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# MICROTURBULENCE <br> IN MAIN SEQUENCE STARS 

by<br>Frederic Henry Chaffee, Jr.

A Dissertation Submitted to the Faculty of the DEPARTMENT OF ASTRONOMY

## In Partial Fulfillment of the Requirements For the Degree of

DOCTOR OF PHILOSOPHY
In the Graduate College

THE UNIVERSITY OF ARIZONA

GRADUATE COLLEGE

I hereby recommend that this dissertation prepared under my direction by $\qquad$ Frederic Henry Chaffee, Jr. entitled Microturbulence in Main Sequence Stars
be accepted as fulfilling the dissertation requirement of the degree of $\qquad$

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\frac{\text { Thelnut } a, \text { aet }}{\text { Dissertation Director }} \frac{\text { Chile } 22,1868}{\text { Date }}
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## ABSTRACT

High dispersion spectra have been obtained for 23 main sequence stars in the spectral range from $A 0$ to $G 0$. Spectra were observed in the green for 17 of these stars at a dispersion of $17.8 \mathrm{~A} / \mathrm{mm}$, and in the blue for 8 of the stars at a dispersion of $8.9 \mathrm{~A} / \mathrm{mm}$. Microturbulent velocities have been determined for each of these stars from a curve of growth analysis of approximately 90 Fe I lines in each stellar spectrum.

The variation of microturbulence with temperature (as indicated by the $b-y$ photometric index) is such that the microturbulent velocity has two maxima--one at spectral class A8 and one at GO. In addition, there are two minima--one in the early $A$ stars and the other in the middle F stars.

Two photometric effects have been examined: a) the variation of microturbulence with age as indicated by the $\Delta c_{1}$ photometric index and b) the variation of the $\Delta_{m_{1}}$ index with microturbulence (the Conti-Deutsch effect). It has been found that there is no significant variation of microturbulence with age for dwarf stars, and that there
is a small correlation of the $\Delta m_{1}$ index with microturbulence. This correlation is 9 per cent for the stars in the present investigation and 15 per cent for eight metallic line stars whose microturbulent velocities are available in the literature. It is thus proposed that the Conti-Deutsch effect can account for only a small percentage of the observed variation of $m_{1}$ in stars--the remaining variation being caused by abundance differences among stars.

Finally, the dependence of microturbulence on temperature is qualitatively explained by a simple model of the shape of the velocity field in and above the convection zone in a stellar atmosphere. In this model, the run of the small-scale velocity progresses from laminar flow deep in the convection zone to turbulent convection near and above the surface of the zone to a decay in the overshooting of the convection to progressive acoustic waves far above the zone. With this model and with the fact that the optical depth at which the atmosphere becomes unstable against convection moves to greater optical depth for later spectral types, the general run of microturbulence with temperature can be explained--with the important
exception that the solar microturbulent velocity is not consistent with this explanation. Nevertheless, it is felt that the model explains the general behavior of microturbulence quite well and that with more elaborate theoretical calculations, the solar microturbulence could be explained in terms of the model presented here.

## I. INTRODUCTION

In 1927 Unsold showed that the theory of radiation damping could be applied to the wings of strong Fraunhofer lines. This theory predicted that the strength (equivalent width) of an absorption line in a stellar spectrum should be proportional to the square root of the number of atoms effective in producing the line. This relationship became known as the square root law, and observational confirmation was immediately attempted. It was found that for a majority of cases the law was obeyed in stars but that there were occasional departures from the law resulting in the equivalent width being proportional to the number of atoms to a higher power than the one half predicted by the square root law.

In a paper which had not been brought sufficiently to the attention of astronomers, Voigt (1912) had derived the expression for the absorption coefficient for atoms which are in thermal agitation. This was followed by a classic paper in modern astrophysics by Struve and Elvey (1934). In this paper they explained the departures from the square root law in terms of the Voigt profile. The
absorption coefficient derived by Voigt is such that the line strength is directly proportional to the number of effective absorbers. Struve and Elvey combined the square root law with the Voigt profile to produce a relationship between the line strength and the abundance of the absorber. This was the first curve of growth which has since become an extremely valuable tool in astrophysical research. (It should be added, however, that a similar study had been carried out by van der Held (1931) in which he derived a more realistic curve of growth than that of Struve and Elvey, and van der Held's curve is still used in some current astrophysical problems.)

Struve and Elvey noticed that the observations of certain stars led to anomalously high Doppler velocities for the absorption lines. This meant that either the temperature giving rise to the thermal agitation was much higher than the temperature indicated by the star's spectral type or there was some other mechanism causing microscopic motions in the stellar atmosphere. Their conclusion was

In order to explain these departures we propose the following working hypothesis: The atmospheres of the stars are agitated not only by thermal
motion of the individual atoms, but also by currents of a macroscopic character, which we shall refer to as 'turbulence.' (1934)

Several stars in their sample showed very high turbulent velocities-up to $67 \mathrm{~km} / \mathrm{sec}$ in the case of 17 Lep. Thus they established the existence of "microturbulence," as we now call this phenomenon, in the stars. They did not, however, make the distinction between microturbulence and macroturbulence which must be made.

As the term is used today, microturbulence is a small scale motion of the atoms in the stellar atmosphere. The scale length of the motion of the atoms between collisions is small compared to the mean free path of the photon to be absorbed by the atoms, and the resulting motion will increase the atomic absorption coefficient. This motion is assumed to have a Gaussian distribution about some mean velocity and is thus indistinguishable from the random thermal motions in the atmosphere. Macroturbulence, on the other hand, is a large scale motion. Because its scale length is large compared to the photon mean free path, it has no effect upon the atomic absorption coefficient. However, as viewed from outside the atmosphere, macroturbulence produces large areas on the
stellar disc with systematic velocities--that is, it produces a pattern of organized large scale motion. This is, in fact, the same kind of effect produced by stellar rotation, and observationally macroturbulence and stellar rotation are difficult to disentangle.

In light of our present knowledge of theoretical
laboratory intensities and observed line strengths, the -original work of Struve and Elvey can be reanalyzed. This has been done by Greenstein (1967) who states

They were...extremely fortunate in their choice, since 17 Lep has a truly exceptional curve of growth. Their observational and theoretical errors were so serious that if so widely different stars had not been chosen, they would not have been able to discover the existence of turbulence!

Nonetheless, their fundamental work pointed the way to nearly all analyses to date of both the abundances of the elements and the velocity fields in stellar atmospheres.

Initially, the curve of growth method was used chiefly as a convenient tool to obtain the relative abundances of the elements in stars. Nevertheless, the microturbulent velocity was always obtained in such analyses and became interesting in its own right.

Goldberg (1939) carried out an analysis of 56
early stars. He found high microturbulent velocities
(greater than $10 \mathrm{~km} / \mathrm{sec}$ ) for a great majority of the supergiants he examined. However, he ignored the Stark broadening of helium lines which is a very important effect, especially in dwarf stars; and his data were taken at a dispersion of $30 \mathrm{~A} / \mathrm{mm}$ which is considered by modern investigators to be too low a dispersion for reliable results to be obtained. Similar analyses were carried out by many investigators in the $1940^{\prime \prime} \mathrm{s}$ (e.g. Unsöld 1941, Aller 1942, Greenstein 1942).

By 1955 it was possible for Wright to present a resumé of the known behavior of microturbulence over a wide range of spectral types and luminosity classes. The picture which emerges from these data is that for high luminosity stars, the microturbulent velocity is very large (greater than $10 \mathrm{~km} / \mathrm{sec}$ ); and in lower luminosity classes, the microturbulence decreases until for dwarfs the velocities are of the order of $1-4 \mathrm{~km} / \mathrm{sec}$. Wright's summary was taken from a rather inhomogeneous collection of data--some of which was of excellent quality and some practically useless. Thus, except for these very general trends, it was not possible to make a more detailed analysis of the behavior of microturbulence among stars.

At this time, a major weakness of the technique was that the laboratory intensities of many spectral lines observed in stars were not known, and because the transition probability (or oscillator strength) is one of the necessary input parameters to the curve of growth analysis, this lack of knowledge led to a fundamental weakness of the method. A second weakness was that except for the van der Held curve of growth no theoretical curves of growth were available to the observer.

These problems resulted in the wide use of the differential curve of growth analysis. In this method some standard star is chosen, and its observed curve of growth is used as a basis upon which to compare its abundances and microturbulent velocity to those of the program star. For $F$ to $K$ stars, the sun is usually chosen as a standard since the equivalent widths of its spectral lines have been measured with a precision far surpassing that obtainable for any other star. This procedure is still the one most widely used in stellar spectrophotometry since for an abundance analysis the atmospheric parameters which appear in the abscissa of the theoretical curve of growth will cancel out in a differential analysis. This method has
the additional advantage "that line blending will produce deviations from the true equivalent width which will be the same for similar stars. Because of this, the differential procedure is not as seriously hindered by line blends as is the absolute curve of growth procedure.

However, the lack of absolute oscillator strengths for stellar spectral lines--one of the basic problems which forced the observer to use a differential analysis--has been nearly eliminated with the publication in recent years of oscillator strengths for most of the important lines of Fe I (Corliss and Warner 1964); Fe II, Ti II, Sc II, V II, Cr II, Mn II, Co II (Warner 1967a); Sc I, Ti I, V I, Cr I, Mn I, Co I and Ni I (Allen 1960, Allen and Corliss 1963).

Similarly, the observational difficulties of the method have been somewhat assuaged. With the advent of faster photographic emulsions, coudé spectrographs with fast cameras, and higher quality diffraction gratings, more accurate high dispersion studies of solar and stellar spectra became possible; and more elaborate theoretical curves of growth were developed to analyze the observational material. To supplement the curves of Struve and Elvey and those of van der Held, Wrubel computed curves
based on Chandrasekhar's exact solutions to a) the MilneEddington model (Wrubel 1949, Chandrasekhar 1947) and b) the Schuster-Schwarzschild model (Wrubel 1954, Chandrasekhar 1950, p. 320). These models are discussed in more detail in Chapter III of the present work.

When very high dispersion data (< $5 \mathrm{~A} / \mathrm{mm}$ ) are available, methods other than the curve of growth can be used to determine the behavior of the velocity fields in stars. In 1952 Huang and Struve derived a correlation between the equivalent widths of lines and their half widths. This method was the first one developed exclusively for the purpose of examining the behavior of velocity fields in stellar atmospheres, and both the microturbulent and macroturbulent velocities were to be obtained using the curve of line width correlation. However, Huang and Struve neglected the fact that the spectrograph broadens the lines, and thus the macroturbulent velocity derived from their method is unreliable. Nevertheless, the microturbulent velocity can still be determined if the instrumental profile does not dominate the shape of the observed stellar lines. Since this method requires very high
dispersion, it has not been applied to any but the brightest stars (e.g. Wehlau 1956, Huang and Struve 1952).

The most recent method developed to determine the microturbulent velocity is that derived by Goldberg (1958), and applied to the sun by Unno (1959a,b). The GoldbergUnno procedure requires observations of the highest possible precision of two spectral lines from the same multiplet of a given element. By measuring the half width of the lines at various values of the residual intensity, the run of microturbulence with optical depth in the atmosphere is obtained directly. This procedure has not been applied to stars other than the sun because the dispersion required for its use is extremely high (< $1 \mathrm{~A} / \mathrm{mm}$ ). However, with the photoelectric scanning devices which are becoming increasingly available for stellar telescopes, the GoldbergUnno procedure may prove to be a valuable technique for stellar astrophysicists in the future.

One of the most extensive investigations of both abundances and microturbulent velocities was carried out by Wallerstein in 1961. He found that a) the metal abundance (as indicated by the iron-to-hydrogen ratio, $\mathrm{Fe} / \mathrm{H}$ ) is high for stars in nearly circular orbits about the
galactic center, and b) the metal abundance is low for stars in highly elliptical orbits--the high-velocity stars (Roman 1955). This seemed to confirm the theory of the origin of the galaxy as proposed later by Eggen, LyndenBell, and Sandage (1962). According to their theory, the stars which were formed early in the initial collapse of the proto-galaxy were composed of the primordial material of the galaxy--presumably pure hydrogen and helium. Some of these stars are those which are presently in highly eccentric orbits about the galactic center. The stars which were formed in the disc were created out of a material which was enriched with heavy elements produced in the interiors of the first generation stars, and thus these stars would be expected to have a higher metal content. This is indeed what was found by Wallerstein in his investigation. In addition, Wallerstein confirmed the expected correlation between the UBV photometric metal indicator (the $U-B$ excess, $\Delta(U-B)$ ) and the $F e / H$ ratio. In the Strömgren four-color system, $m_{1}$ is a photometric index defined on the basis of four narrow-band filters $v, b$, and y centered at 4100A, 4700A, and 5500A, respectively:

$$
\begin{equation*}
m_{1}=(v-b)-(b-y) \tag{1.1}
\end{equation*}
$$

Thus the index is a measure of the energy absorbed in the heavily blanketed blue region of the spectrum as compared to the reasonably line-free green region (Strömgren 1963). In this same system $\Delta m_{1}$ is defined as the difference between the value of the $m_{1}$ index for a zero-age star with a certain $b-y$ index and that of any other star with the same $b-y$. Wallerstein has shown that $\Delta_{m_{1}}$ is also strongly correlated with the $\mathrm{Fe} / \mathrm{H}$ ratio. The importance of these correlations is clear. For many purposes it is essential to obtain estimates of the metal abundances of large numbers of stars, and high dispersion spectrophotometric studies can be carried out only in limited numbers. Thus the proper choice of a metal-sensitive index such as $\Delta(U-B)$ or $\Delta m_{1}$ makes feasible a program such as that carried out by Eggen, Lynden-Bell, and Sandage.

However, in 1966 Conti and Deutsch published an important paper in which they argued that the $\Delta \mathrm{m}_{1}$ index in solar-type stars is much more strongly affected by slight changes in the microturbulent velocity than it is by substantial changes in the metal abundance. This paper, however, contained a mathematical error which was latex corrected (Conti and Deutsch 1967). The end result is
that $\Delta m_{1}$ is approximately twice as sensitive to changes in the microturbulent velocity as it is to changes in metal abundance. Therefore, Conti and Deutsch believe that it is possible that Wallerstein's abundance determinations are in error and that the important parameter being measured by $\Delta m_{1}$ and $\Delta(U-B)$ is microturbulence rather than metal abundance.

Since this important paper, two observational tests of this theory have been published (Barry 1967a, McNamara 1967) both of which seem to confirm the fact that $\Delta m_{1}$ and $\Delta(U-B)$ differ from one star to another of the same spectral type because of metal abundance differences and not because of differences in microturbulent velocity. Their arguments are, however, based on photometric observations; ultimately, the question can be resolved only spectroscopically. Because large-scale studies of our galaxy with respect to element abundances can be carried out only photometrically, it is essential that the ContiDeutsch effect be clarified. Until then the photoelectric photometrist is uncertain of the meaning of his measurements.

From an astrophysical standpoint, a study of microturbulence is also of great interest. It is known that the outer convection zone plays an important role in the energy transport in stars later than FO and that through the F stars this zone increases in importance (Schwarzschild 1961. Böhm-Vitense 1964). It would seem quite reasonable that the observed microturbulence in stars is in some way connected with both the position of the convective zone and the velocity of the material at the surface of the zone. The situation is complicated, however, by the fact that the convective zone is increasing in importance and the equatorial rotational velocity of the stars decreases sharply in this spectral range (Slettebak 1955, Abt and Hunter 1962). The suggestion has been made (Kippenhahn 1959 ) that for rapidly rotating stars microturbulent velocities might originate from meridional circulations set up by the rapid rotation. This would further complicate the picture in two ways. First, it might then be that the observed microturbulent velocity will depend upon the angle at which the star is observed. Since the equivalent widths of lines can be measured only for stars with small values of $V$ sin $i$ (Chapter II), sharp-lined intrinsically
rapid rotators are necessarily seen pole-on. If meridional circulations are more vigorous near the poles, then for rapid rotators they might dominate the microturbulent velocity. Second, the fact that the outer convective zone increases in importance as the rotation decreases would tend to make these two effects difficult to disentangle. A certain amount of information exists, however, about the behavior of microturbulence in certain classes of stars. In his study of G dwarfs, Wallerstein (1961) found that the microturbulent velocity for the metal-poor high-velocity stars is lower than for $G$ dwarfs with normal metal abundance. In fact, he found a direct correlation between the microturbulent velocity and the metal abundance in his sample of stars. Until recently no attempt has been made to place any astrophysical interpretation on this correlation. Fischel (1964), in his theoretical calculations, has been unable to find any correlation between convective velocity (which he expects the microturbulent velocity to reflect) and metal abundance. He explains this as being caused by his inadequate treatment of convection in his models.

However, Strom (1968a) has recently demonstrated that the observed correlation between microturbulence and metal abundance is not a real effect but is a result of Wallerstein's improper application of the differential curve of growth procedure to stars which differ greatly from the sun in metal abundance. By taking account of the difference in temperature structure of the sun and of a metal-poor star resulting from the different degree of line blanketing of the two stars, Strom has shown that the microturbulent velocities derived by Wallerstein are in error for metal-poor stars and that these stars have, in fact, normal microturbulent velocities. On the other hand, the metallic line stars have traditionally been considered to have high microturbulent velocities and to be intrinsically slow rotators (Conti 1965).

It is difficult to extract, from existing observations, the values of the microturbulent velocity over a wide range in spectral types. As has been mentioned previously, many observers use the differential procedure which, as Strom has shown, is subject to serious restrictions, and to apply it to stars over a wide range in spectral type is unacceptable. Even more serious is the fact
that the equivalent widths of lines in a given star measured at different observatories differ significantly from each other; and these differences are often larger than the order of magnitude of the effect which is to be measured (Chapter II). In addition, observers use many different theoretical curves of growth as a basis upon which to derive their values of abundances and microturbulent velocities, making intercomparisons difficult from a theoretical as well as from an observational standpoint.

All of the above emphasizes the need for a systematic study of the behavior of the microturbulent velocity among the normal stars. Traditionally this parameter has often been considered a secondary product of the curve of growth analysis. Most of these analyses have been undertaken to study unusual stars (such as supergiants, Am and Ap stars or high-velocity stars) based on the belief that the normal stars are well behaved and thus are a less important subject for the astrophysicist's scrutiny. The Conti and Deutsch paper has exposed the great need for a systematic study of these normal stars, and it is in order to fulfill this need that the present investigation has been undertaken.
II. DATA ACQUISITION AND REDUCTION

## A. Selection of Stars

For a systematic survey of the microturbulent velocities among normal stars, enough stars must be chosen so that the data are statistically meaningful; at the same time, the stars should cover a sufficiently wide range in effective temperature so that any general dependence of microturbulence upon spectral type may be detected. In order to satisfy these criteria, the following restrictions were placed upon the stars to be observed:
a) All stars are in the Yale Bright Star Catalogue (Hoffleit 1964).
b) MKK spectral types are available for all stars.
c) The stars are all luminosity class V objects.
d) Four-color photometry is available for all stars (Strömgren and Perry 1965).
e) The stars have a small enough projected rotational velocity (V sin i) such that broadening and blending of the lines is not acute.

These restrictions are nearly self-explanatory except that some comment should be made about restriction e).

The value of $V$ sin $i$ at which line broadening and blending become critical depends upon the spectral type of the star in question. In stars later than about $F O$, the crowding of the spectrum is such that the blending of the lines caused by stellar rotation is the chief problem. In earlier stars, whereas the lines are well separated, the weakness of the metallic absorption lines is such that rotation will wash out nearly all but the very strongest lines, making indeterminate the value of the microturbulent velocity as derived from the curve of growth analysis. After careful examination it was found that any star with $V \sin i>40 \mathrm{~km} / \mathrm{sec}$ has its lines either broadened or blended to such an extent as to make its spectrum too difficult to measure. Thus restriction e) above places an upper limit on the value of $V \sin i$ of $40 \mathrm{~km} / \mathrm{sec}$.

Figure 1 is a histogram showing the number of main sequence stars in each tenth of a spectral class interval in the Bright Star Catalogue (Hoffleit 1964). Figure 2 is a histogram identical to Figure 1 except that only those stars with observed values of $V$ sin $i$ less than $80 \mathrm{~km} / \mathrm{sec}$ have been included. The rotational velocities of these stars have been taken from the rotational velocity


Fig. 1. Distribution of Spectral Classes of Dwarf Stars


Fig. 2. Distribution of Spectral Classes for Stars with $V \sin i<80 \mathrm{~km} / \mathrm{sec}$.
catalogue of Boyarchuk and Kopylov (1964). The lack of stars between $A 4 V$ and $F 5 V$ with sufficiently low values of $V \sin i$ to allow them to be studied is immediately apparent. The final list of stars which were observed and which fulfill all of the above restrictions is given in Table 1. Column 1 contains the number which will be used to locate the star on the various figures which will be presented in the present study; column 2, the star's name; column 3, its Harvard Revised number; column 4, its Henry Draper number; columns 5 and 6, its celestial coordinates (epoch 2000); columns 7 and 8, its galactic coordinates; and column 9, its parallax. On the second page of Table 1 the star's name is repeated in column 1; columns 2 and 3 contain the components of its proper motion; column 4, its radial velocity; column 5, its spectral type; column 6, its $V$ magnitude; column 7, its $B-V$ color index; columns 8-10, its Strömgren four-color indices (Strömgren and Perry 1965); and column 11, its V sin $i$ value when available (Boyarchuk and Kopylov 1964). In the case of one star, 9 Com, the classification and four-color photometry were taken from the dissertation of Donald Barry (1967b).

Table 1 Properties of Program Stars

| No. | Name | HR | HD | $\begin{gathered} \alpha(2000) \\ \mathrm{h} \quad \mathrm{~m} \quad \mathrm{~s} \\ \hline \end{gathered}$ |  |  | 8 (2000 | -00) | $\begin{aligned} & \text { II } \\ & 0 \end{aligned}$ |  | $\begin{gathered} \mathrm{b}^{I I} \\ 0 \end{gathered}$ |  | $\pi$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 47 UMa | 4277 | 95128 | 10 | 59 | 28 | +40 | 26 | 175 | 46 | +63 | 22 | +. 073 |
| 2 | $\beta C V n$ | 4785 | 109358 | 12 | 33 | 45 | +41 | 21 | 136 | 5 | +75 | 19 | +. 108 |
| 3 |  | 672 | 14214 | 2 | 18 | 2 | $+1$ | 45 | 162 | 12 | -54 | 24 | +. 036 |
| 4 | $\beta$ Com | 4983 | 114710 | 13 | 11 | 52 | $+4$ | 40 | 43 | 14 | +85 | 24 | +. 120 |
| 5 | 99 Her | 6775 | 165908 | 18 | 7 | 2 | +30 | 34 | 56 | 58 | +22 | 18 | +. 058 |
| 6 | $\beta$ Vir | 4540 | 102870 | 11 | 50 | 42 | $+1$ | 46 | 270 | 27 | +60 | 45 | +. 098 |
| 7 | - | 5691 | 136064 | 15 | 18 | 24 | +67 | 21 | 104 | 36 | +44 | 19 | +. 046 |
| 8 | 9 Com | 4688 | 107213 | 12 | 19 | 29 | +28 | 9 | 202 | 44 | +82 | 51 | +. 021 |
| 9 | $\theta$ Per | 799 | 16895 | 2 | 44 | 12 | +49 | 13 | 141 | 9 | - 9 | 37 | +. 077 |
| 10 | 110 Her | 7061 | 173667 | 18 | 45 | 39 | +20 | 33 | 50 | 48 | +10 | 27 | +. 049 |
| 11 | $\times \mathrm{Cnc}$ | 3262 | 69897 | 8 | 40 | 3 | +27 | 13 | 195 | 47 | +30 | 27 | +.061 |
| 12 | 22 Lyn | 2849 | 58855 | 7 | 29 | 57 | +49 | 41 | 241 | 57 | - 2 | 26 | +. 044 |
| 13 | $\pi^{3}$ Ori | 1543 | 30652 | 4 | 49 | 51 | $+6$ | 57 | 191 | 27 | -23 | 5 | +. 125 |
| 14 | $\sigma$ Boo | 5447 | 128167 | 14 | 34 | 41 | +29 | 45 | 45 | 38 | +67 | 13 | +. 063 |
| 15 | $\gamma \operatorname{VirA}$ | 4825 | 110379 | 12 | 41 | 40 | - 1 | 27 | 297 | 51 | +61 | 20 | +. 101 |
| 16 | $\gamma$ VirB | 4826 | 110380 | 12 | 41 | 40 | - 1 | 27 | 297 | 51 | +61 |  | +.101 |
| 17 | 37 UMa | 4141 | 91480 | 10 |  | 9 | +57 | 5 | 152 | 16 | +51 | -34 | +. 023 |
| 18 | $\eta$ Lep | 2085 | 40136 | 5 | 56 | 24 | -14 | 10 | 219 | 45 | -18 | 29 |  |
| 19 | 9 Aur | 1637 | 32537 | 5 | 6 | 41 | +51 | 36 | 157 | 3 | + 6 | 30 | +.011 |
| 20 | 30 LMi | 4090 | 90277 | 10 | 25 | 55 | +33 | 47 | 191 | 51 | +58 | 6 | ---- |
| 21 | 14 Aur | 1706 | 33959 | 5 | 15 | 25 | +32 | 41 | 173 | 18 | - 3 | 22 | +. 007 |
| 22 | 95 Leo | 4564 | 103578 | 11 | 55 | 41 | +15 | 39 | 251 | 39 | +72 | 41 | -. 004 |
| 23 | $\theta$ Leo | 4359 | 97633 | 11 | 14 | 15 | +15 | 26 | 235 | 20 | +64 | 35 | +. 019 |

Table 1 cont.

| Name | $\mu_{\alpha}$ | $\mu_{\delta}$ | $\mathrm{V}_{r}$ | Sp | V | B-V | $\mathrm{b}-\mathrm{y}$ | $\mathrm{m}_{1}$ | $\mathrm{C}_{1}$ | $V \sin$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 47 UMa | -. 318 | +. 052 | +13 | GOV | 5.06 | . 61 | . 392 | . 203 | . 337 | --- |
| $\beta \mathrm{CVn}$ | -. 705 | +. 284 | + 7 | GOV | 4.29 | . 59 | . 385 | . 182 | . 296 | <15 |
| HR 672 | +. 372 | +.381 | $+27 \mathrm{~V}$ | F9V | 5.58 | . 60 | . 373 | . 192 | . 382 | --- |
| $\beta$ Com | -. 799 | +.876 | + 6 | GOV | 4.28 | . 58 | . 372 | . 193 | . 336 | --- |
| 99 Her | -. 098 | +. 067 | + 1 | F7V | 5.04 | . 52 | . 361 | . 143 | . 326 | --- |
| $\beta$ Vir | +. 742 | -. 277 | + 5 | F8V | 3.61 | . 55 | . 354 | . 190 | . 412 | --- |
| HR 5691 | +. 218 | -. 394 | -47 | F8V | 5.14 | . 53 | . 350 | . 177 | . 422 | --- |
| 9 Com | -. 198 | -. 134 | - 8 | F8Va | 6.20 | --- | . 336 | . 192 | . 451 | --- |
| $\theta$ Per | +. 337 | -. 087 | +25 | F7V | 4.12 | . 48 | . 326 | . 165 | . 373 | $<25$ |
| 110 Her | -. 014 | -. 338 | +24 | F6V | 4.20 | . 46 | . 314 | . 150 | . 484 | 5 |
| $\times$ Cnc | -. 015 | -. 384 | +33 | F6V | 5:13 | . 47 | . 314 | . 146 | . 384 | $<20$ |
| 22 Lyn | +. 116 | -. 085 | -27 | F6V | 5.35 | . 46 | . 308 | . 142 | . 390 | <20 |
| $\pi^{3}$ Ori | +.468 | +. 018 | +24 | F6V | 3.19 | . 45 | . 299 | . 162 | . 413 | 20 |
| $\sigma$ Boo | +. 187 | +. 124 | 0 | F2V | 4.45 | . 36 | . 254 | . 1,35 | . 490 | $<25$ |
| $\gamma$ VirA | -. 567 | +. 005 | -20 | FOV | 3.65 | . 35 | . 245 | . 147 | . 528 | 25 |
| $\gamma$ VirB | -. 567 | +. 005 | -20 | FOV | 3.68 | . 35 | . 245 | . 147 | . 528 | 40 |
| 37 UMa | +. 066 | +. 033 | -12 | FlV | 5.16 | . 34 | . 228 | . 159 | . 574 | 45 |
| $\eta$ Lep | -. 041 | +. 138 | - 2 | FOV | 3.70 | . 33 | . 218 | . 163 | . 625 | $<25$ |
| 9 Aur | -. 024 | -. 175 | - 1 | FOV | 4.94 | . 34 | . 217 | . 152 | . 642 | $<25$ |
| 30 LMi $^{1}$ | -. 068 | -. 069 | +13 | FOV | 4.73 | . 26 | . 151 | . 195 | . 956 | 30 |
| 14 Aur | -. 020 | +.011 | -10V | A9V | 5.06 | . 20 | . 130 | . 180 | . 998 | 33 |
| 95 Leo | +. 010 | -. 007 | -21V | A3V | 5.47 | . 10 | . 067 | . 168 | 1.108 | 20 |
| $\theta$ Leo | -. 059 | -. 085 | + 8 | A2V | 3.31 | -. 01 | . 002 | . 150 | 1.161 | 2 |

In the Strömgren photometric system, $c_{1}$ is an index defined on the basis of three narrow-band filters $u, v$, and $b$ centered at $3500 A, 4100 A$, and 4700A, respectively.

$$
\begin{equation*}
c_{1}=(u-v)-(v-b) \tag{2.1}
\end{equation*}
$$

This index is a measure of the Balmer discontinuity and hence a measure of the luminosity of the star (Strömgren 1963). The index $\Delta c_{1}$ is defined as the difference between the $c_{1}$ index of a star with a given $b-y$ and that of a zero age star having the same $b-y$.

Figure 3 is a $c_{1}, b-y$ diagram and Figure 4 , an $m_{1}$, $b-y$ diagram for the stars in Table 1 , and the locations of the zero age main sequence lines are those given by Crawford (1966). The numbers near each point in the figures correspond to the numbers given in the first column of Table 1 . In the $c_{1}, b-y$ diagram, the stars were selected so that two sequences are outlined. One group of stars lies as close to the zero age line as possible while the other group was chosen as far away from the zero age line as possible with the restriction that the star's luminosity classification be that of a dwarf. These sequences were chosen with the hope that if the microturbulence is a function of the star's age, any changes in


Fig. 3. $c_{1}, b-y$ Diagram for Program Stars


Fig. 4. $m_{1}, b-y$ Diagram for Program Stars
the microturbulence may be detectable in the early evolution of the star.

The lack of stars in the spectral region already mentioned is reflected by the large gap in the region $.100<\mathrm{b}-\mathrm{y}<.200$. In an attempt to find dwarf stars with small values of $V$ sin $i$ in this region, a search program was undertaken to obtain high dispersion plates of all stars in this region of the $c_{1}, b-y$ diagram. The search, which eventually included 38 stars, yielded only two stars whose lines are sharp enough to satisfy restriction e) above. One of these is an Am star and the other a subgiant; thus they were both rejected for failing to satisfy restriction c).

That no sharp-lined stars were found is not startling. The mean value of $V$ sin $i$ for field stars in this range of $\mathrm{b}-\mathrm{y}$ is approximately $90 \mathrm{~km} / \mathrm{sec}$ (Abt and Hunter 1962). For a random orientation of rotational axes, the correction for the mean projection factor is given by

$$
\begin{equation*}
\mathrm{V}=\frac{4}{\pi} \overline{\mathrm{~V} \sin \mathrm{i}} \tag{2.2}
\end{equation*}
$$

(Chandrasekhar and Münch 1950). Therefore, the mean equatorial rotational velocity for these stars is $120 \mathrm{~km} / \mathrm{sec}$. Because for the present investigation the value of $v$ sin $i$
must be less than $40 \mathrm{~km} / \mathrm{sec}$ for such stars, it can be shown that only 1 star in 20 will satisfy restriction e) if the rotational axes are randomly oriented.

## B. The Observations

Observations were obtained at the coude focus of the 84-inch telescope at Kitt Peak National Observatory, Tucson, Arizona. Because of the crowding of the lines in the blue region of the spectrum of stars later than FO , these stars were observed in the green region using the Kodak IIa-D emulsion. Stars earlier than $F O$ were observed in the blue region using the Kodak IIa-0 emulsion. Two stars $\gamma$ Vir A and $\eta$ Lep, were observed in both regions to provide a tie-in between the two regions. In both wavelength regions the $f / 4.1$ Schmidt camera was used. In the green region the spectra were obtained using the 600 line/mm Bausch and Lomb grating in the first order. This yields a dispersion on the plate of $17.7 \mathrm{~A} / \mathrm{mm}$. In the blue region the spectra were obtained in second order which gives a dispersion of $8.9 \mathrm{~A} / \mathrm{mm}$. Because of spectrophotometric scatter which is inherent in all photographic spectroscopy, an attempt was made to obtain three spectra for each star. In order to further reduce scatter, the
spectra were widened to 0.66 mm (twice the normal width), and the slit width was set at $15 \mu$ on the plate. In addition to stellar spectra, a spectrum of the daylight sky was obtained in the green to provide a tie-in with the well-studied solar curve of growth.

The final list of observations obtained is given in Table 2. Column 1 gives the star name or its HR number; column 2, the dispersion used; and column 3, the number of spectra obtained at that dispersion.

A spot sensitometer was used to obtain photographic calibration of the spectrograms. Color filters were used to isolate various wavelength regions of the spectrum in order to compensate for the dependence of the characteristic curve on wavelength. In the green region, spot plates were taken at three different wavelength regions; in the blue, the spot plates were taken at two regions. The properties of the filters used in the spot sensitometer are given in Table 3. The spot sensitometer uses plates $2 \frac{1}{4}$ by $3 \frac{1}{4}$ inches in size. Usually the calibration plates and the plate on which the spectrum was exposed were cut from the same 8 by 10 inch plate. In all cases they were cut from plates in the same box. The plates (stellar and

Table 2 Information on Spectra Obtained

| Object | Dispersion | No. |
| :---: | :---: | :---: |
| Sky | 17.8 | 1 |
| 47 UMa | 17.8 | 3 |
| $\beta \mathrm{CVn}$ | 17.8 | 3 |
| HR 672 | 17.8 | 3 |
| $\beta$ com | 17.8 | 3 |
| 99 Her | 17.8 | 3 |
| $\beta$ Vir | 17.8 | 3 |
| HR 5691 | 17.8 | 3 |
| 9 com | 17.8 | 1 |
| $\theta$ Per | 17.8 | 3 |
| 110 Her | 17.8 | 3 |
| $x$ Cnc | 17.8 | 3 |
| 22 Lyn | 17.8 | 3 |
| $\pi^{3}$ Ori | 17.8 | 4 |
| $\sigma$ Boo | 17.8 | 3 |
| $\gamma$ VirA | 17.8 | 4 |
| $\gamma$ VirA | 8.9 | 4 |
| $\gamma$ VirB | 8.9 | 4 |
| 37 UMa | 8.9 | 3 |
| $\eta$ Lep | 17.8 | 3 |
| $\eta$ Lep | 8.9 | 3 |
| 9 Aur | 17.8 | 3 |
| 30 LMi | 8.9 | 3 |
| 14 Aur | 8.9 | 3 |
| 95 Leo | 8.9 | 3 |
| $\theta$ Leo | 8.9 | 3 |

$\left.\begin{array}{l}\text { Table } 3 \text { Filter Properties for Spot Sensitometer } \\ \text { Filter No. } \\ \begin{array}{ccc}\text { Central } \\ \text { (A) }\end{array}\end{array} \begin{array}{c}\text { Half Width } \\ \text { (A) }\end{array}\right]$

Table 4 Intensity Calibration for Spot Sensitometer

| Spot No. | Relative <br> Intensity |
| :---: | :---: |
| 1 | .067 |
| 2 | .110 |
| 3 | .194 |
| 4 | .243 |
| 5 | .421 |
| 6 | .579 |
| 7 | 1.00 |
| 8 | 1.36 |
| 9 | 2.04 |
| 10 | 3.13 |
| 11 | 5.02 |
| 12 | 7.22 |
| 13 | 9.87 |

calibration) were developed simultaneously in D-76 developer at a temperature of $68^{\circ}$ for 15 minutes.

Relative intensities of the spots for the spot sensitometer were measured photoelectrically at Kitt Peak with an ITT FW-130 photocell. The photocell was masked with a brass plate in which a slit-shaped aperture had been cut; the length of the aperture was $1 / 3$ the spot diameter, and the width was .02 inches. The photocell assembly was mounted on a lathe micrometer crossfeed device to allow the assembly to be positioned accurately in the focal plane of the spot sensitometer. A Keithley electrometer was used to measure the output of the photocell. For all 13 of the $3 / 16$ inch diameter spots, intensity maps were made by moving the photoce11 in .02 inch steps in the focal plane. For each spot, the map is an 8 by 8 grid, each grid point containing the Keithley reading for that point in the spot. The resulting relative intensity calibration for the 13 spots is given in Table 4.

Because of vignetting in the spot sensitometer, some of the spots appearing on the photographic plate are quite non-uniform, having the shape of a gibbous moon. Nevertheless, the photoelectric calibration of the
sensitometer allows an unambiguous interpretation to be placed on the transmissions which result when the calibration plates are scanned with a microphotometer.

In order to check the accuracy of the photoelectric calibration of the Kitt Peak spot sensitometer, a comparison test was carried out between it and the device designed and built by Mr. David Latham at Harvard University. His spot sensitometer is similar to the one used at Kitt Peak except that it has 40 spots instead of 13 and the illumination of the spots is very uniform. The device has been calibrated to an accuracy of 0.1 per cent (Latham 1967). For the comparison, exposures were taken in the same spectral region with each device, and the plates were processed simultaneously. The characteristic curves obtained from each plate are shown in Figure 5. The solid line is the curve obtained using Latham's device. The X's represent the points measured with the Kitt Peak device. The differences between the curves are smaller than the errors associated with measuring each point; thus the Kitt Peak device is in very satisfactory agreement with the more elaborate and presumably more reliable spot sensitometer.


Fig. 5. Comparison of Kitt Peak and Latham Spot Sensitometers

Finally, it was found that the differences are quite small among characteristic curves taken over a period of two years at Kitt Peak. Because every effort was made to maintain the same developing conditions in the dark room, this is not an unreasonable result. Figure 6 is a comparison of several characteristic curves taken in the blue region with a IIa-0 emulsion. The differences among the curves are very small; therefore, a mean characteristic curve has been adopted for each wavelength region. This curve was used to reduce all of the data. The behavior is similar for the IIa-D emulsion, and mean curves were used in the green region of the spectrum as well. By simple numerical experiments with the curves, it was shown that for the worst case--i.e. for the characteristic curve which differs by the largest amount from the mean curve-the difference between the equivalent width of strong lines derived from each curve on the same spectrum is about 12 per cent. This is of the order of the expected spectrophotometric scatter normally encountered in photographic spectroscopy; and because three plates were averaged for each star, the errors introduced by using the mean characteristic curve are felt to be negligible. The


Fig. 6. Intercomparison of Characteristic Curves
figure of 12 per cent has been computed based on a density in the continuum of 0.7. At a greater density than this, the slope of the characteristic curve is very large; slight changes in the characteristic curve can lead to a large difference in the measured equivalent width. Therefore, as a matter of general practice, it seems the continuum density should be 0.7 and no greater--especially when mean characteristic curves are used for the reductions.

## C. Reduction of Spectrograms

Because a large number of spectrograms were to be reduced for this study, it was felt that some labor-saving technique should be attempted in the reductions. With this in mind, the spectra were scanned on the Boller and Chivens microphotometer in the Solar Division at Kitt Peak. This instrument digitally samples the spectral plate as it is scanned, and the resulting transmission readings are output onto a digital magnetic tape recorder. The characteristic curve was then read into the observatory's CDC 3200 computer where the transmissions were converted to intensities and written onto another magnetic tape. This tape was used as input to a computer program largely adapted from the solar photoelectric spectrum reduction
program written by Dr. James Brault of the Solar Division at Kitt Peak. Brault's program smoothes the observed spectrum by taking its Fourier transform; later, the noise cutoff of the transform is determined. The computer then restores the spectrum, leaving out all space frequencies above the cutoff of the transform.

The adapted program fits a fourth-order polynomial to the continuum, and, given the dispersion and a list of wavelengths of lines to be measured, produces equivalent widths of the lines. However, at the relatively low dispersions being used, the blending of the lines is usually such that the observer must complete much of the line profile by hand on the tracing; this problem remains with the computer-produced tracings. It was therefore decided that the computerized technique, although it is indispensible for the analysis of accurate high-resolution photoelectric scans, is more time consuming than conventional means for reductions of low dispersion spectra; and this data reduction procedure was abandoned.

The system which was eventually used to reduce the spectra has three major components: a) a Hilger-Watts microphotometer" b) a Moseley "Autograph" $x-y$ plotter with
a curve-following attachment, and c) a Brown chart recorder. The upper slit of the microphotometer is focused onto the plate. This slit defines that part of the spectrum which is measured as it is moved past the slit by the microphotometer carriage. Light transmitted through the plate then passes through a second slit and is measured by a photocell. The output of this cell is amplified and is used as the input to the $x$-coordinate of the Autograph. A region of clear plate is scanned to establish the 100 per cent transmission point on the $x$-axis, and an opaque screen is placed between the plate and the upper slit to establish the 0 per cent transmission point. The curve follower will then lock onto the characteristic curve placed on the table of the Autograph. The $y$-coordinate is the output which is used as the input to the Brown recorder, and the resulting tracing of the spectrum will be in relative intensity.

Using the above technique, each observed spectrum was scanned to produce a relative intensity tracing. The upper slit was set at a projected width on the plate of $10 \mu$ and at a projected length of 0.66 mm , the latter corresponding to the width of the spectrum on the plate.

In determining the microturbulent velocity from a curve of growth analysis, it is desirable to use the absorption lines of one element in a single stage of ionization. Two difficulties are thereby avoided: a) the abundance determination problem which exists when a composite observational curve of growth is made from several elements, and b) the ionization temperature determination problem which exists when a composite curve is made from lines of differing stages of ionization. The spread in strengths of the lines should be large in order to obtain the least ambiguous fit of the observed curve of growth to the theoretical one, and there should be a sufficient number of lines of the element to subsist throughout a wide temperature range:

The element which best satisfies these criteria is neutral iron (Fe I); therefore, only lines of Fe $I$ were measured in all stars. For stars later than $F O$ which were measured in the green region of the spectrum the Utrecht Atlas (Minnaert, Mulders, and Houtgast 1940) was used to choose the iron lines to be measured. Only those lines which are well isolated from other lines were chosen. For stars earlier than $F O$, the iron lines were selected
systematically by identifying all the lines of multiplets which are in the blue region of the spectrum. Blends were eliminated by the use of the Multiplet Table (Moore 1945) to identify lines of other elements which might interfere with nearby iron lines. The final list of iron lines for the green region contains 93 lines; that for the blue region contains 89 lines.

The equivalent widths for these lines were measured on each stellar spectrum. The continuum was located by choosing high points on the relative intensity tracings and drawing a smooth curve through these points. First, a planimeter was used to measure the area under each absorption line. However, for all stars except 37 UMa and $\gamma$ Vir B, the lines are sufficiently sharp to be considered to be triangles. Therefore, only the depth and the width at half depth (half width) were measured for each line. Because the instrumental profile for the spectrograph is more nearly Gaussian than it is triangular, the triangular area was multiplied by 1.06 to convert it to a Gaussian curve having the same depth and half width. For approximately $1 / 3$ of the lines in every spectrum, however, the half width is unmeasurable because either the noise of
the tracing is too great or the line is nearly unresolved, being blended with a nearby line. To obtain the equivalent width of these lines, the following procedure was adopted. A graph of the equivalent width versus depth was plotted for the lines whose equivalent width could be measured directly. A linear least-squares fit was then calculated to obtain the relation between equivalent width and depth. Subsequently, this relation was applied to those lines whose depth alone could be measured; their equivalent width was thus obtained.

Since the relevant parameter to be used for the observational curve of growth is $\log W / \lambda$ (henceforth called $\log W^{*}$ ) where $W$ is the equivalent width and $\lambda$ is the wavelength of the line (both in Angstroms), the value of $-\log W^{*}$ has been computed. Tables 5 and 6 contain this value for each line in each star. The column labeled $\lambda$ gives the wavelength of the line in Angstroms; that labeled mult, the multiplet number from which the line arises; that labeled log $g f$, the value of log $g f$ (Corliss and Warner 1964); that labeled $X_{1}$, the excitation potential for the lower energy level of the line. The remaining columns give the value of $-\log \mathrm{W}^{*}$ for the star listed at

Table 5 Observed Values of $-\log W^{*}$ (green region)

| $\lambda$ | mult | $\log g f$ | $\chi_{1}$ | sky | 47 UMa | $\beta \mathrm{CVn}$ | HR 672 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6265.14 | 62 | -2.00 | 2.18 | 4.86 | 4.75 | 4.87 | 4.81 |
| 6256.37 | 169 | -2.03 | 2.45 | 4.77 | 4.71 | 4.92 | 5.01 |
| 6252.36 | 169 | -1.29 | 2.40 | 4.73 | 4.65 | 4.93 | 4.77 |
| 6246.32 | 816 | -0.31 | 3.60 | 4.63 | 4.62 | 4.77 | 4.79 |
| 6232.66 | 816 | -0.72 | 3.65 | 5.00 | 4.84 | 5.25 | 4.86 |
| 6230.73 | 207 | -0.93 | 2.56 | 4.60 | 4.60 | 4.74 | 4.68 |
| 6213.44 | 62 | -2.15 | 2.22 | 4.93 | 4.86 | 5.06 | 4.92 |
| 6200.32 | 207 | -1.96 | 2.61 | 4.86 | 4.91 | 5.12 | 4.89 |
| 6188.04 | 959 | -0.89 | 3.94 | 5.09 | 5.16 | 5.67 | 5.15 |
| 6173.34 | 62 | -2.40 | 2.22 | 4.95 | 4.90 | 5.37 | 4.96 |
| 6151.62 | 62 | -2.71 | 2.18 | 5.10 |  |  | 5.07 |
| 6065.49 | 207 | -1.03 | 2.61 | 4.71 | 4.68 | 4.86 | 4.70 |
| 6055.99 | 1259 | 0.00 | 4.73 | 4.90 |  | 5.11 | 4.83 |
| 6027.06 | 1018 | -0.35 | 4.07 | 4.91 | 4.96 | 5.45 | 4.91 |
| 6024.07 | 1178 | 0.45 | 4.55 | 4.64 | 4.77 | 4.91 | 4.67 |
| 6008.50 | 982 | -0.37 | 3.88 | ----- | 4.76 | 4.97 | 4.65 |
| 6003.03 | 959 | -0.42 | 3.88 |  | 4.82 | 4.90 | 4.79 |
| 5987.06 | 1260 | -0.03 | 4.79 |  |  | 5.23 | 4.86 |
| 5984.80 | 1260 | 0.16 | 4.73 |  |  | 5.05 | 4.68 |
| 5983.70 | 1175 | -0.10 | 4.55 |  | 4.87 | 5.16 | 4.97 |
| 5952.75 | 959 | -0.60 | 3.98 |  | 4.81 | 4.98 | 4.86 |
| 5934.66 | 982 | -0.40 | 3.93 | ---- | 4.86 | 5.07 | 4.91 |
| 5930.17 | 1180 | 0.34 | 4.65 | ---- | 4.67 | 4.83 | 4.84 |
| 5862.36 | 1180 | 0.18 | 4.55 | 4.92 | 4.67 | 4.94 | 4.91 |
| 5816.36 | 1179 | -0.05 | 4.55 |  | 4.70 | 4.99 | 4.67 |
| 5809.24 | 982 | -0.83 | 3.88 | 4.76 | 4.87 | 5.11 | 5.00 |
| 5806.73 | 1180 | -0.20 | 4.61 | ---- | 4.85 | 5.03 | 4.88 |
| 5775.09 | 1087 | -0.31 | 4.22 | ---- | 4.99 | 4.96 | 4.88 |
| 5762.99 | 1107 | 0.25 | 4.21 | 4.59 | 4.51 | 4.65 | 4.54 |
| 5752.04 | 1180 | -0.18 | 4.55 | 4.80 | 4.60 | 4.77 | 4.76 |
| 5679.02 | 1183 | -0.02 | 4.65 | 5.14 | 4.90 | 5.04 | 4.99 |
| 5633.97 | 1314 | 0.21 | 4.99 | 4.87 | ---- | 4.89 | 4.85 |
| 5624.55 | 686 | -0.18 | 3.42 | 4.46 | 4.37 | 4.47 | 4.58 |
| 5620.53 | 1061 | -0.79 | 4.15 | ----- | 5.00 | 5.23 | 5.44 |
| 5586.76 | 686 | 0.34 | 3.37 | 4.37 | 4.28 | 4.40 | 4.55 |
| 5576.10 | 686 | -0.31 | 3.43 | 4.71 | 4.56 | 4.64 | 4.74 |
| 5569.63 | 686 | 0.07 | 3.42 | 4.55 | 4.46 | 4.56 | 4.63 |
| 5567.40 | 209 | -1.98 | 2.61 | ---- | 4.84 | 4.96 | 4.91 |
| 5565.71 | 1183 | 0.14 | 4.61 | 4.81 | 4.75 | 4.73 | 4.62 |
| 5554.90 | 1183 | 0.00 | 4.55 | 4.82 | 4.72 | 4.86 | 4.81 |

Table 5 . Values of $-\log W^{*}$ (green region)--cont.

| $\lambda$ | $\beta$ Com | 99 Her | $\beta$ vir | HR 569 | 9 Com | $\theta$ Per | 110 Her |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6265.14 | 4.89 | 4.95 | 4.92 | 4.90 | 4.74 | 4.96 | 4.93 |
| 6256.37 | 4.76 | 5.12 | 4.80 | 4.97 | 4.88 | 4.73 | 4.76 |
| 6252.56 | 4.71 | 4.73 | 4.64 | 4.91 | 4.63 | 4.72 | 4.71 |
| 6246.33 | 4.65 | 4.96 | 4.65 | 4.87 | 4.59 | 4.57 | 4.72 |
| 6232.66 | 4.84 | 5.00 | 4.83 | 5.02 | 4.79 | 4.79 | 4.96 |
| 6230.73 | 4.53 | 4.69 | 4.57 | 4.55 | 4.52 | 4.58 | 4.66 |
| 6213.44 | 4.91 | 5.02 | 4.78 | 4.87 | 4.63 | 4.89 | 4.90 |
| 6200.32 | 5.02 | 5.19 | 4.91 | 5.02 | 4.64 | 4.86 | 4.93 |
| 6188.04 | 5.30 | 5.31 | 5.26 | 5.13 | 5.18 | 5.07 | 5.24 |
| 6173.34 | 4.96 | 5.14 | 4.88 | 5.04 | 4.84 | 4.80 | 5.14 |
| 6151.62 | 5.14 | 5.35 | 5.15 |  |  | 5.12 | 5.09 |
| 6065.49 | 4.74 | 4.77 | 4.65 | 4.74 | 4.62 | 4.64 | 4.65 |
| 6055.99 | 4.78 | 5.07 | 4.80 | 5.01 | 4.74 | 4.76 | 4.89 |
| 6027.06 | 4.99 | 5.14 | 4.89 | 4.96 | 4.85 | 4.89 | 5.12 |
| 6024.07 | 4.77 | 4.82 | 4.59 | ---- | 4.65 | 4.74 | 4.79 |
| 6008.50 | 4.83 | 4.87 | 4.56 | ---- | 4.67 | 4.69 | 4.69 |
| 6003.03 | 4.86 | 5.00 | 4.74 |  | 4.75 | 4.80 | 4.79 |
| 5987.06 | 4.96 | 5.17 | 4.93 |  | 4.86 | 4.89 | 4.99 |
| 5984.80 | 4.82 | 4.92 | 4.68 | ---- | 4.7 .4 | 4.69 | 4.80 |
| 5983.70 | 4.87 | 5.06 | 4.85 | 4.99 | 4.79 | 4.87 | 4.97 |
| 5952.75 | 4.82 | 5.18 | 4.75 | 4.91 | 4.72 | 4.93 | 4.82 |
| 5934.66 | 4.80 | 5.09 | 4.81 | 4.90 |  | 4.80 | 4.93 |
| 5930.17 | 4.67 | 4.95 | 4.64 | 4.79 | 4.66 | 4.72 | 4.70 |
| 5862.36 | 4.72 | 4.94 | 4.75 | 4.84 | 4.84 | 4.78 | 4.77 |
| 5816.36 | 4.72 | 5.15 | 4.66 | 4.85 | 4.67 | 4.74 | 4.83 |
| 5809.24 | 5.08 | 5.15 | 5.00 | 5.23 | 5.22 | 5.06 | 5.20 |
| 5806.73 | 4.92 | 5.14 | 4.95 | 5.10 | 4.94 | 5.05 | 4.99 |
| 5775.09 | 4.89 | 5.02 | 4.86 | 4.91 | 4.83 | 5.02 | 5.07 |
| 5762.99 | 4.49 | 4.83 | 4.50 | 4.53 | 4.46 | 4.67 | 4.65 |
| 5752.04 | 4.60 | 5.03 | 4.64 | 4.72 | 4.58 | 4.63 | 4.69 |
| 5679.02 | 4.81 | 4.99 | 4.92 | 5.01 | 4.94 | 5.00 | 4.96 |
| 5633.97 | 4.78 | 5.08 | 4.99 | 4.83 | ----- | 4.81 | 4.90 |
| 5624.55 | 4.43 | 4.66 | 4.41 | 4.56 | 4.44 | 4.51 | 4.57 |
| 5620.53 | 5.05 | 5.21 | 5.13 | 5.19 | 4.95 | 5.11 | 5.01 |
| 5586.76 | 4.30 | 4.48 | 4.37 | 4.45 | 4.42 | 4.41 | 4.46 |
| 5576.10 | 4.61 | 4.79 | 4.64 | 4.75 | 4.70 | 4.72 | 4.62 |
| 5569.63 | 4.48 | 4.68 | 4.54 | 4.66 | 4.50 | 4.58 | 4.58 |
| 5567.40 | 4.84 | 5.15 | 4.89 | 4.96 | 4.90 | 4.99 | 5.02 |
| 5565.71 | 4.64 | 4.88 | 4.62 | 4.75 | 4.57 | 4.66 | 4.72 |
| 5554.90 | 4.67 | 5.04 | 4.80 | 4.85 | 4.68 | 4.68 | 4.80 |

Table 5 Values of $-\log W^{*}$ (green region)--cont.
$\lambda \quad \mathrm{X}$ Cnc 22 Lyn $\pi^{3}$ Ori $\sigma$ Boo $\gamma \operatorname{VirA} \eta$ Lep 9 Aur

| 6265.1.4 | 4.91 | 4.92 | 5.03 | 5.22 | 5.21 | 5.24 | 5.17 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6256.37 | 4.90 | 4.92 | 4.86 | 5.30 | 5.25 | 5.22 | 5.27 |
| 6252.56 | 4.78 | 4.77 | 4.82 | 5.18 | 4.99 | 4.89 | 4.83 |
| 6246.33 | 4.79 | 4.77 | 4.77 | 4.91 | 5.02 | 4.81 | 4.96 |
| 6232.66 | 5.00 | 4.93 | 5.00 | 5.19 | 5.15 | 5.16 | 5.20 |
| 6230.73 | 4.70 | 4.64 | 4.67 | 4.69 | 4.90 | 4.73 | 4.76 |
| 6213.44 | 4.98 | 4.98 | 5.00 | 5.30 | 5.26 | 5.36 | 5.21 |
| 6200.32 | 5.12 | 5.21 | 5.12 | 5.45 | 5.49 | 5.41 | 5.21 |
| 6188.04 | 5.36 | 5.11 | 5.34 | 5.58 | ----- | 5.42 | ---- |
| 6173.34 | 5.21 | 5.07 | 5.12 | 5.38 | ----- | ---- | 5.53 |
| 6151.62 | 5.61 |  | 5.32 | 5.78 | ---- | ---- | 5.67 |
| 6065.49 | 4.81 | 4.73 | 4.73 | 4.76 | 4.98 | 4.87 | 4.72 |
| 6055.99 | 5.03 | 4.99 | 4.86 | 5.23 | 5.30 | 5.19 | 4.94 |
| 6027.06 | 5.10 | 5.08 | 5.03 | 5.50 | 5.31 | 5.30 | 5.04 |
| 6024.07 | 4.80 | 4.85 | 4.78 | 5.00 | 5.07 | 4.96 | 4.67 |
| 6008.50 | 4.96 | 4.89 | 4.79 | 5.23 | 5.07 | 4.98 | 4.96 |
| 6003.03 | 5.02 | 4.92 | 4.94 | 5.23 | 5.18 | 5.16 | 4.94 |
| 5987.06 | 5.10 | 5.08 | 5.11 | 5.30 | 5.12 | 5.14 | 5.13 |
| 5984.80 | 4.98 | 4.93 | 4.89 | 5.13 | 5.14 | 4.99 | 4.91 |
| 5983.70 | 5.05 | 4.96 | 5.02 | 5.54 | 5.14 | 5.09 | 5.02 |
| 5952.75 | 4.95 | 4.99 | 4.97 | 5.51 | 5.28 | 5.23 | 5.23 |
| 5934.66 | 5.07 | 4.98 | 4.98 | 5.26 | 5.29 | 5.22 | 5.16 |
| 5930.17 | 4.80 | 4.80 | 4.75 | 4.99 | 4.82 | 4.89 | 4.85 |
| 5862.36 | 4.84 | 4.99 | 4.80 | 5.27 | 4.88 | 4.96 | 4.89 |
| 5816.36 | 4.91 | 5.01 | 4.84 | 5.30 | 5.06 | 4.97 | 4.93 |
| 5809.24 | 5.20 | 5.20 | 5.24 | 5.27 | 5.33 | 5.24 | 5.36 |
| 5806.73 | 5.09 | 5.24 | 5.20 | 5.12 | 5.11 | 5.18 | 5.12 |
| 5775.09 | 5.09 | 5.04 | 4.93 | ----- | ----- | 5.10 | 5.33 |
| 5762.99 | 4.69 | 4.64 | 4.66 | 4.86 | 4.77 | 4.75 | 4.93 |
| 5752.04 | 4.84 | 4.74 | 4.75 | 4.98 | 4.88 | 4.94 | 4.89 |
| 5679.02 | 5.15 | 5.16 | 4.90 | 5.06 | 5.24 | 5.13 | 5.19 |
| 5633.97 | 5.04 | 5.12 | 4.93 | 5.24 | 5.03 | 5.15 | ----- |
| 5624.55 | 4.62 | 4.59 | 4.55 | 4.84 | 4.71 | 4.67 | 4.78 |
| 5620.53 | 5.23 | 5.10 | 5.35 | 5.38 | 5.36 | ----- | 5.31 |
| 5586.76 | 4.53 | 4.50 | 4.48 | 4.63 | 4.55 | 4.54 | 4.47 |
| 5576.10 | 4.77 | 4.74 | 4.71 | 4.88 | 4.96 | 4.80 | 4.76 |
| 5569.63 | 4.69 | 4.71 | 4.65 | 4.85 | 4.65 | 4.62 | 4.69 |
| 5567.40 | 5.11 | 5.26 | 5.04 | 5.40 | ----- | 5.24 | ----- |
| 5565.71 | 4.83 | 4.84 | 4.74 | . 4.94 | 4.88 | 4.88 | 4.71 |
| 5554.90 | 4.97 | 4.83 | 4.79 | 4.97 | 4.86 | 4.84 | 4.94 |

Table 5 Values of $-\log W^{*}$ (green region)--cont.

| $\lambda$ | mult | $\log \mathrm{gf}$ | $\mathrm{X}_{1}$ | sky | 47 UMa | $\beta \mathrm{CVn}$ | HR 672 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5525.55 | 1062 | -0.52 | 4.23 | 5.03 | 4.92 | 5.12 | 5.04 |
| 5522.46 | 1180 | -0.78 | 4.21 | ---- |  | 5.23 | 5.07 |
| 5506.78 | 15 | -2.44 | 0.99 | 4.73 | 4.55 | 4.64 | 4.69 |
| 5501.47 | 15 | -2.66 | 0.96 | 4.65 | 4.52 | 4.71 | 4.60 |
| 5497.52 | 15 | -2.49 | 1.0 . | 4.63 | 4.54 | 4.63 | 4.67 |
| 5473.91 | 1062 | -0.21 | 4.15 | 4.74 | 4.60 | 4.84 | 4.73 |
| 5466.40 | 1144 | -0.31 | 4.37 | 4.92 | 4.68 | 4.89 | 4.95 |
| 5445.05 | 1163 | 0.37 | 4.39 | 4.60 | 4.47 | 4.64 | 4.80 |
| 5434.53 | 15 | -1.97 | 1.01 | 4.41 | 4.46 | 4.50 | 4.55 |
| 5424.07 | 1146 | 0.91 | 4.32 | 4.32 | 4.20 | 4.38 | 4.43 |
| 5415.20 | 1165 | 0.89 | 4.39 | 4.49 | 4.40 | 4.52 | 4.60 |
| 5410.91 | 1165 | 0.68 | 4.47 | 4.56 | 4.42 | 4.56 | 4.59 |
| 5405.78 | 15 | -0.16 | 0.99 | 4.31 | 4.23 | 4.40 | 4.42 |
| 5398.28 | 1145 | -0.08 | 4.44 |  | 4.79 | 4.86 | 4.88 |
| 5397.13 | 15 | -1.88 | 0.91 | 4.39 | 4.30 | 4.39 | 4.57 |
| 5393.17 | 553 | -0.10 | 3.24 | 4.60 | 4.46 | 4.61 | 4.69 |
| 5389.48 | 1145 | 0.05 | 4.41 | 4.86 | 4.61 | 4.83 | 4.72 |
| 5383.37 | 1146 | 0.89 | 4.31 | 4.51 | 4.41 | 4.51 | 4.52 |
| 5371.49 | 15 | -1.60 | 0.96 | 4.28 | 4.23 | 4.33 | 4.33 |
| 5367.47 | 1146 | 0.65 | 4.41 | 4.59 | 4.45 | 4.58 | 4.60 |
| 5339.94 | 553 | -0.11 | 3.26 | 4.48 | 4.38 | 4.54 | 4.50 |
| 5324.19 | 553 | 0.47 | 3.00 | 4.31 | 4.22 | 4.32 | 4.40 |
| 5321.11 | 1165 | -0.47 | 4.43 | 5.15 | ---- | 5.13 | 5.29 |
| 5307.37 | 36 | -2.46 | 1.61 | 4.96 | ---- | 4.87 | 4.72 |
| 5302.31 | 553 | -0.04 | 3.28 | 4.51 | 4.41 | 4.59 | 4.61 |
| 5288.54 | 929 | -0.79 | 3.69 |  | 4.87 | 5.05 | 5.01 |
| 5281.80 | 383 | -0.25 | 3.04 | 4.60 | 4.41 | 4.58 | 4.49 |
| 5269.54 | 15 | -1.36 | 0.86 | 4.05 | 4.11 | 4.19 | 4.34 |
| 5266.56 | 383 | 0.09 | 3.00 | 4.27 | 4.21 | 4.33 | 4.38 |
| 5263.31 | 553 | -0.19 | 3.26 | 4.33 | 4.30 | 4.44 | 4.49 |
| 5253.48 | 553 | -0.80 | 3.28 | 4.82 | 4.62 | ---- | 4.73 |
| 5242.50 | 834 | -0.22 | 3.63 | 4.78 | 4.64 | 4.66 | 4.80 |
| 5232.95 | 383 | 0.39 | 2.94 | 4.35 | 4.21 | 4.30 | 4.41 |
| 5229.86 | 553 | -0.19 | 3.28 | 4.77 | 4.54 | 4.62 | 4.70 |
| 5217.40 | 553 | -0.37 | 3.21 | ---- | 4.54 | 4.69 | 4.66 |
| 5216.28 | 36 | -1.65 | 1.61 | 4.65 | ----- | 4.58 | 4.57 |
| 5215.18 | 553 | -0.17 | 3.26 | 4.58 | 4.39 | 4.46 | 4.54 |
| 5162.29 | 1089 | 0.50 | 4.18 | 4.56 | 4.35 | 4.45 |  |
| 5159.07 | 1091. | -0.25 | 4.28 | .4.96 | 4.71 | 4.88 | 5.09 |
| 5151.92 | 16 | -2.83 | 1.01 | 4.81 | 4.58 | 4.68 | 4.80 |

Table 5 Values of $-\log W^{*}$ (green region)--cont.
$\lambda \quad \beta$ Com 99 Her $\beta$ Vir HR 56919 Com $\theta$ Per 110 Her

| 5525.55 | 4.92 | 5.25 | 4.93 | 5.01 | 4.83 | 4.84 | 4.99 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5522.46 | 5.03 | 4.74 | 5.19 | 5.02 | .--- | 5.09 | 5.29 |
| 5506.78 | 4.58 | 4.74 | 4.54 | 4.67 | 4.50 | 4.59 | 4.58 |
| 5501.47 | 4.61 | 4.70 | 4.55 | 4.71 | 4.51 | 4.60 | 4.64 |
| 5497.52 | 4.54 | 4.65 | 4.57 | 4.71 | 4.53 | 4.63 | 4.58 |
| 5473.91 | 4.76 | 4.88 | 4.66 | 4.92 | 4.60 | 4.69 | 4.95 |
| 5466.40 | 4.76 | 4.86 | 4.68 | 4.79 | 4.68 | 4.74 | 4.86 |
| 5445.05 | 4.50 | 4.84 | 4.50 | 4.54 | 4.54 | 4.62 | 4.69 |
| 5434.53 | 4.47 | 4.56 | 4.45 | 4.61 | 4.42 | 4.45 | 4.56 |
| 5424.07 | 4.30 | 4.58 | 4.29 | 4.42 | 4.36 | 4.36 | 4.39 |
| 5415.20 | 4.43 | 4.56 | 4.45 | 4.58 | 4.44 | 4.46 | 4.54 |
| 5410.91 | 4.43 | 4.77 | 4.43 | 4.63 | 4.42 | 4.52 | 4.57 |
| 5405.78 | 4.30 | 4.54 | 4.34 | 4.40 | 4.31 | 4.36 | 4.41 |
| 5398.28 | 4.78 | 5.00 | 4.71 | 4.48 | 4.68 | 4.82 | 5.05 |
| 5397.13 | 4.33 | 4.55 | 4.34 | 4.51 | 4.34 | 4.41 | 4.50 |
| 5393.17 | 4.50 | 4.69 | 4.57 | 4.68 | 4.50 | 4.60 | 4.64 |
| 5389.48 | 4.76 | 4.83 | 4.73 | 4.87 | 4.65 | 4.78 | 4.90 |
| 5383.37 | 4.39 | 4.64 | 4.42 | 4.54 | 4.54 | 4.54 | 4.52 |
| 5371.49 | 4.27 | 4.47 | 4.32 | 4.38 | 4.38 | 4.38 | 4.49 |
| 5367.47 | 4.51 | 4.69 | 4.56 | 4.50 | 4.42 | 4.58 | 4.59 |
| 5339.94 | 4.47 | 4.70 | 4.51 | 4.54 | 4.43 | 4.57 | 4.61 |
| 5324.19 | 4.29 | 4.57 | 4.34 | 4.37 | 4.34 | 4.38 | 4.45 |
| 5321.11 | 4.97 | 5.01 | 5.11 | ---- | 5.04 | 4.82 | 5.29 |
| 5307.37 | 4.78 | 4.86 | 4.71 | 4.73 | 4.68 | 4.79 | 4.72 |
| 5302.31 | 4.51 | 4.64 | 4.55 | 4.52 | 4.48 | 4.67 | 4.64 |
| 5288.54 | 5.06 | 4.72 | 4.99 | 4.98 | 4.83 | 5.11 | 5.08 |
| 5281.80 | 4.47 | 4.65 | 4.52 | 4.52 | 4.56 | 4.57 | 4.61 |
| 5269.54 | 4.11 | 4.33 | 4.17 | 4.19 | 4.25 | 4.28 | 4.39 |
| 5266.56 | 4.30 | 4.48 | 4.29 | 4.29 | 4.31 | 4.36 | 4.44 |
| 5263.31 | 4.35 | 4.50 | 4.40 | 4.50 | 4.25 | 4.36 | 4.44 |
| 5253.48 | 4.77 | 4.91 | 4.88 | 4.81 | 4.69 | 4.84 | 4.82 |
| 5242.50 | 4.71 | 4.89 | 4.64 | 4.53 | 4.69 | 4.68 | 4.77 |
| 5232.95 | 4.24 | 4.41 | 4.29 | 4.33 | 4.31 | 4.35 | 4.43 |
| 5229.86 | 4.66 | 4.77 | 4.61 | 4.63 | 4.54 | 4.56 | 4.90 |
| 5217.40 | 4.58 | 4.72 | 4.68 | 4.55 | 4.60 | 4.64 | 4.70 |
| 5216.28 | 4.54 | 4.81 | 4.51 | 4.51 | 4.48 | 4.53 | 4.59 |
| 5215.18 | 4.47 | 4.79 | 4.52 | 4.46 | 4.48 | 4.55 | 4.62 |
| 5152.29 | 4.49 | 4.61 | 4.42 | 4.49 | ---- | 4.42 | 4.49 |
| 5159.07 | 4.91 | 4.98 | 4.83 | 4.90 | 4.64 | 4.82 | 5.00 |
| 5151.92 | 4.70 | 4.76 | 4.66 | 4.77 | 4.69 | 4.66 | 4.76 |
| 5 | 4.70 |  |  |  |  |  |  |

Table 5 Values of $-\log W^{*}$ (green region)--cont.
$\lambda \quad X$ Cnc 22 Lyn $\pi^{3}$ Ori $\sigma$ Boo $\gamma$ VirA $\eta$ Lep 9 Aur

| 5525.55 | 5.11 | 5.21 | 4.97 | 5.26 | 5.32 | ---- | ---- |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5522.46 | 5.62 | 5.17 | ----- | 5.46 |  |  |  |
| 5506.78 | 4.75 | 4.68 | 4.71 | 4.76 | 4.89 | 4.74 | 4.74 |
| 5501.47 | 4.71 | 4.62 | 4.69 | 4.90 | 4.81 | 4.71 | 4.83 |
| 5497.52 | 4.66 | 4.71 | 4.60 | 4.83 | 4.80 | 4.71 | 4.71 |
| 5473.91 | 5.01 | 4.88 | 4.76 | 5.05 | 5.10 | 4.85 | 5.03 |
| 5466.40 | 4.88 | 4.81 | 4.74 | 5.09 | 5.03 | 4.90 | 4.93 |
| 5445.05 | 4.71 | 4.66 | 4.64 | 4.80 | 4.70 | 4.65 | 4.63 |
| 5434.53 | 4.55 | 4.55 | 4.54 | 4.70 | 4.66 | 4.58 | 4.62 |
| 5424.07 | 4.48 | 4.46 | 4.37 | 4.61 | 4.47 | 4.46 | 4.48 |
| 5415.20 | 4.63 | 4.54 | 4.47 | 4.66 | 4.59 | 4.51 | 4.54 |
| 5410.91 | 4.59 | 4.67 | 4.49 | 4.81 | 4.69 | 4.59 | 4.68 |
| 5405.78 | 4.48 | 4.48 | 4.45 | 4.59 | 4.46 | 4.50 | 4.45 |
| 5398.28 | 4.79 | 4.98 | 4.87 | 5.08 | 5.04 | 5.18 | 4.91 |
| 5397.13 | 4.52 | 4.50 | 4.47 | 4.64 | 4.48 | 4.53 | 4.57 |
| 5393.17 | 4.62 | 4.70 | 4.63 | 4.79 | 4.64 | 4.69 | 4.70 |
| 5389.48 | 4.88 | 4.81 | 4.80 | 4.96 | 4.80 | 4.91 | 4.86 |
| 5383.37 | 4.60 | 4.60 | 4.54 | 4.63 | 4.52 | 4.55 | 4.51 |
| 5371.49 | 4.44 | 4.44 | 4.43 | 4.56 | 4.47 | 4.46 | 4.48 |
| 5367.47 | 4.61 | 4.64 | 4.61 | 4.69 | 4.64 | 4.68 | 4.63 |
| 5339.94 | 4.62 | 4.59 | 4.62 | 4.80 | 4.65 | 4.57 | 4.63 |
| 5324.19 | 4.44 | 4.46 | 4.38 | 4.62 | 4.65 | 4.51 | 4.59 |
| 5321.11 | 5.27 | 5.05 | ---- | 5.27 | ----- | ---- | 5.19 |
| 5307.37 | 4.97 | 4.88 | 4.78 | 4.92 | 4.91 | 4.88 | 4.91 |
| 5302.31 | 4.65 | 4.69 | 4.57 | 4.82 | 4.71 | 4.68 | 4.71 |
| 5288.54 | 5.06 | 5.12 | 5.12 | 5.23 | 5.24 | 5.20 | 5.12 |
| 5281.80 | 4.59 | 4.69 | 4.65 | 4.80 | 4.74 | 4.79 | 4.72 |
| 5269.54 | 4.39 | 4.31 | 4.33 | 4.53 | 4.35 | 4.36 | 4.33 |
| 5266.56 | 4.50 | 4.43 | 4.45 | 4.59 | 4.49 | 4.48 | 4.53 |
| 5263.31 | 4.51 | 4.52 | 4.43 | 4.76 | 4.51 | 4.50 | 4.55 |
| 5253.48 | 4.98 | 4.90 | 4.82 | 5.02 | 5.04 | ----- |  |
| 5242.50 | 4.84 | 4.85 | 4.74 | 4.95 | 5.00 | 4.86 | 4.90 |
| 5232.95 | 4.45 | 4.47 | 4.38 | 4.55 | 4.45 | 4.43 | 4.46 |
| 5229.86 | 4.70 | 4.71 | 4.68 | 4.97 | 4.80 | 4.80 | 4.78 |
| 5217.40 | 4.78 | 4.70 | 4.66 | 4.96 | 4.81 | 4.72 | 4.91 |
| 5216.28 | 4.65 | 4.67 | 4.61 | 4.80 | 4.73 | 4.67 | 4.67 |
| 5215.18 | 4.67 | 4.73 | 4.60 | 4.81 | 4.74 | 4.69 | 4.81 |
| 5162.29 | 4.53 | 4.58 | 4.51 | 4.70 | 4.68 | 4.53 | 4.54 |
| 5159.07 | 5.06 | 4.84 | 5.04 | 5.02 | 5.14 | 4.97 |  |
| 5151.92 | 4.84 | 4.91 | 4.89 | 5.12 | 4.99 | 4.61 | 5.03 |

Table 5 Values of $-\log W^{*}$ (green region)--cont.

| $\lambda$ | mult | $\log 9 f$ | $\mathrm{X}_{1}$ | sky | 47 UMa | $\beta \mathrm{CVn}$ | HR 672 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5150.84 | 16 | -2.70 | 0.99 | 4.79 | 4.54 | 4.66 | 4.66 |
| 5145.11 | 66 | -2.40 | 2.20 | 4.84 | 4.81 | 5.05 | 5.13 |
| 5133.69 | 1092 | 0.63 | 4.18 | 4.59 | 4.42 | 4.54 | 4.58 |
| 5127.36 | 16 | -2.91 | 0.91 | 4.75 | ---- | 4.73 | 4.73 |
| 5123.72 | 16 | -2.79 | 1.01 | - | 4.41 | 4.60 | 4.54 |
| 5110.41 | 1 | -3.34 | 0.00 | 4.45 |  | 4.50 |  |
| 5090.79 | 1090 | -0.10 | 4.26 | 4.87 | 4.67 | 4.83 | 4.82 |
| 5074.76 | 1094 | 0.24 | 4.22 | 4.93 | 4.45 | 4.59 | 4.62 |
| 5068.77 | 383 | -0.59 | 2.94 | 4.53 | 4.45 | 4.59 | 4.64 |
| 5054.65 | 884 | -1.57 | 3.64 | 5.55 | 4.91 | - | ----- |
| 5049.82 | 114 | -1.00 | 2.28 | 4.57 | 4.45 | 4.49 | 4.60 |
| 5044.22 | 318 | -1.61 | 2.86 | 4.81 | 4.85 | 4.93 | 4.81 |
| 5012.07 | 16 | -2.41 | 0.86 | 4.42 | 4.25 | 4.42 | 4.38 |


| $\lambda$ | $\beta$ Com | 99 Her | $\beta$ Vir | HR 5691 | 9 Com | $\theta$ Per | 110 Her |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5150.84 | 4.64 | 4.72 | 4.63 | 4.60 | 4.62 | 5.03 | 4.79 |
| 5145.11 | 5.04 | 5.09 | 4.92 | 5.04 | 4.88 | 5.03 | 4.82 |
| 5133.69 | 4.46 | 4.68 | 4.51 | 4.49 | 4.54 | 4.47 | 4.60 |
| 5127.36 | 4.71 | 4.67 | 4.80 | 4.71 | 4.69 | 4.67 | 4.74 |
| 5123.72 | 4.56 | 4.75 | 4.54 | 4.57 | 4.51 | 4.53 | 4.62 |
| 5110.41 | 4.48 | 4.51 | 4.49 | 4.45 | ----- | 4.47 | 4.47 |
| 5090.79 | 4.75 | 4.92 | 4.87 | 4.65 | 4.67 | 4.72 | 4.93 |
| 5074.76 | 4.50 | 4.74 | 4.59 | 4.62 | 4.65 | 4.66 | 4.70 |
| 5068.77 | 4.50 | 4.71 | 4.56 | 4.50 | 4.55 | 4.56 | 4.63 |
| 5054.65 | 5.17 | 5.42 | 5.19 | --- | - | 5.15 | 5.30 |
| 5049.82 | 4.51 | 4.66 | 4.58 | 4.51 | 4.53 | 4.53 | 4.54 |
| 5044.22 | 4.84 | 4.97 | 4.83 | 4.84 | 4.75 | 4.93 | 4.88 |
| 5012.07 | 4.34 | 4.59 | 4.35 | 4.37 | 4.25 | 4.45 | 4.46 |

Table 5 Values of $-\log W^{*}$ (green region)--cont.

| $\lambda$ | $\chi$ Cnc | 22 Lyn | $\pi^{3}$ ori | $\sigma$ Boo | $\gamma$ VirA | $\eta$ Lep | 9 Aur |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5150.84 | 4.71 | 4.74 | 4.75 | 5.04 | 4.91 | 4.63 | 4.79 |
| 5145.11 | 5.15 | 5.06 | 4.97 | 5.53 |  |  | 5.34 |
| 5133.69 | 4.62 | 4.62 | 4.53 | 4.72 | 4.62 | 4.55 | 4.68 |
| 5127.36 | 4.79 | 4.85 | 4.78 | 5.16 | 5.04 | 5.01 | 4.89 |
| 5123.72 | 4.67 | 4.74 | 4.58 | 4.53 | 4.81 | 4.83 | 4.48 |
| 5110.41 | 4.51 | 4.69 | 4.59 | 4.80 | 4.62 | ----- | 4.65 |
| 5090.79 | 4.79 | 4.77 | 4.72 | 5.18 | 4.84 | 4.74 | 4.90 |
| 5074.76 | 4.59 | 4.64 | 4.55 | 4.84 | 4.67 | 4.61 | 4.69 |
| 5068.77 | 4.58 | 4.69 | 4.59 | 4.90 | 4.75 | 4.66 | 4.64 |
| 5054.65 |  |  | ---- | ---- | ---- | ---- |  |
| 5049.82 | 4.57 | 4.66 | 4.65 | 4.71 | 4.63 | 4.67 | 4.70 |
| 5044.22 | 4.91 | 5.03 | 5.06 | 5.04 | 5.03 | 5.18 | 5.19 |
| 5012.07 | 4.47 | 4.47 | 4.43 | 4.59 | 4.54 | 4.57 | 4.56 |

Table 6 Observed Values of $-\log W^{*}$ (blue region)

| $\lambda$ | mult | $\log 9 f$ | $\chi_{1}$ | $\gamma$ VirA | Virb | 37 UMa | $\eta$ Lep |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4736.78 | 554 | -0.02 | 3.21 | 4.77 | 4.66 | 4.59 | 4.66 |
| 4710.29 | 409 | -0.74 | 3.02 | 5.02 | 4.99 | 4.81 | 5.00 |
| 4707.28 | 554 | -0.23 | 3.24 | 4.74 | 4.72 | 4.73 | 4.66 |
| 4691.41 | 409 | -0.59 | 2.99 | 4.90 | 4.86 | 4.83 | 4.79 |
| 4654.50 | 39 | -2.18 | 1.56 | 4.62 | 4.65 | 4.59 | 4.63 |
| 4625.05 | 554 | -0.63 | 3.24 | 4.98 | 5.01 | 4.90 | 5.02 |
| 4613.21 | 554 | -0.88 | 3.29 | 4.90 | 4.91 | 4.80 | 4.87 |
| 4611.29 | 826 | -0.13 | 3.65 | 4.80 | 4.84 | 4.75 | 4.77 |
| 4602.94 | 39 | -1.46 | 1.48 | 4.87 | 4.82 | 4.71 | 4.72 |
| 4598.12 | 554 | -0.70 | 3.28 | 4.91 | 4.87 | 4.79 | 4.90 |
| 4528.62 | 68 | -0.20 | 2.18 | 4.44 | 4.52 | 4.47 | 4.40 |
| 4525.14 | 826 | 0.03 | 3.61 | 4.62 | 4.71 | 4.61 | 4.60 |
| 4484.22 | 828 | 0.08 | 3.60 | 4.91 | 4.83 | 4.92 | 4.75 |
| 4476.02 | 350 | 0.14 | 2.84 | 4.55 | 4.65 | 4.55 | 4.54 |
| 4469.38 | 830 | 0.19 | 3.65 | 4.58 . | 4.67 | 4.51 | 4.52 |
| 4466.55 | 350 | 0.18 | 2.83 | 4.61 | 4.65 | 4.54 | 4.53 |
| 4447.72 | 68 | -0.58 | 2.22 | 4.70 | 4.73 | 4.70 | 4.59 |
| 4446.84 | 828 | -0.57 | 3.69 | 5.15 | ---- | 4.94 | 5.09 |
| 4443.18 | 350 | -0.22 | 2.86 | 4.67 | ---- | ---- | 4.60 |
| 4442.34 | 68 | -0.50 | 2.20 | 4.69 | 4.62 | 4.57 | 4.57 |
| 4430.62 | 68 | -1.02 | 2.22 | 4.74 | 4.70 | 4.62 | 4.67 |
| 4427.31 | 2 | -2.51 | 0.05 |  | 4.70 | 4.62 | 4.55 |
| 4422.57 | 350 | -0.22 | 2.84 | 4.73 | 4.74 | 4.66 | 4.66 |
| 4415.13 | 41 | -0.13 | 1.61 | 4.29 | 4.33 | 4.24 | 4.31 |
| 4408.42 | 68 | -0.95 | 2.20 | 4.73 | 4.80 | 4.67 | 4.65 |
| 4404.75 | 41 | 0.25 | 1.56 | 4.30 | 4.38 | 4.32 | 4.30 |
| 4388.41 | 830 | 0.02 | 3.60 | 4.84 | 4.79 | 4.64 | 4.71 |
| 4383.55 | 41 | 0.51 | 1.48 | 4.21 | 4.26 | 4.21 | 4.21 |
| 4375.93 | 2 | -2.59 | 0.00 |  | 4.80 |  | 4.65 |
| 4352.74 | 71 | -0.56 | 2.22 | 4.78 | 4.81 | 4.61 | 4.68 |
| 4325.77 | 42 | 0.36 | 1.61 | 4.25 | 4.30 | 4.23 | 4.23 |
| 4298.04 | 520 | -0.56 | 3.05 | 5.20 | 5.07 | ---- | 4.97 |
| 4282.41 | 71 | -0.16 | 2.18 | 4.57 | 4.54 | 4.41 | 4.50 |
| 4271.76 | 42 | 0.20 | 1.48 | 4.29 | ---- | 4.20 | 4.25 |
| 4271.16 | 152 | 0.25 | 2.45 | 4.45 | ---- | 4.47 | 4.45 |
| 4267.83 | 482 | -0.34 | 3.11 | 4.97 | 4.98 | 4.90 | 4.86 |
| 4266.97 | 273 | -0.87 | 2.73 | 5.06 | 5.18 | 4.91 | 5.00 |
| 4260.48 | 152 | 0.63 | 2.40 | 4.24 | 4.31 | 4.22 | 4.23 |
| 4250.79 | 42 | -0.28 | 1.56 | 4.49 | 4.57 | 4.43 | 4.41 |
| 4250.13 | 152 | 0.25 | 2.47 | 4.57 | 4.62 | 4.43 | 4.46 |

Table 6 Values of $-\log W^{*}$ (blue region)--cont.

| 4736.78 | 4.57 | 4.77 | 5.15 | 5.15 |
| :---: | :---: | :---: | :---: | :---: |
| 4710.29 | 4.94 | 5.03 | 5.41 | 5.41 |
| 4707.28 | 4.68 | 4.84 | 5.41 | 5.46 |
| 4691.41 | 4.77 | 5.03 | 5.67 |  |
| 4654.50 | 4.57 | 4.75 | 5.38 | 5.49 |
| 4625.05 | 4.91 | 5.18 | 5.48 | 5.46 |
| 4613.21 | 4.81 | 5.09 | 5.36 | ---- |
| 4611.29 | 4.76 | 4.95 | 5.26 |  |
| 4602.94 | 4.74 | 4.84 | 5.23 | 5.62 |
| 4598.12 | 4.80 | 5.09 | ----- | ----- |
| 4528.62 | 4.37 | 4.45 | 4.83 | 5.02 |
| 4525.14 | 4.60 | 4.66 | 5.19 |  |
| 4484.23 | 4.80 | 4.96 | 5.23 | 5.53 |
| 4476.02 | 4.51 | 4.58 | 4.96 | 5.20 |
| 4469.38 | 4.55 | 4.60 | 5.21 | 5.50 |
| 4466.55 | 4.53 | 4.64 | 5.00 | 5.28 |
| 4447.72 | 4.58 | 4.76 | 5.26 | 5.41 |
| 4446.84 |  |  |  |  |
| 4443.18 | 4.25 | 4.67 | 5.07 |  |
| 4442.34 | ---- | 4.63 | 5.15 | 5.26 |
| 4430.62 | 4.61 | 4.82 | 5.60 | ---- |
| 4427.31 | 4.54 | 4.73 | 5.24 | ---- |
| 4422.57 | 4.48 | 4.72 | 5.18 |  |
| 4415.13 | 4.16 | $\cdot 4.27$ | 4.68 | 4.70 |
| 4408.42 | 4.69 | 4.85 | 5.34 | 5.49 |
| 4404.75 | 4.30 | 4.30 | 4.67 | 4.71 |
| 4388.41 | 4.62 | 4.88 | 5.60 | 5.68 |
| 4383.55 | 4.25 | 4.26 | 4.57 | 4.59 |
| 4375.93 | 4.64 | 4.76 | 5.49 | ---- |
| 4352.74 | 4.65 | 4.76 | 5.52 |  |
| 4325.77 | 4.17 | 4.25 | 4.65 | 4.71 |
| 4298.04 | 5.06 | 5.40 | ---- | ---- |
| 4282.41 | 4.38 | 4.51 | 4.95 | 5.18 |
| 4271.76 | 4.21 | 4.27 | 4.64 | 4.72 |
| 4271.16 | ----- | 4.47 | 4.89 | 5.09 |
| 4267.83 | 5.02 | 5.42 | 5.45 | ---- |
| 4266.97 | --- | ---- | 5.43 | 5.63 |
| 4260.48 | 4.23 | 4.30 | 4.68 | 4.77 |
| 4250.79 | 4.34 | 4.40 | 4.86 | 5.01 |
| 4250.13 | 4.49 | 4.51 | 4.93 | 5.08 |

Table 6 Values of $-\log W^{*}$. (blue region)--cont.

| $\lambda$ | mult | $\log 9 f$ | $\mathrm{X}_{1}$ | A | rB | UM | Lep |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4248.23 | 482 | -0.53 | 3.07 | 5.03 ${ }^{\text {- }}$ | 5.07 | 4.90 | 4.84 |
| 4247.43 | 693 | 0.43 | 3.37 | 4.66 |  | 4.44 | 4.57 |
| 4238.82 | 693 | 0.47 | 3.40 | 4.70 | 4.72 | 4.61 | 4.57 |
| 4235.94 | 152 | 0.31 | 2.42 | 4.45 | 4.50 | 4.38 | 4.38 |
| 4227.43 | 693 | 0.90 | 3.33 | 4.41 | 4.46 | 4.39 | 4.34 |
| 4225.46 | 693 | 0.13 | 3.42 | 4.64 | 4.62 | 4.47 | 4.49 |
| 4222.22 | 152 | -0.35 | 2.45 | 4.79 | 4.77 | 4.61 | 4.55 |
| 4220.35 | 482 | -0.56 | 3.07 | 5.04 | 4.99 | 4.81 | 4.76 |
| 4219.36 | 800 | 0.79 | 3.57 | 4.71 | 4.69 | 4.60 | 4.56 |
| 4217.55 | 693 | 0.12 | 3.43 | 4.83 | 4.81 | 4.67 | 4.69 |
| 4213.65 | 355 | -0.55 | 2.84 | 4.99 | 4.98 | 4.83 | 4.86 |
| 4210.35 | 152 | -0.19 | 2.48 | 4.63 | 4.70 | 4.62 | 4.55 |
| 4203.90 | 355 | -0.21 | 2.84 | 4.72 | 4.76 | 4.75 | 4.61 |
| 4202.03 | 42 | -0.25 | 1.48 | 4.38 | 4.45 | 4.38 | 4.30 |
| 4200.93 | 689 | -0.22 | 3.40 | 4.94 | 5.05 | 4.92 | 4.82 |
| 4199.10 | 522 | 0.81 | 3.05 | 4.58 | 4.55 | 4.45 | 4.48 |
| 4196.22 | 693 | -0.08 | 3.40 | 4.80 | 4.83 | 4.77 | 4.64 |
| 4191.44 | 152 | 0.06 | 2.47 | 4.47 | 4.50 | 4.48 | 4.42 |
| 4187.80 | 152 | 0.13 | 2.42 | 4.49 | 4.50 | 4.43 | 4.40 |
| 4187.04 | 152 | 0.17 | 2.45 | 4.60 | 4.61 | 4.47 | 4.44 |
| 4184.90 | 355 | -0.05 | 2.83 | 4.86 | 4.84 | 4.70 | 4.69 |
| 4181.76 | 354 | 0.46 | 2.83 | 4.44 | 4.45 | 4.35 | 4.33 |
| 4175.64 | 354 | 0.10 | 2.84 | 4.80 | 4.87 | 4.73 | 4.65 |
| 4157.79 | 695 | 0.17 | 3.42 | 4.83 | 4.87 | 4.75 | 4.67 |
| 4153.91 | 695 | 0.33 | 3.40 | 4.53 |  | 4.49 | 4.48 |
| 4152.17 | 18 | -2.45 | 0.96 |  | 4.67 | 4.60 | 4.49 |
| 4150.26 | 695 | -0.71 | 3.43 | 5.01 | 5.07 | 5.00 | 4.90 |
| 4147.67 | 42 | -1.47 | 1.48 | 4.76 | 4.82 | 4.72 | 4.67 |
| 4143.87 | 43 | -0.07 | 1.56 | 4.34 | 4.32 | 4.21 | 4.31 |
| 4134.68 | 357 | 0.18 | 2.83 | 4.55 | 4.63 | 4.49 | 4.46 |
| 4132.90 | 357 | -0.02 | 2.84 | 4.69 | 4.74 | 4.57 | 4.64 |
| 4132.06 | 43 | -0.16 | 1.61 | 4.36 | 4.43 | 4.35 | 4.32 |
| 4114.45 | 357 | -0.47 | 2.83 | 4.98 | 5.07 | 4.89 | 4.89 |
| 4107.49 | 354 | 0.06 | 2.83 | 4.90 | 4.97 | 5.05 | 4.86 |
| 4084.50 | 698 | 0.13 | 3.33 | 4.87 | 4.85 | ----- | 4.82 |
| 4076.64 | 558 | 0.24 | 3.21 | 4.44 | 4.53 | 4.39 | 4.34 |
| 4074.79 | 524 | -0.14 | 3.05 | 4.83 | 4.91 | 4.92 | 4.68 |
| 4073.76 | 558 | -0.14 | 3.14 | 5.01 | 5.06 | 5.11 | 4.86 |
| 4071.74 | 43 | 0.40 | 1.61 | 4.31 | 4.40 | 4.31 | 4.27 |
| 4070.77 | 558 | 0.01 | 3.24 | 4.86 | 4.89 | 4.91 | 4.73 |

Table 6 Values of $-\log W^{*}$ (blue region)--cont.

| $\lambda$ | 30 LMi | l4 Aur | 95 Leo | $\theta$ Leo |
| :---: | :---: | :---: | :---: | :---: |
| 4248.23 | 4.81 | 5.32 | 5.48 | ---- |
| 4247.43 | --- | 4.61 | 5.16 | ---- |
| 4238.82 | 4.55 | 4.63 | 5.18 | 5.45 |
| 4235.94 | 4.36 | 4.41 | 4.86 | 5.10 |
| 4227.43 | 4.30 | 4.38 | 4.80 | 4.89 |
| 4225.46 | 4.50 | 4.68 | 5.28 | ---- |
| 4222.22 | 4.61 | 4.64 | 5.22 | 5.51 |
| 4220.35 | 4.78 | 5.03 | 5.54 | ---- |
| 4219.36 | 4.59 | 4.65 | 5.16 | 5.32 |
| 4217.55 | 4.71 | 4.82 | 5.16 | ---- |
| 4213.65 | 4.94 | ---- | 5.54 | ---- |
| 4210.35 | 4.59 | 4.67 | 5.14 | 5.58 |
| 4203.90 | 4.65 | 4.70 | 5.19 | 5.44 |
| 4202.03 | 4.31 | 4.32 | 4.79 | 4.86 |
| 4200.93 | 4.97 | 5.16 | ---- | 5.58 |
| 4199.10 | 4.36 | 4.46 | 4.89 | ---- |
| 4196.22 | 4.76 | 4.83 | 5.34 | ---- |
| 4191.44 | 4.42 | 4.49 | 5.06 | 5.20 |
| 4187.80 | 4.43 | 4.46 | 4.88 | 4.90 |
| 4187.04 | 4.44 | 4.48 | 4.99 | 5.13 |
| 4184.90 | 4.63 | 4.82 | 5.41 | 5.54 |
| 4181.76 | 4.34 | 4.41 | 4.91 | 5.13 |
| 4175.64 | 4.66 | 4.72 | 5.12 | ---- |
| 4157.79 | 4.73 | 4.88 | 5.27 | ---- |
| 4153.91 | ---- | 4.58 | 5.14 | ---- |
| 4152.17 | 4.64 | 4.72 | 5.36 | ---- |
| 4150.26 | ---- | 5.29 | ---- | ---- |
| 4147.67 | 4.69 | 4.83 | 5.44 | ---- |
| 4143.87 | 4.18 | 4.28 | 4.79 | 4.82 |
| 4134.68 | 4.55 | 4.64 | 5.18 | 5.15 |
| 4132.90 | 4.70 | 4.78 | 5.33 | 5.71 |
| 4132.06 | 4.33 | 4.38 | 4.78 | ---- |
| 4114.45 | 4.82 | ---- | ---- | ---- |
| 4107.49 | 4.87 | 4.99 | ---- | ---- |
| 4084.50 | ---- | 4.75 | ---- | ---- |
| 4076.64 | 4.41 | 4.49 | 5.07 | 5.26 |
| 4074.79 | 4.87 | 4.95 | 5.30 | ---- |
| 4073.76 | 5.03 | 4.98 | 5.83 | $--\cdots-$ |
| 4071.74 | 4.35. | 4.32 | 4.68 | 4.81 |
| 4070.77 | 4.81 | 4.84 | 5.21 | 5.30 |
|  |  |  |  |  |

Table 6 Values of $-\log W^{*}$ (blue region)--cont.

| $\lambda$ | mult | $\log \mathrm{gf}$ | $\chi_{1}$ | $\gamma$ VirA | $\gamma$ Virb | 37 UMa | $\eta$ Lep |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4067.98 | 559 | 0.29 | 3.21 | 4.75 | 4.83 | 4.70 | 4.64 |
| 4063.60 | 43 | 0.43 | 1.56 | 4.21 | 4.27 | 4.21 | 4.15 |
| 4062.45 | 359 | 0.05 | 2.84 | 4.94 | 4.81 | 4.72 | 4.70 |
| 4045.82 | 43 | 0.68 | 1.48 | 4.04 | 4.14 | 4.07 | 4.03 |
| 4044.61 | 359 | -0.17 | 2.83 | 4.85 | 4.84 | 4.64 | 4.65 |
| 4021.87 | 278 | 0.12 | 2.76 | 4.68 | 4.71 | 4.65 | 4.54 |
| 4014.53 | 802 | 0.58 | 3.57 | 4.87 | 4.91 | 4.66 | 4.53 |
| 4009.71 | 72 | -0.44 | 2.22 | 4.73 | 4.75 | 4.73 | 4.62 |
| 3998.05 | 276 | -0.05 | 2.69 | 4.69 | ---- | 4.60 | 4.58 |

Table 6 Values of $-\log W^{*}$ (blue region)--cont.

| $\lambda$ | 30 LMi | 14 Aur | 95 Leo $\theta$ Leo |  |
| :---: | :---: | :---: | :---: | :---: |
| 4067.98 | 4.77 | 4.78 | 5.24 | 5.40 |
| 4063.60 | 4.23 | 4.23 | 4.61 | 4.70 |
| 4062.45 | 4.78 | 4.81 | ---- | $--\cdots-$ |
| 4045.82 | 4.11 | 4.12 | 4.46 | 4.55 |
| 4044.61 | 4.72 | 4.89 | --- | ---- |
| 4021.87 | 4.66 | 4.65 | 5.08 | 5.26 |
| 4014.53 | 4.64 | 4.92 | 5.15 | 5.07 |
| 4009.71 | 4.65 | 4.68 | 5.26 | 5.56 |
| 3998.05 | 4.64 | 4.72 | 5.45 | $-\cdots-$ |

the top of the column. This value was usually computed from the mean of the equivalent widths from all spectra of the star. However, if one value differed significantly (by 30 per cent or more) from the others, it was discarded; and the average was taken from the remaining lines.

## D. Accuracy of Equivalent Widths

Several of the stars included in the present work have been measured by other investigators. Wallerstein (1961) has observed $\beta$ Vir and 99 Her in the green region of the spectrum at a dispersion of $15 \mathrm{~A} / \mathrm{mm}$. Chamberlain and Aller (1951) have observed 95 Leo in the blue at 10 A/mm. Wright et al. (1964) have observed $\theta$ Leo and 11 other stars at very high (< $5 \mathrm{~A} / \mathrm{mm}$ ) dispersion from several major observatories and have published these observations for use as spectrophotometric standards. Finally, the equivalent widths of the daylight sky spectrum can be compared to those quoted in the Rowland Tables (Moore, Minnaert, and Houtgast 1966). These tables give equivalent widths for the center of the solar disc whereas the daylight sky spectrum is an integrated spectrum of the entire solar disc. Nevertheless; it is felt that center-to-limb variations in the solar spectrum are not sufficiently great
to invalidate a comparison between daylight sky and center-of-disc equivalent widths.

Figures 7-11 show comparisons between the $\log W^{*}$ values of the present work and those of the just-mentioned studies. The scatter is fairly large, with systematic differences in some cases. In order to comment on the quality of the data in the present work, Figure 12 has been included. This is a comparison between the $\log W^{*}$ values for $\theta$ Leo obtained at Palomar (4.5 A/mm) and Victoria (3.4 A/mm) (Wright et al. 1964). Both spectrographs use a grating as the dispersive element, and the dispersions obtained are quite high. Nevertheless, the systematic differences between the observations are very pronounced. Whether these differences are caused by the spectrograph or the reduction procedures is unimportant. The important result is that, even at high dispersion, the equivalent widths obtained by one observer can differ markedly from those obtained by another. In light of this, the comparisons shown in Figures 7-ll are well within the accuracy limits which must be expected from even the most accurate photographic spectrophotometry.


Fig. 7. Intercomparison of Equivalent Widths for $\beta$ Vir


Fig. 8. Intercomparison of Equivalent Widths for 99 Her


Fig. 9. Intercomparison of Equivalent Widths for 95 Leo


Fig. 10. Intercomparison of Equivalent Widths for $\theta$ Leo


Fig. 11. Intercomparison of Equivalent Widths between Daylight Sky and Rowland Tables


Fig. 12. Intercomparison of Equivalent Widths for $\theta$ Leo from Victoria and Palomar

In order to make direct comparisons between low and high dispersion plates of the same star, equivalent widths of the same lines at the two dispersions should be measured. However, in the case of both $\gamma \operatorname{Vir} A$ and $\eta$ Lep (the only two stars in the present study whose spectra were obtained at both high and low dispersion), the regions measured at each dispersion did not overlap in wavelength; therefore, no such direct comparison can be made. The ultimate test is whether the two dispersions yield the same microturbulent velocity for a given star. Because these numbers will be discussed in Chapter 4, no further discussion will be presented here.
III. THE CURVE OF GROWTH

The curve of growth is a, relationship between the equivalent width of an absorption line and the number of atoms effective in producing the line. If a reliable model stellar atmosphere is available for the star being investigated, the equation of radiative transfer can be integrated to predict the theoretical profile of any absorption line if the relevant physical parameters which determine the line absorption coefficient are known. Then by changing the abundance of the element in question, the run of equivalent width with abundance can be predicted. Traditionally, this is called the "fine" analysis approach and involves rather lengthy computations which have only recently been made possible through the use of high speed digital computers. The approach which is usually implied by the term "curve of growth" involves making some simplification to the physical structure of the stellar atmosphere which will allow the equation of transfer to be solved analytically. When such simplifications are made, the method is called the "coarse" analysis.

In the coarse analysis, several assumptions are essential to the method (Aller 1960, p. 194):
a) All lines are formed by the same mechanism (scattering or absorption or some combination of the two).
b) The excitation and ionization temperatures and the electron pressure, which are used in the combined BoltzmannSaha equation, can be characterized by one value in the region where the lines are formed.
c) The form of the atomic absorption coefficient is the same for all lines under consideration.
d) The stratification of the atmosphere is specified by some simplified model.

Before describing the two major types of coarse analysis, we must consider the causes of the finite width of stellar absorption lines. Absorption lines in a stellar atmosphere are broadened by three mechanisms:
a) Doppler effect arising from small-scale motions in the atmosphere; these may be either thermal or nonthermal in origin;
b) natural broadening arising from the fact that the lower level from which the transition takes place has a finite lifetime;
c) pressure broadening arising from the interaction of the atom with its neighbors.

The broadening of the line caused by b) and c) are physically similar and they are usually considered to be characterized by one parameter--the damping constant, a. The form of the atomic absorption coefficient which results from these broadening mechanisms is described by an integral which has no analytic solution. However, the integral has been evaluated numerically (Harris 1948), and the resulting atomic absorption coefficient at any point in the line is given by

$$
\begin{equation*}
\alpha_{\nu} / \alpha_{0}=\mathrm{H}(\mathrm{a}, \mathrm{u}) \tag{3.1}
\end{equation*}
$$

which can be expanded in the form

$$
\begin{equation*}
\alpha_{\nu} / \alpha_{0}=\mathrm{H}_{0}(\mathrm{u})+\mathrm{a} \mathrm{H}_{1}(\mathrm{u})+\mathrm{a}^{2} \mathrm{H}_{2}(\mathrm{u})+\ldots \tag{3.2}
\end{equation*}
$$

where

$$
\begin{gather*}
\alpha_{0}=\frac{\pi e^{2}}{m_{e}} f \frac{1}{\Delta \nu_{0}} \frac{1}{\sqrt{\pi}}  \tag{3.3}\\
\cdot \Delta \nu_{0}=\nu_{o} \frac{v}{c}  \tag{3.4}\\
v=\sqrt{\frac{2 k T}{M}+\xi^{2}}  \tag{3.5}\\
u=\frac{\nu-\nu_{0}}{\Delta \nu_{0}} \tag{3.6}
\end{gather*}
$$

The values of $H_{0}, H_{1}, \mathrm{H}_{2} \ldots$ have been tabulated (Harris 1948). $\nu_{o}$ is the frequency at the center of the line of
interest; $e$, the charge on the electron; $m_{e}$, the mass of the electron; $f$, the oscillator strength for the line; $v$, the velocity parameter; $k$, the Boltzmann constant; $T$, the kinetic temperature; $M$, the mass of the atom; $\xi$, the microturbulent velocity; and $\pi$ and $c$ have their usual meaning. The quantity $\Delta \nu_{0}$ is called the Doppler width, and it is through the Doppler width that the value of the microturbulent velocity enters the problem. Thus $u$ is the number of Doppler widths from the center of the line at which the line absorption coefficient is being evaluated.

Knowing the line absorption coefficient, we may now proceed to describe the coarse analysis approach. There are two simplifying models which are usually used in the traditional coarse analysis approach. These are known as the Milne-Eddington (ME) model and the SchusterSchwarzschild (SS) model after the astrophysicists who first introduced them.

In the $S S$ model it is assumed that the atmosphere of the star is made up of two separate regions: the photosphere which gives rise to the continuous spectrum of the star, and the "reversing layer" above the photosphere in which the absorption lines are formed. Using

Chandrasekhar's solution to the equation of transfer resulting from these assumptions, Wrubel (1954) has tabulated the curves of growth on the basis of the SS model. In his treatment of the problem, Wrubel assumed that the lines formed in the reversing layer are formed by a process of pure scattering, and he tabulated curves which are functions of both the limb darkening and the damping constant. The resulting theoretical curves of growth are graphs in which the ordinate is $\log W^{*} \frac{C}{V}$, where $W^{*}$ is as defined in Chapter II; $C$, the velocity of light; and $v$, the velocity parameter. The abscissa of the curves is $\tau_{0}$, the optical depth at the center of the line. By using the Boltzmann equation, it can be shown that

$$
\begin{equation*}
\log \tau_{0}=c_{1}+\log \frac{g £ \lambda}{1000}-\theta_{e x} X_{1}+N_{i}-\log \frac{v}{c} \tag{3.7}
\end{equation*}
$$

where $C_{1}$ is a numerical constant, $g$ is the statistical weight of the lower level, $\theta_{\text {ex }}$ is $5040 / \mathrm{T}_{\text {ex }}$ where $\mathrm{T}_{\text {ex }}$ is the excitation temperature, $X_{1}$ is the excitation potential for the lower level, $N_{i}$ is the number of atoms $/ \mathrm{cm}^{2}$ of the element in the $i^{\text {th }}$ stage of ionization above the photosphere, and the other symbols have been described previously.

In the ME model the other extreme of stratification is assumed--that is, the lines and the continuum are assumed to be formed in the same region, and the ratio of line to continuous absorption ( $\eta$ ) is independent of optical depth. Using Chandrasekhar's solution to the equation of transfer resulting from these assumptions, Wrubel (1949) has tabulated the curves of growth for the ME model. As in the case for the $S S$ model, the resulting curves are tabulated for various values of the limb darkening and the damping constant. The resulting ordinate for the theoretical curves of growth is the same as in the SS curves, and the abscissa is $\eta_{0}$. the ratio of line to continuous absorption at line center. Again using the Boltzmann equation we obtain
$\log \eta_{0}=A+\log \frac{g f \lambda}{1000}-\theta_{e x} \chi_{l}+\log n_{i}-\log \frac{v}{c}-\bar{x}_{\lambda}$
where $A$ is a numerical constant, $n_{i}$ is the number density of absorbing atoms, $\bar{x}_{\lambda}$ is some appropriate mean continuous absorption coefficient, and all other symbols have the same meaning as in equation (3.7).

The general shape of the theoretical curve of growth is determined by the form of the absorption coefficient; and therefore, the various assumptions which are
inherent in the models used will not grossly affect this basic shape. Figure 13 is a schematic theoretical curve of growth in which the various regions have been marked. The abscissa is log Q--some function of the abundance whose exact form depends upon the model used. In region $A$ the equivalent width is directly proportional to the abundance; this is known as the "linear" portion of the curve. Region $B$ is where the line is saturated; this is known as the "transition" or "flat" portion of the curve. Region $C$ is where the damping wings of the line contribute appreciably to the equivalent width; this is known as the "damping" portion of the curve.

The behavior of the ME and SS curves with respect to limb darkening is different. However, it is only at extreme limb darkening values, which seem to be physically unrealistic, that the differences between the two theoretical curves are sufficient to change the velocity parameter which is derived from the models.

Which of the two models should be used in determining the abundances and the velocity parameter in stars depends primarily upon the ion whose lines are being analyzed. The lines of the neutral metals are usually formed

$\log Q$
Fig. 13. Schematic Theoretical Curve of Growth
high in the atmosphere, above the regions in which the continuum is formed, and thus the SS model is probably preferable for neutral metals. The lines of ionized metals are formed deeper in the atmosphere and may be formed with the continuum, in which case the ME model is probably to be preferred.

The choice for the line formation mechanism is considerably more difficult. It depends upon the physical state of the atmosphere and upon the energy level structure "seen" by the atom in the upper state of the transition, and this structure is different for each level in each type of atom. To compute the mechanism for each line would require knowing the details of the physical conditions in the stellar atmosphere and the various radiative and collisional cross-sections for all levels in the atom of interest. Such detailed calculations are precisely what the curve of growth technique is meant to avoid. Fortunately, since the differential effects over a wide range are more important (and more meaningful) than the absolute values of the velocity parameter, an improper choice of the line formation mechanism should not seriously affect the derived relationship between the velocity parameter and spectral type.

The other two parameters which must be used in the theoretical curve of growth are the limb darkening and the damping constant. In the two models considered, the temperature structure of the atmosphere (or equivalently, the limb darkening) is usually given by

$$
\begin{equation*}
\mathrm{B}_{\nu}(\mathrm{T})=\mathrm{B}^{0}+\mathrm{B}^{\mathrm{I}} \tau \nu \tag{3.9}
\end{equation*}
$$

where $B_{\nu}(T)$ is the Planck function and $\tau_{\nu}$ is the optical depth. Wrubel's solutions are tabulated as functions of $B^{0} / B^{1}$ and of the damping constant, a. In the case of the SS models, the theoretical curves are essentially independent of the value of $B^{0} / B^{1}$; and because these are the curves which are physically more reasonable for the lines of the neutral metals, the choice of the limb darkening does not affect the results. Nevertheless, with a reasonable choice of $B^{0} / B^{1}$, the $M E$ theoretical curves are indistinguishable from the SS curves. The value of the damping constant has been found to be reasonably constant for dwarf stars over the spectral range of the present study; and following Greenstein (1948), we adopt a value of $\log a=-1.8$. Similarly we adopt a value of $B^{0} / B^{l}=2 / 3$ since it seems to be a satisfactory representation of the temperature structure in dwarf stars (Aller 1963, p. 378).

Once a theoretical curve of growth--which seems to represent adequately the range of stars of interest-has been adopted, the observed curves can be fitted to the theoretical one and the velocity parameter can be obtained. For the present investigation, the fitting procedure is carried out in the following way.

First, the observed equivalent widths are broken into 8-10 groups with small ranges in the value of $X_{1}$ (a spread of not greater than 0.5 eV ). Then for each of these groups, a separate observational curve of growth is plotted. The observational curve of growth is a plot of $\log W^{*}$ versus $\log \frac{g f \lambda}{1000}$. The differences between the ordinate and abscissa of the observational (o) and theoretical ( $t$ ) curves of growth can be formed such that

$$
\begin{align*}
& {[w]=\left(\log w^{*}\right)_{0}-\left(\log w^{*} \frac{C}{v}\right)_{t}=\log \frac{y}{c}}  \tag{3.10}\\
& {[z]=\left(\log \frac{g f \lambda}{1000}\right)-\left(\log \tau_{0}\right)_{t}=B+\theta_{e x} X_{1}} \tag{3.11}
\end{align*}
$$

where $B$ is a constant which is the same for all groups. In the fitting procedure, a value of $[W]$ is adopted, and all groups of lines are fitted to the theoretical curve at this value of [W].

For each of the groups, the value of $[z]_{i}$ is obtained for each $\bar{x}_{1_{i}}$, where $\bar{X}_{I_{i}}$ is the average $X_{1}$ for all
of the lines in the $i^{\text {th }}$ group. By performing a leastsquares fit of the observed values of $[z]_{i}$ to the equation of condition (3.11), a value for $\theta_{\text {ex }}$ is obtained. At the same time, the mean error for the observed points from the least-squares solution is computed. Then another value of [W] is chosen and the procedure is repeated. Because a single excitation temperature is to be applied to all lines, the value of $\theta_{e x}$ is chosen which corresponds to the solution with the smallest mean error; and this value is used to fit all of the groups into one curve of growth. For the single observational curve of growth, $\log W^{*}$ versus $\log Z$ is plotted where

$$
\begin{equation*}
\log z=\log \frac{g f \lambda}{1000}-\theta_{\text {ex }} x_{1} \tag{3.12}
\end{equation*}
$$

This single curve is then fitted to the theoretical curve, and the value of $[W]$ is the value of $v / c$ which best represents all of the observed lines. Thus from the curve of growth, the velocity parameter, $v, i s$ derived for each star. The difference between the abscissa of the observed curve and that of the theoretical one is related to the abundance of the ion in question, but several other physical quantities must be known before the abundance can be derived. Because the present study is concerned with only
the velocity parameter, no attempt is made to derive the abundance of the Fe I ion for which the lines were measured.

Because the curve of growth is to be used to determine simply the microturbulent-velocity, only a rough estimate of the kinetic temperature is necessary. The velocity parameter is proportional to the square root of the sum of the squares of the thermal and microturbulent velocities, and thus an error in the kinetic temperature does not significantly change the computed microturbulent velocity (Greenstein 1967). Following Huang and Struve (1956), we set the kinetic temperature equal to the excitation temperature derived from the observational curve of growth. Because the mass of the iron atom is known, equation (3.5) can then be solved to determine the microturbulent velocity.

The SS and ME models represent two extremes of stratification. The physical state of a real stellar atmosphere lies somewhere between these extremes. The fine analysis simply takes into account more details in the physical situation. As has been pointed out, the form of the atomic absorption coefficient is the most important
factor in determining the shape of the relationship between abundance and equivalent width, and the form of this coefficient is identical in both the coarse and fine analyses. Therefore, the curve of growth, although involving many assumptions about the physical state of the atmosphere, remains a satisfactory tool when the details of the physical state of the atmosphere are of secondary importance. We are interested, in the present investigation, in the general trends in the behavior of microturbulence among stars. The curve of growth analysis is a relatively simple method to use in handling large amounts of data; and, for the purpose of determining the average microturbulent velocity, it adequately represents the physical conditions in the stellar atmosphere. Therefore, the observations presented in Chapter II can be satisfactorily analyzed using the curve of growth technique; and the results of this analysis are the subject of Chapter IV.

## IV. RESULTS AND DISCUSSION

## A. Observed Behavior of Microturbulence

Observational curves of growth have been constructed by the Calcomp plotter on the CDC 3200 computer at Kitt Peak for all stars included in the present investigation and for the daylight sky by the method described in the previous chapter. These curves are included in Appendix I. In each of the curves, the solid line through the observed points represents the theoretical curve of growth. The different symbols for each line represent different excitation potentials (chi), and these symbols are explained in the upper right-hand corner of each curve of growth.

The physical parameters which have been derived from these curves are included in Table 7. Column l gives the number used in Figures 15 and 16 to represent the star; column 2, the star name or $H R$ number; column 3, the value of $\theta_{\text {ex }}$ derived from observed data; column 4, the value of $-\log v / c ; ~ c o l u m n ~ 5, ~ t h e ~ r e s u l t i n g ~ v a l u e ~ o f ~ t h e ~ v e l o c i t y ~$ parameter (v) in km/sec; column .6, the microturbulent velocity ( $\xi$ ) computed from the velocity parameter in $\mathrm{km} / \mathrm{sec}$;

Table 7 Observed Microturbulence Parameters for Program Stars

| No. | Name | $\theta_{\mathrm{ex}}$ | $-\log \frac{\mathrm{V}}{\mathrm{c}}$ | v | $\xi$ | $\Delta c_{1}$ | $\mathrm{m}_{1}$ | $\Delta \mathrm{v}$ | [v] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 47 UMa | . 90 | 4.84 | 4.3 | 4.1 | +. 020 | -. 025 | +. 40 | +.05 |
| 2 | $\beta \mathrm{CVn}$ | . 89 | 4.90 | 3.7 | 3.5 | -. 022 | -. 032 | . 00 | . 00 |
| 3 | HR 672 | . 85 | 5.05 | 2.7 | 2.4 | +. 057 | -. 011 | -. 75 | -. 11 |
| 4 | $\beta$ Com | . 85 | 4.90 | 3.8 | 3.6 | +. 008 | -. 006 | +. 35 | +. 04 |
| 5 | 99 Her | . 91 | 5.10 | 2.4 | 2.0 | -. 011 | -. 051 | -. 80 | -. 13 |
| 6 | $\beta$ Vir | . 91 | 4.96 | 3.3 | 3.0 | +. 067 | . 000 | +. 20 | +. 01 |
| 7 | HR 5691 | . 93 | 4.94 | 3.5 | 3.3 | +. 072 | -. 010 | +. 50 | +. 07 |
| 8 | 9 Com | . 90 | 5.05 | 2.7 | 2.4 | +.082 | +. 011 | -. 10 | -. 02 |
| 9 | $\theta$ Per | . 85 | 5.00 | 3.0 | 2.0 | -. 011 | -. 011 | +. 35 | +. 05 |
| 10 | 110 Her | . 91 | 5.12 | 2.3 | 1.9 | $+.080$ | -. 023 | -. 15 | -. 03 |
| 11 | $\chi$ Cnc | . 90 | 5.12 | 2.3 | 1.9 | -. 018 | -. 027 | -. 15 | -. 03 |
| 12 | 22 Lyn | . 87 | 5.08 | 2.5 | 2.1 | -. 024 | -. 030 | +. 10 | +. 02 |
| 13 | $\pi^{3}$ Ori | . 89 | 5.06 | 2.4 | 2.0 | -. 015 | -. 008 | $+.10$ | +. 01 |
| 14 | $\sigma$ Boo | . 81 | 5.17 | 2.0 | 1.5 | -. 033 | -. 034 | -. 80 | -. 14 |
| 15g | $\gamma$ Vir A | . 83 | 4.92 | 3.6 | 3.3 | -. 015 | -. 034 | +.75 | +. 10 |
| 15b |  | . 78 | 4.90 | 3.8 | 3.5 | " | " | --- | -_- |
| 16 | $\gamma \operatorname{Vir} \mathrm{B}$ | . 83 | 4.96 | 3.3 | 3.0 | -. 015 | -. 034 | +. 30 | $+.05$ |
| 17 | 37 UMa | . 82 | 4.92 | 3.4 | 3.1 | -. 014 | -. 016 | +. 10 | +. 01 |
| 18 g | $\eta$ Lep | . 87 | 5.00 | 3.0 | 2.7 | +.012 | -. 014 | -. 30 | -. 04 |
| 18b | " | . 84 | 4.96 | 3.3 | 3.0 | " | " |  |  |
| 19 | 9 Aur | . 85 | 4.92 | 3.6 | 3.3 | +. 025 | -. 026 | 4.10 | +. 02 |
| 20 | 30 LMi | . 76 | 4.86 | 4.1 | 3.8 | $+.167$ | -. 006 | . 00 | . 00 |
| 21 | J.4 Aur | . 82 | 4.88 | 4.0 | 3.7 | +. 170 | -. 027 | . 00 | . 00 |
| 22 | 95 Leo | . 80 | 5.10 | 2.4 | 2.0 | +. 125 | -. 038 | -. 70 | -. 12 |
| 23 | $\theta$ Leo | . 79 | 5.12 | 2.3 | 1.8 | --- | -. 015 | +. 45 | +. 09 |

column 7, the value of $\Delta c_{1}$ (Chapter II); column 8, the value of $\Delta m_{1}$ (Chapter $I$ ); column 9, the value of $\Delta v$ in $\mathrm{km} / \mathrm{sec}$; and column 10, the value of $[\mathrm{v}]$ in $\mathrm{km} / \mathrm{sec}$. The meaning of $\Delta v$ and $[v]$ will be explained below. The subscript of "g" and "b" for $\gamma \operatorname{Vir} A$ and $\eta$ Lep mean green and blue, respectively, because the microturbulent velocity for these stars has been determined at both high (blue region) and low (green region) dispersion to check the consistency of the observations.

The errors in the values of $v$ are difficult to estimate. There are theoretical as well as observational uncertainties which contribute to the errors. As a result of fitting the observational to the theoretical curve of growth there is an error of approximately $\pm .05$ in the value of $\log v / c$. This means that the fitting error is approximately 10 per cent of the value of $v$. As an average error, then, the value of $\pm 0.3 \mathrm{~km} / \mathrm{sec}$ has been adopted for all observationally determined values of $v$. With this error estimate, the agreement between the values of $v$ for $\gamma \operatorname{Vir} A$ and $\eta$ Lep is very satisfactory. Thus the errors introduced by using two different dispersions and two different spectral regions seem to be small.

The only case for which the value of $\xi$ is indeterminate, when using the present data, is the daylight sky. The SS theoretical curve of growth, which gives a satisfactory fit to the observations of all of the program stars, is unacceptable for the solar data. According to one of the more recent investigations of the solar curve of growth (Cowley and Cowley 1964), the van der Held curve (Chapter I) gives the most satisfactory fit to the observations; this curve has been used in Figure 19. In the observational curve, the transition from the linear to the flat part of the curve of growth occurs at a value of $\log W^{*}$ of about -5.2. This corresponds to an equivalent width of less than 35 mA , and not enough lines of this strength or weaker were measured in the present investigation to fix the turnover point. Thus the curve of growth for the daylight sky presented here can give only an upper limit to the microturbulent velocity of $\xi \leq 1.4 \mathrm{~km} / \mathrm{sec}$. In their paper, the Cowleys give a value of $\xi=1.4 \mathrm{~km} / \mathrm{sec}$ which is within the limits determined in the present investigation.

Figure 14 is a graph of $v$ versus $b-y$. The error bars on the points are the $0.3 \mathrm{~km} / \mathrm{sec}$ error estimated

previously, and a mean curve has been drawn by eye through the points. The solar velocity parameter (Cowley and Cowley 1964) is shown by the $X$. The value of $\Delta v$ listed in Table 7 is the difference between the observed $v$ and the mean $v$ as indicated by the solid curve in Figure 14; and the value of [v] is the logarithm of the ratio between the observed v and the mean v .

The dependence of microturbulence on temperature (b-y) is very pronounced; and, at first, this behavior is surprising because it has not been detected previously. In his analysis of $G$ dwarfs, Wallerstein (1961) found that the microturbulent velocity for these stars does not differ markedly from that of the sun. He determined the velocity parameter for three of the stars included in the present investigation ( $\beta$ Com, $\beta$ Vir, and 99 Her) differentially with respect to the sun; and he found that they differ little from the solar value, whereas in the present investigation, they differ by a factor of 3. However, Wallerstein did not measure lines weak enough to be on the linear portion of the curve of growth, and this makes the velocity parameter quite uncertain.

On the other hand, Danziger (1966) has determined the microturbulent velocity of $\sigma$ Boo to be $2.0 \mathrm{~km} / \mathrm{sec}$. This is somewhat higher than the value determined in the present investigation, but his is a value which would put $\sigma$ Boo onto the mean curve in Figure 14 rather than below it. Strom and Conti (1968) have found microturbulent velocities for the late A stars in the Pleides to be of the order of $4 \mathrm{~km} / \mathrm{sec}$, which is to be expected according to the mean curve in Figure 14. Warner (1967b) has found a value of $\xi=2.3 \mathrm{~km} / \mathrm{sec}$ for Sirius indicating that the microturbulence for the early A stars is lower by a factor of nearly 2 from that of the late A stars. Thus other investigations which are currently in progress fit in well with the general shape of the observed $v$ versus $b-y$ relation shown in Figure 14.

## B. Photometric Effects

As indicated in Chapter $I$, there are two important photometric effects to be examined--the relationship between the velocity parameter and age (as indicated by $\Delta c_{1}$ ) and that between the velocity parameter and the observed metal index, $\Delta \mathrm{m}_{1}$. The $\Delta \mathrm{c}_{1}$ index has been calibrated in terms of the age of the star (Strömgren 1963), and it is
of great interest to determine whether or not the microturbulence depends upon time. Figure 15 is a graph of $\Delta v$ versus $\Delta c_{1}$. Each star is located by its number in Table 7 , and the solid line is the least-squares solution to the observed points. A strong correlation between these two. parameters would imply a significant time-dependence of microturbulence over a large spectral range. Such a correlation is not indicated by Figure 15.

In order to determine the degree of correlation between two variables, it is convenient to introduce the correlation coefficient which is a measure of the departures of the observed relationship from a purely random sample (Hoel 1947, p. 81). Whether or not the leastsquares solution represents the observed relationship better than the simple average value of $\Delta v$ is indicated by the computed correlation coefficient of 0.11 . The meaning of this coefficient is such that its square is the percentage of the relationship which can be attributed to some non-random connection between the variables. In the case of $\Delta v$ versus $\Delta c_{1}$ only 1 per cent of the observed dependence is non-random. This number means that in the time that a star in the spectral range of $A$ to $G$ spends on


Fig. 15. $\Delta c_{1}, \Delta v$ Diagram for Program Stars ( $\mathrm{km} / \mathrm{sec}$ )
or near the main sequence, its microturbulent velocity remains nearly constant.

The second photometric effect to be examined is the Conti-Deutsch effect. Conti and Deutsch predicted a dependence of $\Delta m_{1}$ on $[\mathrm{V}]$, the logarithm of the ratio of the velocity parameter of the star in question to that of the sun. Their predictions, however, are based on the behavior and position of absorption lines on the solar curve of growth and thus may be applied strictly only to stars having curves of growth similar to that of the sun. For solar-type stars, Conti and Deutsch predict that, at a fixed value of the velocity parameter, a change in the. metal abundance by a factor of 2 will change the $m_{1}$ index by . 040; whereas at a fixed value of the metal abundance, a change in the velocity parameter by a factor of 2 will change the $m_{1}$ index by .068 (Conti and Deutsch 1967). Even the most extreme metallic line stars, for example, have values of $\Delta m_{1}$ of not greater than .080. Thus Conti and Deutsch state that it is possible that stars of a given color differ in their metal indices only because the stars have different velocity parameters.

In order to check the Conti-Deutsch hypothesis, a graph of $\Delta_{m_{l}}$ versus [v] has been plotted in Figure 16. If the stars in the present investigation differ only in their turbulent velocities, then the least-squares solution to the observed points should have a slope of +0.225 with a correlation coefficient of 100 per cent. The slope in Figure 16 is +0.064 and the degree of correlation is 9 per cent. It should be noted, however, that the estimated error in [v] is $\pm .05$, and the observed values of [v] have a total range of only .22 with the majority lying within $\pm .05$ of $[v]=0$. Thus the conclusion can be drawn that among main sequence stars the range in the velocity parameter at a given $b-y$ is insufficient by itself to explain the differences in $m_{1}$ among stars. Therefore, the ContiDeutsch implication that all $\Delta m_{1}$ 's are caused by microturbulence differences is not upheld by the present observations. Nevertheless, the fact that a small correlation exists between $\Delta \mathrm{m}_{1}$ and [v] indicates that some of the differences in $m_{1}$ among stars are caused by variations in microturbulence.

The difficulty in examining the Conti-Deutsch effect is that among normal stars there is no large variation


Fig. 16. $\Delta m_{1},[v]$ Diagram for Program Stars
in microturbulence, and the errors in the observations are too large. On the other hand, the metallic line stars have high microturbulent velocities, and for these stars the 'observational errors should not so seriously mask the Conti-Deutsch effect. $H_{\gamma}$ line profiles must be used to estimate the effective temperatures of Am stars, however, since their $b-y$ colors are affected by the strengthened absorption lines. The information necessary to carry out an analysis is given for eight Am stars in Table 8. Column 1 gives the letter which will be used to designate the star in Figure 17; column 2, the star name; column 3, its velocity parameter in $\mathrm{km} / \mathrm{sec}$; column 4, its effective temperature; column 5, the $b-y$ corresponding to a main sequence star with this same temperature (Crawford 1966); column 6, its value of $[v] ;$ column 7 , its value of $\Delta m_{1}$; and column 8, the reference from which this information has been obtained. Figure 17 is the $\Delta \mathrm{m}_{1}$, [v] diagram for the Am stars, and the solid line is the least-squares solution. The slope of this relation is +0.08 , and the degree of correlation is 15 per cent. Thus, for the Am stars where the range of microturbulence is large, the correlation is somewhat stronger than for the normal stars.

## Table 8 Microturbulence Parameters for Am Stars

| Symbol | Name | v | $\mathrm{T}_{\text {eff }}$ | $\mathrm{b}-\mathrm{y}\left(\mathrm{T}_{\text {eff }}\right)$ | $[\mathrm{v}]$ | $\Delta \mathrm{m}_{1}$ | reference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 60 Tau | 4.75 | 7340 | .228 | .15 | .020 | Conti 1965 |
| B | 15 Vul | 3.5 | 7600 | .178 | .03 | .010 | Miczaika et al. 1956 |
| C | 63 Tau | 5.4 | 7640 | . .170 | .13 | .041 | Van't Veer-Menneret 1963 |
| D | $\zeta$ Lyr A | 6.0 | 8000 | .127 | .18 | .023 | Praderie 1967 |
| E | 81 Tau | 3.8 | 8130 | .114 | .00 | .049 | Conti 1965 |
| F | 16 Ori | 4.75 | 8160 | .105 | .10 | .035 | Conti 1965 |
| G | $\tau$ UMa | 3.8 | 8260 | .092 | .02 | .028 | Greenstein 1948 |
| H | 8 Com | 4.5 | 8830 | .040 | .25 | .067 | Miczaika et al. 1956 |



The fact that only eight stars have been used makes the relation subject to severe statistical uncertainties, since discarding one observed point could significantly alter both the slope and the degree of correlation. Nevertheless, the fact that a weak correlation is present leads to the same conclusion which has been given for the normal stars--variations in microturbulence change the $m_{1}$ index. Therefore, for both the normal stars and the Am stars, there is a weak, but significant, correlation of microturbulence with the $\Delta_{m_{1}}$ index. This confirms the Conti-Deutsch effect and means that approximately 10-20 per cent of the observed scatter in the $m_{1}$, b-y relation for stars is caused by differences in their microturbulent velocities.

## C. Microturbulence and Stellar Convection Zones

The only strong correlation of microturbulence with a photometric index is that with $b-y$ shown in Figure 14. As indicated in Chapter $I$, such a correlation could be related to the behavior of the convection zone in this spectral range. In his review paper on convection in stars, Schwarzschild (1961) combined the results of Unno (1959a,b) to map the variation of microturbulent velocity
with depth in the solar atmosphere. The Goldberg-Unno procedure (Chapter I) was applied to the sun, and the resulting velocity dependence is such that there is a steep drop from an optical depth of 10 to a depth of 0.1 caused by the decay of the overshooting of the convective zone. Beyond 0.1 there is a rise of the microturbulent velocity caused by progressive acoustic waves which increase in amplitude as the density decreases. A possible explanation of the observed dependence of microturbulence on $b-y$ is suggested by the shape of the solar relationship.

To allow even the most elementary analysis of the data to be performed, two parameters must be known--the height of the convection zone and the optical depth at which the microturbulent velocity is measured. In the temperature range from $A$ to $G$ stars, the onset of convection occurs at greater optical depths for later spectral classes. Table 9 gives the parameters which are relevant to the convection zone in stars in this temperature range. Column 1 is the spectral type of the star; column 2, the effective temperature of the star (Johnson 1966); column 3, the value of $b-y$ corresponding to the spectral type (Crawford 1966); column 4, the mean optical depth at which

Table 9
Convection Zone Data and Microturbulence

| Sp. | $T_{\text {eff }}$ | $b-y$ | $\tau_{\mathrm{c}}$ | $\Delta \tau_{c}$ | $\xi$ | $\tau$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 8020 | . 125 | . 22 | $+.37$ | 3.7 | . 67 |
| A8 | 7810 | . 150 | . 28 | $+.31$ | 3.8 | . 61 |
| , | 7610 | . 175 | . 34 | $+.25$ | 3.7 | . 55 |
| F'0 | 7500 | . 200 | . 40 | $+.19$ | 3.4 | . 49 |
|  | 7360 | . 225 | . 45 | $+.14$ | 3.0 | . 44 |
| F2 | 7200 | . 250 | . 51 | $+.08$ | 2.5 | . 38 |
|  | 6750 | . 290 | . 59 | . 00 | 1.9 | . 30 |
| F6 | 6600 | . 300 | . 62 | -. 03 | 1.9 | . 27 |
|  | 6375 | . 325 | . 67 | -. 08 | 2.3 | . 22 |
| F8 | 6150 | . 350 | . 72 | -. 13 | 3.0 | . 17 |
|  | 5900 | . 375 | . 76 | $-.17$ | 3.7 | . 13 |
| GO | 5775 | . 390 | . 80 | -. 21 | 4.0 | . 09 |

the atmosphere becomes unstable against convection (BöhmVitense 1964); and the remaining columns will be explained below.

The mean depth of line formation (and hence the depth at which the microturbulence is measured) could be determined by choosing the optical depth in the appropriate model atmosphere at which the temperature is equal to the excitation temperature determined from the curve of growth. However, the uncertainties in the observed values of $\theta_{\text {ex }}$ are quite large, and this procedure for determining the depth of line formation is unreliable. The method which will be used is to assume a mean depth of line formation which is the same for all lines and for stars of all spectral types. For Fe $I$, the contribution function, which is a measure of how much of the equivalent width of an absorption line is contributed by each layer in the stellar atmosphere (e.g. Underhill 1957), has its maximum at approximately $\tau=0.3$ for these stars.(Strom 1968b).

With the aid of Table 9, an observed dependence of the microturbulence on depth can be constructed, if the assumption is made that the shape of this dependence does not change over. the spectral range being considered but
the position of the curve shifts depending upon where the convection zone ends. Because the minimum in the observed $\boldsymbol{\xi}$ versus $b-y$ curve occurs at. a $b-y$ corresponding to spectral class $F 5$, it is reasonable to assume that the minimum of the $\xi$ versus $\tau$ curve for this spectral type occurs at $\boldsymbol{\tau}=0.3$. The value of $\boldsymbol{\xi}$ at this value of $\tau$ is $1.9 \mathrm{~km} / \mathrm{sec}--$ the value of $\xi$ corresponding to $v_{\text {min }}$ in the solid curve in Figure 14. Once this point has been fixed on the curve, the difference between the optical depth at which the convection zone ends for the various spectral types can be used to construct the rest of the $\xi$ versus $\tau$ curve. Column 5 of Table 9 is the difference between the optical depth at which convection begins at a given $b-y$ and the depth for an F5 star. Column 6 is the optical depth in an F5 star at which $\xi$ has the value given in column 4.

The method of the construction of this curve is made more clear if the curve is used to predict the observed microturbulent velocities. Because the onset of convection occurs at $\tau=0.22$ for an A8 star (Table 9), the $\xi$ versus $\tau$ curve is shifted to the left relative to that of an F5 star by the difference between the $\tau$ values at which convection begins (0.29 in this case). Similarly,
for a GO star, the curve is shifted by 0.23 to the right of that of an F5 star. The curves for these three spectral types are shown in Figure 18. A vertical line has been drawn at $\tau=0.3$, which corresponds to the point at which the microturbulent velocity is measured. The point at which this line intersects each curve is the value of the microturbulent velocity which was observed for that spectral type.

The general shape of the $\xi$ versus $\tau$ curve derived in this way is remarkably similar to the curve which has been observed for the sun (Schwarzschild 1961). There is a decrease of the microturbulent velocity in the region where the convective zone overshoots, and there is a rise above the $\xi_{\text {min }}$ point.

This curve qualitatively explains the behavior of the microturbulent velocity between A8 (the inferred maximum of the $\xi$ versus $b-y$ curve) and $G O$. However, two problems remain--the observed minimum in the early $A$ stars and the known low value of the solar microturbulence.

Two possible answers can be given to the first of these problems. First, because very little of the flux is carried by the convection zone in early A stars, the zone


Fig. 18. Inferred Relation between $\xi$ and $\tau$
may be so weak that it generates very little energy at its surface. In the models of Böhm-Vitense (1964), such a decrease in the velocity at the surface of the zone does not appear in the hottest star considered $\left(7000^{\circ}\right)$. However, this corresponds to an FO star in which the observed microturbulence is still high. Therefore, without the calculations beyond $T_{\text {eff }}=7000^{\circ}$ being available, no theoretical evidence can be used to accept or reject this explanation.

Second, in the early A stars, the onset of convection occurs so high in the atmosphere that the depth at which the lines are formed may actually lie in the convective zone. This is the case in an A8 star where the lines are assumed to be formed at $\tau=0.3$, and the onset of convection occurs at $r=0.22$. Because the flow deep in the convection zone is probably laminar and not turbulent, if the lines are formed sufficiently deep, the measured microturbulent velocity will be small. Near the surface of the zone, the Reynolds number (which is a measure of how nearly turbulent the flow is) becomes larger, and turbulent convection occurs. Thus the observed maximum in the A stars of the $\xi$ versus $b-y$ relation may be the point at
which the lines are formed at the surface of the convective zone where turbulent convection has begun. The decrease of microturbulence in the convective zone is, in fact, predicted by convective theories for the sun (Schwarzschild 1961). However, in the sun the transition from laminar to turbulent flow occurs too deep $(\tau=10)$ to be observed.

Either of these two answers would explain the behavior of the A stars. The low microturbulent velocity of the sun is more difficult to explain. Based on the simple assumptions which have been made, the low velocity does not fit in with the picture which has been presented. However, it is an oversimplification to assume that the $\boldsymbol{\xi}$ versus $\tau$ curve simply moves downward with the convection zone without changing either its shape or scale. In the case of the sun, the observed microturbulent velocity is measured at the minimum of the $\xi$ versus $\tau$ curve (Schwarzschild 1961). In the picture presented above, however, this minimum occurs where the microturbulent velocity is measured for the middle $F$ stars. Thus the scale and amplitude of the $\xi$ versus $\tau$ curve are certainly not fixed, but change as the structure of the atmosphere changes from
the A to the G stars. The theoretical behavior of the velocity fields in this spectral range is a problem which still must be solved. However, the general picture of the progression from laminar flow to turbulent convection to acoustic waves fits the observed variation of microturbulence remarkably well even though this picture is oversimplified.

## D. Summary and Suggestions for Future Work

The present investigation has established the following points:

1) In the early evolution of a star away from the main sequence, there is no detectible change in its microturbulent velocity.
2) The Conti-Deutsch effect can account for a small percentage (10-20 per cent) of the observed scatter in the $m_{1}, b-y$ diagram for main sequence and metallic line stars; the remaining scatter can presumably be attributed to differences in metal abundance.
3) The variation of microturbulence with $b-y$ can be explained in terms of a simplified model of the dependence of $\xi$ on $\tau$ with the important exception that the solar microturbulence does not fit into this scheme.

The most obvious direction for future research-both observational and theoretical--is with respect to the astrophysical interpretation of Figure 14. It is physically unsound to assume that the exact shape of the $\xi$ versus $\tau$ curve does not change over the spectral range considered. The atmospheres of stars in this range are changing significantly in their structure, and Figure 18 should reflect this change. Nevertheless, the qualitative picture presented here should point the way to a more detailed theoretical analysis.

On the observational side, there are several possibilities for future research. To find more sharp-lined $A$ and early $F$ stars, a search program of the type described in Chapter II should be undertaken. The shape of the $v$ versus $b-y$ curve could then be filled in by curve of growth analyses for these stars.

As has been pointed out, the Am stars are ideally suited to examine the Conti-Deutsch effect, because they have a large range in microturbulent velocities. A systematic study of a larger sample of Am stars than is currently available would be extremely helpful in further studies of the significance of this effect.

Finally the run of $\xi$ with $\tau$ can be observed directly through the use of the Goldberg-Unno procedure (Chapter I). These observations would require large amounts of telescope time with high resolution photoelectric coudé spectrum scanners; but these devices are becoming increasingly available to the observational astrophysicist, and such observations should be possible in the future.

## APPENDIX I

## OBSERVATIONAL CURVES OF GROWTH





Fig. 21. Curve of Growth for $\beta \mathrm{CVn}$


Fig. 22. Curve of Growth for HR 672







Fig. 28. Curve of Growth for $\theta$ Per




Fig. 31. Curve of Growth for 22 Lyn



Fig. 33. Curve of Growth for $\sigma$ Boo


Fig. 34. Curve of Growth for $\gamma$ Vir A (green region)



Fig. 36. Curve of Growth for $\gamma$ Vir $B$









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