude ($\sigma = 0.5$) for the high metal abundance group and 0.7 magnitude ($\sigma = 0.5$) for the low metal abundance group. Thus, there is apparently no correlation of reddening and metallicity, and implicitly dust-to-gas ratio, detectable from the optical data alone. We can only note that the reddening of the LSB HII regions is similar to that derived in the same way for metal-poor HII regions in M 101. We believe it unlikely that dust-driven radiation softening is a significant effect in the LSB HII regions as compared with normal galaxies primarily because of the overall low

metallicity of the LSB galaxies, which we take to imply a low dust-to-gas ratio.

Unfortunately, we do not have enough data on any one galaxy to say very much about the presence or absence of abundance gradients in the ionized gas across the galactic disk. The fragmentary data which we do have suggests the presence of a small gradient in $[\text{OIII}]/\text{H}\beta$, of the order of 0.4 dex, from center to outer regions in a typical LSB spiral. In M 33 and M 101, $[\text{OIII}]/\text{H}\beta$ changes by at least 2.0 dex from the center to the outer regions of the galaxy.

If we assume the low value of $W(\text{H}\beta)$ in the LSB HII regions is caused by a lack of massive, hot ionizing stars, what can we learn about the IMF or upper stellar mass limit (M_{upp}) in the ionizing star clusters? Shields and Tinsley (1976) present a quantitative discussion of $W(\text{H}\beta)$ of an HII region as a function of the parameters of its ionizing star cluster. They consider an HII region ionized by a star cluster with a power-law IMF:

$$\frac{dN}{d(\log M)} = CM^{-x} \quad (18)$$

where M is the stellar mass and N is the number of stars per unit mass interval. They find that the outer regions in M 101, such as NGC 5471 ($W(\text{H}\beta) = 165 \text{ \AA}$) require ionizing clusters with T_{upp} , the temperature of their hottest star, of about 55,000 K. They present a graph of $W(\text{H}\beta)$,

normalized to the value for NGC 5471, for various values of T_{upp} and two values of x , representing a flat ($x \approx 0.5$) and a steep ($x \approx 3.0$) IMF. The average LSB HII region has a value of $\log W(\text{H}\beta)$ about 0.7 dex smaller than that for NGC 5471. Using the Shields and Tinsley graph, we find the LSB HII region T_{upp} is about 40,000 K for a steep IMF and 43,000 K for a flat IMF. These T_{upp} can be converted to approximate M_{upp} , by using the main-sequence models of Chiosi, Nasi and Sreenivasan (1978). The corresponding M_{upp} are about $30 M_{\odot}$ for the steep IMF and $40 M_{\odot}$ for the flat IMF. These values are far below the M_{upp} of $\sim 100 M_{\odot}$ required for NGC 5471 ($T_{\text{upp}} = 55,000$ K). Are there other differences between the LSB and M 101 HII regions which can account for this difference, or is M_{upp} actually different in the LSB and M 101 HII regions? One difference between the LSB and M 101 HII regions which could be important is the absolute luminosities of the HII regions. An apparently small value of M_{upp} might be due to the relative rarity of very massive stars, even for a normal IMF, for intrinsically faint clusters of low population. The absolute magnitudes of the 3 brightest HII regions in M 101 are given by Sandage and Tammann (1974b). After applying a correction for extinction, using the values of A_V given by Smith (1975) for these HII regions, we find their average absolute U magnitude, M_U , to be -16.6 ($\sigma = 0.27$). From

Section 2.5, we find the average M_U for the brightest HII regions in the LSB galaxies, (the ones chosen to be observed with the IIDS), to be -13.9 ($\sigma = 0.48$). Below, we present a quantitative discussion of the possible importance of the low absolute magnitude of the ionizing clusters of the LSB HII regions, by means of simple models of these star clusters. The aim is to answer the following question: Is M_{upp} or the IMF significantly different in the LSB HII regions as compared to those in M 101?

The model calculations were carried out as follows:

1. We assumed an IMF with separate power-law sections from $0.1-2 M_{\odot}$, $2-10 M_{\odot}$ and $10-60 M_{\odot}$. This approximates the behavior of the IMF found by Miller and Scalo (1979) who find that the slope of the IMF changes with mass. We found the number of stars in small mass ranges for various combinations of x for the three power law sections. We calculated the number versus mass interval for a cluster containing $10^3 M_{\odot}$ in the lowest mass range. The separate power-law sections were normalized to join smoothly at the mass where the value of x changed.

2. We found the total absolute magnitude M_V and integrated color (U-V) by multiplying the number of stars in each mass interval by the intensity corresponding to the absolute V and U magnitudes for each star, and summing over all mass intervals. The U and V absolute magnitudes were

derived from M_V - $\log(M/M_\odot)$ and $(U-V)$ - M_V relations for main sequence stars. For $\log(M/M_\odot) < 0.0$, data from Allen (1973) was used. For $0.0 < \log(M/M_\odot) < 1.0$, data comes from the models of Kelsall and Stromgren (1969), using the bolometric corrections of Code et al. (1976). Data for the most massive stars ($1.0 < \log(M/M_\odot) < 2.0$) was taken from the models of Chiosi et al. (1978) using the bolometric corrections of Morton (1969). The following simple relations were found to adequately represent the M_V - $\log(M/M_\odot)$ data:

$$M_V = -10.7 \log(M/M_\odot) + 4.8 \quad , \quad \log(M/M_\odot) < 0.0 \quad (19)$$

$$M_V = -6.46 \log(M/M_\odot) + 4.8 \quad , \quad 0.0 < \log(M/M_\odot) < 1.3 \quad (20)$$

$$M_V = -3.43 \log(M/M_\odot) + 0.9 \quad , \quad 1.3 < \log(M/M_\odot) < 2.0 \quad (21)$$

The $(U-V)$ - M_V relation was derived from the main sequence colors listed by Allen (1973). The following adequately represents the $(U-V)$ - M_V data:

$$(U-V) = -1.4 \quad , \quad M_V < -4 \quad (22)$$

$$(U-V) = 0.26 M_V - 0.38 \quad , \quad -4 < M_V < 6 \quad (23)$$

$$(U-V) = 0.62 M_V - 2.55 \quad , \quad M_V < 6 \quad (24)$$

Since the models use only main sequence colors, they will be strictly correct only for young clusters. Since we chose the brightest, and hence probably youngest clusters

to observe spectroscopically, this is adequate, at least for a first look at the properties of the clusters.

The results of three model calculations are listed in Table 15. This table lists, for each set of x values, the number of massive stars in various mass ranges for a cluster with $M_U = -14$, and the cluster's (U-V) color. Model A uses an approximation to the Miller and Scalo (1979) solar neighborhood IMF. Model B uses an approximation to the IMF for massive stars found by Ostriker, Richstone and Thuan (1974). Both these models predict on the order of 100 stars more massive than about $40 M_\odot$. Since the value of $W(H\beta)$ in the LSB HII regions requires a M_{upp} of $40 M_\odot$ or less, there must be a real cutoff in the stellar mass distribution if the model IMFs are valid. Model C has a very steep massive star IMF, for which $W(H\beta)$ would imply $M_{\text{upp}} \approx 30 M_\odot$. This model predicts a significant number of stars (at least 50) above $30 M_\odot$. Thus, there is probably a real cutoff in the number of stars above $30 M_\odot$ or so.

The HII region (U-V) colors can also be used to study the IMF. The average (U-V) for the LSB HII regions is -0.32 ($\sigma = 0.30$). Models A and B have (U-V) significantly bluer than -0.32 . The color of model C agrees well with the average LSB HII region color. Obviously, this model is not unique, but offers a consistency argument for a steep massive star IMF. We note that changing M_{upp} from

Table 15. Model Stellar Clusters with $M_U = -14$

Model	x			Number of Stars in Various Mass Ranges			(U-V)
	0.1-2 M_\odot	2-10 M_\odot	10-60 M_\odot	20-35 M_\odot	35-45 M_\odot	45-60 M_\odot	
A	1.	1.3	2.3	900	150	90	-1.0
B	1.	2.3	2.6	800	130	80	-0.8
C	1.	3.	3.5	350	30	15	-0.3

$60 M_{\odot}$ to $35 M_{\odot}$ changes the integrated (U-V) by only a few hundredths of a magnitude, so the presence of a mass cutoff does not effect this conclusion. It is also of interest to compare the colors of the LSB HII regions with those of the brightest HII region in M 101. The average (U-V) for the M 101 regions, corrected for internal reddening using the A_V values found by Smith (1975), is -0.99 ($\sigma = 0.35$). The normal z test (Section 3.1) shows that the probability that the (U-V) colors of the LSB and M 101 HII regions do not differ is less than 0.1%. It is interesting to note that the (U-V) of the M 101 HII regions agrees with the predicted (U-V) from the model using the Miller and Scalo (1979) IMF.

Although the models presented here are very simplified, we make the following tentative conclusions:

1. If the IMF in the LSB HII regions clusters is similar to that presented by Miller and Scalo (1979) or Ostriker et al. (1978), there is an upper mass limit of about $30-40 M_{\odot}$. Even with a steep massive star IMF, there must be few if any stars with masses above $30 M_{\odot}$ or so to match the $W(H\beta)$ data.

2. The rather red (U-V) colors of the LSB HII regions imply a steep IMF for the massive stars, with x on the order of 3 to 4.

Thus, the low $W(H\beta)$ and red $(U-V)$ colors found for the LSB HII regions require a steep IMF for massive stars, with very few stars above $30-40 M_{\odot}$. There appears to be a real lack of massive stars in the LSB HII region star clusters compared to HII regions in the outer parts of M 101. This suggests some global difference in star-forming activity between the LSB and normal galaxies. It would be interesting to observe intrinsically faint HII regions in M 101. If they also have small $W(H\beta)$, we can assume that M_{upp} is a function of the absolute size of the star-forming region, or that there is some gross error in the simple models presented here.

The steep IMF implied for massive star formation in the LSB galaxies may be inconsistent with the broad-band optical colors. Approximate broad-band colors for the LSB galaxies, found from the average total $(U-R)$ photographic color (1.14) and the color interpolation equations, are $(U-B) = 0.0$, $(B-V) = 0.6$. From the models of Huchra (1977) and Larson and Tinsley (1978), it would appear impossible to produce a color this blue, even with a constant star formation rate, with $x = 3-4$. However, the low metallicity of the LSB galaxies will make the galaxies bluer than the simple models predict. It would be very interesting to produce models of evolving galaxies using stellar models of low metal abundance.

3.4 Spiral Structure

In this section, we present a discussion of some aspects of the spiral structure of the LSB galaxies. We derive estimates of the strength or amplitude of the spiral pattern in a few LSB galaxies, using methods similar to those of Schweizer (1976). We examine the LSB galaxies in the context of the Roberts, Roberts and Shu (1975) picture of the density wave theory of spiral structure.

In the density wave theory of spiral structure, spiral arms are a manifestation of a rigidly-rotating density wave pattern in the disk mass distribution. The wave pattern rotates with a constant angular velocity Ω_p , while the angular velocity of the material in the galaxy as a function of galactic radius, $\tilde{\omega}$, is $\Omega(\tilde{\omega})$. The velocity component of the disk gas (assumed to be in circular motion around the galactic center) normal to a spiral arm is w_\perp :

$$w_\perp = \tilde{\omega} (\Omega - \Omega_p) \sin i \quad (25)$$

where i is the pitch angle of the spiral arms. If w_\perp is larger than the sound speed in the gas, a_s , gas will approach the crest of the density wave at a supersonic velocity, and a shock front will form. In this picture, compression of gas clouds behind the shock front is responsible for the initiation of star formation, and can therefore

account for the presence of newly-formed stars near the crest of the density wave (Shu 1975).

One important recent discovery, discussed by Schweizer (1976) and Jensen (1977), is the existence of broad spiral density patterns in the underlying old (red) disk stars in spirals. This pattern is most readily understood as a manifestation of a density wave in the mass distribution. From visual inspection of R photographs of the LSB spirals we find some show fairly well-developed global density wave patterns. Examples of such galaxies are: NGC 4411A, UGC 06983, UGC 04841 and NGC 4517A. Others, such as UGC 07557 and NGC 3913, show substantial fragments of such a pattern. Others show little evidence of a global spiral pattern.

Photographs of all LSB galaxies with 4-m plates are shown in Figure 6. These were produced by photographing the digital images displayed on a precision CRT. The contrast has been chosen to try to bring out faint spiral features.

Following Schweizer (1976) we define the quantity A to be the amount of light in the arms, defined as all the light above the underlying axisymmetric disk, and D to be all non-arm light, including light from the underlying disk and central bulge. The quantity $A/(D+A)$ measures the fraction of the total light emitted by the arms. We found the

Figure 6. Photographs of LSB Galaxies--

Most photographs are 4 arcmin on a side.

Those showing NGC 5774-5 and NGC 45 are
8 arcmin on a side.

p. 116 NGC 4411A, NGC 4411B

p. 117 NGC 3913, UGC 06922

p. 118 UGC 06983

p. 119 UGC 07557, DDO 144

p. 120 NGC 4517A

p. 121 DDO 142

p. 122 DDO 146

p. 123 NGC 5774-5

p. 124 UGC 04841, DDO 102

p. 125 UGC 11868, UGC 04922, NGC 45, UGC A0333

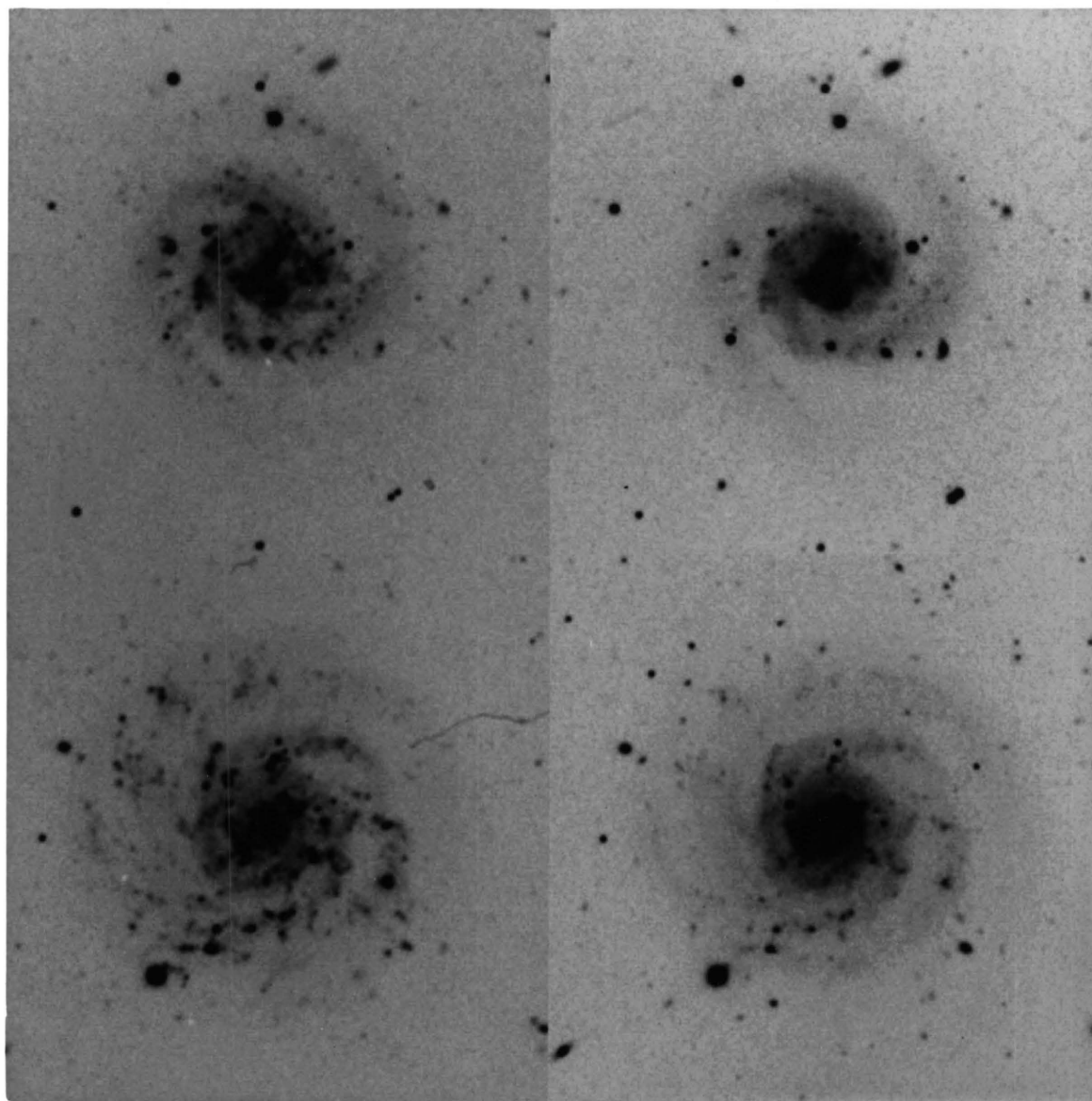


Figure 6. Photographs of LSB Galaxies

upp. left- NGC 4411A (U) , upp. right- NGC 4411A (R)
low. left- NGC 4411B (U) , low. right- NGC 4411B (R)

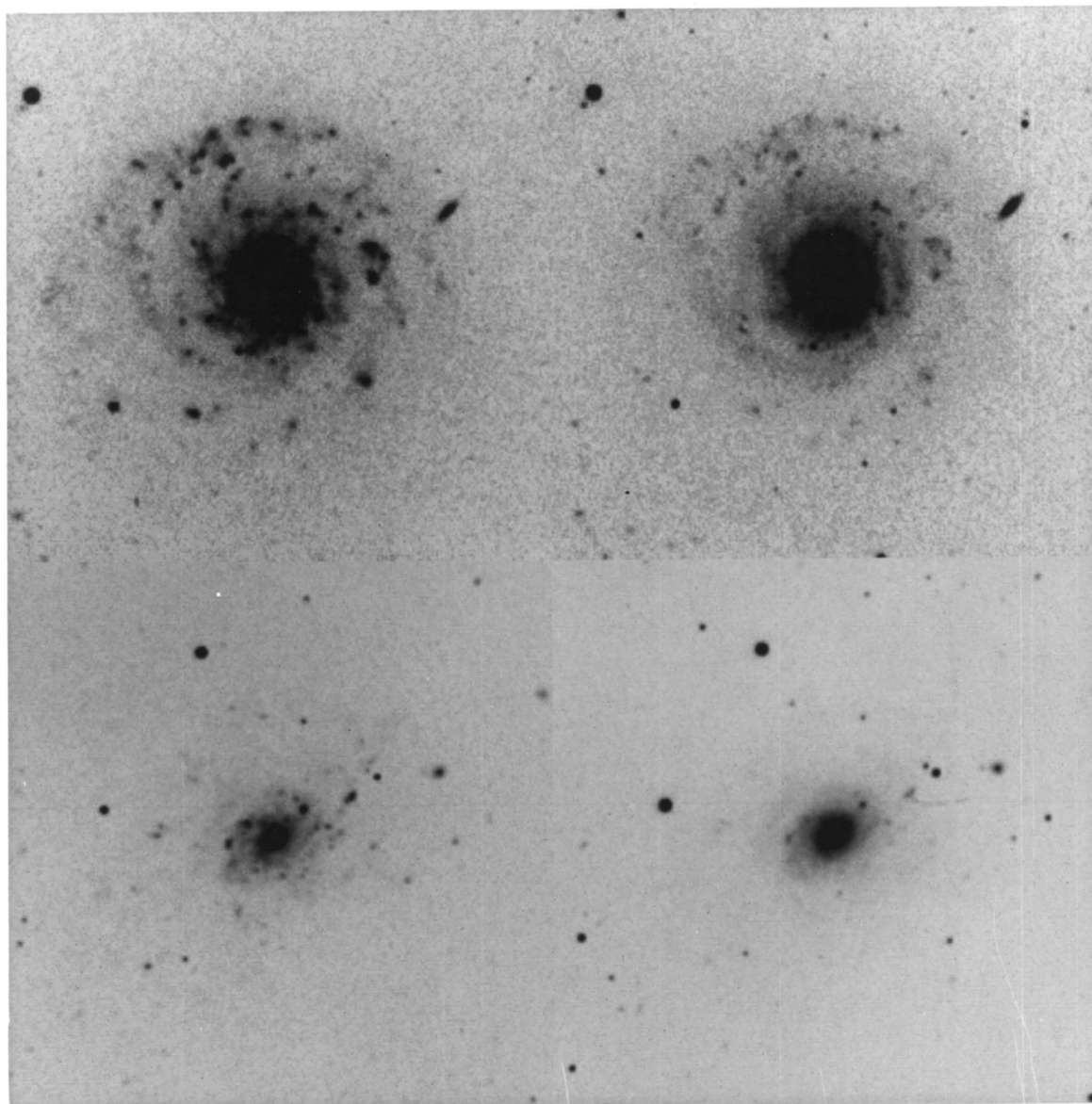


Figure 6. Continued.-- Photographs of LSB Galaxies

upp. left- NGC 3913 (U) , upp. right- NGC 3913 (R)
low. left- UGC 06922 (U) , low. right- UGC 06922 (R)

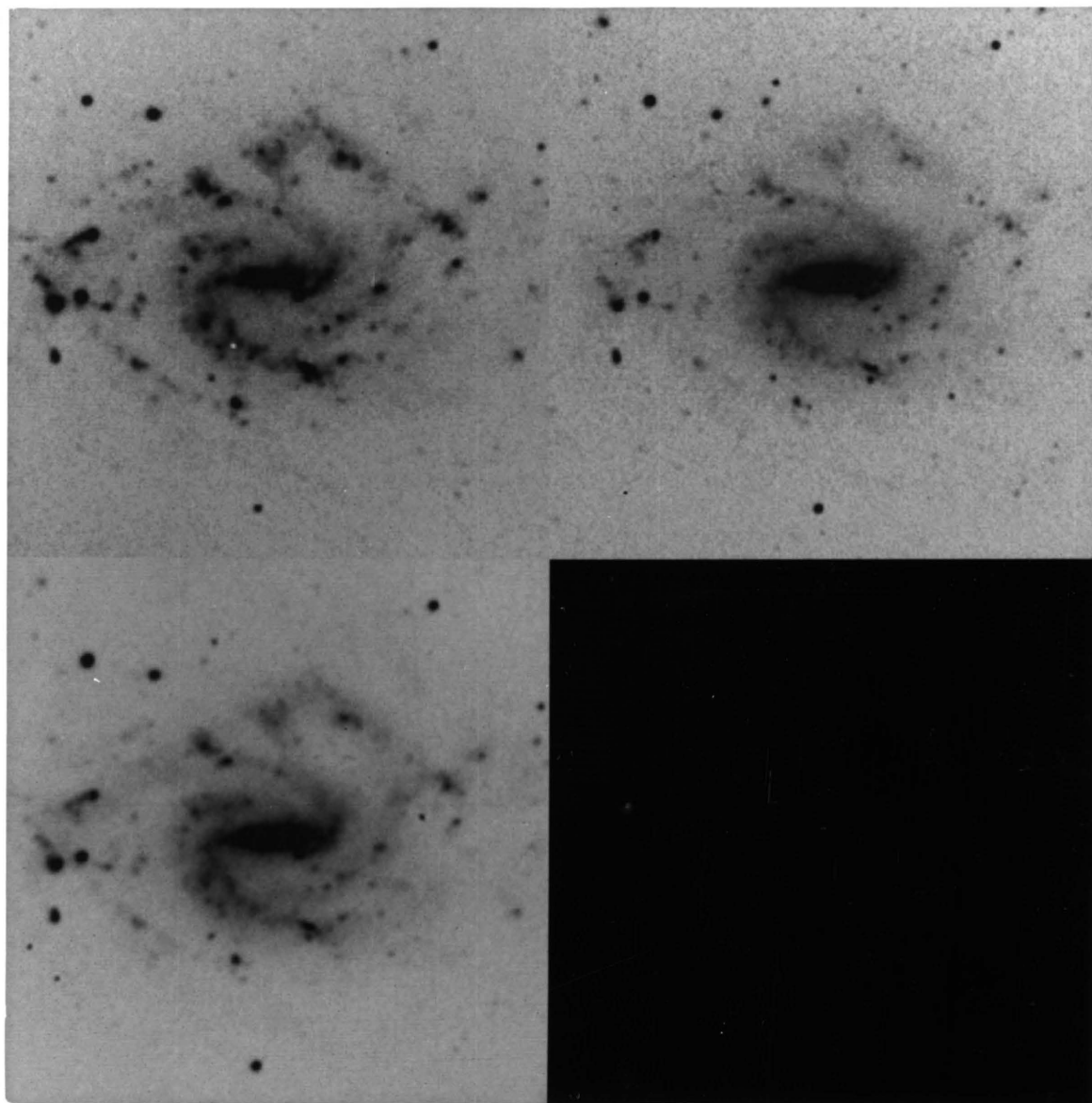


Figure 6. Continued.-- Photographs of LSB Galaxies

upp. left- UGC 06983 (U) , upp. right- UGC 06983 (R)
low. left- UGC 06983 (B)

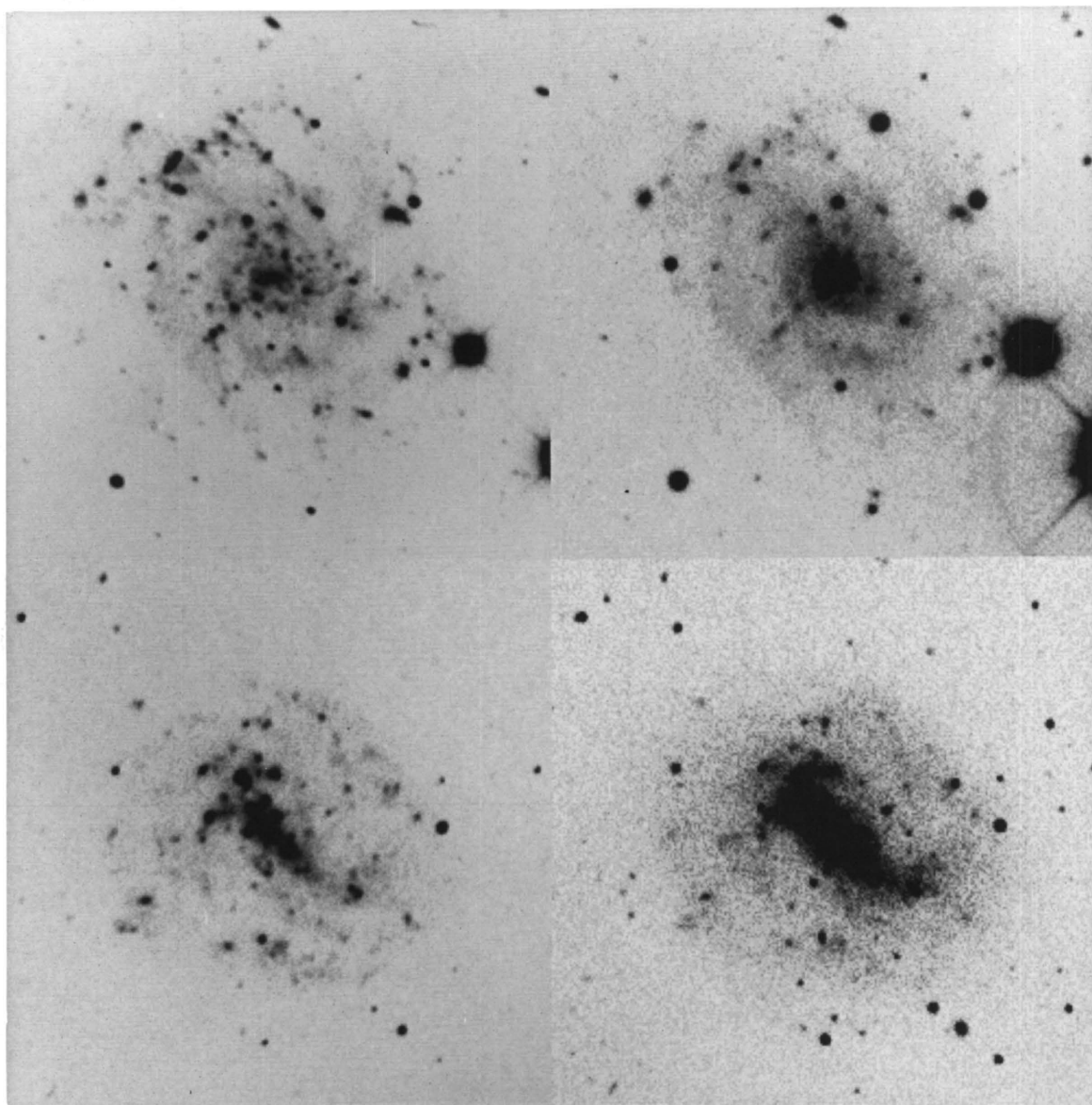


Figure 6. Continued.-- Photographs of LSB Galaxies

upp. left- UGC 07557 (U) , upp. right- UGC 07557 (R)
low. left- DDO 144 (U) , low. right- DDO 144 (R)

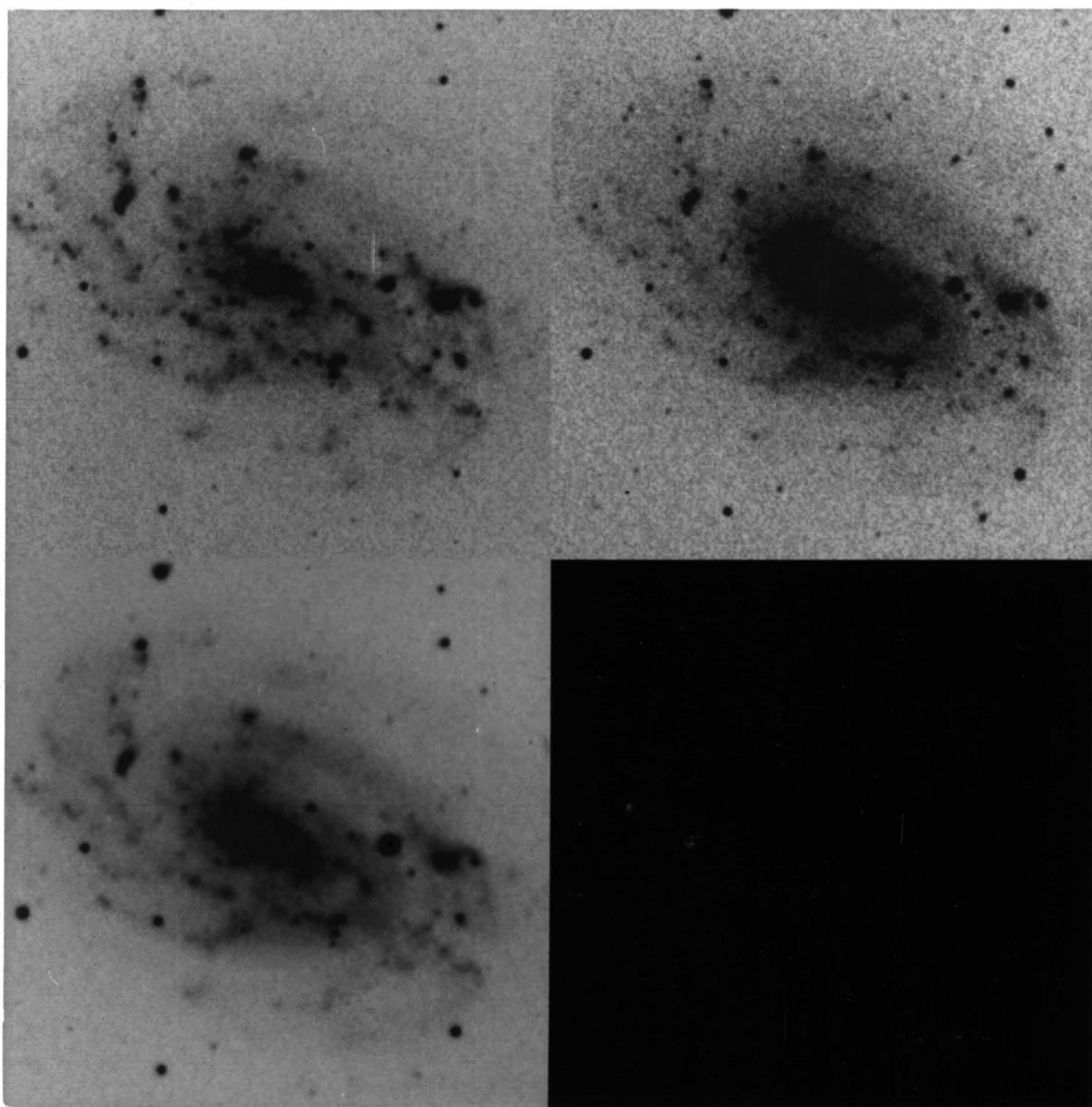


Figure 6. Continued.-- Photographs of LSB Galaxies

upp. left- NGC 4517A (U) , upp. right- NGC 4517A (R)
low. left- NGC 4517A (B)

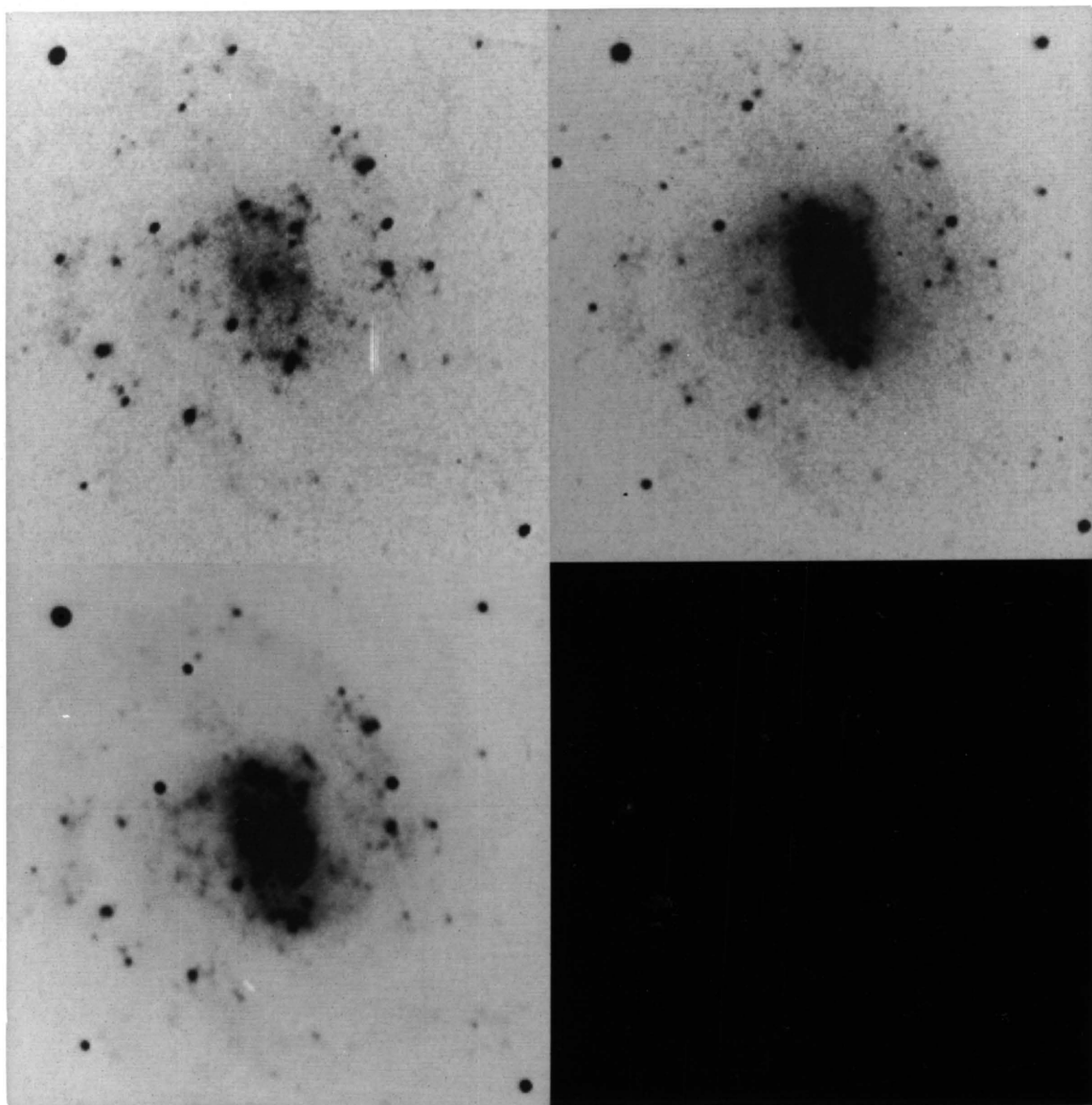


Figure 6. Continued.-- Photographs of LSB Galaxies

upp. left- DDO 142 (U) , upp. right- DDO 142 (R)
low. left- DDO 142 (B)

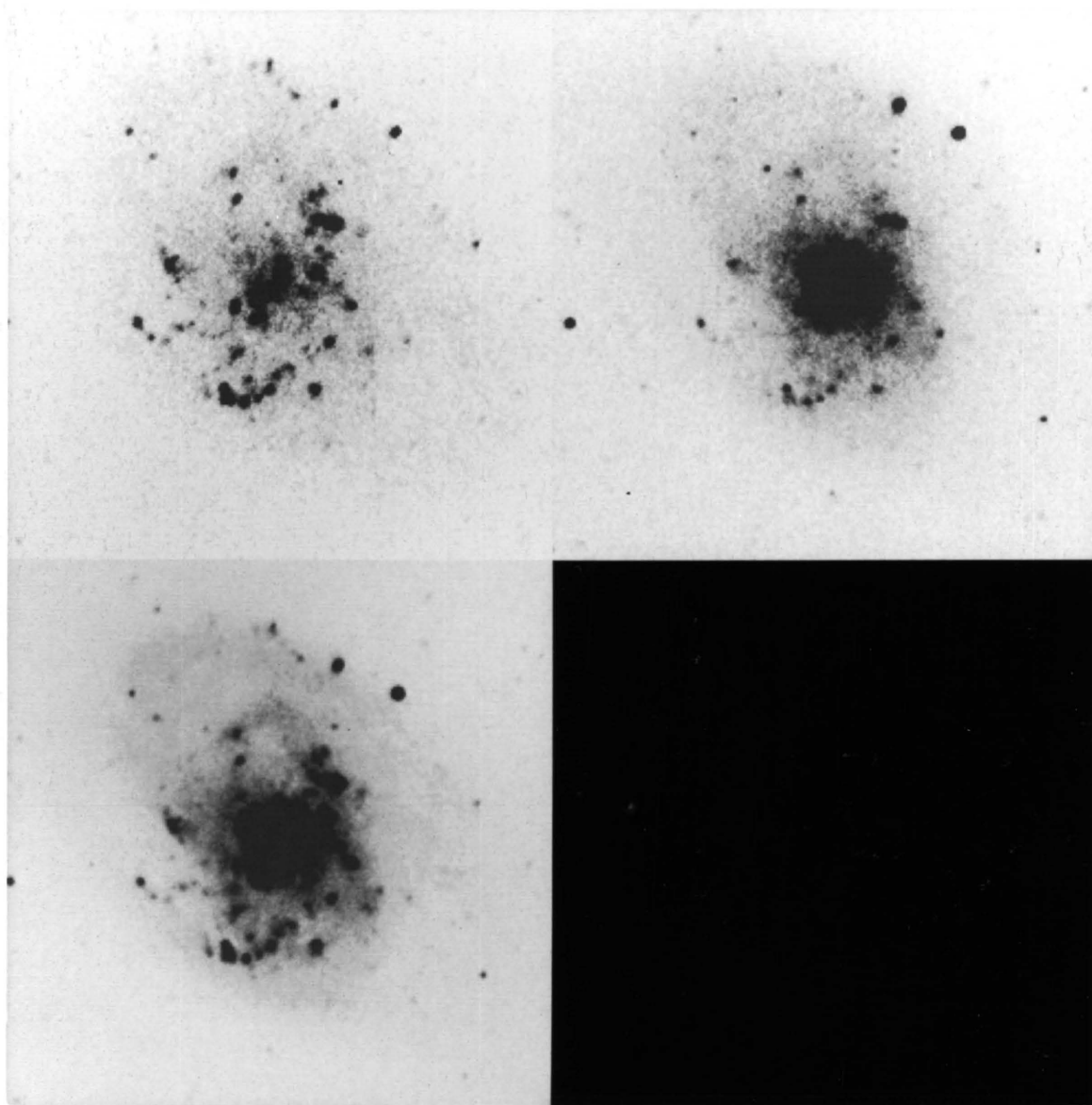


Figure 6. Continued.-- Photographs of LSB Galaxies

upp. left- DDO 146 (U) , upp. right- DDO 146 (R)
low. left- DDO 146 (B)

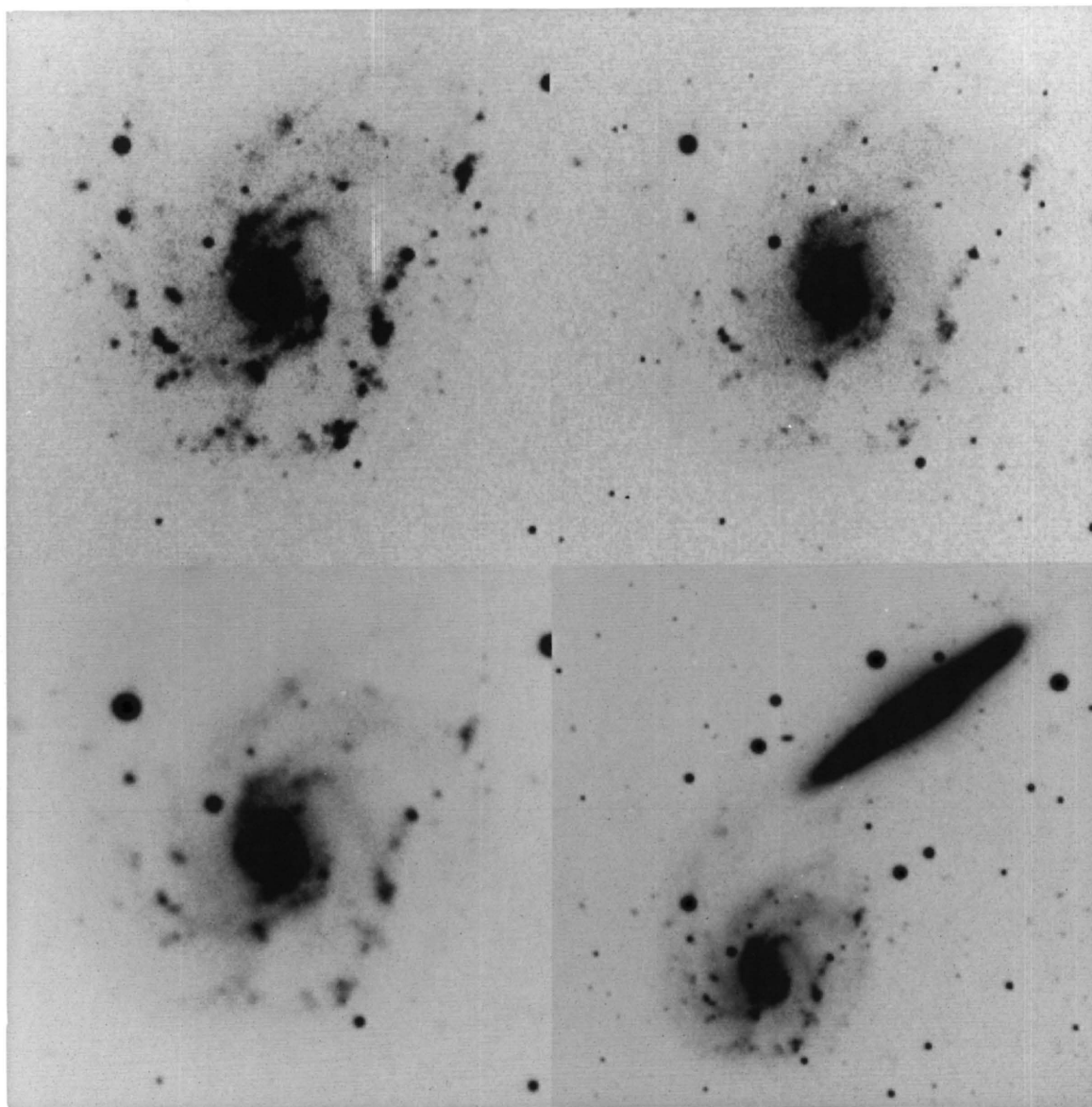


Figure 6. Continued.-- Photographs of LSB Galaxies

upp. left- NGC 5774 (U) , upp. right- NGC 5774 (R)
low. left- NGC 5774 (B) , low. right- NGC 5774-5 (B)

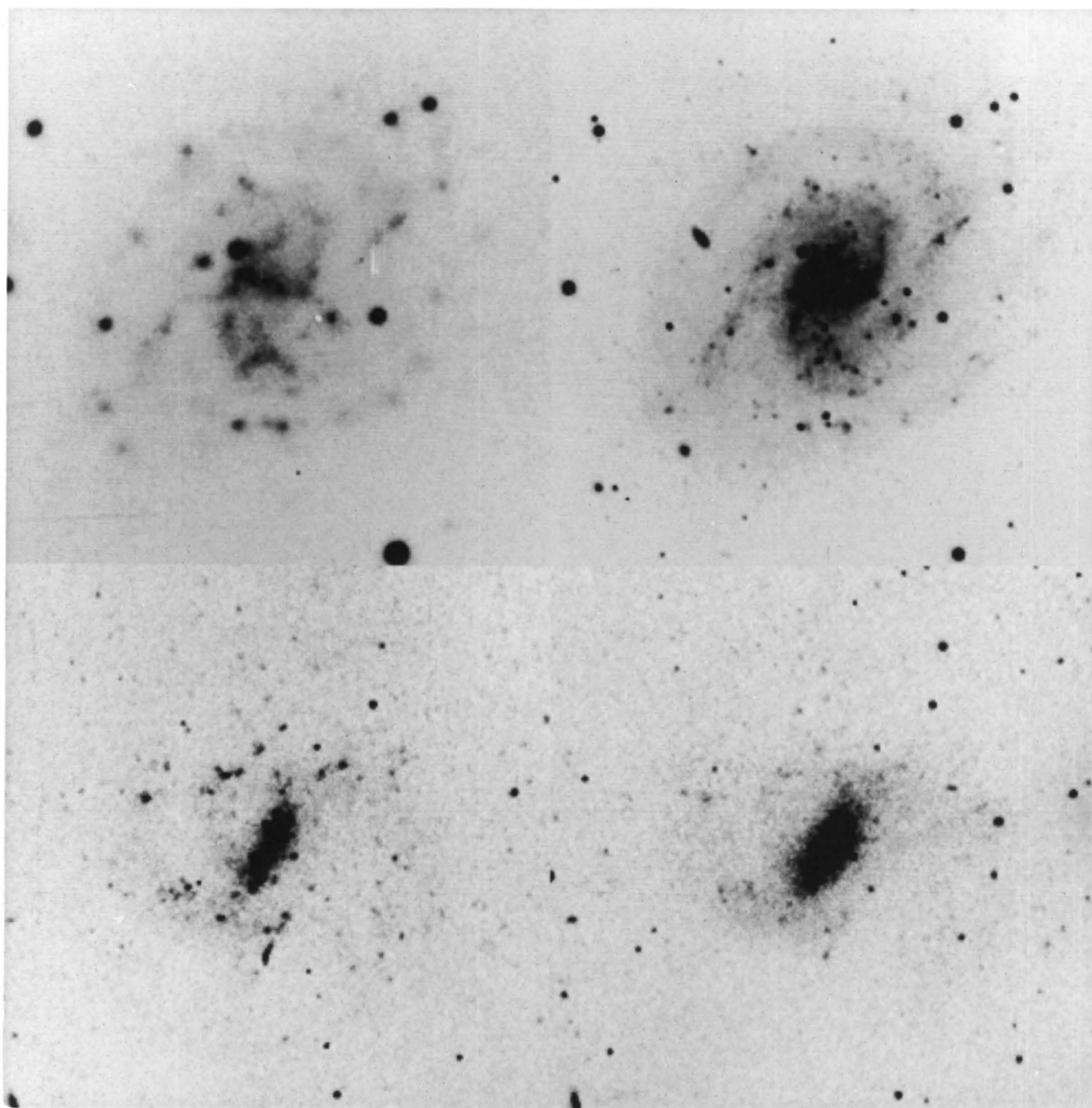


Figure 6. Continued.-- Photographs of LSB Galaxies

upp. left- UGC 04841 (U) , upp. right- UGC 04841 (R)
low. left- DDO 102 (U) , low. right- DDO 102 (R)

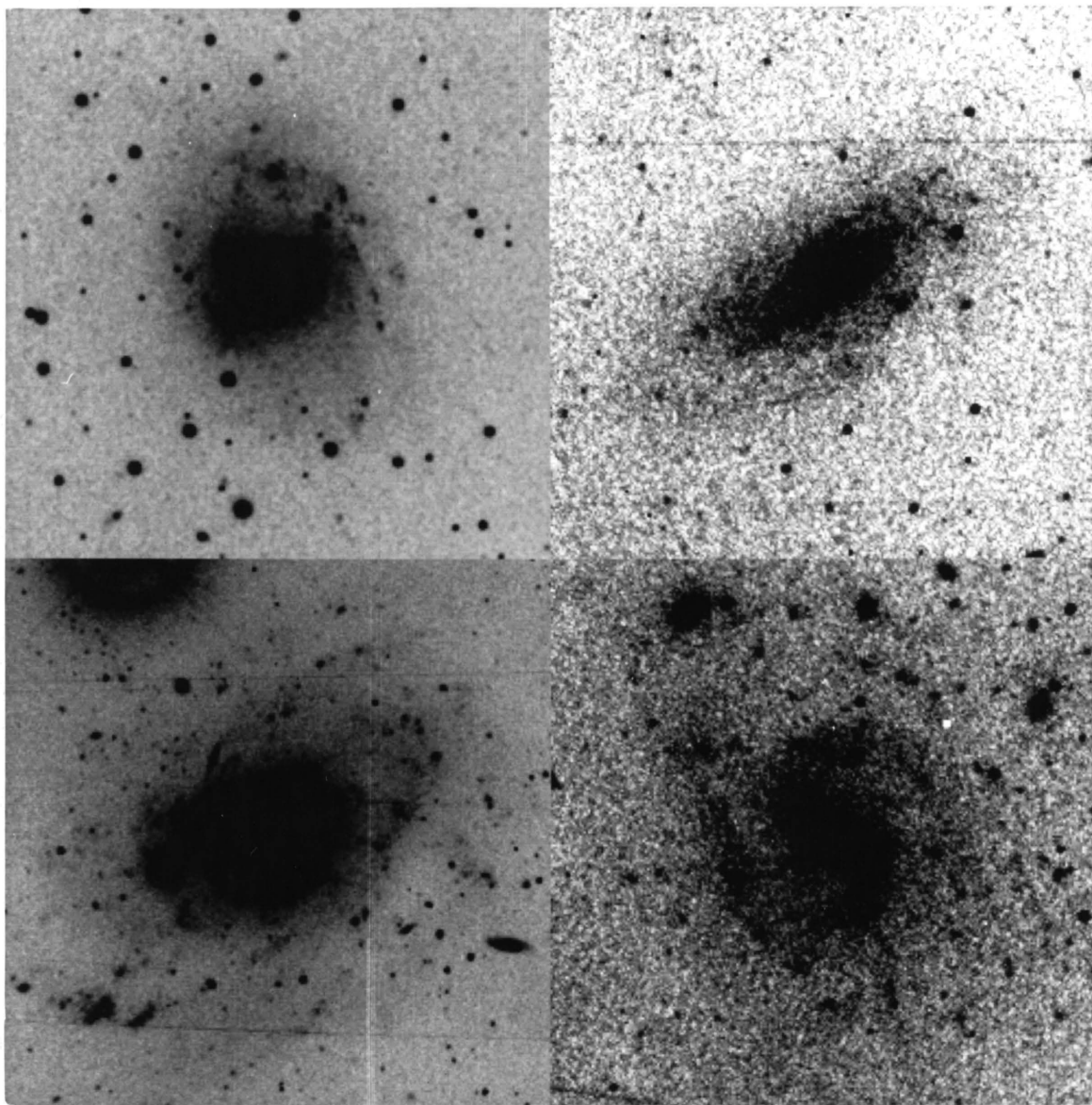


Figure 6. Continued.-- Photographs of LSB Galaxies

upp. left- UGC 11868 (R) , upp. right- UGC 04922 (R)
low. left- NGC 45 (R) , low. right- UGC A0333 (R)

level of the underlying disk by measuring the surface brightness at a number of points chosen to lie between the spiral arms. These points fell roughly on a straight line in the surface brightness versus radius plot, implying an exponential falloff in intensity, as expected for the disk component. We fit a straight line to the points in the surface brightness versus radius plot, to define the underlying disk intensity. By comparing the azimuthal luminosity profiles with the surface brightness of the disk, we could measure $A/(D+A)$, since the azimuthal profiles measure the sum of the light in the arms and disk. This assumes that all the light above the disk level is contributed by the arms. This assumption is probably valid for galaxies with well-developed arms, but becomes invalid as the galaxies become more irregular.

Another parameter measured by Schweizer (1976) is ΔP , the peak brightness of the arms above the disk, expressed as a magnitude difference. This is a measure of the peak-to-trough contrast in the wave pattern. We estimated this quantity for a few LSB spirals by measuring the surface brightness in a number of small areas along the crest of the wave and comparing this to the level of the underlying disk at the same radius.

The quantities $A/(D+A)$ and ΔP , measured for three LSB spirals with well or moderately well-developed spiral

arms, are listed in Table 16, along with the parameters for the four giant spirals measured by Schweizer (1976) which have U measurements. The values of $A/(D+A)$ listed for the LSB galaxies overlap those of the giant spirals. Thus, the LSB spirals can have wave amplitudes comparable to those of the giant spirals studied by Schweizer. However, from visual appearance, some of the LSB galaxies have significantly smaller wave amplitudes than those listed in Table 16.

At a given value of $A/(D+A)$, ΔP appears to be smaller in the LSB galaxies than in the spirals studied by Schweizer (1976). This would be consistent with the presence of only weak shock fronts in the gas in the LSB spirals, as discussed below. For a given amplitude of the broad spiral wave pattern, as measured by $A/(D+A)$, we expect a smaller ΔP for galaxies with weak shocks, as the gas compression and hence star formation rate would be lower.

Roberts et al. (1975) have found that the characteristics of the spiral density wave pattern depend primarily on two global properties of a galaxy: the degree of central concentration of the mass, and the total mass divided by the corotation radius, $\tilde{\omega}_c$, the radius where $\Omega(\tilde{\omega}) = \Omega_p$. Roberts et al. (1975) have studied a selection of 24 spiral galaxies with published rotation curves. For each galaxy in their sample, they estimated w_{10} , the shock velocity at

Table 16. Arm Strength Parameters

	A/(D+A)			ΔP (Magnitudes)		
	U	B	R	U	B	R
<u>LSBs:</u>						
NGC 3913	0.11		0.09	0.39		0.30
UGC 06983	0.37	0.28	0.19	0.79	0.67	0.50
NGC 4411A	0.53		0.34	0.83		0.66
<u>Schweizer Late-type Spirals:</u>						
NGC 3031	0.22	0.12	0.11		1.03	
NGC 5457	0.37	0.28	0.25		1.58	
NGC 4321	0.42	0.31	0.28		1.01	
NGC 4254	0.54	0.46	0.41		1.58	

one-half the corotation radius, as a function of $M/\tilde{\omega}_c$, where M is the total galactic mass, and $\tilde{\omega}_{0.5M}/\tilde{\omega}_c$, where $\tilde{\omega}_{0.5M}$ is the radius containing half of the total mass. The first step in estimating $w_{\perp 0}$ using their method is to determine $\tilde{\omega}_c$. They estimate $\tilde{\omega}_c$ from the maximum radial extent of three indicators: (1) the prominent spiral structure, (2) the "easily visible" disk, and (3) the distribution of HII regions. The radii of the "easily visible" disk listed for galaxies in Roberts et al. (1975) corresponds to about 0.8 times the de Vaucouleurs radius r_{dev} . The radii of the outermost HII regions in the LSB galaxies corresponds to about 1.2 r_{dev} . We have, therefore, adopted r_{dev} as a suitable and well-defined estimate of $\tilde{\omega}_c$ for our galaxies. The other parameter we need for this analysis is the total mass M . Most observed rotation curves do not exhibit a falloff after the initial rise, and thus the total mass is not defined. Roberts et al. (1975) use an analytic continuation of the observed rotation curves, based on the disk mass models of Toomre (1963), to compute a well-defined and finite galactic mass. This fit allows for a falling rotation curve after the peak observed velocity. We estimated the total mass of the LSB galaxies by extending their observed rotation curves from the last observed point to 1.2 Holmberg radii, using a falling velocity with slope equal to 0.3 times the maximum velocity

per Holmberg radii. The slope was estimated from the rotation curve for NGC 157 illustrated in Roberts et al. (1975). The outer integration limit was derived by applying our simple procedure to the rotation curve of NGC 4236 (Shostak 1973), and finding the radius where our mass matched that given by the Toomre (1963) rotation curve fit. The rotation curve of NGC 4236 is very similar in shape, maximum velocity, and observed radial extent to the observed rotation curves for the LSB galaxies. Table 17 lists the parameters for the three LSB galaxies with rotation curves (see Section 3.7) and two galaxies with similar parameters from Roberts et al. (1975), M 33 and NGC 4236. In both the important parameters, the central mass concentration and $M/\tilde{\omega}_c$, the LSB spirals lie between the corresponding parameters for M 33 and NGC 4236. The value of w_{10} for M 33 is 8.3 km/sec, for NGC 4236, 6.9 km/sec. The value of w_{10} for the LSB galaxies should also lie in this range.

Since the sound speed, a_s , probably lies in the range 7-12 km/sec, the value of w_{10} in the LSB galaxies is low. We would therefore expect only a weak shock front, if any, to form in the LSB galaxy spiral arms. Naively, we might try to associate the deficiency of massive stars, found in our HII region analysis, with the weakness of the shock. However, M 33, with a similar value of w_{10} , does appear to contain many more massive stars than the LSB

Table 17. Roberts, Roberts and Shu Parameters

	M ($10^{10} M_{\odot}$)	$\tilde{\omega}_c$ (kpc)	$\tilde{\omega}_{0.5M}$ (kpc)	$M/\tilde{\omega}_c$	$\tilde{\omega}_{0.5}/\tilde{\omega}_c$
NGC 4517A	1.8	9.7	10.0	0.19	1.0
UGC 06983	1.8	7.7	6.5	0.23	0.8
UGC 07557	1.8	5.9	5.5	0.31	0.9
M 33	1.3	2.8	3.5	0.46	1.2
NGC 4236	1.7	9.6	6.9	0.18	0.7

galaxies. This is indicated by the value of the average equivalent width of H β in the M 33 HII regions, which is over three times than in the LSB HII regions (Searle 1971). The density wave theory offers no simple explanation for the difference between the initial mass function in the LSB galaxies and a normal galaxy, M 33, which has a similarly low value of w_{10} .

3.5 Surface Mass Density

In this section, we compare the average surface mass density of LSB and normal late-type galaxies. Galaxies with differing surface mass densities are expected to evolve at different rates, as indicated in the models of Talbot and Arnett (1975). Their models show that galaxies with a low surface mass density should have a higher neutral hydrogen content and bluer colors than galaxies with a high surface mass density. We make no direct comparison between our data and the Talbot and Arnett models, as they assumed an IMF with $x = 1.35$, significantly different from the IMF we believe applicable in the LSB galaxies. The surface mass density is also of interest as a fundamental structural parameter of any disk galaxy. We expect the LSB galaxies to have a low average surface mass density, provided they have a normal mass-to-light ratio.

We calculated the average surface mass density ($\sigma_{0.5M}$) within the radius containing half the total mass as:

$$\sigma_{0.5M} = (1/2 M)/(\pi \tilde{\omega}_{0.5M}^2) \quad (26)$$

where $\tilde{\omega}_{0.5M}$ is the radius containing half the total mass M , as in Section 3.4. Values of $\sigma_{0.5M}$ are listed in Table 18 for the three LSB galaxies with rotation curves (Section 3.6) and all galaxies of type Scd or later from Roberts et al. (1975). As $\sigma_{0.5M}$ is distance-dependent, we corrected all distances for the normal galaxies to the Sandage-Tammann distance scale used for the LSB galaxies, using individual galaxy distances from Sandage and Tammann (1974a and 1974c). For the normal late-type galaxies, the average value of $\sigma_{0.5M}$ is $142 M_{\odot}/\text{pc}$ ($\sigma = 73$), while for the LSB galaxies the average is $64 M_{\odot}/\text{pc}$ ($\sigma = 33$). The normal z test (Section 3.1) shows the probability that the mean of $\sigma_{0.5M}$ for the two samples does not differ is less than 2%. The galaxy NGC 4236, one of the Roberts et al. galaxies, has a value of $\sigma_{0.5M}$ lower than the LSB galaxy average. This galaxy is an excellent example of a galaxy which would have been placed on the LSB observing list had it not been for its large angular size (diameter > 12 arcmin).

The surface mass density, along with the velocity dispersions in the stars and gas, controls the scale heights of the two mass components. For a given velocity dispersion, the lower the surface mass density, the larger the scale height. Thus the LSB galaxies, with their low average

Table 18. Surface Mass Density

	$\sigma_{0.5M} \text{ (M}_{\odot}/\text{pc}^2)$
<u>LSBs:</u>	
NGC 4517A	29
UGC 06983	68
UGC 07557	95
Average	64 ($\sigma = 33$)
<u>Late-type Spirals from Roberts et al. (1975):</u>	
NGC 598	142
NGC 925	56
NGC 2403	181
NGC 4236	57
NGC 4631	88
NGC 5457	186
NGC 6946	267
IC 342	161
Average	142 ($\sigma = 73$)

surface mass density, should have scale heights larger than normal galaxies. It would be very interesting to observationally compare the scale heights and velocity dispersions of the LSB and normal galaxies.

3.6 Neutral Hydrogen Content

One method of measuring the relative neutral hydrogen content of galaxies is to divide the total hydrogen mass, M_H , by some measure of the luminosity, L . Some authors derive luminosities from Holmberg magnitudes, others use de Vaucouleurs magnitudes. Different authors use different corrections to the observed luminosity to account for Galactic and internal extinction, in an attempt to derive a true, or at least a face-on, luminosity. The average M_H/L at a given Hubble type can vary by as much as 0.2 dex (60%) depending on the sample used and the procedure for deriving L (Sullivan and Johnson 1978). We compute luminosities using the de Vaucouleurs B_T magnitude system. We corrected our total B magnitudes for Galactic and internal extinction following the precepts of de Vaucouleurs et al. (1976). For the internal absorption correction, we assumed all galaxies were of type Sd. Any uncertainty caused by this will be negligible, as the galaxies are mostly face-on, so the internal absorption corrections are small. Table 19 lists the fully corrected B magnitude, B_T^O , and M_H/L_B for the LSB galaxies. Also listed

Table 19. Hydrogen Mass to Luminosity Ratio

	B_T^O	M_H/L_B (solar units)
NGC 45	11.13	1.14
UGC 04841	12.83	0.60
UGC 04922	13.50	0.65
NGC 3913	13.09	0.33
UGC 06922	13.97	0.56
UGC 06983	12.71	0.74
NGC 4411A	13.46	0.40
NGC 4411B	12.88	0.36
UGC 07557	13.28	0.87
NGC 4517A	12.55	0.68
DDO 144	13.24	0.46
DDO 142	12.51	0.62
DDO 146	12.60	0.41
NGC 5774	12.37	0.83
UGC 11868	13.68	0.45
LSB Average		0.61 ($\sigma = 0.22$)
Shostak (1978) Scd-Sm Average		0.37
de Vaucouleurs (1977) Scd-Sm Average		0.32

in Table 19 are the average value of M_H/L_B for galaxies of type Scd, Sd, Sdm and Sm from two sources: de Vaucouleurs (1977) and Shostak (1978). These authors use the B_T magnitude system, so our results are directly comparable to theirs. The ratio M_H/L_B is distance-independent.

The average M_H/L_B value for the LSB sample is about 1.8 times that of the other late-type galaxies. Using the Student t test (Langley 1971), we find the probability that the mean M_H/L_B for the LSB sample does not differ from the mean for the Shostak (1978) or de Vaucouleurs (1977) late-type samples is less than 0.2%.

The three galaxies judged from their luminosity profiles to have early Hubble types (Section 3.1) have high M_H/L_B . For their type as defined by their profiles, all three galaxies all hydrogen-rich in the sense used by de Vaucouleurs (1977): their M_H/L_B is greater than the average for their type by a factor of at least 3.2.

Another way of measuring the HI content of galaxies is to find the fraction of the total galactic mass, M_T , in the form of neutral hydrogen. This quantity is more fundamental than M_H/L_B , but the ambiguity in defining the total mass of a galaxy makes it more difficult to readily compare different galaxies using this quantity. We make limited use of this quantity, by calculating it only for the three LSB galaxies with masses determined from rotation curves using

our approximation to the Roberts et al. (1975) analysis (Section 3.4). Table 20 lists M_H , M_T and M_H/M_T for these three galaxies and for five late-type galaxies with masses from Roberts et al. (1975). The hydrogen masses for these five galaxies were taken from Rogstad and Shostak (1972).

The average value of M_H/M_T for the LSB galaxies is roughly twice that of the normal galaxies. However, the normal z test (Section 3.1) does not indicate a significant difference between the mean M_H/M_T for the LSB and normal galaxies.

3.7 Mass-to-light Ratio

The individual HII region velocities measured from the long-slit spectra (Section 2.6) were combined to form a rotation curve for each galaxy. The observed velocities (V_{obs}) were corrected for galaxy inclination and projection effects due to the misalignment of the spectrograph slit and the major axis of the galaxy. Both these corrections are incorporated in the following equation:

$$V(r) = (V_{\text{obs}} - V_{\text{sys}}) \frac{(\sec^2 i - \tan^2 i \cos^2 \theta)^{\frac{1}{2}}}{\sin i \cos \theta} \quad (27)$$

where i is the galaxy inclination, $V(r)$ is the corrected velocity relative to the systemic velocity of the galaxy (V_{sys}) and θ is the angle between the spectrograph slit and the major axis of the galaxy. We adopted the velocities

Table 20. Hydrogen Mass to Total Mass Ratio

	M_{HI} ($10^9 M_{\odot}$)	M_{T} ($10^{10} M_{\odot}$)	$M_{\text{HI}}/M_{\text{T}}$
<u>LSBs:</u>			
UGC 06983	4.8	1.8	0.27
UGC 07557	2.3	1.8	0.13
NGC 4517A	7.4	1.8	0.41
Average			0.27 ($\sigma = 0.14$)
<u>Normal Late-type Spirals:</u>			
M 33	1.64	1.3	0.13
NGC 2403	3.5	2.3	0.15
M 101	18.5	14.5	0.13
NGC 6946	21.2	11.5	0.18
IC 342	14.7	8.8	0.17
Average			0.15 ($\sigma = 0.02$)

determined from the 21-cm line as the systemic velocities. Systemic velocities estimated from the distribution of the individual velocity points agreed with the 21-cm velocities to within 10 km/sec.

The mass of the galaxy as a function of radius was obtained using a standard disk model (cf. Burbidge and Burbidge 1975). The Kuzmin integral was integrated by a combination of Simpson's rule and analytic integration. The computer program used was tested using the rotation curve for NGC 4378 from Rubin et al. (1978). The program results agreed precisely with the values of mass as a function of radius published in that paper.

The rotation curves are shown in Figures 7-9. The individual velocity points are plotted with different symbols for points on either side of the galaxy center. The solid line is the fit to the points used by the program which calculates the mass.

The local mass to blue luminosity ratio (M/L_B) for several annuli are listed in Table 21. Corrections for Galactic and internal absorption were made using the procedures of Rubin, Ford and Thonnard (1978). The local M/L_B increases substantially from the inner kiloparsec outwards in all three galaxies. This phenomena has been observed in several other galaxies for which detailed rotation curves and surface photometry exist. These include NGC 4378

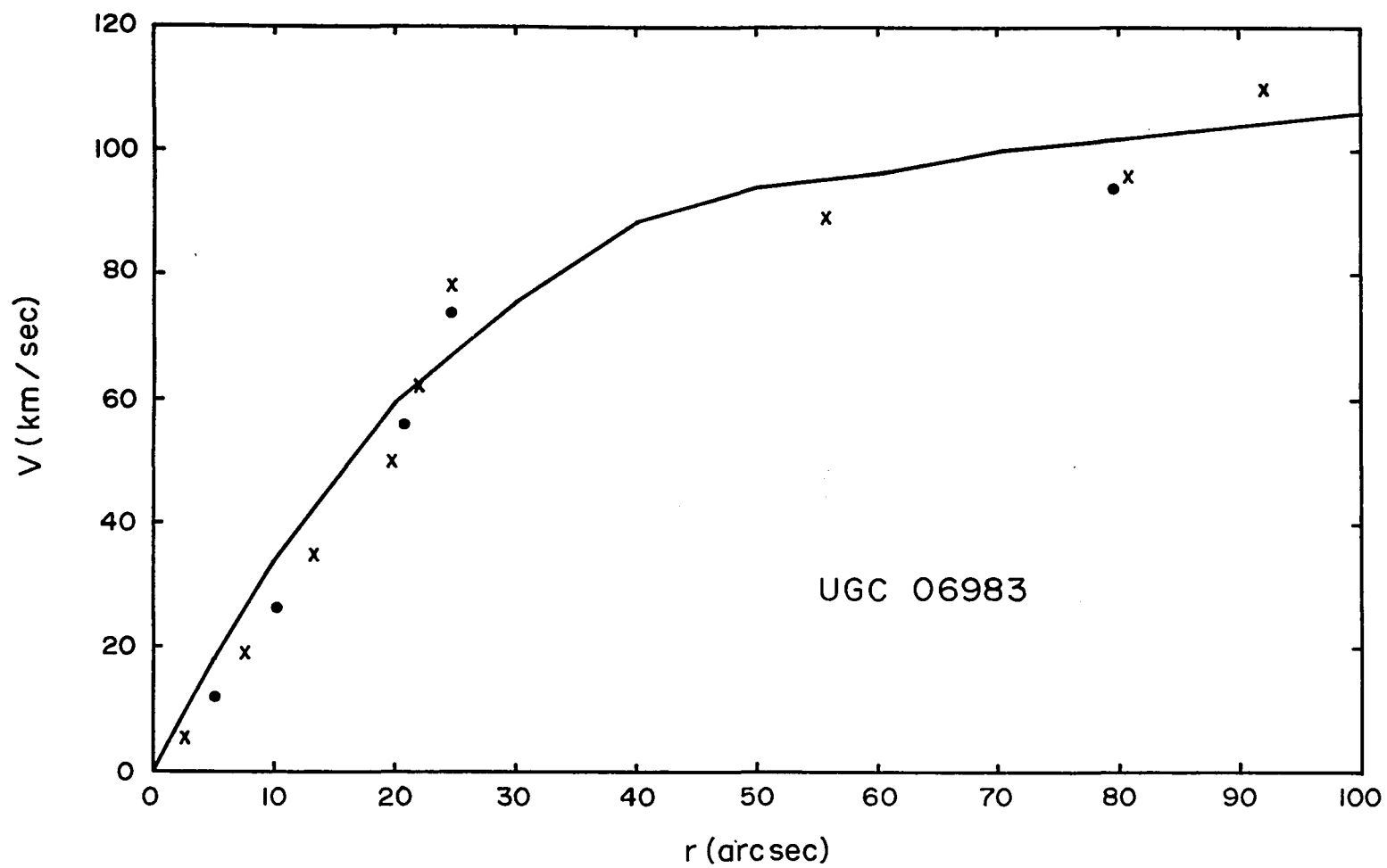


Figure 7. Rotation Curve for UGC 06983.

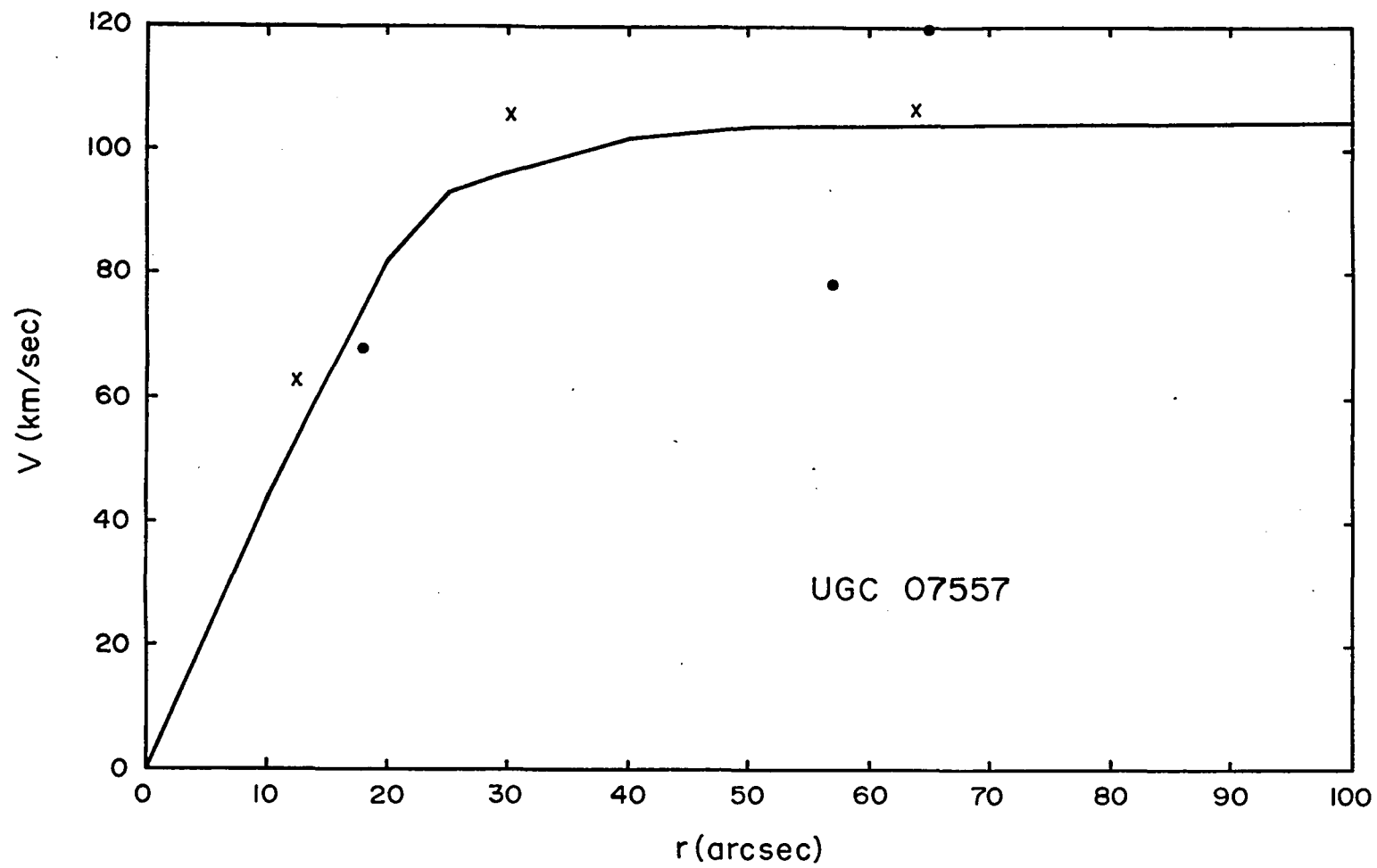


Figure 8. Rotation Curve for UGC 07557.

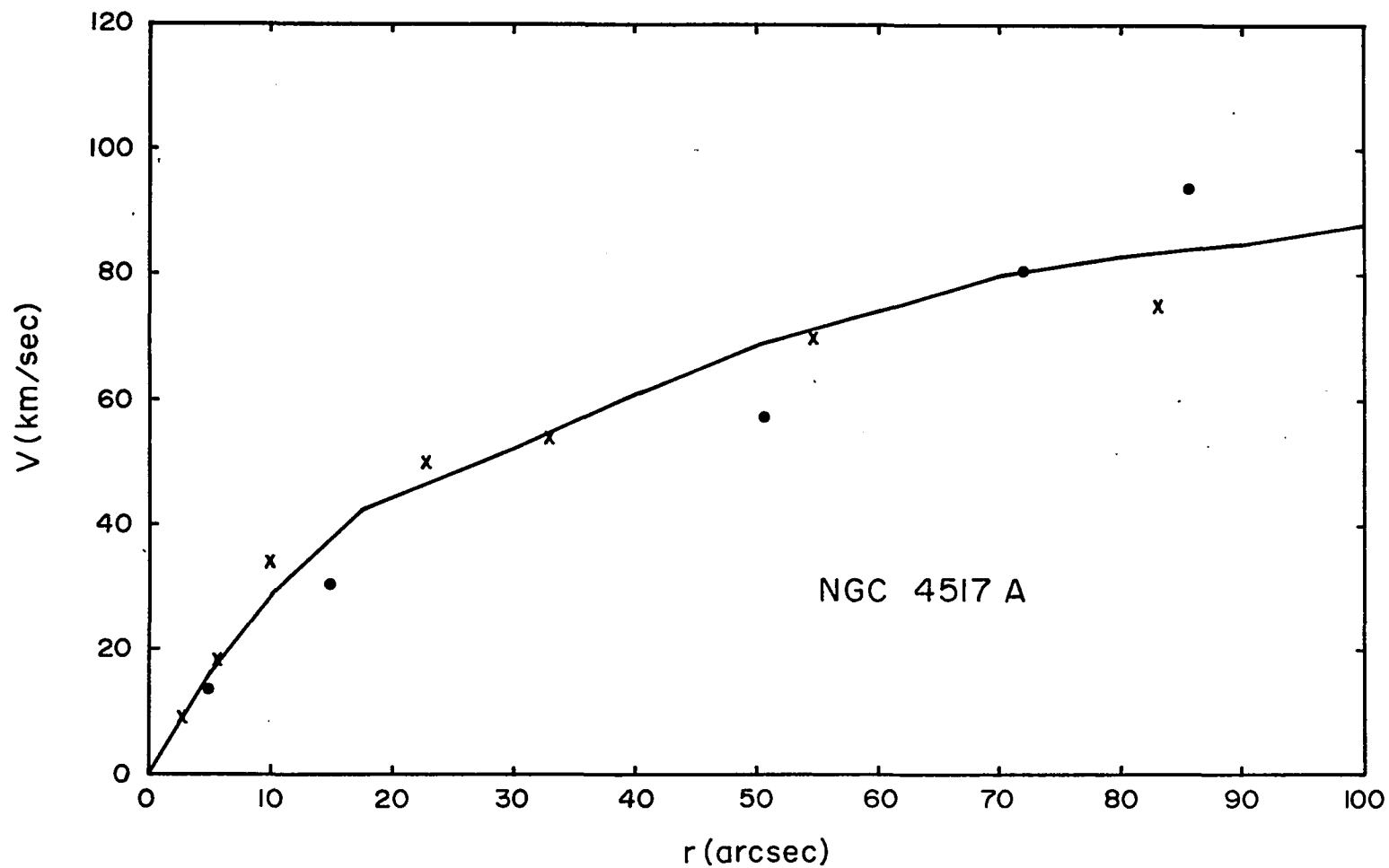


Figure 9. Rotation Curve for NGC 4517A.

Table 21. Annular M/L_B Measurements

Annulus (kpc)	M/L_B (solar units)		
	UGC 06983	UGC 07557	NGC 4517A
0-1.1	1.0	2.4	0.4
1.1-2.1	3.4	8.3	0.9
2.1-4.1	5.6	9.3	1.5
4.1-6.1	6.8	8.3	2.3
6.1-8.1	8.0		2.8
8.1-10.1			3.3

(Rubin et al. 1978), NGC 7217 (Peterson et al. 1978) and NGC 4594 (Schweizer 1978). These three galaxies are all early-type spirals of large mass. The three LSB galaxies are of late type. Our observations suggest that the increase of M/L from the inner kiloparsec to the outer regions is not associated with the presence of a dominant bulge component. Either this increase in M/L is a general property of spiral galaxies or a similar error exists in the interpretation of rotation curves as a mass indicator for both early and late spirals.

The integrated values of M/L_B within the de Vaucouleurs radii (r_{dev}) for UGC 06983, UGC 07557 and NGC 4517A are 5.9, 8.1, and 2.3, respectively, for an average of 5.4 ($\sigma = 2.9$). The average M/L_B within r_{dev} for a sample of eleven spirals observed by Rubin et al. (1978) is 3.7 ($\sigma = 1.9$). The average for the five galaxies of latest type (Sbc-c or Sc) observed by Rubin et al. is 2.6 ($\sigma = 0.8$). Only one of their galaxies (NGC 4378) definitely has M/L_B greater than UGC 06983 and UGC 07557. The normal z test (Section 3.1) shows no significant difference between the average M/L_B for the LSB galaxies and either the entire Rubin et al. sample or only the five latest galaxies. Thus, the very limited nature of our sample makes it impossible to say whether the average M/L_B for the LSB galaxies is higher than for normal spirals. We note that the mass of

UGC 07557 may be in error because of uncertain inclination corrections as it is close to face-on ($b/a = 0.8$).

We can also study the masses of the LSB spirals using their 21-cm line widths. Masses determined from 21-cm line widths depend on assumptions made concerning the shape and extent of the rotation curve. We follow the simple procedure of Faber and Gallagher (1979). They calculate the mass (M_{Ho}) within the Holmberg radius as simply:

$$M_{\text{Ho}} = R_{\text{Ho}} V_{\text{Ho}}^2 / G \quad (28)$$

where V_{Ho} is the rotational velocity at the Holmberg radius R_{Ho} . For galaxies with 21-cm line widths, they calculate V_{Ho} from V_{max} , the velocity width corrected for inclination, as:

$$V_{\text{Ho}}^2 = V_{\text{max}}^2 / 1.25 \quad (29)$$

We calculated M_{Ho} in this way for all LSB galaxies with $b/a \leq 0.86$ ($i \leq 31^\circ$). We calculated L_B in the Faber and Gallagher system, using B_T magnitudes corrected for internal absorption as in the RC2 (see Section 3.5) and the Faber and Gallagher Galactic absorption correction. The resulting M/L_B ratios are listed in Table 22. The average value is 5.7 ($\sigma = 2.1$). This value is almost identical to the average value Faber and Gallagher derive from 21-cm line widths for Scd galaxies (5.8). However, they derive a

Table 22. M/L_B from 21-cm Widths

	M/L_B (solar units)
NGC 45	9.5
UGC 04841	7.3
UGC 04922	6.2
UGC 06983	6.8
UGC 07557	8.0
NGC 4517A	2.9
DDO 144	4.9
DDO 142	4.4
DDO 146	4.1
NGC 5774	2.3
UGC 11868	5.3
UGC 06922	7.0
Average	5.7 ($\sigma = 2.1$)

smaller value, 3.1, for Sdm galaxies. It seems unlikely that the actual M/L_B ratio changes by a factor of 2 from type Scd to Sdm. The variation in M/L_B with type at these late types probably reflects more on uncertainties in measuring M/L_B using this technique. The average 21-cm M/L_B for Sc, Scd and Sdm galaxies derived by Faber and Gallagher is 4.6. We find no statistically significant difference between this mean M/L_B and the LSB data listed in Table 22, using the Student t test (Langley 1971).

As discussed in Section 2.7, our 21-cm line widths may be slightly underestimated. If we assume our line widths are too small by 10% and make an appropriate correction, the LSB average M/L_B becomes 6.8 ($\sigma = 2.5$). The Student t test shows that the probability that this LSB M/L_B average and the Faber and Gallagher Sc, Scd and Sdm M/L_B average do not differ is about 2%. Another factor, which would tend to work in the opposite direction, is the bias towards face-on galaxies in the LSB sample. The 21-cm line widths of nearly face-on galaxies includes a substantial contribution from noncircular motion. This is shown by the substantial line widths (40-80 km/sec) measured for the face-on galaxies NGC 3913, 4411A and 4411B. Without a knowledge of the velocity dispersion perpendicular to the plane (ΔV_z) we cannot accurately evaluate this contribution to the observed line width. A very crude correction for

this effect can be estimated by assuming the actual circular velocity width, ΔV_c , can be derived from the measured velocity width, ΔV_{obs} , and ΔV_z using the following simple relation:

$$\Delta V_c^2 = (\Delta V_{\text{obs}}^2 - \Delta V_z^2)^{\frac{1}{2}} \quad (30)$$

With $\Delta V_z = 40$ km/sec, the average M/L_B ratio for the LSB sample is lowered by about 10%. After making this correction for face-on bias, we find, using the Student t test, no significant difference between the LSB and Sc, Scd and Sdm average M/L_B , whether or not we make the correction for possible line width underestimation.

Thus, we find no conclusive evidence for a difference in M/L between the LSB and normal galaxies. There is a suggestion that the M/L ratio of the LSB spirals may be higher than that of normal spirals of late type, by no more than about 50%. However, the scatter in the values of M/L for both LSB and normal galaxies is large.

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

This work has presented a large quantity of varied observational material on a number of low surface brightness spiral galaxies. The following form the primary conclusions of this work:

1. There exist intrinsically bright spiral galaxies with average surface luminosities about 1/3 that of normal spirals. These galaxies have extrapolated central disk surface brightnesses nearly a factor of 3 fainter than the value found by Freeman (1970) to characterize the majority of spirals. These systems have M_B between -17 and -20. Thus they are brighter than dwarf galaxies.

2. The primary reason these galaxies appear to have low optical surface brightness disks is because they have low average surface mass densities. A higher than normal M/L ratio may also contribute to their low surface brightness. However, with the available data, we cannot definitely tell if the average LSB M/L ratio is significantly higher than normal.

3. The emission-line spectra of the LSB galaxy HII regions suggest a lack of massive stars in their ionizing clusters, compared to HII regions in galaxies such as M 33 and M 101. These spectra provide only an upper limit for the average metal abundance in the gas. The unusual radiation field prevents a firm estimate of the metal abundance.

4. The broad band colors (both the integrated infrared colors and the optical colors of the old disk population) indicate a stellar population of low metal abundance. The estimated metal abundance of a typical LSB spiral is on the order of $1/5$ solar.

5. The spiral density wave theory as presented in Roberts et al. (1975) can perhaps explain the weak shocks and low star formation efficiency (as evidenced by the high M_H/L ratio) in the LSB galaxies. This theory does not offer any simple explanation of the unusual mass function of the massive stars.

Perhaps the most interesting properties of the LSB galaxies are the apparent deficiency of massive stars and the low metal abundance in the stars. The low metal abundance is probably the consequence of the low proportion of massive stars, rather than a consequence of a high gas/star ratio. Although the HI mass fraction is relatively large, the mass of HI still comprises less than half the total mass. In a simple galactic evolution model, (Tinsley

1976), the metallicity of the gas depends only on the logarithm of the inverse of the gas mass fraction. If we assume a HI mass fraction of 25% for the LSB galaxies, and a value half as large for normal late-type galaxies, the difference in metallicity between LSB and normal late spirals predicted by the simple model is only about 30%. The LSB galaxies appear to have an overall metal abundance a factor of two or more lower than normal spirals of late Hubble type. To account for this low metallicity, we are forced to postulate a lower than normal yield of heavy elements. This would be a natural result of an initial mass function with a deficiency of massive stars.

We can make a very crude estimate of the effect of varying the IMF on the average metal abundance of a galaxy by calculating the amount of mass ejected by massive stars, as a fraction of the total mass forming stars, for various IMFs. This should give a crude idea of the relative metal abundance for galaxies with different IMFs, as this ejected gas enriches the interstellar medium with metals. In this context, massive stars are those which recycle enriched matter into the interstellar medium, say stars with masses $2 M_{\odot}$ and larger. We calculated this ejected mass fraction assuming all massive stars leave a $1 M_{\odot}$ remnant. This calculation was performed for two of the IMFs used in Section 3.3, the Miller and Scalo (1979) IMF (model A), and

the steep massive star IMF (model C). The ejected mass fraction will depend somewhat on the details of the low-mass IMF, as changing this changes the proportion of mass locked up in non-evolving stars. Also, the metallicity of the ejected gas will vary with initial stellar mass. We are encouraged that the predicted metallicity ratio, estimated using IMFs we believe appropriate for the normal and LSB galaxies, is similar to the metallicity ratio inferred from our observations.

Our data suggest that the main difference in the evolution of the LSB and normal galaxies is in the massive star IMF, with the LSB galaxies having fewer massive stars. Kormendy (1977b) has discussed another type of galaxy, the filamentary arm spiral, which may be similar to the LSB galaxies in this respect. The filamentary arm spirals, such as NGC 2841, have many short spiral filaments, but lack global spiral structure. Kormendy cites two members of this class as having HII regions much fainter than expected. He speculates that these galaxies may be deficient in massive stars because of a lack of density wave driven star formation. These galaxies have disks of normal surface brightness. Perhaps a strong shock, absent in both LSB and filamentary arm spirals, is needed to make massive stars efficiently. Unfortunately, as pointed out in Section 3.4, the lack of massive stars probably cannot be explained quite

so easily. M 33, a galaxy predicted to have weak shocks, does have an IMF with a larger proportion of massive stars than the LSB galaxies.

As for future observational work, perhaps the most immediate need is to investigate whether factors other than the lack of massive stars may be causing the low $W(H\beta)$ in the LSB HII regions. Observations of a number of HII regions of low absolute luminosity in a normal galaxy such as M 101 should help in disentangling any effects of age or absolute luminosity of the HII region from effects of the IMF. Spectra and colors of HII regions in filamentary arm spirals should also be obtained to determine how they compare with the LSB HII regions.

Also needed are observations of emission lines in the LSB HII regions sensitive to the stellar ionizing temperature and the electron temperature of the gas, such as the He II λ 5876 line or the [OIII] λ 4363 line. These observations will allow measurement of the chemical abundances in the ionized gas in the LSB HII regions. We expect these lines to be very weak. Perhaps the best place to measure these lines will be in the HII regions of NGC 4236, as it is relatively nearby. First, of course, we must verify that the HII regions in NGC 4236 have spectra similar to those of the galaxies in the present sample.

It would be very useful to have a totally independent check on our conclusion that there are relatively few massive stars in the LSB galaxies. Observations of the equivalent width of $H\alpha$ integrated over an entire galaxy might provide such a check. Cohen (1976) has presented such $W(H\alpha)$ data for a number of normal galaxies, finding a decrease of $W(H\alpha)$ with increasing $(B-V)$. She attributes this decrease to the decreasing amount of light emitted by the hottest O stars for galaxies with redder colors. Our interpretation of the LSB colors implies their low metallicity makes them bluer than expected for the number of massive stars they contain. Thus, we expect a lower $W(H\alpha)$ at a given $(B-V)$ for the LSB galaxies.

The study of edge-on LSB galaxies should help elucidate any effects of the scale height of the stars and gas on the evolution of these galaxies. The obvious problem is identifying edge-on LSB galaxies. These systems might be identified from their size/infrared magnitude ratio. If any nearby edge-on LSB galaxies can be found, we can measure the scale height of the neutral gas with the Very Large Array observing in the 21-cm line.

There are several 21-cm observations of face-on galaxies that would be of great interest. One is the measurement of the spiral arm peak-to-trough density contrast in the gas. If the gas scale height is large, perhaps the gas

is not as effected by the density wave as in a normal galaxy. This could be determined by comparing the peak-to-trough arm contrast in the gas with the arm contrast in the stars (Section 3.4). In face-on galaxies, measurement of the velocity dispersion in the gas with high velocity resolution would be useful. This, along with an estimate of the disk surface mass density, would make possible an estimate of the gas scale height.

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