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# OBSERVATIONAL ASPECTS OF CEPHEID EVOLUTION 

by<br>Robert James Havien

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GRADUATE COLLEGE

I hereby recommend that this dissertation prepared under my direction by ___ Robert James Haven entitled Observational Aspects of Cepheid Evolution
be accepted as fulfilling the dissertation requirement of the degree of $\qquad$ Doctor of Philosophy


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## ABSTRACT

Three galactic classical cepheids are studied with respect to their relationship to the surrounding stars and interstellar material. The observations are interpreted within the existing framework of present theories of cepheid evolution and of the origin of stars in groups. The 41.4 period cepheid, RS Pup, has previously been discovered within a symmetrically structured reflection nebula and as a member of Pup III. In the present study, isodensitometry is employed to analyze the reflection nebula for spatial and temporal light variations. No spatial variations in the nebular structure are found, but the intensity from three individual nebular features is found to vary with the same period as the cepheid. On this basis the distance to the cepheid is geometrically determined to be $1.78 \pm .02 \mathrm{kpc}$. The radial color change of the nebula with increasing angular separation from RS Pup goes from blue to red, in support of the reflection character of the nebula. The observations suggest a model of the nebula consisting of several concentric spherically symmetric reflecting shells. In addition, the radial change of the average nebular $B$ magnitude indicates that the probable formation mechanism for the shells was a combination of material ejection from the cepheid and a sweeping up of
the ambient interstellar medium. Two independent methods indicate that the total mass of dust in the nebula is no greater than $.03 M_{0}$. This value $i s, ~ h o w e v e r, ~ s u f f i c i e n t l y$ uncertain to indicate the dominant mechanism in the origin of the nebular material. The red giant phase of evolution between successive transits of the instability strip is suggested as the epoch during which the nebulosity around the cepheid is structured. In a less dusty environment, however, the structured nebula would not be visible.

SU Cas is the one other galactic classical cepheid with a nearby nebulosity. An image tube spectrum reveals that the nebula is a reflection nebula, but there is no visible indication of any interaction of cepheid and nebula. A photometric and spectroscopic study of the $B$ stars within $5^{\circ}$ of $S U$ Cas reveals an increase in the number of stars at the distance of the cepheid. The observations, however, are not conclusive enough to be interpreted as an old association with the cepheid as a member. At present, SU Cas is considered as an independent member of the Orion arm, and the cepheid is undergoing a coincidental meeting with a dust cloud in the region.

A photometric study of the $O B$ stars within $3^{\circ}$ of the 45 . 1 period cepheid, $S V$ Vul, reveals that it is near Vul $O B 1$ and that there is a general concentration of earlytype stars in the region. The interpretation of the cepheid as a member of the association, however, is at
present uncertain since it appears to be older than the association members.

## INTRODUCTION

Recent investigations concerning the evolution of massive stars through the region of the $H-R$ diagram known as the instability strip have given rise to a better understanding of the evolutionary history of the classical cepheids. It is presently thought that stars more massive than about $4 M_{0}$ evolve off the main sequence toward the red giant region, crossing the instability strip several times in the course of their history. Less massive stars either pass through the instability strip too rapidly or never enter it at all in their evolution to the red giant stage. Of the stars that do become cepheids, however, the more massive stars evolve more rapidly than the less massive stars and form cepheids of longer period and higher luminosity. The less massive cepheids are much older, their physical conditions giving rise to lower intrinsic luminosities and shorter periods. Furthermore, there is a uniform transition of physical conditions from those cepheids at the lower mass boundary up to the most massive cepheids, as evidenced by the long established observational period-luminosity relation. The presently accepted theory has developed primarily as a result of independent parallel investigations on the physics of stellar pulsational instability and in the area of stellar evolution
beyond the main sequence hydrogen burning phase. Historically, progress in each area has been based largely upon comparisons between the contemporary theory and available observations.

Eddington (1917) pioneered the study of the pulsational character of cepheids when it became clear that the observations of light and velocity variations in these stars no longer supported the binary star hypothesis. Eddington advanced the view that cepheids were pulsating radially with their energy supplied in the manner of an internal heat engine. The maintenance of the energy source, which was required to supply heat during the star's compression, remained, however, a mystery. Subsequently, Eddington (1918) put forth a linear adiabatic theory based on small perturbations applied to normal stellar structure. It successfully reproduced several of the major observational characteristics which were known about cepheids: the period-density relation, the trend toward later spectral types with increasing luminosity, and the asymmetry of cepheid velocity curves. The dissipation of energy due to the periodic flow of heat from the star, however, resulted in pulsation lifetimes no longer than 1500 years. Although short as measured by present day standards, these lifetimes were not necessarily in contradiction with those predicted on the basis of contemporary stellar evolution theory. Eddington's investigations provided many insights into
cepheid theory which are still basically sound today. Nevertheless, it remained until the discovery of the high abundance of hydrogen in the stars in 1932 (Eddington 1.932; Strömgren 1932) that the physical mechanism responsible for exciting cepheid pulsations was recognized.

The first clear understanding of this mechanism was realized when Zhevakin (1953) succeeded in applying linear nonadiabatic theory to his study of stellar pulsation. Near the stellar surface the adiabatic assumption previously employed breaks down and Zhevakin was able to demonstrate that the partial ionization of $\mathrm{He}^{+}$in this region contributed toward exciting the pulsation. During the compression of these layers in the stellar atmosphere the ionization of $\mathrm{He}^{+}$absorbs energy. In addition, the opacity increases during the ionization leading to a further retention of heat. Upon expansion the process is reversed wherein the release of heat perpetuates the pulsation. Other investigators (Baker and Kippenhahn 1962, 1965) later calculated a similar large contribution to the pulsation from the hydrogen ionization zone.

Linear adiabatic and nonadiabatic pulsation theories, however, offer only limited comparison to the observations of variable stars. Both theories result in eigenvalue equations for the variation of radius, density, and temperature from assumed values of the mass and luminosity, but they lack the sophistication necessary for
a thorough comparison with the observations. Linear adiabatic theory provides confirmation of the periods of the stars, and linear nonadiabatic theory extends the comparison to include the existence of instability and the location of the instability strip, but these comparisons are unable to explain most of the observational details. Linear theories are principally only a satisfactory treatment for small amplitude perturbations on the equilibrium stellar configurations. The calculations, on the other hand, indicate the amplitude of the temperature variation in the hydrogen ionization zone to be so large that the results of the linear theory can not validly be compared to the observed stellar variations. In other words, the assumed initial conditions of the theory are contradicted by the results of the calculations. A much more satisfying treatment of the problem has been made possible through the use of nonlinear theory.

The application of nonlinear theory to the problem of stellar pulsation was initiated by Christy (1962). This theory employs the same basic physics as the preceding linear theories with the one fundamental procedural exception that all of the equations of motion and heat transfer are integrated in their complete nonlinear form as an initial value problem and with a minimization of simplifying assumptions. With the advantage of not being restricted by small amplitude assumptions, Christy's approach then
successfully treats the behavior of an actual large amplitude stellar oscillation. The nonlinear theory encompasses all of the physical interpretations of the prior linear theories while at the same time offering more detailed physical insights into the various aspects of the pulsation over the entire stellar structure as well as the observable surface variätions. The hydrogen and $\mathrm{He}^{+}$ ionization zones are still the major driving force of the pulsations, but there no longer appears to be an abnormal growth in the amplitude of pulsation for a large range of initial conditions.

The principal most valuable contribution of the nonlinear theory to the study of pulsation, however, has been the comparison of theory and observation that the theory allows. The physical situation is handled in such detail as to be able to predict many of the observable variations which are a manifestation of the outermost layers of the stellar envelope. Consequently, observable phenomena such as the amplitudes of light and velocity variations, the phase relations, and the relations between temperature and radius amplitude have come under the scrutiny of the theory. It has even been possible to reproduce and understand the characteristic fine details of both the light and velocity curves. The resulting analysis has led to improved values of the basic stellar parameters of mass, luminosity, effective temperature, and helium content. Two excellent
reviews by Christy (1966, 1968) serve to summarize these advances in pulsation theory.

As far reaching as the advances in pulsation theory have been, however, there are still many weaknesses in the physical theory that remain to be treated satisfactorily. Of major concern in the study of stellar envelopes are the effects of convective energy transport in the static as well as the dynamic envelope. The neglect or inadequate treatment of convection in the study of cepheid pulsation has failed to account for the existence of the low temperature boundary of the instability strip. Whether or not an adequate treatment of the interaction of convection and pulsation will explain the observations is as yet unknown. Also unknown are the effects on the surface layers of the successive shocks associated with pulsation. Not unlikely is the possibility that surface layers are driven off by the passage of such shock waves, however this likelihood is presently uncertain, and its study is left for a more coherent theory of pulsation and hydrodynamics.

Paralleling the developments in pulsation theory, there have been several refined attacks on the evolution of massive stars beyond their main sequence hydrogen burning phase that have added to our knowledge of the cepheid variable stars. These refinements have resulted from the more sophisticated computing techniques now available and from a more complete knowledge of the input physical
parameters which influence the stellar structure. Continuing advances in laboratory astrophysics have supplied more accurate values of nuclear reaction rates, interaction cross sections, and radiative opacities, while modern high speed computers provide researchers with the possibility for following stellar evolutionary models in much greater detail, especially during the more rapid phases of evolution.

Investigations of this nature have been successfully carried out by Hofmeister (1967), Hofmeister, Kippenhahn, and Weigert (1964), Kippenhahn, Thomas, and Weigert (1965), and by Iben (1965, 1966a, b). So far, stellar masses of 3, 5, 7, 9 , and $15 \mathrm{M}_{0}$ have been carried from the initial main sequence through the major portion of core helium burning with a fair degree of certainty. Considering the dissimilar approaches that have been used, all have agreed quite well concerning the general features and time scales of the evolutionary tracks. A full discussion of the configurations of the evolutionary tracks and their physical interpretation has been given by Iben (1967) and will not be given in detail here. Suffice it to say that for stars more massive than about $3 M_{0}$ the rapid transition from hydrogen exhaustion at the end of the main sequence phase to the first phase of helium core burning in the red giant phase is followed by a definite leftward movement in the H-R diagram during which the stellar envelope contracts
and helium burning in the core is firmly established as the major source of energy. The time scale associated with this phase is a steadily decreasing function of the mass as are the time scales of all phases in the star's evolution, but the helium core burning phase is exceeded in duration only by the major phase of hydrogen burning in the core. While approaching helium exhaustion in the core the star again moves to the right. in the $H-R$ diagram thus forming a so-called "loop" in the evolutionary track. The loop is characteristic of stars in the mass range of from 3 to $10 M_{0}$, whereas higher mass stars have been shown to exhibit only a rightward trend to steadily decreasing effective temperatures during their evolution. Transitions to additional major phases of nuclear burning follow analogously up to helium shell burning. These phases have been calculated by Hofmeister (1967) to show similar loops in the H-R diagram. The point of major concern to this discussion, then, involves the passage of these massive stars through the region designated by theory and observation as the instability strip.

Pulsation theory dictates that only when a star falls within a narrow range of luminosity and effective temperature will it be unstable to perturbations in its equilibrium structure and form a cepheid variable star. Observationally, the cepheids have been shown to delineate the equivalent narrow region in the color-magnitude
diagram (based on the techniques of Kraft [1961] in determining intrinsic colors and color excesses). Consequently, this region has been termed the instability strip since all stars within its boundaries exhibit variability. Evolutionary theory has subsequently shown that all stars which evolve through this specific strip in the $H-R$ diagram have one basic physical property in common. All are in the major phase of their evolutionary history during which core helium burning is their source of energy generation. Furthermore, the theory succeeds in predicting several statistical properties of cepheids which have an observational basis.

Theory shows that for stars of a given composition the evolutionary tracks of stars of different mass indicate a fairly tight correlation between mass and luminosity during comparable evolutionary phases. Coupling this with the correlation of period and mass along the instability strip serves to confirm the observed period-luminosity relation for cepheids. The exact location of the helium burning phase with respect to the instability strip governs the relative duration of the phase of stellar pulsation among stars of different masses. Stars less massive than about $4 M_{\odot}$ have a core helium burning phase which carries them just short of the low $T_{e}$ side of the instability strip. Slightly more massive stars are carried into the strip so that the shortest pulsation period is predicted
to be around two days. At the mass where the greatest portion of the star's core helium burning occurs within the instability strip, the longest duration of variability takes place and should correspond to the most prevalent period of all cepheids observed. Higher mass stars pass more rapidly through the instability strip during their evolution to and from the core helium burning phase, thus leading to a decrease in the number of variables with longer periods and higher luminosities.

Kraft (1966) has summarized observational evidence to support the above conclusions about evolution in the Hertzsprung Gap and the instability strip. His presentation includes a comparison of star counts for all stars in the spectral range FO to Go within 1000 pc. of the sun to the counts predicted using Schmidt's (1963) luminosity function and the theoretical evolutionary time scales. He. concludes that the observed counts are too large to be accounted for by stars evolving toward the red giant phase in their first crossing of the Hertzsprung Gap, whereas the counts that are predicted for higher order crossings agree nicely with those observed once the possible incompleteness of the counts is allowed for. Additional supporting evidence is given in a similar comparison of observation and theory with respect to the frequency of cepheids as a function of luminosity. First order crossing theory simply does not predict long enough time scales for the
cepheid phase to explain either the observed high frequency of cepheids with periods near five days or the steadily decreasing frequency function with increasing luminosity into the long period cepheid region.

Kippenhahn and Smith (1969) have investigated the ages of cepheids in order to confirm the evolutionary trends described above. By applying linear pulsation theory to a series of evolving models at each crossing of the instability strip, they were able to show a theoretical relationship between pulsation period and model age which clearly segregates the first crossing of the strip from all subsequent crossings. This isolation of the first crossing is influenced by several factors which become apparent in the theoretical treatment. For any given stellar mass, the pulsation period differs only very slightly between the second crossing and later crossings, whereas the period at the initial crossing is generally a factor of two smaller. The ages, nonetheless, are only very slightly larger from crossing to crossing due to the large portion of a star's lifetime that is spent on the main sequence. As an observational verification of this period-age relation, they relied on cepheids which are known members of clusters or associations in a method similar to that employed originally by Efremov (1964). Under the assumption that all the stars in such a group have the same age and initial chemical composition, the "turn off point" age establishes the age
of each member cepheid. The agreement of the observationally determined ages with the theoretical ages proves adequate for the older clusters which contain the shorter period cepheids, but there is some departure at younger ages, for which, however, RS Pup in Pup III is the only observational example.

That the study of stellar clusters and their number cepheids has great importance to understanding stellar evolution can be seen from the work of Arp (1967) on NGC 1866 in the Large Magellanic Cloud. The cluster is young and very rich (especially in red giants), making it extremely valuable for comparison with theoretical evolutionary tracks of massive stars. Arp has therefore fitted the observed color-magnitude diagram of NGC 1866 with Iben's (1966a) evolutionary track for a $5 \mathrm{M}_{0}$ star to obtain qualitative confirmation of the general trends predicted by theory. Particularly noteworthy is the fact that all of the member cepheids fall nicely at the loop in the evolutionary track where there is a major phase of core helium burning. As a first comparison of the evolutionary tracks to an evolving group of stars, Arp's work showed encouragingly the prospects of accurate evolutionary model calculations.

A more recent study of NGC 1866 by Hofmeister (1969) describes the possibility of a considerable improvement in the comparison with theory. Her study was based on
isochrones in the $H-R$ diagram constructed from evolutionary tracks for stellar models of 3.5 to $5.5 \mathrm{M}_{0}$ distributed according to the "original mass function" of Salpeter (1955) and with a spread in age according to her assumed duration of the star forming epoch of $1.5 \times 10^{7}$ years. Not only does the appearance of the evolved sequence mimic the observations but the distribution of the stars along the sequence also compares favorably with the observed dis-. tribution. The few differences which occur are assumed to be caused by the specific choice of chemical composition for the models.

Encouraging as these results are to pulsation theory and stellar evolution theory, one puzzling discrepancy between the two theories remains to be solved. An investigation of the observational data on cepheids and the use of nonlinear pulsation theory led to values of the mass which are annoyingly lower than those required by evolutionary models which pass through the same cepheid period range. The calculated masses turn out to be only 50 to $60 \%$ of those required by evolutionary models. The tempting solution to this enigma, barring major faults in either calculational theory or procedure, is to invoke some form of mass loss during the star's lifetime. Indeed, this is a distinct possibility when considered in the light of observational evidence for mass loss in the red giant phase
(Deutsch 1960) and direct theoretical evidence for an evolutionary connection between the red giants and cepheids. Several authors have calculated evolutionary models through the onset of core helium burning including the effects of mass loss, but the results are by no means complete. A $15 \mathrm{M}_{0}$ model (Hartwick 1967) with a rate of mass loss directly proportional to the luminosity and radius and inversely proportional to the mass (McCrea 1962) failed to become a red giant. Forbes (1968) limited mass loss in his $5 M_{\odot}$ model to those phases containing a convective envelope (i.e., the red giant phase) but found the detailed configurations of the evolutionary tracks to be quite sensitive to the rate of mass loss assumed. Nevertheless, his models show that there is general agreement between the average luminosity levels of constant mass models with the mass loss models. These are but initial attempts to solve a rather complex problem. What is needed hinges not only on a more complete theoretical approach to the mechanism of mass loss and its application to a larger range of masses but also on direct observational indications that mass has been lost from a star either before or during the cepheid phase.

Since cepheids are now generally accepted as being the advanced stages of evolution of main sequence $O$ and $B$ stars, they should logically reflect some of the observed properties of their early type progenitors. In particular,
current ideas of star formation place the origin of all 0 and $B$ stars in clusters of associations. This was shown by Roberts (1957) on the basis of the observed 0 and $B$ stars within 2 kpc . of the sun compared with those expected on the assumption that they were formed only in clusters and associations. More recently, von Hoerner (1968) came to the same conclusion based on an analysis of lifetimes and formation rates of clusters and associations. His results indicate that the fraction of all stars that are formed in clusters and associations is very close to unity. Consequently, it is likely that cepheids originate in such star groupings and likewise have a common history with the many other early type stars of the group. As was seen. above, the study of these clusters and their member cepheids provides an extremely useful tool for testing stellar evolution theory and the role played by cepheids therein.

This particular point has received strong emphasis by several Russian investigators, who have examined various cepheids as possible members of clusters and associations. The suggestion by Kholopov and Artiukhina (1963) that open clusters are surrounded with extensive halos containing about half of the number of bright stars of the cluster brought about a search for cepheids in such regions. On the basis of coincidences in position on the celestial sphere and rough distance correspondences from a cepheid absolute magnitude calibration (Kraft 1961), they found
nineteen cepheids in these regions. An analysis of the nineteen cepheids (Efremov and Kopylov 1967) confirms the existence of the relationship first reported by Efremov (1964) between the period of a cepheid and the age of the cluster containing it (cf. Kippenhahn and Smith 1969) and in general supports the conclusion that stars repeatedly enter the instability strip to become cepheids.

Not all cepheids are expected to be detectable as members of stellar groupings, however. Based on von Hoerner's (1968) argument the majority of $O$ and $B$ stars are formed in associations, leaving only about 15\% to originate in clusters. Since clusters are gravitationally bound, their member cepheids should be detectable within the observable limits of the cluster. Owing, however, to the expansion of associations into the general field, it is safe to assert that most of the association members, during their evolution from main sequence 0 and $B$ stars, travel away from their point of origin. As cepheids they will not necessarily be found in the associations of their birth. This is particularly true of short period cepheids which evolve from less massive and less luminous $B$ stars and therefore have a greater age than longer period cepheids. In part, this is evidenced by their scattered distribution throughout the galactic plane and apparent failure to delineate structural features of the galaxy. The long period cepheids, on the other hand, are much younger and
should be found much closer to their place of birth. Nevertheless, only one long period cepheid has been firmly located as an OB association member (Westerlund 1963). This is primarily a result of the limited information available about distant associations with respect to not only their distances, content, and extent but also whether or not they actually exist. Assigning cepheid membership to such associations is difficult.

Since one of the natural discriminants of group membership for cepheids are their distances, they are an invaluable aid for any observational test of the role played by cepheids in stellar evolution. With the exception of the work on NGC 1866 and the efforts of the Russian investigators, most of the investigation of cepheids as cluster members has been directed toward a calibration of the period-luminosity relation for cepheids in our galaxy. The resulting distances provided by the calibration are then extremely useful in studies of galactic structure (Kraft and Schmidt 1963) as well as being potentially important for the comparison of observation and theory in evolution studies. Of prime importance to these studies are the long period cepheids which exemplify the evolution of massive stars and delineate the structure of and follow the distribution of young objects in the Galaxy.

The use of the long period cepheids is, however, limited by assumptions made in the calibration procedure.

The five known cluster member cepheids on which the calibration is based (Kraft 1961) span only a small period range from five to ten days, making them useful for determining only the zero point of the period-luminosity relation. With the slope taken from the work of Arp (1960) on cepheids in the Small Magellanic Cloud, the procedure is therefore based upon the assumption that the slope in our Galaxy is identical to that in the SMC. The long period cepheids thus remain the most inadequately calibrated cepheids by way of direct observational evidence in our Galaxy. Their improved calibration would serve, not only to reinforce galactic structure research, but also to extend observational tests of stellar evolution into previously unsampled period and luminosity ranges.

The preceding discussion indicates the broad interface that exists between theory and observation in relation to studies of the cepheid variable stars and their role in stellar evolution. Qualitatively, the comparisons of theory and observation are in good accord, but quantitatively, there are several points of disagreement caused by inadequacies of both theory and observation. Pulsation theory indicates that mass loss may be significant in cepheid evolution, but the detailed mass loss models and the evolutionary calculations are not yet sufficiently complete to decide the issue. At the same time, observations have yet to supply adequate indications of mass loss
before or during the cepheid phase. Conventional evolutionary studies of star clusters and associations have emphasized the importance of finding cepheids in these groups, yet the available information on distant associations is far too incomplete and the luminosity calibration of long period cepheids too insecure to provide accurate membership assignments. Ít is with these inadequacies in mind that the present study has been conducted in an attempt to provide additional observational evidence in support of present cepheid evolutionary theory.

The nebulosity surrounding RS Pup discovered by Westerlund (1961) provides a unique opportunity to study the interaction of a relatively young, luminous, long period ( 41.4 ) cepheid with the interstellar medium and to attempt to understand the relationship of star and nebula in terms of their mutual evolution. The peculiar nature of the nebula, in addition, suggests a direct geometrical method of distance determination with which to test the possible existence of the cepheid in an $O B$ association as suggested by Westerlund (1963) and to examine the presently adopted galactic period-luminosity relations.

The particular properties of RS Pup--its surrounding nebulosity and its membership in an $O B$ association-prompts an investigation of other cepheids that present similar apparent properties. The only other cepheid known having a nearby reflection nebulosity is the short period
(1.95) cepheid SU Cas considered by van den Bergh (1966) and Racine (1968a, b) to be in a region of space occupied by other reflection nebulae. It is similar to RS Pup in its proximity to the nebulosity but otherwise very different because of its relative age, relatively faint luminosity and short period. Another cepheid, SV Vul, is a promising candidate for study since it is the longest period (45.1) cepheid observable in the northern hemisphere and could therefore be a possible $O B$ association member. The information on $O B$ stars and associations in the vicinity of $S V$ Vul is presently poor enough to warrant an investigation of this possibility.

These three selected cepheids, RS Pup, SU Cas, and SV Vul, indicate several similar properties which relate to their individual evolutionary histories and might reflect evolutionary tendencies in classical cepheids as a whole. The present study is thus concerned with the observational properties of these particular cepheids and their resulting influence on present cepheid evolutionary theory.

## RS PUPPIS

## The Nebulosity and Observational Material

With the discovery of the faint nebulosity around the 41.4 day period cepheid RS Puppis (Westerlund 1961) and the later indication of its probable membership in an $O B$ association (Westerlund 1963), it was obvious that a detailed investigation of the interaction of this cepheid with the interstellar matter could lead to both a deeper understanding of the role played by cepheids in stellar evolution and a fundamental calibration of the periodluminosity relation for galactic cepheids.

RS Pup is one of the few galactic cepheids known to have a period greater than forty days. Its photometric parameters include an average integrated visual magnitude of 6.98 and a not uncommon total visual absorption of 1.86 magnitudes. The distance of RS Pup is taken to be 1500 parsecs, as based on Kraft's (1961) period-luminosity relation. The light and color curves as given by Westerlund (1963) are reproduced in Figures 1 through 3 and are based on his period of 41.384 derived from additional previous observations by Eggen, Gascoigne, and Burr (1957) and Irwin (1961). RS Pup is the most evolved member known in the OB association Puppis III, which has an age of only $4 \times 10^{6}$



Figure $1 . \quad R S$ Pup: $V$ magnitude light curve


Figure 3. RS Pup: U-B color curve
years. The association is an important member of the Orion arm or spur of the Galaxy.

The nebulosity around RS Pup was initially discovered by Westerlund (1961) during a survey of the southern Milky Way with the 20-26 inch Schmidt telescope of the Uppsala Southern Station. The blueness of the image indicated at the time that the nebulosity was in all probability a reflection nebula. Larger scale photographs with the 74 -inch telescope of the Mount Stromlo Observatory revealed that the nebula had a very unusual structure consisting of diffuse sections and ring segments which appeared to be concentric and centered on RS Pup (see Figure 4). The nebula extended a total of approximately 1 minute of arc from the cepheid. The symmetrical arrangement of the nebula with respect to the star strongly indicated that the two objects were indeed associated with one another and not simply the result of a chance superposition. Thus a special interest was drawn to the reflection nebula since, unlike most typical reflection nebulae, its light source varies in intensity in a well known fashion. As additional evidence of the reflection nature of the nebula, a spectrum was obtained of the nebula by Herbig using the 36 -inch Crossley nebular spectrograph at Lick Observatory. As shown in Figure 5 there are no indications of emission line features that might invalidate the conclusion that the nebula is indeed a reflection nebula.


Figure 4. RS Puppis and the surrounding reflection nebula -- The exposure for Plate No. 3117 was 46 min with the Mt. Stromlo 74 -inch telescope on a $103 \mathrm{a}-0$ plate behind a 2 mm Schott GG13 filter. (North is up, East is to the left. Scale: $1 \mathrm{~mm}=1: 5$.


Figure 5. Spectra of RS Pup and nebula -- Taken on February 19, 1961 by G. Herbig with the Crossley spectrograph on the Lick 36-inch telescope. The dispersion is $300 \mathrm{~A} / \mathrm{mm}$ on $1 I_{\text {a-0 }}$ emulsion.
Top: 75 min exposure showing bright nebular region of Feature 1 (see Figure 8) with spectra of the two faint stars beneath it. Bottom: 1 min widened exposure of RS Pup.

Therefore, a program of regular photography of RS Pup was begun with the 74 -inch telescope with the purpose of detecting possible variations in nebular intensity distribution due to light variations of the star.

The large scale plates of the region around RS Pup span a time period from late 1960 to early 1967 with a large gap of four years between 1963 and 1967 partly due to poor observing conditions on Mt. Stromlo as an aftermath of a volcanic eruption in Indonesia. The plates were taken in the three color ranges of blue, visual, and red as defined by the plate-filter combinations of Table l. Table 2 lists the pertinent information for each plate including subjective ratings on the general plate quality, as indicated by the stellar images across the plate, and a general statement as to the nature of the nebulosity as seen under a simple hand lens.

Dr. Merle Walker supplied two additional plates taken with the electronic camera attached to the 60-inch telescope of Cerro Tololo Interamerican Observatory. These were a valuable addition to the observational material since the linear response of the spectrocon direct electrographs provides an accurate calibration useful over a large magnitude range for the remainder of the direct plates.

In order to assess the possible variations in nebular intensity with respect to the known light variation of the cepheid, the phase of RS Pup for each plate was

Table 1. Plate-filter combinations

| Plate | Filter |  |
| :--- | ---: | :--- |
| Blue | lo3a-0 | Schott GG13 |
| Visual | $103 \mathrm{a}-\mathrm{D}$ | Chance oY4 |
| Red | $103 \mathrm{a}-\mathrm{E}$ | Schott RG1 |

Table 2. General plate information

| Plate | No. | Date | Phase | Plate Quality | $\begin{aligned} & \text { Nebular } \\ & \text { Structure } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Blue | 1593 | 10/14/60 | : 949 | good | a |
|  | 1617 | 11/23/60 | . 914 | good--underexposed | b |
|  | 1713 | 1/22/61 | . 364 | slightly comatic | b |
|  | 1719 | 2/15/61 | . 943 | soft images | b |
|  | 1723 | 2/16/61 | . 963 | soft images | b |
|  | 1.729 | 2/21/61 | . 087 | slightly comatic | a |
|  | 1990 | 12/8/61 | . 095 | soft images | b |
|  | 2463 | 3/30/62 | . 799 | slightly comatic | a |
|  | 2612 | 5/5/62 | .667 | good--underexposed | c |
|  | 3117 | $2 / 26 / 63$ | . 844 | excellent | a |
|  | 4265 | 4/11/67 | . 212 | soft images | b |
| Visual | 2464 | $3 / 30 / 62$ | . 800 | excellent | a |
|  | 2611 | $5 / 5 / 62$ | . 666 | excellent | a |
|  | 3116 | 2/26/63 | . 843 | excellent | a |
|  | 4264 | 4/11/67 | . 212 | good | b |
| Red | 1584 | 10/12/60 | . 901 | slightly elongated images | c |
|  | 1594 | 10/14/60 | . 949 | good | a |
|  | 1616 | 11/23/60 | . 913 | good | b |
|  | 1722 | 2/16/61 | . 962 | good | b |
|  | 1725 | 2/21/61 | . 083 | elongated images | c |

1. a--outer structure visible; b-only inner structure visible; c--nebula barely apparent.
calculated using the ephemeris of Westerlund (1963) as follows:

$$
\operatorname{Max}=\mathrm{JD} 2434533.37+41.384 \mathrm{E}
$$

The resulting phases are tabulated in Table 2 and a graphical representation of the occurrence of the plates with respect to the visual light curve of RS Pup is shown in Figure 6. A majority of the plates occur within the relatively small range in phase of from 0.8 to 0.2. However, the large amplitude of $R S$ Pup in the visual and especially in the blue should make any ncbular intensity variations observable even under these less than optimum conditions of distribution in phase.

The Method of Plate Reduction
Because of the diverse conditions under which the plate material was accumulated--the different exposure times, the variable seeing conditions, and the variable sky background--a simple visual inspection of the faint nebulosity is not nearly sufficient to detect a regular variation in its light, or even to establish any variation at all. The appearance of the nebulosity from plate to plate, which suggests large variations even between times of equal phase, could conceivably indicate a periodicity much different than that of the cepheid. More than likely, however, with an extended object such as this the seeing conditions strongly effect the concentration of light and


Figure 6. Distribution of the plates
serve to broaden any structural features of the nebula. For this reason, any attempt to analyze intensity distributions from particular nebular features must treat the intensity from the entire extended feature and not simply that from its small central region. One technique particularly suited for an analysis of this type is the use of an isodensitometer. Such an instrument gives quantitative information on an extended region of the plate and enables one to allow for the effects of seeing, etc.

The equipment used to reduce the plates was the Joyce-Loebl microphotometer of the Kitt Peak National Observatory and its isodensitometer modification, the Isodensitracer, manufactured by Technical Operations, Inc. This machine can be used either as a precision microphotometer or in the more sophisticated isodensitometer mode from which the scanned region of the plate is reproduced as a contour diagram. Both modes are required if accurate density calibration of the plate is desired.

The advantage of using the Isodensitracer over conventional techniques involving an iris photometer for measuring stellar intensities is the fact that valid information $c a n$ be extracted from the resulting contour diagrams even though the stellar images may not be circular. The major difference between this procedure and the one discussed by Walker and Kron (1967) and by Ables, Kron, and Hewitt (1969) for use with electronographic plates is the
use of direct photographic plates and the necessary calibration curves. The preceding authors find errors similar to those in photoelectric photometry, whereas the present results for plate to plate variations of different stars show errors not exceeding $\pm 0^{m} .06$ (p.e. $\sim 0^{m} 02$ ) in adjusting all plates to the same scale.

Basically, the Isodensitracer is a dual beam microdensitometer, one beam passing through a moveable reference wedge which has a linear change in density from one end to the other. The moveable wedge carries the pen when the instrument is used in the microphotometer mode but otherwise turns a coded commutator which is sampled electronically when it is operated as an isodensitometer. The electronic logic of the Isodensitracer transfers the signal from the commutator to the fixed ball point pen and instructs it to write different symbols depending on the position of the reference wedge. As the sample is moved in the $x$-direction the record moves under the ball point pen in the same direction while the pen writes the different symbols along a single line. After the edge of the sample is reached both the sample and the record are returned to their starting positions and moved slightly in the y-direction and another single line of different symbols is written. The repetition of this process gradually scans an entire area on the sample plate while constructing a contour
diagram on the record as a combination of numerous single line scans.

Aside from the usual choice of analyzing slit size; the Isodensitracer has many different adjustable parameters which are not available on an ordinary microphotometer. There are several reference wedges available for various density ranges and there are three different commutators for three different rates of pen symbol changes during the excursion of the reference wedge. The final appearance of the contour diagram is also highly dependent on the space allowed between single scans of the plate in relation to the size of the scanning aperture. A larger aperture size and a smaller stepping space provides a more coherent contour diagram but naturally requires a longer time to complete. Also important is the choice of magnification from plate to record which is variable in several steps from 1:1 to $1: 200$. All of these factors come into play when reducing a plate, and unfortunately there is not a unique combination which will provide all of the information from any one plate.

In attempting to detect variations in the intensity distribution of the nebula, there are two types of variation to be considered. Spatial variations in the position of the structural features in the nebula require a high degree of spatial resolution on the plate whereas temporal variations at any one position in the nebula require, in
addition, an adequate calibration of the relation between density and intensity on the plate. The available plates were suitable for both types of investigation since each plate had a corresponding spot calibration plate for comparison.

The calibration plates corresponding to each program plate were traced using the microphotometer mode of the Joyce-Loebl and calibration curves were constructed making use of the known intensities and measured densities in the usual fashion. The resulting calibration curves are given in Figure 7.

In searching for spatial variations in the intensity distribution of the nebula, it was deemed advisable to employ a high degree of magnification of the plates in order to facilitate the detection of any position shifts. It was also necessary to view a large enough region of the nebula so that more than one feature could be observed in relation to the fixed background of stars. Therefore, a magnification of 100:1 was used with the analyzing slit opened to $100 \mu$ square as compared with the typical resolution of the IIaO plates of 20 to $30 \mu$. The desire was to average out grain fluctuations and smooth the structural features. The advantage of this method over optical blinking techniques lies in its quantization of information and high degree of magnification which leads to a more


Figure 7. Representative characteristic curves
precise location of nebular features than is possible with other methods.

The region of the nebula selected for the investigation of possible spatial variations was centered on the faintest of the detectable ring segments to the southwest of RS Pup (see Figure 8). These appear as obvious candidates because of their linearity and proximity to faint reference stars. Because of their faintness, however, these features are not obvious on all of the plates, and no amount of careful work with the Isodensitracer could make them any more obvious on the isodensitracings. The most complete sampling of the nebula during the cepheid light variation was between phases of 0.8 and 0.2 . Isodensitracings of 8 of the plates occurring between these phases were obtained at the magnification of $100: 1$ and transparent contour diagrams of each were made by hand. By carefully overlaying individual transparencies and isodensitracings obtained for different phases it was possible to search for any spatial motions of the nebular features.

The original intent of the above measurements was to be able to use the isodensitracings both for the purpose of positional measures and for intensity measures. Under. the necessary requirements of plate calibration using stellar images as described later, however, it was found desirable for the intensity measures to increase the magnification to 200:1, the limit of the instrument, and to


Figure 8. Features of the nebula chosen for study
decrease the area of the analyzing slit by a factor of 4 to a value of $50 \mu$ square as projected on the plate. In this way a maximum amount of information $c$ an be obtained from individual stellar images while still adequately smoothing the plate grain noise. One disadvantage of the increased magnification, however, is the resulting decrease in the area of the plate that can be scanned during one operation of the Isodensitracer. Consequently, in order to be able to compare information from a small nebular region among several plates, the regions of comparison had to be chosen carefully to insure that at least two background reference stars were included and yet that the reference stars themselves did not contaminate the nebular region of interest.

Four regions of the nebula were therefore chosen according to the above criteria. These areas were along features suggesting ring-like structure and were not in the diffuse areas of the nebula. The four regions chosen are indicated on Figure 8 where it can be seen that two of the regions are those regions also studied for positional variation and were recorded on the same Isodensitracing. The regions consist of a bright feature near RS Pup and almost due south of the cepheid, a region of intermediate brightness further away from and to the east of RS Pup, and the two faint features at the southwest edge of the nebula. One unfortunate restriction on the selection of these specific regions was the existence on several of the
plates of an additional diffraction spike from RS Pup due to an obstruction which was in the beam during approximately half of the exposures. The four regions chosen were uncontaminated by diffraction spikes on all exposures.

Another very important step taken in the plate reduction procedure was the use of a nonvariable source for the relative comparison of all plates. This was intended as a measure of the different exposures and varying seeing conditions to which each plate was subjected. To serve as standard comparison sources, three stars were chosen in uncrowded regions of the background star field and completely separated from the nebulosity surrounding RS Pup. Isodensitracings of each star were made and reduced in the manner described• below and detailed in Appendix A. The factors necessary for adjusting the intensities on all plates to the same scale were then obtained from a comparison of the three stars on all plates. The use of three stars in this process served not only to average out small differences in the adjustment indicated by different stars but also to ascertain that the standard stars were truly nonvariable themselves. In fact, one of the stars consistently differed from the results of the other two and was therefore disregarded in the adjustment procedure. The final relative adjustments ranged up to a factor of four in intensity for some of the weaker plates.

One important problem in the reduction was the proper selection of the correct sky brightness level near each region of study as compared to the sky near the calibration stars. This was of some concern because of the extreme brightness of the central cepheid relative to the rest of the field. The resulting scattered light in the earth's atmosphere and in the telescope during the long exposures causes a radial dependence of sky brightness away from the cepheid which could give a false indication of nebular variability. In order to minimize this effect on the resulting reductions of the nebular brightness, various regions around the cepheid which were apparently devoid of nebulosity were sampled as a function of distance from the cepheid. The darkest area found at the same radial distance from the cepheid as the nebular region of interest was then used as the apparent sky value for the reductions. In all cases these sky values were brighter than the sky value used for reduction of the calibration stars and were found to decrease outward from the cepheid. Because of the possibility that these darkest regions still contain residual nebulosity, however, the sky values used were upper limits.

The procedure used to evaluate the nebular intensity was simply to integrate the total volume under the three dimensional density profile bounded by a circular aperture slightly greater than 5 seconds of arc in diameter and
centered on the region of interest. Densities were converted to intensities with the help of a calibration curve and the resulting intensity volume was converted to a magnitude relative to one of the plates which served as a standard. The size of the aperture was chosen large enough to allow for the vagaries of seeing yet small enough to include only one nebular feature. Certainly it would be advantageous to investigate more spatially restricted regions of the nebula, but the disturbances due to seeing effects make this impossible. In the present circumstances, nevertheless, the size of the aperture used will not appreciably dilute the possible light variations as long as the aperture is larger than the apparent angular width of the nebular feature sampled. Increasing the aperture beyond the width of the feature wide not introduce additional phase mixing and resulting light curve dilution (refer to the section of the present chapter on distance determination).

Conversion of the relative intensity values into absolute intensity values was done through the use of a standard magnitude sequence and the reduction of electronographic plates of the nebular region. Lodén (1969) has published a photometric sequence of stars extending from 9th to 13 magnitude in $V$ and which are included on all the plates of RS Pup. These images, however, were too dense to allow reduction by the Isodensitracer method. The sequence
stars were therefore reduced by ordinary iris plotometer techniques to establish magnitudes in the immediate region of RS Pup and, in particular, in the limited narrow strip covered by several electronographic plates of the nebula. The several exposure ranges of the electronographic plates and their additionally valuable property of linearity in response over a 5 to 6 magnitude range from the plate limit made the electronographic plates extremely useful in providing sufficiently accurate extrapolated magnitudes for the faint stars in the field. Once established, these magnitudes were used to calibrate the contours of the Isodensitracings and therefore to set the zero point for the nebular light curves. To supplement the above discussion and as an aid in understanding the procedures involved, the details of a sample plate reduction including the details of treating the Isodensitracings are presented in Appendix A. In addition, the magnitude calibration of the plates using the photoelectric sequence and the electronographic plates is given in Appendix B.

## Results of the Plate Analysis

The search for spatial movements of the nebular features using the Isodensitracer has confirmed the preliminary conclusions which can be drawn from a cursory inspection of the appearance of the nebula on the available plates. No spatial motion was found between any of the
contoured plates even between those with maximum phase separation or those taken in different colors. In all the cases which were examined, the errors involved in superposing the fixed background reference stars and the errors introduced due to seeing effects on the nebular features were sufficiently large to produce all of the minor dissimilarities in the structural features which were noted from plate to plate.

The fact that not all of the plates were adaptable to the above detailed search for spatial motions is a statistical disadvantage of the technique. Nevertheless, the plates that were measured had well defined features that were easily detectable above the sky background, and yet, compared to the size of the features and their angular separation, no motion was observed to occur within a 0.4 phase interval. The lack of such spatial motion must, therefore, certainly be real.

The search for temporal variations in the intensity of individual nebular features was more successful. Figure 9 illustrates, for the four nebular regions investiw gated, the varying blue magnitude of each feature with respect to the phase of the cepheid light curve. The magnitudes expressed in this figure are integrated over the entire analyzing aperture and therefore are equal to magnitudes per 19.63 square seconds of arc. Only the blue plates were numerous enough to warrant a complete reduction

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Figure 9.--Continued RS Pup nebula: B magnitude vs. cepheid phase -- c: Feature 3


Figure 9.--Continued RS Pup nebula: B magnitude vs. cepheid phase -- d: Feature 4
throughout the cepheid light curve, and even these show extended intervals lacking in data. In an attempt to provide information on the color distribution in the nebula, a few of the high quality visual plates were reduced. Table 3 contains the information obtained for each plate including the visual data which is not displayed in any of the figures.

The error bars designated in each of the above mentioned figures are indicative of the effects of an error of $\pm 0.10$ in the intensity level of the sky background at each nebular feature as will be discussed in Appendix $C$. Considering the effects of the various characteristic curves on the conversion of plate density to intensity this corresponds to an error in assigned sky density from the tracings of about one-half of the density resolution element or approximately 0.02 to 0.03 in density. This is undoubtedly the largest source of error in the reduction procedure, and its display in the figures therefore serves to set the scale of the uncertainties present.

Although the errors appear quite large for some plates, there is still a detectable regular light variation, especially in the two brighter nebular features, and the amplitude of the variation exceeds the size of the errors involved. That the light variation is quite real and not a random alignment of points $c$ an be seen by an inspection of each of the nebular light curves in comparison

Table 3. Numerical results of RS Pup plate reductions

| Plate | No. | Feature 1 | $\Delta \mathrm{m}$ <br> Feature 2 | Feature 3 | Feature 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Blue | 1593 | . 00 | . 00 | . 00 | . 00 |
|  | 1713 | -. 64 | $+.50$ | -- | -- |
|  | 1719 | - 117 | -. 05 | -- | -- |
|  | 1723 | -. 09 | -. 27 | -- | -- |
|  | 1729 | -. 37 | $-.34$ | -. 24 | $+.20$ |
|  | 1990 | $-.30$ | -. 18 | -. 19 | -. 14 |
|  | 2463 | +.01 | -. 01 | - 28 | $+.24$ |
|  | 2612 | -. 65 | $+.03$ | -- | -- |
|  | 3117 | +. 25 | $+.28$ | -. 29 | +. 12 |
|  | 4265 | -. 21 | -- | -. 10 | +. 01 |
| Visual | 2464 | . 00 | . 00 | . 00 | . 00 |
|  | 3116 | $+.36$ | +. 20 | +. 11 | -. 02 |

with the light curve of RS Puppis itself, reproduced here in the $B$ magnitude as Figure 10. Each of the two brighter nebular regions has a light curve which is quite similar to the cepheid light curve in that there is a slow steady decline followed by a much more abrupt increase in intensity. The points for Feature 3 of the nebula are much less distinct in their variation, and the well determined points cover a more restricted range of phase. A few points from other plates outside this range are indicated as upper limits so that the other more defined plates may be seen to form a maximum in the light curve. This is still subject to a great deal of uncertainty, however. Feature 4 is far too faint to provide any kind of acceptable light curve.

Since only two visual plates were fully reduced, the information on the color distribution in the nebula is limited. However, the two visual plates that were treated fully have corresponding blue plates available that were taken on the same night. The colors for these phases are then easily obtained once the magnitude calibrations, as illustrated in Appendix $B$, are completed. A tabulation of the resulting color" distribution through the four nebular features for the two phases is given in Table 4 . Since the phases are only separated by 0.04 , it is hardly possible to draw any conclusions from these two color distributions that might relate to the cepheid light variation. Therefore, the results from the two phases have been averaged as


Figure 10. RS Pup: B magnitude light curve

Table 4. Nebular (B-V) color data

| Feature | B | V | B-V | $\langle\mathrm{B}-\mathrm{V}\rangle$ |
| :---: | :---: | :---: | :---: | :---: |
|  | phase . 844 | phase . 843 |  |  |
| 1 | 16.85 | 15.91 | $+.94$ | $+.89$ |
| 2 | 18.26 | 16.55 | +1.71 | +1.76 |
| 3 | 19.12 | 17.02 | +2.10 | +2.04 |
| 4 | 19.71 | 17.47 | +2.24 | +2.19 |
|  | phase . 799 | phase . 800 |  |  |
| 1 | 17.09 | 16.25 | $+.84$ |  |
| 2 | 18.55 | 16.75 | +1.80 |  |
| 3 | 19.11 | 17.13 | +1.98 |  |
| 4 | 19.59 | 17.45 | $+2.14$ |  |

shown in Table 4 as a more meaningful indication of the color distribution.

Ordinarily, in a conventional reflection nebula, the color throughout the nebula can be directly compared with that of the central star to determine the reddening produced by the nebula. In this case, however, where the cepheid regularly changes color itself, a direct comparison of the color of star and nebula must take into consideration the light travel time to the outer nebular regions. A more thorough discussion of this effect will be presented in the section of the present work concerned with an interpretation of the nebula. At this point it is sufficient to note that the cepheid color variation covers about a 0.7 range from $B-V=1{ }^{m} 1$ at maximum light to $B-V=1^{m} \cdot 7$ at minimum light. It is clear from the color distribution of Table 4 that the nebula at Feature 1 is bluer than the cepheid and at Feature 3 and 4 is redder than the cepheid irregardless of the cepheid phase.

## A Model of the Nebula

Several working models of the RS Pup nebula can be put forth in an attempt to explain its observed properties. The first most tempting model, termed the homogeneous model, suggests that RS Pup is simply a variable light source surrounded by a homogeneous medium which reflects the cepheid's maxima and minima as they travel outward from
the star through the medium at the speed of light. As observed from the earth, then, the expanding light spheres would be most visible when viewed near their edges and would appear as rings. Although this model successfully explains the ring structure of the nebula it is in apparent conflict with other observations of the nebula.

One criticism of the homogeneous model is based upon the known approximate distance of RS pup based upon the period-luminosity relation or its suggested association membership compared to the distance calculated by the angular size of the nebula, the number of concentric rings in it, and the period of the cepheid. Since there are only five apparent rings extending to 1 minute of arc from the cepheid and if these rings are considered to be caused by successive light maxima of $R S$ Pup separated by 41.4 days, the derived distance is about 600 parsecs. This is only about one-third to one-half of the value which would be implied by the present period-luminosity relation. In order for the distances to be more compatible, either there should appear more rings in the nebula or a revision in the period-luminosity relation is needed.

This difficulty of distance compatibility could conceivably be overcome by postulating a different type of reflecting medium still based on the reflection of expanding light spheres. This model, called the screen model, suggests a uniform reflecting screen in front of the
cepheid as the point where the rings might originate when the expanding light spheres impinge upon it. The flexibility of the model with respect to distance and tilt of the screen then makes the previous criticism against the homogeneous model invalid. The appearance of the rings would be more dependent on these small geometrical details than upon the distance of the cepheid.

There is, however, one basic criticism which argues very strongly against the previous two models and any further models that might be based on the premise that the rings are light maxima from the cepheid. In all such models the light maxima radiate outwards at the speed of light and necessarily require the nebular features to move. If the rings are successive light maxima from the cepheid, each ring must move to replace the previous one in a period equal to the period of the cepheid. The positional measurements of the nebular features have shown, however, that no such motion is observed. The appearance of the nebula is unchanging and has remained that way at least from 1961 to 1967, the period of time sampled by the available plates.

A more tenable model of the nebula is based on the assumption that the nebular features actually exist in space around RS Pup as regions which are more dense in light reflecting material than the dark regions. This model, labeled the inhomogeneous model, would not be expected to show any indication of positional shifts of its
features on the time scale of the cepheid's period. Mass motions could, of course, occur on a much longer time scale with velocities around $10 \mathrm{~km} . / \mathrm{sec}$. and yet still be undetectable with the present time base line. The density features in such a model would, furthermore, not necessarily have to be distributed in any particular fashion with respect to the period of the cepheid, thus avoiding the distance compatibility criticism of the homogeneous model. The inhomogeneous model must, however, reflect the periodic variability of its central light source since each density feature receives different amounts of light to reflect at various times during the cepheid's cycle. This is, in fact, supported by the observations and is therefore the single most compelling argument in favor of this model.

Once it has been accepted that the inhomogeneous model is best supported by the observations, the detailed appearance of the nebula leads to further inferences concerning its physical nature. The most striking characteristic of the nebula is its high degree of circular symmetry about the cepheid. The nebular features are arranged in arcs about the cepheid and are generally circular and concentric to a first approximation. Although the nebular arcs are not perfectly circular, concentric, or continuous in extent, the slight apparent ellipticity and the lack of continuity can be attributed to inhomogeneities
in the surrounding interstellar material. In the interpretation of the nebula, which is developed later in this chapter and in, the concluding chapter, the surrounding ambient interstellar matter either is concentrated by physical processes originating from the star or impedes the outflow of matter from the star. In any case, its initial distribution and velocity in space will ultimately affect the appearance of the visible nebula. The possibility that the observed configuration is merely a chance coincidence in space of the cepheid and a circularly structured nebula seems remote. In addition, if the cepheid and the nebula are physically associated with one another, then any physical process that might cause such a degree of observed circular symmetry is in all probability more generally spherically symmetric. The specific case of circular symmetry in one plane only would have to be viewed in very restricted directions to maintain its circular appearance and not be presented as elliptical. Spherical symmetry which gives rise to the observed ring features then strongly suggests an inhomogeneous density model of spherical shells surrounding RS Pup.

A very approximate impression of the nature of the nebula in relation to what is projected on the plane of the sky can be obtained by an approach analogous to interpretations of other ring-shaped nebulae. With an assumed spherically symmetric shell around the star, the shell
width is taken to be considerably smaller than its corresponding distance from the central star. Based on the observed ring structure, the shell thickness must then be assumed to be optically thin in its radial dimension. The opposite case of optical thickness would be visible as a more uniformly bright nebula homogeneously distributed in front of the star and not in the form of a ring. For the case of an optically thin nebula, then, its appearance is governed by two factors: the increasing optical depth along the line of sight from center to edge and the angular phase function which governs the light scattering process. For most reasonable assumptions about the ratio of shell thickness to distance from the central star, the observed path length through the shell increases very slowly out from the center, remaining less than a factor of three from its central value until the line of sight is nearly $70^{\circ}$ from the nebula-star radius. This means that if the observed optical depth is to become at all thick, it does so only near the edge of the nebula. Even in the case when the optical depth in the line of sight never becomes thick, the intensity again reaches a maximum near the edge of the nebula as will be seen below.

As an indication of these trends, consider a
simplified nebular model consisting of a spherical shell of relative half thickness of $1 / x$ as measured against the distance from star to shell (see Figure 16). The optical
depth in the line of sight can then be given as a function of $x$ and $\alpha$, the angle between the line of sight and the nebula-star radius, as

$$
\begin{gathered}
\tau(\alpha)=\left(2 x+1+x^{2} \cos ^{2} \alpha\right)^{1 / 2}-\left(1-2 x+x^{2} \cos ^{2} \alpha\right)^{1 / 2} \\
\text { for } \alpha<\sin ^{-1}(1-1 / x)
\end{gathered}
$$

or,

$$
\begin{aligned}
& T(\alpha)=\left(2 x+1+x^{2} \cos ^{2} \alpha\right)^{1 / 2} \\
& \quad \text { for } \alpha>\sin ^{-1}(1-1 / x)
\end{aligned}
$$

For the scattering phase function, consider that adopted by van Houten (1961) in a study of the.reflection nebula NGC 7023,

$$
s(\alpha)=\left[\left(1-g^{2}\right) / 4 \pi\right]\left(1+g^{2}-2 g \cos \alpha\right)^{-3 / 2}
$$

where $g$ is the scattering parameter ranging from $g=0$ for a completely symmetric angular distribution to $g=1$ for the case of complete forward scattering. Van Houten found the best values of $g$ to lie between 0.3 and 0.5 .

The product $\tau(\alpha) S(\alpha)$ has been calculated as a function of $\alpha$ for various choices of the parameters' $x$ and $g$ and can be seen tabulated in Table 5 and displayed graphically in Figure 1l. Also tabulated is the function $T(\alpha)$ alone for two values of $x$. The calculations have not included the scattering contribution from the portion of the nebula which lies behind the star for $\alpha$ greater than

Table 5. $T(\alpha)$ and $T(\alpha) S(\alpha)$ for different values of $x$ and $g$

|  | $\begin{gathered} 10 \\ -- \\ \tau(\alpha) \end{gathered}$ |  | $\begin{gathered} 20 \\ \cdot 3 \\ \tau(\alpha) S(\alpha) \end{gathered}$ | $\begin{gathered} 20 \\ .4 \\ \tau(\alpha) S(\alpha) \end{gathered}$ | $\begin{gathered} 20 \\ \cdot 5 \\ \tau(\alpha) S(\alpha) \end{gathered}$ | $\begin{gathered} 20 \\ .6 \\ \tau(\alpha) S(\alpha) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\circ}$ | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 5 | 1.00 | 1.00 | -99. | . 99 | . 98 | . 95 |
| 10 | 1.02 | 1.02 | . 99 | . 97 | . 93 | . 87 |
| 15 | 1.04 | 1.04 | . 97 | . 93 | . 85 | .74 |
| 20 | 1.07 | 1.07 | . 96 | . 89 | . 77 | .61 |
| 25 | 1.11 | 1.11 | .94 | . 83 | . 68 | . 50 |
| 30 | 1.16 | 1.16 | . 92 | . 78 | .61 | . 41 |
| 35 | 1.23 | 1.22 | . 90 | .74 | .54 | .34 |
| 40 | 1.32 | 1.31 | . 90 | . 70 | . 49 | . 29 |
| 45 | 1.43 | 1. 42 | . 89 | .67 | . 44 | . 25 |
| 50 | 1.59 | 1.57 | . 91 | . 65 | .41 | . 22 |
| 55 | 1.81 | 1.76 | . 95 | . 65 | . 40 | . 21 |
| 60 | 2.17 | 2.03 | . 99 | . 66 | . 39 | . 20 |
| 65 | 3.12 | 2.45 | 1.10 | . 71 | . 41 | . 20 |
| 70 | 2.83 | 3.29 | 1.36 | . 85 | . 48 | . 23 |
| 75 | 2.63 | 4.12 | 1.57 | . 95 | . 52 | . 24 |
| 80 | 2.45 | 3.65 | 1.27 | . 76 | . 41 | . 19 |
| 85 | 2.33 | 3.32 | 1.08 | . 63 | . 33 | . 15 |
| 90 | 2.29 | 3.20 | . 96 | . 55 | . 29 | .13 |



Figure ll. $T(\alpha) \cdot S(\alpha)$ for different values of the scattering parameter, $g$
$90^{\circ}$. For the forward scattering parameters used here the back scattering contribution is quite small. There are two points to be noted from these calculations. Firstly, the maximum intensity at the periphery of the nebula originates at values of $\alpha$ between $65^{\circ}$ and $75^{\circ}$ and has a fairly narrow width at half maximum of around $10^{\circ}$ independent of the values of $x$ and $g$. The second point to note is that for the cases of extreme forward scattering (i.e., for $g$ greater than 0.4), the intensity maximum at the periphery of the nebula is exceeded in magnitude by the broad central maximum at $0^{\circ}$ scattering angle.

The first of the above points confirms in a general way the assumption that the observed ring structured nebula is very close to the plane of the sky. The second point indicates that the nebulosity should also have a central maximum whose size and intensity relative to the outer intensity maximum are strongly dependent upon the nature of the scattering phase function. This feature, however, would in most cases be undetectable simply because of the dominating intensity from the central star. In the case of the RS Pup nebula, it would be extremely difficult to separate out such a central feature from the similarly structured sky background caused by atmospheric scattering of the light from RS Pup itself.

## A Geometrical Determination of the

## Distance to RS Pup

Once it has been established that the cepheid light variation causes a detectable variation of intensity in the nebula, it should be possible to determine geometrically the distance of the system with a high degree of accuracy. In principle, this requires a detailed knowledge of exactly how the nebulosity is distributed in space and not simply how it is projected onto the plane of the sky as observed. Without this detailed knowledge the observed radial distances in the nebula can be quite ambiguous and misleading (similar to the uncertainties imposed in stellar rotational velocity measurements by the lack of a knowledge of the projection factor sin i). In this case, however, the calculations show that the assumption of spherical symmetry indicates that any ringlike feature observed is at most $15^{\circ}$ to $25^{\circ}$ away from the plane of the sky and the equivalent sin $i$ projection factor is always between 0.9 and 1.0. The apparent separation of each ring feature from the cepheid can thus be meaningfully interpreted as being very nearly equal to the true angular separation of each spherical shell from the cepheid.

Geometrically determining the cepheid's distance is then, in principle, a simple task. Basically, for each ring feature that has an observed light curve there are two measured quantities of importance: the angular separation
of the ring from the cepheid and the phase difference of any specific point in the light curve of the cepheid from that in the ring. The primary unknown for each ring, however, is the integer number of periods, measured in light days, that separate it from the cepheid. These integers must be found for each ring such that the resulting distance of the cepheid from the earth in each case is the same.

Determining the angular separation of each ring from the cepheid entails a straightforward measurement from the plates, but the determination of the phase difference poses additional problems. It is apparent from an inspection of the cepheid light curve and that of the inner three rings, that the amplitudes and shapes of the light curves differ considerably. In principle, any variation in intensity of the central light source of a reflection nebula should be exactly reproduced by the light curve of the nebula itself. In this case, however, there are several effects which will tend to dilute and distort the appearance of the reflected light curve. For a nebular ring of finite thickness and a finite sampling resolution element, the observed light curve will be a sum of many light curves from parts of the ring that are at slightly different distances from the cepheid than the measured average distance. Also, since the observable portion of each ring is not exactly in the plane of the sky but
extends in front of this plane with angles up to $25^{\circ}$ and has a finite extent of up to $10^{\circ}$ to $15^{\circ}$, the light from the nebula lying further from the observer will arrive at the observer later than the light from the nebula lying closer to the observer. Thus, according to the model, the light from all parts of the observed section of nebular ring will. be slightly out of phase üpon reaching the observer. The effect is identical to observing the cepheid light curve through a time averaged window that is not small compared to the period of the cepheid. It is also, therefore, to be expected that the ring features which are furthest from the cepheid will suffer the greatest degree of phase mixing simply because the same angular extent as seen from the cepheid will correspond to a much larger spatial extent. To allow for these effects, then, several artificial reflected light curves were computer generated for various values of the size of a hypothetical reflection nebula. The known cepheid light curve was arbitrarily divided into forty equal time intervals, converted from magnitudes into intensities, and new light curves were formed by adding the intensities from more than one time interval. The new time intervals were chosen to equal from one-tenth to ninetenths of the cepheid's period which correspond equivalently to the reflected light curves from ring features with sizes of from one-tenth to nine-tenths of the period as measured in light days. These results are shown in Figure 12 and


Figure 12. Artificially computed B magnitude light curves
illustrate several points worth noting. As the size of the reflecting feature increases the amplitude of its light curve decreases and the separation in phase of the maximum and minimum increases while the maximum moves to larger values and the minimum to lower values. The effect of phase separation of maximum and minimum is so pronounced for the large sizes, that the reflected light curve loses all resemblance of the primary cepheid light curve.

The calculated curves give the phase at which the nebular minimum light should occur if the nebula were an integral number of periods measured in light days away from the cepheid. Fitting these curves to the observed curves then gives the true phase of the minimum and the difference between the two gives the desired phase difference. Each observed curve, however, can be fitted to several calculated curves, and since there is no way of determining which fit is best, an average phase difference from several fits was obtained. Table 6 gives the details of these fits, and it is to be noted that for each position in the nebula the range in phase difference is only about o.l even though the range in acceptable fits is considerably larger especially for the faint nebular region. The faintest nebular region was not fitted at all simply because of the ill-defined nature of the observed curve. These results, nevertheless, illustrate roughly the expected increasing dilution of the nebular light curves as ring features

Table 6. Numerical data for nebular light curve fitting

|  | Width of nebular feature | Acceptable <br> range of <br> phase at <br> minimum light | Adopted phase at minimum light | $\Delta \emptyset$ | $\langle\Delta \phi\rangle$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Feature 1: | . 1 P | . 52-. 62 | . 57 | . 80 |  |
|  | . 2 | . . 52-. 60 | . 56 | . 76 | . 81 |
|  | . 3 | $.47-.57$ | . 52 | . 80 |  |
|  | . 4 | . $49-.55$ | . 52 | . 87 |  |
| Feature 2: | . $1 P$ | .01-. 23 | . 12 | . 35 |  |
|  | . 2 | .99-. 23 | . 11 | . 31 |  |
|  | . 3 | . $99-.21$ | . 10 | .38 |  |
|  | . 4 | .99-.17 | . 08 | . 43 | .38 |
|  | . 5 | .99-.09 | . 04 | .38 |  |
|  | . 6 | .97-.09 | . 03 | . 41 |  |
|  | . 7 | . $91-.05$ | . 98 | . 40 |  |
| Feature 3: | . 4 P | . $58-.66$ | . 62 | . 97 |  |
|  | . 5 | . $52-.66$ | . 59 | . 93 |  |
|  | . 6 | . 4 7-.65 | . 56 | . 94 | .98 |
|  | . 7 | .43-.78 | . 61 | . 03 |  |
|  | . 8 | $.37-.74$ | . 56 | . 01 |  |

further from the central cepheid are sampled. It becomes increasingly difficult to fit the light curves of the outer ring features with calculated curves which include less phase mixing.

Determining the cepheid's distance then consists of solving the following equation in a consistent manner for the three adequately measured nebular regions:

$$
\begin{equation*}
\left(n_{i}+\Delta \emptyset_{i}\right) P=5.7688 s_{i}^{d} \tag{1}
\end{equation*}
$$

In equation (1) the index $i$ denotes the nebular feature under consideration and ranges from 1 to 3 for the bright, intermediate, and faint nebular features. The known quantities of equation (1) are $p$, the period of the cepheid in days, $\Delta \emptyset_{i}$, the phase difference determined above, and $s_{i}$, the angular separation of the nebular feature and the cepheid measured in seconds of arc. The constant, 5.7688, is the necessary conversion factor from seconds of arc to light days. The integer $n_{i}$ in equation (1) refers to the unknown integral number of periods measured in light days between the cepheid and the nebular feature and $d$ is the unknown distance in kiloparsecs. Obviously, no matter how many nebular features are measured, there will always be one more unknown than there are equations, since each feature has an unknown $n_{i}$ and $d$ is the unknown distance common to every feature. A trial and error iterative
scheme is, however, adaptable for determining all of the unknowns.

The iterative scheme used has been to assume the integer value, $n_{1}$, for the bright nebular feature, thereby determining a provisional value for the distance of the cepheid, d. This distance is then used for the intermediate nebular feature to derive $n_{2}$. The derived value of $n_{2}$ must necessarily be close to an integral value to be meaningful. In any case, it is then rounded off to the nearest integer and the integer $n_{2}$ is used to compute another value of $d$. The comparison of the two values of $d$ thus obtained is a measure of the consistency of the solution. For this purpose, both the absolute and the percentage change in the derived distances are included. This has been done for several choices of the integer $n_{1}$ and the results are tabulated in Table 7. As the table indicates, $n_{1}$ was chosen to include a wide range of possible distances from 1 kpc . to 3 kpc ., since other independent distance estimates of RS Pup range from 1.5 to 2.0 kpc . A. further measure of the consistency of the results can be seen in Table 8 , where calculations for the faint nebular region have been included. In this table the derived distances are shown for various choices of the integers $n_{i}$.

Table 7 shows quite convincingly that the first two nebular features are best fitted by integers of 4 and 8

Table 7. Geometrical distance determination for RS Pup based on the two brightest nebular features

| Assumed ${ }^{n_{1}}$ | $d\left(n_{1}\right)$ | $\mathrm{n}_{2}$ | $\begin{gathered} \mathrm{n}_{2} \\ (\text { integer ) } \end{gathered}$ | $d\left(n_{2}\right)$ | $\begin{gathered} \Delta d= \\ d\left(n_{1}\right)-d\left(n_{2}\right) \end{gathered}$ | $\Delta \mathrm{d} / \mathrm{d}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 1.038 | 4.53 | 4 | . 926 | .112 | . 108 |
|  |  |  | 5 | 1.138 | -. 100 | -. 096 |
| 3 | 1.408 | 6.277 | .. 6 | 1.349 | . 059 | . 042 |
| 4 | 1.778 | 8.024 | 8 | 1. 772 | .006 | . 003 |
| 5 | 2.147 | 9.771 | 10 | 2.195 | -. 048 | -. 022 |
| 6 | 2.517 | 11.519 | 11 | 2.407 | . 110 | . 044 |
|  |  |  | 12 | 2.618 | -. 101 | -. 040 |
| 7 | 2.886 | 13.226 | 13 | 2.830 | . 056 | . 019 |

Table 8. The inclusion of Feature 3 in the distance determination

respectively and the resulting distance of 1.775 kpc . The other integer combinations have much larger percentage dispersions in distance. To see how this result is affected by the third nebular feature in Table 8, Table 9 has been set up using the best integer combinations of Table 8 and the corresponding integer $n_{3}$ which gives a similar distance. The table gives the average distance for the three integer combination and the standard deviation from the average in kiloparsecs. Although the standard deviation for all choices is small, the smallest standard deviation still results from the best integer combination of Table 7. Due to the greater errors involved in fitting the observed nebular light curve for the faint feature, the relative sizes of the deviations in Table 7 are more significant than in Table 9. Tables 8 and 9 simply serve to confirm the conclusions of Table 7. The distance to RS Pip is therefore taken to be $1.78 \pm .02 \mathrm{kpc}$.

## Interpretation of the RS Pup Nebula

The previous sections have shown how the observations of the RS Pup nebula support the conclusion of the existence of a series of higher density, spherically symmetric, reflecting shells around RS Pup. The spectrum and color distribution of the nebula indicate its interpretation as a reflection nebula, and the periodic variation in intensity and lack of spatial motion of several of the

Table 9. Average distance of RS Pup for several integer combinations

| $n_{1}$ | $n_{2}$ | $n_{3}$ | $d\left(n_{i}\right)$ | $\sigma$ |
| :---: | :---: | :---: | :---: | :---: |
| 3 | 6 | 8 | 1.392 kpc. | $\pm .025 \mathrm{kpc}$. |
| 4 | 8 | 10 | 1.762 | .019 |
| 5 | 10 | 13 | 2.184 | .027 |
| 7 | 13 | 17 | 2.852 | .024 |

nebula's features indicate its inhomogeneous structure and association with the cepheid. On the basis of the above described working model of the nebula, then, it is possible to further interpret the observations to provide more of an insight toward the physical mechanisms which lead to such a structure.

The method used to explain the shape of the nebular light curves in the determination of the cepheid's distance can be put to further use strictly as an additional confirmation of the adopted working model. The observations show that the outer nebular features have smaller light curve amplitudes than the inner features. When these light curves are then fitted with computed light curves the outer features necessarily can be fitted only with those artificial curves which are based on larger reflecting regions. This is interpreted to mean that the outer nebular features
have more of an extent along the line of sight than the inner features do. In fact, if the working model is correct, the extent of the observed reflecting region should increase linearly with $r$, the distance of the region from the central cepheid. If each successive nebular shell is composed of similar reflecting particles, the observed range of scattering angles will be equal for every shell, and the corresponding extent of the reflecting region will increase outward with r. Table lo exhibits the relative constancy of the derived angular extent of the observed reflecting regions for the first three nebular features. In this table $W$ is the average width of the feature taken from Table 6, $r$ for each feature is based on the previous solution for the cepheid's distance, and $\theta$ is the resulting angular extent of the feature. Even though there is a relatively large range of fits for each light curve, the overall trend supports the present scattering model.

Table 4 gives the colors derived for each of the observed nebular features and as previously discussed provides general support for the expected radial increase in reddening found to occur in all reflection nebulae. The actual determination of the star minus nebula color as a function of $r$, however, depends again on the results of the light curve fitting employed in the distance determination. If each nebular feature were separated from the

Table 10. A comparison of the derived angular extent for the three brightest nebular features

|  | Feature 1 | Feature 2 | Feature 3 |
| :---: | :---: | :---: | :---: |
| $W$ | .25 P | .40 P | .60 P |
| r | 4.8 P | 8.4 P | 11 P |
| $\theta$ | .052 rad | . | .048 |

cepheid by an integral number of periods expressed as light days, no phase shift correction to the cepheid colors would be necessary. The actual nebular features, however, are phase shifted from the cepheid 15 an amount, $\Delta \varnothing$, and have finite extents, $W$, which dilute the cepheid light and color curves. The cepheid color curve must therefore be artificially diluted in a manner similar to that used for the light curves of Figure 12. The results of such an artificial dilution are shown in Figure 13 where, as expected, the more extended features give color curves with much lower amplitudes. Since the average cepheid phase for the available nebular color observations is 0.82 , the phase at each nebular feature is ( $0.82-\Delta \varnothing$ ). Table ll then displays the colors of the cepheid at the phases for the appropriate $W$ previously determined for each feature. Even though the observations for the outermost feature are far too uncertain to indicate $\Delta \emptyset$ or $W$, it has been assumed that


Figure 13. Artificially computed $B-V$ color curves

Table 11. The calculation of ( $B-V$ ) star-nebula

| Feature | W | $.82-\Delta \emptyset$ | $(\mathrm{B}-\mathrm{V})_{*}$ | $(\mathrm{~B}-\mathrm{V})_{\mathrm{neb}}$ | $(\mathrm{B}-\mathrm{V})_{*-n e b}$ | r |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | .25 P | .01 | +1.18 | +.89 | +.29 | 1.0 |
| 2 | .4 P | .44 | +1.62 | +1.76 | -.14 | 1.7 |
| 3 | .6 P | .84 | +1.51 | $+2.04_{4}$ | -.53 | 2.3 |
| 4 | -- | - | $\sim+1.52$ | +2.19 | -.67 | 2.8 |

$W$ continues its increasing trend to this feature and Figure 13 shows that the corresponding diluted cepheid color should be very close to +1.52. This value has been used in Table 11 for Feature 4. Figure 14 shows how the star minus nebula color varies radially, where the radial index has been normalized to unity for Feature 1. A positive value of the star minus nebula color indicates an intrinsically blue nebula whereas a negative value indicates an intrinsically red nebula.

In a reflection nebula the colors are generally blue near the central star and redden with angular displacement from the star. This is a consequence of the well-known Mie scattering theory whereby the blue wavelengths are preferentially scattered out of the light from the star and into the observer's line of sight. The red wavelengths are selectively transmitted to the outer nebular regions and the light scattered into the observer's line of sight at large angular displacements is therefore redder than at small angular displacements. Figure 14 reproduces this expected trend in the RS Pup nobula. The intrinsic redness of the outer nebular features is quite large and is most likely due to the specific physical. characteristics of the nebula: its geometry and particle size distribution.

Theoretical colors and polarizations of reflection nebulae have been calculated for different compositions and

geometries most recently by Hanner (1969) for comparison with observations of the Merope nebula. The results of the calculations have been shown to be quite sensitive to the particular geometry chosen. None of the calculations, however, allow the possibility of nebular colors, with respect to the central star, greater than +0.3 and seem to be in conflict with the observations presented here. On the other hand, the specific geometry of concentric shells has not been treated, especially in the case of large particle sizes, which can give redder nebular colors. Specifically, for the RS Pup nebula, the light scattered from successive nebular shells must pass through greater thicknesses of material since the outer reflecting regions have a greater extent than the inner regions. Also, the range of scattering angles observed is limited to $70^{\circ}$ to $80^{\circ}$ and there is no contribution from extreme forward scattering angles close to $0^{\circ}$ where the scattered light is very blue. This combination of large extinction path lengths and limited scattering angles near $80^{\circ}$ might give rise to the red colors observed here. In order to have complete confidence that the presently observed colors are consistent with the geometrical model, the detailed Mie calculations need to be done for an acceptable range of grain sizes and compositions. Polarization measures would also be extremely helpful in isolating the particle sizes and the acceptable range of scattering angles.

Particularly, the variation of polarization with wavelength is quite sensitive to the dominant scattering angles involved. For large particles, however, the sensitivity decreases considerably. The small angular size of the nebula and the proximity to the extreme relative brightness of RS Pup combined with the need for good angular resolution of individual nebular features, however, would make polarization measurements extremely difficult. The distribution of the nebular surface brightness as a function of radial distance from the cepheid can be obtained from the individual light curves for each nebular feature. Each of the artificially diluted light curves of Figure 12 is normalized to the magnitude level which corresponds to its intensity average $(\Delta m=.86$ in Figure 12). The absolute value of this level was read from the observed light curves when they were fitted with the average diluted curve determined from Table 6. This magnitude level for each light curve was then adopted as the mean magnitude of each feature integrated over the analyzing diaphragm, and its variation from feature to feature has been tabulated in Table 12 versus the radial distance normalized to Feature 1. The mean magnitudes observed are also plotted in Figure 15 along with the mean magnitudes which would result from two different assumptions about the distribution of material in the nebula as described below.

Table 12. Average $B$ magnitude as a function of $r$

| Feature | $r / r_{1}$ | $\bar{B}$ | $\overline{\Delta B}$ |
| :---: | :---: | :---: | :---: |
| 1 | 1.0 | 17.5 | 0 |
| 2 | 1.7 | 18.4 | .9 |
| 3 | 2.3 | 19.1 | 1.6 |
| 4 | 2.8 | 19.8 | 2.3 |



Figure 15. Variation of the average $B$ magnitude with $r$

Consider the nebular surface brightness which would be observed under the assumption that each nebular shell was composed of the same number of reflecting particles and that each shell was of the same radial. thickness. In addition to the incident light intensity decreasing as $1 / r^{2}$, the density of material would fall off from shell to shell as $1 / r^{2}$ per unit volume, and the combined effect would mean that the intensity per unit volume would fall off as $1 / r^{4}$. The present method of observing the intensity variation from a constant sized diaphragm, however, is similar to observing the same fractional volume of an expanding toroid of constant cross sectional area. The total volume observed would therefore increase for larger toroids approximately proportional to r. The resultant observed intensity variation would be proportional to $1 / r^{3}$ and has been plotted on Figure 15. Next, consider the case of each nebular shell having the same density of reflecting particles. For this case the only radial effect observable would be the $1 / r^{2}$ dependence of the incident intensity. The reflected intensity per unit volume would therefore drop off as $1 / r^{2}$. Combined with the increasing volume of the sampled region at larger radial distances, then, the observed intensity variation would be proportional to $1 / r$. This has also been plotted on Figure 15.

A comparison in Figure 15 of the observed radial intensity variation in the RS Pup nebula to the variation expected from the previous two assumed nebular models shows that the distribution of material is intermediate between constant number of particles per shell and constant particle density per shell. With the nebula symmetrically distributed as a series of spherical shells around the cepheid there are several possible formation mechanisms involving the cepheid and the interstellar medium which could give rise to the above discussed surface brightness distribution. If there is postulated to be a series of material ejections from the cepheid into a region where there is no initial medium then it is reasonable to assume that each shell would be made up of equal amounts of matter and would have similar thicknesses caused by small internal velocity dispersions in each shell. This very crude and simplified model would then give rise to the $1 / r^{3}$ intensity distribution calculated above for the case of constant particle number per shell. At the other extreme of possible formation mechanisms is the possibility that an initial uniform interstellar material surrounded the cepheid and was swept up into concentric shells by a series of shock waves from the cepheid. If the shock waves are postulated only as a means of initiating the outward motion of a shell of the initial medium and each shell sweeps up a fraction of the surrounding medium as it expands, the successive expanding
shells would always maintain a constant density if the initial medium were of constant density. The observed nebular surface brightness would then be expected to go as $1 / r$.

Since the observed intensity distribution in the RS Pup nebula falls somewhere between the above extremes of $1 / r^{3}$ and $1 / r$, it is reasonable to conclude that the physical mechanism responsible for the formation of the nebula must contain some aspects of both of the above models. Specifically, it is unreasonable to assume that there would be no initial interstellar medium surrounding the cepheid since it is a member of a young association and there is evidently some absorbing material in its volume of space (Westerlund 1961, 1963). Also, it is rather artificial to assume that a shock wave would sweep up the surrounding interstellar matter without having a similar effect upon the outer layers of the cepheid. A reasonable combination of the above two models is the assumption that the interstellar matter around RS Pup has been swept up into a series of shells by matter ejected from the cepheid, possibly associated with a shock wave phenomenon.

There are several possible effects which have not been taken into consideration in postulating the above simplified model of the nebula. Firstly, the density of the ambient interstellar material will be periodically decreased by the successive passage of nebular shells and
therefore each shell cannot be considered to be swept up from the same density of ambient material. This can be considered to be a second order effect, however, provided that only a small fraction of the medium goes into each shell. Secondly, the well-known nonuniform distribution of the interstellar material can not be disregarded in this region. Its irregular nature would force the swept up shells to be irregular in form which is indeed how they are observed. The influence of this phenomena on the derived intensity distribution has been minimized by observing only in the brightest region of each apparent nebular ring. In addition, the interpretation has involved the assumption that the brightest regions of each ring have sinilar physical properties from shell to shell. Thirdly, in considering shock wave phenomena with associated ejected matter, it has been assumed that each successive process would be identical. This will be further discussed in the final chapter of the present work where the possible formation mechanism will be treated with regard to the evolution of the cepheid. Lastly, and perhaps most basically, is the assumption that some of the matter which is postulated to be ejected from the cepheid is ejected in particle form that has reflecting properties similar to the dust in the interstellar medium. Hoyle and Wickramasinghe (1962) and later Donn et al. (1968) have investigated the production of graphite grains in the atmospheres of cool stars as a
source of interstellar particles but the production of similar grains in other stars has not been fully studied. It is still quite uncertain as to whether or not such particle emission from stars could be important to problems of this nature.

An important quantity, of which there is only very limited knowledge, is the mass of the nebula. Given the amount of material confined in the nebula, it may be possible to estimate the relative contribution from the ambient medium and from the cepheid itself.

One approach to determining the mass of a reflection nebula involves making use of the relative brightness of star and nebula and reasonable assumptions about the physical properties of the scattering particles. Based on the derived properties of the shell model of the RS Pup nebula, the simplest method of finding the particle density in each shell is to consider the observable section of each nebular shell as being of finite volume situated in the plane of the sky as in Figure 16. Since the effective nebular extent $W$ is very much smaller than the distance $r$, the scattering angle for all points along the extent of the region can be taken as constant. Consider a small element in the nebula of extent dw, of observed cross-sectional area $d A$, and at a distance $w$ from the front of the nebula. For each particle in this element the incident intensity from the star is $I_{0} / r^{2}$. According to van de Hulst (1957)


## OBSERVED NEBULA

$\oplus$

Figure 16. Schematic of a simple nebular model
the intensity scattered by particles of radius a in a scattering angle $\theta$ is given by

$$
\begin{equation*}
\frac{1 / 2\left[i_{1}(a, \theta)_{2}+i_{2}(a, \theta)\right]}{k^{2} r^{2}} I_{0}=\frac{F(a, \theta)}{k^{2} r^{2}} I_{0} \tag{2}
\end{equation*}
$$

where $k=2 \pi / \lambda$ and the functions $i_{1}(a, \theta)$ and $i_{2}(a, \theta)$ are given by van de Hulst. In the volume element dwdA there are ndwdA scattering particles and the resulting intensity received by an observer at a distance d is given by

$$
\begin{equation*}
d I(r)=n d w d A \frac{F(a, \theta)}{k^{2} r^{2}} \frac{I_{o}}{d^{2}} e^{-n \sigma w} \tag{3}
\end{equation*}
$$

where $\sigma=Q \pi a^{2}$ is the extinction cross-section per particle with $Q$ the scattering efficiency. Integrating over the extent of the reflecting region, the intensity received becomes

$$
\begin{align*}
I(r) & =\int_{0}^{W} d I(r)=n d A \frac{F(a, \theta)}{k^{2} r^{2}} \frac{I_{o}}{d^{2}} \int_{0}^{W} e^{-n \sigma w} d w \\
& =d A \frac{F(a, \theta)}{k^{2} r^{2}} \frac{I_{o}}{d^{2}} \frac{1}{\sigma}\left(1-e^{-n \sigma W}\right) \tag{4}
\end{align*}
$$

The intensity of starlight received by the observer is $I_{*}=I_{o} / d^{2}$ and therefore the intensity ratio between nebula and star is given as

$$
\begin{equation*}
\frac{I(r)}{I_{*}}=10^{-.4 \delta m}=\frac{F(a, \theta)}{k^{2} r^{2}} \frac{d A}{\sigma}\left(1-e^{-n \sigma W}\right) \tag{5}
\end{equation*}
$$

This can be directly solved for $n$, the number of scattering particles per unit volume, and then the mass of the nebular shell is given by

$$
\begin{equation*}
M=n\left(4 \pi r^{2} \Delta r\right) \rho_{g} 4 / 3 \pi a^{3} \tag{6}
\end{equation*}
$$

where $\Delta r$ is the shell thickness and $\rho_{g}$ is the material density of each scattering particle of radius a. The observed cross-sectional area dA of a nebular feature is related to $\Delta r$ by $d A=\Delta r h$ with $h$ the dimension of the observed feature which is set by the size of the analyzing diaphragm. The 5 second of arc diaphragm corresponds to $h=1.32 \times 10^{17} \mathrm{~cm}$. at the distance of RS Pup, but there is an uncertainty in $\Delta r$ caused by the seeing conditions.

Nevertheless, $\Delta r$ is at least less than one-half of $h$. For typical dielectric scattering particles with $a=.5 \mu$ and $\rho_{g} \sim 1 \mathrm{gm} / \mathrm{cm}^{3}$ the function $F(a, \theta)$ has been given by Hanner (1969) and $F(.5 \mu . \theta)$ is quite dependent on the scattering angle and $F\left(.5 \mu, 80^{\circ}\right) \sim 5$. A representative value for the extinction efficiency for these type particles is $Q \sim 4$. The model of the nebular shells is based on the assumption that the optical depth along the periphery of each shell is greater than unity. Because of the uncertainty of the parameters of equation (5), however, the exact value of $T=n_{\sigma W}$ is difficult to determine. The mass of a shell in terms of $\tau$ can be given as

$$
\begin{equation*}
M=16 r^{2} \Delta r a \rho_{g} T / Q W \tag{7}
\end{equation*}
$$

Table 13 shows the derived shell masses based on the assumption that $\Delta r \sim .5 \times 10^{17} \mathrm{~cm}$ and $\tau \sim 1$. The average mass per shell in the form of light scattering particles is $.08 M_{0}$ and the total of five shells is $.4 M_{0}$. This value must be considered an upper limit primarily due to the possibility that $\Delta r$ could be very much smaller than assumed.

Table 13. Shell masses derived from eq. (7)

| Feature | $r(\mathrm{~cm})$ | $W(\mathrm{~cm})$ | $\delta \mathrm{m}$ | M |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $5.1 \times 10^{17}$ | $2.7 \times 10^{16}$ | 9.0 | $.05 \mathrm{M}_{0}$ |
| 2 | $8.9 \times 10^{17}$ | $4.3 \times 10^{16}$ | 9.9 | .10 |
| 3 | $12.0 \times 10^{17}$ | $6.4 \times 10^{16}$ | 10.6 | .11 |
| 4 | $14.3 \times 10^{17}$ | $\ldots$ | 11.3 | $\ldots$ |

Another possible method for estimating the total mass of the particles in the nebula is based on the procedure outlined by Lynds (1968) for obtaining masses of dark nebulae. For a spherical nebula with total observed cross-sectional area, A, given in square degrees the mass can be expressed as

$$
M=\left[\begin{array}{lllll}
1.2 & (\Delta m) & a \rho_{g} A d^{2} / Q \tag{8}
\end{array}\right] M_{0}
$$

where $d$ is in parsecs and $\Delta m$ is the extinction in the visual suffered by starlight upon its passage through the center of the nebula. Since the color excess of RS Pup is .62, and assuming the cepheid to lie at the center of the nebula the extinction must lie in the range $0<\Delta \mathrm{m}<3.72$. Based on this information alone, however, it is impossible to determine how much of this extinction is due to the nebula, the interstellar material in Pup III, or the general interstellar material in the 1760 parsecs between the observer and Pup III. A comparison of the nine nearest association neighbors to RS Pup shows a range in color excess of . 26 to .79 with an average value of . 48 . Using this value as representative of the non-local excess for RS Pup gives an average extinction due to the nebula alone of $\Delta m=.84$. Inserting this along with the typical parameters for interstellar particles and the one minute of arc angular radius of the nebula into equation (8) then gives a mass of $.03 M_{0}$. Because of the large range in color excess of the nearest association members, however, this value of the mass must be considered only as an upper limit.

Equations (7) and (8) lead to values of the nebular mass which differ by a factor of 10 from one another. This can be attributed to the large range of uncertainty in the parameters $\Delta \mathrm{m}, \Delta \mathrm{r}$, and T . It is interesting to note, nonetheless, that both equations have the same dependence
on the values of $a, \rho_{g}$, and $Q$ so that a comparison of the results should not be affected by uncertainties in these quantities. Also, the value of $a / Q$ remains reasonably constant for all particle sizes less than $.5 \mu$ with the consequence that its assumed value will not significantly affect the results.

Taking $.03 M_{0}$ as the upper limit of the particle mass of the nebula, it is interesting to compare this value to the total mass of particles expected in an average region of the interstellar medium of the same volume. For the standard HI cloud with a density of $10 / \mathrm{cm}^{3}$, as designated by Spitzer (1968), the mass density in hydrogen alone corresponds to $0.25 \mathrm{M}_{0} / \mathrm{pc}^{3}$. For a region the size of the RS Pup nebula the corresponding mass of hydrogen would be $0.13 \mathrm{M}_{0}$. At present our knowledge of the dust/gas ratio for the interstellar medium is quite uncertain, but using a value of $1 \%$ based on O'Dell's (1965) observations of HII regions the total mass of dust in a region the size of the RS Pup nebula would be on the order of $10^{-3} M_{0}$. This is consistent with the upper limit of the particle mass determined above, since there should reasonably be at least as much dust in the nebula as in a typical region of the interstellar medium.

The previous discussion still does not settle the question of how much of the nebular material has been contributed by the cepheid and how much is due to the
ambient interstellar material. Depending on how close the upper limit of $.03 M_{0}$ is to the total mass of the nebular dust, the majority of the mass of the nebula could have been supplied by the cepheid. This would correspond to about $3 M_{0}$ for no deviation from the upper limit and a $1 \%$ contribution from the dust. In order for the majority of the mass of the nebula to have been supplied by the interstellar medium, the presently calculated upper limit would have to be a factor of 30 too large. The present observations are unable to differentiate adequately between the two cases.

The RS Pup nebula can thus be very simply interpreted as a reflection nebula whose material distribution in space has been strongly influenced by its centrally located cepheid. Whether the material of the nebula originated from the cepheid itself or was already a part of the interstellar material is a problem which is at present unsolved. In either case, however, the observational evidence indicates that the cepheid has played some role in the formation of the presently observable structure of the nebula. If the cepheid itself was not the source of the mass of the nebula, it has at least influenced the structure of the already existing interstellar material. In this sense the direct association of the cepheid and the interstellar material is unique. The discussion of how this interaction might be related to evolutionary
characteristics of the cepheid will be postponed until the final chapter.

## Introduction

- The classical cepheid SU Cas with a period of only 1.95 days has one of the shortest periods known of any cepheid in our Galaxy. In this respect it is quite unlike the long period cepheid RS Pup. Aside from its categorical similarity to RS Pup as a classical cepheid, however, SU Cas has one other characteristic which makes it important to any discussion of cepheid evolution. It, also, is associated with a reflection nebula, making it and RS Pup the only cepheids known to be located in reflection nebulae.

The publication of the Palomar Sky Survey and the subsequent catalogues of reflection nebulae (Dorschner and Gürtler 1964, 1965, Lynds 1965, van den Bergh 1966) resulted in the discovery of the faint reflection nebula near SU Cas. Previously undiscovered, the nebula appeared brighter on the blue plate than on the red plate of the survey, a characteristic of most reflection nebulae. The nebula appears as a faint wisp about 4' in length and separated from the cepheid by approximately 2'. Unlike the case of RS Pup, the cepheid is not imbedded in the nebula and there is no apparent structure. Figure 17 shows the nebula as it appears with the Steward Observatory


Figure 17. SU Cas and the nearby reflection nebula -- The exposure was 2 hr with the Steward Observatory 36-inch telescope automatically guided on a IIa-0 plate behind a 2 mm Schott GGl3 filter. (North is up, East to the left. Scale: lmm = 4:3.)

36-inch telescope. The lack of any stars as bright as SU Cas in the vicinity of the nebula and the orientation of the nebula, roughly perpendicular to the line joining it to $S U$ Cas, point conclusively to $S U C$ as as the illuminating star. In this case, however, there is no apparent evidence which favors any particular geometrical orientation of star and nebula in space.

As a further confirmation of the reflection nature of the nebula, an image tube spectrogram was taken by Dr. C. R. Lynds at Kitt Peak National Observatory. The spectrum, shown in Figure 18, is essentially a recording of the night sky spectrum consisting of several identifiable night sky emission lines with only a very faint continuum attributable to the nebula. In the continuum there is a suggestion of the faint absorption lines of $H$ and $K$ of calcium which are strong in the spectrum of $S U C$ as and in all cepheids.

The distance of $S U$ Cas has been calculated as 325 pc on the basis of Fernie's (1967) period-luminosity relation and as 300 pc by Sandage and Tammann (1968). If we accept a distance of SU Cas of 300 pc , the angular distance of light travel in the cepheid's 1.95 day period corresponds to about 142 . The relative width of the reflection nebula and its extreme distance from the cepheid thus makes it virtually impossible to detect any intensity variation in the nebula, especially considering the small


Figure 18. Image tube spectrum of the SU Cas nebula -The exposure time was 70 min., taken by C. R. Lynds with the Kitt Peak 84 -inch telescope at $180 \mathrm{~A} / \mathrm{mm}$ 。 The brightest night sky line is [OI] at 5577 A and the brightest comparison line at the left end is at 3888 A .
amplitude of the cepheid's light variation ( $V=0^{m!} \cdot 37$ ). Thus, it is impossible to study the interaction of cepheid and nebula in any detail as was the case with RS Pup. One comparison between the cepheids SU Cas and RS Pup which is observationally possible, however, is their location as possible members of $O B$ associations. As discussed in the preceding chapter, RS Pup is a member of a young $O B$ association. Because of the short period of $S U$ Cas, however, it is almost a certainty that $S U C$ Cas is one of the oldest cepheids in the Galaxy. Since the shortest period cepheids have a mass of about $4 M_{\rho}$, the age of such stars according to Iben (1967) would be at least 1 or $2 \times 10^{8}$ years when they reach the instability strip. It is therefore extremely doubtful that $S U$ Cas is a member of a typical $O B$ association containing much younger stars. In fact, $S U$ Cas is not known to be member of any galactic cluster or $O B$ association. Its nearest neighboring association, II Per, is located about $20^{\circ}$ away in galactic longitude.

On the other hand, if $S U C a s$ originated in an association, the association would no longer display the characteristics of a typical young $O B$ association, but would be composed exclusively of stars later than about B5 in spectral type. The earlier and more massive stars of the association would have already evolved from the main sequence to the later stages of their evolution, but the
less massive stars would retain their initial identities located on or near the main sequence for a much longer period of time. Whether or not these stars would still be loosely associated with one another is, of course, highly dependent upon their initial velocities and the character of the surrounding medium. In view of the nebulosity in the region, it is informative to determine whether or not SU Cas is physically related to any of the surrounding $B$ type stars.

## Observational Material

To insure that a survey of the $B$ stars near SU Cas extends out to and beyond the distance of the cepheid, it is necessary to observe at least down to the 9 th magnitude to allow for a moderate amount of intervening interstellar absorption. Also, because of the relative closeness of the cepheid to the sun, a loosely associated group in the cepheid's vicinity would be spread over a fairly large angular portion of the sky. For these reasons, all stars of spectral type B9 or earlier and within $5^{\circ}$ of SU Cas were selected from the Henry Draper Catalog as candidates for observation. The distribution of these stars on the plane of the sky can be seen in Figure 19, in which it is apparent that over half of the $B$ stars are concentrated in the part of the region nearest to the galactic equator. The fact that $S U$ Cas is located near $b^{I I}=+8^{\circ}$ contributes


Figure 19. Distribution of $B$ stars near $S U C$ Cas
to this apparent paucity of nearby $B$ stars. In addition, Weaver (1953) and Blaauw (1956) have pointed out the apparent lack of early $B$ stars between $\ell^{I I}=132^{\circ}$ and $e^{I I}=182^{\circ}$. A number of the lower latitude $B$ stars within the present region of interest have published obsedvational material available because of their logical inclusion in previous surveys near the galactic plane. For those candidate stars which had no available data in the literature, therefore, the present observations consisted of photoelectric photometry and spectroscopy.

UBV photometry of all but one of the stars was obtained on two different nights in September of 1.968 with the Steward Observatory 36 -inch telescope located at the Kitt Peak Field Station. On each night extinction and transformation coefficients were determined from the observation of several of the bright UBV standard stars. Reduction of the data was completed by means of a photoelectric reduction program written for Steward Observatory by Dr. W. S. Fitch and based on the Hardie (1962) reduction procedure.

Widened spectra suitable for spectral classification and the measurement of radial velocities were obtained at a dispersion of $63 \AA^{\circ} / \mathrm{mm}$ with the Cassegrain spectrograph of the No. 1 36-inch telescope of the Kitt Peak National Observatory. Up to three spectra of each of twenty-five of the candidate stars were obtained during the summer of
1968. The spectra were measured on the Grant Line Profile Comparator, also of the Kitt Peak National Observatory, and the measurements reduced with the aid of a radial velocity program written for the KPNO CDC 6400 computer by Dr. N. Sanwal. The spectra were also traced on the Steward Observatory Hilger-Watts microphotometer, and the tracings were combined with visual inspection of the spectra and of MKK standard stars to assign spectral types.

The results of both the photometry and the spectroscopy can be seen in Table 14 for all of the candidate stars including additional data obtained from the literature. Star No. $832=\mathrm{HD} 17443$ was added to those stars chosen from the HD Catalog on the basis of its B9V classification by Racine (1968b). Other data from the literature are available besides that indicated in Table 14 and provide a good basis of comparison for the accuracy of the photometry presented here. In Figure 20 are presented the photometric comparisons for the few overlapping stars, and it is evident that the agreement is quite good.

## The B Stars Near SU Cas

The two color diagram for the 33 stars, given in Figure 21 , indicates that the majority of the stars are reddened $B$ stars, and as expected from such a wide area of the sky, the range of reddening is quite large. The six stars which appear'beneath the unreddened main sequence

Table 14. Photometric and spectroscopic data for Cas B stars

| No. | HD | V | B-V | U-B | SpT | R.V. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | km/sec |
| 800 | 11529 | 4.96 | -. 09 | -. 42 | B8V | -9.1 |
| 801 | 11744 | 7.81 | $+.37$ | -. 32 | B2III 5 | +12.7 |
| 802 | 12301 | 5.62 | $+.37$ | -. 28 | B8Ib 3 | -15.06 |
| 803 | 12509 | 7.10 | $+.33$ | -. 56 | BlIII 3 | -12.06 |
| 804 | 12567 | 8.32 | +.38 | -. 57 | B05III 3 | -24.6 |
| 805 | 12882 | 7.57 | $+.36$ | -. 52 | B6Ia 3 | -10.5 2 |
| 806 | 13590 | 8.00 | $+.38$ | -. 40 | B2III 3 | -27.5 2 |
| 807 | 13630 | 8.79 | $+.34$ | $+.05$ | B9V | -10.1 |
| 808 | 14010 | 7.15 | +. 59 | -. 10 | B9Ia 3 | -43.0 6 |
| 809 | 14863 | 7.76 | $+.06$ | -. 39 | B6V | +11.5 |
| 810 | 14980 | 9.10 | $+.42$ | $+.02$ | B9V | +14.8 |
| 811 | 15472 | 7.874 | $+.074$ | -.61 4 | B3Ve 4 | -21.0 |
| 812 | 15727 | 8.23 | $+.49$ | -. 23 | B6V | -3.9 |
| 813 | 16036 | 8.20 | $+.44$ | +. 26 | AOV | +32.9 |
| 814 | 16393 | 7.58 | $+.03$ | -. 30 | B8V | +4.1 |
| 815 | 16440 | 7.87 | +.72 | +.05 | B7II 8 | -16.0 |
| 816 | 16907 | 8.35 | +.11 | -. 06 | AOV | -100.5 |
| 817 | 17179 | 7.88 | $+.22$ | -. 42 | B2V | -2.3 |
| 818 | 17327 | 7.48 | $+.34$ | -. 04 | B8III 1 | +13.5 |
| 819 | 17706 | 8.47 | $+.35$ | -. 18 | B5V 1 | -14.0 |
| 820 | 17857 | 7.71 | +.74 | -. 01 | B8Ib 3 | -38.0 2 |
| 821 | 17929 | 7.86 | +. 29 | -. 15 | B8V | +7.8 |
| 822 | 17982 | 8.03 | $+.40$ | $+.38$ | AOV | +21.5 |
| 823 | 19065 | 5.87 | -. 02 | -. 14 | B9V | +14.0 |
| 824 | 19856 | 8.85 | +. 19 | -. 27 | B8V | +5.5 |
| 825 | 20226 | 8.61 | +. 22 | -. 18 | B7V | -18.0 |
| 826 | 20336 | 4.85 | -. 15 | -. 75 | B2Ve 9 | +23.3 |
| 827 | 20566 | 8.08 | +.38 | -. 25 | B3Ve 7 | +1.8 |
| 828 | 20710 | 7.60 | $+.08$ | -. 17 | B9V | +12.0 |
| 829 | 21.267 | 7.98 | $+.00$ | -. 28 | B8V | $+3.7$ |
| 830 | 21725 | 9.10 | +. 21 | $+.10$ | AOV | +2.0 |
| 831 | 21930 | 8.42 | $+.17$ | +. 02 | A2V | $+4.6$ |
| 832 | 17443 | 8.741 | +.29 1 | $+.171$ | B9V 1 | -- |

References: 1--Racine 1968b; 2--Bonneau 1967; 3-Morgan, Code, and Whitford 1956; 4--Mendoza 1958; 5-Guetter 1968; 6--Wilson 1953; 7--Merrill and Burwell 1949; 8--Hardorp et al. 1959; 9--Blaauw 1956.


$$
B-V
$$



Figure 20. Photometric comparisons


Figure 21. Cas B stars: two-color diagram
line are probably also reddened late $B$ and early $A$ stars as indicated by their spectra.

The intrinsic colors and absolute magnitudes for the observed stars were obtained using Schmidt-Kaler's (1965) calibration and are presented in Table 15. Part a of Table 15 is based primarily on the photometry and Johnson's (1958) nomogram as a comparison to part b, which is taken more directly from the observed spectral types.

The frequency distribution of the distance moduli for both calibrations has been plotted as Figure 22. This figure indicates an obvious preponderance of stars with moduli in the range 6.4 to 8.5 , corresponding to a distance interval from 200 to 500 pc .

Of the stars which fall beyond the above distance interval, most are also at large angular separations from SU Cas and lie near the lower galactic latitude range of this investigation (see Figure 19). Although the survey is by no means complete for the late $B$ stars beyond about 500 pc it is interesting to note a slight concentration of stars near 900 pc . Their galactic positions near $\ell^{I I}=$ $131^{\circ}, b^{I I}=+3^{\circ}$ at this distance indicate that they may be members of Cam OBl. Also in Figure 22 a few stars appear even further away than those stars considered as candidates for membership in Cam OBI. These more distant stars are all very luminous and probably can be assigned distances compatible with that of the Pereus arm.

Table 15. $(B-V)_{0}$ and $\left(V_{o} M_{v}\right)$ for Cas B stars

|  | Pub. |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| No. | Spt | $(B-V)_{o}$ | $E_{B-V}$ | $V_{0}$ | $M_{v}$ |$V_{o-M_{v}}$

(a) Calibrated from obseived colors and Johnson's (1958) nomogram

| 800 |  | -. 12 | . 03 | 4.87 | 0.0 | 4.87 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 801 | B2III 5 | -. 20 | . 57 | 6.10 | -3.6 | 9.70 |
| 802 | B8Ib 3 | -. 06 | . 43 | 4.33 | -5.6 | 9.93 |
| 803 | BlIII 3 | -. 27 | . 60 | 5.30 | -4.4 | 9.70 |
| 804 | BO.5III 3 | -. 28 | . 66 | 6.34 | -4.7 | 11.04 |
| 805 | B6Ia 3 | -. 06 | . 42 | 6.31 | -7.0 | 13.31 |
| 806 | B2III 3 | -. 23 | . 61 | 6.17 | -3.6 | 9.76 |
| 807 |  | $-.07$ | .41 | 7.56 | +0.7 | 6.86 |
| 808 | B9Ia 3 | . 00 | . 59 | 5.38 | -7.0 | 12.38 |
| 809 |  | -. 1.14 | . 20 | 7.16 | -0.4 | 7.56 |
| 810 |  | -. 1.0 | . 52 | 7.54 | +0. 3 | 7.24 |
| 811 | B3e 4 | -. 22 | . 29 | 7.00 | -1. 7 | 8.70 |
| 812 |  | -. 20 | . 69 | 6.16 | -1. 4 | 7.56 |
| 813 |  | -. 02 | . 46 | 6.82 | $+1.0$ | 5.82 |
| 814 |  | -. 11 | . 14 | 7.16 | +0.1 | 7.06 |
| 815 | B7II 8 | -. 12 | . 84 | 5.35 | -4.0 | 9.35 |
| 816 |  | -. 05 | . 41 | 6.65 | -1.2 | 7.07 |
| 817 |  | -. 19 | . 41 | 6.65 | -1.2 | 7.85 |
| 818 | B8III 1 | $-.10$ | .44 | 6.16 | -1.0 | 7.16 |
| 819 | B5V 1 | -. 1.14 | . 49 | 7.00 | -1.0 | 8.00 |
| 820 | B8Ib 3 | -. 06 | . 80 | 5.31 | -5.6 | 10.91 |
| 821 |  | -. 12 | .41 | 6.63 | 0.0 | 6.63 |
| 822 |  | . 00 | . 40 | 6.83 | +1.0 | 5.83 |
| 823 |  | -. 04 | . 02 | 5.81 | +0.8 | 5.01 |
| 824 |  | -. 14 | . 33 | 7.86 | -0.4 | 8.26 |
| 825 |  | -. 11 | . 33 | 7.62 | +0.1 | 7.52 |
| 826 | B2Ve 9 | -. 21 | . 06 | 4.67 | -2.5 | 7.17 |
| 827 | B3e 7 | -. 18 | . 56 | 6.40 | -1.0 | 7.40 |
| 828 |  | -. 04 | . 12 | 7.24 | +0.8 | 6.44 |
| 829 |  | -. 09 | . 09 | 7.71 | +0.4 | 7.31 |
| 830 |  | -. 02 | .23 | 8.41 | +1.0 | 7.41 |
| 831 |  | -.03 | . 20 | 7.82 | +0.9 | 6.92 |
| 832 | B9V 1 | -. 02 | . 31 | 7.81 | +0. 5 | 7.31 |

(b) Calibrated from observed spectral types and SchmidtKaler (1965)

| 800 | B8V | -.12 | .03 | 4.87 | 0.0 | 4.87 |
| :--- | :--- | :--- | :--- | ---: | ---: | ---: |
| 801 | B2III 5 | -.24 | .61 | 5.98 | -3.6 | 9.58 |
| 802 | B8Ib 3 | -.06 | .43 | 4.33 | -5.6 | 9.93 |

Table 15.--Continued

| 803 | BIITI 3 | -. 27 | . 60 | 5.30 | -4.4 | 9.70 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 804 | BO.5III 3 | -. 28 | . 66 | 6.34 | -4.7 | 11.04 |
| 805 | B6Ia 3 | -. 06 | . 42 | 6.31 | -7.0 | 13.31 |
| 806 | B2III 3 | -. 24 | . 62 | 6.14 | -3.6 | 9.74 |
| 807 | B9V | -. 08 | . 42 | 7.53 | +0.5 | 7.03 |
| 808 | B9Ia 3 | . 00 | . 59 | 5.38 | -7.0 | 12.38 |
| 809 | B6V | -. 16 | . 22 | 7.1 .0 | -0.7 | 7.80 |
| 810 | B9V | -. 08 | . 50 | 7.60 | +0.5 | 7.10 |
| 811 | B3e 4 | -. 22 | . 29 | 7.00 | -1.7 | 8.70 |
| 812 | B6V | -. 16 | . 65 | 6.28 | -0.7 | 6.98 |
| 813 | AOV | -. 02 | . 46 | 6.82 | +1.0 | 5.82 |
| 814 | B8V | -. 12 | . 15 | 7.13 | 0.0 | 7.13 |
| 815 | B7II 8 | -. 12 | . 84 | 5.25 | $-4.0$ | 9.35 |
| 816 | AOV | -. 02 | . 13 | 7.96 | $+1.0$ | 6.96 |
| 817 | B2V | -. 26 | . 48 | 6.44 | -2.5 | 8.94 |
| 818 | B8III 1 | -. 12 | . 46 | 6.10 | -1.0 | 7.10 |
| 81.9 | B5V 1 | -. 18 | . 53 | 6.88 | -1.0 | 7.88 |
| 820 | B8Ib 3 | -. 06 | . 80 | 5.31 | -5.6 | 10.91 |
| 821 | B8V | -. 12 | . 41 | 6.63 | 0.0 | 6.63 |
| 822 | AOV | -. 02 | . 42 | 6.77 | $+1.0$ | 5.77 |
| 823 | B9V | -. 08 | . 06 | 5.69 | +0.5 | 5.19 |
| 824 | B8V | -. 12 | . 31 | 7.92 | 0.0 | 7.92 |
| 825 | B7V | -. 14 | .36 | 7.53 | -0.4 | 7.93 |
| 826 | B2Ve 9 | -. 26 | . $11{ }^{\text { }}$ | 4.52 | -2.5 | 7.02 |
| 827 | B3e 7 | -. 22 | . 60 | 6.28 | -1.7 | 7.98 |
| 828 | B9V | -. 08 | . 16 | 7.12 | +0.5 | 6.62 |
| 829 | B8V | -. 12 | . 12 | 7.62 | 0.0 | 7.62 |
| 830 | AOV | -. 02 | . 23 | 8.41 | +1.0 | 7.41 |
| 831 | A2V | $+.05$ | . 12 | 8.06 | +1.6 | 6.46 |
| 832 | B9V 1. | -. 08 | .37 | 7.63 | +0.5 | 7.13 |

References: 1--Racine 1968b; 2--Bonneau 1967; 3-Morgan, Code, and Whitford 1956; 4-Mendoza 1958; 5-Guetter 1968; 6--Wilson 1953; 7--Merrill and Burwell 1949; 8--Hardorp et al. 1959; 9--Blaauw 1956.


Figure 22. Distribution of Cas B stars with distance - (a) Calibrated from observed colors and Johnson's (1958) nomogram. (b) Calibrated from observed spectral types and Schmidt-Kaler (1965).

Figure 22 clearly indicates a concentration of $B$ stars in the volume of space outlined by the vertical dashed lines in the figure. The question of whether or not these stars are physically related to one another with a common origin, history, and motion in space can only be answered by investigating the motion of each star.

Figure 23 shows the distribution of radial velocities among the B stars studied. The velocities with respect to the Local Standard of Rest have been plotted versus the distance moduli listed in Table l5(b) and shown in Figure 22(b). Also shown is the velocity-distance relation for $\ell^{I I}=133^{\circ}$ based on the circular orbit model of galactic rotation (Schmidt, 1965). In general, the stars seem to follow the velocity-distance relation with only a few exceptions, as expected. Star 816 (HD16907) has a radial velocity considerably larger than the surrounding stars. It could be either a binary, a runaway star, or not a physical associate of the other $B$ stars around SU Cas. Allowing for errors in the velocities of between 5 and 10 km/sec, Figure 23 merely shows that the dispersion in velocities for the $B$ stars around $S U C$ C does not rule out the existence of a physical association, nor does it suggest that such an association exists. A similar analysis may be carried out for the proper motions of the stars in question, and the same conclusions follow concerning the existence or nonexistence of a physical association.


Figure 23. Radial velocities with respect to the LSR

The color-magnitude diagram for all of the $B$ stars within the above distance limits is presented in Figure 24, where a segment of the zero age main sequence for the distance modulus of $S U$ Cas $\left(V_{0}-M_{v}=7.5\right)$ has been included for reference. The stars form what appears to be a fairly well-defined sequence between $v_{0}=6.2$ and $v_{0}=8.4$ with the ZAMS situated on the leftmost envelope. Furthermore, there is a suggestion of a turn off from the ZAMS near $V_{0}=7.0$ which indicates that the relatively young star 826 may not be physically associated with the other B stars. The position of star 827 is fairly uncertain since an accurate luminosity classification of the star is prevented by emission filled lines. Star 816 has also been included and identified in Figure 24.

The position of SU Cas in Figure 24, above and to the right of the other $B$ stars, is an argument in favor of its having evolved from the position of a normal $B$ star near the top of the existing sequence. The motions of SU Cas as seen in Figure 23 are not unlike those of the surrounding $B$ stars, which have been shown by all available evidence to be located in the same volume of space as the cepheid. To further test the possibility that the cepheid may be a physical associate of the above $B$ stars, the more complete color-magnitude diagram of Figure 25 has been drawn to include Iben's (1965, 1966a) evolutionary tracks for stars of 3 and $5 M_{\odot}$ as diagrammed by Kraft (1966).


Figure 24. Cas B stars: color-magnitude diagram


Figure 25. Color-magnitude diagram with $3 M_{0}$ and $5 M_{0}$ evolutionary tracks

In Figure 25, the position of $S U$ Cas at the lower end of the instability strip in relation to the two evolutionary tracks of 3 and $5 M_{\odot}$ stars indicates that the cepheid has most likely evolved from a $B$ star of between 4 and $5 M_{0}$. In this mass range a star's evolutionary track would carry it within the instability strip on the leftmost portion of its core helium burning loop. The position of its origin on the ZAMS would then correspond fairly well to the point in Figure 25 where the $B$ star sequence departs from the ZAMS. This correspondence argues quite strongly in support of the conclusion that the cepheid is similar in age to those $B$ stars around it that are still near the main sequence in their evolution.

## Discussion'

The present observations of selected B stars surrounding the cepheid SU Cas have indicated that a number of these stars coexist in the same volume of space as the cepheid. It is not clear, at present, whether this concentration of $B$ stars between 200 and 500 pc can be considered as the late evolutionary stage of an evolved $O B$ association or as a result of their common existence as members of a spiral arm. The observations indicate a definite confinement of the stars to the above distance range, but no attempt has been made to investigate their
possible confinement to a restricted range of galactic longitudes.

The present sample of $B$ stars may be interpreted as a group of rather old stars of which $S U C$ C is a member. The cepheid is not unlike the surrounding $B$ stars in most of its past history with the exception that, being slightly more massive, it has undergone its evolution from the main sequence into the cepheid stage more rapidly than other members of the group. Undoubtedly more massive stars of the group have previously undergone the same evolution but are no longer identifiable as $B$ stars or as cepheids. Undoubtedly, also, some of the present late B stars will populate the lower end of the instability strip in the future, depending, of course, on their exact evolutionary paths.

Some objections, however, may be raised against the existence of an old $O B$ association. Ordinary $O B$ associations are not gravitationally bounded and the member stars are recognized as partaking in a general expansion from the group. With ages not much older than $10^{7}$ years and expansion velocities around $10 \mathrm{~km} / \mathrm{sec}$, the member stars remain within a 100 to 200 pc sized volume of space and are identifiable as association members. Only the late B stars will remain near the main sequence when the age of an association exceeds $10^{8}$ years. In order to remain in such a confined volume of space for $10^{8}$ years, however,
these late $B$ stars would have to have random space motions on the order of only $2 \mathrm{~km} / \mathrm{sec}$ or less. The observed radial velocities of the Cas B stars, however, do not show a sufficiently small enough dispersion to have kept them in the same volume of space for so long a time.

These objections, of course, must ultimately be tested by detailed calculations of the orbits and past history of the $B$ stars in question. It is expected from the Lin Theory that the $B$ stars will remain in spiral arms much longer than recently accepted. Consequently, calculations such as those done by Lin, Yuan, and Shu (1969) on the migration of $B$ stars must also be done in this case to determine if $S U C$ as and its surrounding $B$ stars originated together.

At this s.tage of the analysis SU Cas can be considered as an ind ependent member of the Orion arm, having evolved from a point on the main sequence near the upper limit of those present $B$ stars observed. Moreover, the appearance of the reflection nebula is suggested as a coincidental meeting of the cepheid and a denser portion of the interstellar dust cloud in the region. The fact that other such encounters occur within the area and also appear as reflection nebulae, places no special uniqueness on this particular encounter of nebula and cepheid.

## SV VULPECULAE

## Introduction

The classical cepheid $S V$ Vul, with a period of 45.1. days, is the longest period galactic cepheid presently observable from the northern hemisphere. It is also sufficiently bright to be accessible for adequate observational investigation. Not only have its light and color variations been studied photometrically (Eggen 195l, Fernie, Demers, and Marlborough 1965), but also spectroscopic work has been carried out in sufficient detail to provide complete coverage of its variations in velocity (Sanford 1956), in CaII emission (Kraft 1957), and in several line profiles at isolated phases (Kraft et al. 1959). In these respects, it is not the only cepheid so studied, but its position as the longest period cepheid examined makes it unique and extremely important as a basis for interpreting the trend of the observations from the short into the long period cepheids.

Table 16 separates the presently known information on SV Vul into two parts: (1) for that data directly observed both photometrically and spectroscopically and (2) for that information directly obtainable from a knowledge of the cepheid's complete light curve and the adopted period-color-luminosity relation of Fernie (1967).

Table 16. Photometric properties of SV Vul
(a) Observed data

|  | V | $\mathrm{B}-\mathrm{V}$ | $\mathrm{U}-\mathrm{B}$ | Sp T |
| :--- | :---: | :---: | :---: | :---: |
| Max. | 6.72 | +1.12 | +0.82 | F7Iab |
| Min. | 7.78 | +1.78 | +1.62 | KOIab |
| Amplitude | 1.06 | .66 | .80 |  |

(b) Computed data


| $\mathrm{E}_{\mathrm{B}-\mathrm{V}}$ | 0.58 |
| :--- | :---: |
| $\left\langle\mathrm{~V}^{\circ}\right\rangle$ | 5.48 |
| $\left\langle\mathrm{~V}^{\circ}-\mathrm{M}_{\mathrm{V}}^{\circ}\right\rangle$ | 11.61 |
| d | 2100 pc |

The average $V$ magnitude is obtained from the usual intensity averaging procedure while the mean colors are obtained from a straight average over the color light curves expressed in magnitudes.

As one of the longest period cepheids in the Galaxy, SV Vul is an excellent candidate for membership in an $O B$ association, similar to that found around RS Pup. In addition, its apparent brightness makes it more practical to detect surrounding fainter stars, with which it could be physically associated. Located at $\ell^{I I}=63: 9, b^{I I}=+0: 3$ and with a distance of 2.1 kpc according to the periodluminosity relation of Fernie (1967), the cepheid is situated in a region of space considered to be part of the Cygnus arm of the Galaxy. As such, the region is heavily populated by the young stars which make up a spiral arm. Several $O B$ associations have becn noted in this direction (Ruprecht 1966) but only the brightest possible members have been adequately observed and there is still considerable doubt concerning membership or even the existence of some of the suggested groupings. In general, the observational material available is too sparse for an adequate analysis of the spatial distribution of all but the brightest of the proposed association members.

The aim of this chapter, therefore, is to analyze the observational material, both old and new, on the earlytype stars in the direction of $S V$ Vul in order to determine
whether or not the cepheid $c$ an be considered a member of a physical star grouping.

The Early-Type Stars Near SV Vul
The early $O B$ surveys of Morgan, Whitford, and Code (1953) and of Hiltner (1957) provided the identification, UBV photometry, and spectral types of the most luminous early-type stars in the northern Milky Way including those in the direction of $S V$ Vul identified in Table 17. The spectral types of these stars have been used in conjunction with Schmidt-Kaler's (1965) calibration to determine their intrinsic colors, $(B-V)_{o}$, and absolute magnitudes. Combining the observed and calibrated photometric parameters with a correction for extinction of $-3 \times E_{B-V}$ leads to the distance moduli and distances of Table 17.

A search for much fainter $O B$ stars was later carried out jointly by the Hamburg and the Warner and Swasey Observatories (Stock, Nassau, and Stephenson 1960) with an approximate completeness down to the 12 photographic magnitude. This survey supplemented the earlier work in that many more distant stars were sampled in addition to closer less luminous early B stars. The survey, however, only gives a rough indication of the luminosities of the early-type stars and is primarily a finding list for these stars. To learn more about the distribution of the less luminous $B$ stars in the direction

Table 17. Derived data for Morgan, Whitford, and Code (1953) OB stars near SV Vul

| $\begin{aligned} & \text { Hiltner } \\ & \text { (1957) } \\ & \text { No. } \end{aligned}$ | $\ell^{\text {II }}$ | $(B-V)_{0}$ | $E_{B-V}$ | ${ }^{\text {A }} \mathrm{V}$ | $\mathrm{V}_{0}$ | $\mathrm{M}_{\mathrm{V}}$ | $\left(\mathrm{V}_{0}-\mathrm{M}_{\mathrm{v}}\right)$ | $r(k p c)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 806 | 60.8 | -. 24 | . 41 | 1.23 | 6.82 | -3.6 | 10.42 | 1.21 |
| 807 | 59.1 | -. 05 | . 76 | 2.28 | 5.92 | -5.5 | 11.42 | 1.92 |
| 808 | 69.0 | -. 26 | . 15 | . 45 | 9.71 | -2.5 | 12.21 | 2.77 |
| 809 | 53.6 | -. 29 | . 51 | 1.53 | 5.92 | -3.9 | 9.82 | . 92 |
| 810 | 60.9 | -. 24 | . 94 | 2.82 | 7.39 | -2.1 | 9.49 | . 79 |
| 811 | 59.7 | -. 06 | 1.05 | 3.15 | 6.13 | -5.6 | 11.73 | 2.22 |
| 812 | 56.7 | -. 20 | . 59 | 1.77 | 4.72 | -6.4 | 11.12 | 1.68 |
| 813 | 64.2 | -. 20 p | . 40 | 1.20 | 7.30 | -5.7 | 13.00 | 3.98 |
| 814 | 64.2 | -. 25 | . 26 | . 78 | 8.15 | -3.1 | 11.25 | 1.78 |
| 815 | 59.5 | -. 22 | . 88 | 2.64 | 6.09 | -5.8 | 11.89 | 2.39 |
| 816 | 59.4 | -. 30 | . 73 | 2.19 | 7.56 | -4.8 | 12.36 | 2.97 |
| 817 | 59.5 | -. 27 | . 88 | 2.64 | 7.73 | -4.4 | 12.13 | 2.67 |
| 818 | 59.4 | -. 33 | . 89 | 2.67 | 6.67 | -5.2 | 11.87 | 2.37 |
| 819 | 62.1 | -. 27 | . 97 | 2.91 | 7.50 | -3.0 | 10.50 | 1.26 |
| 820 | 60.3 | -. 28 | 1.05 | 3.15 | 5.62 | -5.4 | 11.02 | 1.60 |
| 821 | 60.6 | -. 28 : | . 94 | 2.82 | 6.01 | -5.4 | 11.41 | 1.91 |
| 822 | 60.2 | -. 02 | . 95 | 2.85 | 4.18 | -7.2 | 11.38 | 1.89 |
| 823 | 60.3 | -. 28 : | 1.04 | 3.12 | 6.17 | -4.7 | 10.87 | 1.49 |
| 824 | 61.5 | -. 32 | . 95 | 2.85 | 7.30 | -5.0 | 12.30 | 2.88 |
| 825 | 61.2 | -. 30 f | 1.48 | 4.44 | 5.13 | -5.0 | 10.13 | 1.06 |
| 826 | 59.6 | -. 24 : | . 57 | 1.71 | 7.90 | -3.6 | 11.50 | 2.00 |
| 827 | 60.4 | -. 18 | . 95 | 2.85 | 5.00 | -6.6 | 11.60 | 2.09 |
| 828 | 64.0 | WN | - | . 8 | 5.00 | WN | 11. | 2.0) |
| 829 | 67.3 | -. 32 | . 40 | 1.20 | 6.28 | -5.1 | 11.38 | 1.89 |
| 830 | 64.1 | -. 32 | . 52 | 1.56 | 7.21 | -5.1 | 12.31 | 2.90 |
| 831 | 61.2 | -. 30 f | . 99 | 2.97 | 6.15 | -5.4 | 11.55 | 2.04 |
| 832 | 63.9 | -. 02 | . 41 | 1.23 | 7.08 | -7.5 | 14.58 | 8.24 |
| 833 | 69.5 | -. 20 : | . 16 | . 48 | 10.11 | -4.6 | 14.71 | 8.75 |

Table 17.-- Continued Derived data for Morgan, Whitford, and Code (1953) ob stars

| $\begin{aligned} & \text { Hiltner } \\ & \text { (1957) } \\ & \text { No. } \end{aligned}$ | $\ell^{\text {II }}$ | $(B-V)_{0}$ | $\mathrm{E}_{\mathrm{B}-\mathrm{V}}$ | ${ }^{\text {A }} \mathrm{V}$ | V | $\mathrm{M}_{\mathrm{v}}$ | $\left(\mathrm{V}_{\mathrm{o} .}-\mathrm{M}_{v}\right)$ | $r(k p c)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 834 | 55.7 | WR | -- | -- | -- | WR | -- | -- |
| 835 | 61.4 | -. 27 | . 67 | 2.01 | 7.62 | -3.0 | 10.62 | 1.33 |
| . 836 | 64.8 | -. 26 : | . 36 | 1.08 | 8.32 | -2.5 | 10.82 | 1.46 |
| 837 | 68.8 | -. 22 | . 42 | 1.26 | 5.18 | -5.8 | 10.98 | 1.57 |
| 838 | 68.4 | -. 28 : e | . 38 | 1.14 | 7.94 | -3.6 | 11.54 | 2.03 |
| 839 | 65.5 | -. 30 f | . 80 | 2.40 | 7.70 | -5.2 | 12.90 | 3.80 |
| 840 | 60.0 | -. 26 | . 75 | 2.25 | 6.95 | -5.0 | 11.95 | 2.46 |
| 841 | 65.5 | -. 18 | . 74 | 2.22 | 7.71 | -5.7 | 13.41 | 4.81 |
| 842 | 69.1 | -. 20 | . 75 | 2.25 | 7.62 | -5.7 | 13.32 | 4.61 |
| 843 | 64.1 | -. 26 : | . 39 | 1.17 | 6.57 | -2.5 | 9.07 | . 65 |
| 844 | 61.9 | -. 03 | . 74 | 2.22 | 3.36 | -6.6 | 9.96 | . 98 |
| 845 | 59.7 | -. 26 e | . 60 | 1.80 | 8.05 | -2.5 | 10.55 | 1.29 |
| 846 | 60.1 | -. 18 | . 41 | 1.23 | 8.35 | -2.2 | 10.55 | 1.29 |
| 847 | 65.0 | . 00 | . 98 | 2.94 | 6.00 | -7.0 | 13.00 | 3.98 |
| 848 | 66.7 | -. 30 | . 75 | 2.25 | 7.93 | -4.2 | 12.13 | 2.67 |
| 849 | 71.3 | -. 23 | 1.08 | 3.24 | 5.65 | -5.8 | 11.45 | 1.95 |
| 850 | 72.2 | -. 28 | . 44 | I. 32 | 7.80 | -4.7 | 12.50 | 3.16 |
| 851 | 71.5 | -. 33 | . 71 | 2.13 | 6.86 | -5.2 | 12.06 | 2.58 |
| 852 | 67.1 | -. 24 | .34 | 1.02 | 7.20 | -3.6 | 10.80 | 1.45 |
| 853 | 67.5 | -. 28 : | . 77 | 2.31 | 7.80 | -3.6 | 11.40 | 1.91 |
| 854 | 60.8 | -. 19 | . 37 | 1.11 | 5.37 | -6.2 | 11.57 | 2.06 |
| 855 | 72.1 | -. 33 | . 95 | 2.85 | 6.89 | -5.2 | 12.09 | 2.62 |
| 856 | 73.3 | -. 30 | . 53 | 1.59 | 9.00 | -4.8 | 13.80 | 5.75 |
| 857 | 66.4 | -. 29 : | 1.31 | 3.93 | 5.50 | -5.0 | 10.50 | 1.26 |
| 858 | 64.5 | -. 28 | . 91 | 2.73 | 6.21 | -4.4 | 10.61 | 1.32 |
| 859 | 72.9 | -.16 : | . 33 | . 99 | 7.22 | -4.4 | 11.62 | 2.11 |
| 860 | 69.4 | -. 20 | . 66 | 1.98 | 6.72 | -5.7 | 12.42 | 3.05 |

Table 17.- $\frac{-C o n t i n u e d}{\text { near } S V V u I}$ Derived data for Morgan, Whitford, and Code (1953) ob stars

| $\begin{aligned} & \text { Hiltner } \\ & \text { (1957) } \\ & \text { No. } \end{aligned}$ | $\ell^{I I}$ | $(B-V)_{0}$ | $\mathrm{E}_{\mathrm{B}-\mathrm{V}}$ | ${ }^{\text {A }} \mathrm{V}$ | $V_{0}$ | $\mathrm{M}_{\mathrm{v}}$ | $\left(\mathrm{V}_{0}-\mathrm{M}_{\mathrm{v}}\right)$ | $r(k p c)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 861 | 70.7 | -. 33 : | . 80 | 2.40 | 7.88 | -5.2 | 13.08 | 4.13 |
| 862 | 69.5 | -. 16 | . 69 | 2.07 | 3.58 | -6.8 | 10.38 | 1.19 |
| 863 | 72.4 | -. 34 | . 54 | 1.62 | 6.14 | -5.4 | 11.54 | 2.03 |
| 864 | 72.7 | -. 30 e | . 65 | 1.95 | 6.87 | -4.2 | 11.07 | 1.64 |
| 865 | 72.6 | -. 29 | . 48 | 1.44 | 7.41 | -3.9 | 11.31 | 1.83 |
| 866 | 72.6 | -. 23 | . 48 | 1.44 | 6.47 | -5.8 | 12.27 | .2.84 |
| 867 | 72.3 | -. 23 | . 64 | 1.92 | 6.12 | -5.4 | 11.52 | 2.01 |
| 868 | 76.2 | -. 30 | . 44 | 1.32 | 6.90 | -4.8 | 11.70 | 2.19 |
| 869 | 73.1 | -. 20 | . 53 | 1.59 | 8.09 | -4.6 | 12.69 | 3.45 |
| 870 | 72.0 | -. 29 | . 63 | 1.89 | 6.71 | $-5.0$ | 11.71 | 2.20 |
| 871 | 73.2 | -. 31 | . 48 | 1.44 | 7.76 | -4.5 | 12.26 | 2.83 |
| 872 | 73.4 | -. 27 | . 65 | 1.95 | 7.94 | -4.4 | 12.34 | 2.94 |
| 873 | 74.8 | -. 27 | . 52 | 1.56 | 6.57 | -5.2 | 11.77 | 2.26 |
| 874 | 78.5 | -. 31 | . 47 | 1.41 | 6.62 | -4.7 | 11.32 | 1.84 |
| 875 | 74.3 | -. 28 | . 54 | 1.62 | 7.63 | -3.6 | 11.23 | 1.76 |
| 876 | 77.4 | -. 24 | . 49 | 1.47 | 8.97 | -3.6 | 12.57 | 3.27 |
| 877 | 72.8 | -- | -- | -- | -- | -- | -- | -- |
| 878 | 73.6 | -. 28 | .65 | 1.95 | 6.64 | -4.7 | 11.34 | 1.85 |
| 879 | 72.8 | -. 29 | .65 | I. 95 | 7.02 | -3.9 | 10.92 | 1.53 |
| 880 | 73.8 | -. 26 | . 73 | 2.19 | 6.63 | -5.0 | 11.63 | 2.12 |
| 881 | 81.0 | -. 23 | . 87 | 2.61 | 6.93 | -5.8 | 12.73 | 3.52 |
| 882 | 74.5 | -. 28 | . 35 | 1.05 | 7.39 | -3.6 | 10.99 | 1.58 |
| 883 | 73.2 | -. 28 | . 56 | 1.68 | 7.16 | -4.0 | 11.16 | 1.71 |
| 884 | 77.8 | -. 32 | . 46 | 1.38 | 6.64 | -5.0 | 11.64 | 2.13 |
| 885 | 73.6 | WR |  | -- | -- | WR | -- | -- |
| 886 | 74.9 | -. 28 | . 44 | 1.32 | 8.10 | -3.6 | 11.70 | 2.19 |
| 887 | 76.8 | -- | -- | - | -- | -- |  | -- |

Table 17.-- Continued $\frac{\text { near SVVul }}{\text { SVived data for Morgan, Whitford, and Code (1953) OB stars }}$

| Hiltner <br> (1957) <br> No. | $\ell^{\text {II }}$ | $(B-V)_{0}$ | $E_{B-V}$ | ${ }^{\text {A }} \mathrm{V}$ | $\mathrm{V}_{0}$ | $\mathrm{M}_{\mathrm{V}}$ | $\left(\mathrm{V}_{\mathrm{O}}-\mathrm{M}_{\mathrm{v}}\right)$ | $r(k p c)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 888 | 77.1 | -. 30 | . 68 | 2.04 | 5.51 | -5.6 | 11.11 | 1.67 |
| 889 | 72.8 | -. 33 | . 82 | 2.46 | 5.93 | -5.2 | 11.13 | 1.68 |
| 890 | 77.7 | -. 28 | . 51 | 1.53 | 8.48 | -3.6 | 12.08 | 2.61 |
| 891 | 75.9 | -. 22 | . 72 | 2.16 | 4.93 | -5.7 | 10.63 | 1.34 |
| 892 | 70.8 | -- | -- | -- | - |  | 10.6 | 1. |
| 893 | 73.2 | -. 22 | . 88 | 2.64 | 7.13 | -1.7 | 8.83 | . 59 |
| 894 | 70.4 | -. 24 | . 37 | 1.11 | 6.18 | -3.6 | 9.78 | . 91 |
| 895 | 74.4 | -. 25 | . 50 | 1.50 | 8.36 | -3.1 | 11.40 | 1.91 |
| 896 | 75.4 | -. 23 | . 63 | 1.89 | 7.58 | -4.8 | 12.38 | 2.99 |
| 897 | 71.6 | -. 26 | . 95 | 2.85 | 7.03 | -2. 5 | 9.53 | . 81 |
| 898 | 74.3 | WR | - | , | 7.0 | WR | 9.53 | -- |
| 899 | 74.9 | -. 30 | . 65 | 1.95 | 5.16 | -5.0 | 10.16 | 1.08 |
| 900 | 77.3 | -. 22 | . 85 | 2.55 | 4.99 | -6.2 | 11.19 | 1.73 |
| 901. | 78.6 | -. 27 | 1.10 | 3.30 | 6.90 | -5.2 | 12.15 | 2.63 |
| 902 | 77.1 | --- |  |  | -- | -- | -- |  |
| 903 | 74.4 | -. 26 | 1.06 | 3.18 | 6.89 | -5.0 | 11.89 | 2.39 |
| 904 | 74.6 | -. 27 | . 84 | 2.52 | 7.32 | -4.4 | 11.72 | 2.21 |
| 905 | 79.1 | -. 08 | . 97 | 2.91 | 5.71 | -7.0 | 12.71 | 3.48 |
| 906 | 73.2 | -. 20 | - 90 | 2.70 | 7.26 | -4.6 | 11.86 | 2.36 |
| 907 | 75.5 | -. 29 | . 56 | 1.68 | 7.57 | -3.9 | 11.47 | 1.97 |
| 908 | 75.3 | -. 30 | . 68 | 2.04 | 6.95 | -4.2 | 11.05 | 1.62 |
| 909 | 76.4 | -. 29 | . 64 | 1.92 | 6.40 | -5.0 | 11.40 | 1.91 |
| 910 | 78.0 | -. 20 | 1.33 | 3.99 | 4.66 | -6.4 | 11.06 | 1.63 |

of $S V$ Vul, therefore, requires a more thorough observational investigation of the stars.

All stars in The Luminous Stars in the Northern
Milky Way II within $3^{\circ}$ of $S V$ Vul and with objective prism spectral classifications of $O B, \mathrm{OB}^{-}$, or OBce were selected for UBV photoelectric observation. Observations were made for each star on at least three different nights using the Steward Observatory 36 -inch telescope and the reduction procedure previously outlined under SU Cassiopeae. The repeatability of the measures from night to night and the solutions based on the UBV standard stars indicated that the probable errors of measurement were $\pm 0.04$ in the $V$ magnitude and $\pm 0.03$ in the colors $B-V$ and U-B.

Table 18 displays the photometric results for the stars observed. The intrinsic colors, $(B-V)_{o}$, have been determined from the observed $B-V$ and $U-B$ colors with the aid of Johnson's (1958) nomogram for main sequence stars earlier than AO. The color excesses, $E_{B-V}$, were obtained simultaneously, and the unreddened visual magnitudes, $V_{0}$, were derived by correcting the observed magnitudes, $V$, with $-3 \times \mathrm{E}_{\mathrm{B}-\mathrm{V}}$.

It is evident from the two-color diagram of Figure 26 that most of the stars are highly reddened early $B$ stars. The large range of reddening in the area is due to the combined effects of a large spread in distance and the spatial variation in the distribution in the absorbing

Table 18. Photometric data for $O B$ stars near SV Vul

| Case No. | Program No. | $e^{\text {II }}$ | $\mathrm{b}^{\text {II }}$ | $r(k p c)$ | V | B-V | U-B | $(\mathrm{B}-\mathrm{V}){ }_{0}$ | $\mathrm{E}_{\mathrm{B}-\mathrm{V}}$ | $\mathrm{V}_{0}$ | $\mathrm{M}_{\mathrm{v}}$ | $\mathrm{V}_{\mathrm{o}} \mathrm{MM}_{\mathrm{v}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25-3 | 500 | 60.5 | +1.5 | -- | -- | -- | -- | -- | -- | -- | -- | -- |
| 25-5 | 501 | 61.0 | +1.1 |  | -- | -- |  | -- | -- |  |  |  |
| 25-9 | 502 | 61.4 | 0.0 | 2.44 | 11.52 | . 87 | -. 20 | -. 29 | 1.16 | 8.04 | -3.9 | 11.94 |
| 25-11 | 503 | 61.7 | -0.1 | 2.22 | 11.16 | . 73 | -. 29 | -. 28 | 1.01 | 8.13 | -3.6 | 11.73 |
| 25-13 | 504 | 62.4 | 0.0 | 1.84 | 10.79 | 1.17 | -0.7 | -. 32 | 1.49 | 6.32 | -5.0 | 11.32 |
| 25-14 | 505 | 62.6 | -0.1 | 1.93 | 11.25 | . 95 | -. 13 | -. 29 | 1.24 | 7.53 | -3.9 | 11.43 |
| 25-15 | 506 | 62.7 | -0.3 | 1.08 | 12.12 | . 59 | $+.03$ | -. 13 | . 72 | 9.96 | -0.2 | 10.16 |
| 25-16 | 507 | 62.7 | -0.3 | 1.49 | 11.42 | . 70 | -. 23 | -. 25 | . 95 | 8.57 | -2.3 | . 10.87 |
| 25-18 | 508 | 62.7 | -2.1 | 3.40 | 10.91 | . 64 | -. 43 | -. 31 | . 95 | 8.06 | -4.6 | 12.66 |
| 25-19 | 509 | 63.2 | -1.8 | 2.86 | 12.14 | . 47 | -. 40 | -. 25 | . 72 | 9.98 | -2.3 | 12.28 |
| 25-20 | 510 | 62.9 | -2.3 |  |  |  |  | -- |  |  |  | -- |
| 26-2 | 511 | 61.2 | $+3.5$ | 1.61 | 11.96 | . 52 | -. 18 | -. 19 | . 71 | 9.83 | -1.2 | 11.03 |
| 26-4 | 512 | 62.3 | +1.5 | 1.72 | 11.51 | . 84 | -. 18 | -. 27 | 1.11 | 8.18 | -3.0 | 11.18 |
| 26-5 | 513 | 62.3 | +1.5 | 1.37 | 11.63 | . 83 | -. 13 | -. 25 | 1.08 | 8.39 | -2.3 | 10.69 |
| 26-6 | 514 | 63.4 | +0.3 | 2.32 | 11.41 | . 78 | -. 25 | -. 28 | 2.06 | 8.23 | -3.6 | 11.83 |
| 26-7 | 515 | 62.9 | 0.0 | -- | -- | -- | -- | -- | -- | -- |  | -- |
| 26-8 | 516 | 63.4 | +0. 3 | 4.06 | 12.14 | . 71 | -. 31 | -. 29 | 1.00 | 9.14 | -3.9 | 13.04 |
| 26-9 | 517 | 62.7 | -0. 3 | . 93 | 12.14 | . 70 | +.13 | -. 13 | . 83 | 9.65 | -0.2 | 9.85 |
| 26-10 | 518 | 63.8 | -0.5 | . 70 | 11.37 | . 79 | +.09 | -. 16 | . 95 | 8.52 | -0.7 | 9.22 |
| 26-11 | 519 | 63.9 | -1.0 | 2.11 | 11.50 | . 88 | -. 16 | -. 28 | 1.16 | 8.02 | -3.6 | 11.62 |
| 27-8 | 520 | 63.3 | +1.9 | 2.05 | 11.31 | . 49 | -. 42 | -. 26 | . 75 | 9.06 | -2.5 | 11.56 |
| 27-9 | 521 | 63.6 | +1.8 | 2.25 | 11.66 | . 37 | -. 42 | -. 23 | . 60 | 9.86 | -1.9 | 11.76 |
| 27-11 | 522 | 63.6 | +1.0 | 1.17 | 11.84 | . 71 | -. 05 | -. 19 | . 90 | 9.14 | -1.2 | 10.34 |
| 27-12 | 523 | 64.0 | +1.0 | 1.21 | 11.31 | . 76 | -. 14 | -. 24 | 1.00 | 8.31 | -2.1 | 10.41 |
| 27-1.3 | 524 | 64.0 | $+1.0$ | 1.90 | 11.81 | . 35 | -. 33 | -. 19 | . 54 | 10.19 | -1.2 | 11.39 |
| 27-19 | 525 | 64.1 | +0.3 | -- | -- | -- |  |  |  |  |  |  |
| 27-21 | 526 | 64.6 | +0. 5 | 2.52 | 11.11 | . 71 | -. 31 | -. 29 | 1.00 | 8.11 | -3.9 | 12.01 |
| 27-22 | 527 | 64.6 | +0. 5 | 1.09 | 9.94 | . 84 | -. 21 | -. 28 | 1.12 | 6.58 | -3.6 | 10.18 |
| 27-23 | 528 | 64.2 | 0.0 | 1.29 | 11.63 | . 57 | -. 17 | -. 19 | .76 | 9.35 | -1.2 | 10.55 |

Table 18.--Continued Photometric data for $O B$ stars near SV Vul

| Case P <br> No. | Program No. | $2^{\text {II }}$ | $\mathrm{b}^{\text {II }}$ | $r(k p c)$ | V | B-V | U-B | $(B-V)_{0}$ | $E_{B-V}$ | $\mathrm{V}_{0}$ | $\mathrm{M}_{v}$ | $\mathrm{V}_{\mathrm{o}} \mathrm{MM}_{\mathrm{V}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 27-24 | 529 | 64.2 | 0.0 | 1.12 | 11.33 | . 57 | -. 17 | -. 19 | . 76 | 9.05 | -1.2 | 10.25 |
| 27-26 | 530 | 64.4 | -0.1 | . 43 | 10.04 | . 44 | +. 01 | -. 10 | . 54 | 8.42 | +0.3 | 8.17 |
| 27-27 | 531 | 64.4 | -0.1 |  |  | -- | -- | -- | -- |  | -- |  |
| 27-28 | 532 | 64.0 | -0.3 | 1.38 | 11.42 | . 70 | -. 19 | -. 24 | . 94 | 8.60 | -2.1 | 10.70 |
| 27-29 | 533 | 65.4 | -0.7 | . 92 | 10.97 | . 73 | -. 11 | -. 22 | . 95 | 8.12 | -1.7 | 9.82 |
| 27-30 | 534 | 65.4 | -0.7 | 1.33 | 11.77 | . 73 | -. 10 | -. 22 | . 95 | 8.92 | -1.7 | 10.67 |
| 27-31 | 535 | 64.9 | -1.9 |  |  |  | -- | -- | -- |  |  | -- |
| 28-8 | 536 | 64.6 | +1.8 | 2.57 | 11.40 | . 30 | -. 52 | -. 25 | . 55 | 9.75 | -2.3 | 12.05 |
| 28-11 | 537 | 64.4 | +0.9 | 1.83 | 11.50 | . 41 | -. 37 | -. 22 | . 63 | 9.61 | -1.7 | 11.31 |
| 28-12 | 538 | 64.4 | +0.9 | 2.84 | 10.83 | . 44 | -. 51 | -. 28 | . 72 | 8.67 | -3.6 | 12.27 |
| 28-13 | 539 | 65.0 | +0. 8 | 5.06 | 11.45 | . 23 | -. 66 | -. 28 | . 51 | 9.92 | -3.6 | 13.52 |
| 28-16 | 540 | 65.9 | +0.4 | 1.43 | 11.72 | . 83 | -. 14 | -. 25 | .1.08 | 8.48 | -2.3 | 10.78 |
| 28-17 | 541 | 65.5 | 0.0 | 1.28 | 11.65 | . 66 | -. 13 | -. 21 | . 87 | 9.04 | -1.5 | 10.54 |
| 28-18 | 542 | 66.1 | 0.0 | 1.49 | 12.04 | . 68 | -. 12 | -. 21 | . 89 | 9.37 | -1. 5 | 10.87 |
| 28-19 | 543 | 66.1 | 0.0 | 1.09 | 10.37 | . 79 | -. 22 | -. 27 | 1.06 | 7.19 | -3.0 | 10.19 |
| 28-22 | 544 | 66.4 | -1.4 | 1.63 | 11.09 | . 93 | -. 12 | -. 28 | 1.21 | 7.46 | -3.6 | 11.06 |
| 29-8 | 545 | 64.4 | $+5.0$ | 2.17 | 10.22 | . 03 | -. 73 | -. 25 | . 28 | 9.38 | -2.3 | 11.68 |
| 29-13 | 546 | 65.4 | +2.1 | 1.96 | 11.64 | .27 | -. 39 | -. 19 | . 46 | 10.26 | -1.2 | 11.46 |
| 29-16 | 547 | 65.9 | +1.9 |  |  |  |  |  |  |  |  |  |
| 29-20 | 548 | 66.2 | +1.5 | 2.19 | 12.10 | . 48 | -. 31 | -. 22 | . 70 | 10.00 | -1. 7 | 11.70 |
| 29-22 | 549 | 65.8 | +1.0 | . 79 | 11.14 | . 58 | -. 04 | -. 15 | . 73 | 8.95 | -0.6 | 9.50 |
| 29-24 | 550 | 66.2 | +0.7 | 2.49 | 11.44 | . 28 | -. 50 | -. 24 | . 52 | 9.88 | -2.1 | 11.98 |
| 29-25 | 551 | 66.3 | +0.6 | 2.44 | 10.95 | . 40 | -. 50 | -. 27 | .67 | 8.94 | -3.0 | 11.94 |
| 29-28 | 552 | 67.0 | +0.5 | 2.27 | 10.88 | . 43 | -. 47 | -. 27 | . 70 | 8.78 | -3.0 | 11.78 |
| 29-29 | 553 | 66.1 | 0.0 | 1.39 | 11.80 | . 71 | -. 12 | -. 22 | . 93 | 9.01 | -1.7 | 10.71 |
| 29-30 | 554 | 66.6 | +0.2 | 1.92 | 11.81 | . 59 | -. 27 | -. 24 | . 83 | 9.32 | -2.1 | 11.42 |
| 29-32 | 555 | 67.0 | -0.3 | 1.04 | 11.94 | . 73 | +. 02 | -. 17 | . 90 | 9.24 | -0.9 | 10.09 |
| 29-33 | 556 | 67.0 | -0.5 | 2.23 | 10.77 | . 90 | -. 23 | -. 31 | 1.21 | 7.14 | -4.6 | 11.74 |
| 29-35 | 557 | 67.4 | -0.7 | 2.16 | 11.80 | . 39 | -. 37 | -. 22 | . 61 | 9.97 | -1.7 | 11.67 |

Table 18.--Continued Photometric data for $O B$ stars near $S V$ Vul

| Case <br> No. | ogram | $\ell^{\text {II }}$ | $\mathrm{b}^{\text {II }}$ | $r(k p c)$ | V | B-V | U-B | $(B-V)_{0}$ | $E_{B-V}$ | $V_{0}$ | $\mathrm{M}_{\mathrm{v}}$ | $\mathrm{V}_{0}-\mathrm{M}_{\mathrm{v}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 29-36 | 558 | 66.9 | -1.3 | 1.45 | 11.63 | . 79 | -. 16 | -. 25 | 1.04 | 8.51 | -2.3 | 10.81 |
| 29-37 | 559 | 66.9 | -1.3 | 3.27 | 11.76 | . 65 | -. 33 | -. 28 | . 93 | $8.97{ }^{\circ}$ | -3.6 | 12.57 |
| 29-38 | 560 | 66.9 | -1. 3 | 1.66 | 11.68 | . 71 | -. 22 | -. 25 | . 96 | 8.80 | -2.3 | 11.10 |
| 29-39 | 561 | 66.9 | -1.3 |  | 10.90 | 1.00 | -. 25 |  | -- | -- | -- | -- |
| 29.40 | 562 | 66.9 | -1.3 | 1.41 | 11.22 | . 73 | -. 21 | -. 26 | . 99 | 8.25 | -2.5 | 10.75 |
| 30-6 | 563 | 66.8 | +0.9 | 1.78 | 10.76 | . 41 | -. 48 | -. 26 | . 67 | 8.75 | -2.5 | 11.25 |
| 30-9 | 564 | 67.6 | +1.1 | 2.14 | 11.47 | . 95 | -. 15 | -. 29 | 1.24 | 7.75 | -3.9 | 11.65 |
| 30-10 | 565 | 67.6 | +0.9 | 1.32 | 11.82 | . 52 | -. 14 | -. 17 | . 69 | 9.75 | -0.9 | 10.60 |
| 30-11 | 566 | 67.2 | +0.6 | . 98 | 10.84 | . 45 | -. 22 | -. 18 | . 63 | 8.95 | -1.0 | 9.95 |
| 30-13 | 567 | 67.4 | +0.5 | 1.69 | 11.36 | . 59 | -. 29 | -. 25 | . 84 | 8.84 | -2.3 | 11.14 |
| 30-14 | 568 | 67.4 | +0.5 | 2.08 | 11.88 | . 50 | -. 31 | -. 23 | . 73 | 9.69 | -1.9 | 11.59 |
| 30-15 | 569 | 67.9 | $+0.3$ | . 49 | 11.43 | 1.10 | +. 30 | -. 17 | . 1.27 | 7.62 | -0.9 | 8.47 |
| 30-17 | 570 | 67.5 | +0.1 | . 77 | 10.80 | . 53 | -. 10 | -. 16 | . 69 | 8.73 | -0.7 | 9.43 |
| 30-19 | 571 | 67.9 | 0.0 | 1.74 | 11.19 | . 57 | -. 34 | -. 26 | . 83 | 8.70 | -2.5 | 11.20 |
| 30-20 | 572 | 67.9 | 0.0 | 3.57 | 10.55 | . 61 | -. 48 | -. 32 | . 93 | 7.76 | -5.0 | 12.76 |
| 30-21 | 573 | 67.6 | -0.1 | . 75 | 10.97 | . 41 | -. 06 | -. 12 | . 53 | 9.38 | 0.0 | 9.38 |
| 30-22 | 574 | 68.2 | 0.0 |  | 11.86 | . 86 |  |  |  |  |  |  |
| 30-23 | 575 | 67.8 | -0.3 | 1.27 | 11.08 | 1.20 | $+.05$ | -. 29 | 1.49 | 6.61 | -3.9 | 10.51 |
| 30-24 | 576 | 67.9 | -0.5 | 1.19 | 11.04 | 1.14 | +. 02 | -. 28 | 1.42 | 6.78 | -3.6 | 10.38 |
| 25-4 | 577 | 60.9 | +1.2 |  |  |  |  |  |  |  |  |  |
| 27-15 | 578 | 63.9 | +0.4 | 3.03 | 10.79 | . 47 | -. 51 | -. 29 | . 76 | 8.51 | -3.9 | 12.41 |
| 29-31 | 579 | 66.1 | +0.3 | 1.11 | 10.44 | . 36 | -. 37 | -. 21 | . 57 | 8.73 | -1.5 | 10.23 |
| 30-8 | 580 | 67.7 | +1.2 |  | 10.72 | 1.19 | -. 15 |  |  |  |  |  |
| 30-12 | 581 | 67.8 | +1.0 | 2.83 | 10.43 | . 49 | -. 52 | -. 30 | .79 | 8.06 | -4.2 | 12.26 |
| 30-18 | 582 | 67.5 | +0.1 | 2.67 | 10.92 | . 52 | -. 45 | -. 28 | . 80 | 8.52 | -3.6 | 12.12 |
| 25-12 | 012 | 62.5 | +0.1 | . 77 | 10.31 | . 87 | -. 13 | -. 26 | 1.13 | 6.92 | -2.5 | 9.42 |
| 25-17 | 013 | 62.7 | -0.3 | 1.82 | 9.92 | .64 | -. 41 | -. 30 | . 94 | 7.10 | -4.2 | 11.30 |
| 28-10 | 020 | 65.0 | +1.4 | 1.20 | 10.12 | . 48 | -. 42 | -. 26 | .74 | 7.90 | -2.5 | 10.40 |

Table 18.--Continued Photometric data for $O B$ stars near $S V$ Vul

| Case Program <br> No. <br> No. | $\ell^{I I}$ | $\mathrm{~b}^{I I}$ | $\mathrm{r}(\mathrm{kpc})$ | V | $\mathrm{B}-\mathrm{V}$ | $\mathrm{U}-\mathrm{B}$ | $(\mathrm{B}-\mathrm{V})_{\mathrm{o}}$ | $\mathrm{E}_{\mathrm{B}}-\mathrm{V}$ | $\mathrm{V}_{\mathrm{o}}$ | $\mathrm{M}_{\mathrm{V}}$ | $\mathrm{V}_{\mathrm{o}}-\mathrm{M}_{\mathrm{V}}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $28-20$ | 023 | 65.3 | -0.6 | 1.97 | 10.39 | .56 | -.41 | -.28 | .84 | 7.87 | -3.6 | 11.47 |
| $30-3$ | 026 | 64.6 | +6.8 | 1.80 | 9.97 | -.03 | -.71 | -.23 | .20 | 9.37 | -1.9 | 11.27 |
| $30-5$ | 028 | 65.5 | +3.9 | 1.46 | 10.40 | .15 | -.54 | -.21 | .36 | 9.32 | -1.5 | 10.82 |
| 29.9 | 037 | 64.5 | +4.3 | 1.29 | 9.16 | -.06 | -.78 | -.23 | .17 | 8.65 | -1.9 | 10.55 |



Figure 26. Two-color diagram for $O B$ stars within $3^{\circ}$ of $S V$ Vul
interstellar dust clouds. The interstellar extinction is quite irregular in the area, as can be seen from the examination of a high contrast photograph of the region. A "splotchy" appearance due to the interstellar dust clouds associated with the Cygnus arm is quite evident. In general, however, there is less obscuration in a direction slightly north of the galactic plane than in a corresponding southerly direction. This can be seen plainly in Figure 27 where the galactic coordinates are plotted accompanied by the respective observed color excesses. A line drawn roughly separating the large color excess values from the small ones coincides very crudely to the apparent boundary between the obscured and unobscured areas in that part of the sky.

The color-magnitude diagram of Figure 28 displays the general properties expected from a sample of early B stars distributed at different distances. In this region of the diagram the main sequence is too nearly vertical to determine which stars define a possible association from among the many stars distributed at different distances.

Slettebak and Stock (1957) have shown that the $O B$ and $\mathrm{OB}^{-}$stars as classified in the above objective prism survey are predominantly Bo to $B 2$ in spectral type with lower luminosity classes. The $O B$ group has more of a tendency for higher luminosity than does the $\mathrm{OB}^{-\quad}$ group.


Figure 27. Distribution of OB stars near SV

$$
-+2^{\circ}
$$



519
-1.16
$--1^{\circ}$

ars near $S V$ Vul including color excesses


Figure 28. Color-Magnitude diagram for $O B$ stars near SV Vul

In the absence of higher dispersion slit spectra for classification of the program stars, therefore, their individual distances were derived under the assumption that they are of luminosity class $V$ with spectral types corresponding to their intrinsic colors, (B-V) o. The absolute magnitudes of Table 18 were then obtained from Schmidt-Kaler's (1965) tables. In this manner a rough idea can be obtained of the relative distribution of the stars in this direction of the Galaxy.

Using the above rudimentary knowledge of the distance moduli, Figure 29 displays the frequency of stars occurring in $0^{m} \cdot 2$ modulus intervals from the sun. Slight increases in frequency can be noted around moduli of 10.6 and $11^{m} \cdot 5$. This is more realistically shown as a projection of the individual stars onto the plane of the galaxy as in Figure 30. In this diagram the more luminous early-type stars of the earlier surveys have also been included. In addition, the youngest clusters of the region have been included in this diagram. These clusters were chosen on the basis of their blue spectral types of B3 or earlier, and their distances were taken from the compilation by Hoag (1965b).

## Discussion

The distribution of the early-type stars and clusters in Figure 30 shows several characteristic


Figure 29. Distribution of Vulpecula $O B$ stars with distance


e 30. Distribution of early type stars, clusters, and associations near SV V

properties that are worth noting. First of all, the most luminous $O B$ stars of the surveys of the early 1950's show tendencies to outline a few groupings. These groupings are the $O B$ associations Vul $O B 1$ and Vul $O B 4$ as designated by Ruprecht (1966). The use here of Schinidt-Kaler's (1965) calibrations has altered the positions of a few of the stars which Ruprecht designated as association members but the apparent groupings have not been dissolved. Table 19 gives the pertinent information for those stars presently included as members of Vul $O B 1$ and Vul $O B 4$. The existence of the doubtful $O B$ association $C y g$ OB5 at a distance of 1610 pc between $\ell^{\mathrm{II}}=64 \% 2$ and $70 \% 0$ may be questioned on the basis of Figure 30. The present calibration confirms Ruprecht's analysis of Cyg OB5 as doubtful since the previously assigned members are more dispersed in distance than the members of the two better established associations mentioned above.

The early B stars selected from The Luminous Stars in the Northern Milky Way II and photoelectrically observed in the present work show a general tendency to group in two regions of Figure 30 , confirming the preliminary conclusions drawn from the histogram of Figure 29. Unfortunately the observed $B$ stars were taken only between $\ell^{I I}=61^{\circ}$ and $67^{\circ}$ whereas the more luminous $O B$ stars cover a larger range of longitudes from $57^{\circ}$ to $72^{\circ}$. Comparison of the two distributions is therefore fairly limited, but in general

Table 19. Photometric and spectroscopic data for Vul obl and Vul OB4 based on data from liiltner (1957)

Hiltner
(1.957)

No. $\begin{aligned} & \mathrm{HD} / \mathrm{BD} \quad \mathrm{SpT}(\mathrm{B}-\mathrm{V}) \\ & 4 \mathrm{Vul} \text { OBI }\end{aligned}$

| 807 | HD 184943 | B9Ib. | -.05 | -5.5 | 11.42 | 1920 | 59.1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 821 | $+23^{\circ} 3761$ | BOII | -.28 | -5.4 | 11.41 | 1910 | 60.6 |
| 822 | HD $186745-6$ | B8Ia | -.02 | -7.2 | 11.38 | 1890 | 60.2 |
| 826 | $+22^{\circ} 3800$ | B2III | -.24 | -3.6 | 11.50 | 2000 | 59.6 |
| 827 | HD 186841 | B1Ia | -.18 | -6.6 | 11.60 | 2090 | 60.4 |
| 831 | +2403881 | O6f | -.30 | -5.4 | 11.55 | 2040 | 61.2 |
| 854 | HD 190066 | BIIab | -.1 .9 | -6.2 | 11.57 | 2060 | 60.8 |

Vul OB4

| 806 | HD 183561 | B2III | -.24 | -3.6 | 10.42 | 1210 | 60.8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 819 | $+25^{\circ} 3941$ | B1.5V | -.27 | -3.0 | 10.50 | 1260 | 62.1 |
| 835 | $+24^{\circ} 3893$ | B1.5V | -.27 | -3.0 | 10.62 | 1330 | 61.4 |
| 845 | $+21^{\circ} 3959$ | B2V | -.26 | -2.5 | 10.55 | 1290 | 59.7 |
| 846 | $+22^{\circ} 3847$ | B5III | -.18 | -2.2 | 10.55 | 1290 | 60.1 |
| 858 | $+25^{\circ} 4083$ | BIIII | -.28 | -4.4 | 10.61 | 1320 | 64.5 |

the early $B$ groupings appear to lie at distances similar to those of the two $O B$ associations. A concentration of the early $B$ stars in longitude is not apparent from the diagram because of the limited nature of the survey, so that it is impossible to tell whether or not the $O B$ associations might be located at the center of much larger concentrations of early $B$ stars. At present the distribution of the early $B$ stars must be viewed as a particular clumpiness of the spiral structure in that direction.

The locations of the clusters in Figure 30 are fairly uncertain as evidenced by a comparison of the distances derived by Johnson et al. (1961) and by Hoag (1965b). In some cases the distances vary by as much as $50 \%$ between the two investigations. Hoag (1965b) averaged the results of several independent photometric investigations as well as the results of spectral classifications to arrive at the presently adopted distances. There still exist disparities yet to be resolved, however, between these distances and those based upon $H Y$ equivalent width absolute magnitudes. The only one of the clusters which does not seem to suffer this incongruity is NGC 6830 which appears to be located in the same volume of space as Vol OB4.

Also indicated in Figure 30 is the position of SV Vul based on its photometric parameters and the periodluminosity relation of Fernie (1967). If this location is
correct it appears that the cepheid is situated in the same volume of space as the association Vul OBl, the cluster NGC 6834 and one of the general regions of higher density of early B stars. Under these conditions it is tempting to conclude that the cepheid may be physically related to the association or the cluster or even that all three may be physically related to one another. More evidence than simple proximity in space, however, is needed to test such possible assertions. Although the cepheid is located (using the present calibrations) no more than 75 pc from the cluster and 150 pc from the center of the association, this general position is part of the Cygnus arm with its many young objects complicating the physical interpretation.

Figure 31 exhibits the combined color-magnitude diagrams of Vul OBI and NGC 6834. The photometric data for the members of NGC 6834 (Table 20) were taken from Hoag et al. (1961), and Schmidt-Kaler's (1965) calibrations were applied under the assumption that the stars were of class $V$ except those stars with available MK spectral classifications from Hoag (1965a). Also included in this diagram are the positions of SV Vul and two of Iben's evolutionary tracks for 9 and $15 M_{0}$ based on the modulus of SV Vul. The procedure for transforming Iben's tracks to the coordinates of the observational color-magnitude diagram was the same as that outlined by Kraft (1.966).


Figure 31. Color-magnitude diagram of Vul OBl, NGC 6834 with $9 \mathrm{M}_{0}$ and $15 \mathrm{M}_{0}$ evolutionary tracks

Table 20. Photometric data for NGC 6834 based on Hoag et al. (1961)

| Star No. <br> (Hoag et al. 1961) | $(\mathrm{B}-\mathrm{V})_{0}$ | E | 3E | $V_{0}$ |
| :---: | :---: | :---: | :---: | :---: |
| 3 | -. 13 | . 46 | 1.38 | 8.97 |
| 4 | -. 02 | 1. 42 | 4.26 | 6.64 |
| 5 | --. 08 | . 50 | 1.50 | 9.65 |
| 6 | -. 11 | . 77 | 2.31 | 9.36 |
| 7 | -. 14 | . 75 | 2.25 | 9.69 |
| 8 | -. 17. | . 77 | 2.31 | 10.10 |
| 9 | -. 16 | . 75 | 2.25 | 10.30 |
| 10 | -. 16 | . 62 | 1.86 | 10.71 |
| 11 | -. 16 | . 72 | 2.16 | 10.45 |
| 13 | -. 08 | . 68 | 2.04 | 10.75 |
| 15 | -. 09 | . 79 | 2.37 | 10.81 |
| 16 | -. 20 | . 79 | 2.37 | 10.84 |
| 17 | -. 13 | . 61 | 1.83 | 11.39 |
| 18 | -. 12 | .69 | 2.07 | 11.54 |
| 19 | -. 14 | . 76 | 2.28 | 11.54 |
| 20 | -. 10 | . 63 | 1. 89 | 12.28 |
| 21 | -. 10 | . 64 | 1. 92 | 12.34 |
| 22 | -. 15 | . 81 | 2.43 | 11.87 |
| 23 | -. 01 | . 65 | 1. 95 | 12.69 |
| 24 | -. 13 | . 79 | 2.37 | 12.33 |
| 25 | -. 02 | . 88 | 2.64 | 12.27 |
| 26 | -. 06 | . 67 | 2.01 | 13.02 |
| 27 | -. 04 | .68 | 2.04 | 13.73 |

As a basis for discussion, Figure 31 provides additional information on the possible association of the cepheid with the other star groupings in the area. The evolutionary tracks indicate that when a star becomes a cepheid by entering the instability strip it has evolved to a point about two and one-half magnitudes brighter than its original position on the zero age main sequence. Clearly SV Vul, which appears to be of an intermediate mass between 9 and $15 M_{0}$, falls at least one magnitude fainter than would be expected if it were one of the most evolved members of Vul OBI. The luminous $O B$ stars of Vul OBI appear to be evolving through a region of the colormagnitude diagram on paths similar to those which would be taken by stars more massive than $15 \mathrm{M}_{0}$. The fact that the less massive cepheid has already evolved further than the Vul OBI stars leads only to the conclusion that it must be considerably older than the association stars.

Also apparent from Figure 31 is the fact that $S V$ Vul is far too luminous with respect to the brightest stars of NGC 6834 to be one of its most luminous members. The positions of the most luminous members of NGC 6834 in the color-magnitude diagram are indicative of the early evolutionary stages of stars with masses near $5 M_{0}$. At this stage in the evolution of a cluster when the "turn off" point from the main sequence has reached the $5 M_{0}$ stars, the higher mass stars would not be expected to
remain visible due to the rapidity of their later evolutionary stages. SV Vul must therefore be considerably younger than the stars of NGC 6834.

The question of whether or not there exists any physical connection between $S V$ Vul and the two groupings or between the two groupings themselves must therefore be answered negatively. If a physical relationship between objects is thought of entirely as a proximity in position and a rough equality in age then the three present objects are apparently unrelated. In a broader sense, however, the region in which these objects are located can be thought of as an area which has contributed substantially to star formation in the past and may continue to do so in the future. This is supported by the generally high density of young objects in the region: Vul OBl, NGC 6834, SV Vul, and the several early B stars of Figure 30. Ages for these objects range from a few times $10^{6}$ years to a few times $10^{7}$ years. In addition, Dieter (1960) has noted the reasonable association of neutral hydrogen with the stars around Vul OBl, and Sharpless (1959) has pointed out the HII region NGC 6820 as part of the $O B$ association. The region in general has the characteristics of a star forming section of a spiral arm, and it is in this manner that the objects contained within it can be considered to be related.

In a similar fashion, the slightly closer region occupied by Vul $O B 4$ and NGC 6830 also appears to contain a generally high density of young objects. Although a long period cepheid such as $S V$ Vul has not been identified in the region, the apparent concentration of early B stars in coincidence with the $O B$ association and the cluster indicates a young region. In general, however, it must be noted that Vul OB4 has slightly different characteristics than Vul OBl. Table 19 shows that the members of Vul OBl are considerably more luminous than those of Vul oBt even though such members should be easier to detect in the closer association. The color-magnitude diagram for Vul OBl in Figure 32 also illustrates this point, and, in general, it appears that Vul $O B 4$ "turns off" from the main sequence at a lower mass than Vul OBl. If this is the case, Vul oB4 would be the older of the two associations and its lack of more luminous members is explained. It is, however, young enough to be associated with long period cepheids, and the absence of such stars in the region must be attributed to the relative rapidity of their evolution and the low probability of observing them during this stage in their lifetime.

In conclusion, therefore, $S V$ Vul, although relatively young, is apparently not a physical member of a presently-existing early-type star grouping. The cepheid's apparent age is not compatible with the ages of such


Figure 32. Color-magnitude diagram of Vul OB4
groupings in the area. Whether or not $S V$ Vul was formed in a previously existing group which has dispersed into the general field, however, is at present an unanswerable question.

SUMMARY AND CONCLUSIONS

In the preceding chapters, specific observations of the regions surrounding three classical cepheids have been presented and discussed. In each case, conclusions about the cepheids themselves were drawn on the basis of these observations. It is important, however, to attempt to deal with the cepheid class of stars as a whole and to see how their individual properties define or contrast with their group properties. It is, therefore, the purpose of this chapter to bring together for inter-comparison the results from the three cepheids of the preceding chapters.

## General Properties of RS Pup and <br> SV Vul, and SU Cas

Both RS Pup and SV Vul can be considered as relative rarities among classical galactic cepheids because of their long periods and high luminosities. Coincidently they are both situated at similar distances, within a factor of $20 \%$, from the sun, and although their distances are large, they are still bright enough to be well observed. Therefore, if the two cepheids were both similarly related to their environment, no particular observational selection effects would be expected to interfere with the detection of specific properties of one and not of the other.

The fact is, nevertheless, that RS Pup has previously been discovered as the most evolved member of a young $O B$ association, whereas the present observations and a reanalysis of data available in the literature fail to confirm a similar property for SV Vul. RS Pup is located in a region of space heavily populated by dust and gas, typified by emission and heavy obscuration. As was discussed earlier, this location is probably partly responsible for the present nebulosity around the cepheid in addition to the probable role played by the cepheid itself in forming and structuring the nebula. Nevertheless, it is especially interesting that no such nebula has been detected around SV Vul. This must be attributed either to intrinsic differences between the structure and evolutionary histories of the two cepheids (if the material of the reflection nebula was formed with or by the cepheid) or to basic differences in the interstellar medium in the vicinity of these stars. The region in which $S V$ Vul is located is neither directly within a young $O B$ association nor does it exhibit such pronounced evidence for the existence of gas and dust as does the region around RS Pup.

SU Cas, in comparison, must be interpreted on the basis of evolutionary models as being considerably older than either of the above two cepheids. It, together with a group of old B stars, exist in the same volume of space, which is not typified by active star forming regions or
young $O B$ associations. The concentration of $B$ stars in the region may reflect the tendency of these stars to remain in the spiral arm as indicated by the Lin Theory. In general, therefore, the $B$ stars, including the cepheid, can only be considered as independent members of the spiral arm, and these stars may have originated from different OB associations existing in the past. In particular, also, the several reflection nebulae appearing in the same volume of space as the $B$ stars are indicative of a region of dust in the spiral arm. The SU Cas reflection nebula itself is interpreted entirely differently than that surrounding RS Pup. Showing no evidence of having been structurally influenced by $S U C$ Cas, the nebula is too linear to be anything other than a coincidental meeting of cepheid and dust cloud.

## Implications for the Theory of Cepheid Evolution

By far the most intriguing aspect of the present work is the apparently unique location of RS Pup and its surrounding reflection nebula in a young $O B$ association. So far, neither of these characteristics has been detected for any of the other luminous cepheids in the Galaxy. The question of the uniqueness of these two properties must consequently be examined in the light of the present findings and with respect to the existing theory of cepheid evolution.

The RS Pup nebula contains enough symmetry and structure to imply that the cepheid itself influenced the formation of the observable nebula. The detection of periodic light variations in the nebula supports the conclusion that the nebula is primarily a density structure and suggests that the material of the nebula originated from both the cepheid and the surrounding material. The observations and the analysis, however, are not yet complete enough to indicate which physical mechanism exerted the dominant influence in the formation of the nebula. Consider the implications involved in assuming that the nebula originated solely from the ejection of material from the cepheid. Since, in this case, no other long period cepheids have been detected with an accompanying nebulosity, RS Pup would have to be treated either as a unique example or simply as being in a particular evolutionary state in which the remains of such an ejection are still detectable. It is possible that RS Pup could be in the rapid state of its evolution in which it is passing through the instability strip for the first time on its way to the red giant phase. Presumably, then, other detectable long period cepheids are in higher crossings of the strip and the evidences of their initial crossings are no longer present. Such an interpretation would also imply that RS Pup is considerably younger than other long period cepheids. This would nicely explain the fact that

RS Pup falls, in the Kippenhahn and Smith (1969) period-age relation, at an age considerably less than that expected from theoretical considerations and generally confirmed by observations of shorter period cepheids. At present, however, there is no indication of why the first passage of the instability strip should be any different than successive passages in any physical respect other than the duration of the crossing.

Considerations as above, however, neglect several important problems. Matter ejected in any process from a cepheid or other type star would probably be in gaseous form. Present investigations on the ejection of particletype matter from stars have dealt successfully with graphite emission from the atmospheres of cool stars and probably would not apply during the cepheid stage. Nevertheless, the existence of a reflection nebula means reflecting dust particles, and it is unclear at present as to how effectively dust grains can form from gas atoms and molecules, especially during the short time intervals ( $10^{6}-10^{7}$ years) for the evolution of such young stars. One other problem with the ejection hypothesis is the existence of several ejected shells as opposed to a uniform sphere or a single shell. This will be considered further in some of the following discussion, but at this stage it would seem difficult to eject so many shells on a single first passage through the instability strip. The
ejection hypothesis is attractive in its obvious solution to the problem of mass inequality between the calculations of pulsation theory and the theory of stellar evolution, but it is by no means free of problems itself. As seen previously, the nebula can not be considered naively to be the result of a coincidence of a cepheid and a dust cloud of the structure presently observed. If, however, the assumption is made that the majority of the nebular material is composed of dust from the interstellar matter either originally surrounding the cepheid, or having been encountered during the space motion of the cepheid, the structure of the present nebula remains to be explained. Exactly how the cepheid has influenced the nebular structure is not clear, since several possible mechanisms could be postulated equally well at present. Not enough is known about the possible interaction either of shock phenomena or of ejected gaseous matter with the surrounding interstellar material to adequately differentiate between the two processes in this case. If, however, the interstellar material can be sufficiently concentrated into shells upon interaction with some phenomenon from the cepheid, the question remains as to what physical interpretation can be made from the number and position of the existing nebular shells.

It is not likely that the individual shells were created during successive pulsations of the stellar
atmosphere, if the pulsation period were similar to its present value. The shell spacing, size, and relative distance from the cepheid are too great for this to have occurred. Distinctive physical occurrences other than the individual pulsations must be sought to explain the nebular structure. The small number of reflecting shells around RS Pup might suggest that the shells were created during previous transits of the cepheid across the instability strip. The structuring of the shells would then be the result of initiating or terminating stellar pulsation as the star entered or left the instability strip. This possibility arises due to the present knowledge that cepheids traverse the strip several times during their stellar lifetimes. The actual physical mechanism, which might cause the onset or the termination of pulsation to trigger the formation of a shell, is, of course, uncertain. The physical act of entering or leaving the instability strip, however, is a significant enough event in the evolution of the star to allow the suggestion that a shell may be structured at this time. Consequently, under this line of reasoning, RS Pup could no longer be considered to be only on its first passage of the strip. This is more reconcilable with the very small probability of detecting a first crossing cepheid, but, then too, the implied age of the cepheid would be increased.

Another possible era in the star's lifetime during which material ejection might have occurred and structured the surrounding dust is the red giant phase. Just as cepheids cross the instability strip several times during their lifetime, they also enter the red giant region more than once. Several stars in this region have been observed to exhibit mass loss characteristics, so that it is not unlikely that evolving cepheids could do the same. The successive nebular shells could then be due to individual ejection events during one passage to the red giant region or during several successive passages, depending on the cepheid's age. Recent calculations by Donn et al. (1968) have shown that the condensation of graphite grains can occur in the atmospheres of cool red giants as early as type M3. Their calculations indicate that $2 \mathrm{x} \mathrm{Jo}^{25} \mathrm{~g} \mathrm{yr}^{-1}$ would be ejected in the form of carbon grains and that this would approach a total ejection of $10^{30} \mathrm{~s}$ during the red giant stage. Although this quantity is only about $10^{-3} \mathrm{M}_{0}$ of reflecting grains, the value does not exceed the previously determined upper limit for the mass of each shell in the RS Pup nebula. Furthermore, this value does not exclude the possibility of a significant contribution to the nebular mass from the ambient interstellar medium. In this case, the ejected grains serve as nucleii for further condensation of material from the ambient interstellar medium.

In all of the above discussion, it is difficult to see how RS Pup could be the only long period cepheid with an observable nebula, since there is no reason to believe that other such stars would not undergo similar physical events during their evolution. The unique location of $R S$ Pup in an extremely dusty volume of space must therefore be the dominant factor in its particular appearance. The cepheid alone, in a less dusty environment, perhaps, would undergo the same physical events, but only in the present environment, in which the density of material is high enough for efficient condensation on the ejected grains, are the events recorded for future observation as a structured reflection nebula.

Although RS Pup is the only long period cepheid yet detected as member of a young $O B$ association, there is no reason to believe that $R S$ Pup may be unique in this respect. Associations are still among the most ill-defined configurations in the Galaxy, and a considerable amount of work remains to be done on the more distant of the known associations. Therefore, the possible existence of young cepheids in $O B$ associations should be considered very carefully. According to the recent work by Blaauw (1964) summarizing the work on nearby $O B$ associations, dispersions in age of the association members may be as large as four or five million years. Under these conditions, $S V$ Vul may yet be
considered as a member of Vul OBl although a more detailed investigation of the region is necessary.

## Effects on the Existing PeriodLuminosity Relations

In analyzing the observational material of the previous chapters, one very important parameter for each of the three cepheids was its distance. In each case a rough idea of the distance to each cepheid was known initially on the basis of existing period-luminosity calibrations (Fernie 1967). As has been already discussed, however, the three cepheids studied have periods near the limits for known galactic cepheids. They can therefore serve very importantly as calibration points themselves, provided more accurate distances for them are determined independent of the existing calibrations.

SV Vul was not found to be a member of any stellar group nor was its possible relationship indicated to any of its neighboring groups. Unfortunately, therefore, no independent means of distance determination was possible.

The presence of $S U C$ Cas in a concentration of rather old $B$ stars suggests that the mean distance of these $B$ stars might be used as a better indication of the location of the cepheid. A similar technique has been used by Racine (1968b) in which he used the mean distance of all the stars surrounding $S U C$ Cas which exhibit reflection nebulae. If the dust near $S U C$ as is more localized than
the surrounding $B$ stars, then Racine's distance determination should be more precise than that resulting from the present method. Indeed, the present method gives a distance of $r=280 \pm 60 \mathrm{pc}$ whereas Racine obtains $r=310$ $\pm 30 \mathrm{pc}$. Neither method relies upon an assumption of common origin and past history but only upon the indications that the cepheid exists in the same volume of space as some of its surrounding stars. The results of the two methods differ significantly only in their degree of scatter about the mean.

RS Pup is the one long period cepheid which now has more than one independent distance determination available. Westerlund's (1963) distance for the Pup III association was 1.74 kpe. This was modified on the basis of the ( $M_{v}, U-B$ )-diagram to 1.82 kpc by Sandage and Tammann (1969). The present geometrical determination based on the detectable light variations in the surrounding nebula gives a distance of $1.78 \pm .02 \mathrm{kpc}$.

In the most recent period-luminosity calibration of Sandage and Tammann (1969) both SU Cas and RS Pup were partially used as important calibrating points. The present results on both stars compare quite favorably to the previous distances mentioned above and which were used by Sandage and Tammann. These results, therefore, serve most satisfactorily to confirm the previous distances. In this manner, more confidence can be placed in the existing
period-luminosity calibrations, since their long and short period portions are now more firmly established.

## Suggestions for Future Work

The present study has primarily been concerned with the interpretation of several observational properties of three galactic cepheids and the relationship of these cepheids to the surrounding stars and interstellar medium. The basic questions considered in this study were the following: (1) How can the present observational evidence for the interaction of cepheids with the interstellar medium be interpreted in view of the cepheid's role in stellar evolution? and (2) To what extent do the cepheids studied indicate a possible origin in associations? By the limited nature of the observations, this study has only approached satisfactory answers to the above questions, and in this respect there is much room for future rescarch. The most obvious area for future study concerns the detailed physical structure and dynamics of the RS Pup nebula. A much more thorough investigation of the light variation of the nebula is needed using a large aperture, large scale telescope. Preferably, the study should be undertaken making use of the linear response of electronographic plates and in the visible wavelength range, where the intensity of the nebula is nearly two magnitudes brighter than in the blue. Such an investigation would
provide more precise parameters for the nebular model and would provide an additional confirmation of the geometrical distance analysis of the present study. To further test the model, the inclusion of polarization data would be desirable. Several problems must be overcome for these observations, however, primarily because of the small size of the nebula and its extreme relative faintness compared to RS Pup.

Further attempts should also be made to detect possible nebulosity near other cepheids. The present study has suggested that this may be possible only for those cepheids which are already located in dusty regions of space. In these cases, it would be extremely useful to see if the nebulosity has been influenced by the cepheid in a manner similar to that suggested for RS Pup.

Another area for future research is the continuation of work on $O B$ associations and the possible membership of long period cepheids. Preferably, narrow band photometry should be carried out for the members of faint and distant associations, for which there is a nearby cepheid. These observations would allow the fairly accurate determination of stellar ages, which could be compared to the cepheid age, determined on the basis of evolutionary models. The comparison of distance, age, and motion would then serve as the criteria for membership in the association. Furthermore, once a cepheid is established as a member of an

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association, the cepheid can then be considered as funda-
mental to the calibration of the galactic period-luminosity
relation.
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## APPENDIX A

## A SAMPLE ISODENSITRACING REDUCTION

As an example of the reduction procedure used in the analysis of the RS Pup nebula, the details will be presented here for Feature 1 on the blue plate No. 2463. Throughout this procedure, all plates including No. 2463 were normalized to plate No. 1593 , so that the present discussion assumes that the results of plate No. 1593 are also available.

Figure 33 presents an exact scale reproduction of a 200:1 magnification isodensitracing of plate No. 2463. The two roughly equal sized bull's eyte patterns in the lower left and in the upper left of the figure are the density contours formed by very faint stellar images. These two stars, visible on each plate, were used as reference points to insure that the proper region of the nebula was measured. The circle in the central portion of the figure isolates the brightest portion of the nebula, to which all of the following measures of Feature 1 were restricted. The circle corresponds to an approximate angular diameter in the sky of $5^{\prime \prime}$ and on the isodensitracing it has an area of $20 \mathrm{~cm}^{2}$.


Figure 33. Sample isodensitracing of plate 2463

Within the circular analyzing area there are seven different contour intervals made up of the three symbols: $\mathbf{s}=$ space, $\mathrm{d}=$ dot, $\ell-$ line. The settings of the Isodensitracer were selected so that each symbol covers a range in density of .059 D . In Table 21 , an $x$ just to the left of column 1 indicates that the corresponding contour interval is identical to that space which is similarly marked with an $x$ in Figure 33. The symbols in column 2 of Table 21 refer to the junctions of succeeding contour intervals in column 1 and will be henceforth referred to as contour lines. Column 3 gives the density above fog of these contour lines and i.s transformed into the log of the intensity above fog in column $l_{1}$ by means of the characteristic curve of Figure 5. Column 5 is simply the average of two successive values of log I' in column 4. Columns 6 through 8 are, for each contour interval respectively, the average intensity above sky, the area in square centimeters, and the intensity $x$ area product. $S^{11}$ is the sum of all these products, and, except for a few corrections, is a measure of the total intensity of light from the circular analyzing aperture. The individual areas of the contour intervals should sum to $20 \mathrm{~cm}^{2}$, but errors in measurement of so many areas require the use of a small correction factor. In this case, the areas sum only to 19.88 , and $S^{\prime \prime}$ must be multiplied by $20 / 19.88$ to produce $S$ '.

Table 21. Sample isodensitracing reduction for plate 2463

| Region | Contour line | ${ }^{D}>$ fog | $\operatorname{logI'}$ | $\overline{\operatorname{logI}}$ | I | $A\left(\mathrm{~cm}^{2}\right)$ | IA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | $\ell / s$ | 1.112 | . 999 |  |  |  |  |
|  |  |  |  | 1.007 | 4.54 | . 32 | 1.453 |
|  | s/d | 1.172 | 1.015 |  |  |  |  |
| d |  |  |  | 1.026 | 5.00 | 1.18 | 5.900 |
|  | d/l | 1.231. | 1.037 |  |  |  |  |
| $\ell$ |  |  |  | 1.047 | 5.52 | 3.70 | 20.424 |
|  | $\ell / s$ | 1.291 | 1.057 |  |  |  |  |
| s |  |  |  | 1.068 | 6.07 | 6.30 | 38.241 |
|  | s/d | 1.351 | 1.078 | 1.088 | 6.63 | 5.70 | 37.791 |
| d | d/l | 1.410 | 1.098 |  |  | 5.70 |  |
| $\ell$ |  |  |  | 1.108 | 7.20 | 2.04 | 14.688 |
|  | \&/s | 1.470 | 1.117 |  |  |  |  |
| $\times \quad \mathbf{s}$ | s/d |  |  | 1.126 | 7.75 | . 64 | 4.960 |
|  |  | 1.530 | 1.135 |  |  | . |  |

$$
\begin{aligned}
& S^{\prime \prime}=\Sigma I A=123.457 \\
& \Sigma A=19.88 \\
& S^{\prime}=\frac{20}{\Sigma A}(\Sigma I A)=124.202 \\
& S^{\prime}=(.917) S^{\prime}=113.893 \\
& S_{2463} / S_{1593}=1.012
\end{aligned}
$$

$$
\Delta m_{B}=+.01
$$

A further correction to $S '$ requires the measurement on both plates No. 2463 and No. 1593 of the two nonvariable calibration stars. The measures of these stars follow exactly the above procedure except that the contour lines are measured right down to the level of the sky. A comparison of the resulting intensity $x$ area products on the two plates then indicates that the intensities on plate No. 2463 must be multiplied by .917 to make the calibration stars equal. This factor multiplied by $S$ ' then leads to $S$ for the nebular region. Plate No. 1593 has been similarly measured and the ratio $2463 / 1593$ is then 1.012 , which corresponds to a magnitude difference $\Delta m=$ $\left(m_{1593}-m_{2463}\right)=+.01$.

The above procedure is, of course, dependent on the identification of the highest contour line with the density level $D_{>f o g}=1.530$ in Table 21 . This is done quite simply using the microphotometer mode of the JoyceLoebl. Initially, the sky around one of the calibration stars is measured with reference to the plate fog. After an isodensitracing is made, one scan is rerun through a known portion of the isodensitracing as in Figure 34, where the scan was taken through the brightest portion of the nebulosity near the $x$ in Figure 33. Since this can be identified in a one to one fashion with the symbols of the isodensitracing, the sky density level calibrates all other contour levels. Also indicated in Figure 34 is the density


Figure 34. Calibration scan for isodensitracing reduction
level of the sky at the same radial distance from RS Pup as the bright nebular region but in a region devoid of nebulosity. This is the sky level to which the intensities of column 6 in Table 21 are compared.

## APPENDIX B

## THE PROCEDURE FOR MAGNITUDE CALIBRATION

The relative magnitudes of an individual nebular feature on several different plates were all obtainable using the methods of Appendix A. To compare the magnitudes of one feature to another or to obtain color information, however, requires a knowledge of the actual magnitudes of these features. This would be easily obtainable if the magnitudes of the calibration stars were known, but this is not the case. The only photoelectric information available for the region is that done by Lodén (1969). About ten of these stars appear on the present plates but unfortunately their images are too dense for reduction with the Isodensitracer. These stars are adequately distributed in magnitude, however, to use as a sequence in determining the magnitudes of other stars in the region. Furthermore; the electronographic plate $S D-79$, supplied by Walker, is sufficiently underexposed so that stars comparable in magnitude to those in Lodén's sequence can be reduced using the Isodensitracer technique. In addition, the linear response of plate $S D-79$ allows the reduction on still fainter stars within its field. These fainter stars are then measurable on the ordinary photographic plates with
the Isodensitracer, where they can be directly compared to the nebular regions.

Table 22 presents the average $B$ and $V$ magnitudes which were derived on the basis of iris photometry using Lodén's sequence applied to star $P$ on $S D-79$. Star $P$ is bright enough to be just barely of $f$ the faint end of Lodén's sequence yet not too bright to have a saturated electronographic image. Table 23 then shows the results of the isodensitracings of the three stars $D, G$, and $P$ in the $B$ and $V$ magnitudes on SD-79 and of stars $D$ and $G$ on plates No. 1729 and No. 3116. Stars $D$ and $G$ are easily seen to be considerably fainter than star $P$ which was too bright to be traced on the two ordinary photographic plates. These results are further compared in Table 24 , which tests roughly the invariability of stars $D$ and $G$ as indicated by their magnitude differences from the electronographic plates to the photographic plates. The last two columns of Table 24 allow the calculation of the actual magnitudes of stars $D$ and $G$ compared with star $P$ and these magnitudes are given in the first sections of Tables 25 and 26.

Finally, in Tables 25 and 26 the completed calibration is presented. The results of isodensitracings of stars $D$ and $G$ are compared to the results from the four nebular features to establish the magnitudes of the four features at the phase of the plate in question. In each case the magnitudes derived from the separate comparisons

Table 22. Derived $B$ and $V$ magnitudes from Lodén's (1969) sequence for star $P$ on plate $S D-79$

| Plate | B | Plate | V |
| :--- | :---: | :---: | :---: |
| 1593 | 14.27 | 3116 | 13.63 |
| 1719 | 14.38 | 2611 | 13.76 |
| 1723 | 14.46 | $\ldots$ | 2464 |
| 1729 | 14.42 |  | 13.78 |
| 1990 | 14.40 |  |  |
| 2612 | 14.36 |  |  |
| $\mathbf{B}=14.38 \pm .06$ |  |  |  |

Table 23. Isodensitracing results for stars D, G, and $P$ on plates SD-79, 1729, and 3116

| Plate | Mag. | Star |  |
| :--- | :--- | :--- | ---: |
| SD-79 | B | IA |  |
|  |  | G | 2.830 |
|  |  | P | 1.669 |
| SD-79 | V | D | 21.731 |
|  |  | G | 1.410 |
|  |  | P | .998 |
| 1729 | B | D | 21.075 |
|  |  | G | 59.34 |
| 3116 | V | D | 36.44 |
|  |  | G | 65.67 |
|  |  |  | 51.22 |

Table 24. Comparison of stars $D$ and $G$ and of stars $P$ and $D$

| Mag. | Plate | $I A_{\mathrm{D}} / I A_{\mathrm{G}}$ | $\Delta \mathrm{m}$ | $I A_{\mathrm{P}} / I A_{\mathrm{D}}$ | $\Delta \mathrm{m}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| B | $\mathrm{SD}-79$ | 1.70 | .57 | 7.68 | 2.21 |
|  | 1729 | 1.63 | .53 |  |  |
|  |  |  |  |  |  |
| V | SD-79 | 1.41. | .38 | 14.95 | 2.94 |
|  | 3116 | 1.28 | .27 |  |  |

Table 25. V magnitude calibration for isodensitracings of plate 3116

|  | IA | $\mathrm{IA}^{\text {/ }} \mathrm{SA}_{\text {tar }} \mathrm{D}$ | ${ }^{\Delta \mathrm{m}_{\mathrm{V}}}$ | V | IA/IA ${ }_{\text {Star }} \mathrm{G}$ | $\Delta \mathrm{m}_{\mathrm{V}}$ | V | $\stackrel{\rightharpoonup}{\mathrm{V}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Star D | 65.67 | -- | -- | -- | -- | -- | -- | 16.66 |
| Star G | 51.22 | -- | -- | -- | -- | -- | -- | 17.04 |
| Feature 1 | 137.09 | 2.09 | . 80 | 15.86 | 2.68 | 1.07 | 15.97 | 15.91 |
| Feature 2 | 76.20 | 1.16 | .16 | 16.50 | 1.49 | . 43 | 16.61 | 16.55 |
| Feature 3 | 49.46 | . 75 | -. 31 | 16.97 | . 97 | -. 04 | 17.08 | 17.02 |
| Feature 4 | 32.64 | . 50 | -. 76 | 17.42 | . 64 | -. 49 | 17.53 | 17.47 |

Table 26. B magnitude calibration for isodensitracings of plate 1593

|  | IA | IA/IA ${ }_{\text {Star }} \mathrm{D}$ | ${ }^{\Delta m_{B}}$ | B | $\mathrm{IA}_{\text {/ }} \mathrm{A}_{\text {Star }}{ }_{\text {G }}$ | $\mathrm{Am}_{\mathrm{B}}$ | B | $\overline{\mathrm{B}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Star D | 176.35 | -- | -- | -- | -- | -- | -- | 16.59 |
| Star G | 108.31 | -- | -- | -- | -- | -- | -- | 17.16 |
| Feature 1 | 112.54 | . 64 |  | 17.08 | 1.04 |  | 17.12 | 17.10 |
| Feature 2 | 29.89 | . 17 |  | 18.51 | . 28 |  | 18.56 | 18.54 |
| Feature 3 | 22.90 | .13 |  | 18.81 | . 21 |  | 18.85 | 18.83 |
| Feature 4 | 9.10 | . 05 |  | 19.80 | . 08 |  | 19.85 | 19.83 |

with stars $D$ and $G$ are averaged and presented in the final column of each table. In Table 26 , the information is presented for plate No. 1593, whereas the data for stars D and $G$ were obtained on plate No. 1729. The transformation was made using the calibration stars and the multiplicative factor from 1729 to 1593 was 2.972.

## APPENDIX C

## A DISCUSSION OF THE EXPERIMENTAL ERRORS

It is the purpose of this section to specify the sources of error which significantly affect the Isodensitracer plate analysis technique. To a large extent, the interpretation of the RS Pup nebular light curves depends on the proper understanding of these details of the plate reductions.

Undoubtedly, the largest source of error is the use of the reduction technique on ordinary photographic plates. The reduction to intensity from plate density requires the use of different calibration curves for each plate. For faint light sources, the problem arises that a large intensity change corresponds to only a small density change on the plate. Therefore, a typical uncertainty in the measurement of the density level gives rise to a much larger uncertainty in the intensity. This greatly affects the intensity values derived for sky level on all plates and especially affects the nebular measurements themselves in the faint regions.

In general, the use of a calibration curve must be looked upon as a necessary intermediate step in the reduction procedure and which introduces its share of
uncertainties. To a certain extent, the use of calibration stars to adjust the zero point of each plate helps minimize the errors introduced due to irregularities in the calibration curves themselves. The stars must be reduced in the same manner as the nebular regions, with the exception that the stellar reductions make more use of the higher density portions of the calibration curves.

Another major uncertainty arises in determining the various reference levels which are used in the reduction procedure. The most basic reference level was, of course, the fog level, which was assumed to be identical on both calibration and object plates. From this, the secondary reference level was the sky level surrounding one of the calibration stars. Once the average sky density was known, it was continually used as a reference level while tracing the nebular regions. Naturally, there is a certain degree of error involved in using such an ill-defined reference level, but there were no alternatives. In addition, there arose similar uncertainties in attempting to determine if the sky in the vicinity of the nebular feature was significantly brighter than the sky near the calibration star.

In all of the above, it is particularly difficult to assign formal probable errors. The errors in the relative nebular magnitudes are influenced by so many factors, which differ from plate to plate and region to region, that
it would be impossible to derive the formal probable error for each experimental point individually. Nevertheless, an attempt has been made to estimate the probable errors for all the data by treating only a few plates. A typical uncertainty in density of $\pm .03 \mathrm{D}$ was propagated through the reductions of Feature 2 on several plates. Generally, the smallest probable errors obtained were on the order of $\pm 0^{m} \cdot 2$ for Feature 2 (refer to Figure 9). The various errors for different plates and for other features on the same plate are primarily a function of their absolute intensity. Faint features and plates taken with lower exposure times or under less than optimum observing conditions had to be reduced on the shallow portions of the characteristic curves and therefore suffered greater errors by at least a factor of two. The reductions for the bright Feature 1 were the least uncertain, simply because most of the densities were located on the linear portion of the calibration curves.

One additional point that should be mentioned here concerns the reduction of the electronographic plate, SD-79. Since this plate was reduced without the use of a calibration curve, its probable error has been estimated as roughly half that of the other plates. The accuracy of its position depends solely on the correct determination of the magnitude scale and its direct comparison to the secondary comparison stars $D$ and $G$ (see Appendix B).

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