DEVELOPMENT OF A CONDITIONAL SIMULATION MODEL OF A COAL DEPOSIT

by

Harvey Peter Knudsen

A Dissertation Submitted to the Faculty of the

DEPARTMENT OF MINING AND GEOLOGICAL ENGINEERING

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY
WITH A MAJOR IN MINING ENGINEERING

In the Graduate College

THE UNIVERSITY OF ARIZONA

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As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Harvey P. Knudsen

entitled Development of a Conditional Simulation Model of a Coal Deposit

and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

	7/8/81 Date
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Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copy of the dissertation to the Graduate College.

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Young C. Kim Dissertation Director	July 10, 1981 Date

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ABSTRACT

One of the important factors in developing an emission control strategy for a coal fired steam generator is the characterization of the insitu variability of the coal being used in the furnace. Development of a model to correctly capture the insitu variability of the coal is thus fundamental to the analyses of emission control strategies.

A simulation model of a portion of the Upper Freeport coal seam in Western Pennsylvania was developed using the recently developed technique called conditional simulation. This model was constructed so that it has the same mean, variance, and distribution of values as the real deposit, and most importantly, has the same spatial correlations as the real deposit.

Validation of the model confirmed that the statistical characteristics of the model closely matched the characteristics of the real deposit. A second validation of the model showed that when the model is "mined" according to an actual daily mining sequence, the resulting daily variability corresponded extremely well to what was observed during the actual mining. This second verification served not only to validate the model but also served as a practical

demonstration that the model can be successfully used to predict day by day variation in the quality of run of mine coal.

One potential use of conditional simulation to "test" how well a mine plan works in actual mining was illustrated by an example where four mine plans were tested on their ability to correctly estimate coal production and sulfur content on a yearly basis. In each case, the simulated deposit was mined out according to the mine plan. The resulting comparison of "actual" production and estimated production clearly shows the adequacy or inadequacy of each one.

CHAPTER 1

INTRODUCTION

Coal is the dominant source of fuel for the electric utility industry. Almost 70% of the U.S. production of coal is burned for steam generation. When coal is burned, the major products of combustion are a large amount of heat, large quantities of carbon dioxide and nitrogen oxide, ash, and lessor quantities of SO_2 (sulfur dioxide). In order to limit the amount of SO_2 entering the atmosphere, the Environmental Protection Agency has imposed strict standards on the amount of SO_2 that can be emitted from coal fired steam generation plants.

Current emission standards specify that emissions of SO_2 not exceed 1.2 lbs of SO_2 per million Btu when averaged over a 3 hour period. This standard has forced electric utilities and other coal users to carefully evaluate the quality of coal used in their plants. The chief parameters governing the amount of SO_2 emissions are the amount of sulfur in the coal and the calorific or Btu content of the coal. Since much of the coal mined in the Eastern United States has too high a sulfur content to be burned and meet emission standards, utilities have also investigated various methods to reduce the emissions of SO_2. Major options

to reduce SO_2 emissions are; the removal of SO_2 from the flue gas, removal of sulfur from the coal by cleaning the coal, and reducing the average sulfur content of the coal by blending of high sulfur coal with low sulfur coal.

Development of a cost effective emission control strategy is difficult due to the large number of variables that must be considered and the uncertainties that exist in many of the variables. The quality of the coal reserves is a most important factor in developing an emission control strategy. The coal must be characterized not only by mean values of sulfur and Btu content, but also by the expected variation in these quality characteristics. Variation of coal quality over short time periods (a few hours) is important because most emission regulations set a limit that cannot be exceeded at any time. Variation in coal quality over longer periods such as months or years can influence the amount of flexibility needed in the emission control strategy.

Variability in coal quality entering a generating station is influenced by many factors. Among these are the number of mines supplying coal to the plant, the nature of any blending and/or cleaning that occurs at the plant, and the serial variability of the coal produced from each mine. The variability of run of mine coal from each mine is, in turn, affected by the particular mining methods used to

extract the coal and by the inherent insitu variability of the coal.

Clearly the evaluation of an emission control strategy must start with a careful evaluation of the coal reserves that are going to be burned. This evaluation of coal reserves must include both estimates of average values of the coal quality parameters and quantification of the extent and pattern of variation exhibited by each variable. Conventional methods of modeling insitu coal characteristics such as contouring or kriging are designed to estimate mean values at all locations in the deposit. The techniques used to make these models produce a model that is smoother than the real deposit; thus, these models are unsuitable as models of the true variability exhibited by the deposit. A second model is needed to study the variation in coal quality within a deposit. The second model must possess the same extent and pattern of variability as the real deposit. Such a model can be produced by simulation.

Objective of Study

The purpose of this study is to develop a simulation model of a coal deposit. The simulation model produced must reproduce the mean and variance of assay values in the real deposit. It must have the same spatial correlations and it must reproduce the correct distribution of assay values. The methodology used to make a simulation of a

three-dimensional ore deposit has recently been developed by Journel (1974).

This work is one portion of a research program being done at the University of Arizona in conjunction with the owners of the Homer City generating station in Pennsylvania. The overall purpose of this research program is to develop and evaluate emission control strategies for the Homer City plant. During initial work on this study, it became apparent that to meet emissions limits, it would be necessary to develop the technology to predict and control the variability of coal entering the burners of the generating station. The conditional simulation model of a coal deposit developed in this study is the basic model for studying insitu coal characteristics and determining what the variability of run of mine coal is on an hour by hour, day by day, or year by year basis.

Organization

This study is composed of six chapters. Chapter Two reviews the theoretical basis of conditional simulation. The actual construction and validation of the conditional simulation model are discussed in Chapters Three and Four. Use of the conditional simulation model is illustrated in Chapter Five. Summary and conclusions are presented in Chapter Six.

CHAPTER 2

THE THEORETICAL BASIS OF CONDITIONAL SIMULATION

This chapter describes the methodology used to make a simulation of a three-dimensional ore deposit.

The basis of conditional simulation is that grade values in a three-dimensional ore deposit can be represented by a three-dimensional random function. Until the method of turning bands was proposed by G. Matheron in 1973, simulation of a three-dimensional random function was extremely difficult, if possible at all, at any reasonable cost. The techniques available to simulate one-dimensional random functions were not suitable or realistic to use on three-dimensional problems. Matheron's solution to making three-dimensional simulations was to reduce the three-dimensional problem to a set of one-dimensional simulations that are easily made. These one-dimensional simulations are then combined by a technique called the turning bands method, to form the desired three-dimensional simulation.

The explanation of conditional simulation will start with a description of how one-dimensional simulation can be made. Next, the transition to three dimensions is explained. Finally, conditioning of the simulation to the sample data is explained. The explanations presented in this chapter

are, when possible, made at a low level of mathematical sophistication so that mining engineers who have not had courses in stochastic processes, time series analysis, or functional analysis, can understand the methodology used.

Generation of Correlated Random Variables in One Dimension

A simple way of generating a sequence of correlated random variables is to use a moving average process as illustrated below.

A series of correlated random variables denoted by X(t) can be calculated from a set of independent random variables denoted T(t) by the moving average process defined by Equation 2.1.

$$X(t) = \sum_{k=-m}^{m} f(k)\ T(t+k) \qquad (2.1)$$

$$f(k) = k \qquad -m \leq k \leq m$$

$$f(k) = 0 \qquad \text{otherwise.}$$

The smoothing effect of this moving average is easily shown. Start with a sequence of independent random numbers drawn from a uniform distribution between -0.5 to 0.5. The mean of this distribution is zero and the variance is one-twelfth (0.083). Ten such random numbers are listed below.

0.19	-0.26	-0.30	-0.14	+0.20	+0.37	-0.37	-0.00	+0.15	-0.28
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

Figure 2.1 shows a plot of the above numbers plus the next twenty random numbers.

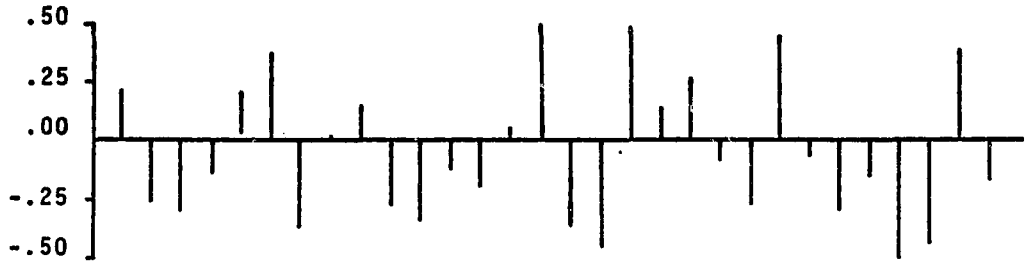


Figure 2.1. Thirty Plotted Random Variables

Now, by using the above simple moving average and letting m equal 3, the random variable X(5) is calculated as follows.

$$X(5) = (-3)(-0.26)+(-2)(-.30)+(-1)(-14)+(0)(.20)$$
$$+(1)(.37)+(2)(-.37)+(3)(0.0)$$
$$X(5) = 1.15$$

The first ten correlated variables are listed below.

0.08	1.15	1.06	-.89	-2.4	-1.69	-.19	-.37	1.83	1.59
(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)

Figure 2.2 below shows the results for the first thirty values of X(t).

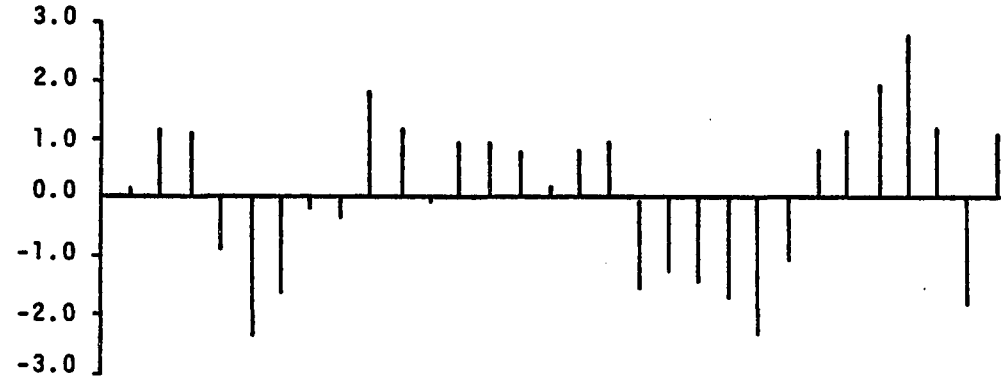


Figure 2.2. Thirty Correlated Random Variables

Notice that there is a tendency for similar values to be grouped together, hence, a certain amount of correlation has, in fact, been introduced by the moving average procedure. This is, of course, what was desired. For instance, X(5) and X(6) should be related because four of the six random numbers used in their calculations are the same for each. For the same reasoning, X(5) and X(16) need have no similarity since none of the same random numbers were used to calculate each one.

Confirmation of the correlations between the simulated values is demonstrated by the variogram shown in Figure 2.3 which was calculated for the sequence of values shown in Figure 2.2.

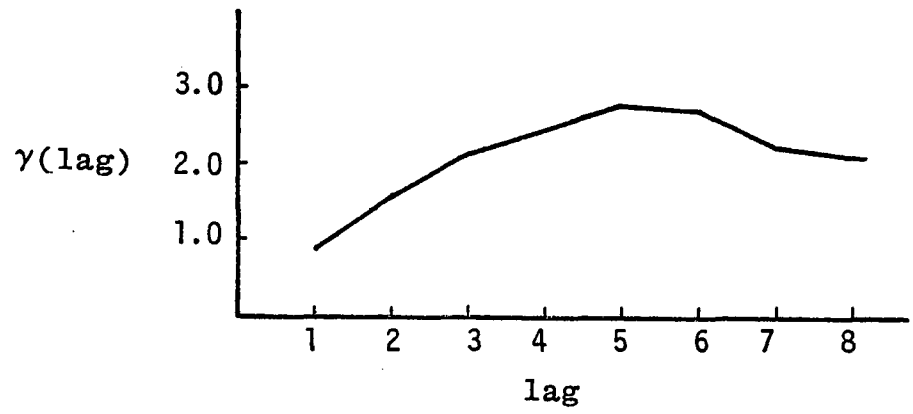


Figure 2.3. Variogram of One-Dimensional Values

The variogram shows that the correlation between variables decreases with increasing lag, until at lag 7, the variables are independent.

These experimental results can be easily verified. The covariance between X(t) and X(t+h) is given by Equation 2.2. Note that E[X(t)] equals zero.

$$C(h) = E[X(t) \cdot X(t+h)] \tag{2.2}$$

By substituting Equation 2.1 into Equation 2.2, we have the following equation.

$$C(h) = E\left[\left(\sum_{k=-3}^{+3} f(k)\ T(t+k)\right) \cdot \left(\sum_{j=-3}^{+3} f(j)\ T(t+h+j)\right)\right] \tag{2.3}$$

Now, since T(t+k) and T(t+h+j) are independent, the expectation E[T(t+k) · T(t+h+j)] equals zero unless t+k = t+h+j, in which case $E[T(i)^2] = \sigma^2$. This only happens when k = h+j.

Thus, we can simplify Equation 2.3 to get the following equation.

$$C(h) = \sigma^2 \sum_{k=-3}^{+3} f(k) \cdot f(k-h) \qquad (2.4)$$

Using this equation, the values C(0) to C(7) were calculated and listed below. Remember, the value of σ^2 is one-twelfth (0.083) and $f(k) = k$ for $-3 \leq k \leq 3$.

C(0) = 2.33	C(4) = -1.16
C(1) = 1.33	C(5) = -1.0
C(2) = .42	C(6) = -.75
C(3) = -.33	C(7) = 0.0

The variogram and covariance function are related by Equation 2.5, hence, we can compute the theoretical variogram and compare it with the experimental one.

$$\gamma(h) = C(0) - C(h) \qquad (2.5)$$

Figure 2.4 shows the excellent agreement between the theoretical and experimental variograms.

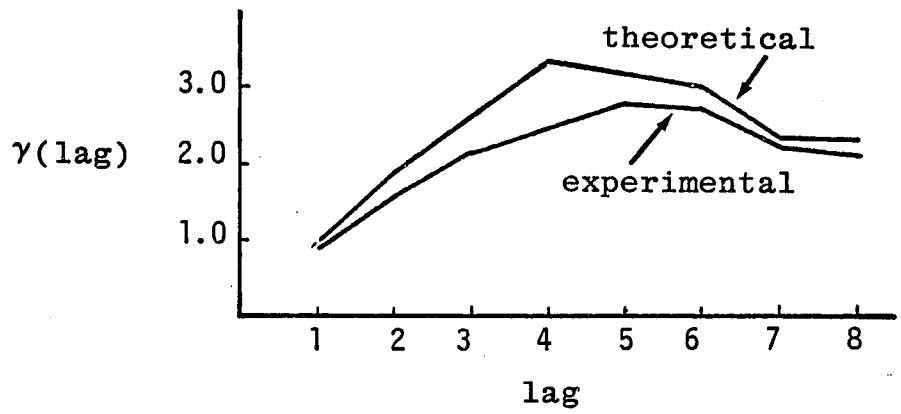


Figure 2.4. Comparison of Theoretical and Experimental Variograms

Other characteristics of the simulated values can be seen in Figure 2.5. Since the simulated values are calculated basically by a weighted sum of independent random variables, the distribution of the simulated values should approximate a Gaussian distribution. As seen in Figure 2.5, the simulated values do appear to approximate a Gaussian distribution.

Since the moving average process simply sums independent random variables, the distribution of X(t) should tend towards a normal distribution due to the Central Limit Theorem. The histogram shown below is symmetric and visually appears to be quite similar to a normal distribution.

This example shows that the moving average process generates a series of random variables that are serially correlated and that the experimental results compare closely

to the theoretical prediction of the covariance and the distribution type.

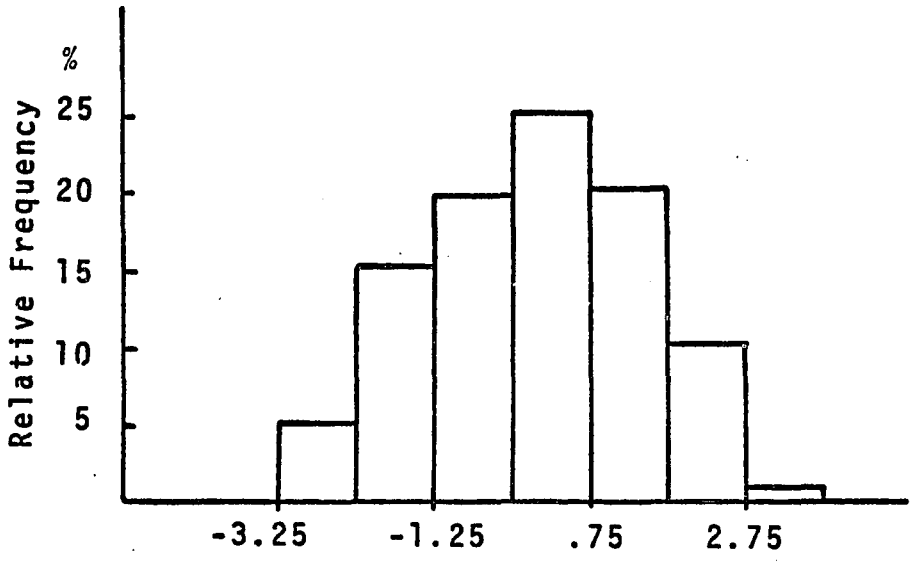


Figure 2.5. Histogram of One-Dimensional Simulated Values

What is really needed to simulated ore deposits is a procedure that will reproduce both the distribution and the particular form of the variogram for a specific deposit. By rewriting Equations 2.1 and 2.4 slightly, more general forms of the moving average process and the covariance function are obtained.

$$X(t) = \sum_{k=-\infty}^{\infty} f(k) \cdot T(t+k) \qquad (2.6)$$

$$C^1(h) = \sigma^2 \sum_{k=-\infty}^{\infty} f(k) \cdot f(k-h) \qquad (2.7)$$

Both X(t) and $C^1(h)$ are now seen to depend upon the weighting function f(k). The problem of generating random variables X(t) having a particular covariance function C(h) can be solved if the appropriate function f(k) can be determined.

A theorem from Bochner (Journel and Huijbregts, 1978) provides a basis for determining f(k) for a particular covariance function. This theorem says essentially that a covariance function can be expressed as Fourier transform. In Equation 2.7, the covariance function is expressed as the discrete form of a convolution product of the weighting function f(k) and its transpose (Journel and Huijbregts, 1978, p. 504). By expressing Equation 2.7 in a continuous form and using the property that a Fourier transform of a convolution is the product of the Fourier transform of f(u) and itself, the following equation can be formed.

$$f\left[C^1(h)\right] = \left[f(f(u))\right]^2 \tag{2.8}$$

Now, by finding the inverse Fourier transform denoted f^{-1}, the weighting function f(u) can sometimes be determined.

$$f(u) = f\left[\left[f^{-1}\left(C^1(u)\right)\right]^{-\frac{1}{2}}\right] \tag{2.9}$$

Although the theoretical solution exists, the practical problem of actually computing f(u) is not easy and,

in some cases, not possible. Fortunately, for the three most common covariance (or variogram) models, e.g. spherical, exponential, Gaussian, the weighting functions have been calculated by Journel and Huijbregts (1978, p. 507).

Examples of One-Dimensional Covariance Models

The one-dimensional covariance function corresponding to the three-dimensional spherical variogram model is given below.

$$C^1(h) = K\left[1 - \frac{3h}{a} + \frac{2h^3}{a^3}\right] \qquad h < a \qquad (2.10)$$

$$= 0 \qquad h \geq a$$

How $C^1(h)$ is derived is explained in the next section. The weighting function used to generate simulated values having the above covariance function is given below (Journel and Huijbregts, 1978, p. 507).

$$f(u) = \sqrt{\frac{12K}{a^3}}\, u \qquad -\frac{a}{2} < u < \frac{a}{2} \qquad (2.11)$$

$$= 0 \qquad \text{otherwise.}$$

In this function, a refers to the range of the spherical variogram and K is the desired variance of the simulated values.

Verification that the above weighting function is appropriate is easily shown. Start with the definition of

the one-dimensional covariance as a convolution and simply perform the integration using the weighting function shown in Equation 2.11.

$$C^1(h) = \int_{-\infty}^{\infty} f(u) \cdot f(u+h)du$$

$$= \frac{12K}{a^3} \int_{-a/2}^{a/2 - h} (u)(u+h)du$$

$$= \frac{12K}{a^3} \int_{-a/2}^{a/2 - h} \left(u^2+uh\right)du$$

$$= \frac{12K}{a^3} \left[\frac{u^3}{3} + \frac{u^2}{2} h\right]_{-a/2}^{a/2 - h}$$

$$= \frac{12K}{a^3} \left[\frac{2a^3}{24} - \frac{a^2h}{4} + \frac{h^3}{6}\right]$$

$$= K \left[1 - \frac{3h}{a} + \frac{2h^3}{a^3}\right]$$

The one-dimensional covariance model corresponding to the exponential variogram model is given below (Journel and Huijbregts, 1978, p. 507).

$$C^1(h) = K(1-\lambda h)e^{-\lambda h} \qquad (2.12)$$

The appropriate weighting function is given in Equation 2.13 (Journel and Huijbregts, 1978, p. 508).

$$f(u) = 2\left(\sqrt{K\lambda}\right)\left(1-\lambda u\right)e^{-\lambda u} \qquad u \geq 0 \qquad (2.13)$$

$$f(u) = 0 \qquad u < 0$$

The final model that has been worked out is the one-dimensional Gaussian covariance

$$C^1(h) = K\left(1 - \frac{2h^2}{a^2}\right)e^{-h^2/a^2} \qquad h > 0 \qquad (2.14)$$

which has the following weighting function (Journel and Huijbregts, 1978, p. 508).

$$f(u) = \left(\frac{16K}{a^3\sqrt{\pi}}\right)u\ e^{-\frac{2u^2}{a^2}} \qquad -\infty < u < \infty$$

Going from One-Dimensional to Three-Dimensional Random Variables

In the previous section, it was shown that a simple moving average could be used to generate one-dimensional correlated random variables for the three variogram models commonly found in ore deposits.

In this section, Matheron's unique solution to the problem of simulating three-dimensional random variables with imposed covariance using one-dimensional correlated random variables is explained.

When making a variogram study of a deposit, it is always useful to study the geology of the deposit and the mechanisms that formed the ore deposit in order to understand the spatial characteristics of the deposit. An ore deposit is not formed by just one process, but instead, may be the result of many different processes acting in various

manners and at various times. Take a fairly simple example, for instance, a placer deposit formed in a river channel. As shown in Figure 2.6, the deposit may consist of numerous interbedded sand lenses.

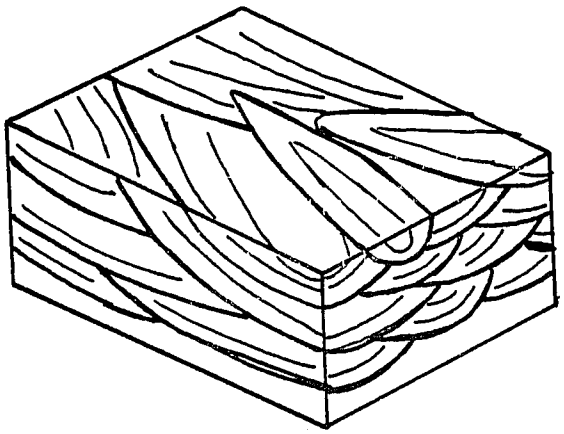
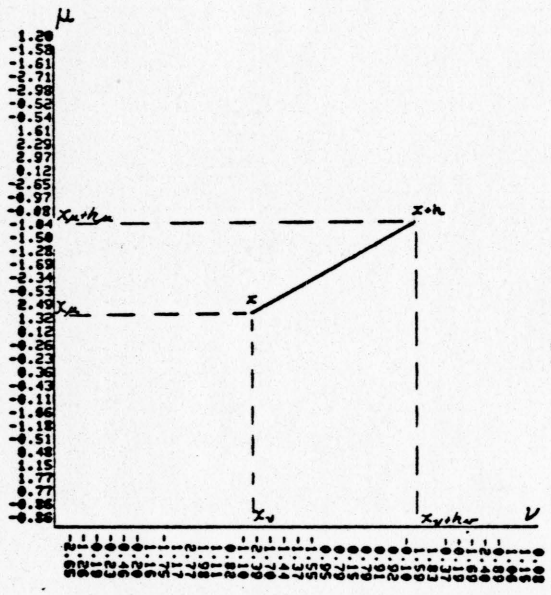


Figure 2.6. Idealistic Placer Deposit

The grade of ore within a sand lens should show more continuity or correlation parallel to the direction of deposition than perpendicular to it or vertically within the lens. The continuity of the grade of the lenses should be greater within a horizontal zone of the deposit than vertically within the deposit. The overall spatial variability of grade is thus the sum total of all the directional characteristics due to the manner of deposition of the ore minerals within the sand lenses, and the different characteristics of each sand lens deposited during the formation of the placer. We can thus view the three-dimensional covariance (or variogram) as the sum of the various directional

covariances for each process that aided in the formation of the deposit.

The above idea can be explained by considering two points x and x+h as shown in Figure 2.7. For the moment, restrict the discussion to two dimensions. Let $y(\mu)$ and $y(\nu)$ be independent realizations of one-dimensional random functions with covariance $C^1(h\mu)$ and $C^1(h\nu)$, respectively, on vectors μ and ν.



$$Z(x) = \frac{1}{\sqrt{2}} (-2.39+1.32) = -.75$$

$$Z(x+h) = \frac{1}{\sqrt{2}} (1.59-1.04) = .777$$

Figure 2.7. Using Two One-Dimensional Lines to Make a Two-Dimensional Simulation

Assume for now that the grades at x and x+h are a function of the two random functions $y(\mu)$ and $y(\nu)$. Specifically, let Z(x), the grade at point x, be given by the sum

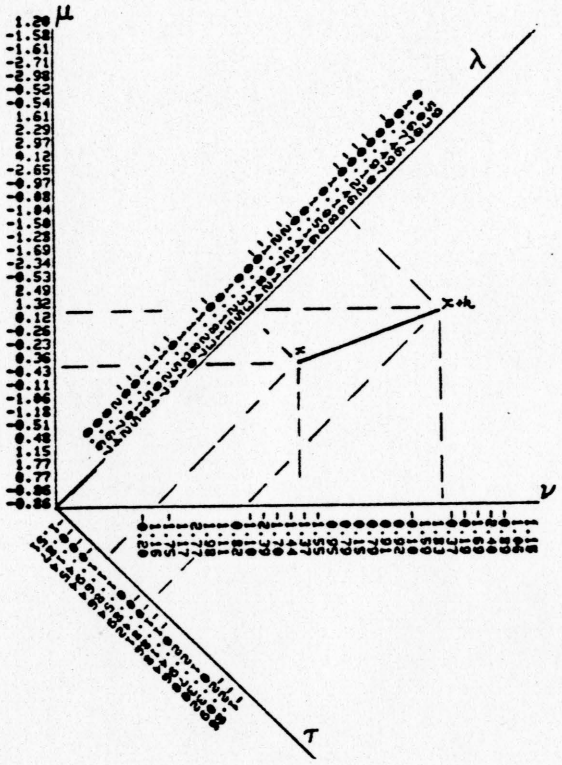
$$Z(x) = \frac{1}{\sqrt{2}} [y(<x,\mu>) + y(<x,\nu>)] \qquad (2.15)$$

where $\langle x,\mu \rangle$ refers to the projection of point x on vector μ, and $y(\langle x,\mu \rangle)$ the value of the random variable at $\langle x,\mu \rangle$. The projection $\langle x,\mu \rangle$ is used to indicate which value from line μ is to be included in the sum given by Equation 2.15. Sample calculations are shown in Figure 2.7. Notice that if x and x+h were exactly East-West of one another, the difference in the grades Z(x) and Z(x+h) would be entirely due to the process that is represented by the values on line ν. The covariance of Z(x) is a function of the one-dimensional covariances $C^1(h\mu)$ and $C^1(h\nu)$ and is given by the simple summation of

$$C(h) = C^1(\langle h,\mu \rangle) + C^1(\langle h,\nu \rangle) \qquad (2.16)$$

where $\langle h,\nu \rangle$ is the projection of vector ν onto vector h. Thus, if h is parallel to v, the covariance between x and x+h will be mainly due to the process acting in the direction v, and vice versa.

More lines can be used to improve the simulation. For instance, in Figure 2.8, four lines are included.



$$Z(x) = \frac{1}{\sqrt{4}} (-0.43+0.25-1.37$$

$$+1.94) = .195$$

$$Z(x+h) = \frac{1}{\sqrt{4}} (0.12+0.16+1.83$$

$$-0.42) = .845$$

Figure 2.8. Using Four One-Dimensional Lines to Make a Two-Dimensional Simulation

Z(x) is again defined as the sum of the perpendicular projections of x onto each of the one-dimensional lines.

$$Z(x) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} y(<k,i>) \qquad (2.17)$$

Since Equation 2.17 is a sum of independent random variables having covariance $C^1(0)$, the term $\frac{1}{\sqrt{n}}$ is included to insure the variance of Z(x) will be also $C^1(0)$. The covariance of Z(x) is given by Equation 2.18.

$$C(h) = \frac{1}{n} \sum_{i=1}^{n} C^1(\langle h,i \rangle) \qquad (2.18)$$

As the number of lines used goes to infinity, the covariance C(h) tends toward the desired three-dimensional covariance.

$$C(h) = \frac{1}{2\pi} \int_{1/2 \text{ sphere}} C^1(\langle h,\mu \rangle)du \qquad (2.19)$$

The simulation will more accurately reflect the imposed covariance C(h) the greater the number of lines that are used. There are, however, practical limits to the number of lines used. The lines should be uniformly distributed over the unit sphere. In three dimensions, it turns out that 15 is the largest number of lines that can be exactly evenly distributed over the sphere. These 15 lines are obtained by connecting the midpoints of opposing edges of an icosahedron (Journel and Huijbregts, 1978, p. 503). The icosahedron is shown in Figure 2.9. It is sufficient to integrate over half the sphere since μ and $-\mu$ determine the same line.

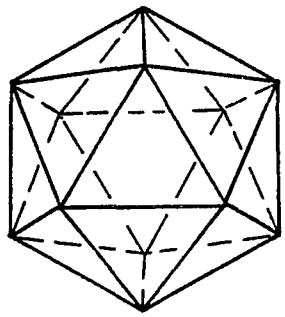


Figure 2.9. Icosahedron

The method of simulation outlined above is referred to as the Method of Turning Bands (Journel and Huijbregts, 1978, p. 500). The name requires some explanation. The term "bands" refers to the distance between the values simulated on the one-dimensional lines. The term "turning" refers to rotating the lines in space so that they are evenly distributed in space.

One last point needs to be considered. Given a three-dimensional covariance C(h), what is the one-dimensional covariance $C^1(t)$ that, when integrated over the half sphere, will give C(h)? In the three-dimensional case, Equation 2.19 can be rewritten as shown below.

$$C(h) = \frac{1}{2\pi} \int_0^{2\pi} d\theta \int_0^{\pi/2} C^1\Big(|h \cos \phi|\Big) \sin \phi \, d\phi \qquad (2.20)$$

Making the substitution $t = |h \cos \phi|$ and integrating out ϕ gives

$$C(h) = \frac{1}{h} \int_0^{h} C^1(t)dt \qquad (2.21)$$

Now, by taking the antiderivative of Equation 2.21, the means of finding the desired one-dimensional covariance $C^1(h)$ is given.

$$C^1(t) = \frac{\partial[h\ C(h)]}{\partial h} \qquad (2.22)$$

Example of Determining One-Dimensional Covariance

The covariance corresponding to the common spherical variogram is given below.

$$C(h) = K\left[1 - \frac{3h}{2a} + \frac{h^3}{2a^3}\right] \qquad h < a$$

$$= 0 \qquad \text{otherwise.}$$

Applying Equation 2.22 gives the one-dimensional covariance $C^1(h)$ corresponding to the spherical variogram.

$$C^1(h) = \frac{\partial\left[h\left(K\left(1 - \frac{3h}{2a} + \frac{h^3}{2a^3}\right)\right)\right]}{\partial h}$$

$$= \frac{K\,\partial\left(h - \frac{3h^2}{2a} + \frac{h^4}{2a^3}\right)}{\partial h}$$

$$C^1(h) = K\left[1 - \frac{3h}{a} + \frac{2h^3}{a^3}\right] \qquad h < a$$

$$= 0 \qquad \text{otherwise.}$$

In the previous section, it was shown that this covariance is the convolution of the following weighting function:

$$f(u) = \sqrt{\frac{12K}{a^3}}\,u \qquad -\frac{a}{2} < u < \frac{a}{2}$$

$$= 0 \qquad \text{otherwise.}$$

Summary of Turning Bands Method

The last two sections have explained how the turning bands method produces a simulation of a stationary three-dimensional random function. The three-dimensional random function being simulated has the following characteristics:

1. Imposed covariance
2. Mean equal zero
3. Gaussian distribution
4. Stationary

The method can be used to simulate any stationary three-dimensional random function as long as:

1. the one-dimensional covariance $C^1(h)$ corresponding to the specified three-dimensional covariance can be determined, and
2. the weighting function f(u) can be determined.

In practice, most ore deposits exhibit covariance functions that can be modeled by either the spherical, exponential, or Gaussian models, or by a combination of these models; thus, the above limitations usually pose no problems. Controlling these simulations so that they have a specified mean and correct distribution is explained in the next section.

Controlling the Distribution of Simulated Values

One of the necessary requirements of the simulation model of a deposit is that it correctly reflect the distribution of the assay grades in the deposit. The Turning Bands Method produces simulated values by adding together 15 independent realizations defined on lines (Equation 2.13). Since the values making up the lines are approximately normally distributed, the sum of 15 independent values should also be normally distributed due to the Central Limit Theorem. More specifically, the simulated values produced by Equation 2.13 should be normally distributed with zero expected value and variance equal to the variance of the one-dimensional lines, $C^1(0)$.

For deposits having a normal distribution of assay grades, the simulation model must be adjusted to have the correct mean grade. This is done by simply adding the mean to each simulated value.

In most cases, however, the distribution of assay grades is not well approximated by a normal distribution. In these cases, the simulated values have to be transformed to reproduce the correct distribution of assay grades. This can be done by finding the transformation function that converts the original assay grades to a standard normal distribution. Let $Z_0(x)$ represent the real assay grades, and $\mu(x)$

the corresponding normally distributed value. The transformation function, $\phi(\mu)$, shown below, transforms the standard normal variable $\mu(x)$ to $Z(x)$.

$$Z(x) = \phi[\mu(x)] \tag{2.23}$$

Likewise, the inverse of $\phi(\mu)$ transforms $Z(x)$ to $\mu(x)$.

$$\mu(x) = \phi^{-1}[Z(x)] \tag{2.24}$$

For example, certain deposits have assay grades that are lognormally distributed. The inverse transformation function $\phi^{-1}(Z)$ relating $\mu(x)$ to $Z(x)$ is thus

$$\mu(x) = \log[Z(x)]$$

and, therefore, $\phi(\mu)$ is simply

$$Z(x) = e^{\mu(x)}.$$

In many cases, the appropriate transformations may not be obvious and more generalized methods are used to make the transformations. Simple graphical means can be used if the histogram of the available data is representative of the distribution of the grades in the deposit. Figure 2.10 shows an example of the one-to-one correspondence between the cumulative frequency of the assay values and the cumulative normal distribution function. Another approach is to

approximate $\phi[\mu(x)]$ by Hermite polynomials (Kim, Myers, and Knudsen, 1977).

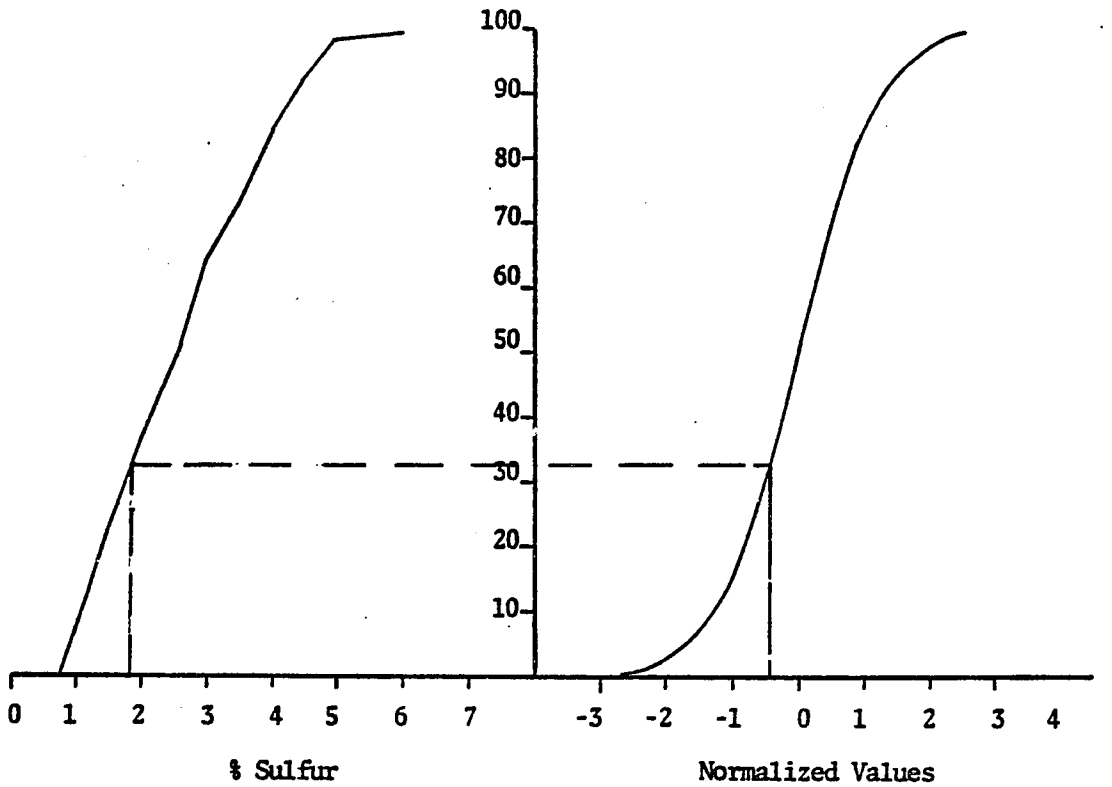


Figure 2.10. Graphical Transformation Function

In general, the Turning Bands Method will be used to simulate the transformed grades $\mu(x)$ rather than the grades $Z(x)$. The covariance imposed on the simulation must, therefore, be the covariance of the transformed grades $\mu(x)$. Then, when the simulation is finally transformed to represent $Z(x)$, the covariance of $Z(x)$ will be correctly reproduced.

If the simulation is to be conditioned to the sample data, the conditioning must be done before the final transformation to Z(x), and the conditioning of $\mu(x)$ must be done with the transformed sample grades.

Conditioning the Simulation to the Available Sample Points

The Turning Bands Method provides realizations denoted $Z_s(x)$ of a random function RF Z(x) that has the same mean, variogram, and distribution as the real deposit. These simulations thus model the most important characteristics of the deposit. However, by conditioning the model so that the simulated values match the sample values at the data locations, certain additional characteristics of the deposit are added to the model (Journel, 1974, p. 674). Chief among these is credibility, since the model now looks like the deposit at all the places where both values can be observed, i.e., the sample points. Conditioning also adds a certain robustness to the model because certain local characteristics otherwise unmodeled are imparted to the model.

The effects of conditioning the simulation can be seen in the plots shown on Figure 2.11. The upper plot shows the exact thickness [Z(x)] of coal measured along a hypothetical coal seam. For illustration purposes, assume that this hypothetical coal seam is the real deposit. A

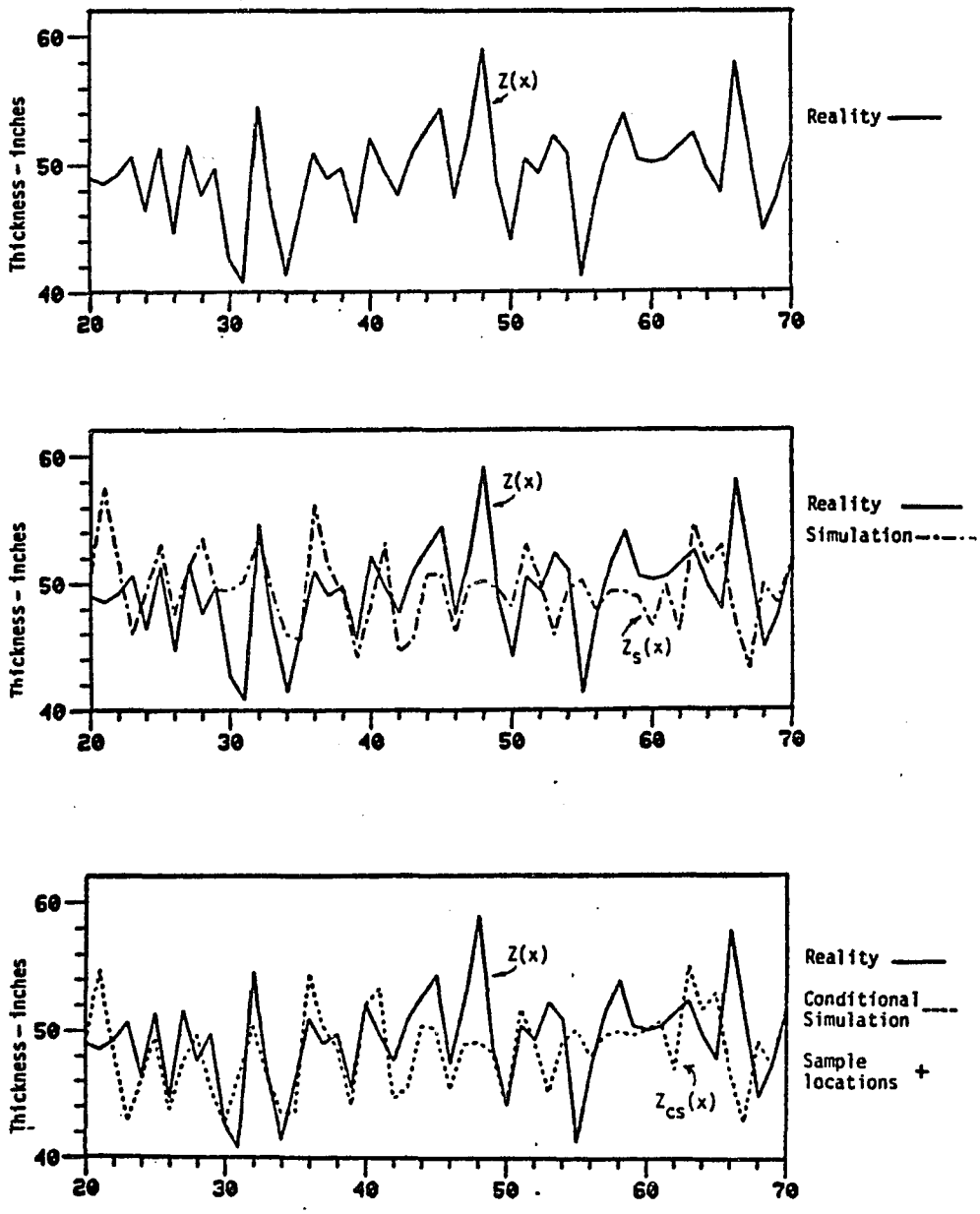


Figure 2.11. Comparison of Reality and Conditional Simulation Along a Hypothetical Coal Seam

simulation of this coal seam was made and is shown on the middle plot together with the "real" deposit. The real values are denoted Z(x) and the simulated values $Z_s(x)$. The simulated data appear to model the general characteristics of the deposit fairly well, but of course, do not match the real values at each and every point. The last plot shows the real deposit again and the simulation after conditioning to the six data points shown.

The conditioning essentially forces the simulation to pass through the data points. The resulting conditional simulation $[Z_{cs}(x)]$ looks more like the real deposit than the nonconditional simulation $[Z_s(x)]$. In addition to lending relevance to the model, other characteristics are imparted to the model by conditioning. For example, local drifts in the sample data will be reproduced within the same zone in the conditional simulation model. This fact can be used to advantage if some specific characteristics need to be modeled. For instance, assume we want the thickness of coal to be very thin in one area of the model, say to test the influence of an unexpected low coal area due to channeling on the production capacity of the mine. The low coal area can be included by using fictitious conditioning points in the area where the low coal is desired. Figure 2.12 shows an example of this.

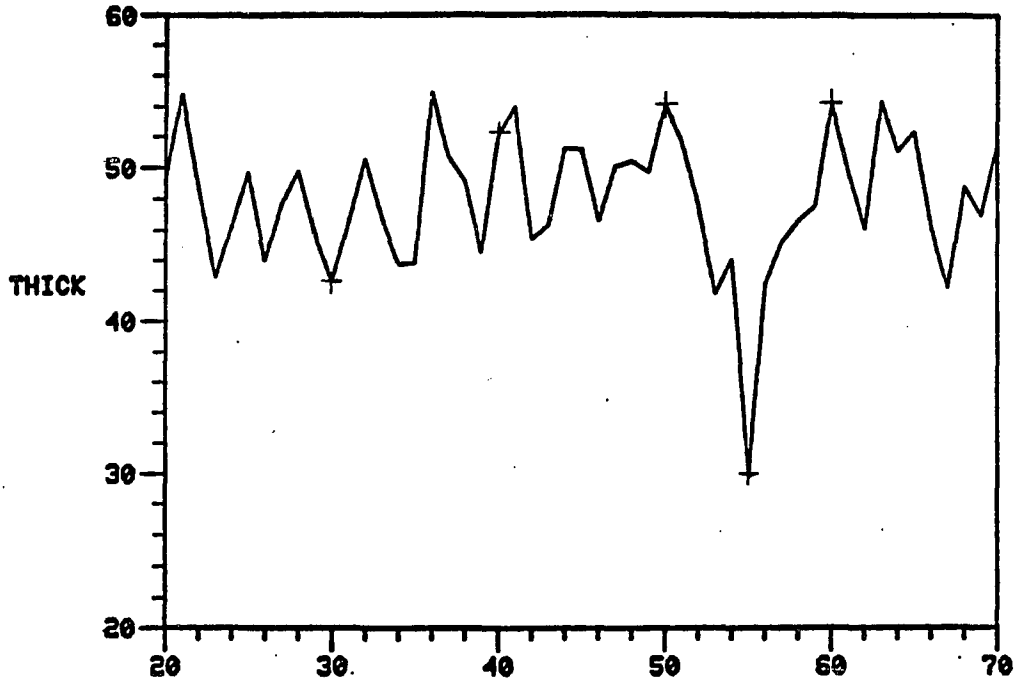


Figure 2.12. Conditioning with Pseudo Sample Values

Requirements of the conditional simulation model $Z_{cs}(x)$ are that it has the same mean, variogram, and distribution as the real deposit, and that at the sample points x_a, it has the same values as the sample values, i.e., $Z_{cs}(x_a) = Z(x_a)$. To condition the simulation model, several properties of kriging are utilized. Some of these can be shown by examining Figure 2.13, which shows a profile of true thickness, $Z(x)$, and a profile of kriged estimates $Z_k^*(x)$. The first property to note is that the kriged estimates pass through the sample values $Z(x_a)$ at the sample positions x_a. This is due to kriging being an exact estimator, hence, the best estimate of a sample grade is the

sample grade. The second property to note from the figure is that the kriged profile is much smoother than the true thickness and that the two profiles meet at the known sample points. The difference between the profiles, $[Z(x)-Z_k^*(x)]$, is of special interest because $[Z(x)-Z_k^*(x)]$, the kriging error, is orthogonal to (independent of) the estimate $Z_k^*(x)$ (Journel and Huijbregts, 1978, p. 495). This fact suggests that one way to make a conditional simulation is to simulate the error $[Z(x)-Z_k^*(x)]$ and add it to the kriged values $Z_k^*(x)$. To be valid, however, the error that is simulated would have to be isomorphic to the real error $[Z(x)-Z_k^*(x)]$.

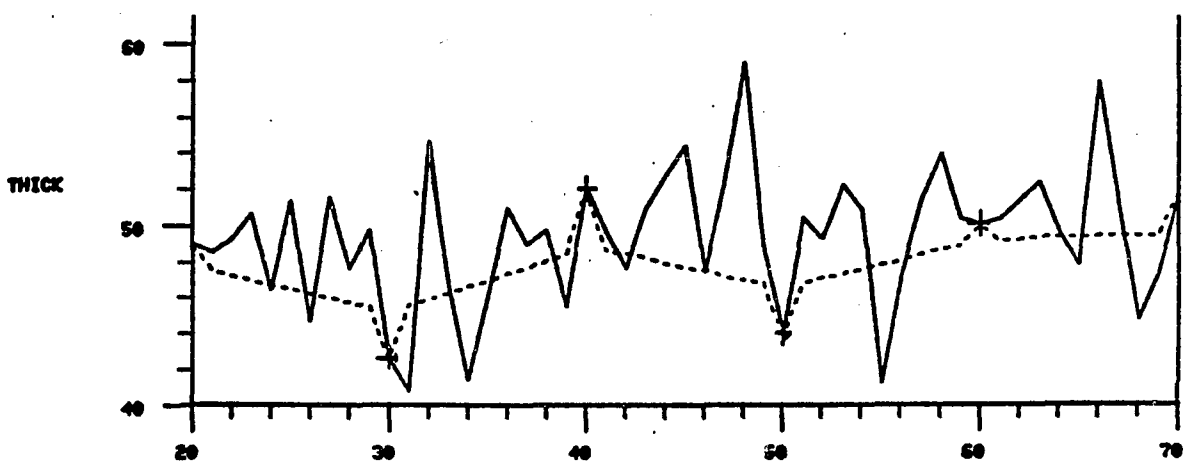


Figure 2.13. Profiles of True Thickness and Kriged Thickness

Simulating $[Z(x)-Z_k^*(x)]$ can be done by taking a non-conditional simulation $Z_s(x)$ and constructing a kriged

estimate $Z^*_{sk}(x)$ using the simulated values at the same sample positions, x_a, as the real data.

This error $[Z_s(x)-Z^*_{sk}(x)]$ has the same properties as $Z(x)-Z^*_k(x)$, and in addition, $Z_s(x)-Z^*_{sk}(x)$ is an independent realization of $Z(x)-Z^*_k(x)$.

The conditional simulation, $Z^*_{cs}(x)$ is formed by the equation below.

$$Z^*_{cs}(x) = Z^*_k(x) + [Z^*_s(x)-Z^*_{sk}(x)] \qquad (2.25)$$

It can be easily seen that $Z^*_{cs}(x)$ passes through the known data points x_a. At each data point x_a, the kriged value $Z^*_k(x_a)$ equals the value of the data point $Z(x_a)$,

$$Z(x_a) = Z^*_k(x_a)$$

and the kriged value $Z^*_{sk}(x_a)$, resulting from the simulated values, equals the simulated value at the points x_a,

$$Z_s(x_a) = Z^*_{sk}(x_a).$$

Thus, the conditionally simulated value equals the value of the data point x_a,

$$Z^*_{cs}(x) = Z(x_a).$$

This process is illustrated by the plots shown in Figure 2.14. The top plot shows a nonconditional simulation, $Z_s(x)$, and a kriged profile, $Z^*_{sk}(x)$, made from the simulated

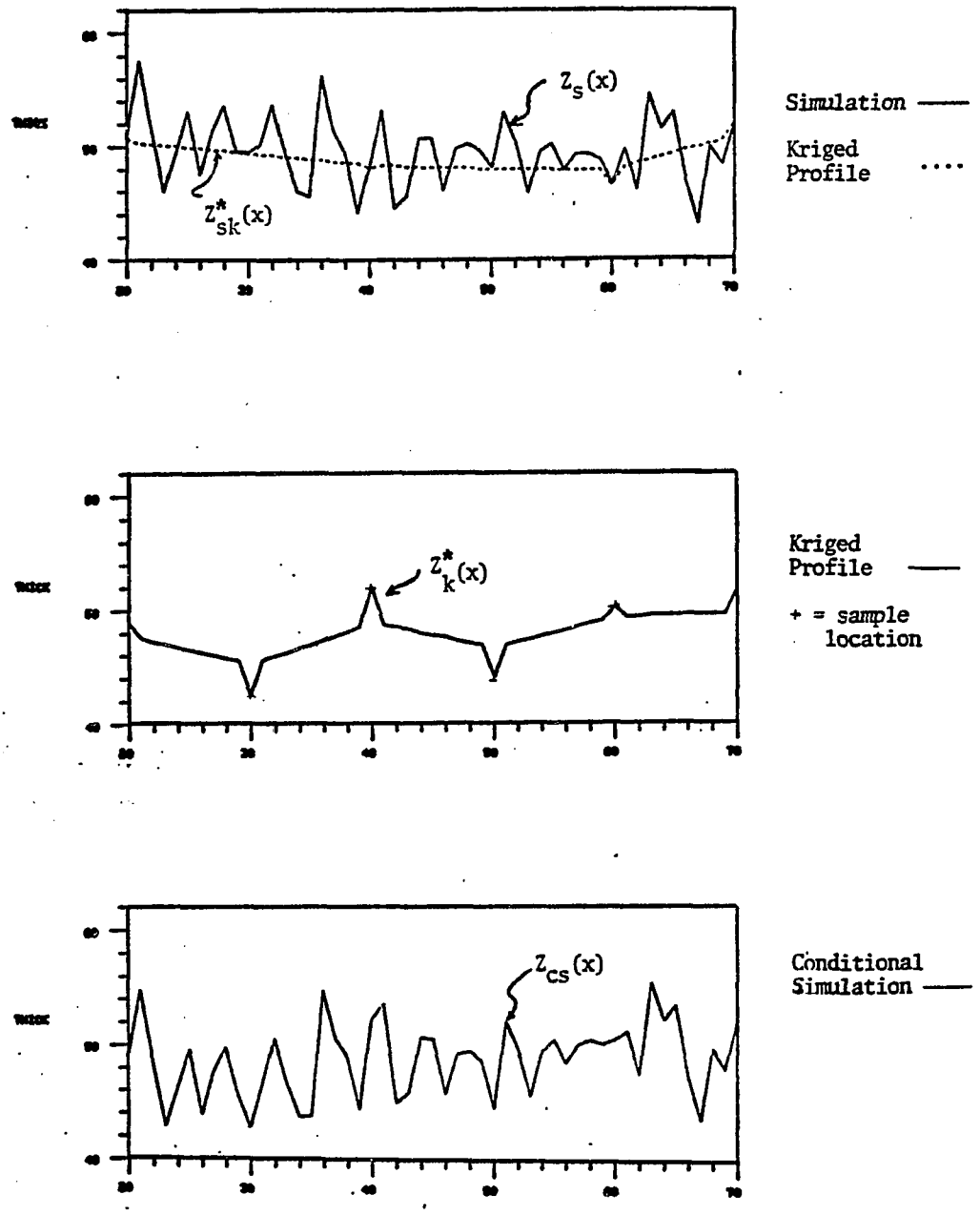


Figure 2.14. Conditioning the Simulation

values located at the six sample points shown. The middle plot shows a kriged profile, $Z_k^*(x)$, made from the real sample values located at the six sample points shown on the plot. The difference $[Z_s(x)-Z_{sk}^*(x)]$ from the upper plot is added to the profile $Z_k^*(x)$ shown in the middle plot to get the final conditional simulation $Z_{cs}^*(x)$ shown in the bottom plot. The resulting conditional simulation is the same as shown earlier in Figure 2.11.

Simulation of Anisotropic Covariance Functions

The Turning Bands Method produces a simulation that has an isotropic three-dimensional covariance function. Most ore deposits, however, do not exhibit isotropic covariance functions. Two solutions are available to solve this apparent problem. The general solution depends on the fact that any three-dimensional covariance (anisotropic or isotropic) can be modeled as a nested sum of isotropic covariances in dimensions one, two, or three. This is possible because every linear combination of covariances with positive coefficients is a covariance (Journel and Huijbregts, 1978, p. 162). Thus, for example, an anisotropic covariance may be modeled by Equation 2.26.

$$C(h) = C_1\left(\sqrt{hx^2+hy^2+hz^2}\right) + C_2(hz) \qquad (2.26)$$

where C_1 is an isotropic covariance in three dimensions and $C_2(hz)$ is isotropic in only one dimension. The random

function Z(x) in this example would be interpreted as the sum of two independent random functions

$$Z(x) = \sum_{i=0}^{2} T_i(x)$$

where T_1 refers to a realization of a random function with isotropic three-dimensional covariance and T_2 represents a realization of a one-dimensional random function. The simulation of this random function would thus be done in two individual steps and the results simply added together.

The second solution is possible if the deposit has a geometrical anisotropy, and a linear transformation of coordinates can transform the geometrical anistropy to isotropic. In this special case, the deposit is modeled using an isotropic covariance, and then the models' coordinates are transformed to restore the geometrical anisotropy.

Simulation of Non-stationary Random Functions

Some ore deposits display a definite trend or drift in the assay values. For example, some deposits show a progressive decrease in grade with depth. In such cases, the deposit should probably be considered as a realization of a non-stationary random function.

Non-stationary random functions can be simulated if the random function Z(x) can be interpreted as a sum of a

drift component, M(x), and a stationary residual, R(x), (Journel and Huijbregts, 1978, p. 500) as shown by Equation 2.27.

$$Z(x) = M(x) + R(x) \tag{2.27}$$

The deposit can then be simulated by modeling the drift and simulation of the residuals using the Turning Bands Method. Practical problems exist with this approach, due to the difficulty of modeling the drift.

A second approach is possible if the drift is localized or restricted to a certain section of the deposit. In such cases, the drift may be simulated by conditioning the simulation to true or fictitious data in the zone (Journel and Huijbregts, 1978, p. 500). An example of this technique was shown in Figure 2.11 earlier. This conditioning will force the simulation to follow the pseudo or fictitious data points.

CHAPTER 3

DEVELOPMENT OF COMPUTER PROGRAMS AND PROCEDURE TO CONSTRUCT A CONDITIONAL SIMULATION MODEL

In this chapter, the structure and design of the computer programs written to construct a conditional simulation model are described. The description of the computer programs is limited to a general discussion of the specific functions each program performs in the construction of the model.

Desired Characteristics of the Model

The conditional simulation model must reproduce the most essential statistical characteristics of the real deposit. Specifically, the model should have:

1. the same mean as the real deposit,
2. the same variance and variogram function,
3. the same empirical distribution, and
4. be conditioned to the real data.

The Turning Bands Method produces a simulation that is nonconditional to the sample data and that has approximately a Gaussian distribution of simulated values. Clearly, the sequence of construction of a conditional simulation must involve several steps if the model is to reproduce the above desired characteristics of the true deposit. For

simple deposits, the procedure of making a conditional simulation can be divided into the following basic steps.

1. Make a nonconditional simulation of the deposit. This model has values that have a Gaussian distribution with mean equal to zero and variance equal to one.
2. Transform the model to correct mean and variance.
3. Condition the model to the sample data.
4. Transform the model so the simulated values have the appropriate distribution.

For simple deposits, the above sequence should be sufficient. For more complex models, the above sequence might have to be expanded to correctly model the deposit.

Development of Computer Programs Used to Make Conditional Simulation Models

Development of a set of computer programs used to make a conditional simulation model was guided by the general need to divide the construction into a sequence of discrete steps as outlined above. The particular tasks assigned to each program were carefully chosen to provide flexibility in modeling, so that even very complex deposits can be modeled.

The final design of the computer programs resulted in a set of seven programs that are jointly referred to as the CSIM system. These programs are listed and briefly described below.

Programs for CSIM Model Construction

Program CSIM01 - This program initializes the model.

Program CSIM03 - This program makes a noncondi-tional simulation.

Program CSIM05 - Three types of transformations can be made with this program. In addition, the nugget affect can be added to the model with this program.

Program CSIM07 - This program conditions the model to the available sample data.

Program CSIM08 - This program adds two models to-gether. It is used for complex deposits that cannot be described with a simple spherical model.

Programs for Calculation of Statistics and Display of the Model

Program CSIM04 - This program calculates basic sta-tistics, plots histograms, and calculates variograms of the model.

Program CSIM06 - This program makes single digit printer plots of the CSIM model.

The model actually consists of a three-dimensional array where each element in the array represents a precise physical volume of ground in the deposit. The value of each

element is the simulated attribute, such as the % ash or the % sulfur of that volume of ground. Detail in the model is a function of the volume of ground represented by each element of the array. The smaller the volume of ground represented by each element, the more detail the model has.

The number of simulated values used to model a deposit depends on the size of the deposit and on the spacing between grid points of the model. The physical relationship between the grid of simulated values for a two-dimensional simulation and the actual deposit is shown in Figure 3.1. The area shown in this figure to be modeled is 4,000 ft by 4,000 ft. Using a grid spacing of 20 ft, a total of 40,000 grid points would be necessary to model the area shown.

The large number of grid points needed to model even a very small portion of a deposit presents practical problems in the design of the computer programs to construct the model and to access an entire model in core; therefore, some method of partitioning the model must be used so that only a small portion of the model need be in core at any one time. The option chosen for the computer programs developed in this study is to partition the model into blocks where each block represents 100 by 100 by 1 arrays of grid points (10,000 points). Each of these blocks is then stored on a direct access disk file and only one block at a time is held in core during program execution.

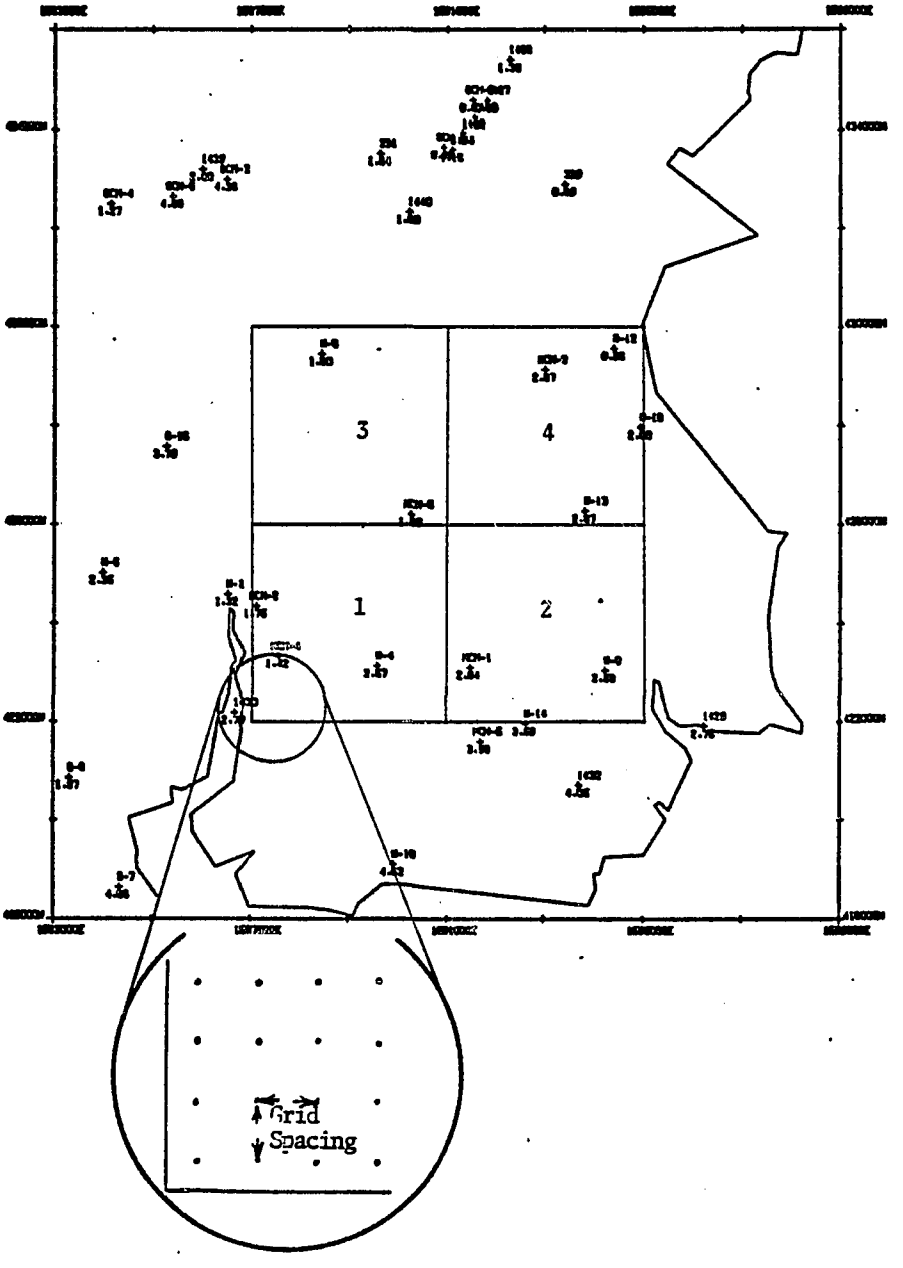


Figure 3.1. Plan Map of Area Modeled

Dividing the deposit into blocks having a given volume, rather than rows or columns, was done primarily because one of the end uses of the model will be to determine

the average quality of coal mined within a certain volume. This calculation can most efficiently be done if the grid points representing the block are all in core at the same time. If row or column storage had been chosen, much more input/output time would be used during program execution.

Description of Program CSIM03

Program CSIM03 is the only program in the CSIM system that is totally unique to the system. The other major programs, CSIM05 and CSIM07, are adaptations of standard geostatistical programs. Program CSIM07 is basically a kriging program and Program CSIM05 performs simple transformations. These programs will not be described here.

Program CSIM03 (see Appendix A) generates a non-conditional simulation by the Turning Bands Method. The computer program was written so that the simulation produced by CSIM03 would have the following properties.

1. Second order stationarity
2. Simulated values having an approximate Gaussian distribution with mean equal to zero and variance equal to one
3. Model exhibiting a spherical variogram with specified range and sill value equal to one

There are several reasons for producing a simulation model that can be considered a unit model. The most important reason is that most deposits exhibit assay values that

do not have a Gaussian distribution. Thus, at some stage, the values in the simulation model must be transformed to the appropriate distribution. This is most easily done if the simulation model, before transforming, has a mean of zero and variance equal to one. A second reason to construct a unit model is the possibility that the model can be saved and utilized for several deposits having the same variogram type but with different means, variances, and distribution types.

Program CSIM03 is programmed to produce a three-dimensional simulation, but as presently coded, only two-dimensional simulations are produced. Full three-dimensional simulations can be obtained by increasing the array size of the Fortran variable IB, which is presently (100, 100, 1). The third index of this array determines how many points will be simulated in the vertical direction.

Programming the Turning Bands Method is relatively straightforward because the method can be divided into two distinct steps. The first step is to generate fifteen lines of one-dimensionally correlated random variables. In the second step, the one-dimensional random variables are combined to form a three-dimensional simulation.

Step One. Algorithm to Generate Correlated Random Variables in One Dimension

One-dimensionally correlated random variables denoted X(t) can be generated by the moving average procedure described in Chapter Two and repeated in the following equation.

$$X(t) = C \cdot \sum_{k=-NR}^{NR} f(k) \cdot T(t+k) \qquad (3.1)$$

The number of random variables X(t) to generate for each of the fifteen lines depends on the size and shape of the deposit and on the grid spacing used in the model. Since each line has a different orientation in space, a different number of variables X(t) are needed for each line.

For each line, the number of random variables to generate can be determined by first calculating the length of the line needed to completely span the deposit. This length is then divided by the grid spacing to give the number of random variables needed. For example, in Figure 3.2, the length of the line necessary to span the deposit is the distance between points A and B. These points are the perpendicular projections of the vertices labeled 1 and 3 onto the one-dimensional line.

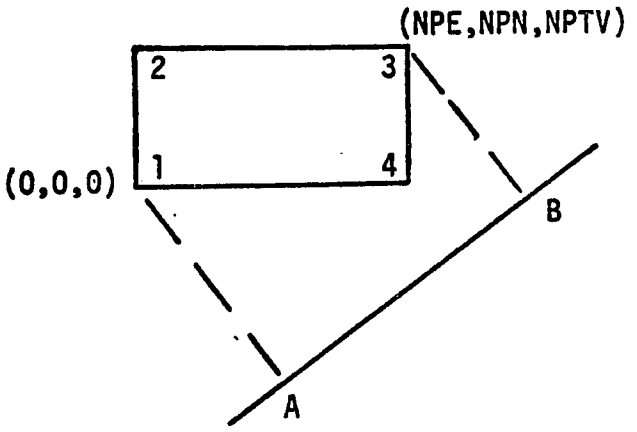


Figure 3.2. Determining Length of Vector that Spans the Deposit

The algorithm used in Program CSIM03 to calculate the number of random variables can be summarized by the following steps.

1. Calculate the coordinates of each vertice of the model. The coordinates are calculated in units corresponding to the grid spacing. For instance, in Figure 3.2, the lower left vertice would have coordinates (0,0,0) while the upper right coordinate would be (NPE, NPN, NPTV) where NPE refers to the number of grid points in the East-West direction, NPN is the number of points in the North-South direction, and NPTV is the number of points vertically.
2. For each of the fifteen vectors, Steps 3 and 4 below are done.

3. Calculate the perpendicular projection of each vertice of the model onto the vector.
4. The number of points to simulate is given by the difference between the maximum and minimum values of the projections calculated in Step 3.

The Fortran code for the above algorithm is shown in Figure 3.3.

```
C-------------------------------------------------------
C     MATRIX V CONTAINS COORDINATES OF THE VERTICES OF THE
C     MODEL BEING SIMULATED.
C-------------------------------------------------------
        V(1,5)=PE
        V(1,6)=PE
        V(1,7)=PE
        V(1,8)=PE
        V(2,3)=PN
        V(2,4)=PN
        V(2,7)=PN
        V(2,8)=PN
C-------------------------------------------------------
C     FIND THE MIN AND MAX INTERSECTIONS OF VECTOR A WITH
C     THE MODEL
C-------------------------------------------------------
        DO 70 L=1,15
           MIN(L)=20000
           MAX=-20000
           DO 10 I=1,8
            LR=(A(1,L)*V(1,I)+A(2,L)*V(2,I)+A(3,L)*V(3,I))
              IF(LR.LE.MIN(L)) MIN(L)=LR
              IF(LR.GE.MAX) MAX=LR
10            CONTINUE
C-------------------------------------------------------
C     CALCULATE THE NUMBER OF POINTS TO GENERATE FOR THIS LINE
C       AND GENERATE THAT MANY 1-D POINTS
C-------------------------------------------------------
           NUM(L) =MAX-MIN(L) +2
           NTOT=NUM(L) +2*NR
```

Figure 3.3. Fortran Code for Determining Number of Correlated Random Variables to Calculate

Having determined the number of values of X(t) to calculate, the values are then calculated using Equation 3.1. In this equation, the number of random numbers T(t) included in the calculation of each value of X(t) is two times the variable NR. This variable is a function of the range of influence (see Equation 2.11) of the variogram and the grid spacing and is calculated as shown in Equation 3.2.

$$NR = \frac{\text{Range}}{2 \cdot \text{Gridspacing}} \tag{3.2}$$

The random numbers T(t) are generated by a Fortran algorithm developed by Schrage (1979, p. 132). This random number generator has excellent characteristics and an extremely large cycle length. The Fortran code is portable and can be used to generate the same sequence of random numbers on a wide variety of computers.

The constant C in Equation 3.1 is included to ensure that the variance of the random variables X(t) is equal to one. In Chapter 2, it was shown that the variance of X(t) calculated without the constant C is given by Equation 2.4, which is repeated here as Equation 3.3.

$$\text{Var } X(t) = \sum_{k=-NR}^{NR} f(k) \cdot f(k) \tag{3.3}$$

For example, if NR = 3 and f(k) = k, then the variance of X(t) is calculated as shown below.

$$\text{Var } X(t) = \sigma^2 \sum_{k=-NR}^{NR} k^2 \qquad \text{and } \sigma^2 = 1/12$$

$$= 1/12\ [9+4+1+0+1+4+9]$$

$$= 28/12$$

$$= 7/3$$

For this example, the value of C needed in Equation 3.1 would thus be $\sqrt{3/7}$. The general formula for the constant C is given in Equation 3.4.

$$C = \sqrt{\frac{36}{(NR)\ (NR+1)\ (2NR+1)}} \tag{3.4}$$

The actual calculation of the random variables X(t) is done by simply evaluating Equation 3.1. The Fortran code used to evaluate Equation 3.1 is shown in Figure 3.4. The Fortran variable DLINE refers to the random variables X(t). After the variables for each line are calculated, they are written sequentially to a temporary storage file.

Step Two. Forming the Three-Dimensional Simulation

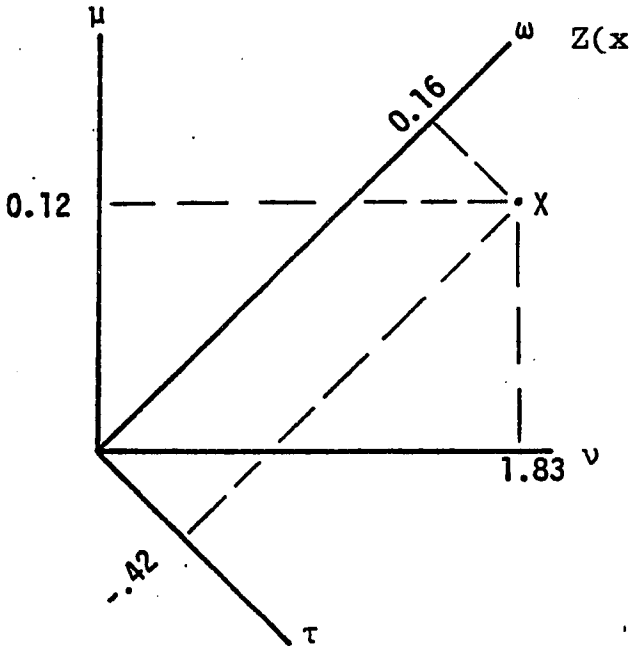
The second step of the Turning Bands Method is to combine the one-dimensional random variables into a three-dimensional simulation. Each simulated value in the three-dimensional model is formed by summing the perpendicular projections of each one-dimensional line onto the individual point x as shown by Equation 3.5.

```
20        DO 30 I=1,NTOT
          RANVAL(I)=RAND(ISEED)-0.5
30        CONTINUE
40           N1=NR+1
             NT1=NTOT-NR
             DO 50 J=N1,NT1
                JN=J+NR
                JN1=J-NR
                DO 50 I=JN1,JN
                   DIS=J-I
                   DLINE(J)=DLINE(J)+DIS*RANVAL(I)*CONT
50                 CONTINUE
             WRITE(ICOR)(DLINE(I),I=N1,NT1)
             DO 60 I=1,10000
                DLINE(I)=0.0
60              CONTINUE
70           CONTINUE
```

Figure 3.4. Fortran Code for Calculation of One-Dimensionally Correlated Random Variables

$$Z(x) = \frac{1}{\sqrt{15}} \sum_{i=1}^{15} Y<k,i> \qquad (3.5)$$

In this equation, Y<k,i> is the k^{th} value of line i. The value of k is determined by calculating the perpendicular projection of the point x onto the vector i. Figure 3.5 illustrates the mechanics of Equation 3.5 when only four lines are included.



$$Z(x) = \frac{1}{\sqrt{4}} [0.12+0.16+1.83-0.42]$$

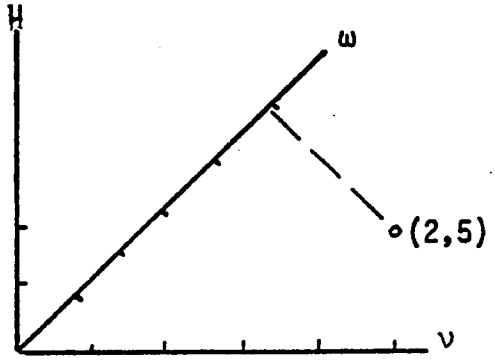
$$= .845$$

Figure 3.5. Example of Calculations Used in Turning Bands Method

The perpendicular projection of point x onto a line can be found by using the dot product of two vectors. Let C be the vector from the origin to point x, and A be the unit vector of the line. The perpendicular projection of x on line a is given by Equation 3.6.

$$L = A \cdot C \tag{3.6}$$

This is illustrated in Figure 3.6. In this example, when x is projected onto ω, it intersects ω at 4.9 units from the origin.



$$A = \begin{bmatrix} .707 \\ .707 \end{bmatrix}$$

$$C = \begin{bmatrix} 2.0 \\ 5.0 \end{bmatrix}$$

$$L = A^T \cdot C$$

$$L = \begin{bmatrix} .707 \\ .707 \end{bmatrix}^T \begin{bmatrix} 2.0 \\ 5.0 \end{bmatrix}$$

Figure 3.6. Perpendicular Projection of a Point to a Line

Implementing Step Two is quite easy. Since the model is divided into blocks, each block can be read into memory individually and all the simulated values for that block calculated before the next block is read in. The Fortran code implementing Step Two is shown in Figure 3.7.

In the code shown in Figure 3.7, the variable A contains the unit vectors for each of the 15 one-dimensional lines. Variable C contains the coordinates of the point being simulated. The array B contains the final simulated values. Each element of array B is formed by the sum of 15 values, one from each of the 15 lines. This sum is divided by the square root of 15 so that the variance of the simulated values is equal to one.

```
C-----------------------------------------------------------------
C       LOOP THROUGH THE MODEL BLOCK BY BLOCK
C-----------------------------------------------------------------
        DO 100 NL=1,NBN
           DO 100 NE=1,NBE
              KBLOCK=NE+(NL-1)*NBE
C-----------------------------------------------------------------
C       READ IN ONE BLOC FROM CSIM MODEL
C-----------------------------------------------------------------
              CALL  TAKOUT(KBLOCK,IVAR)
C-----------------------------------------------------------------
C     READ IN "NUM(L)" VALUES FOR VECTOR A.
C-----------------------------------------------------------------
              DO 90 L=1,15
                 READ(ICOR)(DLINE(I),I=1,NUM(L))
C-----------------------------------------------------------------
C       NOW LOOP THRU POINTS IN THE BLOCK
C-----------------------------------------------------------------
                 DO 80 K=1,NPTV
                    DO 80 J=1,NPT
                       DO 80 I=1,NPT
                          C(1)=I+(NE-1)*NPT
                          C(2)=J+(NL-1)*NPT
                          C(3)=K
                          RL=A(1,L)*C(1)+A(2,L)*C(2)+A(3,L)*C(3)
                          LR2=RL + SIGN(0.5,RL)
                          LR=LR2-MIN(L)+2
                          B(I,J,K)=B(I,J,K)+DLINE(LR)/SQ15
80                        CONTINUE
90              CONTINUE
              REWIND ICOR
```

Figure 3.7. Fortran Code for Turning Bands Algorithm

CHAPTER 4

CONDITIONAL SIMULATION OF A COAL DEPOSIT

The coal deposit simulated in this study is a portion of the Upper Freeport Coal Seam located in Western Pennsylvania near the town of Homer City. The general geology and statistical characteristics of the Upper Freeport Coal Seam will be reviewed prior to describing the construction and validation of the simulation model.

Geology of the Upper Freeport Coal Seam

The Upper Freeport Coal Seam is one of the many coal seams forming the bituminous coal field of the Appalachian Plateau Province in Western Pennsylvania. The Upper Freeport Coal Seam is Pennsylvanian (Allegheny) in age.

The Upper Freeport Coal Seam was originally deposited as peat in a large swamp either in an estuary or possibly a large freshwater lake (Clark, 1979, p. 30). Several events during the deposition of peat have affected the characteristics of the coal seam throughout the deposit. During deposition of the peat in the swamp, a channel system was intermittently active and caused sediments to be deposited in the swamp from time to time. The result of this intermittent sedimentation is pronounced splitting and partings

in the coal seam, especially in the central zone of the swamp. The splitting is less pronounced away from the central channel system.

Peat formation was stopped when almost the entire swamp was flooded and sediments were deposited over the peat. This first peat bed is referred to as the main bench of the Upper Freeport Coal Seam and is present throughout the area studied. The coal varies from 24 to 60 inches thick, averaging about 48 inches.

Although most of the swamp remained flooded, peat continued to accumulate along the margins of the swamp. Coal formed from this peat is referred to as the upper bench of the Upper Freeport Coal Seam. This bench is present only in the Eastern portion of the area studied and varies from 8 inches to 30 inches (Clark, 1979, p. 20).

Finally the entire swamp was flooded or submerged and covered with fine grained sediments resulting in a shale or sandy-shale roof in most areas of the deposit. The last event to affect the coal seam was the start of a channel system. In certain areas, the channel system scoured into the shales forming the roof of the coal and, in some cases, scoured into the coal itself. The end result of the channeling is a sandstone roof in some areas and want areas if the coal had been completely eroded away.

The portion of the Upper Freeport Coal Seam is shown in Figure 4.1. The coal reserves controlled by the Homer City Owners are shown by the polygonal outline. The pluses indicate locations of drillholes intersecting the coal seam. The hachured line is the approximate edge of the upper bench of the coal seam. To the northeast, both the main and upper benches are present and result in a thicker coal seam. Inasmuch as this thicker coal has mostly been already mined out, this area is not included in further analyses or in the simulation.

Statistical Analysis

Three characteristics of the coal seam were analyzed in this study. The thickness of coal and the sulfur content of the coal are the only variables that will be included in the conditional simulation model constructed. For completeness, however, the ash content of the coal was also included in the statistical analyses. Industry practice assumes that the BTU content of coal is directly proportional to the ash content, therefore, the statistics of the BTU content of the coal can be directly determined from the ash statistics.

Basic statistics of the thickness, ash, and sulfur values for the drillhole samples are summarized in Table 4.1.

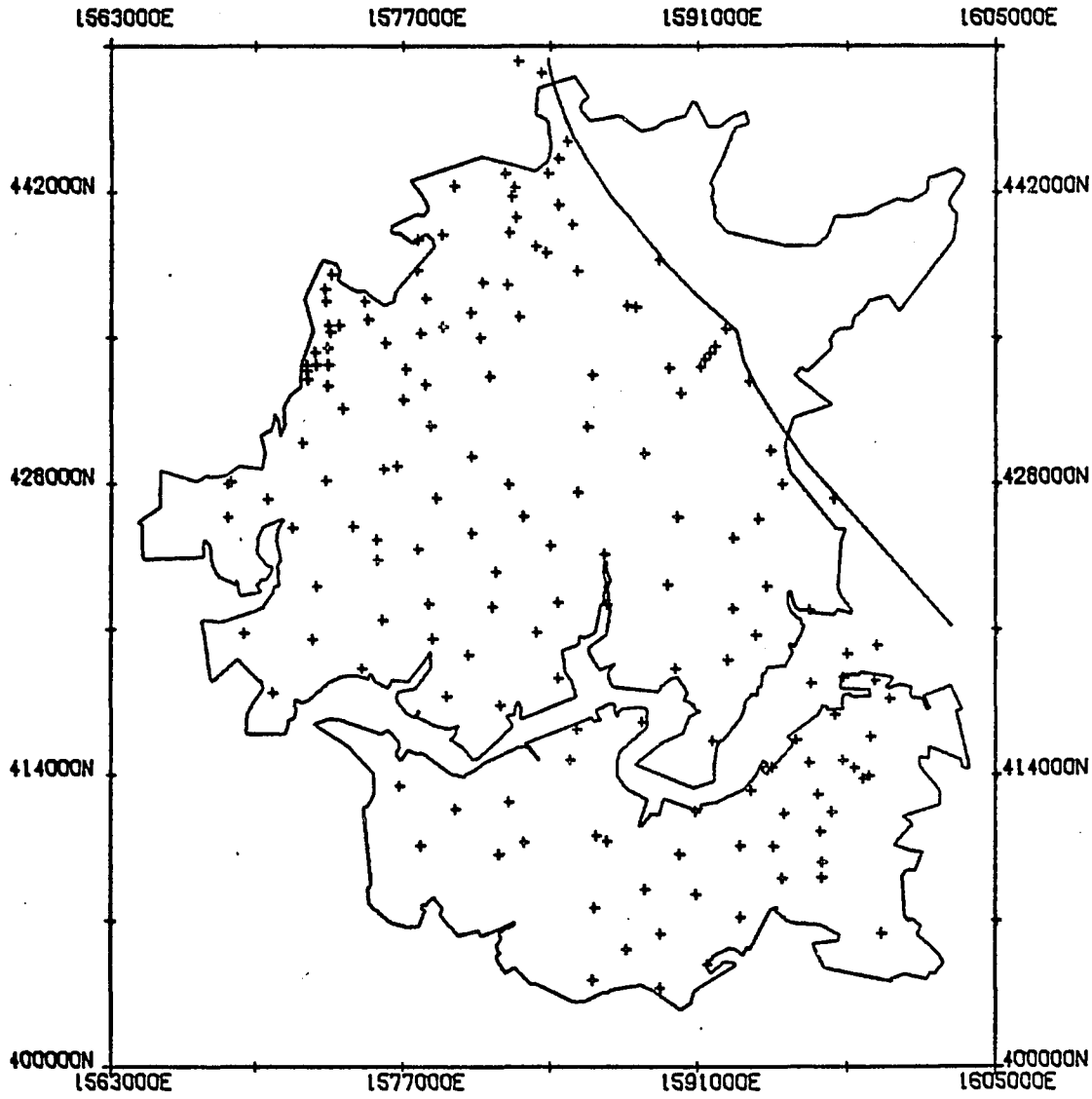


Figure 4.1. Plan Map of Upper Freeport Coal Seam

Table 4.1. Summary Statistics for Coal Characteristics

Characteristic	Number of Assays	Sample Mean	Sample Variance	Sample Std. Dev.
Thickness	170	49.3	36.0	6.0
Sulfur	135	2.62	1.49	1.22
Ash	126	16.2	17.4	4.2

The histogram of coal thickness shown in Figure 4.2 appears almost symmetric except for an apparent truncation of the data at 60 inches. This truncation results mainly from the subjective determination of the boundary of the upper bench of the coal seam and the possible misclassification of whether one or two benches of coal were present at any particular location.

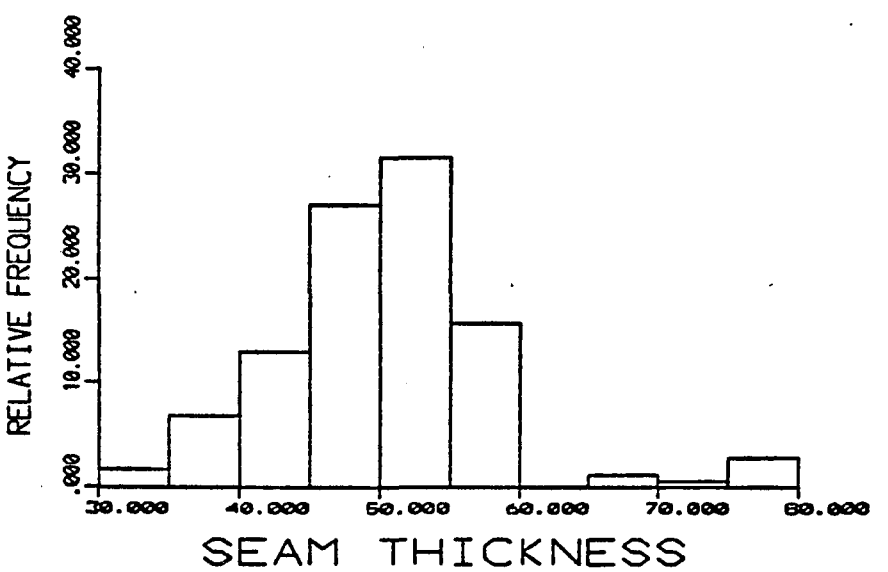


Figure 4.2. Histogram of Seam Thickness Values

The histogram of sulfur values in Figure 4.3 shows several important aspects of the distribution of sulfur. First, the distribution appears to be truncated at about 0.85% S. Secondly, there are several modes, one at about 1.25% S, and at 3.0% S, and a minor one at 4% Sulfur. Finally, the data is skewed to the right. These aspects can possibly be accounted for by the relative differences in amounts of sulfur in organic materials and the amount of sulfur from inorganic materials.

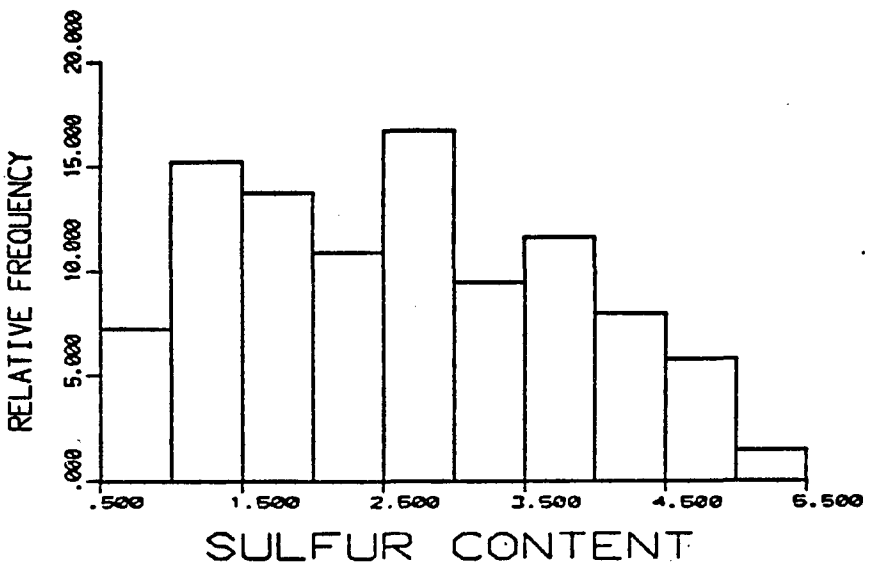


Figure 4.3. Histogram of Sulfur Content

Plants use sulfur in their growth processes and much of this sulfur is bound organically during peat accumulation and coal formation (Cecil et al., 1978, p. 42). The amount of organically bound sulfur is probably between 0.5 and 1.0%.

The amount of this sulfur is important because organically bound sulfur cannot be removed from the coal by coal cleaning processes.

Inorganic sulfur occurs predominantly as pyrite or marcasite and, to a lesser extent, in the form of other metal sulfides. The amount of inorganic sulfur in the coal is sometimes affected by the nature of the roof. Areas having a sandstone roof sometimes show a higher sulfur content due to the downward perculation of solutions rich in iron sulfides (Clark, 1979, p. 21). Due to several distinct origins of sulfur in coal, it can be argued that sulfur from each source is likely to have different distributions and, thus, the combined sulfur distribution may be bimodal as implied in the histogram shown in Figure 4.3.

The ash content of coal is largely a function of the amount of detrital sediments washed into the swamp during peat formation (Staub, 1979). The histogram of ash content is shown in Figure 4.4. Clark (1979, p. 21) regards ash values of 10% to 13% as normal or usual values for the Upper Freeport Coal Seam. Values greater than 13% probably reflect areas of the coal seam where shale partings are thicker or more abundant. In certain portions of the Upper Freeport Coal Seam, a definite relationship between the amount of ash and thickness of coal exists. This is shown by Figure 4.5 which compares the ash content for various

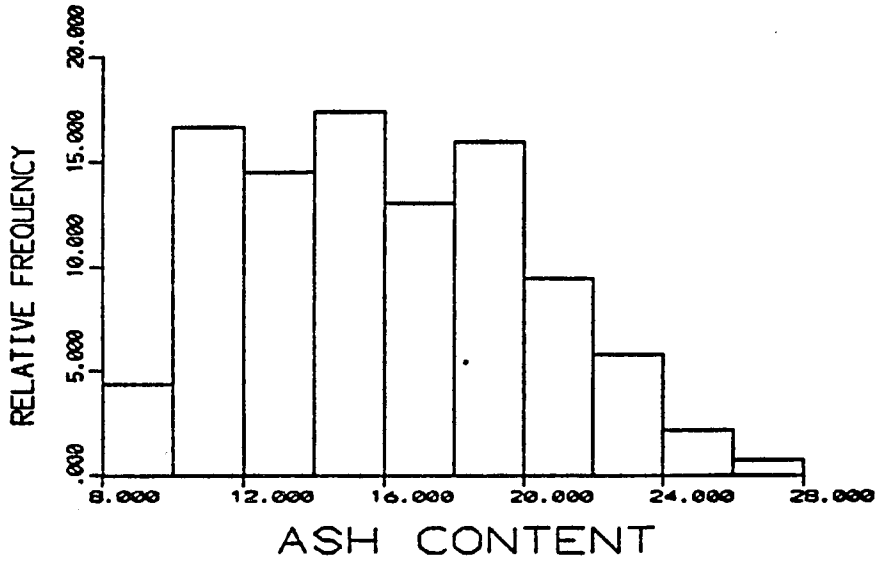


Figure 4.4. Histogram of Ash Content

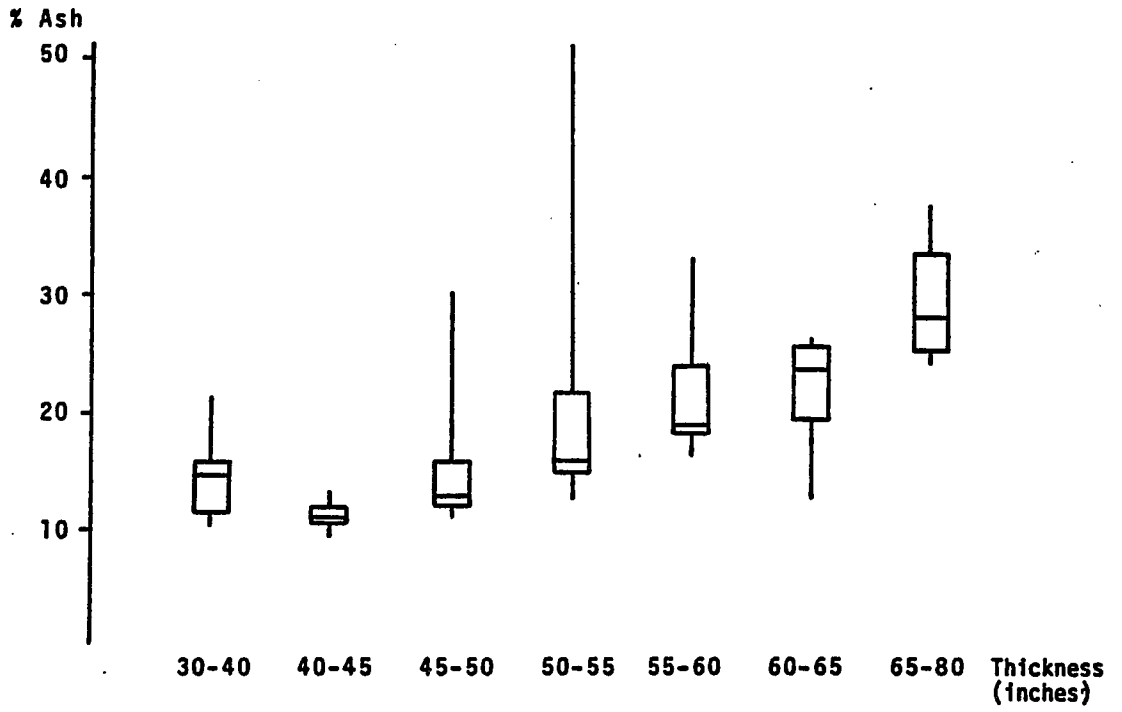


Figure 4.5. Comparison of Ash Content Versus Thickness of Coal for Lucerne #8 Mine

coal thicknesses in an area to the west of the study area. In the study area, the tendency to have a larger ash content in areas of thick coal is not as apparent as shown by Figure 4.6. This lack of a relationship is further evidenced by the low linear correlation shown in Table 4.2.

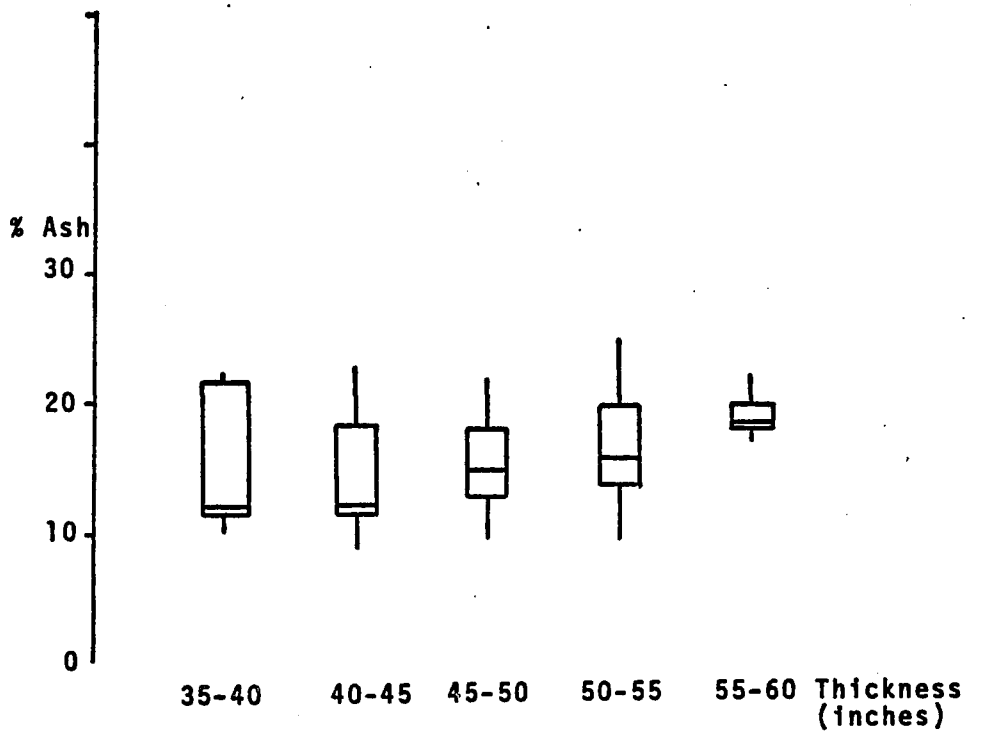


Figure 4.6. Comparison of Ash Content Versus Thickness for Area Shown on Figure 4.1

Table 4.2. Correlation Matrix

Thickness	1		
Ash	0.276	1	
Sulfur	0.099	0.614	1
	Thickness	Ash	Sulfur

In Table 4.2, sulfur and ash show the greatest amount of correlation. This can be partially explained by the fact that shale partings in the coal often contain pyrite. A coal that has high ash content is, therefore, likely to have a high sulfur content. This is shown in Figure 4.7 which compares the sulfur content of coal having various amounts of ash.

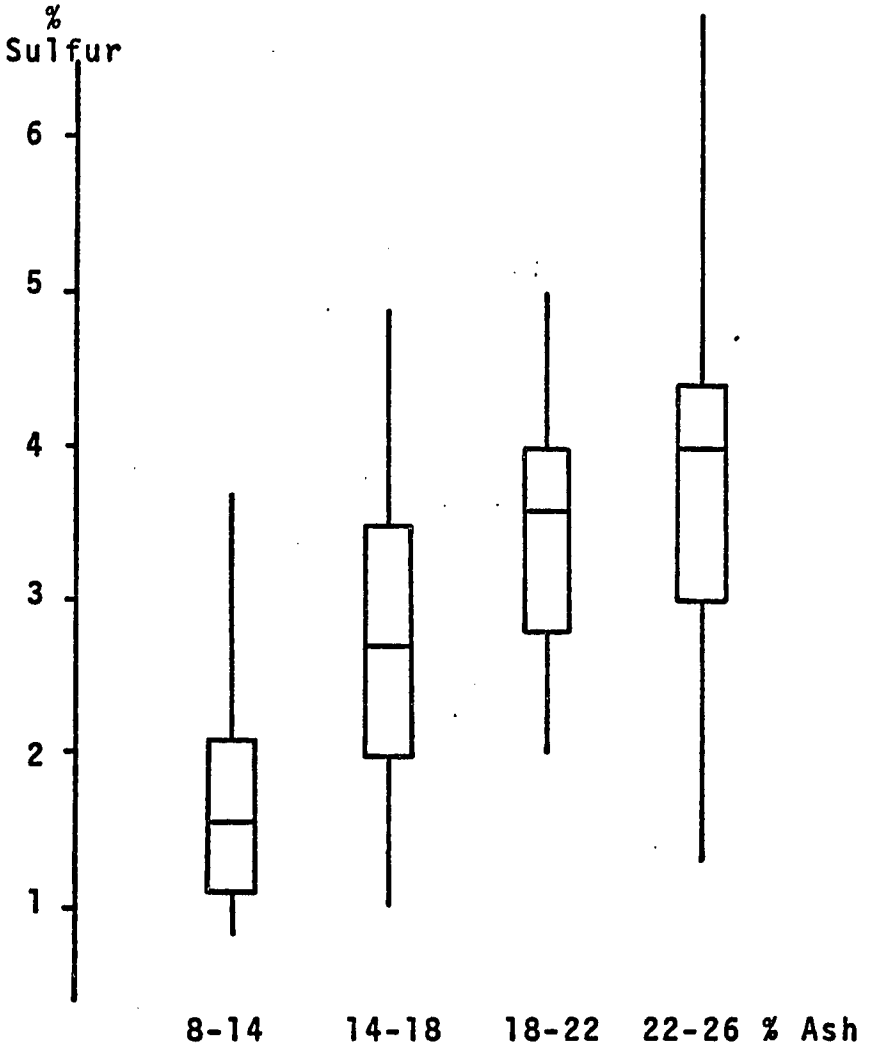


Figure 4.7. Comparison of Ash Content Versus Sulfur Content for Area Shown on Figure 4.1

Normalization of the Sulfur Values

The conditional simulation model must reproduce the distribution of each variable being simulated. Since the Turning Bands Method produces only normally distributed values, if the variable being simulated has some other form of distribution, a transformation must be found to reproduce the correct distribution.

The thickness of coal can be approximated by a normal distribution, thus, no transformation is needed. Sulfur, however, displays a skewed distribution. A logarithmic transformation was tried, but the transformed values were not well modeled by a normal distribution. A transformation function, consisting of Hermite polynomials, was next attempted (Kim, Myers, and Knudsen, 1977, p. 70). Figure 4.8 shows the resulting histogram of transformed sulfur values. An inverse of this transformation function is used to transform the normally distributed simulation values to the correct distribution of the original values.

Variogram Analysis

Experimental variograms were calculated to determine the spatial continuity of each of the variables. No anisotropy was observed in any of the variables. Figures 4.9, 4.10, and 4.11 show the resulting experimental variograms for thickness, sulfur, and ash, respectively.

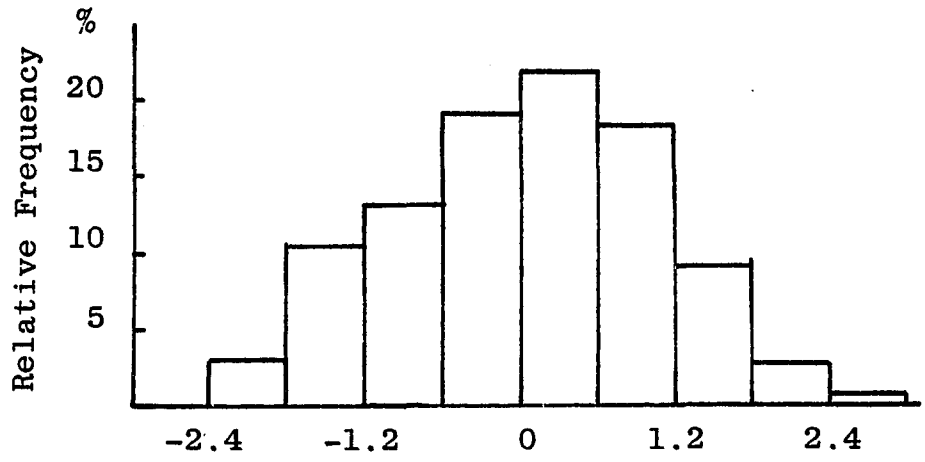


Figure 4.8. Normalized Sulfur Values

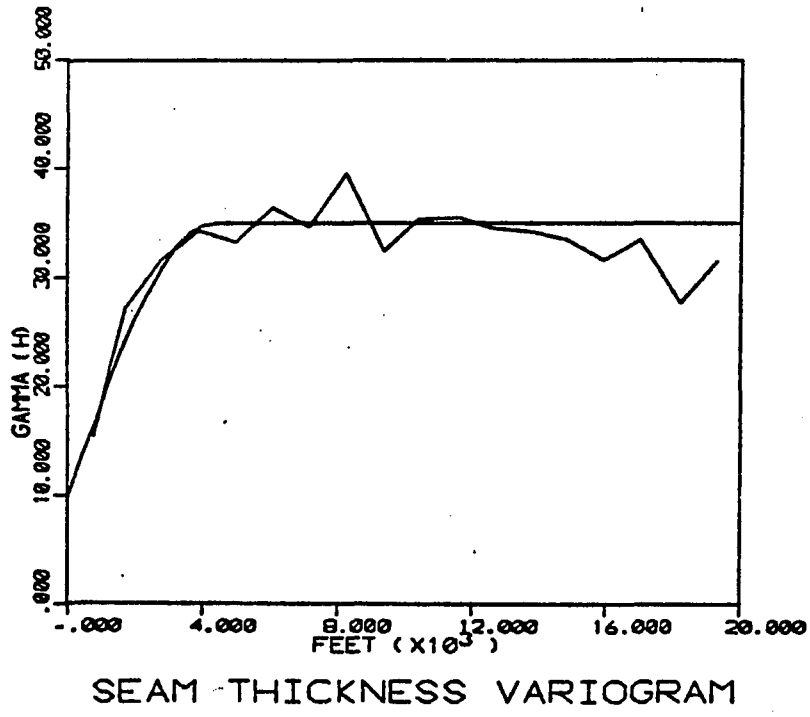


Figure 4.9. Variogram of Seam Thickness

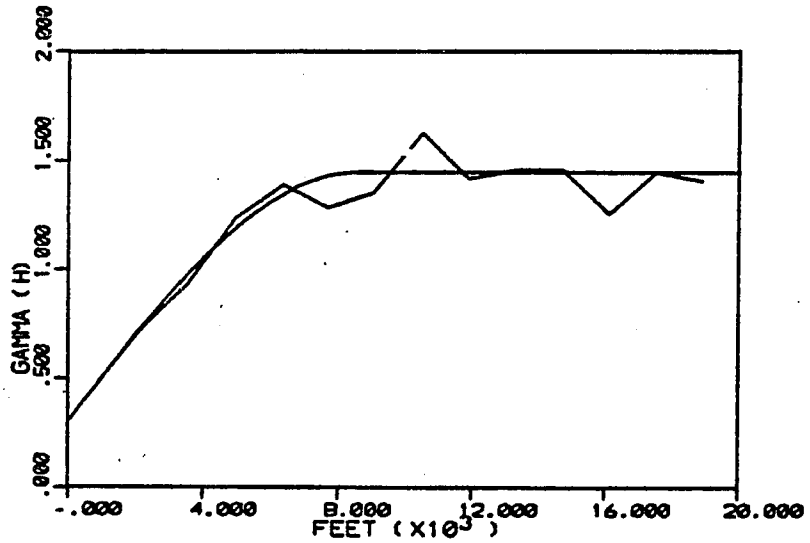


Figure 4.10. Variogram of Sulfur Content

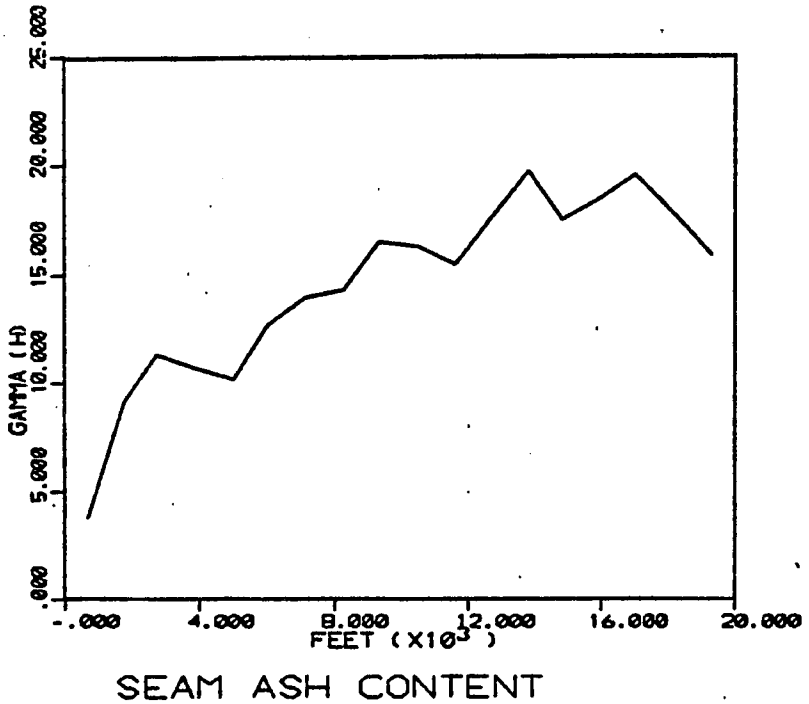


Figure 4.11. Variogram of Ash Content

The experimental variograms each show quite distinctive features. The variogram of thickness shows a maximum range of influence of 4,300 feet, while the variogram for sulfur shows a range of about 8,500 feet. The nugget effect is quite large for all three variables. The true nugget may be smaller than indicated, but no closely spaced data is available to more accurately determine the nugget. The ash variogram is not as nicely behaved as the thickness or sulfur variograms. The jagged appearance may be due to the fact that ash is largely related to the amount of shale included in the coal, both as partings and at the top or bottom of the seam. The characteristics of the ash should be quite complex because each of the partings probably has different spatial characteristics, depending on what caused the parting. In some cases, difficulty in defining the top and bottom of the seam can cause eratic amounts of shale to be included. This may cause an increase in the nugget effect.

Spherical variogram models were fitted to the thickness and normalized sulfur variogram. The fitted models are plotted with the experimental variograms in Figure 4.9 and 4.10. Parameters of the fitted spherical variogram models are given in Table 4.3.

A theoretical variogram was not fitted to the ash variogram because ash will not be modeled in the simulation models developed in this chapter.

Table 4.3. Parameters of Spherical Variogram Models

Variable	Sill Co + C	Nugget Co	C Value C	Range a
Thickness	35.	10.	25.0	4300'
Sulfur	1.45	.30	1.15	8500'

Construction of the Conditional Simulation Model

A conditional simulation model of the Upper Freeport E Seam was made for an area encompassing a particular coal mine. The model was specifically sized to include areas of the mine actually in production during 1978 and 1979, so that the model could be validated with actual production records of that period. The model covers an area of 12,000' x 12,000', as shown in Figure 4.12, and consists of 90,000 simulated values on a 40' x 40' grid.

Details of model construction and model validation will only be presented for the simulation of sulfur content, although the conditional simulation models used in this chapter and in Chapter 5 contain both sulfur and thickness values.

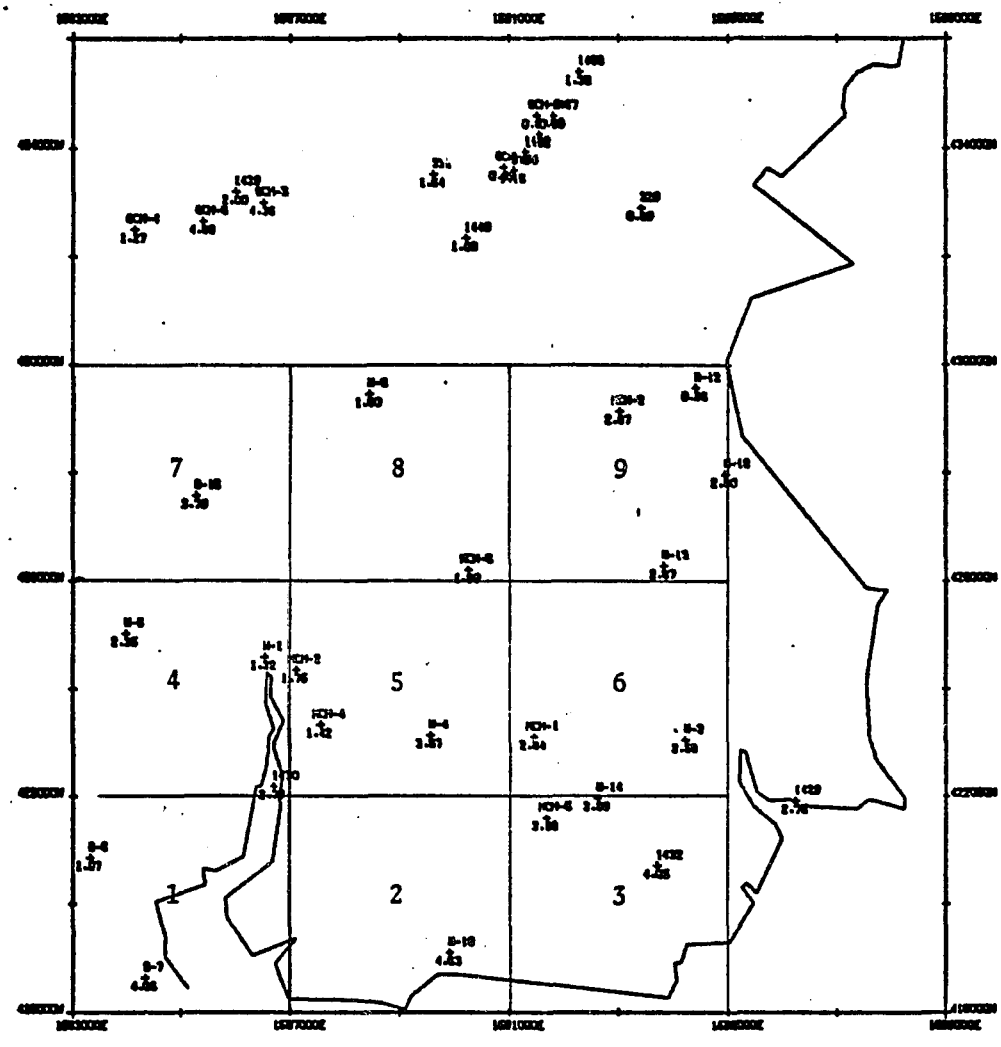


Figure 4.12. Map Showing Area Modeled by Conditional Simulation

Outline of Model Construction

The conditional simulation of sulfur is fairly straightforward because the variogram of normalized sulfur values can be modeled by a simple spherical variogram. Parameters of the variogram of normalized sulfur values are given below.

Sill Value = 1.00

Nugget Effect = 0.23

C Value = 0.77

Range = 8500'

The above parameters, together with the transformation function, are the main input items needed for the model construction. The sequence of steps used in construction of the model are discussed next.

Step One. Nonconditional Simulation of Sulfur

The nonconditional simulation of normalized sulfur values was made by executing Program CSIM03. The major input parameter to this program is the range of influence (8500').

Step Two. Adjusting the Variance and Adding the Nugget Effect

This step has two purposes. First, the nonconditional model is transformed so that the variance of the simulated values is equal to the C value of the normalized variogram. Secondly, a nugget effect is added to the model. Input parameters to Program CSIM05 are given below.

C Value of Variogram = 0.77

Nugget Effect = 0.23

Mean (of normalized sulfur values) = 0.00

At the end of this step, the simulated values should have a mean equal to zero and variance equal to one.

Step Three. Conditioning the Model

The model is conditioned to the normalized sulfur data by executing Program CSIM07. Input to CSIM07 consists of the variogram parameters of the normalized sulfur data, and the normalized sample data.

Step Four. Final Transformation of the Model

The final step in the model-building is to transform the simulated values to the original distribution of sulfur values by executing Program CSIM05 again. The inverse of the transformation function originally used to transform the sulfur values to a normal distribution is input to Program CSIM05.

Sample Output of the Final Model

The completed model contains 90,000 simulated sulfur values. Since the model is so large, it is partitioned into a series of blocks, each containing 10,000 simulated values and corresponding to a 3,000' x 3,000' area of the coal deposit. Summary statistics and histograms for each of these blocks are shown in Figure 4.13. The average value of sulfur in these blocks varies from a low of 1.99% to a high of 3.61%. The variance of the individual, simulated values varies from $0.54(\%)^2$ to $1.66(\%)^2$. The histograms are all

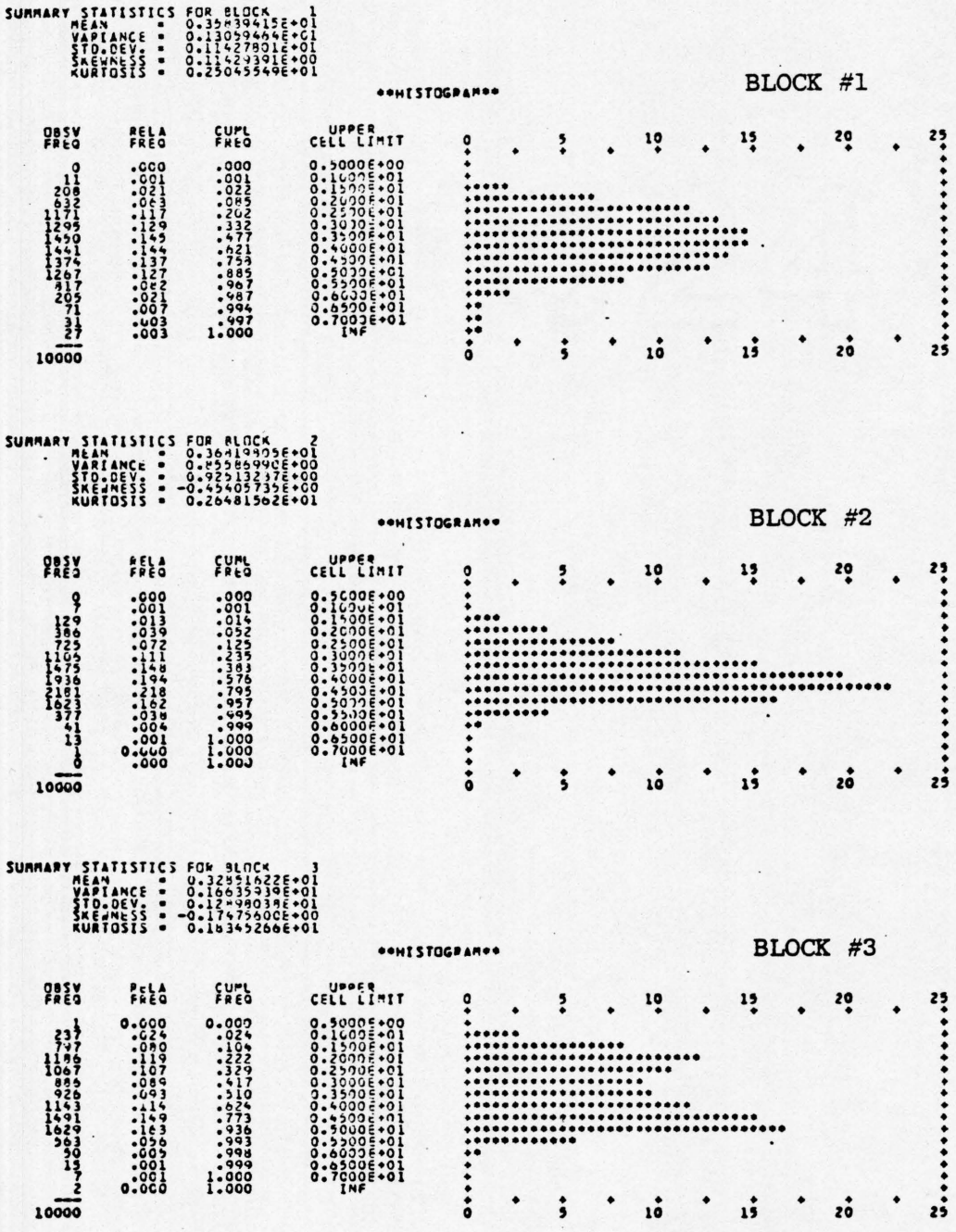


Figure 4.13. Summary Statistics for Each Block of CSIM Model

```
SUMMARY STATISTICS FOR BLOCK     4
      MEAN      =  0.19906949E+01
      VARIANCE  =  0.5431X1498E+00
      STD.DEV.  =  0.73743785E+00
      SKEWNESS  =  0.60451145E+00
      KURTOSIS  =  0.29399176E+01

                        **HISTOGRAM**                                BLOCK #4

 OBSV     RELA      CUML        UPPER
 FREQ     FREQ      FREQ     CELL LIMIT      0    .    10   .    20   .    30   .    40   .    50
                                              +    .    +    .    +    .    +    .    +    .    +
    1    0.000     0.000    0.5000E+00       +
  682     .068      .068    0.1000E+01       +********
 2202     .220      .288    0.1500E+01       +**********************
 2612     .261      .550    0.2000E+01       +***************************
 2138     .214      .763    0.2500E+01       +**********************
 1312     .131      .895    0.3000E+01       +**************
  695     .070      .964    0.3500E+01       +********
  272     .027      .991    0.4000E+01       +***
   74     .007      .999    0.4500E+01       +*
   12     .001     1.000    0.5000E+01       +
    0     .000     1.000    0.5500E+01       +
    0     .000     1.000    0.6000E+01       +
    0     .000     1.000    0.6500E+01       +
    0     .000     1.000    0.7000E+01       +
    0     .000     1.000       INF           +
                                              +    .    +    .    +    .    +    .    +    .    +
10000                                         0    .    10   .    20   .    30   .    40   .    50
```

```
SUMMARY STATISTICS FOR BLOCK     5
      MEAN      =  0.27969381E+01
      VARIANCE  =  0.10529063E+01
      STD.DEV.  =  0.10250884E+01
      SKEWNESS  =  0.28398582E+00
      KURTOSIS  =  0.22026298E+01

                        **HISTOGRAM**                                BLOCK #5

 OBSV     RELA      CUML        UPPER
 FREQ     FREQ      FREQ     CELL LIMIT      0    .    5    .    10   .    15   .    20   .    25
                                              +    .    +    .    +    .    +    .    +    .    +
    0     .000      .000    0.5000E+00       +
  127     .013      .013    0.1000E+01       +****
  870     .087      .100    0.1500E+01       +*****************
 1557     .156      .255    0.2000E+01       +*******************************
 1828     .183      .438    0.2500E+01       +************************************
 1593     .158      .597    0.3000E+01       +*******************************
 1364     .136      .733    0.3500E+01       +**************************
 1135     .114      .846    0.4000E+01       +**********************
  905     .090      .937    0.4500E+01       +******************
  534     .053      .990    0.5000E+01       +**********
   91     .009      .999    0.5500E+01       +***
    9     .001     1.000    0.6000E+01       +
    0     .000     1.000    0.6500E+01       +
    0     .000     1.000    0.7000E+01       +
    0     .000     1.000       INF           +
                                              +    .    +    .    +    .    +    .    +    .    +
10000                                         0    .    5    .    10   .    15   .    20   .    25
```

```
SUMMARY STATISTICS FOR BLOCK     6
      MEAN      =  0.3276X2X3E+01
      VARIANCE  =  0.72X04993E+00
      STD.DEV.  =  0.85325942E+00
      SKEWNESS  = -0.2X217590E+00
      KURTOSIS  =  0.25307005E+01

                        **HISTOGRAM**                                BLOCK #6

 OBSV     RELA      CUML        UPPER
 FREQ     FREQ      FREQ     CELL LIMIT      0    .    5    .    10   .    15   .    20   .    25
                                              +    .    +    .    +    .    +    .    +    .    +
    0     .000      .000    0.5000E+00       +
   19     .002      .002    0.1000E+01       +
  238     .024      .026    0.1500E+01       +******
  969     .097      .123    0.2000E+01       +************
 1970     .197      .320    0.2500E+01       +************************
 1753     .175      .495    0.3000E+01       +*********************************
 2060     .206      .701    0.3500E+01       +****************************************
 2089     .209      .910    0.4000E+01       +****************************************
 1343     .134      .044    0.4500E+01       +*************************
  608     .061      .105    0.5000E+01       +***********
   50     .005     1.000    0.5500E+01       +*
    2    0.000     1.000    0.6000E+01       +
    0     .000     1.000    0.6500E+01       +
    0     .000     1.000    0.7000E+01       +
    0     .000     1.000       INF           +
                                              +    .    +    .    +    .    +    .    +    .    +
10000                                         0    .    5    .    10   .    15   .    20   .    25
```

Figure 4.13--Continued

```
SUMMARY STATISTICS FOR BLOCK     7
      MEAN      =  0.21505072E+01
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HISTOGRAM BLOCK #7

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2748	.275	.477	0.2000E+01
2284	.229	.705	0.2500E+01
1541	.154	.859	0.3000E+01
852	.085	.944	0.3500E+01
405	.041	.985	0.4000E+01
123	.012	.997	0.4500E+01
27	.003	1.000	0.5000E+01
0	.000	1.000	0.5500E+01
0	.000	1.000	0.6000E+01
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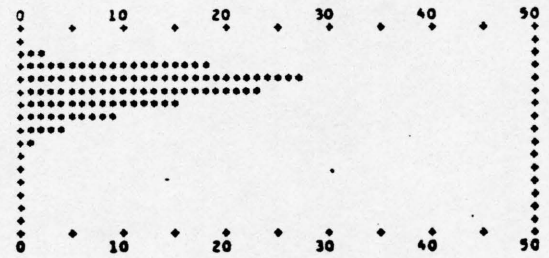


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HISTOGRAM BLOCK #8

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1476	.148	.171	0.1500E+01
2319	.232	.402	0.2000E+01
2191	.219	.622	0.2500E+01
1696	.170	.791	0.3000E+01
1114	.111	.903	0.3500E+01
653	.065	.968	0.4000E+01
270	.027	.995	0.4500E+01
50	.005	1.000	0.5000E+01
1	0.000	1.000	0.5500E+01
0	.000	1.000	0.6000E+01
0	.000	1.000	0.6500E+01
0	.000	1.000	0.7000E+01
0	.000	1.000	INF
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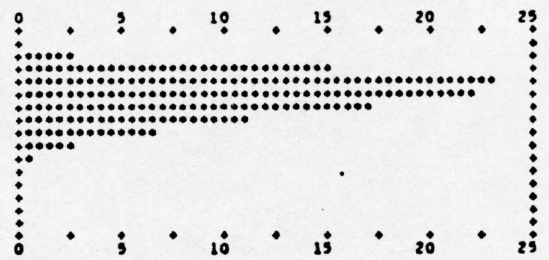


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HISTOGRAM BLOCK #9

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2107	.211	.302	0.2000E+01
2555	.256	.558	0.2500E+01
2132	.213	.771	0.3000E+01
1313	.131	.902	0.3500E+01
675	.067	.970	0.4000E+01
250	.025	.995	0.4500E+01
51	.005	1.000	0.5000E+01
2	0.000	1.000	0.5500E+01
0	.000	1.000	0.6000E+01
0	.000	1.000	0.6500E+01
0	.000	1.000	0.7000E+01
0	.000	1.000	INF
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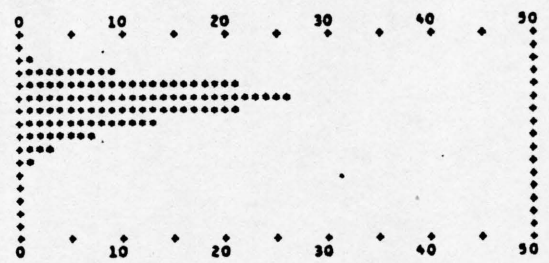


Figure 4.13--Continued

quite different from one another. Most show positive skewness, and Block #3 is bimodal.

Validation of the Model

Since the conditional simulation model should reproduce the essential characteristics of the deposit, the results of the geostatistical study also serve as a base for comparison of the model.

Figure 4.14 shows the basic statistics and histograms for both the original sample data and the CSIM model. The mean grade of the model, 2.83, is slightly higher than the mean of the sample data, 2.62. The variance of the simulated values is 1.24 versus 1.49 for the samples.

Neither of these apparent discrepancies is significant. The CSIM model covers an area of 4 square miles, whereas the sampling data comes from an area of about 50 square miles. The variance of simulated values should correspond more closely to the variance of sulfur grades within a 4 square mile area, than to the variance of samples within a 50 square mile area. Using the variogram model of the deposit, the value of the variance of a sample within a 12,000' x 12,000' area was calculated to be $\sigma^2_{(0/12,000' \times 12,000')} = 1.22$. This value is extremely close to the variance shown by the model, hence, there is no discrepancy in the variance of the model.

STATISTICS FOR SULFUR DATA

MEAN = 2.62
VARIANCE = 1.49
STD.DEV. = 1.22
SKEWNESS = 0.45
No. Assays 136

STATISTICS FOR CSIM MODEL

MEAN = 2.83
VARIANCE = 1.24
STD.DEV. = 1.11
SKEWNESS = 0.36

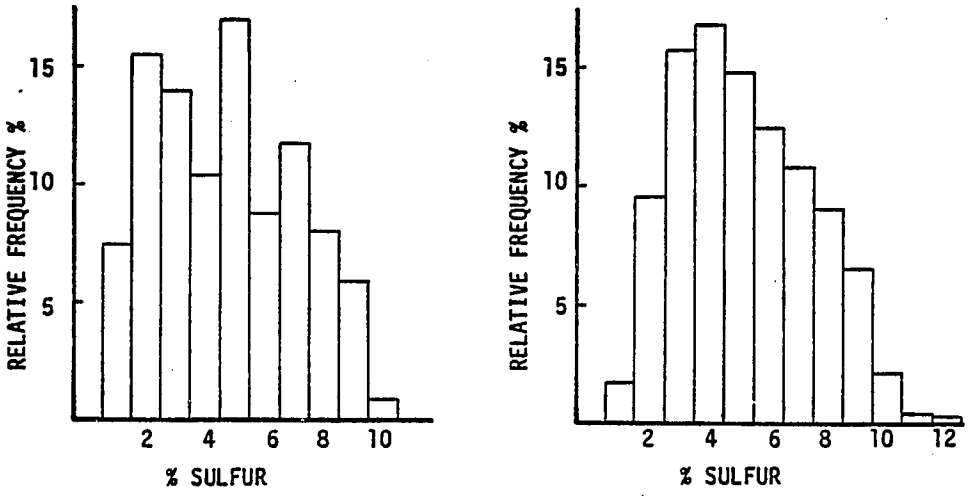


Figure 4.14. Comparison of Sample Values with Simulated Values

Similar reasoning can be used to test the difference shown by the mean of the model. If the entire deposit was split into 12,000' x 12,000' blocks, each block would likely have different mean grades and the mean grades would be distributed about the overall mean grade of the deposit. The variance of the mean grade of 12,000' x 12,000' blocks is $\sigma^2_{(12,000' \times 12,000'/\text{deposit})} = 0.28$. Making the assumption that the mean grades are approximately normally distributed, it follows that about 68% of the grades should be within one standard deviation of the overall mean grade.

Since $\sigma = \sqrt{.28} = .52$, the mean of the CSIM model is well within one standard deviation of the mean.

The histogram of simulated sulfur values is smoother than the histogram of sample values. The bimodal nature of the sample values is not reproduced by the histogram of the entire CSIM model, although the bimodal characteristics are reproduced in some local areas of the simulation model (see Block #3 in Figure 4.13). The differences between the distribution of sample values and the simulated values is probably not significant because the model area is much smaller than the entire deposit, and it is not known whether the bimodal nature of the sulfur distribution is really apparent throughout the deposit.

One of the most important characteristics which the CSIM model should reproduce is the spatial correlations exhibited by the sulfur. Variograms calculated for the model are compared against the variogram calculated from the sample data in Figure 4.15.

The N-S variogram of the CSIM model corresponds closely to the variogram of the sulfur sample, while the E-W variogram of the model has greater slope than the variogram of sulfur samples. The difference in the E-W variogram from the sample data variogram is probably not significant, although no convenient statistical test is available to test the significance of the difference.

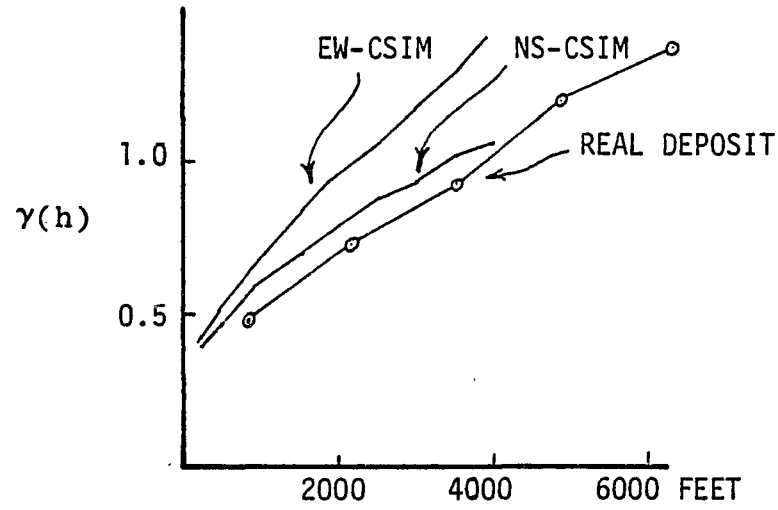


Figure 4.15. Experimental Variograms for Real Data and Simulated Data

Based on the general excellent agreement of the model to the observable statistical characteristics of the data, the conditional simulation model appears to have adequately simulated the real deposit.

Validation of the Model by Simulation of Mining

Production data on the average daily sulfur content of run of mine coal produced during 1978 and 1979 provides another base for validation of the simulation model. If the conditional simulation model is a reasonable representation of the true deposit, then the production results obtained by applying the same mining system as actually used in the real deposit should be very similar to the actual production data.

In order to make the comparison between actual production data and that obtained from the conditional simulation model, the existing mining system had to be modeled. E. Baafi (Kim, Knudsen, and Baafi, 1980, p. 61) developed a mining simulation for this purpose and performed the actual validation reported on here.

The existing mining system of the coal mine was simulated on the conditional simulation model for about two (2) months.

Figure 4.16 portrays serial plots of sulfur grades of actual run-of-mine coal and for the simulation results for sixty (60) consecutive days. A visual inspection of Figure 4.16 suggests that the pattern of variability of sulfur levels at the mine head and the results from the conditional simulation model are not significantly different. The first order autocorrelations for the data depicted in Figure 4.16 are 0.66 for the ROM data and 0.64 for the simulated data. The first order autocorrelation is a measure of correlation that exists between a sulfur value at a time t (S_t) and one following it, i.e., at time (t+1) (S_{t+1}).

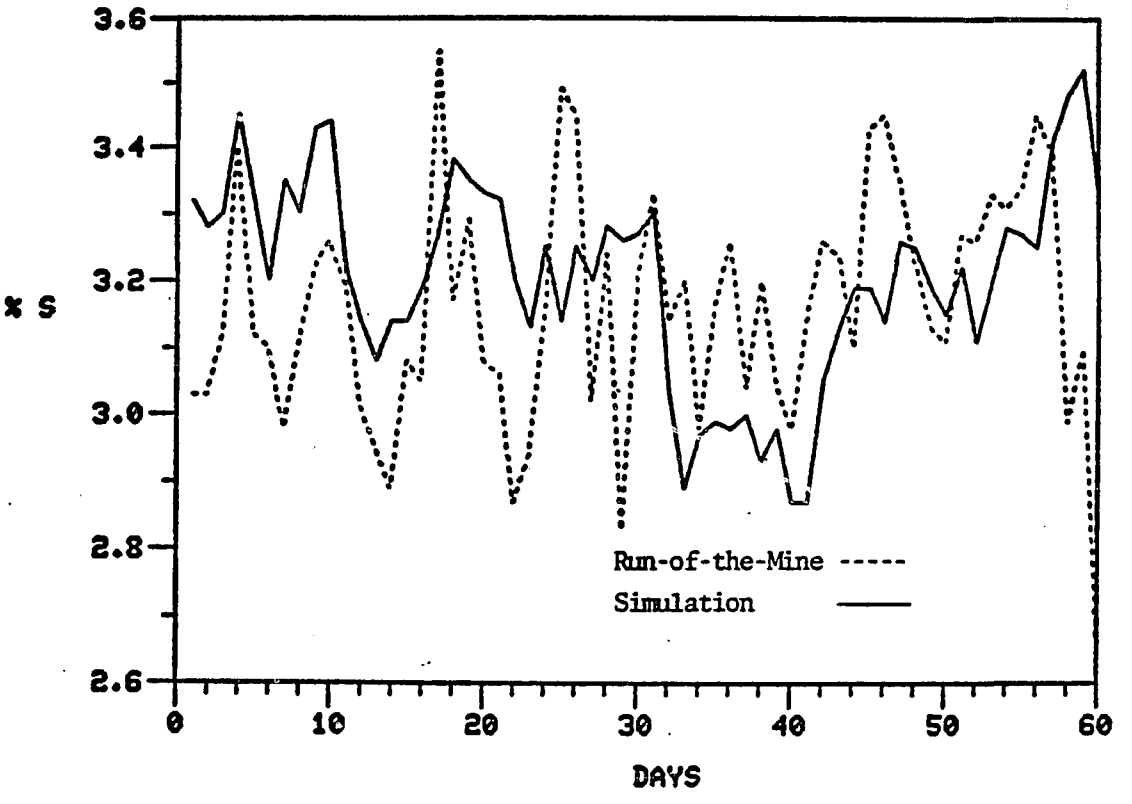


Figure 4.16. Serial Plots of ROM and Simulated Coal Sulfur Grades

The statistical summary of ROM and the simulation sulfur data are given in Table 4.4.

Table 4.4. Summary of ROM and Simulated Coal Sulfur Statistics

Source of Data	No. of Samples	Minimum Value	Maximum Value	Sample Mean	Sample Standard Deviation
Simulation	62	2.87%	3.52%	3.21%	0.152%
Run-of-the-mine	80	2.67%	3.55%	3.12%	0.177%

Table 4.4 and Figure 4.16 suggest that the statistics of the simulated sulfur model grade values are practically indifferent from the ROM values as provided by the company.

Both methods of validation of the conditional simulation model indicate that the model has effectively captured the essential characteristics of the true deposit.

CHAPTER 5

USE OF CONDITIONAL SIMULATION IN MINE PLANNING

Conditional simulation shows great potential to help engineers solve difficult mine planning problems. For example, assume a utility produces about 60% of the coal for its generating station from captive mines that happen to be high sulfur mines. The remaining 40% is purchased from independent suppliers and must be a low sulfur coal, so that when the coals are blended, the generating station meets emission regulations. Obviously, the ability to predict the quality of coal from the captive mine on a monthly basis, for example, can give the utilities much flexibility in purchasing the remaining amount of coal.

Variations in coal quality over shorter time frames can pose equally difficult problems. Coal cleaning plants perform most efficiently when given feed having uniform characteristics. The variability actually observed in run of mine coal can affect the efficiency and performance of the plant, especially if the plant was not designed to handle the variability or the variability was greater than expected.

This chapter presents one example of the use of conditional simulation.

Prediction of Year to Year Sulfur Variability

Motivation of this example was to show how the amount of development drilling affects the yearly predictions of coal quality. Accurate prediction of year to year variation of run of mine coal can give a utility flexibility in contracting for the additional coal necessary to operate a generating station at full capacity.

Methodology

The simulated coal deposit used in this study covers 30,000' x 30,000' in plan, or about 20,000 acres. The simulation grid used was 100' x 100', resulting in 90,000 simulated values. One hundred and fifteen drillhole samples were used to condition the deposit.

Three stages of development drilling were simulated using the CSIM model. After the first stage of drilling, a hypothetical mine plan was prepared for six years with a yearly production of 800,000 tons. Production was to come from one shortwall section, and two room and pillar sections. In addition, three sections were included for shortwall development.

After the second and third stages of development drilling, new year by year estimates of coal production and sulfur content were prepared using the results of the new

drilling. The mine plan was not altered after the first stage of drilling.

The final step was to mine out the deposit to determine how well each of the mine plans actually performed.

Stage One Drilling. The initial drilling was chosen to be on a 5000' x 5000' grid. Thirty-six holes were necessary to cover the coal deposit. In addition to the initial drilling grid, an additional two fences of drillholes were drilled for the sole purpose of variogram determination. Figure 5.1 shows a plan map of the first stage drilling. The fences are probably longer than necessary. In actual practice, the holes would be sequentially drilled, and the drilling stopped when a variogram was obtained.

Variograms calculated using the results of stage one drilling are shown in Figure 5.2. Spherical models were fitted to these variograms and included on the plots of Figure 5.3

Stage One Kriging. The deposit was divided into 2000' x 2000' blocks. The thickness and sulfur content of each block were kriged using the variograms derived from the Stage One drilling. The resulting kriged model was used as the basis of the hypothetical mine plan. Figure 5.3 shows the resulting yearly plans superimposed on the kriged map of sulfur values. The individual mining sections were placed so that uniform sulfur content would be maintained for the six years.

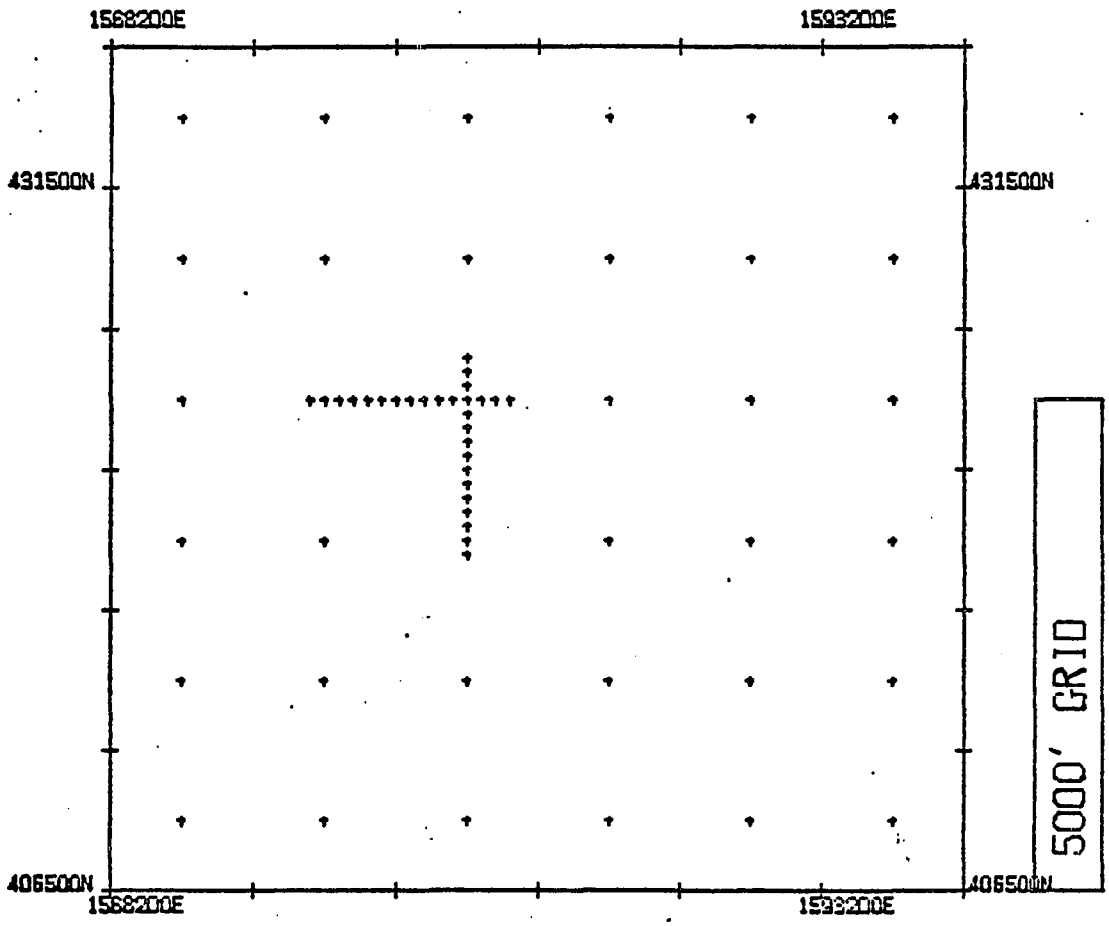
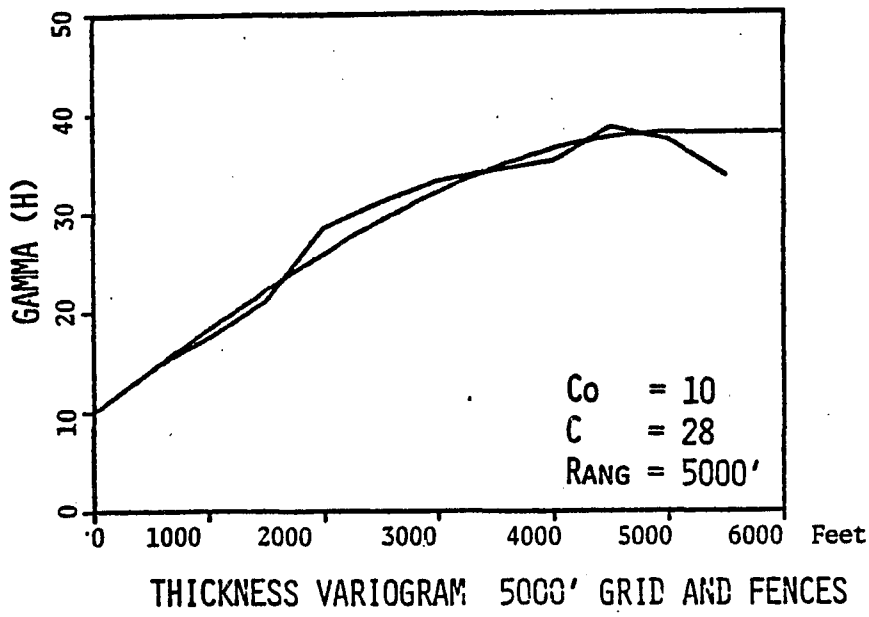


Figure 5.1. Stage One Drilling



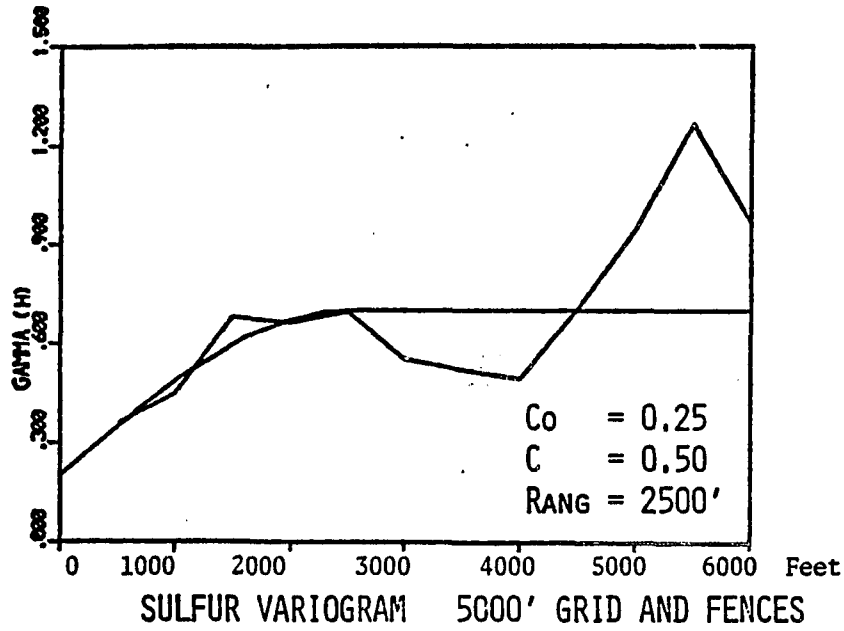


Figure 5.2. Variograms from Stage One Drilling

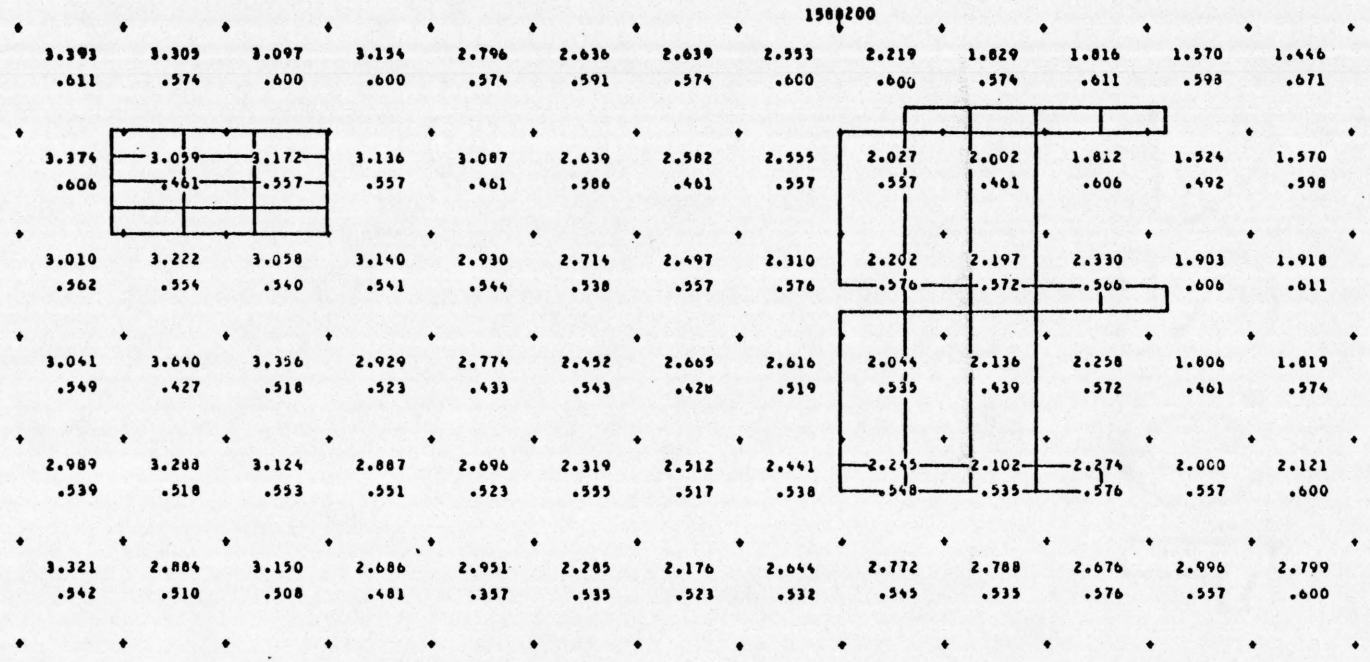


Figure 5.3. Mine Plan Superimposed on Kriged Model

Table 5.1 shows the resulting yearly estimates. The mine plans based on the 5000' drilling grid and 2000' kriged blocks will be referred to as Case 1.

Table 5.1. Estimated Coal Production for Case 1

Year	Sulfur	Thickness	Tons
1	2.36	49.9	842100
2	2.32	49.0	827100
3	2.31	48.7	822100
4	2.24	50.2	846000
5	2.30	50.5	850900
6	2.32	48.4	817700

Stages Two and Three Drilling. The second drilling campaign consisted of drilling an additional twenty-five holes. The holes were placed in the center of the first grid, thus resulting in an approximately 3500' x 3500' grid.

The final drilling campaign consisted of an additional eighty-three drillholes needed to complete a 2500' x 2500' grid. Figure 5.4 shows the completed 2500' drilling grid.

At the end of each stage of drilling, variograms were calculated using all available drilling. The final variograms after Stage Three are shown in Figure 5.5.

The thickness variogram changed very little from the Stage One drilling to the Stage Three drilling. The sulfur variogram, however, changed significantly. At the end of

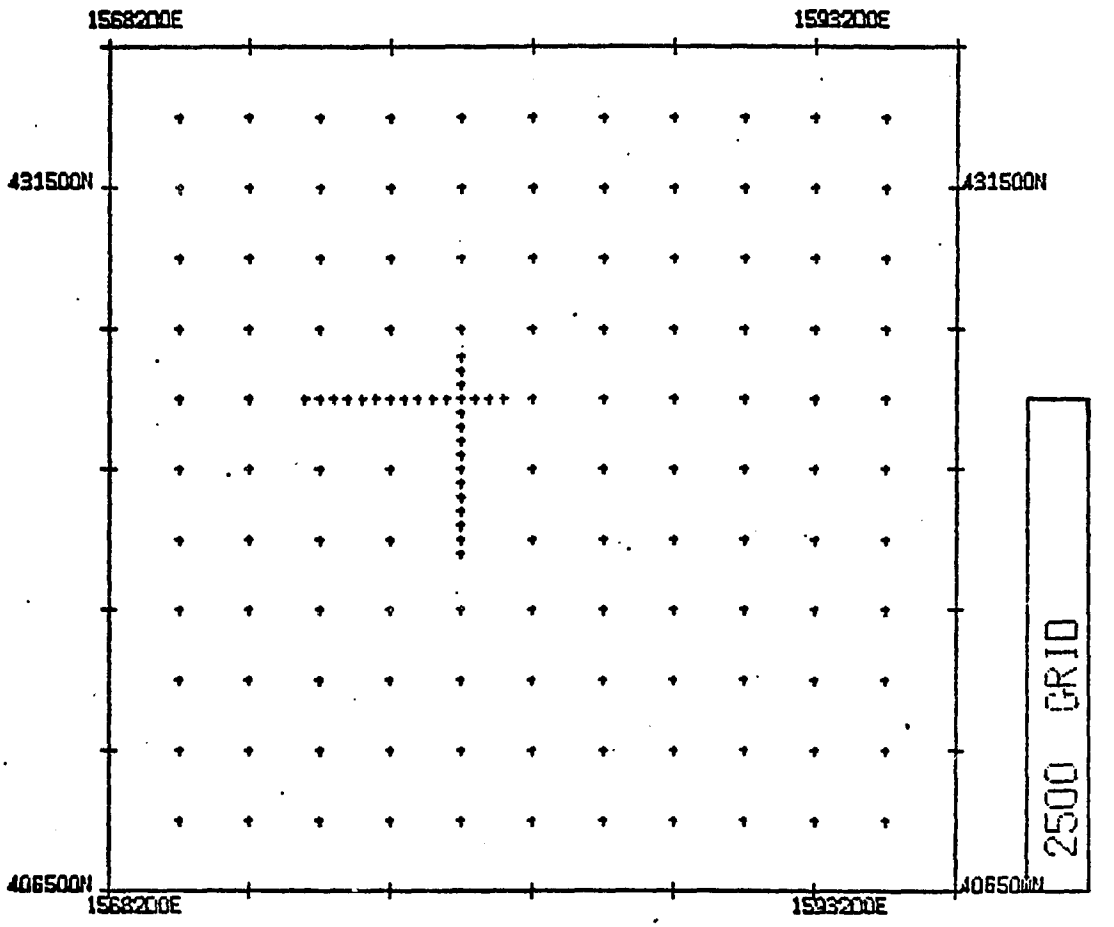


Figure 5.4. Stage Three Drilling

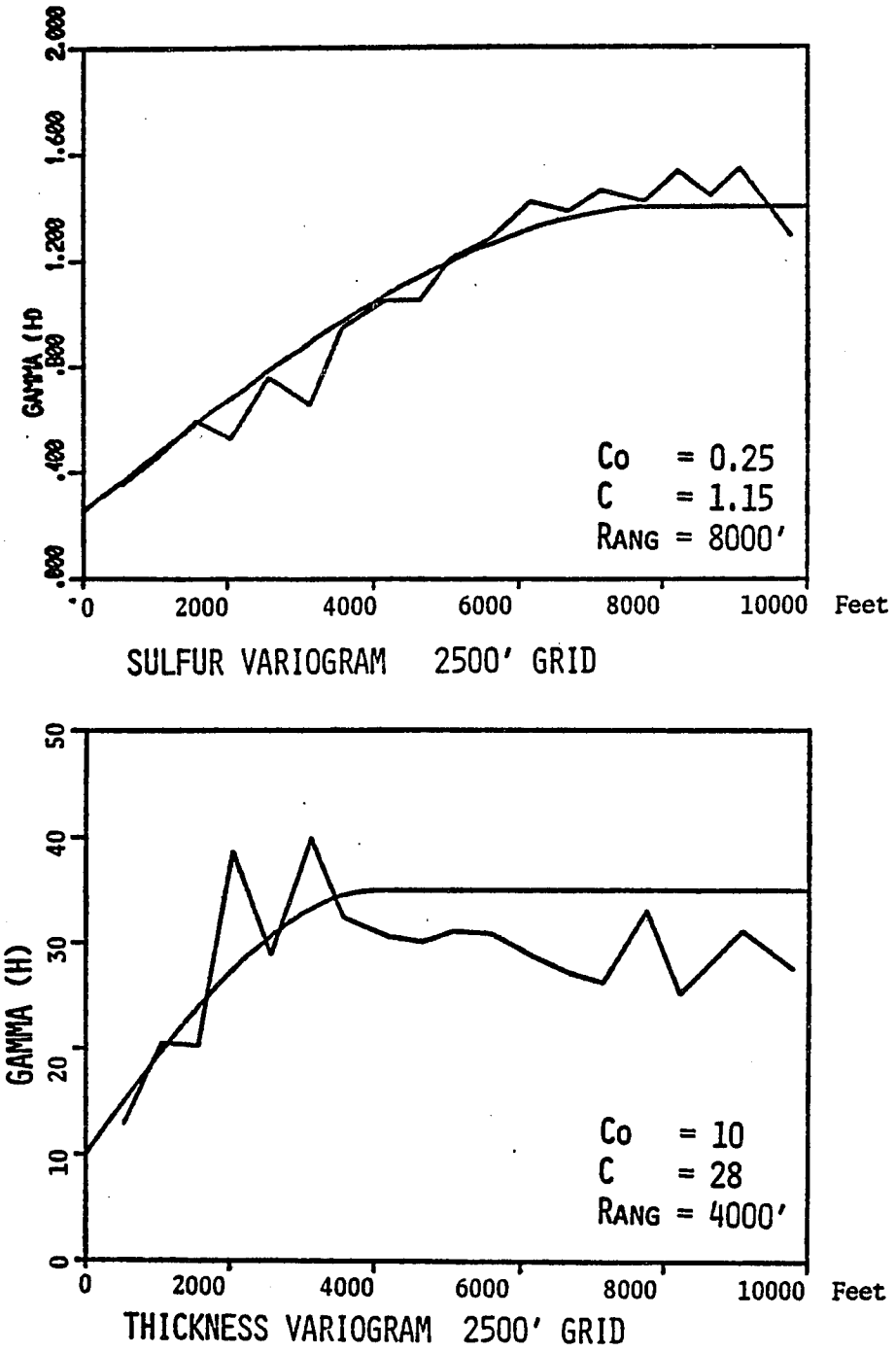


Figure 5.5. Variograms from Stage Three Drilling

Stage One drilling, the sill of the sulfur variogram was estimated to be 0.75, while at the end of Stage Three, the sill value had increased to 1.4 and the range of the variogram had increased from 2500 feet to 8000 feet.

Stages Two and Three Kriging. At the end of each stage of drilling, the deposit was then kriged using the new variogram models. The individual kriged blocks were again 2000' x 2000' in plan.

The mine plan was not developed at the end of each stage of drilling in order to allow meaningful comparison of the effect of additional sample information on the accuracy of yearly estimates. Yearly estimates of mine production were, therefore, updated after each drilling campaign. The estimates resulting from Stage Two drilling will be referred to as Case 2, and the estimates from Stage Three as Case 3.

Stage Three Kriging of 1000' Blocks. The 2000' x 2000' block was initially chosen on a fairly arbitrary basis. A general rule of thumb is that the block size should be greater than 1/4 the grid size. With the initial drilling being done on a 5000' grid, the 2000' block seemed suitable.

Rather than change the block size at each drilling campaign, it was held constant to allow meaningful comparison of the resulting mine plans. At Stage Three, however, a second kriged model was prepared using a 1000' x 1000' block size. The 1000' blocks have a certain appeal because the

size of the yearly production blocks corresponds more closely to the 1000' block size. Yearly estimates made using the 1000' blocks are referred to as Case 4.

Accuracy of Yearly Estimates

At each stage of drilling, estimates of yearly coal production and sulfur content were made using the kriged models. Error bounds were calculated for each yearly estimate using the variogram models obtained at each drilling stage. Figure 5.6 shows the relationship between the computed error bound for yearly estimates and the size of the drilling grid.

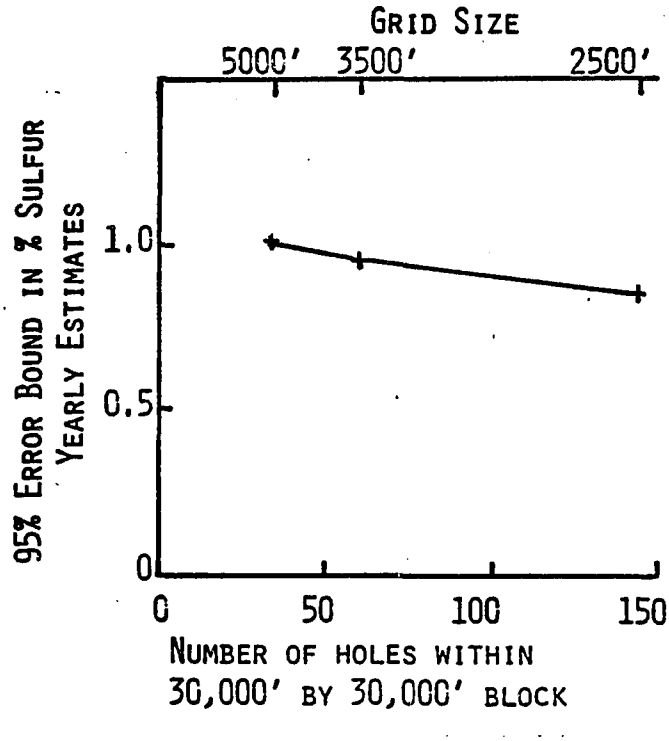


Figure 5.6. Error Bound for Yearly Estimates

Testing of the Mine Plan

The real test for any mine plan is how well the plan works in actual mining. With the development of conditional simulation, the mine plan can now be tested on the simulated deposit.

Each of the four mine plans developed during this study were tested on their ability to correctly estimate the production and sulfur content on a yearly basis. In each case, the simulated deposit is mined out according to the mine plans. The resulting "actual" production can then be compared to the estimated production and estimated quality characteristics.

In Figure 5.7, the year by year estimates of sulfur content in the run of mine coal are presented for the four mine plans prepared in this study. The estimates developed from the 5000' grid show a quite uniform sulfur content over the six years of production. The reason for this uniformity is that the mine plan was designed using the kriged model based on the 5000' grid. The design goals were a constant production and uniform sulfur content.

Case 2 also predicts a uniform sulfur content, but notice that the grade is greatly different from Case 1. The reason for this is the chance occurrence of two low sulfur assays right in the middle of the shortwall section. Since the shortwall accounts for about 60% of production, the

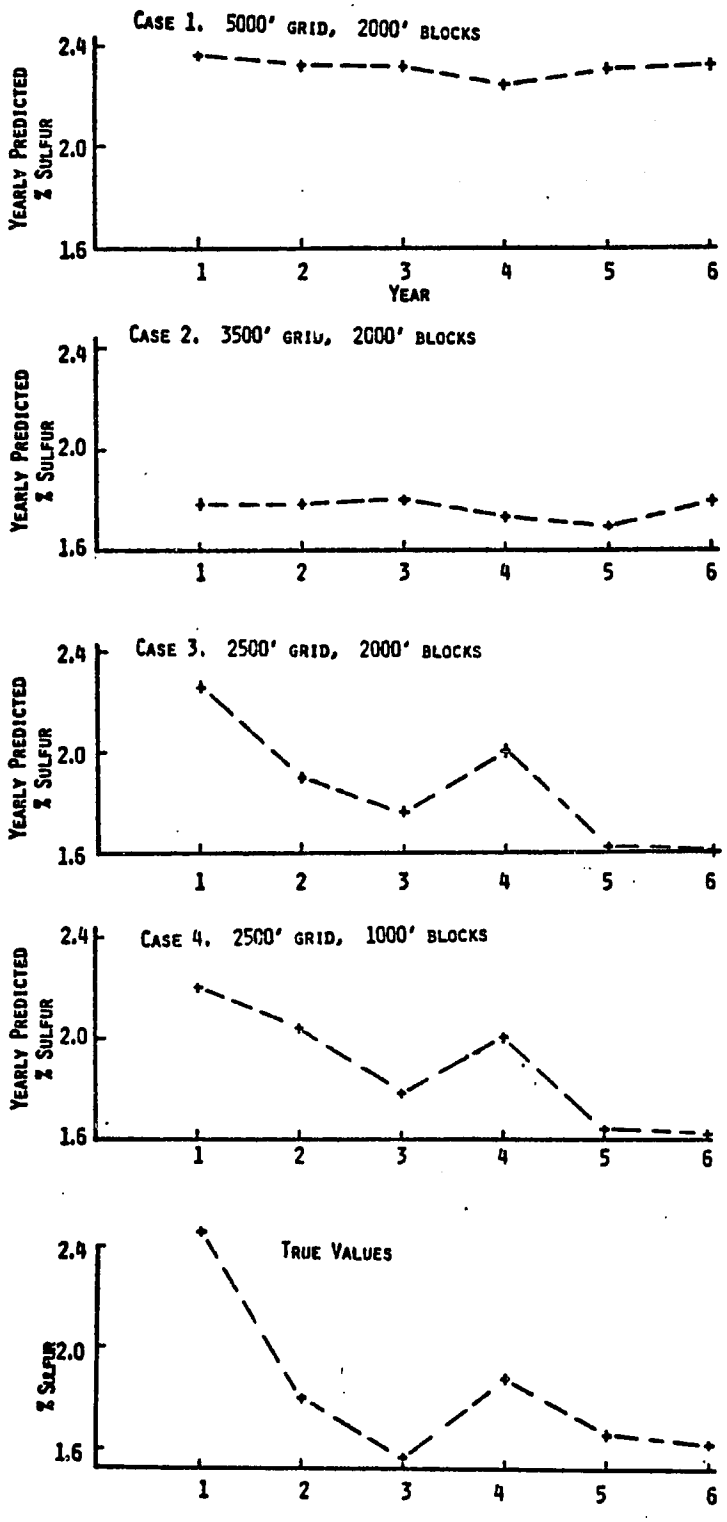


Figure 5.7. Comparison of Yearly Estimates to Actual Values

lower sulfur assays greatly affected the estimation of overall sulfur content.

Starting with Case 3, the predicted yearly sulfur content showed increased variation. Notice that the predictions made in Case 3 and Case 4 are quite alike, as they should be, since both used the same drillhole data.

The final graph, shown in Figure 5.7, is the actual year to year sulfur content produced when the deposit was mined according to the mine plan. From Figure 5.7, it is easily seen that Cases 3 and 4 produced much better year to year estimates than Cases 1 or 2.

A comparison, such as shown in Figure 5.7, gives a clear picture of the adequacy or inadequacy of each case. The amount and extent or pattern of errors is quickly realized.

Figure 5.8 shows a summary of the errors made on a year by year basis for each case for both thickness of coal and sulfur content.

In all cases, the thickness of coal, and hence, the estimates of yearly production had very small average errors. The increased drilling of Stages Two and Three produced only minor improvements in the accuracy of the yearly estimates.

Prediction of yearly sulfur content was less successful. Errors in prediction of sulfur ranged from an average of 30% for Case 1 to about 8% for Case 3. The improvement

in yearly estimates of sulfur was substantial as the density of sampling increased.

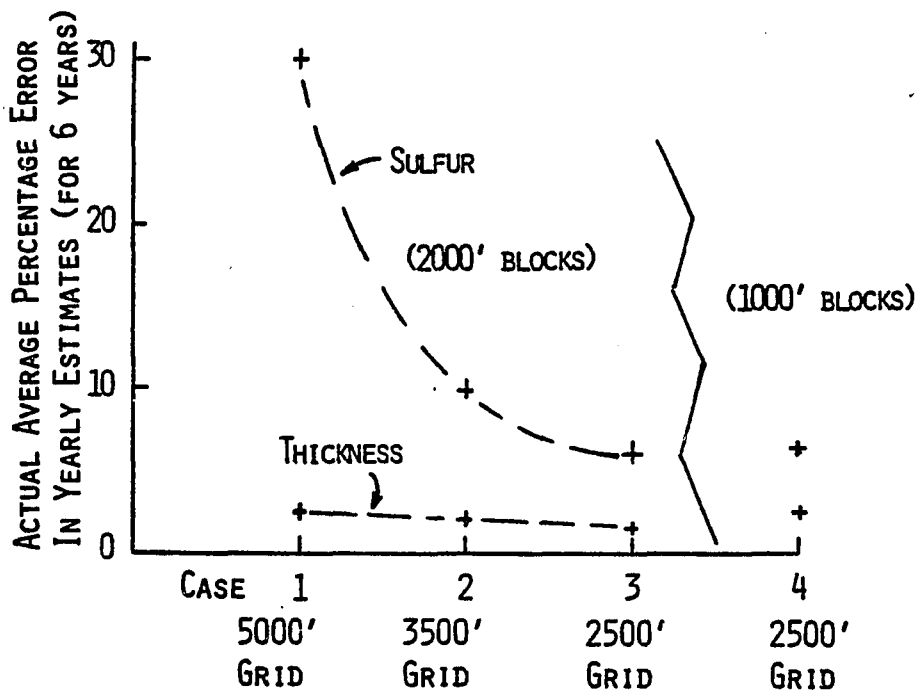


Figure 5.8. Summary of Yearly Prediction Errors

Notice that Case 4 gave slightly poorer estimates of both sulfur and thickness than Case 3. The hoped for increase in accuracy as a result of using 1000' x 1000' blocks was not only unachieved, but slightly poorer results were obtained. This emphasizes the point that, although it appears that more detail or definition is being included in the kriged model when small blocks are used, in fact, the smaller blocks are less well known due to the higher kriging

variance for a 1000' x 1000' block than a 2000' x 2000' block.

CHAPTER 6

CONCLUSIONS

The major objective of this research was to develop a simulation model of a coal deposit that captures the inherent insitu variability of the coal. This model would subsequently be used to determine the variability of run of mine coal on a day by day, week by week, or year by year basis.

Results of the validation of the model clearly indicate that the conditional simulation model produced by the Turning Bands Method does correctly model the insitu variability of the deposit. The model displays the same mean, variance, and distribution of values as the real deposit. Most importantly, the model reproduced the same spatial correlations as the real deposit as indicated by the fact that the model displays the same variogram as the real deposit.

A second validation of the model showed that when the model is "mined" according to an actual daily mining sequence, the resulting daily variability corresponded extremely well to what was observed during the actual mining. This second verification served not only to validate the model but also served as a practical demonstration that the

model can be used to predict day to day variation in the quality of run of mine coal.

In Chapter 5, one example was presented to illustrate the use of conditional simulation. This example showed that the model could be used to simulate the differences between estimated coal quality and actual coal quality on a year by year basis. The unique aspect of the results was that differences were given year by year so that any correlations or patterns in the differences are easily observed.

Suggestions for Future Research

The conditional simulation model produced by the Turning Bands Method is a good representation of the coal quality in a coal deposit. One potential limitation to the method is that the method only reproduces the first two moments of the deposit being simulated. For mining problems, this limitation is very minor from a practical point of view because in almost no cases is there sufficient information to deduce the higher moments. Further research in methods of simulating a three-dimensional process may be warranted if the correct representation of higher moments becomes necessary.

The immediate need for further research is to expand the applications and usefulness of this tool. A few specific suggestions for future research are given below.

1. Risk analysis is an important tool in economic analysis. The conditional simulation model could be used to provide simulated coal quality on a daily, monthly, or yearly basis. Such input would reflect the serial correlation of the coal quality and, therefore, provide a more realistic model of coal quality which is an important variable in the economic analysis.
2. The efficiency of long term mine plans is difficult to analyze. The conditional simulation model could be mined out according to the long range plans. If the results were not satisfactory, then the plan could be modified or, if necessary, new data might have to be acquired before completing the new plan. The new mine plan could then be tested again using a second simulation of the deposit.
3. The conditional simulation model is an ideal model for teaching and mine planning. For example, a student's mine plan could be graded not only by the usual subjective means, but also by how well it works on the simulated deposit. Several universities have already incorporated the use of simulated deposits in their coursework.

APPENDIX A

FORTRAN LISTING OF PROGRAM CSIM03

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      PROGRAM CSIM03(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
     1TAPE7,TAPE8,TAPE9)
C*************************************************************
C       THIS PROGRAM WAS WRITTEN BY
C                       HARVEY P.KNUDSEN
C
C          DEPARTMENT OF MINING AND GEOLOGICAL ENGINEERING
C                     UNIVERSITY OF ARIZONA
C                     TUCSON, ARIZONA  85712
C
C       LAST MODIFICATION: JULY 1979
C
C       PROGRAM CSIM03 GENERATES  A 3-D SIMULATION
C       WITH DESIRED COVARIANCE FUNCTION.
C
C       PRIOR TO EXECUTING THIS PROGRAM PROGRAM CSIM01 MUST
C       HAVE BEEN EXECUTED IN ORDER TO INITIALIZE THE CSIM MODEL.
C
C       INPUT DATA CONSISTS OF THE FOLLOWING:
C       1.NAME OF MASTER FILE
C       2.DESIRED RANGE OF INFLUENCE OF THE VARIOGRAM
C
C       OUTPUT CONSISTS OF THE FOLLOWING:
C       1. THREE DIMENSIONAL SIMULATION. SIMULATED DATA IS
C          NORMALLY DISTRIBUTED N(0,1), AND HAS VARIOGRAM WITH
C          USER SPECIFIED RANGE.
C       2. PRINTOUT OF INPUTTED PARAMETERS
C*************************************************************
      COMMON /CSIMOD/ B(100,100,1),IB(100,100,1)
      COMMON /PAREM/ NBLOC,NBN,NBE,NPT,NPTV,YNORT,XEAST,YLPT,XLPT,
     1 NSIM,NVAL,NPACK(3),NAME(3),NDEC(3)
      COMMON /INOUT/ IPT,IOUT,MASTR,ICOR,IMOD
      COMMON /FILES/ MASFIL,CSFIL
      EQUIVALENCE (RANVAL,B),(DLINE,IB)
      DIMENSION DLINE(10000),RANVAL(10000)
      DIMENSION A(3,15),C(3),V(3,8)
      DIMENSION NUM(15),MIN(15)
      CHARACTER *7 CSFIL,MASFIL
      CHARACTER *10 DAY,DATE,CDAY
      DATA IPT,IOUT,MASTR,ICOR,IMOD/5,6,7,9,8/
      DATA V,SQ15,DLINE/24*1.0,3.89729883346,10000*0.0/
      DATA A/0.000000,1.000000,0.000000,
     1 1.000000, 0.000000, 0.000000,
     10.000000,0.000000,1.000000,-0.809017,-0.500000, 0.309017,
     20.809017,0.500000,0.309017,0.309017, 0.809017, 0.500000,
     3 0.500000,0.309017,0.809017,0.500000,-0.309017,0.809017,
     40.309017,-0.809017,0.500000,-0.309017,-0.809017,0.500000,
     5-0.500000,-0.309017,0.809017,-0.500000,0.309017,0.809017,
     6-0.309017,0.809017,0.500000,-0.809017,0.500000,0.309017,
     70.809017,-0.500000, 0.309017/
C-------------------------------------------------------------
C      DESCRIPTION OF INPUT CARDS
C        COL.  FORMAT NAME       DESCRIPTION
C        ****  ****** ****       ***********
```

```
C      CARD ONE. FILE SPECIFICATION  CARD.
C        1-10  A10     MASFIL      MASTER FILE NAME
C      CARD TWO. WEIGHTING FUNCTION PARAMETERS
C        1      I1      IVAR        VARIABLE BEING SIMULATED
C                                   (1,2,OR 3)
C        2      I1      IDBUG       DEBUG OUTPUT? (1=YES,0=NO)
C        11-20 F10.0    RANGE       DESIRED RANGE OF VARIOGRAM
C        21-30 F10.0    GRDSP       GRID SPACING (USUALLY SAME AS XLPT)
C        31-40 I10      ISEED       BEGINNING RANDUM NUMBER SEED
C-------------------------------------------------------------
C     MAJOR VARIABLE USED IN CSIM PROGRAMS
C        TITLE    TITLE OF MODEL
C        CSFIL    FILE NAME OF CSIM MODEL
C        MASFIL   FILE NAME OF MASTER FILE
C        NVAL     NUMBER OF VALUES FOR EACH DATA POINT
C        NPACK    PACKING MULTIPLIERS
C        NBN      NUMBER OF BLOCKS IN NORTH DIRECTION
C        NBE      NUMBER OF BLOCKS IN EAST DIRECTION
C        NBLOC    NUMBER OF BLOCKS IN MODEL
C        YNORT    COORDINATE OF SOUTHERNMOST ROW OF MODEL
C        XEAST    CORRDINATE OF WESTERNMOST COL OF MODEL
C        YLPT     DISTANCE BETWEEN MODEL POINTS IN NORTH DIR.
C        XLPT     DISTANCE BETWEEN MODEL POINTS IN EAST DIR.
C        NPT      NUMBER OF POINTS IN BLOCK IN E-W DIR
C        NPTV     NUMBER OF POINTS IN BLOCK IN VERT DIR.
C        A        MATRIX OF 15 VECTORS OF ICOSAHEDRON
C        V        COORD. OF CORNERS OF CSIM MODEL
C        RANVAL   UNIFORM RANDUM NUMBERS
C        DLINE    CORRELATED RANDOM 1-D VARIABLES
C        NUM(L)   NUMBER OF VARIABLES ON LINE L
C        RANGE    RANGE OF THE SPHERICAL VARIOGRAM
C        C        C VALUES OF THE SPHERICAL VARIOGRAM
C        GRDSP    GRID SPACING OF THE SIMULATED POINTS
C        CONT     CONSTANT TO CORRECT FOR BIAS IN 1-D SIMULATION
C-------------------------------------------------------------
C-------------------------------------------------------------
C       READ IN NAME OF MASTER FILE
C-------------------------------------------------------------
        READ(IPT,2000)MASFIL
2000    FORMAT(A7)
        OPEN(MASTR,ACCESS='SEQUENTIAL',FILE=MASFIL)
        CALL  RMAST (CDAY)
C-------------------------------------------------------------
C       READ IN PARAMETERS  N, AND CONST
C-------------------------------------------------------------
        READ(IPT,2010)IVAR,IDBUG,RANGE,GRDSP,ISEED
2010    FORMAT(2I1,8X,2F10.0,I10)
C-------------------------------------------------------------
C       CALCULATE  CONSTANTS
C-------------------------------------------------------------
        NR = RANGE/(2.0*GRDSP)
        CONT=SQRT(36.0/(NR*(NR+1)*(2*NR+1)))
C****   CALL  DATE(DAY)
        DAY = DATE()
```

```
      WRITE(IOUT,2020)DAY,MASFIL,CSFIL,CDAY
2020  FORMAT('1EXECUTION OF PROGRAM CSIM03',//' TODAYS DATE ',
     1 A10,// ' FILE NAMES ACCESSED IN TODAYS RUN'/,
     2 ' MASTER FILE --',A7,/,' MODEL FILE ---',A7,/,
     3 ' FILE CREATION DATE ',A10)
      WRITE(IOUT,2030)NAME(IVAR)
2030  FORMAT(1H0,A10,'IS BEING SIMULATED IN THIS RUN')
      WRITE(IOUT,2040)RANGE,GRDSP,NR,CONT,ISEED
2040  FORMAT('0INPUT PARAMETERS'/,' RANGE    =',F7.0,/
     1 ' GRID SPACING =',F7.0,/,' CALCULATED CONSTANTS',/,
     2 ' NR       =',I5,/,' CONSTANT =',E11.5,/,
     3' BEGINNING RANDOM NUMBER SEED =',I15)
C-------------------------------------------------------------
C     CALCULATE NUMBER OF POINTS IN N-S, E-W, AND VERT
C     DIRECTION
C-------------------------------------------------------------
      PE=NBE*NPT
      PN=NBN*NPT
      PZ=NPTV
C-------------------------------------------------------------
C     MATRIX V CONTAINS COORDINATES OF THE VERTICES OF THE
C     MODEL BEING SIMULATED.
C-------------------------------------------------------------
      V(1,5)=PE
      V(1,6)=PE
      V(1,7)=PE
      V(1,8)=PE
      V(2,3)=PN
      V(2,4)=PN
      V(2,7)=PN
      V(2,8)=PN
C-------------------------------------------------------------
C     FIND THE MIN AND MAX INTERSECTIONS OF VECTOR A WITH
C     THE MODEL
C-------------------------------------------------------------
      OPEN(ICOR,ACCESS='SEQUENTIAL',FORM='UNFORMATTED')
      DO 70 L=1,15
         MIN(L)=20000
         MAX=-20000
         DO 10 I=1,8
          LR=(A(1,L)*V(1,I)+A(2,L)*V(2,I)+A(3,L)*V(3,I))
            IF(LR.LE.MIN(L)) MIN(L)=LR
            IF(LR.GE.MAX) MAX=LR
10          CONTINUE
C-------------------------------------------------------------
C     CALCULATE THE NUMBER OF POINTS TO GENERATE FOR THIS LINE
C       AND GENERATE THAT MANY 1-D POINTS
C-------------------------------------------------------------
         NUM(L) =MAX-MIN(L) +2
         NTOT=NUM(L) +2*NR
         IF(NTOT.LE.10000)GO TO 20
         WRITE(IOUT,2045)L,NTOT
2045     FORMAT('EXCEEDING MODEL CAPACITY-LINE',I5,' NTOT',I6)
         STOP
```

```
C****** PUT RANDOM NUMBER GENERATOR IN HERE
20       DO 30 I=1,NTOT
         RANVAL(I)=RAND(ISEED)-0.5
30       CONTINUE
40          N1=NR+1
            NT1=NTOT-NR
            DO 50 J=N1,NT1
               JN=J+NR
               JN1=J-NR
               DO 50 I=JN1,JN
                  DIS=J-I
                  DLINE(J)=DLINE(J)+DIS*RANVAL(I)*CONT
50                CONTINUE
            WRITE(ICOR)(DLINE(I),I=N1,NT1)
            DO 60 I=1,10000
               DLINE(I)=0.0
60             CONTINUE
C        WRITE(IOUT,2015)ISEED
70          CONTINUE
         REWIND ICOR
         IF(IDBUG.EQ.1) WRITE(IOUT,2070)(I,NUM(I),MIN(I),I=1,15)
2070     FORMAT(3I5)
         WRITE(IOUT,2015)ISEED
2015     FORMAT(' ENDING RANDOM NUMBER SEED =',I12)
         OPEN(IMOD,ACCESS='DIRECT',FILE=CSFIL,RECL=10000)
C-----------------------------------------------------------
C        LOOP THROUGH THE MODEL BLOCK BY BLOCK
C-----------------------------------------------------------
         DO 100 NL=1,NBN
            DO 100 NE=1,NBE
               KBLOCK=NE+(NL-1)*NBE
C-----------------------------------------------------------
C        READ IN ONE BLOC FROM CSIM MODEL
C-----------------------------------------------------------
               CALL  TAKOUT(KBLOCK,IVAR)
         WRITE(IOUT,2016)KBLOCK
2016     FORMAT(' BLOCK',I5,'TAKEN FROM MODEL')
C-----------------------------------------------------------
C      READ IN "NUM(L)" VALUES FOR VECTOR A.
C-----------------------------------------------------------
               DO 90 L=1,15
                  READ(ICOR)(DLINE(I),I=1,NUM(L))
C-----------------------------------------------------------
C        NOW LOOP THRU POINTS IN THE BLOCK
C-----------------------------------------------------------
                  DO 80 K=1,NPTV
                     DO 80 J=1,NPT
                        DO 80 I=1,NPT
                           C(1)=I+(NE-1)*NPT
                           C(2)=J+(NL-1)*NPT
                           C(3)=K
                           RL=A(1,L)*C(1)+A(2,L)*C(2)+A(3,L)*C(3)
                           LR2=RL + SIGN(0.5,RL)
                           LR=LR2-MIN(L)+2
```

```
      IF(LR.LE.0)GO TO 80
                        B(I,J,K)=B(I,J,K)+DLINE(LR)/SQ15
80                      CONTINUE
90              CONTINUE
              REWIND ICOR
C-------------------------------------------------------------
C       PUT THIS BLOCK BACK IN THE MODEL
C-------------------------------------------------------------
              CALL  PUTIN(KBLOCK,IVAR)
C****         WRITE(IOUT,2080)KBLOCK
C2080   FORMAT(' BLOCK ',I5,' WRITTEN TO DISK')
100           CONTINUE
      CLOSE(MASTR)
      CLOSE (IMOD)
      STOP
      END



      SUBROUTINE RMAST(CDAY)
C-------------------------------------------------------------
C       SUBROUTINE RMAST ACCESSES THE MODELS MASTER FILE
C       AND READS IN THE MODEL PARAMETERS.
C-------------------------------------------------------------
      COMMON /PAREM/ NBLOC,NBN,NBE,NPT,NPTV,YNORT,XEAST,YLPT,XLPT,
     1 NSIM,NVAL,NPACK(3),NAME(3),NDEC(3)
      COMMON /INOUT/ IPT,IOUT,MASTR,ICOR,IMOD
      COMMON /FILES/ MASFIL,CSFIL
      COMMON /MISC/ TITLE(8)
      CHARACTER *7 CSFIL,MASFIL
      CHARACTER *10 DAY,DATE,CDAY
      READ(MASTR,2000)TITLE
2000  FORMAT(8A10)
      READ(MASTR,2000) CDAY
      READ(MASTR,2010)CSFIL,MASFIL
2010  FORMAT(A7,3X,A7)
      READ(MASTR,2020)NBLOC,NBN,NBE,NPT,NPTV,NSIM
2020  FORMAT(6I10)
      READ(MASTR,2030)YNORT,XEAST,ZELEV,YLPT,XLPT,ZLPT
2030  FORMAT(6F10.0)
      READ(MASTR,2040)NVAL,NDEC,NPACK,NAME
2040  FORMAT(4I5,3I10,3A10)
      RETURN
      END
```

```
      SUBROUTINE TAKOUT(KBLOCK,IVAR)
C-----------------------------------------------------------
C     SUBROUTINE TAKOUT REMOVES FROM THE CSIM MODEL ONE
C     BLOCK OF DATA AND PUTS IT IN TO ARRAY IB.
C-----------------------------------------------------------
      COMMON /PAREM/ NBLOC,NBN,NBE,NPT,NPTV,YNORT,XEAST,YLPT,XLPT,
     1 NSIM,NVAL,NPACK(3),NAME(3),NDEC(3)
      COMMON /INOUT/ IPT,IOUT,MASTR,ICOR,IMOD
      COMMON /FILES/ MASFIL,CSFIL
      COMMON /CSIMOD/ B(100,100,1),IB(100,100,1)
      CHARACTER *7 CSFIL,MASFIL
      DIV=10**NDEC(IVAR)
      READ(IMOD,REC=KBLOCK)IB
C***** FOR DEBUGGING.
C     WRITE(IOUT,4774) KBLOCK, IB(1,1,1), IB(2,1,1)
C4774 FORMAT(1X,' I = ',I10,2X,'IB = ',2I10)
      DO 10 K=1,1
         DO 10 I=1,100
            DO 10 J=1,100
               IB1=IB(I,J,K)
               DO 5 L=NVAL,IVAR,-1
                  IB2=IB1/NPACK(L)
                  IB1=IB1-IB2*NPACK(L)
5                 CONTINUE
               B(I,J,K)=IB2/DIV
10             CONTINUE
      RETURN
      END



      SUBROUTINE PUTIN(KBLOCK,IVAR)
C-----------------------------------------------------------
C     SUBROUTINE PUTIN TRANSFERS THE BLOCK OF DATA IN
C     ARRAY IB AND PUTS IT INTO THE CSIM MODEL.
C-----------------------------------------------------------
      COMMON /PAREM/ NBLOC,NBN,NBE,NPT,NPTV,YNORT,XEAST,YLPT,XLPT,
     1 NSIM,NVAL,NPACK(3),NAME(3),NDEC(3)
      COMMON /INOUT/ IPT,IOUT,MASTR,ICOR,IMOD
      COMMON /FILES/ MASFIL,CSFIL
      COMMON /CSIMOD/ B(100,100,1),IB(100,100,1)
      CHARACTER *7 CSFIL,MASFIL
      NPAC= 10**NDEC(IVAR)
      NPAC1=NPACK(IVAR)
      DO 10 K=1,1
         DO 10 J=1,100
            DO 10 I=1,100
               ITEMP=B(I,J,K)*NPAC
               IB(I,J,K)=ITEMP*NPAC1
10             CONTINUE
C**** FOR DEBUGGING.
C     WRITE(IOUT,4774) KBLOCK, IB(1,1,1), IB(2,1,1)
C4774 FORMAT(1X,' I = ',I10,2X,'IB = ',2I10)
      WRITE(IMOD,REC=KBLOCK)IB
      RETURN
      END
```

```
      FUNCTION RAND(IX)
C-----------------------------------------------------
C     RAND IS A PORTABLE RANDOM NUMBER GENERATOR BASED
C     ON THE RECURSION IX = IX*A MOD P.
C
C     FORTRAN CODE WAS WRITTEN BY LINUS SCHRAGE
C     REFERENCE AND DOCUMENTATION IS IN
C     ACM TRANSACTIONS JANUARY 1980
C-----------------------------------------------------
      INTEGER A,P,IX,B15,B16,XHI,XALO,LEFTLO,FHI,K
      DATA A,B15,B16,P/16807,32768,65536,2147483647/
C     GET 16 HI ORDER BITS OF IX
      XHI = IX/B16
C     GET 16 LO ORDER BITS OF IX AND FORM LO PRODUCT
      XALO=(IX-XHI*B16)*A
C     GET 16 HI ORDER BITS OF LO PRODUCT
      LEFTLO = XALO/B16
C     FORM THE 31 HIGHEST BITS OF FULL PRODUCT
      FHI = XHI*A + LEFTLO
C     GET OVERFLO PAST 31ST BIT OF FULL PRODUCT
      K = FHI/B15
C     ASSEMBLE ALL THE PARTS AND PRESUBTRACT P
C     THE PARENTHESES ARE ESSENTIAL
      IX=(((XALO-LEFTLO*B16)-P)+(FHI-K*B15)*B16)+K
C     ADD P BACK IN IF NECCESSARY
      IF(IX.LT.0)IX =IX+P
C     MULTIPLY TY 1/(2*31-1)
      RAND =FLOAT(IX)*4.656612875E-10
      RETURN
      END
```

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