

## POLARIZATION IN REFLECTION NEBULAE

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I hereby recommend that this dissertation prepared under my direction by BENJAMIN H. ZELLNER III
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## ABSTRACT

Photoelectric measurements of color and polarization in seven reflection nebulae were made between November 1967 and December 1969. Detailed observations were made in NGC 2068, IC 5076, and NGC 7023 with the $154-\mathrm{cm}$ Catalina reflector and the $229-\mathrm{cm}$ Steward reflector. The observations were restricted to one or two regions in each nebu1a in order to attain higher precision and wider wavelength coverage than would have been possible in a broader survey. Six filters were used, ranging from $0.33 \mu$ to $0.83 \mu$, and probable errors were on the order of $\pm 0.3 \%$ in polarization and $\pm 0.03$ in color. Smaller amounts of data were obtained in the nebulae Cederblad 201, NGC 2245, IC 4601b, and IC 4603.

Equations were derived for calculations of the intensity, color, and polarization of light emerging from a reflection nebula. Exact Mie-scattering computations were made for homogeneous spherical grains with a wide variety of refractive indices and in two types of size distributions. The star-cloud geometry and the scale radii of the size distributions were left as free parameters to be determined by comparisons with the observations. A strict treatment of transmission losses and multiple scattering could not be made.

Polarizations in NGC 2068 and NGC 7023 increase smoothly toward longer wavelength, but in IC 5076 the polarization peaks in visible light. These wavelength dependencies were used to define the scattering geometry; in NGC 2068 and NGC 7023 the illuminating star is behind the
visible nebulosity, but in IC 5076 the star is in front of the dust cloud. In one region of NGC 2068 the polarization position angle departs significantly, at short wavelengths, from symmetry with respect to the plane of scattering; this rotation is probably attributable to the effects of multif 'e scattering. The polarization of the illuminating star HD 38563B in NGC 2068 drops steeply toward shorter wavelengths, unlike the wavelength dependence found on any other star.

No agreement between theory and observation was found for metallic or graphitic grains. For dielectric grains, good fits to the polarization data were found in all three nebulae. A detailed fit to both color and polarization data was possible only for IC 5076; the colors in NGC 2068 are distorted by irregular foreground extinction, and those in NGC 7023 by high optical depth.

Equally good agreement between theory and observation is found for ice or silicate grains, provided the imaginary component of the refractive index is less than 0.05 for ices or less than 0.10 for silicates. The grain sizes derived in this study vary only about $25 \%$ from nebula to nebula, but are a factor of two or three smaller than the values which give the best agreement with interstellar extinction and polarization data.

## CHAPTER I

## INTRODUCTION

Most of our information about the interstellar grains has come from studies of the wavelength dependence of interstellar extinction, and theories as to their nature have required revision whenever a new region of the spectrum has been opened up for such studies. The grains produce at least three other observable effects, each of them more difficult to observe and to interpret than the interstellar extinction. In order of increasing astrophysical neglect they are interstellar polarization, the diffuse galactic light, and reflection nebulae.

In the case of interstellar extinction and polarization the grains are seen only at zero scattering angle; that is, the light that we see was not deviated in direction by its encounter with the grains. But in the diffuse galactic light and in reflection nebulae we see grains illuminated from an angle to the line of sight. This angular scattering gives additional observational constraints on the nature of the grains, an advantage offset by the fact that the geometry of illumination is a priori unknown. Also, in reflection nebulae we are able to sample a relatively tiny volume of interstellar space, whereas other types of data are averaged over tens or hundreds of parsecs.

In the following sections I will briefly discuss previous studies of interstellar grains and of reflection nebulae, with emphasis on recent developments and on results which will be needed in
subsequent chapters. No attempt will be made to give a complete historical review.

### 1.1 Interstellar Grains

Summaries of the known observational and theoretical properties of interstellar grains have recently been given by Wickramasinghe (1967), Greenberg (1968), and Lynds and Wickramasinghe (1968). A concise but thorough historical review has been given by Shah (1967).

Metallic grains, characterized by a refractive index $\mathrm{m}^{*}=$ $m^{\prime}-i m^{\prime \prime}$ with $m^{\prime} \sim m^{\prime \prime}$ (Table 10 of Greenberg 1968), were one of the first types to be proposed (Schalén 1939). With sizes on the order of $0.01 \mu$, they can explain the linear part of the interstellar extinction curve reasonably well. Today such grains are usually dismissed on the grounds that, in order to provide the observed opacity of the galaxy, the abundance of metals in the interstellar medium would have to be anomalously high compared with their solar abundance.

The theory that the interstellar grains are ices condensed from the gaseous component of the interstellar medium has survived for more than three decades. The chemical composition of ice grains has generally been taken to be about $60 \% \mathrm{H}_{2} \mathrm{O}$ ice with smaller amounts of solid $\mathrm{H}_{2}, \mathrm{CH}_{4}$, and $\mathrm{NH}_{4}$, and a trace of metals and metallic hydrides. The real part of the refractive index would probably fall between 1.25 and 1.35, with an imaginary component not exceeding 0.05 (Greenberg 1968, page 249).

Such "dirty ice" grains of a single radius a $\sim 0.3 \mu$ give a fair fit to the visible part of the interstellar extinction curve. For
grains condensed from the interstellar gas, two rather similar types of size distributions have been proposed on physical grounds. Oort and van de Hulst (1946) derived an equilibrium distribution of grain radii by considering the detailed mechanisms of grain growth by accretion and destruction by evaporation in cloud-cloud collisions. They tabulated a distribution which has been fitted analytically by Greenberg (1968) with a function of the form

$$
f(a)=\exp -5\left[a / a_{0}\right]^{3} .
$$

I will refer to this size distribution as the OHG (Oort-van de HulstGreenberg) distribution. The best fit to the visible part of the interstellar extinction curve is found with a scale radius $a_{0} \simeq 0.5 \mu$. Wickramasinghe (1967) showed that a size distribution similar to that above is a consequence of the particular destruction mechanism used by Oort and van de Hulst, which gives a destruction probability per unit time which increases roughly as the cube of the grain radius. But if the dominant destruction mechanism is sputtering in $H^{I I}$ regions, the probability of destruction is independent of grain radius (essentially all ice grains in a volume of space are destroyed) and the resulting size distribution is of the form

$$
f(a)=\exp -\left[a / a_{0}\right] .
$$

A slightly different form of this distribution, which I shall refer to as the EXP distribution, will be defined in Section 2.1.3.

The classical ( $\mathrm{a}_{0}=0.5 \mu$ ) Oort-van de Hulst distribution of ice grains predicts an extinction curve which rises to a peak at about $1 / \lambda=3.5 \mu^{-1}$ and then drops toward larger wavenumbers. Data in the rocket ultraviolet, which began to beconc available about six years ago (e.g., Boggess and Borgman 1964, Stecher 1965) show that the interstellar extinction continues to increase monotonically to $1 / \lambda=7 \mu^{-1}$ and beyond except for a small hump near $1 / \lambda=4.4$. It has been generally assumed that no size distribution of ice grains can explain the extinction data throughout the wavelength range now accessible. However Greenberg and Shah (1969) have found a distribution (Section 4.5) of cylindrical, perfectly oriented ice grains which fits the observed data from $1 / \lambda=1.0$ to at least $1 / \lambda=6.0$, except for the small peak near $1 / \lambda=4.4$, and also explains the observed wavelength dependence of interstellar polarization.

Graphite flakes as a constituent of the interstellar medium were proposed by Cayrel and Shatzman (1954) on the grounds that their highly anisotropic electromagnetic properties might be necessary to account for the observed degree of interstellar polarization. Hoyle and Wickramasinghe (1962) suggested that graphite particles of sizes appropriate to explain the interstellar extinction may be able to condense in the atmospheres of certain cool stars and be ejected into space by radiation pressure. No size distribution has been derived for grains so formed; however Stecher and Donn (1965) found that an OHG-like distribution of graphite particles with mean radius $0.028 \mu$ fits the extinction curve rather well out to $1 / \lambda \sim 5$ but not beyond.

The hump at $1 / \lambda=4.4$ can be identified with an electronic transition in graphite crystals, providing rather strong evidence that graphite is present.

The extinction curve can be well matched throughout its range (Section 10.6 of Wickramasinghe 1967) by graphite grains of radius $0.05 \mu$ which have accreted mantles of dirty ices of outer radius $0.16 \mu$, and such composite grains have been very fashionable since about 1965. Coyne and Wickramasinghe (1969) have made semi-quantitative calculations of the wavelength dependence of interstellar polarization for composite grains with graphite-flake cores of radius $0.05 \mu$ and spheroidal ice mantles of largest radius $0.3 \mu$. The variety of observed polarization curves are explained if the ratio of major to minor mantle axes ranges from 0.7 to 0.9 . That is, given an effective alignment mechanism, the grains do not need to depart very much from spherical shape to explain the interstellar polarization. The "graphite hump" in the extinction curve at $1 / \lambda=4.4$ is not preserved with such a thick coating of dirty ice, however. For this reason among others it has been proposed (Wickramasinghe and Reddish 1968, Wickramasinghe and Krishna Swamy 1968) that graphite cores might accrete mantles of solid molecular hydrogen; such grains can explain the whole extinction curve including the graphite hump. Solid $\mathrm{I}_{2}$ has a refractive index which is closer to unity than for most other solid materials, and which is strongly anisotropic. Wickramasinghe and Krishna Swamy suggested that $m^{\prime}$ in the range 1.05 to 1.10 and $\mathrm{m}^{\prime \prime}$ in the range zero to 0.05 would be appropriate for isotropic-sphere Mic calculations.

A grain temperature $<7^{\circ} \mathrm{K}$, which would probably be found inside thick dust clouds, is necessary for the accretion of solid $\mathrm{H}_{2}$. There is some doubt, however (Wickramasinghe 1969), whether $\mathrm{H}_{2}$ mantles could survive in interstellar space; a pure graphite grain exposed to starlight would take up a temperature $\dot{\sim} 40^{\circ} \mathrm{K}$, and even ice mantles might be unstable. Hence composite grains with graphite cores may be of only historical interest.

An old theory due to Platt (1956) that the interstellar medium contains not grains but large, quantum-mechanically scattering molecules was revived by Witt (1968) to explain his new measurements of the intensity and distribution of the diffuse galactic light. Witt's interpretation of his data required scatterers with albedo close to unity and a phase function that is much more nearly isotropic than can be obtained with Mie-scattering grains. His analysis depended upon the Eddington approximation for solution of the radiative transfer problem; an exact solution by van de Hulst and de Jong (1969) led to quite different results. They found that Witt's observations, and older ones of Henyey and Greenstein (1941) are commensurate with dirty ice or icecoated graphite grains, but not with Platt particles or pure graphite flakes. Polarization measurements of the diffuse galactic light would be of great value; the brightness and polarization of the Zodiacal light, however, will probably require that such data be taken from deep-space probes.

Silicate grains came on the scene about a year ago. Whereas graphite grains are expected to condense in the atmospheres of
carbon-rich stars at temperatures below $2000^{\circ} \mathrm{K}$, oxygen-rich stars should provide (Gilman 1969) silicate grains (e.g., $\mathrm{Al}_{2} \mathrm{SiO}_{5}, \mathrm{Mg}_{2} \mathrm{SiO}_{4}$ ) at temperatures below about $1700^{\circ} \mathrm{K}$. For stars with approximately cqual amounts of oxygen and carbon, the most abundant condensate is expected to be silicon carbide. Broad absorption features in the spectrum of 119 Tauri have been identified (Knacke, Gaustad, Gillett, and Stein 1969) with $(\mathrm{Mg}, \mathrm{Fe}) \mathrm{SiO}_{3}$, and infrared excesses in the spectra of several cool stars have the right shape (Woolf and Ney 1969) to be caused by silicate emissions. No size distributions have been derived for such grains, and there is no consensus as to what refractive index would be most appropriate. Dr. N. J. Woolf (1969, personal communication) suggested $m^{*}=1.65-0.15 i$, but Dr. J. M. Greenberg (1969, letter to Dr. Woolf) pointed out that for olivine the imaginary component is known to be negligible. Wickramasinghe (1969) has found good fits to the entire extinction curve for mixtures of graphite and silicate grains, or graphite grains with mantles of ice or solid $H_{2}$, using $m^{*}=1.66$ for the silicate component.

Searches for the fundamental vibrational absorption band of water ice at $3.07 \mu$ have been unsuccessful (Danielson, Woolf, and Gaustad 1965; Knacke, Cudaback, and Gaustad 1969), providing strong evidence that the interstellar grains contain little ice. Gaustad (1970) has pointed out that forms of such common substances as water and carbon have recently been discovered which have optical properties quite different from the ordinary forms.

Thus there is no lack of models for the interstellar grains. Similarly, there is no paucity of new observational data on the extinction curve in the far ultraviolet. Carruthers (1969) found that for large values of $1 / \lambda$ the extinction of 0 Orionis drops several magnitudes rather than increasing monotonically according to the pattern seen in older rocket data. A large body of data from the OAO Spacecraft (Code 1970a) has shown that for most stars the extinction curve reaches a maximum at $1 / \lambda=5.5$ to 6.0 , and some striking differences from star to star are seen in the far ultraviolet. Code (1970b) has suggested that the more complex curves may require a mixture of $\mathrm{MgSiO}_{3}$ dominating the extinction in the far UV, graphite dominating the near UV, and silicon carbide dominating at longer wavelengths.

The field of interstellar grain studies is a very active one, and models which were introduced in recent years, though seemingly definitive at the time, have tended to be short-lived. The true nature of the grains may be quite different from anything yet proposed.

### 1.2 Reflection Nebulae

As in the section above, I will not attempt a complete historical review of astronomical literature on reflection nebulae. This has been well done by Johnson (1968) for pre-1966 literature, and more recently by Hanner (1969). I will only discuss briefly the more significant quantitative studies which have been made. Reflection nebulae have been a favorite topic for Soviet astronomers (e.g., Dombrovsky 1958; Rozhkovsky 1960, 1969; Parsamian 1963; Minin 1964, 1967;

Glushkov 1965; Kurchakov 1968). Their work will inevitably be somewhat neglected because of the language barrier.

### 1.2.1 Observations

The relevant observations in one region of a reflection nebula are the polarization as a function of wavelength and the intrinsic color, i.e., the ratio of nebular brightness to the brightness of the illuminating star as a function of wavelength. In surface brightness relection nebulae are usually comparable to or fainter than the night sky, and photographic studies have been notoriously subject to systematic errors. The first photoelectric data became available about twelve years ago, with photometry and polarimetry in several nebulae by Martel (1958) and Johnson (1960), and polarimetry in NGC 7023 at three widely spaced wavelengths by Gehrels (1960). In these studies it was established that reflection nebulae are generally bluer than their illuminating stars, by up to a magnitude in $\underline{B}-\underline{V}$, and that the polarization generally increases monotonically toward longer wavelengths.

More recently Vanýsek and Svatos (1964) reported colorimetry in NGC 7023 by scans through the nebula, and 0'Dell (1965) measured brightnesses and colors in the Merope nebula in four narrow-band filters between $0.34 \mu$ and $0.56 \mu$. Rather complete areal coverage of polarization and color in three filters was provided by R. C. Hall (1965) in NGC 2261, and by A. Elvius and J. H. Hall $(1966,1967)$ in the Merope nebula, NGC 7023, and NGC 2068. Additional measures in
the Pleiades nebulosities have been provided by Roark (1966), Dahn (1967), and Artamonov and Efimov (1967), and 0'Dell (1969).

Some representative results are plotted in Figures 1-1 (color) and 1-2 (polarization). All data are plotted at the same scale in each figure. (My new measurements are here included only for purposes of illustration, and in some cases the figures were drawn from preliminary data; improved data will be given in Chapter 3). The color data are plotted so that a line rising to the right represents a blue intrinsic nebular color. It can be seen that intrinsic colors vary widely, from extremely blue in NGC 2068, through neutral in one region of NGC 7023, to slightly red in a region of the Merope nebula. With the single exception of IC 5076, polarizations invariably increase toward longer wavelength. This property will be used in Chapters 2 and 4 to rule out certain types of grains.

For an element of a dust cloud illuminated by a single star, the plane of scattering is the plane containing the star, the scattering element, and the observer. When spherical, isotropic grains or grains of arbitrary shape but random orientation are illuminated by unpolarized light, symmetry requires that the plane of polarization (strongest electric vector) of the scattered light either coincides with the plane of scattering, or lies perpendicular to it. Projected against the plane of the sky, all polarizations should then have position angles that show a radial symmetry about the illuminating star, either along lines drawn from the star to the nebular regions (negative polarization), or perpendicular to such lines (positive polarization).


Figure 1-1. Color Data in Four Nebulae


Figure 1-2. Polarizations in Five Reflection Nebulae

A well-established case of negative polatization has never
been found, though predicted (Chapter 2) in many models. Substantial deviations from perfect positive symmetry, however, are found in Martel's data for several nebulae and in the data of Elvius and Hall, and that of Artamonov and Efimov, in the outer regions of the Merope nebula. The grain properties necessary to give such results have not been quantitatively explored, and will be only briefly discussed herc (Section 3.4.6); in several aspects the necessary theoretical basis seems to be lacking.

### 1.2.2 Models

The first reflection nebula models which were able to handle a realistic star-cloud geometry were those of Schalén (1953). His interpretations of the color of the Merope nebula in terms of iron spheres with radii $\sim 0.05 \mu$ suffered, unfortunately, from a zero-point error in his measured (Schalén 1948) colors. A qualitative but perceptive interpretation of the early observations in NGC 7023 was given by van Houten (1961). Most subsequent studies assumed an idealized shape for the dust cloud, usually in the form of a sphere with the illuminating star at the center (Sobolev 1960; Minin 1962, 1965; Vanýsek and Svatos 1964). Before the observations of 0'Dell and Elvius and Hall, all theoretical studies depended upon inadequate observational data, often restricted to measures of the nebular surface brightness as a function of offset angle; the elegant models of Sobolev and Minin, for example, yielded little in the way of physical information about the grains.

It was suggested by Krishna Swamy and 0'De11 (1967) that, for a nebula which is belicved to lic entirely behind its illuminating star, extrapolation of the intrinsic colors to zero offset angle could be compared with the "radar" scattering properties of spheres to choose between competing grain models. However Greenberg and Hanner (to be published) pointed out that the method is of doubtful validity. Extrapolations to zero offset angle are unreliable. (They are likely to depend upon the direction from which the extrapolation is made!) Also, the backscattering crossection of spherical grains is severely dominated by resonances which would be modified or absent for grains which are not perfect spheres. Finally, it is doubtful that in any particular case we can be sure that no nebulosity lies in front of the star; because of the forward-throwing phase functions of Mie scattering grains, a layer of dust in front of the star too small to give a noticeable amount of reddening could still make a substantial contribution to the total nebular luminosity. Solutions to the problem of reflection nebulae will not come so easily.

Roark (1966), Roark and Greenberg (1967), and Dahn (1967) constructed models of the Merope nebula in the form of plane-paraliel slabs, with the thickness and tilt of the slab and the position of the illuminating star adjusted to give the best match to the observed colors. They obtained no conclusive results concerning the properties of the grains, but found that dielectric or composite grains seemed to be more consistent with the data than small graphite particles. Hanner (1969) expanded the treatment of Roark by using more detailed Mie
calculations and exact corrections for second-order scattering and by including polarization effects. She also made models for grains which are not spheres but randomly oriented infinite cylinders. She was able to match approximately the observed offsct dependence of color and polarization for icc grains in the OHG distribution with $a_{0}=0.5 \mu$ to $0.6 \mu$, but found no such fit for graphite grains.

Gehrels (1967) used a quite different theoretical approach for interpretation of his polarization observations in NGC 7023. The geometry of the dust cloud and the position of the illuminating star were left as free parameters, and using exact Mie calculations for grains of various types, the range of scattering angles and the grain radii were adjusted to give the best agreement with the observed polarization as a function of wavelength. He found that the range of scattering angles was determined with an uncertainty of no more than $10^{\circ}$ at each end. No agreement between theory and observation could be found for pure graphite or metallic grains. The best consistency with interstellar extinction and polarization requirements was found for dirty dielectric grains with $m^{\prime}$ in the range 1.2 to 1.8 and $m^{\prime \prime}$ in the range 0.1 to 0.4 , and diameter $\sim 0.3 \mu$, or composite grains with an absorptive nucleus of diameter $0.05 \mu$ and an icy shell of diameter $0.3 \mu$.

While completely free of geometrical constraints, the method introduced by Gehrels is strictly applicable only to optically thin nebulae; there is no way to confidently evaluate the effects of transmission losses and multiple scattering. Gehrels concluded (incorrectly: Section 4.4) that the optical depth of NGC 7023 is less than 0.2 in
visible light, so that these second-order effects should be relatively unimportant. The advantages and limitations of Gehrels' approach versus that of Roark, Hanner, etc. will be further discussed at the beginning of Chapter 2.

### 1.3 Structure of This Study

In Chapter 2 equations for computation of the intensity, color, and polarization of light emerging from a reflection nebula will be developed from first principles, and representative results for several types of grains will be presented. Certain types of grains will there be ruled out on very general grounds. The observational techniques developed in this work, and results for selected regions in a few nebulae, will be described in Chapter 3. In Chapter 4 comparisons between the new observations and successful theoretical models will be made.

Throughout, the observational and theoretical philosophy will follow that of Gehrels (1967); the emphasis will be placed upon (1) improvec observations, and (2) more detailed Mie calculations, including color effects and with a more quantitative treatment of size distributions of grains.

Until recently in most studies of reflection nebulae, the optical properties of the grains have been described by arbitrary analytic functions containing such parameters as the albedo and the asymmetry factor $<\cos \theta>$. While useful for qualitative discussions, such parameters have little to do with the physical nature of the grains, and are inadequate to fully describe their s: tering properties. Hence I will
describe the grains only by their sizes and refractive indices. Since the refractive index appropriate for a given chemical composition is in some cases open to question, identifications of various grain models as "ices", "silicates", etc., will be kept to a minimum.

Nowhere will I require that my grain models also fit the observed interstellar extinction and polarization; rather I will attempt to establish the range of grain properties that can be derived from the reflection nebula observations alone.

### 1.4 Selection of Nebulae

More than 200 reflection nebulae are known. Catalogs have been compiled by Cederblad (1946), Dorschner and Gurtler (1964), and Van den Bergh (1966). Most of these nebulae, however, are too faint or otherwise unsuitable for astrophysical study.

The night sky is usually somewhat brighter than magnitude 22 per square second of arc in blue light, and few reflection nebulae are brighter than magnitude 21. With proper techniques it is possible to do surface photometry four magnitudes or more below the sky, and in fact accurate photoelectric work has been done (e.g., Sandage and Visvanathan 1969) beyond $\underline{B}=25$ per square arcsecond. For such faint work, however, it becomes uncertain where the "dark" sky should be measured. Also, the sky is much brighter in the near infrared. For colors and polarizations over a wide wavelength range, work fainter than about $\underline{B}=22$ is very time-consuming.

Many desirable properties of reflection nebulac other than brightness can be listed. The nebula must be illuminated by a single
(and unambiguously identifiable) star, and one which is not so bright that instrumental halation (Section 3.2.4) overwhelms the nebular signal. The nebula should be large enough so that focal-plane diaphragms not smaller than $15^{\prime \prime}$ or $20^{\prime \prime}$ diameter can give good spatial resolution, and it should, of course, be reasonably frec of atomic emissions. The polarization should be high enough (> $5 \%$ ) that its wavelength dependence can be measured. It is essential that dark sky regions be available close by and easily located at the telescope; here it is very useful to have a telescope with rapid drift motions. Finally, it is my experience that work should never be begun on a nebula until a large-scale, low-density photograph is available for choice of suitable regions.

Since the $154-\mathrm{cm}$ Catalina reflector at which most of my work was done cannot be pointed northwards of about $+67^{\circ}$ declination, several suitable nebulae (e.g., NGC 7023, Cederblad 201) were not available. I chose NGC 2068 in Orion because of its brightness and because the extensive work of Elvius and Hall (1966) was already available. No such attractive nebula seems to be present in the summer sky. I avoided the bright nebula IC 348 in Perseus because of anticipated halation from the nearby (unrelated) star 38 Persei; it is now clear that halation problems are not insurmountable. After a good bit of unprofitable work (Section 3.8) on IC 4603, I settled on the rather faint but otherwisc ideal nebula IC 5076 in Cygnus. Some work was also done on NGC 7023 at the Steward $229-\mathrm{cm}$ reflector, and a bit of reconnaisance was done in several other nebulae (Section 3.8).

## CIIAPTER 2

## THEORETICAL TECHNIQUES

Previous observational studies of brightness, color, and polarization in reflection nebulae have followed two distinct approaches. Programs of the first type, which I shall call extensive, emphasize wide areal coverage in each nebula, either by working many discrete regions or by making several scans across the nebula. Only limited telescope time can be spent on any one part of the nebula; thus the range of wavelength must be restricted, usually to three filters in ultraviolet and visible light, and only modest precision can be obtained in any one region. The run of the observed quantities across the face of the nebula, however, should be well-determined. Photographic studies are inherently of this type, and most photoelectric investigations have followed the same pattern.

The observational program reported here is of the.second type, which I shall call intensive. Broad areal coverage is sacrificed for higher precision (more integration time) and wider wavelength range in only one or two regions of a nebula. To my knowledge the only previous example is the work of Gehrels (1967) in NGC 7023; O'Dell's (1965) work in the Merope nebula might be considered a transition case. Though Gehrels' observations were taken in 1959, they remain the only published data on reflection nebulac to contain polarimetry in infrared light. My work seems to be the first to include infrared photometry.

To each type of data there corresponds a type of theoretical model which is best suited for comparisons between theory and observation. For comparison with observations of the extensive type, models are constructed with the assumption that the nebula has uniform space density of grains and is bounded by some regular geometrical figure such as a plane-parallel slab or a segment of a sphere. Then the geometrical parameters of the model (e.g., thickness and tilt of the slab and position of the illuminating star) are varied along with the optical properties of the grains in order to best match the observed offset dependence of color and polarization with apparent angular distance from the illuminating star. Good examples of extensive models have been provided by Vanýsek and Svatos (1964), Roark (1966), Dahn (1967), and Hanner (1969).

The theoretical approach used in this work is quite different; it follows that introduced by Gehrels (1967). No assumptions are made about the overall geometry of the nebula. Instead, the optical properties of the grains are varied and the range of scattering angles is freely adjusted for each separate region of any nebula to find the best fit to the observed wavelength dependence of brightness and polarization.

Each type of model has obvious advantages and limitations. While well-adapted for treatment of multiple scattering and space extinction within the nebula, extensive models are quite likely to suffer fron over-idealization of the geometry. They seem to work for the Merope nebula, but would hardly be suited for nebulae of chaotic
structure such as NGC 7023. Intensive models are free of geometrical restraints but are strictly applicable only to optically 'hin nebulae; there is no obvious way to allow for multiple scattering and space absorption between the illuminating star and the region of interest.

The power of the extensive approach at its best is seen in Figures 33 through 35 of Hanner (1969), where plane-parallel-slab models of the Merope nebula are fitted to the observations of Roark (1966) and of Elvius and Hall (1967). The overall dependence of color on offset angle is reproduced rather well, but systematic deviations of up to 0.2 magnitudes (ten times the probable error of a good observation) are present. The dependence of polarization on offset angle is reproduced in a general way but its predicted wavelength dependence is much stronger than is observed. Using the intensive approach and the assumption of an optically thin nebula, Gehrels (1967) was able to fit perfectly his observed wavelength dependence of polarization in two regions of NGC 7023. As will be shown in Chapter 4, however, his model will not explain the nebular colors and brightness in these regions.

### 2.1 Computation of Emergent Intensities

In this section equations for computation of the intensities in two planes of polarization emerging from a segment of a reflection nebula will be developed from first principles. The treatment will be largely original, though necessarily parallel to earlier work in its general outline. No attempt is made to follow the formalism of previous investigators.

The following assumptions are made:

1) The grains are spheres, with bulk optical properties that can be described by an isotropic, wavelength-independent complex refractive index, and which scatter light according to the classical laws of electricity and magnetism (Maxwell's Equations). Quantum effects are ignored.
2) The scattering propertics of each grain are not affected by the presence of other grains. That is, we have independent scattering.
3) Multiple scattering, that is, the illumination of a grain by other grains, is ignored.

Aside from these assumptions and others explicitly listed below, the treatment will be kept general even in cases where observational limitations do not allow the full power of the theory to be applied. The condition of a wavelength-independent refractive index will be relaxed for graphite grains (Section 2.3). Throughout, parentheses will be used to indicate not multiplication but functional dependence.

### 2.1.1 The Mie theory

The interaction of a plane monochromatic wave with a homogeneous sphere of any radius and any composition, situated in a homogeneous medium, is in principle described completely by Maxwell's Equations for the appropriate boundary conditions. Exact solutions of this problem were obtained by Mie (1908) and by Debye (1909). In Debyc's notation, the amplitudes and phases of successive orders of electric and magnetic multipole vibrations in the scattered wave are
described by complex quantities $A_{n}\left(m^{*}, x\right)$ and $B_{n}\left(m^{*}, x\right)$, where $m^{*}$ is the complex refractive index,

$$
m^{*}=m^{\prime}-i m^{\prime \prime},
$$

and x is a dimensionless size parameter

$$
x=2 \pi a / \lambda,
$$

where $a$ is the grain radius and $\lambda$ is the wavelength. The equations for computation of $A_{n}$ and $B_{n}$ are given by van de Hulst (1957) and by Wickramasinghe (1967), and will not be repeated here; the formulae involve several kinds of Riccati-Bessel functions with complex arguments, which can be obtained only by recursion relations.

Extinction and scattering efficiencies are given by:

$$
\begin{aligned}
& Q_{e x t}(x)=\frac{2}{x^{2}} \sum_{1}^{\infty}\{2 n+1\} \operatorname{Re}\left(A_{n}+B_{n}\right), \\
& Q_{s c a}(x)=\frac{2}{x^{2}} \sum_{1}^{\infty}\{2 n+1\}\left\{\left|A_{n}\right|^{2}+\left|B_{n}\right|^{2}\right\} .
\end{aligned}
$$

The fundamental quantities for angular scattering are the van de Hulst dimensionless intensities defined below. They are given by:

$$
\begin{aligned}
& i_{1}(x, \theta)=\left|\sum_{1}^{\infty} \frac{2 n+1}{n\{n+1\}}\left(A_{n} \Pi_{n}(\theta)+B_{n} \tau_{n}(\theta)\right)\right|^{2}, \\
& i_{2}(x, \theta)=\left|\sum_{1}^{\infty} \frac{2 n+1}{n\{n+1\}}\left(B_{n} \Pi_{n}(\theta)+A_{n} \tau_{n}(\theta)\right)\right|^{2},
\end{aligned}
$$

where $\theta$ is the scattering angle,

$$
\begin{aligned}
& \Pi_{n}(\theta)=\csc \theta p_{n}^{l}(\cos \theta) \\
& \tau_{n}(\theta)=\frac{d}{d \theta} p_{n}^{1}(\cos \theta)
\end{aligned}
$$

and $\mathrm{p}_{\mathrm{n}}^{1}$ is an associated Legendre function, related to the ordinary Legendre polynomials $\mathrm{P}_{\mathrm{n}}$ by

$$
P_{n}^{1}(t)=\left\{1-t^{2}\right\}^{\frac{1}{2}} \frac{d}{d t} P_{n}(t)
$$

The computer program which I used for calculation of these quantities is described in Section 2.4 below.

### 2.1.2 Intensity components from a single grain

Van de Hulst (1957) describes the light scattered from a grain by two dimensionless intensities, $i_{1}\left(m^{*}, x, \theta\right)$ and $i_{2}\left(m^{*}, x, \theta\right)$. Suppose that a plane wave of unpolarized light of intensity $I_{0}(\lambda)$ watts/meter ${ }^{2}$ is incident upon a single grain of radius a. Then the (spherical) scattered wave at distance $r$ from the grain has intensity components

$$
\begin{aligned}
& I_{1}(\lambda)=I_{0}(\lambda) \frac{\lambda^{2}}{8 \pi^{2} r^{2}} i_{1}\left(m^{*}, x, \theta\right) \\
& I_{2}(\lambda)=I_{0}(\lambda) \frac{\lambda^{2}}{8 \pi^{2} r^{2}} i_{2}\left(m^{*}, x, \theta\right)
\end{aligned}
$$

where $I_{1}$ is the intensity polarized with electric vector normal to the plane of scattering, and $I_{2}$ is the intensity polarized with electric vector in the plane of scattering. The total intensity of the scattered light is

$$
I=I_{1}+I_{2}
$$

Circular polarization cannot arise (see van de Hulst 1957, page 36) when a spherical grain is illuminated by unpolarized light. It should be emphasized that $I_{1}$ and $I_{2}$ are physical intensities, measured in units like watts/meter ${ }^{2}$, rather than specific intensities as are often used in astrophysics. Beyond this point only equations for $I_{1}$ will be written out. In all cases the equivalent expression for $I_{2}$ can be obtained by replacing $i_{1}$ with $i_{2}$.

If a grain is not in clear space but is imbedded in an attenuating medium of opacity $k(\lambda, r)$ per meter, the optical depth is

$$
\tau(\lambda, r)=\int_{0}^{r} \kappa(\lambda, t) d t
$$

and the intensity received at distance r will be

$$
I_{1}(\lambda)=I_{0}(\lambda) \frac{\lambda^{2}}{8 \pi^{2} r^{2}} \exp (-\tau) i_{1}(x, \theta)
$$

### 2.1.3 Integration over a size distribution

Suppose that in volume $d V$ we have many grains in a size distribution. Let $\eta(a) d a$ be the number of grains per cubic meter with radii between a and a+da. By the assumption of independent scattering, the intensity from an ensemble of grains is the sum of the intensities from each of the grains. Thus the intensity received at distance $r$ from volume $d V$ is

$$
d I_{1}(\lambda)=I_{0}(\lambda) \frac{\lambda^{2}}{8 \pi^{2} r^{2}} \exp (-\tau) d V \int_{0} i_{1}(x, \theta) \eta(a) d a
$$

Both types of size distribution that have been proposed on physical grounds for interstellar grains (Section 1.1 above) can be expressed analytically as

$$
n(a) d a=n_{0} f\left(\frac{a}{a_{0}}\right) d a
$$

The quantity $n_{0}$, with dimensions grains/meter ${ }^{4}$, has no particular physical significance; it is introduced only to make $f$ dimensionless. A more meaningful quantity is the total number $N$ of grains per cubic meter,

$$
N=\int_{0}^{\infty} \eta(a) d a=n_{0} \int_{0}^{\infty} f\left(-\frac{a}{a_{0}}\right) d a
$$

We can write

$$
N=n_{0} a_{0} \gamma
$$

where $\gamma$ is a numerical constant depending only on the nature of the size distribution,

$$
\gamma=\int_{0}^{\infty} f\left(\frac{a}{a_{0}}\right) d\left(\frac{a}{a_{0}}\right)
$$

For grains all of the same size, $f$ is the Dirac Delta function,

$$
\begin{aligned}
\eta(a) d a & =N \delta\left(a-a_{0}\right) d a \\
& =N \delta\left(\frac{a}{a_{0}}-1\right) \frac{d a}{a_{0}}, \\
f\left(\frac{a}{a_{0}}\right) & =a_{0} \delta\left(a-a_{0}\right)=\delta\left(\frac{a}{a_{0}}-1\right),
\end{aligned}
$$

for which $\gamma=1$. The Oort-van de Hulst distribution as described analytically by Greenberg (OHG distribution) is

$$
\eta(a) d a=n_{0} \exp \left(-5\left\{a / a_{0}\right\}^{3}\right) d a,
$$

for which

$$
\gamma_{\text {OHG }} \simeq 0.5222 \text {. }
$$

The exponential size distribution (EXP distribution) is usually written with $f=\exp \left(-a / a_{0}\right)$. However comparisons with the OHG distribution are facilitated if we write instead

$$
n(a) d a=n_{0} \exp \left(-5 a / a_{0}\right) d a .
$$

This change amounts to a re-definition of $a_{0}$, so that $\eta(a)=n_{0} \exp (-5)$ at $a=a_{0}$ in both distributions. This gives $\gamma_{E \times P}=1 / 5$. The two size distributions are plotted in Figure 2-1. A logarithmic ordinate has been chosen so that the EXP distribution comes out to be linear. For the same value of $a_{0}$, the EXP distribution has fewer small grains but more large grains than the OHG distribution. The mean radii of grains in the two distributions are:


Figure 2-1. Size Distributions

$$
\begin{aligned}
& \langle a\rangle_{O H G} \\
& \left\langle a .2956 a_{0},\right. \\
& \langle a \times P
\end{aligned}
$$

and the total volume of grains per cubic meter is

$$
\begin{aligned}
& V_{O H G} \simeq 0.2792 \mathrm{Na}_{0}^{3}, \\
& V_{E X P} \simeq 0.2010 \mathrm{Na}_{0}^{3}
\end{aligned}
$$

From the above equations we have for the scattered intensity:

$$
d I_{1}=I_{0} \frac{\lambda^{2}}{8 \pi^{2}} \exp (-\tau) \frac{d v}{r^{2}} \frac{N}{\gamma} \int_{0}^{\infty} f\left(\frac{a}{a_{0}}\right) i_{1}(x, \theta) d\left(\frac{a}{a_{0}}\right)
$$

Now as this equation stands it would be necessary to evaluate the integral for every value of the scale-size $a_{0}$ to be tested. The amount of work can be greatly reduced by introducing another dimensionless size parameter

$$
x_{0}=2 \pi a_{0} / \lambda
$$

This makes it possible to write

$$
d I_{1}(\lambda, r)=\frac{1}{2} I_{0}(\lambda) N a_{0}^{2} F_{1}\left(x_{0}, \theta\right) \exp (-\tau) \frac{d V}{r^{2}}
$$

where

$$
F_{1}\left(x_{0}, \theta\right)=\frac{1}{\gamma x_{0}^{3}} \int_{0}^{\infty} f\left(\frac{x}{x_{0}}\right) i_{1}(x, \theta) d x
$$

Recall that the grain refractive index is taken to be independent of wavelength. Then $F_{1}\left(x_{0}, \theta\right)$, which contains all terms involving Mie calculations and integration over the size distribution, can be computed for unspecified $\lambda$ and hence arbitrary $x_{0}$. The scale size $a_{0}$ is left as a free parameter. Comparisons between theory and observation will fix the value of $\lambda$ that corresponds to an arbitrarily chosen value of $x_{0}$, and hence determine $a_{0}$.

Neither the scale radius $a_{0}$ nor the mean radius 〈a〉 should be interpreted as an "effective" grain radius. The concept of an effective grain size is often encountered in astronomical literature. However, it seems to have been used in at least three different ways.

1) The most common conception of an effective grain size is the radius of grains in the $\delta$-distribution which, for some scattering process, most closely reproduces the effects of a size distribution. A single grain size with this property may not even exist.
2) The product $\eta(a)$ times the extinction crossection $C(a, \lambda)=$ $\pi a^{2} Q_{\text {ext }}(a, \lambda)$ is often plotted (e.g., Greenberg 1968, page 310 ), showing a bell-shaped curve with a peak at some radius a. This radius is simply the value of a that gives the most extinction at the wavelength $\lambda$.
3) Finally, one can compute, for some $\lambda$, the mean radius with respect to the function $\phi$ that describes some scattering process, as

$$
\langle a\rangle_{\phi}=\frac{\int a \phi(a, \lambda) n(a) d a}{\int \phi(a, \lambda) \eta(a) d a}
$$

None of these definitions give a quantity that has any direct relationship to the physical distribution of grain sizes, as do $a_{0}$ and〈a〉. Thus the concept of an effective grain size can lead to confusion, and to my taste it would best be avoided; I will describe my solutions in terms of the scale size $a_{0}$. There is, of course, nothing sacred about the OHG and EXP distributions as written above, and it may be unrealistic to work with a one-parameter distribution, but at least we have a quantity $a_{0}$ that is well defined in terms of functions that can be easily handled.

### 2.1.4 Computation of opacity

For a single grain with radius a and at wavelength $\lambda$, that is, with size parameter $x=2 \pi a / \lambda$, the effective extinction crossection is

$$
C(x)=\pi a^{2} Q_{e x t}(x)
$$

where the extinction efficiency $Q_{\text {ext }}$ is obtained from the Mie theory. In a size distribution of the type described above, the contribution to the opacity from each increment da of grain radius is

$$
d k=\pi a^{2} Q(x) \eta(a) d a
$$

Introducing $x_{0}=2 \pi a_{0} / \lambda$ as above, and integrating over all grain sizes, we can write

$$
\kappa\left(x_{0}\right)=N \pi a_{0}^{2} q\left(x_{0}\right)
$$

where $q\left(x_{0}\right)$ can be computed for unspecified values of $a_{0}$ or $\lambda$ :

$$
q\left(x_{0}\right)=\frac{1}{\gamma x_{0}^{3}} \int_{0}^{\infty} x^{2} Q(x) f\left(\frac{x}{x_{0}}\right) d x
$$

Note that $\mathrm{q}\left(\mathrm{x}_{0}\right)$ is the mean over the size distribution not of $Q(x)$ but of the product $Q(x)$ times $\left(a / a_{0}\right)^{2}$.

### 2.1.5 Description of the scattering volume

The plane of scattering is sketched in Figure 2-2. The illuminated dust cloud is assumed to be sharply bounded and simply connected in the mathomatical sense. Otherwise the geometry is completely arbitrary; in particular, the illuminating star is not required to be inside the dust cloud. The focal-plane diaphragm cuts through the nebula a cylinder of radius $b$, centered at perpendicular distance $L$ from the illuminating star. Within the cylinder scattering angles range from $\theta_{1}$ to $\theta_{2}$, and a differential volume element $d V$ at distance $R$ from the star is at scattering angle 0 . The optical depths involved are $\tau$ between the scattering element and the front of the dust cloud, $\tau^{\prime}$ between the illuminating star and the scattering element, and $\tau^{\prime \prime}$ between the star and the front of the dust cloud. In general there will be a further attenuation in interstellar space between the reflection nebula and the observer. This interstellar extinction may be ignored if the additional assumption holds:


Figure 2-2. The Scattering Plane
4) The foreground interstellar extinction (fur both planes of polarization) is the same for the nebular region studied and for the illuminating star.

Recall that $I_{0}$ is the intensity of the star as seen by grains in the volume dV. Let $I_{*}$ be the intensity of the star as seen from the earth. Then

$$
I_{0}=I_{*} \frac{r^{2}}{R^{2}} \exp \left(\tau^{\prime \prime}-\tau^{\prime}\right)
$$

and, as seen from the earth,

$$
I_{1}(\lambda)=\frac{1}{2} I_{*} \exp \left(\tau^{\prime \prime}\right) \int_{V} N a_{0}^{2} F_{1}\left(x_{0}, \theta\right) \exp \left(-\tau-\tau^{\prime}\right) \frac{d V}{R^{2}}
$$

where the integral is taken over the whole cylinder. As might be expected, the ratio of nebular to stellar intensity for a fixed volume of nebula is independent of the distance $r$ to the dust cloud. (The ratio of the nebular brightness per square arcsecond to the stellar brightness actually increases with the square of the distance.) One more assumption is made:
5) The grain size distribution, including the concentration $N$, and the optical properties of the grains are constant throughout the cylinder of interest (but not necessarily throughout the dust cloud.)

Then the quantity $\mathrm{Na}_{0}^{2}$ can be removed from the volume integral. At this point my formulation can conveniently be compared to that of Hanner (1969). Adding the above equation to a similar equation for $\mathrm{I}_{2}$,
that is, ignoring polarization effects, gives an expression equivalent to Hanner's Equation II-1. In her notation,

$$
I(\lambda)=I_{*}(\lambda) \pi^{2} \int_{z} e^{-k(\lambda) L(z)} \int_{a} F_{\lambda}(a, 0) \eta(a) d a \frac{d z}{R^{2}},
$$

where

$$
\begin{aligned}
& \lambda=\lambda / 2 \pi \\
& F_{\lambda}(a, \theta)=\frac{1}{2}\left[i_{1}(a, \theta)+i_{2}(a, \theta)\right] .
\end{aligned}
$$

The two formulations differ only in the following respects: Hanner's $I(\lambda)$ is a surface brightness of the nebula, e.g., watts $/ \mathrm{m}^{2} / \mathrm{m}^{2}$ if $\mathrm{I}_{\text {* }}$ is in watts $/ \mathrm{m}^{2}$ and $n(\mathrm{a}) \mathrm{da}$ is in grains $/ \mathrm{m}^{3}$, whereas my $\mathrm{I}(\lambda)$ refers to the total intensity emerging from the cylinder. Thus instead of my integral over the volume of the scattering cylinder, Hanner has an integral over path length; by ignoring the finite size of the observed nebular region, she avoids the problems of volume integration described below. In her formulation the distance to the nebula must be known before $I(\lambda)$ can be compared to an observed quantity. Since Hanner constructs models for only one value of $a_{0}$ at a time, my scale size $x_{0}$ does not enter into her analysis. Finally, in her notation the optical depths $\tau$ and $\tau^{\prime}$ are combined, and $\tau^{\prime \prime}$ is implicitly contained in her quantity $\mathrm{I}_{*}$.
2.1.6 Integration over volume

The exact value of the integral

$$
\int F_{1}\left(x_{0}, \theta\right) \exp \left(-\tau-\tau^{\prime}\right) d V / R^{2}
$$

is not easily obtained. Since the integrand is computed at discrete scattering angles $\theta_{j}$, let us break the cylindrical volume $V$ into discrete elements $\Delta V_{j}$ bounded by surfaces at $\theta_{j} \pm \frac{1}{2} \Delta \theta$. Then, rigorously,

$$
\int_{V}\{\ldots\} \frac{d V}{R^{2}}=\sum_{j} \int_{\Delta V_{j}}\{\ldots\} \frac{d V}{R^{2}}
$$

where each integral in the sum is taken over one volume element. We may now approximate that the integrand is constant over each volume element, that is, that all grains within a volume element $\Delta V_{j}$ scatter at angle $\theta_{\mathbf{j}}$. Then

$$
\int_{V}\{\ldots\} \frac{d V}{R^{2}} \simeq \sum_{j}\{\ldots\}_{\theta_{j}} \int_{\Delta V} \frac{d V}{R^{2}}
$$

The smaller we make $\Delta \theta$ and $b / L$ the more realistic will be this description. I have used focal-plane diaphragms such that $b / L \leq 0.2$. Gehrels (1967) used this approximation with $\Delta \theta=10^{\circ}$; I have used $\Delta \theta=5^{\circ}$ throughout. We are now left with the geometrical factors

$$
\int_{\Delta v_{j}} \frac{d v}{R^{2}}
$$

This integral over the required segment of a cylinder, despite its innocuous appearance, seems to be peculiarly resistant to analytic evaluation, and I have been unable to obtain an exact solution. It has been assumed (see the discussion reported by Greenberg and Roark 1967, page 107) that the integral is independent of $R$ or $\theta$, i.e., that $d V / R^{2}$ is constant simply as a solid-angle effect. This argument, however, will not stand close scrutiny. It is true that in polar coordinates $d V=R^{2} \sin \Theta d \theta d \phi d R$, so that $R^{2}$ cancels out of the integrand, but it still appears in the limits of the integral. As shown in Appendix $I$, the integral can be approximated by

$$
\int_{\Delta V_{j}} d V / R^{2} \simeq \pi b \Delta \theta \operatorname{Tanh}^{-1}(b / L)
$$

which indeed is independent of $\theta_{j}$. Thus the integral over volume is replaced by a simple (unweighted) sum over scattering angle. This method of integration over volume is far from elegant, but physical effects such as spatial variation of the grain concentration $\eta$ (a)da are likely to contribute much more systematic error than this mathematical approximation.

We now have complete equations for the intensities emergent from a reflection nebula in orthogonal planes of polarization. In terms of the functions

$$
T_{1}\left(x_{0}\right)=\frac{1}{\pi} \int_{V} e^{-\tau-\tau^{\prime}} F_{1}\left(x_{0}, \theta\right) d V / R^{2}
$$

$$
\simeq b \Delta \theta \operatorname{Tanh}^{-1}(b / L) \sum F_{1} e^{-\tau-\tau^{\prime}},
$$

and

$$
\begin{aligned}
T_{2}\left(x_{0}\right) & =\frac{1}{\pi} \int e^{-\tau-\tau^{\prime}} F_{2}\left(x_{0}, \theta\right) d V / R^{2} \\
& \simeq b \Delta \theta \operatorname{Tanh}^{-1}(b / L) \sum F_{2} e^{-\tau-\tau^{\prime}},
\end{aligned}
$$

the emergent intensities are given by

$$
\begin{aligned}
I_{1}\left(x_{0}\right) & =\frac{1}{2} I_{*} e^{\tau^{\prime \prime}} N \pi a_{0}^{2} T_{1}\left(x_{0}\right), \\
I_{2}\left(x_{0}\right) & =\frac{1}{2} I_{*} e^{\tau \prime \prime} N \pi a_{0}^{2} T_{2}\left(x_{0}\right), \\
I & =I_{1}+I_{2}
\end{aligned}
$$

Equations for the case of a $\delta$-distribution (grains all of the same size $a_{0}$ ) may be readily obtained by working directly from Section 2.1.2, or by substituting in the above equations

$$
f\left(a / a_{0}\right)=\delta\left(a / a_{0}-1\right)
$$

where $x_{0}=2 \pi a_{0} / \lambda$. This gives

$$
\begin{aligned}
& F_{1}\left(x_{0}, \theta\right)=x_{0}^{-2} i_{1}\left(x_{0}, \theta\right), \\
& F_{1}\left(x_{0}, \theta\right)=x_{0}^{-2} i_{2}\left(x_{0}, \theta\right) .
\end{aligned}
$$

### 2.2 Computation of Observable Quantities

### 2.2.1 Polarization

The polarization of light emerging from a cylindrical volume as described above is

$$
\begin{aligned}
P\left(x_{0}\right) & =\frac{I_{1}-I_{2}}{I_{1}+I_{2}}=\frac{T_{1}-T_{2}}{T_{1}+T_{2}} \\
& =\frac{\sum_{\theta}\left[F_{1}-F_{2}\right]_{\theta}\left[e^{-\tau-\tau^{\prime}}\right]_{\theta}}{\sum_{\theta}\left[F_{1}+F_{2}\right]_{\theta}\left[e^{-\tau-\tau^{\prime}}\right]_{\theta}} .
\end{aligned}
$$

Note that $I_{*}\left(x_{0}\right)$ and $\tau^{\prime \prime}$ cancel out of the equations when polarization is computed, and that $\tau$ and $\tau$ ' only affect the relative weights of different scattering angles; for a nebula in the form of a thin sheet with negligible geometrical depth, internal attenuations would have no effect at all on the polarization.

### 2.2.2 Color

The description of a color is not so straightforward. Hanner (1969) computed $\underline{U} \underline{B} \underline{V}$ colors using specific functions for the stellar intensity $I_{*}(\lambda)$, the filter-detector responses, etc. It is much easier and more precise, however, to obscrve not $\underline{U} \underline{B} \underline{V}$ magnitudes in the usual sense of stellar astronomy, but simply the brightness ratio between the nebular region and the illuminating star as a function of wavenumber (Section 3.3.3 below). I will express such ratios in magnitudes for their convenient additive property. Thus the observational color function that I use is actually

$$
\Delta M(1 / \lambda)=-2.5 \log \frac{I(1 / \lambda)}{I_{*}(1 / \lambda)} .
$$

The slope of this function gives the color difference between the star and the nebula, that is, the intrinsic color of the grains, independent of the color of the star or any properties of the detector system. If the observational data were restricted to two broad, closely-spaced filters such as $\underline{V}$ and $\underline{B}$, detailed response functions for each filter might be necessary. Given data extending from the ultraviolet to the infrared in several filters, however, only the effective filter wavenumbers are of any consequence.

Let us choose a fixed but completely arbitrary wavelength $\lambda^{\prime}$. Then for any wavelength $\lambda$, we can define magnitudes for the nebula as

$$
\begin{aligned}
& M(\lambda)=-2.5 \log I(\lambda), \\
& M\left(\lambda^{\prime}\right)=-2.5 \log I\left(\lambda^{\prime}\right)
\end{aligned}
$$

The observed color function of the nebula is

$$
C_{n e b}\left(\lambda, \lambda^{\prime}\right)=M(\lambda)-M\left(\lambda^{\prime}\right)
$$

Similarly, for the illuminating star,

$$
C_{*}\left(\lambda, \lambda^{\prime}\right)=M_{*}(\lambda)-M_{*}\left(\lambda^{\prime}\right)
$$

Then the intrinsic color function of the nebula is

$$
C=C_{\text {neb }}-C_{\star}
$$

The sign convention here is the usual one of astronomy; a positive $C$ means an intrinsic color that is red if $\lambda<\lambda^{\prime}$, blue if $\lambda>\lambda^{\prime}$. Some investigators of reflection nebulae have used the opposite sign convention. In this work all graphs will be plotted with $C$ increasing toward the bottom of the page, so that the intensity increases upward. Note especially that choosing a different $\lambda$ ' has no effect on the shape of the color function $C\left(\lambda, \lambda^{\prime}\right)$, but only shifts its zero-point. We may substitute

$$
\begin{aligned}
& x_{0}=2 \pi a_{0} / \lambda, \\
& x_{0}^{\prime}=2 \pi a_{0} / \lambda^{\prime},
\end{aligned}
$$

to obtain the theoretical color function

$$
\begin{aligned}
C\left(x_{0}, x_{0}^{\prime}\right) & =2.5 \log \left[\frac{\left.I\left(x_{0}\right)^{\prime}\right) / I_{*}\left(x_{0}{ }^{\prime}\right)}{I\left(x_{0}\right) / I_{*}\left(x_{0}\right)}\right] \\
& \left.=2.5 \log e^{\tau^{\prime \prime}\left(x_{0}\right.}\right)-\tau^{\prime \prime}\left(x_{0}\right)\left[\frac{\sum\left[\left\{F_{1}+F_{2}\right\} e^{-\tau-\tau^{\prime}}\right]_{x_{0}}}{\sum\left[\left\{F_{1}+F_{2}\right\} e^{-\tau-\tau^{\prime}}\right]_{x_{0}}}\right]
\end{aligned} .
$$

Clearly, attenuations have much more effect on color data than on polarization data.

The actual size (b or $L$ ) or distance of the nebula need not be known for computation of colors and polarizations, unless the transmission losses are to be computed from some assumed value of the nebular opacity.
2.2.3 A graphical method for comparisons between theory and observation

The theoretical polarization and color functions $P\left(x_{0}\right)$ and $C\left(x_{0}\right)$ constructed for particular values of $\mathrm{m}^{*}, \theta_{1}$, and $\theta_{2}$ must be compared to observed data points $P(1 / \lambda)$ and $\Delta M(1 / \lambda)$. We must determine if the theoretical and observational data are in agrement, and determine the scale size $a_{0}$ at which the agreement, if any, takes place. The following graphical method was found to be very useful.

On separate sheets of semi-logarithmic graph paper, $P\left(x_{0}\right)$ and $P(1 / \lambda)$ are plotted on the linear ordinate scales versus $x_{0}$ and $1 / \lambda$ on the logarithnic abscissa scales. Since

$$
\begin{aligned}
& x_{0}=2 \pi a_{0} / \lambda \\
& \log x_{0}=\log 2 \pi a_{0}+\log (1 / \lambda),
\end{aligned}
$$

the theoretical and observational curves must be identical except for a horizontal shift of $\log \left(2 \pi a_{0}\right)$. Since polarization is an absolute quantity, no vertical shift is allowed. The two sheets may be superimposed and shifted along the $x_{0}$ or $1 / \lambda$ axis until the best fit is found between the theoretical curve and the observed data points. Then one may note the $x_{0}$ that corresponds to a given $1 / \lambda$, giving immediately the scale size $a_{0}$. A similar procedure is followed for color data, except that there will also be a vertical shift, giving the ratio, in magnitudes, of stellar to nebular brightness at $x_{0}{ }^{\prime}=2 \pi a_{0} / \lambda^{\prime}$. The graphical method is denonstrated in figure 2-3 for hypothetical polarization data and in Figure 2-4 for hypothetical color data.


Figure 2-3. A Graphical Method, Polarization Data


Figure 2-4. A Graphical Method, Color Data
(Hypothetical data are plotted in order to avoid giving the impression that the method is applicable to only a specific nebula. Actually the data in Figure $2-3$ are similar to real polarization data in NGC 7023 and the data in Figure 2-4 are similar to real color data in IC 5076.)

A successful model is defined as one in which good fits to the color and polarization data fall at the same scale size $a_{0}$, and in which the vertical shift of the color curve gives a value of the grain concentration and hence an opacity commensurate with the value of nebular opacity used for the model.

### 2.2.4 Treatment of attenuation losses

The above equations contain the optical depths $\tau(\lambda, \theta)$ between and element of scattering volume and the front of the dust cloud, $\tau^{\prime}(\lambda, \theta)$ between the star and the scattering element, and $\tau^{\prime \prime}(\lambda)$ between the star and the front of the dust cloud. The wavelength dependence of $\tau(\lambda, \theta)$ is given in Section 2.1.4, and the interstellar reddening of the illuminating star puts an upper limit on $\tau^{\prime \prime}$. Otherwise nothing at all is known a priori about these attenuations; my models leave the scattering geometry as a free parameter. Given observations in only one or two regions we can say nothing about the distribution of opacity within the nebula or about the overall cloud geometry. A proper investigation of these matters would require a much more extensive observing program than was undertaken here.

We could fit a model with $\tau=\tau^{\prime}=\tau^{\prime \prime}=0$ to the observations in order to obtain preliminary values for $\mathrm{a}_{0}, \theta_{1}, \theta_{2}$, and the grain concention N. Then the equations in section 2.1 .4 could be used to compute a corrected value for the opacity within the scattering cylinder. Thus an iterative process could in principle determine $\tau\left(x_{0} \theta_{1}\right)$ through the length of the cylinder, but no information about $\tau^{\prime}$ or $\tau^{\prime \prime}$ could be obtained. A rough appraisal of the effects of transmission losses can be made with the help of two approximations: 1) that the opacity is constant throughout the dust cloud, and that 2) the dust cloud is bounded by some simple geometric figure such as a segment of a sphere about the illuminating star. It cannot be emphasized too strongly that such assumptions have no physical justification. On small-scale, long-exposure photographs such as the Palomar Sky Survey prints, a typical reflection nebula appears as a great amorphous blob with all internal structure obscured by overexposure. A much more complex structure with cavities, condensations, and streamers is usually revealed by low-density highresolution photographs.

In view of these difficulties, attenuation losses will be ignored in most of the work reported here. The validity of such simplified models may be judged by their success in fitting the observed color and polarization data, without regard to the actual brightness ratio of nebula to star. In some cases a reconnaisance will be made of the effects of non-zero optical depth, using a cloud geometry that seems appropriate for the reflection nebula in question.

### 2.3 Treatment of Graphite Grains

For the computation of van de llulst intensities $i_{1}$ and $i_{2}$, four parameters are necessary: $m^{*}(\lambda), a, \lambda$, and $\theta$. The computation of an extinction efficiency $Q$ requires the three parameters $m^{*}(\lambda)$, $a$, and $\lambda$. In Sections 2.1.2 through 2.1.4 the utility of a single Mie calculation was greatly enhanced by the assumption that $m^{*}$ was independent of wavelength. This made it possible to substitute $x=2 \pi a / \lambda$ so that individual values of $a$ and $\lambda$ need not be specified in advance. The additional assumption that the size distribution could be written in the form

$$
\eta(a) d a=N F\left(a / a_{0}\right) d a
$$

made it possible to introduce a second dimensionless size parameter $x_{0}=2 \pi a_{0} / \lambda$ and hence to integrate over a size distribution without specifying individual values of $a_{0}$ and $\lambda$.

No such simplifications are possible in the case of graphite grains, for which the refractive index varies strongly over the wavelength range of my observations. Actually, graphite grains would probably take the form of thin flakes with a highly anisotropic refractive index (Wickramasinghe 1967, page 110). Since an exact scattering theory for such grains is not available, I treated them as isotropic spheres. I used the refractive indices given in Table 2-1, interpolated from Table VII of Shah (1967).

For graphite grains no particular type of size distribution has been suggested on physical grounds. Using an Oort-van de Hulst size distribution with mean radius $0.028 \mu$ ( $a_{0}=0.096 \mu$ in my notation)

Table 2-1. Refractive indices for graphite grains


Stecher and Donn (1965) found a fit to the interstellar extinction curve for wavelengths longward of about $0.2 \mu$. Wickramasinghe (1969) found a similar fit using a normal size distribution peaked at $a=0.05 \mu$ with standard deviation $0.01 \mu$ or $0.02 \mu$. In this work I used no size distributions but only discrete sizes of grains, with radii $0.03 \mu, 0.06 \mu$, $0.12 \mu$, and $0.24 \mu$.

### 2.4 Computer Programs

My computer routines were based on a FORTRAN program for Mie scattering developed and kindly made available by Dr. Ghanshyam A. Shah. For computation of $Q_{e x t}(x), Q_{a b s}(x), Q_{s c a}(x)$, and $i_{1}(x, 0)$ and $i_{2}(x, 0)$, the Shah program obtains the Mie Coefficients $A_{n}(x)$ and $B_{n}(x)$ for successive values of $n$, terminating the series when a single additional term does not change $Q_{\text {ext }}$ by more than one part in $10^{8}$. Most of my com. putations were made on the IBM 1130 of the Lunar and Planetary Laboratory. On this small computer the Mie coefficients are obtained in about three seconds for each value of $x$, with roughly one additional second required for computation of $i_{1}(x, \theta)$ and $i_{2}(x, \theta)$ at each value of $\theta$.

The Shah program was checked by comparison with results from an older Mie scattering program written by Dr. B. M. Herman and Maj. Gen. (ret.) S. R. Browning of the Institute of Atmospheric Physics of The University of Arizona. The mean difference between results from the two programs, for about a dozen test cases, was less than one part in $10^{3}$.

Because of the limited core storage ( 8000 words) of the IBM 1130, the computations had to be broken up into several programs. The output from my version of the Shah program (GSSPZ) was punched on IBM cards, with one set of $m^{*}, x, \theta, i_{1}$, and $i_{2}$ on each card. Except for graphite grains, I always used intervals of 0.1 in $x$, and without exception I used intervals of $5^{\circ}$ in $\theta$ from $0^{\circ}$ to $180^{\circ}$. When no size distribution was applied, these cards werc read directly into a program SAMSZ which computed colors and polarizations after summing over scattering angles.

For integration over the OHG and EXP size distributions, the cards from GSSPZ were read into programs SADOG and SADEX. These programs computed the quantities $\mathrm{F}_{1}\left(\mathrm{x}_{0}, \theta\right)$ and $\mathrm{F}_{2}\left(\mathrm{x}_{0}, \theta\right)$ defined in Section 2.1 .3 and transferred them to SAMSZ via a CALL LINK statement, for summing over scattering angles. Being limited to 32 values of $x_{0}$ in order to work entirely in core, I used intervals of either $0.1,0.2$, or 0.5 in $x_{0}$ according to the largest desired value. The integration over a size distribution was performed by Simpson's Rule, that is,

$$
\begin{aligned}
\int g(x) d x=\frac{0.1}{3}\left[g\left(x_{0}\right)\right. & +4 g\left(x_{1}\right)+2 g\left(x_{2}\right)+4 g\left(x_{3}\right) \\
& \left.+\ldots \ldots+g\left(x_{\ell}\right)\right]
\end{aligned}
$$

where $x_{\ell}$ is the largest value of $x$ used in the numerical integration. The quantity $x_{\ell}$ obviously must be well out on the tail of the size distribution, but it is not immediately clear how large it should be. By trial and error, I determined the value of $x_{\ell}$ necessary for fourdigit accuracy in $\mathrm{F}_{1}$ and $\mathrm{F}_{2}$. In the OllG distribution, a value 1.25 times the largest value of $x_{0}$ was generally quite adequate; with its
much longer tail (Figure 2-1), the EXP distribution required that $\mathrm{x}_{\ell}$ be at least twice the largest value of $x_{0}$.

For models including attenuation losses, the wavelength dependence of the opacity was computed as described in Section 2.1.4 above; again the integral over the size distribution was evaluated by Simpson's rule with $\Delta x=0.1$. Then the intensities from each scattering angle were weighted according to their attenuation path lengths for some specific geometrical configuration of the dust cloud.

### 2.5 Representative Theoretical Results

The results from the large number of models which I explored are too numerous to be reproduced in detail. Hence in this section I will give only selected examples for representative grain refractive indices, sizes or size distributions, and ranges of scattering angles.

As was pointed out in Chapter I, observed reflection nebula colors vary widely, but observed polarizations are always positive and less than about $25 \%$, and usually show a smooth drop toward shorter wavelength. On these grounds some types of grains and some ranges of scattering angles will be ruled out in this section. The detailed comparisons between theoretical and observational data will be given in Chapter 4.

### 2.5.1 Diclectric grains of a single size

For each of the refractive indices $m^{*}$ listed in the first column of Table 2-2, van de Hulst intensities $i_{1}\left(m^{*}, x, \theta\right)$ and $i_{2}\left(m^{*}, x, \theta\right)$ were computed at intervals $\Delta \theta=5^{\circ}$ from $\theta=0^{\circ}$ to $180^{\circ}$ and at intervals

Table 2-2. Inventory of Mie calculations

| m* | Maximum $x$ at $\Delta x=0.1$ |
| :---: | :---: |
| 1.05 | 8.0 |
| 1.10 | 10.0 |
| 1.30 | 10.0 |
| 1.50 | 7.0 |
| 1.80 | 4.0 |
| 1.10-0.05i | 11.0 |
| 1.10-0.10i | 8.0 |
| 1.30-0.05i | 8.0 |
| 1.30-0.10i | 12.0 |
| 1.30-0.20i | 9.0 |
| 1.30-0.40i | 8.0 |
| 1.50-0.10i | 4.0 |
| 1.65-0.10i | 6.0 |
| 1.80-0.10i | 3.5 |
| 2.20-0.10i | 3.0 |

$\Delta x=0.1$ from $x=0.1$ to the limiting value of $x$ listed in the second column. In most cases the computations were extended at $\Delta x=0.5$ to $x=15$ or larger. The computer output amounted to about 40000 IBM cards .

If all grains are of the same size and the nebula is optically thin, the quantities $i_{1}$ and $i_{2}$ may be converted directly into color and polarization functions by integrating over a range of scattering angles. This was done for almost all combinations of the smallest and largest scattering angles $\theta_{1}$ and $\theta_{2}$ that can be formed at increments of $10^{\circ}$ in each. Some representative results are plotted in Figures 2-5 through 2-8. Here a linear abscissa was necessary to avoid excessive compression of the structure at large values of $x$. Solid curves indicate data computed at $\Delta x=0.1$; dashed curves are used for coarser intervals in $x$, usually $\Delta x=0.5$. All color curves were normalized to $C\left(x^{\prime}\right)=0$ at $x^{\prime}=2$. For grains of any sort the Rayleigh-like domain ( $x$ very small) gives colors that are very blue and polarization curves that start out wavelength-independent at an anount of polarization determined only by the range of scattering angles.

Figure 2-5 gives colors and polarizations for three values of purely dielectric refractive indices, all for the range of scattering angles $10^{\circ} \rightarrow 50^{\circ}$. The curves are very similar except that a given feature (e.g., neutral color, or zero polarization) occurs at a smaller value of $x$ for a larger $m^{\prime}$, and that curves with larger $m^{\prime}$ exhibit more ripples. The ripples, of course, will disappear when a size distribution is applied. Characteristic of forward-scattering dielectric


Figure 2-5. Theoretical Color and Polarization Functions: $10^{\circ}+50^{\circ}$ for Pure Dielectric Grains


Figure 2-6. Theoretical Color and Polarization Functions: $10^{\circ} \rightarrow 50^{\circ}$ for Dirty Dielectric Grains



Figure 2-7. Theoretical Color and Polarization Functions: $60^{\circ} \rightarrow 120^{\circ}$


Figure 2-8. Theoretical Color and Polarization Functions: $140^{\circ}+170^{\circ}$
grains is the rapid, almost linear drop of polarization with increasing $x$ from the Rayleigh value to zero and negative values. If plotted against a logarithmic abscissa, all three curves would be almost parallel in this domain. At larger values of $x$, dielectric grains show a broad domain of negative polarization, and a red (downwardsloping) branch of the color curve occurs only when the polarization is negative. This seems to be a rather general property of angular scattering from predominately dielectric grains comparable in size to the wavelength; very few examples can be found of a strong red color over an appreciable range of $x$ occuring together with positive polarization.

Figure 2-6, still for $10^{\circ} \rightarrow 50^{\circ}$, illustrates the effects of an increasing imaginary part of the refractive index for real part 1.30. The domain of negative polarization disappears, and the range over which the colors are almost neutral is extended. As we approach a metallic grain ( $m^{\prime} \simeq m^{\prime}$ ), the wavelength dependence of polarization tends to disappear.

The case of star centered in the nebula with scattering angles $60^{\circ} \rightarrow 120^{\circ}$ (Figure 2-7) gives polarizations that generally have negative or large positive values, contrary to all observations.

Backscattering at $140^{\circ} \rightarrow 170^{\circ}$ (Figure 2-8) is characterized by very rapid oscillations in both color and polarization. It is easy to see how Roark (1966) went astray (Hanner 1969, page 29) by using Simpson's Rule with only five points for integration over a size distribution. Hanner used $\Delta a=0.013 \mu$ which corresponds to $\Delta x=0.16$ at $1 / \lambda=2.0$; my use of $\Delta x=0.10$ throughout seems to adequately resolve the structure
in these functions. Unlike the ripples found in forward-scattering curves, the spiky polarization structure is not suppressed by adding an imaginary part to the refractive index; it is present even for metallic grains. A close examination of Figure 2-8, however, shows that, without exception, the largest polarization peaks occur near intensity minima. Hence they will be almost completely suppressed when a size distribution is applied.

### 2.5.2 Graphite grains

Theoretical color and polarization functions for an optically thin reflection nebula of pure graphite spheres were computed for the values of $a, x$, and $m^{*}$ listed in Table 2-1 above. Examples of the results for radii $0.03 \mu, 0.06 \mu$, and $0.12 \mu$ are given in Tables $2-3$ through 2-5. Tabulated are polarizations at the effective wave numbers of the standard $\underline{U} \underline{B} \underline{V}$ filters, and intrinsic $\underline{B}-\underline{V}$ and $\underline{U}-\underline{B}$ colors. Ranges of scattering angles that gave polarization at the $\underline{V}$ filter greater than $+50 \%$ are not listed; polarizations more negative than $\mathbf{- 2 0 \%}$ are indicated only by an asterisk. The listed quantities for $a=0.03 \mu$ and $a=0.06 \mu$ were obtained by linear interpolation between two values of $x$. Quantities for $a=0.12 \mu$, which generally showed stronger wavelength dependence, were obtained by quadratic interpolation over three values of $x$. Quantities for which the second-order interpolation term exceeded 0.001 in color or $0.1 \%$ in polarization are given with colons.

For the larger sizes of graphite grains, both the polarization and color functions tend to show an irregular wavelength dependence.

Table 2-3. Computations for graphite grains of radius $0.03 \mu$.

| $\Theta_{1}$ | $\Theta_{2}$ | $\mathrm{P}_{\mathrm{V}}$ | $\mathrm{P}_{\mathrm{B}}$ | $\mathrm{P}_{\mathrm{U}}$ | $\mathrm{B}-\mathrm{V}$ | $\mathrm{U}-\mathrm{B}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 5 | 15 | 1.7 | 1.7 | 1.7 | -1.040 | -0.954 |
| 5 | 25 | 4.1 | 4.0 | 3.9 | -1.040 | -.953 |
| 10 | 30 | 6.7 | 6.6 | 6.5 | -1.039 | -0.952 |
| 10 | 50 | 15.5 | 15.2 | 15.0 | -1.037 | -.950 |
| 10 | 70 | 26.6 | 26.2 | 25.6 | -1.034 | -.947 |
| 10 | 90 | 37.5 | 36.9 | 36.1 | -1.031 | -.942 |
| 10 | 110 | 44.9 | 44.2 | 43.2 | -1.027 | -.937 |
| 10 | 130 | 46.4 | 45.8 | 45.0 | -1.021 | -.929 |
| 10 | 150 | 43.0 | 42.6 | 42.0 | -1.015 | -.922 |
| 10 | 170 | 37.4 | 37.2 | 37.0 | -1.009 | -.914 |
|  |  |  |  |  |  |  |
| 30 | 60 | 32.5 | 32.0 | 31.5 | -1.034 | -0.945 |
| 50 | 170 | 48.4 | 48.7 | 49.0 | -0.995 | -0.897 |
| 80 | 170 | 44.3 | 44.8 | 45.3 | -0.986 | -0.884 |
| 110 | 150 | 41.6 | 42.0 | 42.4 | -0.982 | -0.880 |
| 110 | 170 | 28.2 | 28.5 | 28.7 | -.979 | -.876 |
| 140 | 150 | 20.3 | 20.4 | 20.6 | -0.977 | -0.873 |
| 140 | 160 | 15.1 | 15.3 | 15.4 | -.976 | -.872 |
| 140 | 170 | 11.2 | 11.4 | 11.5 | -.976 | -.871 |
| 160 | 175 | 2.9 | 2.9 | 3.0 | -0.974 | -0.868 |

Table 2-4. Computations for graphite grains of radius $0.06 \mu$.

| $\theta_{1}$ | $\theta_{2}$ | $\mathrm{P}_{\mathrm{V}}$ | $P_{B}$ | $\mathrm{P}_{\mathrm{U}}$ | B-V | U-B |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 15 | 1.6 | 1.5 | 1.4 | -0.895 | -0.519 |
| 5 | 25 | 3.8 | 3.6 | 3.3 | . 893 | . 515 |
| 10 | 30 | 6.2 | 5.9 | 5.4 | -0.891 | -0.512 |
| 10 | 50 | 14.3 | 13.4 | 12.2 | . 885 | . 501 |
| 10 | 70 | 24.4 | 22.7 | 20.3 | . 878 | . 489 |
| 10 | 90 | 34.2 | 31.7 | 27.6 | . 869 | . 478 |
| 10 | 110 | 40.9 | 37.8 | 32.4 | . 858 | . 463 |
| 10 | 130 | 42.8 | 39.6 | 34.0 | . 844 | . 444 |
| 10 | 150 | 40.6 | 37.8 | 32.7 | . 827 | . 421 |
| 10 | 170 | 36.0 | 34.1 | 30.1 | . 811 | . 396 |
| 30 | 50 | 23.7 | 22.4 | 20.5 | -0.878 | -0.488 |
| 30 | 70 | 36.6 | 34.4 | 31.0 | . 868 | . 473 |
| 30 | 90 | 48.5 | 45.3 | 40.0 | . 857 | . 459 |
| 30 | 170 | 43.5 | 41.9 | 38.0 | . 790 | . 363 |
| 50 | 170 | 49.1 | 48.0 | 44.4 | -0.767 | -0.325 |
| 110 | 150 | 42.7 | 41.7 | 37.6 | -0.723 | -0.252 |
| 110 | 170 | 29.3 | 28.8 | 26.5 | . 711 | . 228 |
| 140 | 150 | 20.8 | 20.2 | 18.2 | -0.703 | -0.212 |
| 140 | 160 | 15.5 | 15.1 | 13.7 | . 698 | . 202 |
| 140 | 170 | 11.6 | 11.6 | 10.3 | . 695 | . 195 |
| 130 | 170 | 16.9 | 16.5 | 15.1 | -0.700 | -0.204 |
| 155 | 165 | 6.7 | 6.6 | 5.9 | -0.691 | -0.185 |
| 155 | 175 | 4.4 | 4.3 | 3.9 | . 688 | . 181 |

Table 2-5. Computations for graphite grains of radius $0.12 \mu$.

| $\theta_{1}$ | $\theta_{2}$ | $\mathrm{P}_{\mathrm{V}}$ | $P_{B}$ | $\mathrm{P}_{\mathrm{U}}$ | B-V | U-B |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 30 | 5.2 | 6.4 | 6.4 | -0.385 | -0.298 |
| 10 | 50 | 10.8 | 12.3 | 11.6 | -. 318 | -. .236 |
| 10 | 70 | 15.7 | 15.4 | 13.2 | - . 261 | -. 185 |
| 10 | 90 | 18.4 | 14.3 | 12.0 | - . 226 | -. 147 |
| 10 | 110 | 18.8 | 11.5 | 11.1 | - . 196 | - . 116 |
| 10 | 130 | 18.2 | 9.6 | 11.6 | -. 160 | -. 102 |
| 10 | 150 | 17.4 | 9.0 | 12.3 | -. 119 | -. 112 |
| 10 | 170 | 16.8 | 8.9 | 12.3 | -. 079 | -. 142 |
| 30 | 50 | 18.4 | 22.1 | 22.0 | -0.216 | -0.122 |
| 30 | 70 | 24.6 | 25.5 | 23.3 | - . 143 | -. 046 |
| 30 | 90 | 27.3 | 22.0 | 19.6 | - . 102 | $+.011$ |
| 30 | 110 | 26.9 | 16.4 | 17.4 | - . 068 | +. 056 |
| 30 | 130 | 25.2 | 12.9 | 18.2 | -. 019 | +. 045 |
| 30 | 150 | 23.6 | 11.8 | 19.3 | $+.039$ | +..054 |
| 30 | 170 | 22.3 | 11.6 | 18.8 | +. 095 | -. 006 |
| 50 | 70 | 35.0 | 83.9 | 29.1 | -0.005 | +0.136 |
| 50 | 90 | 36.5 | 24.0 | 18.2 | +. 030 | +.21: |
| 50 | 110 | 33.9 | 13.0 | 13.4 | $+.066$ | +.21: |
| 50 | 130 | 30.3 | 6.9 | 15.5 | $+.132$ | +. 297 |
| 50 | 150 | 27.3 | 5.1 | 18.0 | $+.217$ | +. 237 |
| 50 | 170 | 25.0 | 4.9 | 17.1 | +. 299 | +.11: |
| 80 | 110 | 30.9 | * | -18.3 | +0.161 | +0.52: |
| 80 | 140 | 22.0 | * | + 1.2 | $+.358$ | +.46: |
| 80 | 170 | 17.2 | * | + 6.7 | +. 574 | +.07: |
| 110 | -140 | 8.9 | * | +39.0: | +0.70: | +0.37: |
| 110 | -170 | 5.8 | * | +26.1: | +.44: | -.42: |
| 130 | 150 | 3.1 | * | +47.8: | +1.32: | -0.54: |
| 130 | 170 | 3.2 | * | +26.6 | +1.63: | -1.18: |
| 140 | 150 | 2.0 | * | +40.9: | +1.62: | -1.0: |
| 140 | 160 | 1.5 | * | +29.5 | +1.8: | -1.4: |
| 140 | 170 | 1.2 | * | +20.8 | +1.9: | -1.6: |
| 150 | 170 | +0.7 | . -16.0 : | +13.9 | +2.2: | -2.0: |

In no case could I find polarization functions rescmbling the observations of reflection nebulae.

For graphite grains with $a=0.03 \mu$, and to a lesser extent at $a=0.06 \mu$, the colors are strongly blue and at all scattering angles the polarization is very nearly independent of wavelength; the angular scattering properties of such small grains are essentially those of Rayleigh scattering. Equations given by Hanner (1969, page 50) show that even for anisotropic, highly non-spherical graphite grains with random orientation, the approximation a << $\lambda$ gives intensity proportional to $(1 / \lambda)^{4}$ and wavelength-independent polarization at all scattering angles.

Even Rayleigh-scattering models can be contrived in which internal attenuation causes the polarization to vary with wavelength. For example, consider the case of a cylinder in which scattering angles range from $20^{\circ}$ to $80^{\circ}$. The deeper (closer to $90^{\circ}$ ) scattering angles, which have higher polarization, will suffer less attenuation at longer wavelengths. Thus the mean scattering angle, and hence the polarization, will increase with wavelength. If the star were in front of the nebula, e.g., with scattering angles $100^{\circ}$ to $160^{\circ}$, attenuation losses would bring about the opposite wavelength dependence. Optically thick models for graphite grains will not be quantitatively explored here, since, as will be shown in Chapter 4, dielectric grains give a natural fit to the observations of polarization without the necessity of introducing high optical depth.

Hanner (1969, page 87) states that the polarization of the Merope nebula, about $10 \%$, is "much too small to be fit by the polarization of small graphite particles, no matter what the geometry may be." Such a statement refers to a restricted range of models and cannot be taken literally. For exanple, Table $2-4$ shows that for pure graphite grains scattering at $155^{\circ} \rightarrow 165^{\circ}$, a range that is found in Chapter 4 to be about right for IC 5076, the polarization is well under $10 \%$. It is the wavelength dependence of polarization that most clearly rules out graphite grains, not its amount.

### 2.5.3 Metallic grains

Colors and polarizations for grains with $\mathrm{m}^{*}=1.4-1.4 \mathrm{i}$ were computed for $x$ from 0.1 to 10.0 in steps of 0.1 and for a complete set of ranges of scattering angles. No size distributions were applied.

The Rayleigh-like domain, where colors are very blue and polarizations are wavelength-independent, holds up to about $x=1.0$. For larger sizes with forward scattering ( $\theta$ less than $90^{\circ}$ ) the colors show a smooth transition through neutral to red, and the polarization generally increases slowly toward larger values of $x$. A few examples are plotted in Figure 2-9. The case $30^{\circ} \rightarrow 50^{\circ}$ illustrates one of the few places in my models where a strong red color is found together with a positive polarization. For backscattering (e.g., $140^{\circ} \rightarrow 170^{\circ}$ ) both polarizations and colors oscillate rapidly between positive and negative values. No case was found of a polarization less than $20 \%$ and dropping toward larger values of $x$.


Figure 2-9. Theoretical Color and Polarization Functions:
Metallic Grains
2.5.4 Dielectric grains in a size distribution

Both the EXP and the OHG size distributions (Section 2.1.3) were applied to the Mie data for most of the refractive indices in Table 2-2, and colors and polarizations were computed for all ranges of scattering angles as described above. The results are illustrated from $m^{*}=1.30$ in Figures $2-10$ through 2-12. All color curves are normalized to $C=0$ at $x=2$. The symbol " $\delta$ " is used to indicate the Dirac Delta distribution, i.e., grains all of the same size. It is clear that in the Rayleigh domain the colors and polarizations are affected not at all by the presence of a size distribution.

Figure 2-10 is constructed for a range $\theta_{1} \rightarrow \theta_{2}$ of $10^{\circ} \rightarrow 50^{\circ}$; a range of angles similar to this is frequently encountered in reflection nebulae. The differences between the OHG, EXP and $\delta$ distributions are surprisingly small, and are qualitatively explained by the fact that the OHG emphasizes small grains and the EXP large ones (Figure 2-1). For example, the OHG distribution gives a bluer color than the $\delta$ or EXP distributions, and requires a larger value of $x_{0}$ before the polarization begins to drop from its Rayleigh value. The power of a logarithmic abscissa is well demonstrated here; if the polarization plot were made on a linear scale, the essential parallelism of the two solid curves would no longer be evident.

Figure 2-12, for $150^{\circ} \rightarrow 170^{\circ}$, shows that the violent oscillations characteristic of back-scattering grains survive only as relatively gentle undulations and inflections when a size distribution is applied.


Figure 2-10. Theoretical Functions for Size Distributions: $10^{\circ} \rightarrow 50^{\circ}$



Figure 2-11. Size Distributions: $\quad 60^{\circ} \rightarrow 120^{\circ}$


Figure 2-12. Size Distributions: $150^{\circ} \rightarrow 170^{\circ}$

Figure 2-13 illustrates the effect of a changing refractive index at $10^{\circ} \rightarrow 50^{\circ}$ for grains in the OHG distribution. At $x_{0}=0$ all refractive indices give $P=15.9 \%$, the Rayleigh value for this range of scattering angles. Once the polarization drops below about $13 \%$, all four polarization curves are essentially parallel, that is, almost identical except for a shift in $x_{0}$. The parallelism can be further improved by slight adjustments in the range of scattering angles and addition of a small imaginary component to the larger refractive indices.

Hanner (1969) has computed colors and polarizations for silicate grains ( $\mathrm{m}^{*} \simeq 1.65-0.10 \mathrm{i}$ ) in an OHG distribution with $\mathrm{a}_{0}=0.25 \mu$. She reported that such grains give colors and polarizations "quite different" from ice grains with $m^{*}=1.30$ and $a_{0}=0.50 \mu$. In particular, negative polarizations would be found at many scattering angles. Figure 2-13, however, shows that the qualitative differences between ice and silicate grains are simply a matter of the scale sizes $a_{0}$ chosen. If she had used a smaller $a_{0}$ for silicates, the negative polarizations would not have appeared.

### 2.5.5 Effects of internal attenuation losses

The effects of non-zero optical depth depend altogether on the overall configuration of the nebula, which is not determined in the present models. I can only assume a geometry that seems appropriate for a particular nebula. Here $I$ will give a single example, which, as will be shown in Section 4.2, should apply to the region I observed in IC 5076.


Figure 2-13. Polarization Functions for Various Refractive Indices in the OHG Distribution

Suppose that we have grains with $\mathrm{m}^{*}=1.30$ and an OHG size distribution in a backscattering nebula with $150^{\circ} \leq \theta \leq 165^{\circ}$. Assume that the total attenuation of each $\theta$, before and after scattering, is proportional to the geometrical depth in the nebula at which the scattering occurs. Figure $2-14$ gives results from $\tau=0$ and $\tau(2)=0.5$, where $\tau(2)$ is the total optical depth at the deepest point in the nebula ( $\theta=165^{\circ}$ ) and at $x_{0}=2$. The $x_{0}$-dependence of the opacity was computed by integration over $\mathrm{Q}_{\text {ext }}(\mathrm{x})$ as described in Section 2.1.4. Multiple scattering is ignored.

The polarization function is affected only by the different extents to which light from different scattering angles is attenuated. The case $\tau(2)=0.5$ for $150^{\circ} \rightarrow 165^{\circ}$, for example, is very similar to the case $\tau=0$ for $150^{\circ} \rightarrow 160^{\circ}$. The colors are affected not at all for small $x_{0}$, but are reddened by nearly a magnitude between $x_{0}=2$ and $x_{0}=4$.


Figure 2-14. Effect of Transmission Losses at $150^{\circ} \rightarrow 165^{\circ}$

## CHAPTER 3

## OBSERVATIONS OF REFLECTION NEBULAE

In most observational studies of reflection nebulae, the emphasis has been placed on collecting a limited amount of data in each of a large number of regions within one nebula. By contrast, I have tried to measure colors and polarizations within only a few selected nebular regions but over the widest practible wavelength range and to the highest precision attainable with a rather generous allotment of telescope time. It remains uncertain which of the two strategies is the more profitable. As pointed out at the beginning of Chapter 2, each is tailored to a distinct type of theoretical model, and a choice between the two is, in part, a matter of taste. A really definitive model is not likely to emerge until the two observational approaches can be combined.

The best-known reflection nebula is that associated with the star Merope in the Pleiades. Since this nebula has been extensively studied in the past ( $O^{\prime}$ Dell 1965, Elvius and Hall 1966, 1967, Roark 1966, Dahn 1967, Artamonov and Efimov 1967), it was not included in this work.

### 3.1 Instrumentation

Unless otherwise noted, all observations were taken at the 154-cm (61-inch) Catalina reflector of the Lunar and Planetary Laboratory. The NGC 7023 data (Section 3.6) were taken at the new $229-\mathrm{cn}$ reflector of Steward Obscrvatory. My observations ranged in time from November 1967 through the fall of 1969.

Both colors and polarizations were measured with the two-channel photoelectric polarimeter which has been described by Gehrels and Teska (1960) and by Coyne and Gehrels (1967). The configuration is essentially unchanged since the latter report. Two photomultiplier tubes, simultaneously feeding separate current integrators, measure the orthogonally polarized beams energing from a Wollaston prism. At the end of a preset integration time the signals in each channel are sequentially read by a digital voltmeter and punched on paper tape. The tubes and prism are rotated together about the photometer axis to analyze the light at various position angles. Immediately before or after each integration, the relative sensitivities in the two channels are calibrated by an additional integration with a Lyot depolarizer in the beam.

Also punched on the tape are the object identification code, the orientation angle of the Wollaston prism, the universal time, the integration time, and codes for the filter, star/sky, and depolarizer states. The integrated signals are also recorded on a strip chart, which provides a diary for the observer and a redundancy in case of failure of the digital gear. Two types of paper tape punches have been used, a Friden SP-2 and a Tally Model 420. The Friden punch has been found to require less maintenance under extended service.

Two sets of detectors were used, with RCA 7102 photomultipliers (S1 response) for the red and near infrared and a variety of 13-dynode EMI tubes ( $6255 \mathrm{~S}, 6255 \mathrm{~B}, \mathrm{D} 205 \mathrm{R}$, and 9634 QR ) with S13 response (S11 cathode with a quartz envelope) for the blue end of the spectrum. All detectors were refrigerated with dry ice. The S 13 tubes were operated at 900 to

1200 volts. The 7102 tubes were found to give the best signal-to-noise ratio at $1450 \pm 50$ volts, at the expense of a generally short tube life; 1250 volts is probably the maximum safe potential for these tubes.

For colors and polarizations of reflection nebulae, I used six intermediate- to wide-band filters, which have been described in detail by Coyne and Gehrels (1967). Their letter designations and effective wave numbers in waves per micron for reddened B stars are listed in Table 3-1. The filter I is included for completeness; it was used in this program only for a bit of reconnaissance. An additional blue filter $B_{m}$, described by Coyne and Pellicori (1970), was used for some of the photometry. It has effective wave number almost identical to the $B$ filter of Table 3-1, but with less than half its bandwidth. The filter N is limited on the short wavelength side by atmospheric ozone absorption; its red leak is suppressed by a solution of nickel sulfate, and that of the $U$ filter by a solution of copper sulfate.

The filters $U, B, G, 0$, and $R$ span a range similar to that of the standard $\underline{U} \underline{B} \underline{V} \underline{R} \underline{I}$ system (Johnson 1965), but the responses are not identical. (The standard filter system will here be distinguished by underlining.) Where comparisons with data taken in the standard $\underline{U} \underline{B} \underline{V}$ system are required, effective wave numbers of $2.78,2.27$, and 1.82 per micron will be adopted for the standard filters; for all filter systems of this type, the effective wave numbers will vary perhaps $\pm 0.03 \mu^{-1}$ according to the apparent color of the source.

The well-known ability of a two-channel polarimeter to suppress seeing noise (liiltner 1962) is of little value on objects as faint

Table 3-1. Filters and instrumental polarization.

| Detector | Filter | $1 / \lambda^{\text {a }}$ | Color | Instrumental | tion |
| :---: | :---: | :---: | :---: | :---: | :---: |
| S13 | N | 3.03 | UV | $0.07 \pm 0.02$ | 180 |
| S13 | U | 2.78 | UV | 0.10 . 01 | 179 |
| S13 | B | 2.33 | Blue | 0.12 . 01 | 182 |
| S13 | G | 1.93 | Green | 0.12 . 00 | 184 |
| S1 | 0 | 1.56 | Orange-red | 0.11 . 00 | 176 |
| S1 | R | 1.21 | IR | 0.08 . 01 | 176 |
| S1 | I | 1.06 | IR | 0.08 . 02 | 176 |

a. Effective wave number, waves per micron.
$(\underline{V}=13.5)$ as selected regions of reflection nebulae, and its ability to work through clouds is limited to objects so bright that no sky measures need be made. However, a considerable advantage derives from the property that every polarization measurement made at any analyzer orientation with a two-channel instrument is completely independent from all other measures. Thus data from different nights, different seasons, and even, in principle, different telescopes can be combined into one solution for polarization. Much of the data presented here were obtained over many nights in two observing seasons before being combined for the final solutions.

### 3.2 Instrumental Effects

### 3.2.1 Instrumental polarization

In each filter more than fifty measures of unpolarized stars ( $\mathrm{P} \leq 0.03 \%$ ) have been made since the realuminization of the telescope optics in October 1967. The results were averaged to give the instrumental polarizations listed in Table 3-1. The probable errors, which are on the order of $\pm 0.01 \%$, give the scatter between averages taken at three epochs between December 1967 and May 1969; no systematic time dependence has been found.

Actually, the measured instrumental-polarization quantities do not strictly obey the Malus Law, and cannot be fully described by a degree of polarization at a position angle; the corrections must be applied individually for each Wollaston angle. This behavior means that at least part of the apparent instrumental polarization is mechanical or
electronic rather than optical in origin. The differences in instrumental polarization using this equipment on various telescopes give the impression that perhaps $80 \%$ of the amounts listed in Table 3-1 arise in the polarimeter rather than from the telescope optics. Possible sources are mechanical flexure or differential reflection losses of orthogonal planes of polarization at the last quartz-to-air surface of the depolarizer.

### 3.2.2 Precision

Individual measures on bright stars with this equipment routinely give internal probable errors less than $\pm 0.02 \%$. This high precision, however, is illusory. At all levels of polarization, repeated measures of bright stars disagree with a RMS scatter of about $0.07 \%$. The source of this instability is unknown, but it does not seem to be of atmospheric origin; any correlation of measured polarization with zenith distance does not exceed $0.03 \%$ at two air masses.

Stellar polarization measures made with this equipment have been compared by Coyne and Wickramasinghe (1969) with measures made elsewhere. The differences are inevitably somewhat inflated by the variety of filter systems used, but show that systematic errors in the LPL data are probably less than $\pm 0.05 \%$.

Since my reflection nebula measures generally have internal probable errors larger than $\pm 0.30 \%$, I will list only the internal errors.

### 3.2.3 Depolarizer performance

All polarizations are multiplied by 1.004 to correct for the measured polarization deficiency obtained (Coyne and Gehrels 1966) when a picce of Polaroid is placed in front of the photometer.

Since a lyot depolarizer works by modifying the waveform of polarized light as a function of wavelength, it will fail to depolarize effectively if used with too narrow a filter. Contrary to a popular impression, however, it does not introduce polarization to unpolarized emission lines; being simply a series of linear retarders, it can have no effect (aside from design flaws) on natural light of any spectral character. I checked this property by measures on Vega ( $\mathrm{P} \leq 0.02 \%$ ) with the green filter and with a combination of interference filters that gave a passband of less than $10 \AA$ at roughly the same effective wavelength. The two measures agreed to within $\pm 0.01 \%$. Thus, unpolarized emission lines from a reflection nebula can at most dilute the polarizations produced by the grains. Night-sky emissions are subtracted out along with the sky measures.

### 3.2.4 Halation

For nebular regions that are close to a bright star, some starlight will be scattered into the focal-plane diaphragm by the earth's atmosphere and by the telescope and photometer optics. This symmetrical flare about a star will here be called halation, by analogy with the turbidity effects in a photographic emulsion. It should not be confused with ghost images of an off-axis star or with diffraction spikes from the secondary-mirror support vanes, which I avoided for all nebular regions. Elvius and Hall applied large corrections for halation on the inner regions of the Merope nebula, and smaller ones on NGC 7023.

With the instrumentation used here the halation can be readily detected at several minutes of arc from a third-magnitude star, despite careful
baffling in the telescope and the photometer. Its intensity is enhanced when the direct starlight is able to strike the depolarizer; this contribution could have been removed by placing the depolarizer slide below the diaphragm slide.

The halation intensity as a function of offset angle, as measured on $\delta$ Leonis in February 1968, is plotted in Figure 3-1 for the blue filter. The ordinate is magnitudes per square second of arc below the star. The halation is a bit stronger in the ultraviolet and nearly a magnitude fainter in the infrared, with much the same offset dependence; the wavelength dependence is discussed in detail in Section 3.5.3 below. It seems to be polarized around $5 \%$, but no detailed polarization measures were made.

Also plotted in Figure 3-1 are the nebular intensities of my Region I in NGC 2068 (Section 3.4.4) and my region in IC 5076 (Section 3.5.2). Due to the faintness of the stars in NGC 2068, the halation intensity was negligible compared to the nebular intensity, and no corrections were applied. Substantial corrections had to be applied for the region in IC 5076 and two regions in NGC 7023; details are given below along with the nebular data.

Hanner's (1969, page 4) emphasis on the difficulties of correction for halation is probably overstated. I found the time variation of halation, either night-to-night or as a function of zenith distance, to be not more than a few percent. The observations necessary to correct for the effect are straightforward and in most cases require only about half an hour per filter.


Figure 3-1. Instrumental Halation at the Catalina Reflector

### 3.3 Observational Techniques

### 3.3.1 Guiding

The nebular regions were located by setting a faint field star to the edge of a larger focal-plane diaphragm, then changing to the program diaphragm size and trusting the telescope drive for the duration of each integration. This procedure is less subject to observer error than the conventional blind-offset technique, and it gives a repeatability not worse than two arcseconds, which is competitive with most offset guiders. The position of the region with respect to the illuminating star was then obtained from large scale direct photographs kindly supplied by Dr. E. Roemer. The sky regions could be located with sufficient precision by visual inspection.

### 3.3.2 Polarization

At each Wollaston angle the observing sequence was ordinarily as follows: sky-neb-neb-sky-sky-neb-neb-sky, each with depolarizer in and out. The integration time was usually 30 seconds at each setting, for a total integration time of eight minutes per Wollaston angle. Then a computer program (originally written by Dr. D. L. Coffeen for twilight polarimetry of Venus) fitted a least-squares straight line to each of the four kinds of observations and interpolated values at the mid-point of the sequence, for further analysis. With this procedure, five or six polarization measures can be obtained per hour. The sequence seems to be short enough to handle variations in airglow intensity and atmospheric transparency; no particular problems were encountered with a nebular
brightness only a few percent of the sky brightness. If the sky was stable and no brighter than the nebula, the above sequence was cut in half and an integration time of 60 seconds was used, for a yorth-while saving in the time spent moving the telescope.

Such polarization measures were taken at six setting angles of the Wollaston prism 30 degrees apart, and the sequence of angles was repeated, often over several nights, until adequate precision was achieved. This meant 30 to 50 angles for the bright filters and 80 to 120 for the ultraviolet and infrared, for a total integration time of around 40 hours per region. This figure should be kept in mind when comparisons are made with data by Elvius and Hall (1966, 1967), who were limited to no more than three hours of telescope time per region (Perkins reflector) in order to survey many regions in each nebula.

A set of computer programs derived from the raw data the degree of polarization and the position angle that give the best least-squares fit to the Malus Law, as described by Gehrels and Teska (1960). The internal probable errors in polarization and in position angle were computed independently.

### 3.3.3 Color

The relevant observable quantities in one region of a nebula are (1) the polarization as a function of wavelength and (2) the ratio of nebular brightness to the brightness of the star as a function of wavelength. A plot of (2), with the brightness ratios expressed in magnitudes, will here be called a color curve; it contains more information than a
tabulation of intrinsic nebular colors such as $(\underline{B}-\underline{V})_{n}-(\underline{B}-\underline{V})_{*}$, which gives only the shape of the color curve and not the brightness level. The actual color of the illuminating star, apparent or intrinsic, is irrelevant. We must assume the foreground reddening and extinction to be the same for the star and the nebula; since the apparent colors of both are often dominated by interstellar reddening, this assumption can in special cases break down completely. Only the effective wavelength of a filter-detector combination matters; its shape and width make little difference so long as there are no shifts in effective wavelength between the star and the nebula. Converting to a standard filter system such as $\underline{U} \underline{B} \underline{V}$, while useful for comparisons with other data, adds no physical information.

To construct an observational color curve, we measure in each filter the brightness of the illuminating star and the brightness of the nebula, and take the difference in magnitudes:

$$
\Delta M(\lambda)=-2.5 \log \frac{I_{n}(\lambda)}{I_{*}(\lambda)}
$$

As usual, a more positive $\Delta M$ means a fainter object. Some writers have given reflection nebula intrinsic colors in the sense star-minusnebula, which is opposite to the sign convention adopted here. Here all color curves will be plotted with $\Delta M$ increasing downward, so that a curve rising toward the ultraviolet will represent a nebula of blue intrinsic color. For reasons explained in 2.2.3, wavelength dependencies will usually be plotted with wavenumber $1 / \lambda$ on a logarithmic scale as the abscissae.

Magnitude differences with respect to the star will be given for the whole area of the region worked, but can be converted to magnitudes per square arcsecond fainter than the star by addition of a constant. Such absolute brightness ratios may be uncertain by $\pm 0.02$ magnitude duc to errors in the adopted diaphragm sizes and the focal-plane scale, but this source of error has no effect on the shape of the color curve, since for each region the same diaphragm was used for all filters.

The color observations were made as the polarization observations described above, except that the Wollaston prism was fixed and the depolarizer was kept in the beam. Measures were taken on the illuminating star immediately before and after each set of nebular measures, with changes in the integrator gains and integration time where necessary. Since all regions were within four minutes of arc of the illuminating star, no corrections for atmospheric extinction were applied other than linear interpolation in time between the star measures. No significant differences between the brightness ratios in the two photometer channels were ever found, which speaks well for the effectiveness of the depolarizer. As many as twelve independent transfers between star and nebula were made per filter, for a total integration time of around ten hours of photometry per region.

### 3.3.4 Sky measurements

Since the surface brightness of a reflection nebula may be four magnitudes or more below that of the night sky, especially in the near infrared, careful sky corrections are essential. Atmospheric emissions are particularly troublesome at wavelengths longer than about $0.77 \mu$,
beyond which a series of bright oll emission bands, together with a strong continuum, appear. Quantitative spectrophotometry of the airglow from $0.31 \mu$ to $1.0 \mu$ has been provided by Broadfoot and Kendall (1968). The R filter used here, with effective wavelength near $0.83 \mu$, contains the $(5,1),(6,2),(7,3)$, and part of the $(8,4)$ bands of of within its halfintensity limits. There are relatively dark regions near $0.82 \mu$ and $0.91 \mu$, but a filter designed to reject all the major emission bands would probably be too narrow for good depolarizer performance.

For sky readings I selected nearby regions that seem to be free of stars and nebulosity on the Palomar Observatory Sky Survey prints. Such regions were always in the same dust-cloud complex as the reflection nebula, so that even the foreground and background diffuse galactic light should be largely taken out.

Elvius and Hall (1967) reported that reliable observations of the Merope nebula could not be made with the moon above the horizon or even during lunar twilight. This may be a consequence of the large angular extent of the Pleiades nebulosity, which requires that dark sky regions be a considerable distance away and hence measured infrequently. I found no degradation of data with a moon four or five days from new in a different part of the sky. The advent of solar (astronomical) twilight can immediately be seen as a change in the sky polarization, but the observing sequence allows observations to be continued for 15 to 20 minutes into twilight. Even though all my observations were taken at 2.5 air masses or less, I did find loss of precision with increasing zenith distance, both in the ultraviolet, duc to atmospheric extinction, and in the infrared, duc to sky emissions.

### 3.3.5 Interstellar polarization

To obtain intrinsic nebular polarizations, the interstellar (foreground) polarization effects must be taken out. The best one can do is to take the interstellar component to be the same as the polarization observed on the illuminating star. This assumption can fail since interstellar polarization can change abruptly over small distances. Also, intrinsic stellar polarization is more common than was realized until recently. Fortunately, interstellar polarizations are generally much smaller than intrinsic nebular polarizations, and small uncertainties in the interstellar component do not cause much damage.

Even if the precise interstellar component were known, however, its exact decoupling from an observed polarization is not a trivial matter. An analytical solution for the polarization produced by two partial analyzers, each with zero optical depth in one plane of polarization, has been obtained after considerable labor by Loden (1961). In the interstellar medium, attenuation is present in both planes of polarization, and the problem has not been solved rigorously to my knowledge; while the principles of combining partial polarizers are simple, the algebra rapidly becomes intractable. The common practice for small polarizations (e.g., Coyne and Kruszewski 1968) has been to resolve the observed and interstellar polarizations into orthogonal components

$$
\begin{aligned}
& P_{x}=P \cos 2 \theta, \\
& P_{y}=P \sin 2 \theta,
\end{aligned}
$$

and subtract the interstellar components from the observed components. It is shown in Appendix II that, while not strictly valid, this procedure is a good approximation for small polarizations, and it will be adopted
here. Note that this is not a vectorial subtraction. Nlso derived in Appendix II are formulae for the propagation of probable errors through the analysis.

### 3.4 NGC 2068

NGC 2068 is imbedded in a large, thick dust cloud a few degrees northeast of the belt of Orion. Among the brightest of reflection nebulae, it is the only one (M78) included in the Messier catalog. At my request, Dr. E. Roemer kindly obtained a direct photograph of NGC 2068 at the Catalina reflector (Figure 3-2). The exposure was 30 minutes, reaching roughly magnitude 18. The large-scale plate reveals a mottled and wispy structure; apparently the foreground extinction is very uneven. There seems to be a bubble or clear space around one of the stars-or is it a foreground dust globule? We will see in Section 3.4 .4 that the latter explanation is probably correct.

### 3.4.1 Illumination

The nebula contains two eleventh-magnitude B stars, HD 38563 A and B, and HD 38563 C a few magnitudes fainter. Four or five others are near the limit of visibility in the 6l-inch. Sharpless (1952) suggested that, although Star A is the brighter and more centrally located, it is actually Star B, at the northern edge of the nebula, that provides the illumination. A heavy foreground dust lane cuts the nebula almost in half, and in fact a bit of nebulosity is visible beyond the dust lane on long-exposure photographs. This conjecture was confirmed by photographic polarimetry by Glushkov (1965) and photoelectric polarimetry by


Figure 3-2. NGC 2068

Elvius and Hall (1966). Throughout the nebula, even very close to Star A, the position angles of polarization show a marked tendency toward positive radial symmetry about Star B. Star A, having much less color excess than Star B, is apparently in the foreground.

Sharpless established the spectral type of Star B as B1V, early enough that some emission is to be expected from the nebula. A. Stockton and G. Chapman (personal communication) found the reflected spectrum to be characteristic of an early B star, but with weak emissions at $\lambda 3727$ of ( $0^{\mathrm{II}}$ ) and at HB , and central emission in $\mathrm{H} \gamma$. H. M. Johnson (1960) found the $H B$ emission in NGC 2068 to be measurable but less intense than in the reflection nebulae NGC 7023 and IC 348. From wide- and narrowband photographic photometry near $H \alpha$, Glushkov concluded that not more than $5 \%$ of the continuous spectrum of the nebula can be attributed to recombination and two-photon emission of hydrogen.

Star $B$ is a visual binary with two seconds of arc separation and $\Delta M_{V} \simeq 1.5$ magnitudes. Individual spectral types are not known, but the brightness difference means that the fainter component is probably not later than B3. Together with Star A, the system is listed as number 2964 in Burnham's (1906) double-star catalog and number 4374 in that of Aitken (1932). The reader should be cautioned that the letter designations introduced by Burnham were not followed by Johnson or Glushkov, and both differ from the notation of Sharpless, which was adopted by Elvius and Hall and is used here. Star B seems to be behind about five magnitudes of extinction, with a $\underline{B}-\underline{V}$ color excess of about 1.3 magnitudes (Lee, 1968). Star A was classified as B5 by Hubble (1922), but
is 0.2 magnitudes brighter than Star B. If both stars are dwarfs, then roughly 2.8 magnitudes of the extinction in front of Star $B$ must be relatively local. It was pointed out by Sharpless and confirmed by Lee that the interstellar reddening law in the direction of NGC 2068 is much closer to the "normal" law, with $R \leqslant 4$, than for the Orion aggregate as a whole.

The field stars on the Roemer plate were located with respect to Star B with a traveling microscope; since this binary star was overexposed, distances were measured from the light center of the system. The positions of the field stars are listed in Table 3-2 and plotted in Figure 3-3. The lettering system is an extension of that used by Sharpless. Distances should be precise to within $\pm 0.3$ arcsecond, but the orientation of the plate was uncertain by $\pm 1^{\circ}$; in my coordinate system the position angle of Star A with respect to Star B is $202.6^{\circ}$, versus $202.0^{\circ}$ as the mean of five measurements listed by Aitken.

### 3.4.2 Regions

Also plotted in Figure 3-3 are the two regions which I studied. The regions are described in Table 3-3; the areas are given in square seconds of arc, but expressed in magnitudes for easy computations of surface brightness. The quantity $\theta_{\text {geom }}$ is the position angle (measured eastward from north) of the normal to the plane through Star B, the observer, and the center of the region. The dashed circles in Figure 3-3 illustrate the region " $7.2 \mathrm{~mm} \mathrm{~S}, 7.1 \mathrm{~mm}$ E" of Elvius and Hall (1966) and the region worked by II. M. Johnson (1960). Region I was originally

Table 3-2. Stars in NGC 2068.
Positions are in seconds of arc from Star B; $X$ is measured eastward, $Y$ northward, and $R$ is the radial distance.

| Star | X | Y | R |
| :---: | :---: | :---: | :---: |
| A | -19.5 | -46.8 | 50.7 |
| B | 0. | 0. | 0. |
| C | +9.9 | -163.2 | 163.5 |
| D | +83.2 | -43.9 | 94.1 |
| E | +44.6 | -50.5 | 67.4 |
| F | -16.0 | +5.4 | 16.9 |
| G | -65.6 | -28.7 | 71.6 |
| Region I | +63.9 | -47.2 | 79.4 |
| Region II | +9.9 | -114.5 | 114.9 |



Figure 3-3. Stars and Regions in NGC 2068

Table 3-3. Regions in NGC 2068.

|  | Region I | Region II |
| :---: | :---: | :---: |
| Location | Halfway ( $\pm 1^{\prime \prime}$ ) between Stars D and E | 49" ( $\pm 2^{\prime \prime}$ ) north of Star C |
| Diameter | 30.8" | 40.7" |
| Area (magnitudes) | 7.18 | 7.79 |
| Radial distance from Star B | $79.4{ }^{\prime \prime}$ | 115" |
| $\theta_{\text {geom }}$ | $36.5{ }^{\circ}$ | $85.1{ }^{\circ}$ |

intended to duplicate exactly the Elvius and Hall region, but was moved over to avoid Star E.

The sky was measured about 1.5 arcminutes dircctly north of Star B, in a region (the foreground dust belt) apparently free of stars and ncbulosity on the Palomar prints. In wavelength bands where the signal was dominated by sky emissions ( 0 and $R$ ), this sky region was alternated with one 6 minutes north of the nearby nebula NGC 2071, though no systematic difference was found.

No corrections for halation need be applied for my regions in NGC 2068. Even in the infrared, where the nebula is faint, the scattered light is five magnitudes fainter.

### 3.4.3 Polarization

Polarizations in Region I were measured on fifteen clear nights between 30 November 1967 and 30 March 1968, and on four nights in September and October 1968, for a total of 39.2 hours of actual integration time. The results, as yet uncorrected for interstellar polarization, appear in Table 3-4 and Figures 3-4 and 3-5. The polarization is positive and increases uniformly toward longer wavelength. There is a small but definite rotation of position angle with wavelength, and at the shorter wavelengths the position angles depart significantly from the normal to the plane of scattering.

Also plotted in Figures 3-4 and 3-5 are the polarizations observed by Elvius and Hall for their partially overlapping region (Figure 3-3). Their ultraviolet position angle of $76^{\circ}$ is far outside the range of the plot. My polarizations are systematically larger than theirs,

Table 3-4. Polarizations in NGC 2068 Region I.

| Filter | $1 / \lambda$ | $P \% \pm p e$ | $\theta \pm \mathrm{pe}$ | $\mathrm{T}^{\mathrm{a}}$ | Dates UT |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| U | 2.78 | 5.15 | 0.28 | 41.9 | 1.6 | 536 |
| B | 2.33 | 7.66 | 0.21 | 41.3 | 0.8 | 122 |
| G | 1.93 | 9.04 | 0.48 | 39.5 | 1.5 | 404 |
| 0 | 1.56 | 10.16 | 0.41 | 37.1 | 1.2 | 432 |
| R | 1.21 | 14.28 | 0.67 | 36.7 | 1.4 | 862 |

a. Total integration time in minutes.


Figure 3-4. Polarization in Region I of NGC 2068
Filled circles with error bars are new data; open circles are data from Elvius and hall (1966), crosses are from Johnson (1960), and the triangle is from data by Glushkov (1965).


Figure 3-5, Position Angles of Polarization in Region I of NGC 2068 Symbols are as in Figure 3-4.
by up to ten times my internal probable errors. The discrepancy is much too large to be attributed to contamination by Star E , or to a poor choice of sky region; if due to non-uniformity of the nebula, the effect is surprisingly large. Actually, the polarizations of Table 3-4 are larger than those reported by Elvius and Hall for any region of NGC 2068. In a region of NGC 7023, Gehrels (1967) found polarizations larger, by nearly this same factor, than those reported by Elvius and Hall for the same region (Section 3.6.2). This makes one wonder if a systematic error has slipped in somewhere, but it is difficult to imagine what it might be.

The crosses in Figures 3-4 and 3-5 give the polarizations reported by Johnson in his much larger region. The agreement is surprisingly good considering the modest equipment with which his work was done. His position angles, however, varied widely over wavelength. Data by Glushkov (1965, taken from his Figure 1), for a region of diameter 12" located entirely inside my Region $I$, are plotted as triangles, assuming an effective wavenumber of $2.3 \mu^{-1}$.

Due to limited telescope time, no color or brightness data were taken in Region II, and its polarizations were obtained only in the blue and red. The signal-to-noise ratio is better for this region, and five hours of integration time, mostly in December 1968, gave good results in these two filters. The results are listed in Table 3-5. The position angles are quite close to that of the normal to the scattering plane $\left(85^{\circ}\right)$. This region is more distant from the illuminating star than any area studied by Elvius and Hall; we see that, at least in this direction,

Table 3-5. Polarizations in NGC 2068 Region II.

| Filter $1 / \lambda$ | $\mathrm{p} \% \pm \mathrm{pe}$ | $\theta \pm \mathrm{pe}$ | T |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| B | 2.33 | $11.0 \pm 0.2$ | $85.8 \pm 0.5$ | 112 |  |
| 0 | 1.56 | 14.7 | 0.4 | 83.9 | 0.8 |

the polarization continues to increase with offset angle instead of flattening at about 75 arcseconds distance as suggested in their Figure 8 (1966). For this part of the nebula Glushkov reported about $9.5 \%$ polarization, somewhat smaller than my value in blue light.

### 3.4.4 Color

The brightness of Region I was measured relative to Star B on six nights between November 1968 and February 1969, for a total integration time of 7.0 hours. The differential results are given in Table 3-6 and the color curve is plotted in Figure 3-6. The values refer to the whole area of the region, but may be converted to magnitudes per square arcsecond below Star $B$ by adding $7.18 \pm 0.02$ magnitudes. Taking the $\underline{V}$ magnitude of Star B as 10.73 (Sharpless 1952), interpolation on the color curve gives $\underline{V}=13.51$ for the whole of Region $I$, or $20.69 \pm 0.03$ per square second of arc, in good agreement with the value $\underline{V}=20.6 \pm 0.1$ reported by Johnson for his larger area.

Figure 3-6 shows an extremely blue intrinsic color, with $\underline{B}-\underline{V}$ =-1.05; this is bluer by far than any other reflection nebula for which photoelectric data are available. (Since the star is highly reddened, the apparent $\underline{B}-\underline{V}$ is actually positive.) Because of the strong wavelength dependence, one might be wary of shifts in effective wavelength. The widest of my filters is the blue one, and its differential magnitude was checked with a narrower blue filter $B_{m}$ of the same effective wavelength. No difference was found.

The color data given by Johnson and by Elvius and llall are also plotted in Figure 3-6. Since Elvius and Hall did not give an actual

Table 3-6. Differential photometry in NGC 2068 Region $I$.

| Filter | $1 / \lambda$ | $\mathrm{M}^{\mathrm{a}} \pm \mathrm{pe}$ | $\mathrm{T}^{\mathrm{b}}$ | Dates UT |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| U | 2.78 | 0.90 | 0.01 | 128 | $69.02 .10+69.02 .12$ |
| B | 2.33 | 1.69 | .02 | 32 | 68.11 .19 |
| $\mathrm{~B}_{\mathrm{m}}$ | 2.33 | 1.70 | .02 | 60 | 69.02 .12 |
| G | 1.93 | 2.62 | .01 | 60 | 68.11 .18 |
| 0 | 1.56 | 3.15 | .01 | 60 | $68.12 .17+69.02 .11$ |
| R | 1.21 | 4.26 | .04 | 80 | 69.02 .11 |
| I | 1.06 | 4.65 | .09 | 104 | $68.12 .17+69.02 .11$ |

a. Magnitudes below Star B.
b. Total integration time in minutes.


Figure 3-6. Color Data in Region I of NGC 2068 Symbols are as in Figure 3-4.
brightness level for the nebula, their data were plotted by interpolating a standard B point between my G and B filters. The differences between the three sets of data can probably be attributed to uncertainties in the effective wavenumbers of the various filters used.

Note the great brightness of the region with respect to the illuminating star. At short wavelengths the whole nebula actually looks brighter than the star; indeed, Rozhkovsky and Kurchakov (1968) give the integrated blue magnitude of the nebula as 8.28. Mie scattering can give such a result only in the case of a very high optical depth, in which case it would be hard to account for the strong polarization and color effects.

The excess brightness of the nebula is not due to atomic emissions. As pointed out above, the emission lines are faint and the atomic continuum makes at most only a very minor contribution. Moreover, the excess brightness of the nebula with respect to the star is, in blue light, about a factor of 30 ; if even half of this excess were due to unpolarized atomic emissions, the polarization of the grain-scattered light would have to be over $100 \%$ to give the observed values. Finally, while atomic emissions could distort individual color indices such as $\underline{B}-\underline{V}$, they could hardly give a color curve which is smooth over wavelength from the ultraviolet to the infrared.

Thus the brightness excess is probably an artifact of the small dark cloud in front of Star B. Unless this dust cloud is made of neutral scatterers (c.g., very large grains), the strong blue color of the region must then also be spurious, and, as pointed out by Elvius and

Hall, we have no reliable way of measuring its true color. It will be shown in Section 4.1 that, for models which best explain the polarizations, the theorctical color curves do tend to be somewhat redder than the color data in Table 3-6.

### 3.4.5 Interstellar polarization.

The polarization of Star $B$ was measured for an evaluation of the interstellar polarization component. A ten-arcsecond diaphragm was used to include both components but avoid contamination by the nebulosity; nights of poor seeing could not be employed. The presence of nebulosity within the diaphragn could affect the polarizations no more than about $0.05 \%$ in the ultraviolet, less in the other filters. All available data on the polarization of this star are collected in table 3-7 and plotted against wavenumber in Figure 3-7. My results from about the same time as the nebular measures were made are given in the first part of Table 3-7; previous photoelectric measures by Hall (1958) and Elvius and Hall (1966) are listed in the second section, and my more recent results are given in the third section of Table 3-7. Note the wide scatter in the blue and ultraviolet data; the evidence from my data alone, while far from conclusive, suggests variable polarization. Unfortunately the star is so faint $(\underline{B}=11.8)$ that a definitive investigation of polarization variability, i.e., working to an internal probable crror $<0.05 \%$, would require a large amount of telescope time.

Two other lines of evidence suggest that intrinsic polarization may be present: (1) As pointed out by Ilall (1958), Star A and Star B, though only $50^{\prime \prime}$ apart, differ by nearly $90^{\circ}$ in polarization position

Table 3-7. Polarization of NGC 2068 Star B.

| Filter | 1/ג | $P ¢ \pm p o$ |  | 0 | $\pm \mathrm{pe}$ | $\mathrm{T}^{\mathbf{a}}$ | Dates UT |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| U | 2.78 | $0.71 \pm$ | 0.55 |  |  | 61 | 67.12 .06 | $\rightarrow 68.02 .22$ |
| B | 2.33 | 1.18 | .09 | 13.5 | 2.1 | 27 | 67.12 .06 | $\rightarrow 68.03 .30$ |
| G | 1.93 | 1.31 | .10 | 8.5 | 2.2 | 48 | 67.12 .01 | + 67.12.08 |
| 0 | 1.56 | 2.05 | .10 | 10.2 | 1.5 | 48 | 68.09 .23 | $\rightarrow 68.10 .24$ |
| R | 1.21 | 2.62 | . 07 | 15.0 | 0.8 | 58 | 67.12 .03 | + 67.12.07 |
| None | 2.3: | 2.4 |  | 9. |  |  |  | 958) |
| UG1 | 2.66 | 0.5 |  |  |  |  |  | 964 |
| BG12+2B | 2.24 | 1.3 |  | -5. : |  |  |  | 964 |
| OG5 | 1.74 | 2.4 |  | -1. : |  |  |  | 964 |
| U | 2.78 | $0.03 \pm$ | . 45 |  |  | 72 | 68.03 .27 | +68.03.29 |
| $\mathbf{U}$ | 2.78 | 1.96 | . 37 | 15. $\pm$ |  | 92 | 68.12 .16 | +69.03.19 |
| B | 2.33 | 1.53 | . 07 | 12.2 | 1.3 | 84 | 69. | 03.29 |
| B | 2.33 | 0.99 | . 10 | -35. | 3. | 20 | 69. | 08.20 |
| B | 2.33 | 1.25 | . 08 | 8.5 | 1.8 | 56 | 69. | 09.02 |
| R | 1.21 | 2.40 | . 05 | 13.6 | 0.6 | $46^{\text {b }}$ | 70. | 01.03 |
| 1 | 1.06 | 2.58 | . 05 | 11.4 | 0.5 | $60^{\text {b }}$ | 70. | 01.03 |

a. Total integration time in minutes.
b. Observations made at Steward 90 -inch reflector


Figure 3-7. Polarization of NGC 2068 Star B
Filled circles are new data; open circles are from Elvius and Hall (1966), and the crossed circle is from Hall (1958). The dashed curve represents the mean wavelength dependence of interstellar polarization, fitted to the mean of the new data at $1 / \lambda=2.33 \mu^{-1}$.
angle, and, though Star $B$ has ncarly twice the color excess of Star $A$, Star A has by far the larger polarization ( $24 \%$ ). (2) The wavelength dependence of polarization for the lower envelope of points in Figure 3-7 is unique. It is vastly different from the mean wavelength dependence of interstellar polarization (Coyne and Gehrels 1967), which is plotted as a dashed line after normalization to the mean of my blue measures. The increase toward longer wavelengths is nearly twice as strong as that of HD 147889, which has been considered (Serkowski, Gehrels, and Wiśniewski 1969, Coyne and Wickramasinghe 1969) to be an extreme case of interstellar polarization. Of the $O B$ stars suspected by Coyne and Gehrels (1967) to show polarization variability, 48 Persei (HD 25940) and $\theta_{2}$ Orionis (HD 37041) have wavelength dependence of polarization similar to that of Star B, though in neither case is the effect so strong. Both of these stars are dwarfs with spectrum peculiarities, neither is known to show light variability, and both are believed (in part on the basis of unpublished Arizona data) to exhibit polarization variability by roughly $0.4 \%$ in the blue end of the spectrum only. To my knowledge no astrophysical model for this class of polarization variables has been ventured.

There is only slight evidence that $S t a r \operatorname{B}$ is variable in intensity as well as in polarization. Photoelectric photometry of Star B by Lee (1968) differs from the results of Sharpless (1952) by $0 . \mathrm{m}_{17}$ in V and 0.09 in $B-\underline{V}$, but by less than half these amounts on Star A. Obviously a proper job of photometry should be done on these stars in the future.

Ordinarily the discovery of intrinsic polarization in an earlytype star would be of considerable astrophysical interest. In this case
however, it is a decided nuisance since it precludes the possibility of confidently decoupling interstellar polarization from the nobular measures. We ean only assume that the non-variable component observed on Star $B$ is largely interstellar. A first-order decoupling (Section 2.3.5) was carried out using the values in the first part of Table 3-7. Since the position angles in the blue, green, and red show no significant deviations from an average value of $10.7^{\circ} \pm 1.4^{\circ}$, this value was adopted for these filters and also for filter $U$. The position angle in the infrared (filter $R$ ) appears to be anomalous, and the observed value was used. The derived intrinsic nebular polarizations are given in Table 3-8. The probable errors have been propagated through the analysis; the large derived probable error in the $U$ polarization of Region $I$ is a consequence of the highly uncertain interstellar component.

### 3.4.6 Polarization position angles

The last column in Table 3-8 gives $\Delta \theta$, the derived position angle minus that of the normal to the plane of scattering through the center of the region and Star B. In all cases $\Delta \theta$ comes out to be positive; the position angles are more easterly than expected. In Region II the discrepancies are small, but in Region I we have both a strong departure from the geometrical angle ( $36.5^{\circ}$ ) and a strong rotation of angle with wavelength; the smoothness of the rotation over wavelength leaves little doubt that the effect is real.

The anomalies can not be instrumental in origin. The equipment has position angles calibrated absolutcly to within a few tenths of a degree (Gehrels and Teska 1960) and good observations of Mars over the

Table 3-8. Rectified polarizations in NGC 2068.

| Region | Filter | $1 / \lambda$ | $\mathrm{P} \% \pm \mathrm{pe}$ | $\theta \pm$ pe | $\Delta \theta$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I | U | 2.78 | $4.9 \pm 0.6$ | $45.6 \pm 3.6$ | 9.1 |
|  | B | 2.33 | 7.20 .2 | 45.40 .9 | 8.9 |
|  | G | 1.93 | 8.40 .5 | 43.31 .6 | 6.8 |
|  | 0 | 1.56 | 9.10 .4 | 42.31 .4 | 5.8 |
|  | R | 1.21 | 12.50 .7 | 40.81 .8 | 4.3 |
| II | B | 2.33 | $12.1 \pm 0.2$ | $87.2 \pm 0.5$ | 2.1 |
|  | 0 | 1.56 | $16.4 \quad 0.5$ | $85.8 \quad 0.8$ | 0.7 |

period when Region I was worked show a mean deviation of only $0.5^{\circ} \pm 0.3^{0}$ from symmetry about the plane of scattering for all filters.

Incorrect values for the interstellar polarization could cause the derived nebular position angles to be anomalous. Actually, however, an attempt to compute the interstellar polarization by assuming the intrinsic nebular polarization to be exactly radial at all wavelengths yields no solution; no interstellar polarization (at a fixed position angle) could have shifted the nebular position angles to where they are observed to be in the raw data. Thus the misbehavior of position angles is probably intrinsic to the nebula.

Non-radial polarization could be due to partial illumination by other stars. Star A, however, is in the wrong direction to account for the angle shift in either region.

The observed effect could be due to a non-uniform, wavelengthdependent distribution of scattered intensity across the finite size of the region. Indeed the Roemer photograph shows the western half of Region I to be slightly brighter than the eastern, and this is in the right direction to explain the shift in mean angle. Region II, however, has an even stronger brightness asymmetry but in the wrong direction.

For the outer regions of the Merope nebula, Elvius and Hall (1967) found that the position angle of polarization "does not depend so strongly on the direction to the illuminating star, but is more closely related to the direction of nebulous filaments in this area. This indicates that the light is scattered by non-spherical particles oriented in a magnetic field associated with the filaments." The same effect was noted independently, and the same interpretation was suggested, by

Artamonov and Efimov (1967). Such an explanation of non-radial polarization, while an obvious one, may be on shaky ground theoretically. To my knowledge it has never been explicitly shown that even infinite cylinders, perfectly aligned at some oblique angle to the plane of scattering, can give polarization position angles that are non-symmetrical with respect to the plane of scattering. (I an indebted to Dr. D. L. Coffeen for a helpful discussion on this point.) The matter is badly in need of theoretical investigation.

Perhaps the most satisfactory explanation of the anomalous position angles is multiple scattering, i.e., illumination by another part of the nebula. The brightest part of NGC 2068 is directly east of Star $B$, in the right place to explain the angle shift in both regions. Also, the observed wavelength dependence of the rotation, stronger toward shorter wavelength, arises naturally since scattered intensity increases rapidly towards the blue end of the spectrum at most scattering angles. Without a detailed knowledge of the geometry involved, it is impossible to say how much multiple-scattered intensity would be required, but a crude estimate puts it at around $15 \%$ of the direct starlight.

For comparisons with the theory, I will use the rectified polarizations in Table 3-8 without regard to their position angles.

### 3.5 IC 5076

IC 5076 is a little-known reflection nebula in northern Cygnus. Van den Berg (1966) identified IID 199478, an MK standard of spectral type B8Ia, as the illuminating star, and classified the nebula as Type II; that is, the main body of the nebula is not in contact with the star.

The nebula is larger and fainter than NGC 2068, much fainter relative to the illuminating star, and its apparent color is much bluer. On the Palomar Sky Survey prints the nebula seems to be the illuminated edge of a thin, oblong dust cloud running to the west. To my knowledge there has been no previous astrophysical study of this nebula.

A direct photograph of IC 5076, made by Dr. Roemer at the Catalina reflector, is reproduced in Figure 3-8. The nebulosity is irregular but shows no evidence of patchy foreground extinction as seen in NGC 2068. The illuminating star is marked " A ". Except for a bit of reconnaissance, I have worked only the circled region, which has diameter 40.7 arcseconds and was centered $49 \pm 1$ arcseconds eastward from the star designated "B". The region was chosen before the large-scale plate was taken, and contains a field star not visible at the telescope. The sky was measured 3.4 minutes of arc west of Star B in a region apparently free of stars and nebulosity on the Palomar prints. Measurement of the Roemer plate with a traveling microscope, but without an exact knowledge of the plate orientation, placed the center of the region 211 arcseconds from HD 199478, and the normal to the plane of scattering at position angle $146^{\circ}$.

Five-color photoelectric photometry of this star has been reported by Iriarte et al. (1965). They found $\underline{V}=5.69$ and $\underline{B}-\underline{V}=+0.46$. Taking the unreddened color $(\underline{B}-\underline{V})_{0}=-0.02$ for spectral type B8Ia and 4.2 for the ratio of total to selective absorption for this region (II. L. Johnson 1968), the color excess is 0.48 and the total visual extinction is 2.0 magnitudes. The absolute magnitudes of $B$ supergiants


Figure 3-8. IC 5076
are poorly known, but taking $M_{V}=-7.0$ as adopted by Keenan (1963), the distance comes out to be 1400 parsecs.

### 3.5.1 Polarization

Polarizations in IC 5076 were measured on 21 nights between April 1968 and July 1969, for a total integration time of 43 hours. Only four filters were used; the infrared surface brightness is so low as to make polarimetry unprofitable. The results are given in Table 3-9. The precision here is not so good as that in NGC 2068, but unmistakably the polarization declines into the red, with a maximum in the blue filter. Such a wavelength dependence has never before been found on a reflection nebula, except perhaps in the data of H. M. Johnson (1960) on the very faint nebula Cederblad 44.

The observed polarizations in Table 3-9 must still be corrected for the interstellar polarization observed on $H D$ 199478. This star was included in a 1967 polarimetric survey by Dr. G. V. Coyne, S. J. His results for the filters of interest, as yet unpublished, were combined with my own 1969 observations to give the polarizations in Table 3-10. The listed values have internal probable errors of about $\pm 0.03 \%$ polarization and $\pm 1^{\circ}$ in position angle; the close agreement of position angles in different filters must be fortuitous. These polarizations differ in amount but not in angle from the $1.8 \%$ at $11^{\circ}$ reported by Hiltner (195G) and the $1.9 \%$ at $10^{\circ}$ found by Hall (1958). The wavelength dependence is typical of interstellar polarization.

Table 3-9. Polarization in IC 5076.

| Filter | $1 / \lambda$ | $\mathrm{T}^{\text {a }}$ | Observed |  |  | Corrected ${ }^{\text {b }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathrm{p} \% \pm \mathrm{pe}$ | $\theta$ | $\pm$ pe | $\mathrm{p} \% \pm \mathrm{pe}$ | $\theta$ |
| U | 2.78 | 860 | $8.04 \pm 0.43$ | 151.9 | $\pm 1.5$ | $9.2 \pm 0.5$ | 147.5 |
| B | 2.33 | 370 | 9.64 . 39 | 142.5 | 1.1 | 10.9 . 4 | 138.2 |
| G | 1.93 | 430 | 8.66 . 31 | 143.4 | 1.0 | 9.7 . 3 | 138.2 |
| 0 | 1.56 | 910 | 8.31 . 84 | 153.7 | 2.5 | 8.7 . 9 | 148.3 |

a. Total integration time in minutes.
b. Corrected for both halation and interstellar polarization.

Table 3-10. Polarization of HD 199478.

| Filter | $1 / \lambda$ | $\mathrm{P} \%$ | $\theta$ |
| :---: | :---: | :---: | :---: |
| U | 2.78 | 1.24 | $10.7^{\circ}$ |
| B | 2.33 | 1.49 | 10.3 |
| G | 1.93 | 1.60 | 10.5 |
| 0 | 1.56 | 1.57 | 10.4 |

Color-curve data were taken on thirteen nights between August 1968 and August 1969 for a total integration time of 11.2 hours in six filters. The raw data and their internal probable errors appear in the third and fourth columns of Table 3-11. The colors here are much weaker than in NGC 2068, being slightly blue at long wavelengths and almost neutral at short wavelengths.

### 3.5.3 Halation

As has been noted, the halation intensity from the stars in NGC 2068 was found to be completely negligible compared to the nebular intensity. In IC 5076, however, even though the illuminating star is at the very edge of the photometer ficld at 211 arcseconds offset, it casts a radiance amounting to nearly $30 \%$ of the nebular brightness in the ultraviolet. The halation color-curve as measured on several second-magnitude stars, averaged and smoothed over wavelength, is plotted in Figure 3-9 along with the observed nebular intensities. The halo photometry was done exactly as if working the nebular region, so the two curves are directly comparable. As a function of wavelength the scattered light obeys almost perfectly a power law:

$$
I_{\text {halo }} \propto I_{\text {star }}(\lambda)^{-2.2} .
$$

The halation intensities showed no correlation with zenith distance. Apparently the effect is mostly instrumental rather than atmospheric in origin; its wavelength dependence is much weaker than would be expected from Rayleigh scattering.

Table 3-11. Intensity data in IC 5076.
Intensities are given in magnitudes below HD 199478 for a region of dianeter 40.7 arcseconds.

| Filter | 1/ $\lambda$ | Nebula <br> Uncorrected | Halation <br> Depol. Out | Halation <br> Depol. In | Nebula <br> Corrected |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N | 3.03 | $7.69 \pm 0.04$ | 9.2 | 9.0 | $8.07 \pm 0.06$ |  |
| U | 2.78 | 7.76 | .03 | 9.4 | 9.2 | 8.09 |
| B | 2.33 | 7.82 | .03 | 9.9 | 9.6 | 8.05 |
| G | 1.93 | 7.89 | .04 | 10.4 | 10.1 | 8.04 |
| 0 | 1.56 | 8.16 | .02 | 10.8 | 10.6 | 8.28 |
| R | 1.21 | 8.80 | .07 | 11.3 | 11.2 | 8.92 |



Figure 3-9. Color Data in IC 5076

The last two columns of Table 3-11 give color-curve data corrected for the halation intensities in the sixth column. The probable errors are propagated assuming a RMS uncertainty of $10 \%$ in the corrections. Although the brightness level is reduced by as much as 0.38 by the correction for scattered light, the shape of the color curve is only slightly affected and the increase of probable error is insignificant. Taking $\underline{V}=5.69$ for the star, the interpolated $\underline{V}$ magnitude of the nebulosity comes out to be $13.80 \pm 0.06$ for the whole region, or 21.6 pcr square arcsecond.

The effect of halation on the polarizations is not immediately evident. If we assume the halo to be seen by the polarimeter exactly as an unpolarized celestial luminosity, it is readily shown that the nebular polarization is affected not at all by the halation obtained with the depolarizer in the beam. Also, unless the nebula is highly polarized, its polarization is simply diluted by the halation obtained with depolarizer out. These properties are demonstrated, and an approximate formula for computation of the true nebular polarization is derived, in Appendix III. For a region in NGC 7023 (Section 3.6.2), I obtained exact halation corrections for polarization data by making individual measures at each Wollaston angle. Such a procedure was not justified by the level of precision in the IC 5076 raw data.

Polarizations corrected for both the halation and the interstellar polarizations are given in Table 3-9 along with the uncorrected values. The formal probable errors in interstellar polarization are so small as to have little effect on the derived error levels, due to the combination
of position angles, the uncertainty is in some cases actually decreased. As in the case of the color curve, the polarization curve is displaced a bit, but its shape is little changed and its probable errors are only slightly inflated by the correction for halation.

As in NGC 2068, the position angles do not come out to be as close to the normal to the scattering plane ( $\theta=146^{\circ}$ ) as we might have expected. In this case, however, the wavelength dependence of position angle seems to be irregular and hence is not easily attributed to multiple scattering.

### 3.6 NGC 7023

NGC 7023 is a large bright reflection nebula in Cepheus. Figure
3-10 is taken from a 30 -minute exposure at the Steward $229-\mathrm{cm}$ reflector kindly provided by J. J. Schreur. This nebula has a complex internal structure with conspicuous condensations, cavities, and streamers, but no true filaments such as are found in the Merope nebula. On the Palomar Sky Survey prints the luminosity extends over as much as 15 minutes of arc, and is imbedded in a thick dust cloud of diameter about half a degree. The whole dust cloud may be faintly luminous with respect to sky regions entirely outside its borders. The nebula is known (Weston 1953, Rosino and Romano 1962) to contain a major clustering of faint irregular variables. Since NGC 7023 is too far north for the yoke mount of the Catalina reflector, only a small amount of new data was obtained with other telescopes.

NGC 7023 is illuminated by the shell star HD 200775. Determinations of its spectral type by several investigators have been compiled

## $N$



Figure 3-10. NGC 7023
by Gehrels (1967); the consensus is B2Ve. Photoclectric $\underline{U} \underline{B} \underline{V}$ photometry by Elvius and Hall (1966), by R. I. Mitchell (communicated by Gehrels 1967), and by Racine (1968) are all in good agreement and were averaged to give the following values:

$$
\begin{aligned}
\underline{V} & =7.41 \pm 0.02, \\
\underline{B}-\underline{V} & =+0.38 \pm 0.01, \\
\underline{U}-\underline{B} & =-0.41 \pm 0.01,
\end{aligned}
$$

### 3.6.1 Color

Photoelectric intrinsic colors in NGC 7023 have been obtained in six regions by Martel (1958), in three scans across the nebula by Vanysek and Svatos (1964), and in fifteen regions by Elvius and Hall (1966). Recent photographic photometry at four wavelengths has been reported by Kurchakov (1968). Because of the variety of diaphragm sizes used and the chaotic structure of the nebula, comparisons among the various sets of data are difficult. Neither are comparisons facilitated by the almost universal practice of giving offset distances in instrumental units. There is unanimity on two points: (1) The nebula is bluer than the star in $\underline{B}-\underline{V}$ in all regions within $2.5^{\prime}$ of HD 200775, and (2) A region about $1^{\prime}$ north of the star is almost neutral in color. Otherwise the agreement is generally poor. For similar regions the measures of Elvius and Hall usually agree within 0.1 magnitudes with those by Martel, but the colors by Vanýsek and Svatos are often systematically redder by $0 . \mathrm{m}_{3}$ or more. An example is provided in Figure 3-11, in which are plotted all available $\underline{B}-\underline{V}$ colors as a function of offset angle for


Figure 3-11. Color Data in NGC 7023
All data are for regions directly east of HD 200775. The filled circles are from Elvius and Hall (1966), the triangles from Martel (1958), the solid line from Vanysek and Svatos (1964), and the dashed line from Kurchakov (1968).
regions directly east of the star. (I have reduced Kurchakov's data to the $\underline{B}$ - $\underline{V}$ scale by multiplying his brightness ratio between 4650 A and 6750 A by 0.57 , assuming a linear color curve.) Clearly, existing data cannot be used uncritically for the construction of detailed theoretical models. In particular, the often-quoted assertion (e.g., Wickramasinghe 1967) that the intrinsic color becomes redder with offset distance is by no means well-established.

### 3.6.2 Polarization

Elvius and Hall also measured polarizations of NGC 7023 in three filters. The polarization was found to be positive, radial with respect to HD 200775, and increasing toward the infrared in all fifteen regions. At the McDonald $208-\mathrm{cm}$ reflector in 1959, Gehrels (1967) measured the polarizations of three regions in ultraviolet, yellow, and infrared light. Both sets of data include a region centered 38" NE of HD 200775, for which Gehrels used a focal-plane diaphragm of diameter 22.6", Elvius and Hall a larger one of diameter $26.6^{\prime \prime}$. The results for this region are compared in Figure 3-12; Gehrels found systematically higher polarizations than Elvius and Hall, but with the same wavelength dependence. The difference of about $4 \%$ polarization is more than four times Gehrels' internal probable error.

This discrepancy is very similar to that found between my data and that of Elvius and Hall in NGC 2068. In the case of NGC 7023, the disparity has been attributed (Gehrels 1967, page 632; Hanner 1969) to the fact that the sky measures were taken at only $2.6^{\prime}$ offset from IID 200775. Such regions are indecd well inside the nebulosity; Elvius and


Figure 3-12. Polarization in NGC 7023
All data are for a region centered 38 arcseconds NE of HD 200775. Focal-plane diaphragm diameters in arcseconds were: Elvius and Hall, 26.6; Gehrels, 22.6; Zellner, 20.0.

Hall performed photometry and polarimetry at greater distances. llowever, the ratios of region to sky given by Elvius and llall, or extrapolation of Kurchakov's data, show that the nebulosity is two magnitudes fainter at $2.6^{\prime}$ offset than at $38^{\prime \prime}$; the residual luminosity could account for most but not all of the discrepancy.

In order to explore this matter, I observed this same region (offset $38.0^{\prime \prime} \pm 0.5^{\prime \prime} \mathrm{NE}$, diaphragm diameter $20.0^{\prime \prime} \pm 0.1^{\prime \prime}$ ) at the Steward $229-c m$ reflector on 8 December 1969. Polarization was measured in the blue filter $(1 / \lambda=2.33)$ only, for a total integration time of one hour. Particular attention was paid to possible sources of systematic error. The sky was measured approximately 5' south and 15 ' west of HD 200775, in a region which appears to be free of stars and nebulosity on the Palomar prints. I applied exact, angle-by-angle corrections for instrumental halation, derived from intermittent observations offset from the nearby star $\beta$ Cephei. The halation intensity was found to vary only about $5 \%$ over Wollaston angle and to increase not more than $2 \%$ with air mass up to an hour angle of five hours. Taking into account the effect on the observed polarization of the residual uncertainties in halation $( \pm 0.03 \%)$, instrumental polarization ( $\pm 0.02 \%$ ), instrumental depolarization $( \pm 0.01 \%)$, the polarimeter instability $( \pm 0.07 \%)$, and choice of sky ${ }^{1}$ region $( \pm 0.1 \%)$, any systematic errors should be unlikely to exceed $\pm 0.2 \%$.

1. For example, suppose that a region apparently free of nebulosity on the Palomar Sky Survey prints actually has a surface brightness of magnitude 22.5 per square arcsecond. This is about $6 \%$ of the surface brightness of NGC 7023 (Section 3.6 .3 below) at this distance from the star. If the nebula is polarized about $15 \%$, the residual luminosity of the sky region will dilute the polarization by $6 \% \times 15 \% \simeq 0.10 \%$.

I obtained for the polarization in blue light a value $12.41 \%$ with an internal probable error of $\pm 0.08 \%$, at position angle $130.9^{\circ}$ $\pm 0.2^{\circ}$. The absolute calibration of this position angle is uncertain by $\pm 0.5^{\circ}$; it may be compared with the value $126^{\circ}$ given by Elvius and Hal1, the value $129^{\circ}$ (mean of G and U filters) given by Gehrels, and $135^{\circ}$ as the normal to the plane of scattering. As shown in Figure 3-12, my value for the polarization falls well within Gehrels' listed probable error. No explanation suggests itself for the discrepancy between my data and that of Elvius and Hall. If due to the difference in diaphragm sizes, the effect is surprisingly large.

The interstellar polarization of HD 200775 has been previously measured without filter by Hall (1958), in three filters by Elvius and Hall (1966) and in two filters by Gehrels (1967). The results, plotted in Figure 3-13, differ substantially even though the star is bright and all data were taken at large telescopes. (The contiguous nebulosity, even in a $20^{\prime \prime}$ diaphragm, could hardly affect the measures more than $0.02 \%$.) I expended several hours of integration time on this star at the Steward 21 -inch reflector, in the blue filter only, with the result $\mathrm{P}=0.95 \% \pm 0.02 \%$. This value is corrected for instrumental polarization, but the position angle was not calibrated. The dashed curve in Figure 3-13 gives the best fit of the mean wavelength dependence of interstellar polarization (Coyne and Gehrels 1967) to my point and the two points of Gehrels. This curve will be taken to represent the foreground interstellar polarization of NGC 7023.


Figure 3-13. Polarization of HD 200775
The filled circle is a new data point; the open circles with error bars are from Gehrels (1967), the triangles from Elvius and Hall (1966), and the square from Hall (1958). The dashed curve represents the mean wavelength dependence of interstellar polarization.

Table 3-12 gives polarizations for this region and for the region $38^{\prime \prime}$ north of 111200775 worked by (iehre1s, after correction for the apparent interstellar polarization.

### 3.6.3 Brightness

At the 1965 Troy conference on interstollar grains (Greenherg and Roark 1967), considcrable discussion was attracted by (ichrels' conclusion that NGC 7023 is optically thin. His amalysis (Gehrels 1967) relied upon a photovisual surface brightness of magnitude 21.3 per square aresccond, averaged from Martel's (1958) measures at 48' offsct. llowever, photographic photometry by Kecnan (1936) and by Grygar (1959), and the ratios of nebula to sky listed by Elvius and llall, argue that the inner regions cannot be this faint.

During a short run at the Steward reflector in October 1969, I measured the brightness ratio of 111$) 200775$ to a region of diameter $20.0^{\prime \prime}$ $\pm 0.1^{\prime \prime}$ centered $38.0^{\prime \prime} \pm 0.5^{\prime \prime}$ directly north of the star. The limited observing time permitted measures in the $U$ and (; filters only. Sky readings were taken at two spots near the edge of the dark dust cloud. Corrections for halation were obtaincd by offsctting from several nearby third-magnitude stars. The results are given in Table 3-13 in magnitudes below the star for the whole region. They may be converted into magnitudes per square arcsecond below the star by adding $6.24 \pm 0.01$ magnitudes. At this telescope the instrumental scattered light turned out to be redder than the star.

A linear interpolation between the $\underline{B}$ and $\underline{V}$ magnitudes of III) 200775 (above) gives $G=7.50$. Thus the (; magnitude of the nebula, in

Table 3-12. Rectified polarizations in NGC 7023.
The observed polarizations are by Gehrels (1967) except for the measure at $1 / \lambda=2.33$ in the $N E$ region, which is a new observation at the Steward reflector. The quantity $\Delta \theta$ is the derived position angle minus that of the normal to the planc of scattering through the center of the region.

| Region | $1 / \lambda$ | Polarization |  | Position Observed | Angle <br> Corrected | $\Delta \theta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Observed | Corrected |  |  |  |
| NE 38' | 1.20 | 20.5 | 20.5 | 130 | 131 | - 4 |
|  | 1.82 | 16.0 | 16.1 | 134 | 134 | - 1 |
|  | 2.33 | 12.4 | 12.45 | 131 | 133 | - 2 |
|  | 2.78 | 10.1 | 9.9 | 123 | 125 | -10 |
| N 38' | 1.20 | 21.6 | 20.9 | 86 | 86 | - 4 |
|  | 1.82 | 19.2 | 18.5 | 87 | 87 | - 3 |
|  | 2.78 | 13.6 | 12.8 | 91 | 91 | $+1$ |

Table 3-13. Intensity of NGC 7023.
Intensities are tabulated in magnitudes below HD 200775, for a $20^{\prime \prime}$ diaphragm centered $38^{\prime \prime}$ north of the star.

|  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Filter | $1 / \lambda$ | $\mathrm{T}^{\mathrm{a}}$ | Nebula <br> Uncorrected | Halation | Nebula <br> Corrected |
| U | 2.78 | 48 | $5.486 \pm .002$ | $8.38 \pm .12$ | $5.56 \pm .01$ |
| G | 1.93 | 30 | $5.592 \pm .005$ | $8.06 \pm .04$ | $5.71 \pm .01$ |

a. Integration time in minutes.
the region described above, comes out to be 19.45 per square arcsccond, with a probable error not exceeding $\pm 0.05$ magnitudes. The consequences of this high surface brightness in terms of the optical depth of the nebula will be explored in Section 4-4.

The intrinsic color of the nebula in this region was found to be $U-G=-0.15 \pm 0.02$ magnitudes. This is a smaller color excess than Elvius and Hall found for any region except $67 \prime$ north of the star, and it agrees with the earlier conclusion that the north region of the nebula is weak in intrinsic color. The wavelength dependence of polarization in NGC 7023 is very similar to that in NGC 2068. However, its intrinsic color, at least in this region and over this wavelength range, is much more like that of IC 5076. It would be very interesting to obtain color data in the infrared.

### 3.7 Cederblad 201

Like NGC 7023, Cederblad 201 is too far north to be reached by the Catalina reflector. Containing the ninth-magnitude star $\mathrm{BD}+69^{\circ} 1231$, it seems to be the illuminated southwestern edge of a thick dust lane. Racine (1968) found the illuminating star to be of spectral type B9.5V, only slightly reddened with a color excess $E_{B-V}=0.21$. Racine identified Ced 201 as a member of the reflection nebula association Cep R2; assuming a ratio of total to selective absorption of 3.0 , this $R$ association is at a distance $400 \pm 80$ parsecs.

Racine (unpublished) has carried out $\underline{U} \underline{B} \underline{V}$ photometry of the nebula in 34 regions extending more than three arcminutes from the star and reaching beyond $V=23$ per square arcsecond. He found the nebula to
be redder than the star in all regions except just to the south of the star, with intrinsic $\underline{B}-\underline{V}$ exceeding +0.3 magnitudes in the remote areas. His isophotes show that the nebulosity has very smooth intensity gradients with peak brightness northeast of the star.

At the Steward $229-\mathrm{cm}$ reflector in October 1969, I made rough measures of the polarization of Ced 201 in two filters. The single region was of diameter $20.0^{\prime \prime} \pm 0.1^{\prime \prime}$, centered $38^{\prime \prime} \pm 0.5^{\prime \prime}$ north of the star. The results are given in Table 3-14. No corrections have been applied for instrumental halation, which (with depolarizer out) amounted to about $3 \%$ of the nebular intensity.

Interpolating in Racine's data, the nebulosity at this point has surface brightness $\underline{V}=20.8$ per square arcsecond, with intrinsic colors

$$
\begin{aligned}
& \underline{B}-\underline{V}=+0.08 \pm 0.02, \\
& \underline{U}-\underline{B}=+0.10 \pm 0.03 .
\end{aligned}
$$

The nebula is unquestionably redder than the star. Again we see that nebulae which are very similar in the wavelength dependence of polarization can differ widely in intrinsic color. Racine is currently using his data in construction of a theoretical model of the nebula, and it will not be discussed further here.

### 3.8 Other Nebulae

A brief survey was made of polarization in three reflection nebula for which no photoelectric data have been published. No special effort was made to attain high precision; sky regions were chosen with care, but no corrections for halation were applied, and the total

Table 3-14. Polarization in Ced 201.

| Filter | $1 / \lambda$ | $\mathrm{p} \% \pm \mathrm{pe}$ | $\theta^{\mathrm{a}} \pm \mathrm{pe}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| B | 2.33 | 12.4 | 0.4 | 93.2 | 0.3 |
| 0 | 1.56 | 18.7 | 0.8 | 92.7 | 1.2 |

a. Calibration uncertain by $\pm 0.5^{\circ}$.
integration time was usually less than one hour per filter. The regions are described in Table 3-15, and the results are given in Table 3-16. In all three nebulae the polarization was found to be small and positive, with position angles within a few probable errors of radial symmetry. In IC 4603, the nebular polarization is hardly larger than that of the star, and scens to have a strange wavelength dependence. The polarization in IC 4601 b appears to be much smaller than that reported by Martel (1958) for the same region; this nebula contains two eighth-magnitude stars, and the position angle suggests that both contribute to the illumination.

The only nebula of the three that seems attractive for further work is NGC 2245, a sinall bright fan-shaped object in Monoceros. Cederblad's (1946) classification of the nebula as pure reflection was confirmed by Herbig (1960). While not good enough for comparisons with theory, the data in Table 3-16 show the wavelength dependence of polarization to be typical of reflection nebulae. Parsamian (1963) has reported multispectral photographic photometry and polarimetry of this nebula. His polarization in blue light for approximately the region worked here is $12 \%$, in good agreement with the value in Table 3-16. Parsamian's photometry of the nebula, combined with $\underline{U} \underline{B} \underline{V}$ photometry of Lick H $\alpha 215$ by Dibai (1969), gives for this region quite blue intrinsic colors of $\underline{B}-\underline{V}=0.32, \underline{U}-\underline{B}=0.77$.

NGC 2245 has been classified with the poorly-understood objects known as cometary nebulae from their distinctive conical form; in the case of NGC 2245, the bright southern lobe is mirrored by fainter nebulosity to the north of the star.

Table 3-15. Other nebulae worked.

|  | IC 4603 | IC 4601b | NGC 2245 |
| :--- | :---: | :---: | :---: |
| Nebula | HD 147889 | HD 147103,4 | Lick Ha 215 |
| Star | B2V | AO + AO | B1p |
| Spectrum | 7.89 | 7.3 | 10.64 |
| mV | $85^{\prime \prime} \mathrm{E}$ | $50^{\prime \prime} \mathrm{W}$ of <br> N star <br> Offset | $40.7^{\prime \prime}$ |

Table 3-16. Polarizations in three nebulae.

| Nebula | Filter | $\mathrm{p} \% \pm \mathrm{pe}$ |  | $\theta \pm \mathrm{pe}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| IC 4603 | B | $4.7 \pm 0.4$ |  | $9 \pm 3$ |  |
|  | 0 | 2.9 | . 7 | 6 | 6 |
| IC 4601b | B | 7.6 | . 8 | 171 | 3 |
| NGC 2245 | B | 11.6 | . 5 | 96 | 1 |
|  | 0 | 13.0 | 1.3 | 83 | 3 |

lubble's variable nebula NGC 2261, illuminated by the diffuse object $R$ Monocerotis, is the best known example of the cometary-nebula phenomenon. Its spectrum is predominately of the reflection type but is not a true copy of the spectrum of $R$ Monocerotis (Herbig 1968), and there is no clear correlation between the light variability of the nebula and that of the star. Various exotic mechanisms have been proposed (Gurzadian 1959, 1960; Johnson 1966) for the luminance of the nebula. A long series of studies at Byurakan Observatory (e.g., Khatchikian and Parsamian 1964), and observations by R. C. Hall (1965), however, have shown that in color and polarization the nebulosity closely resembles an ordinary reflection nebula. I have not observed the nebula, but have reported elsewhere (Zellner 1970) that the illuminating object R Monocerotis has more than $10 \%$ intrinsic polarization and can possibly itself be interpreted as a miniature reflection nebula. I also observed the associated stars in eight other cometary nebulae, but found no such high polarizations; even among the nuclei of cometary nebulae, $R$ Monocerotis would appear to be exceptional.

## CIIAPTER 4

## COMPARISONS BETWEEN THEORY AND OBSERVATION

Techniques were developed in Chapter 2 for computation of the brightness, color and polarization emerging from an idealized reflection nebula. It was seen in Chapter 3 that stringent observational requirements for real nebulae can be obtained, and that real nebulae are likely to be much more complex than the idealized models.

Grains with a high imaginary component of the refractive index, such as graphite and metallic particles, were ruled out on rather general grounds in Chapter 2. This conclusion is reinforced by the new data; in no case could I find a scattering geometry or a grain size which would reproduce the observed polarization data for graphite or metallic grains. In this chapter I will make detailed comparisons between the observations and models for predominately dielectric grains. I will not use an arbitrary criterion for agreement between theory and observation, but will plot the observed and theoretical data for representative cases, and leave evaluation of the quality of the fit to the reader.

The obscrved polarizations in NGC 2068 and NGC 7023 can be fitted rather well by grains of a single size, but the situation is confused by the ripples which become increasingly strong (Figure 2-5) for higher refractive indices. Thus I will give solutions only for the OIIG and EXP size distributions, with emphasis on the former. Also,
the grains are not likely to be perfect spheres, and it is hoped that the application of a size distribution will minimize any shape effects. The solutions will be described by the scale radius a (Section 2.1.3) of the size distribution, which is not the same as the mean radius and should not be interpreted as an "effective" grain radius.

Multiple scattering is ignored in all my models. Unless otherwise stated, internal extinction and reddening will also be neglected. Since the only effect of transmission losses on polarization is a distortion of the relative contributions from different scattering angles, the values of $\theta_{1}$ and $\theta_{2}$ (Figure 2-1) which $I$ list may be regarded as "optically thin equivalents" of the true geometry. Colors are expected to be more strongly affected by internal attenuations, and the probable effects will be discussed qualitatively for each nebula. Only for IC 5076, where the scattering geometry seems to be particularly simple, will the effects of internal attenuations be explored quantitatively.

### 4.1 NGC 2068

It was pointed out in Chapter 3 that the intensely blue color observed in NGC 2068 is at least partly spurious, due to the presence of a small dust globule in front of the illuminating star. Hence I will not require a detailed fit between observational and theoretical color data for this nebula. Since internal reddening could partly compensate for this artificial blueing, I will interpret the observed color as a maximum intrinsic color to be explained by optically thin
models. Also the observed color curve is not perfectly smooth over wavelength but shows irregularities of about one tenth of a magnitude, which the theoretical color functions in my idealized models cannot be expected to duplicate in detail.

Figures 4-1 and 4-2 give a few examples of simultaneous fits to my color and polarization data in Region I of NGC 2068, for grains in the OHG size distribution. For each refractive index the range of scattering angles and the scale size $a_{0}$ were adjusted (Section 2.2.3) to give the best fit to the polarization data. Then the observed and theoretical color data were matched by only a vertical shift with no further adjustments in $a_{0}$ or in the range of scattering angles.

The optimum range of scattering angles comes out to be about $10^{\circ} \rightarrow 50^{\circ}$; smaller scattering angles (e.g., $10^{\circ} \rightarrow 40^{\circ}$ ) do not give high enough polarization in the infrared, and larger angles (e.g., $20^{\circ} \rightarrow 50^{\circ}$ or $10^{\circ}+60^{\circ}$ ) generally give too steep a wavelength dependence of the polarization.

The plots show that the agreement between theory and observation is satisfactory, and surprisingly good considering the crudeness of the models. As expected, the theoretical color curves tend to be not quite so blue as the observed colors.

Table 4-1 gives, for a variety of refractive indices $m^{*}$, the scale sizes $a_{0}$ which give the best fit to the polarization data. It can be seen that ambiquities in the range of scattering angles (e.g., $20^{\circ} \rightarrow 40^{\circ}$ instead of $10^{\circ} \rightarrow 50^{\circ}$ ) have little effect on the derived


Figure 4-1. Solutions for Region I of NGC 2068
The solid curves are for $m^{*}=1.50$, the dashed curves for $m^{*}=1.10$; the OHG distribution is used. Details of the solutions are given in Table 4-1.


Figure 4-2. Solutions for Region I of NGC 2068, for Ice Grains The solid curves are for $\mathrm{m}^{*}=1.30$, the dashed curves for $m^{*}=1.30-0.10$; the OHG size distribution is used. Details of the solutions are given in Table 4-1.

Table 4-1. Fits to polarization data in NGC 2068 Region I.
Where multiple solutions are given for one refractive index, the best solution is indicated by an asterisk. Poor fits are indicated by a colon.

scale sizes. Unless the scale size is given with a colon, all the solutions in Table 4-1 are as close as those in Figures 4-1 and 4-2. Only a marginal fit is found for $\mathrm{m}^{*}=1.80$; for large purely dielectric refractive indices, both the color and the polarization curves become too stecp for a good fit. Both can be flattened by the addition of a moderate imaginary component $\mathrm{m}^{\prime \prime}$. The wavelength dependence of polarization is lost if $\mathrm{m}^{\prime \prime}$ bccomes too large; grains with $\mathrm{m}^{*}=1.30-0.20 \mathrm{i}$ give a poor fit.

Elemental carbon both in the form of graphite ( $\mathrm{m}^{*} \simeq 2.4$ - 1.4i) and in the form of diamonds ( $m^{*} \simeq 2.42$ ) seems to be ruled out, the former because of an imaginary component that is too large and the latter because of one that is too small. The possibility was not explored, but a mixture of the two carbon allotropes might do rather well.

Also listed in Table 4-1 are a few fits for grains in the EXP distribution. An example is given in Figure 4-3, where the curves serve for either $m^{*}=1.30$ or $m^{*}=1.65-0.10$ i; at $20^{\circ} \rightarrow 50^{\circ}$, these two refractive indices give indistinguishable results. The polarization solutions are as good as the best of the OHG fits, but at short wavelengths the color curves are not blue enough, a consequence of this distribution's emphasis on large grains. Again, the slightly redder predicted colors may be closer to the true nebular color than the observations indicate.

In summary, equally good fits can be found for a rather wide range of refractive indices, but for a given $m^{*}$ the scale size $a_{0}$ can


Figure 4-3. Solutions for Region I of NGC 2068, the EXP Distribution The curves serve equally well for $m^{*}=1.30$ or $m^{*}=1.65-$ $0.10 i$. Details of the solutions are given in Table 4-1.
be determined to a precision of about $\pm 15 \%$. In terms of the chemical composition of the grains, the observations do not distinguish between solid $H_{2}\left(m^{*} \simeq 1.10\right)$, dirty ice $\left(m^{*} \simeq 1.30-0.05 i\right)$ and silicates ( $m^{*} \simeq 1.65-0.10 i$ ). In the OIIG distribution the scale sizes are about $0.38 \mu, 0.28 \mu$, and $0.18 \mu$ respectively for the three types of particles. Grains with a high imaginary component ( $m^{\prime \prime}>m^{\prime}-1$ ) are ruled out.

Table 4-2 gives a few examples of fits to my polarization data in Region II of NGC 2068; no color data are presently available for this region. All of the listed solutions fall well within two probable errors of the observations. If the numbers are meaningful, the scale sizes are about $25 \%$ smaller for this outer region than in Region $I$ of the same nebula.

### 4.2 IC 5076

The region that I worked in IC 5076 is unique in having polarizations that increase from red to blue, then drop into the ultraviolet. It will be assumed that this peaked polarization curve is genuine ${ }^{1}$ and is not a statistical fluke in the observations.

1. The observations in IC 5076 allow about a $10 \%$ probability that the polarization is constant over wavelength. Such a polarization curve is given by grains of any sort in the Rayleigh-like domain, e.g., up to about $x_{0}=0.5$ for $m^{*}=1.30$ in the OHG distribution (Figure 2-10). The steep color curve then produced could be flattened by internal reddening, but it is hard to see how the "knee" observed in the color curve at about $1 / \lambda=1.8$ could be produced. Grains with a high imaginary component can also give a wavelength-independent polarization at larger grain sizes; this possibility was not fully explored.

Table 4-2. Fits to polarization data in NGC 2068 Region II.
Where multiple solutions are given for one refractive index, the best solution is indicated by an asterisk. The list is not exclusive; solutions are possible for other refractive indices.

| Distribution | $\mathrm{m}^{*}$ | $\theta_{1} \rightarrow \theta_{2}$ |  | $\mathrm{a}_{0}$ microns |
| :---: | :---: | :---: | :---: | :---: |
| OHG | 1.30 | $10^{\circ}$ | $65^{\circ}$ | 0.14 |
|  |  | 20 | 60 | 0.18 |
|  |  | 20 | 55 | 0.15 * |
| OHG | 1.30-0.05i | 10 | 60 | 0.20 * |
|  |  | 20 | 55 | 0.24 |
| OHG | 1.50 | 20 | 50 | 0.15 |
|  |  | 30 | 40 | 0.15 * |
| OHG | 1.65-0.10i | 20 | 50 | 0.13 |
|  |  | 10 | 60 | 0.14 |
| EXP | 1.30 | 20 | 60 | 0.15 |
| EXP | 1.65-0.10i | 20 | 60 | 0.15 |

In my models such a peaked polarization curve is found only for predominately dielectric grains backscattering at angles greater than $90^{\circ}$; as was illustrated in Figure 2-11, the polarization function then increases slightly from its Rayleigh value to a peak before dropping rapidly to negative values. The peak becomes sharper with higher refractive index, while $\mathrm{m}^{*}=1.10$ it is barely detectable. In the EXP distribution this structure is almost completely washed out, ${ }^{2}$ so my attention here will be confined to the OHG distribution.

Figure 4-4 gives examples of simultaneous fits to the color and polarization data for the OHG distribution and three refractive indices with real part $m^{\prime}=1.30$. The range of scattering angles $150^{\circ} \rightarrow 165^{\circ}$ was found to give the best fit to the polarization data. Then for each $m^{*}$ the scale size $a_{0}$ was adjusted to fit the color data at long wavelengths (where the color curve is essentially independent of refractive index) and the polarization curve was plotted for this value of $a_{0}$. The best fit to both polarization and color seems to fall at about $m^{*}=1.30-0.05 i$.

Figure 4-5 gives a similar fit for $m^{*}=1.65-0.10 i$. Again, the scale size was adjusted to fit the color data; a much better fit to the polarization data could have been obtained for a slightly smaller scale size. The details of these solutions and a few others are listed in Table 4-3.
2. This may be taken as an indication that the distribution of grain sizes is not so wide as in the EXP distribution. However it will be shown below that an effect of internal transmission losses is to sharpen the polarization peak. It would be pushing the data rather hard to exclude the EXP distribution on this evidence alone.


Figure 4-4. Solutions for Data in IC 5076
The OHG size distribution is used. Details of the solutions are given in Table 4-3.


Figure 4-5. A Solution for IC 5076, for Silicate Grains
The OHG distribution is used. Details of the solution $\mathrm{m}^{*}=$ 1.65-0.10i are given in Table 4-3.

Table 4-3. Fits to color and polarization data in IC 5076.
All solutions are for the OHG size distribution. The scale sizes $a_{0}$ were chosen to give the best fit to the color data. The list is not exclusive; solutions are possible for other refractive indices.

| $\mathrm{m}^{*}$ | $\theta_{1} \rightarrow \theta_{2}$ | $\mathrm{a}_{0}$ microns | Notes |
| :---: | :---: | :---: | :---: |
| 1.10 | $150^{\circ}$ | $165^{\circ}$ | 0.17 |
| 1.30 | 150 | 165 | 0.17 |
| $1.30-0.05 i$ | 150 | 165 | 0.16 |
| $1.30-0.20 i$ | 150 | 165 | 0.15 |
| 1.50 | 150 | 165 | 0.15 |
| $1.50-0.10 i$ | 150 | 165 | 0.15 |
| $1.65-0.10 i$ | 155 | 160 | 0.145 |
| 2.42 | 155 | 160 | 0.15 |

a. Fit to color data only.
b. Internal reddening is needed to fit the whole color curve.
c. A smaller imaginary component is needed if internal reddening is present.
d. A poor fit.
e. Fit to polarization data only.

Figure 4-6 illustrates the effects of transmission losses for $m^{*}=1.30$. The main body of IC 5076 does not seem to be in contact with the illuminating star (Figure 3-8). Hence I have assumed that, at any scattering angle, the internal losses before scattering and after scattering are equal, and are proportional to the geonetrical depth within the ncbula. Let $2 \tau_{Q}(2)$ be the total optical depth traversed at $x_{0}=2$ for scattering at the deepest point in the nebula. The plotted solutions are for $\tau_{0}=0$ and $2 \tau_{0}(2)=0.5$; with $a_{0}=0.152 \mu, x_{0}=2$ falls at $1 / \lambda=2.09$ and the total optical depth reaches $2 \tau_{0}=1.46$ at the N filter, $1 / \lambda=3.03$. The addition of attenuation losses gives a stronger polarization peak and hence better agreement with the observed data. An optical depth $2 \tau_{0}(2) \simeq 0.3$ would fit the observations nicely. Increasing the optical depth above this value (I have made calculations up to $2 \tau_{0}(2)=6$ ) gives progressively worse fits to the color data with relatively little effect on the polarization.

Since attenuation losses have the effect of depressing the blue end of the color curve, it can be seen in Figure $4-4$ that $m^{\prime \prime}=0.05 i$ must be taken as an upper limit to the imaginary component for a refractive index with real part 1.30. Similarly, Figure 4-5 for $m^{*}=1.65$ $0.10 i$ shows that the imaginary component for $m^{\prime}=1.65$ must be somewhat less than 0.10.

If my models are correct, the IC 5076 obscrvations can be explained only by a much more restricted range of refractive indices than was the case in NGC 2068. Further observations in this faint nebula should be very interesting.


Figure 4-6. IC 5076 Solutions with Two Values of the Optical Depth.
Scattering angles range $150^{\circ}+165^{\circ}$. The plots are for $\mathrm{m}^{*}=$ 1.30 , wîth $a_{0}=0.16 \mu$ at $\tau_{0}=0$ and $a_{0}=0.15 \mu$ at $2 \tau_{0}(2)=0.5$.

Table 4-4 lists a few polarization solutions for the region of NGC 7023 38" northeast of HD 200775. The observational data were taken from Table 3-12, where the measures of Gehrels (1967) in three filters and a measure of my own in one filter were corrected for the interstellar polarization observed on HD 200775. All the fits in Table 4-4 fall within one probable error of the observed polarizations.

The last column gives the intrinsic $\underline{U}-\underline{V}$ colors predicted by the optically thin models which best fit the polarization data. The theoretical colors are much bluer than the $\underline{U}-\underline{V}=-0.39$ observed in this region by Elvius and Hall (1966). If my polarization models are meaningfu1, there is about one magnitude of internal reddening in $\underline{U}-\underline{V}$. Since this is actually the difference in reddening between the direct starlight and the scattered light, the optical depths in this nebula must be quite high. The polarization solutions in Table 4-4 give a maximum scattering angle of about $60^{\circ}$; it may be that the nebula is actually symmetrical about the star, but that scattering angles larger than $60^{\circ}$ are not seen because of transmission losses.

Table 4-5 gives a few fits to Gehrels' polarization data in the region $38^{\prime \prime}$ north of HD 200775. Due to the non-linearity of the observed polarizations as a function of wavenumber, none of the fits in this region are as good as those given above. Still, however, the theoretical curves all fall within twice Gehrels' probable error ( $\pm 0.9 \%$ ) of his data points. For this region $I$ found (Section 3.6.3) an intrinsic color $U-G=-0.15$; again, if the polarization models are meaningful, the

Table 4-4. Fits to polarization data in NGC 7023 38" NE.
Where multiple solutions are given for one refractive index, the best solution is indicated by an asterisk. The list is not exclusive; solutions are possible for other refractive indices.

| Distr. | $m^{*}$ | $\theta_{1} \rightarrow \theta_{2}$ | $a_{0}$ microns | Predicted <br> U-V |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| OHG | 1.30 | $20^{\circ} 5^{55^{\circ}}$ | $0.22 *$ | -1.31 |  |
|  | $1.30-0.05 i$ | 20 | 60 | 0.20 | -1.12 |
| OHG | $1.65-0.10 i$ | 20 | 55 | 0.14 | -1.50 |
| OHG | 1.10 | 20 | 60 | 0.37 | -1.01 |
| EXP | $1.65-0.10 i$ | 20 | 60 | 0.096 | -1.30 |
| EXP | 1.30 | 20 | 60 | 0.15 | -1.28 |
|  |  |  |  |  |  |

Table 4-5. Fits to polarization data in NGC 7023 38" N.
Where multiple solutions are given for one refractive index, the best solution is indicated by an asterisk. The list is not exclusive; solutions are possible for other refractive indices.

| Distribution | $m^{*}$ | $\theta_{1} \rightarrow \theta_{2}$ | $a_{0}$ microns |
| :---: | :---: | :---: | :---: |
| OHG | 1.10 | $20^{\circ} 60^{\circ}$ | $0.30:$ |
| OHG | 1.30 | 20 | 55 |
| OHG | $1.30-0.05 i$ | 15 | 60 |
| OHG | $1.65-0.10 i$ | 15 | 60 |
| EXP | 1.30 | 20 | 60 |
| EXP | $1.65-0.10 i$ | 20 | 60 |

internal reddening in NGC 7023 is seen to be very strong. The optical depth of this nebula will be further discussed in the following section.

### 4.4 Nebular Surface Brightnesses

Modern astrophysical studies have concentrated on colors and polarizations in reflection nebulae; surface brightnesses have often been neglected. For the Merope nebula, Hanner (1969) gave considerable attention to the offset dependence of the brightness ratio of nebula to star, without mentioning the actual value ${ }^{1}$ of this ratio. She used a nebular opacity obtained from previous studies of interstellar extinction in the Pleiades region. However the space density of grains and hence the optical depth of the nebula can, in principle, be obtained from the brightness ratio $I_{\text {neb }} / I_{*}$. The procedure is as follows: From Section 2.1 .6 we have

$$
I_{\text {neb }}=\frac{1}{2} I_{*} \exp \left(\tau^{\prime \prime}\right) N \pi a_{0}^{2}\left[T_{1}+T_{2}\right],
$$

where

$$
\mathrm{T}_{1}+\mathrm{T}_{2}=\mathrm{b} \Delta \theta \operatorname{Tanh}^{-1}(\mathrm{~b} / \mathrm{L}) \sum_{\theta}\left[\mathrm{F}_{1}+\mathrm{F}_{2}\right]_{\theta} \exp \left(-\tau-\tau^{\prime}\right)_{\theta} .
$$

From Section 2.1.4, the opacity is given by

$$
k\left(x_{0}\right)=N \pi a_{c}^{2} q\left(x_{0}\right)
$$

1. It should be mentioned in this connections that there is an error of a factor of 10 in Table $V$ of Elvius and Hall (1967). The third footnote to that table should read "multiply tabular data by 400" rather than "by 40". I am indebted to Dr. John S. Hall (1969, personal communication) for assistance with this point.
and the total optical depth of the nebula is $\tau_{0}=\kappa D$, where $D$ is the geometrical thickness of the ncbula, obtained from the range of scattering angles and the distance to the nebula. Otherwise the symbols here are as defined in Chapter 2. We now have

$$
I_{\text {neb }}=\frac{1}{2} I_{*} \exp \left(\tau^{\prime \prime}\right) \frac{\tau_{0}}{D q}\left[T_{1}+T_{2}\right] .
$$

If the overall geometrical configuration of the nebula is known, $\tau(\theta)$, $\tau^{\prime}(\theta)$, and $\tau^{\prime \prime}$ can be expressed in terms of $\tau_{0}$, which can then be found from the observed ratio $I_{n e b} / I_{*}$.

For IC 5076 the dust cloud does not seem to extend in front of the star, so $\tau^{\prime \prime}=0$. Then neglecting $\tau(\theta)$ and $\tau^{\prime}(\theta)$ in the exponential gives an approximate value for $\tau_{0}$. The result for $m^{*}=1.30$ is $\tau_{0} \sim 4$ in green light. (The exact value can be obtained only by building a complete multiple-scattering model.) This result is quite different from the value $2 \tau_{0} \sim 0.3$ for twice the nebular thickness derived above on the basis of the observed colors.

This procedure for calculation of the optical depth cannot be directly applied to NGC 2068 because of the dust globule in front of the illuminating star. If the thickness of the globule is about 2.8 magnitudes as suggested in Section 3.4 .1 on the basis of the differential reddening between Star A and Star B, the corrected ratio of total nebular brightness to the brightness of Star B still comes out to be near unity, a result that is possible in Mie scattering only in the case of a high optical depth.

Thus in all three nebulae the surface brightnesses are far too high to be commensurate with my assumption that the nebulae are optically thin. It is then puzzling that the colors and polarizations fit as well as they do, and that at least in the case of IC 5076 the fit is made worse by the addition of more than a slight optical depth. Either there is a fundamental error in my brightness calculations, or else the effects of multiple scattering, completely neglected here, are largely to compensate for transmission losses. This matter needs to be explored by construction of more sophisticated models.

### 4.5 Derived Grain Properties

It was shown in the sections above that satisfactory fits to my observations of reflection nebulae can be obtained for a rather wide range of refractive indices $m^{*}=m^{\prime}$ - im'. With $m^{\prime}$ as small as 1.10 one may have trouble explaining the polarizations and colors in IC 5076, and with $m^{\prime}$ much larger than 1.50 a small imaginary component is needed. In no case can solutions be found for $\mathrm{m}^{\prime \prime}$ as large as in graphite or metallic grains. The IC 5076 solutions suggest that $\mathrm{m}^{\prime \prime}$ should not exceed 0.05 for $m^{\prime}=1.30$ or 0.10 for $m^{\prime}=1.65$; this conclusion contradicts that of Gehrels (1967) that an imaginary component in the range 0.1 to 0.4 is needed for the best fit to his observations in NGC 7023.

In Table 4-6 are collected the scale sizes derived above for three of the more fashionable refractive indices, $\mathrm{m}^{*}=1.30$ for pure ices, $m^{*}=1.30-0.05 i$ for dirty ices, and $m^{*}=1.65-0.10$ i for silicates. The mean values and their standard deviations are obtained

Table 4-6. Derived grain sizes.
The listed values are scale sizes ao (Section 2.1.3) in the OHG size distribution. The listed uncertainties are standard deviations.

| m* | Nebula | $\mathrm{a}_{0}$ microns | Mean $\mathrm{a}_{0}$ |
| :---: | :---: | :---: | :---: |
| 1.30 | 2068 I | 0.26 | $0.20 \pm 0.04$ |
|  | 2068 II | 0.15 |  |
|  | 5076 | 0.17 |  |
|  | 7023 NE | 0.22 |  |
|  | 7023 N | 0.16 |  |
| 1.30-0.05i | 2068 I | 0.27 | $0.22 \pm 0.05$ |
|  | 2068 II | 0.20 |  |
|  | 5076 | 0.16 |  |
|  | 7023 NE | 0.26 |  |
|  | 7023 N | 0.17 |  |
| 1.65-0.10i | 2068 I | 0.18 | $0.15 \pm 0.02$ |
|  | 2068 II | 0.13 |  |
|  | 5076 | 0.14 |  |
|  | 7023 NE | 0.14 |  |
|  | 7023 N | 0.12 |  |

by assigning triple weight to data from IC 5076 and Region I of NGC 2068 (fits to both color and polarization data), double weight to data from 2068 II and 7023 NE (color data lacking or not fitted) and single weight to 7023 N (marginal fit to polarization data.)

In view of the optical-depth difficulties discusscd above, my models are far from definitive, and I doubt that the deviations among the various regions can be taken to be significant. However I do believe that the mean scale sizes are significant. It is not clear that the derived values would be affected very much by a better treatment of optical depth; at least they could not be much larger without obtaining negative polarizations.

It is often assumed (e.g., Greenberg 1968) that silicate grains, with $m^{\prime}-1$ twice as large as ice grains, will fit the interstellar extinction curve as well as ice grains if the scale size is taken to be a factor of two smaller. This scaling of refractive indices is not exact for any scattering circumstances, and Table 4-6 shows that it does not hold in my reflection nebula models; the ratio of ice to silicate scale sizes comes out to be more like 4 to 3 .

Comparisons of the values in Table 4-6 with grain sizes obtained from interstellar extinction studies are interesting. The classical scale size (Greenberg 1968) for dirty ice spheres to give the best fit to the extinction curve in the visible part of the spectrum is $a_{0}=$ $0.5 \mu$, much larger than the values that $I$ found in these nebulae. This is a startling result, and one conflicting with the conclusion of

Greenberg (1968, page 355) and of Hanner (1969) that the best fit to the observed colors and polarizations in the Merope nebula is found for ice grains with $a_{0} \simeq 0.5 \mu$ or slightly larger.

If my models are meaningful, wo are forced to one of three conclusions: Either 1) ice grains in these nebulac are considerably smaller than those in the general interstellar medium, or 2) ice grains have a shape, structure, size distribution, or refractive index that is significantly different from the grains used in my models, or 3) the grains are not ices.

Here I would like to add that the results should not be overinterpreted. The scale size $a_{0}$ is left as a completely free parameter in my models, and it is perhaps not too surprising that it should come out to differ by a factor of two or three from the value which best fits the extinction data; at least the two kinds of observations can be explained by the same typc of grain. Also, in reflection nebulae we are sampling a comparatively tiny volume of space, whereas extinction data are averaged over hundreds of parsecs.

Recently Greenberg and Shah (1969) have found a good fit to the whole interstellar extinction curve (pre-OAO data) for cylindrical grains of dirty ice, $m^{*}=1.33-0.05 i$, with a size distribution of radii

$$
f(a)=49 \exp \left(-5[a / 0.2]^{3}\right)+\exp \left(-5[a / 0.6]^{3}\right)
$$

Disregarding the $2 \%$ admixture of large grains, the value $a_{0}=0.2 \mu$ is in excellent accord with my values. However cylindrical and spherical
grains camnot be directly compared, so this agreement may have no significance.

To my knowledge no detailed fit to the extinction curve for silicate grains has been published. There is no consensus as to exactly what refractive index would be appropriate for silicate grains, and, since they do not condense from the interstellar medium, there is no physical justification for use of the OHG or EXP distributions. But using $\mathrm{m}^{*}=1.65-0.10 \mathrm{i}$ and the OHG distribution, I find the best fit in visible light to the extinction data (taken from Table VI of Shah 1967) to fall at about $a_{0}=0.22$ microns, significantly larger than the values in Table 4-6. Wickramasinghe (1969) has found a good fit to the whole extinction curve (pre-OAO data) for mixture of graphite and single-sized silicate ( $\mathrm{m}^{*}=1.66$ ) grains with radii in the range $0.07 \mu$ to $0.09 \mu$ for the silicate component. This is roughly equivalent (Greenberg 1968, page 247) to $\mathrm{a}_{0}=0.26 \mu$ in the OHG distribution. Thus, as in the case of ices, silicate grains come out to be significantly smaller in my models for reflection nebulae than in the general interstellar medium.

It was remarked in Chapter 1 that the subject of the observed interstellar extinction curve and its interpretation it is much more of a state of confusion today than it was a few years ago. In any case extinction studies alone are not likely to recult in an unique interpretation, and other types of observations will be needed to choose between competing grain models. Hopefully, the grain sizes which I have obtained will be helpful in making such a choice.

## CIIAPTER 5

CONCLUSIONS

Here I will summarize briefly the results obtained in the chapters above, and suggest which lines of future work might be most profitable.

### 5.1 Summary

In Chapter 1 it was shown that a number of quite different models for the interstellar grains are currently viable, and that much more work must be done before a consensus can be reached concerning their nature. High-quality observations of reflection nebulae are few, and a beginning toward definitive models has been made only for the Merope nebula. Existing observations can be used to conclude that, while the intrinsic colors of reflection nebulae vary from slightly red to extremely blue, polarizations are invariably positive and less than about $25 \%$, and usually have a characteristic wavelength dependence in which the polarization drops smoothly toward shorter wavelength.

Two distinct approaches to theoretical models have been used, one suffering from over-idealization of the nebular geometry, and the other, which I used, free of geometrical constraints but poorly adapted for treatment of transmission losses and multiple scattering.

In Chapter 2 equations were developed for the intensity, color, and polarization of light emerging from a segment of an illuminated
dust cloud. The models were general, except for the assumptions of isotropic spherical grains and a certain degree of spatial homogencity, and except for a total neglect of light scattered more than once. Two types of one-parameter size distributions, similar to those which have been proposed for grains accreted from the interstellar medium, were defined, and a new dimensionless quantity was introduced which allowed a single set of Mie calculations to serve for any scale radius in either distribution. A simple but original graphical method was described for comparison of theoretical and observational data.

Representative theoretical color and polarization functions were presented for various types and size distributions of grains. It was shown that, within the limitations of my models, the characteristic wavelength dependence of polarization in reflection nebulae cannot be explained by highly absorptive grains. The differences between predicted observable properties for various kinds of predominately dielectric grains, however, were found to be slight.

In Chapter 3 techniques were described for photometry and polarimetry of objects of low surface brightness. It was pointed out that no exact solution is available for decoupling a known interstellar polarization from an intrinsic nebular polarization. The problems of instrunental halation were found to be less serious than was anticipated.

Colors and polarizations in one region of NGC 2068 and one region of IC 5076 were measured in several filters from the ultraviolet to the infrared. Smaller amounts of polarization data were obtained in
a second region of NGC 2068, a region in NGC 7023, and a region in Cederblad 201. In two cases where comparisons could be made, the polarizations that I obtained were substantially higher than those reported by Elvius and llall (1966) for the same regions.

The illuminating star in NGC 2068 shows a wavelength dependence of polarization unlike that found on any other star, and its polarization may be variable over time. The nebula is much brighter than the illuminating star, apparently due to a small foreground dust globule in the line of sight to the star. In one of the regions of NGC 2068 a wavelength-dependent departure of the position angle of polarization from radial symmetry about the illuminating star was found. The colors in this region of NGC 2068 agreed with those reported by Johnson (1960) and Elvius and Hall (1966).

The wavelength dependence of polarization in IC 5076 is different from all other nebulae for which good data are available. Its intrinsic color was found to be bluish at long wavelengths but almost neutral at short wavelengths.

The measured polarization of NGC 7023 in a region worked by Gehrels (1967) was in excellent agreement with his values. In a second region which he worked the surface brightness was found to be much higher than the value that he used (from Martel 1958); thus his conclusion that NGC 7023 is optically thin was found to be incorrect.

Comparisons between theory and observation were presented in Chapter 4. In no casc could the observations be explained by graphite or metallic grains. Neglecting internal attenuations, good fits to
polarization data in NGC 2068 and NGC 7023 were found for dielectricgrain models with the star behind the nebula. The observed colors in NGC 2068 departed from those predicted by the models in a direction attributable to the effects of the foreground dust globule, and in NGC 7023 in a direction attributable to internal reddening. For refractive indices with real part larger than 1.5 , good fits to the polarization data could be found only if a small ( $\sim 0.10$ ) imaginary component is included.

Successful models for both color and polarization in IC 5076 were found for the nebula well behind the star. Taking into account the effects of internal reddening, the color data required that the imaginary component $\mathrm{m}^{\prime \prime}$ be not greater than 0.05 for real part $\mathrm{m}^{\prime}=$ 1.30 , or not greater than $m^{\prime \prime}=0.10$ for $m^{\prime}=1.65$. Poor fits were found for pure dielectric refractive indices as small as 1.10 or higher than 2.0. For refractive index 1.30 , the best fit to the color data was obtained with a nebular optical depth $\sim 0.15$ in green light.

The scale radii of grains in a size distribution similar to that of Oort and van de Hulst (1946) varied only about $25 \%$ from nebula to nebula, and in all cases were smaller by a factor of two or three than the values which best fit the interstellar extinction curve. It is not clear whether this result should be taken as evidence that the grains are anomalously small in these nebulae, or that the grain models are incorrect.

In all of the nebulae the surface brightnesses were found to be far too high to be consistent with the optically-thin models. It is
then puzzling that the models explain the colors and polarizations as well as they do.

The appropriate refractive index for any chemical composition of grains in interstellar space may be open to question. If dirty ices may be described by $m^{\prime} \simeq 1.30$ and $m^{\prime \prime}<0.05$, and silicates by $m^{\prime} \simeq 1.65, \mathrm{~m}^{\prime \prime}<0.10$, then the observations are consistent with either material. Their optical properties are much alike, and I doubt that a choice between the two can be made by other than spectrographic methods.

### 5.2 Future Work

In the words of Middlehurst and Aller (1968), in all branches of astronomy "we witness an ever-growing number of theories chasing an inadequate number of observations." This is especially true of reflection nebulae; even in the case of the Merope nebula, which has been the subject of at least five major photoelectric studies in the past ten years, observations at wavelengths longer than $0.6 \mu$ are still lacking.

Dr. Hanner expressed to me the opinion that the most valuable measurement in a reflection nebula is the wavelength dependence of polarization as a function of offset angle. I concur, with the warning that such data come slowly; a proper job in three widely spaced filters (e.g., ultraviolet, green, and near infrared) requires about 30 hours of integration time at a 61 -inch telescope for each region. This figure may be reduced by the use of more efficient instrumentation, such
as multi-chamel photometers which simultancously work in several filters or regions, but large observing programs will still be required.

My conclusion that IC 5076 is behind the star needs to be chocked by additional observations. I obtaincd more information about the scattering grains from this nebula than from the brighter, betterknown ones. At least two more regions should be worked to a precision equal to or better than that obtained in this study. This would require six to ten clear nights at a telescope comparable to the Catalina reflector; fortunately, the nebula is well placed for northern observatories and is best worked in August, when demands on telescope time are ordinarily at a minimum.

There are many interesting sidelights to observations of reflection nebulac. Are there genuine departures of the plane of polarization of the scattered light from radial symmetry about the illuminating star, and how strong can they be? Can the existence of a single case of negative polarization be established? Are atomic emissions spatially correlated with fine structure in the intensity of the reflected light? The color curve observed in NGC 2068 was not perfectly smooth over wavelength. Are the irregularities real, and can they be attributed to the scattering grains? Do diffuse interstellar absorption lines show as emission in reflection nebulae?

Is a reflection nebula simply an ordinary dust cloud which is accidentally illuminated by a passing star, or is it a very special region of space, where stars are being formed, grains accreting or losing mantles, etc.? The direct photographs of NGC 2068 and IC 5076 show
structures which, with little effort of the imagination, look like fans or shells symnetric about the illuminating star. Do we here actually see interactions between stars and dust grains?

A search should be made for nebulae which, in addition to the criteria listed in Chapter 1, have smoother isophotes than NGC 2068 and NGC 7023, and, if possible, have lower optical depth. The Palomar Sky Survey prints, while essential for location of dark sky regions, are too heavily exposed and have too small a scale for such evaluations. A photographic survey of likely candidates should be undertaken with a plate scale of not more than 20 arcseconds per millimeter.

Definitive theoretical models are not likely to emerge until the "intensive" and "extensive" approaches described at the beginning of Chapter 2 can be combined, i.e., until complete multiple-scattering models can be built using a detailed nebular geometry which is not arbitrarily assumed but determined from observational data. There are, however, many unsolved theoretical problems which are applicable to existing data. Under what circunstances can grains give non-radial polarizations? Is there some fundamental reason why negative polarizations, or, for that matter, circular polarizations, are not found? What parameters of composite grains would fit my observations?

Is it true that shape effects are minimized when a wide size distribution is used? Could a better agreement between grain sizes obtained in reflection nebulae and those obtained from interstellar extinction and polarization be found for a different type of size distribution? llow is it that nebular surface brightnesses seem to
require high optical depths, but my models fit the data best for small opacities? Finally, are my models, with their at best somi-quantitative treatment of optical depth effects, really meaningful?

For more sophisticated models my inclination would be to use the inefficient but straightforward "Monte Car10" approach rather than more elegant analytic methods.

Even if the properties of interstellar grains are established by other methods, reflection nebulae will still be of interest; only here can the grains in a small volume of three-dimensional space be studied. Reflection nebulae afford problems enough, and fascination enough, for years of work.

## APPENDIX I

THE VOLUME INTEGRAL

In the integration of scattered intensity over volume, one encounters a geometrical integral of the form

$$
\int_{\Delta V_{j}} d V / R^{2},
$$

where $R$ is distance from the illuminating star, and the volume element $\Delta V_{j}$ is that segment of a cylinder (Figure 2-2) of radius $b$ and perpendicular distance $L$ from the illuminating star bounded by planes at scattering angles $\theta_{j} \pm \frac{1}{2} \Delta \theta$. No exact solution of this integral has been found, but it has generally been taken to be independent of $\theta_{j}$.

If $\Delta \theta$ and $b / L$ are small, the integral may be approximated in two ways. Replacing the $\mathrm{R}^{-2}$ in the integral by the central distance $\mathrm{R}_{0}$ of the volume element gives

$$
\int \mathrm{dV} / \mathrm{R}^{2} \simeq 2 \pi \mathrm{~b}^{2} \mathrm{~L}^{-1} \tan ^{\frac{1}{2}} \Delta \theta\left[1-\tan ^{2} \frac{1}{2} \Delta \theta \cot ^{2} \theta\right]^{-1} .
$$

which indeed is nearly constant over $\theta$. If $\Delta \theta=5^{\circ}$, it varies only about six percent between $\theta=10^{\circ}$ and $\theta=170^{\circ}$. A more interesting case arises if we approximate

$$
\int d V / R^{2} \simeq\left\langle R^{-2}\right\rangle_{j} \cdot \Delta V_{j}
$$

where the mean is taken not over the volume element but over its projection $A_{j}$ in the plane of scattering. This gives

$$
\int d V / R^{2} \simeq \pi b \Delta \theta \operatorname{Tanh}^{-1}(b / L)
$$

which is completely independent of the scattering angle. This is the approximation used in my models.

## APPENDIX II

## DECOUPLING OF POLARIZATIONS

Suppose that the combination of an unknown intrinsic polarization $P_{1}$ at position angle $\theta_{1}$ and a known intersteilar polarization $P_{2}$ at position angle $\theta_{2}$ gives an observed polarization $P$ at position angle $\theta$. How can $P_{1}$ and $\theta_{1}$ be determined?

Lodên (1961) found, in the form of six simultaneous transcendental equations, a solution for the inverse problem of finding $P$ and when $P_{1}, \theta_{1}, P_{2}$, and $\theta_{2}$ are known. His solution is rigorous for the case of zero attenuation in the plane of polarization of the interstellar component.

The usual approach for small polarizations is to divide each polarization into orthogonal components:

$$
\begin{aligned}
& P_{1_{x}}=P_{1} \cos 2 \theta_{1}, \\
& P_{1_{y}}=P_{1} \sin 2 \theta_{1}, \\
& P_{2_{x}}=P_{2} \cos 2 \theta_{2}, \\
& P_{2_{y}}=P_{2} \sin 2 \theta_{2}, \\
& P_{x}=P \cos 2 \theta, \\
& P_{y}=P \sin 2 \theta,
\end{aligned}
$$

The array $\left(P_{2}, P_{2}\right)$ is not part of a Stokes' vector or even part of
a normalized Stokes' vector, since it represents not a separate beam of light but an analyzer. If, however, we assume that these quantities behave like the components of a Stokes' vector, we may write

$$
\begin{aligned}
& P_{x}=P_{1_{x}}+P_{2 x}, \\
& P_{y}=P_{l_{y}}+P_{2 y},
\end{aligned}
$$

so that

$$
\begin{equation*}
\tan 2 \theta=P_{y} / P_{x} \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
P^{2}=P_{x}^{2}+P_{y}^{2} \tag{2}
\end{equation*}
$$

The proof that such a procedure is valid for small polarizations is as follows: From his general solution, Lodén obtained approximate equations for small polarizations. In terms of the functions

$$
\begin{aligned}
& x=\theta_{2}-\theta_{1}, \\
& \psi=\theta-\theta_{1},
\end{aligned}
$$

Lodén obtained

$$
\begin{gather*}
\tan 2 \psi=\sin 2 x /\left[\cos 2 x+\left\{P_{1} / P_{2}\right\}\right],  \tag{3}\\
P=\cos 2(x-\psi)\left[P_{2}+P_{1} \cos 2 x\right]+P_{1} \sin 2(x-\psi) \sin 2 x . \tag{4}
\end{gather*}
$$

By the method of orthogonal components, Equation 1 above gives

$$
\tan 2\left(\psi-\theta_{1}\right)=\left[P_{1} \sin 2 \theta_{1}+P_{2} \sin 2 \theta_{2}\right] /\left[P_{1} \cos 2 \theta_{1}+P_{2} \cos 2 \theta_{2}\right] .
$$

Substituting the trigononetric identity for $\tan 2\left(\psi-\theta_{1}\right)$, multiplying out the denominators, and collecting terms with the help of the usual trigonometric identities for the sinc and cosine of the difference between two angles, one obtains an expression identical to Equation 3 above from Lodén. Thus the method of orthogonal components is valid as to position angles for small polarizations.

For the degree of polarization, expanding Equation 2 in terms of $\theta_{1}$ and $\theta_{2}$ and collecting terms gives

$$
\begin{equation*}
\mathrm{p}^{2}=\mathrm{P}_{1}^{2}+2 \mathrm{P}_{1} \mathrm{P}_{2} \cos 2 x+\mathrm{P}_{2}^{2}, \tag{5}
\end{equation*}
$$

whereas Equation 4 from Lodén's analysis reduces, after considerable labor, to

$$
P=P_{2} \sin 2 \chi \csc 2 \psi .
$$

Squaring this expression and using the identity $\csc ^{2} 2 \psi=1+\cot ^{2} 2 \psi$, and substituting Equation 3 for $\cot ^{2} 2 \psi$ gives exactly Equation 5. Thus, for small polarizations, Loden's exact solution and the method of orthogonal components are equivalent. Q.E.D.

Given any function $z=f(x, y)$ and measured values $x$ and $y$ with uncertainties (standard deviations) $\delta x$ and $\delta y$, the uncertainty in $z$ is

$$
\delta z=\delta x \cdot \partial z / \partial x+\delta y \cdot \partial z / \partial y .
$$

Then if we have a measured polarization $P$ at position angle $\theta$, with standard deviations $\delta \mathrm{P}$ and $\delta \theta$, the orthogonal components as defined above have uncertainties

$$
\begin{aligned}
& \delta \mathrm{P}_{\mathrm{x}}=\left[(\delta \mathrm{P})^{2} \cos ^{2} 2 \theta+(\delta \theta)^{2} 4 \mathrm{P}^{2} \sin ^{2} 2 \theta\right]^{\frac{1}{2}} \\
& \delta \mathrm{P}_{\mathrm{y}}=\left[(\delta \mathrm{P})^{2} \sin ^{2} 2 \theta+(\delta \theta)^{2} 4 \mathrm{P}^{2} \cos ^{2} 2 \theta\right]^{\frac{1}{2}}
\end{aligned}
$$

If orthogonal components $P_{x}$ and $P_{y}$ are reconstituted into a degree of polarization at a position angle, the standard deviations are

$$
\begin{aligned}
& \delta P=p^{-1}\left[P_{x}^{2}\left(\delta P_{x}^{2}\right)+P_{y}^{2}\left(\delta P_{y}^{2}\right)\right]^{\frac{1}{2}}, \\
& \delta \theta=\frac{\cos ^{2} 2 \theta}{2 P_{y}}\left[\tan ^{2} 2 \theta\left(\delta P_{x}\right)^{2}+\left(\delta P_{y}\right)^{2}\right] .
\end{aligned}
$$

## APPENDIX III

## EFFECT OF HALATION ON POLARIZATIONS

For a region of NGC 7023, exact corrections for instrumental halation were obtained by measuring the brightness ratio of halation to star at each Wollaston angle and depolarizer state. Such a procedure was not justified by the level of precision attained in IC 5076. Instead, approximate polarization corrections were obtained for each filter from the photometry of the halation, assuming it to be seen by the polarimeter exactly as an unpolarized celestial object.

At any Wollaston angle $\phi$, let $\mathrm{I}_{1}$ and $\mathrm{I}_{2}$ be the intensities in the two channels arising from the nebulosity when the depolarizer is out, $I_{d}$ the nebular intensity observed in either channel when the depolarizer is in, and $I_{h d}$ and $I_{h}$ the halation intensities arising respectively when the depolarizer is in and out of the beam.

The true value of the polarization in magnitudes is computed
as

$$
\Delta M_{p}(\phi)=-2.5 \log G(\phi),
$$

where

$$
G(\phi)=\frac{\alpha_{1} I_{1} / \alpha_{1} I_{d}}{\alpha_{2} I_{2} / \alpha_{2} I_{d}}
$$

and $\alpha_{1}$ and $\alpha_{2}$ are the instrumental sensitivities in the two channels.

Now in the presence of a halation we actually observe

$$
G^{\prime}(\phi)=\frac{\alpha_{1}\left\{I_{1}+I_{h}\right\} / \alpha_{1}\left\{I_{d}+I_{h d}\right\}}{\alpha_{2}\left\{I_{2}+I_{h}\right\} / \alpha_{2}\left\{I_{d}+I_{h d}\right\}}
$$

The effect of the halation with the depolarizer in the beam cancels completely. We are left with

$$
G^{\prime}(\phi)=G(\phi)\left\{1+R_{1}\right\} /\left\{1+R_{1} G(\phi)\right\},
$$

where $R_{1}=I_{h} / I_{1}$. Expanding the denominator in a power series and keeping terms through second order, we obtain

$$
G^{\prime}(\phi)=G(\phi)\left[1+R_{1}\{1-G\}\left\{1-R_{1} G\right\}\right] .
$$

The effect of a non-zero $R_{1}$ is always to make $G^{\prime}$ closer to unity than G, that is, to make the observed polarization smaller than the true polarization.

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