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## EFFECTS OF MAGNETIC FIELDS ON MAIN SEQUENCE STARS

The University of Arizona
PH.D. 1981

# EFFECTS OF MAGNETIC FIELDS ON MAIN SEQUENCE STARS 

by
Eugene Norman Hubbard
A Dissertation Submitted to the Faculty of the DEPARTMENT OF ASTRONOMY

In Partial Fufiliment of the Requirements For the Degree of DOCTOR OF PHILOSOPHY<br>In the Graduate College<br>THE UNIVERSITY OF ARIZCNA

## THE UNIVERSITY OF ARIZONA

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Effects of Magnetic Fields on Main Sequence Stars
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#### Abstract

A number of effects of low to medium strength (<2000 gauss photospheric) magnetic fields on otherwise normal stars are proposed and examined. We consider magnetic perturbations to the standard stellar structure and evolutionary calculations in the core, the deep envelope, and the extreme outer envelope in intermediate to high mass stars.

In the stellar core the gas pressure probably far exceeds the ( $\mathrm{B}^{2} / 8 m$ ) magnetic field pressure term so that the only effect of such a field may come from its inhibiting convection in the core. We present isochrones of both convective and radiative core models of 2 $5 M_{0}$.

In the deep envelope, we may expect to see mixing of partially nuclear processed material driven by rising and falling magnetic flux tubes. The effects of this mixing will be brought to the surface during the deep convection phase of the star's tenure as a red giant. We use this model to predict a signature for magnetic mixing based on the CNO isotope and abundance ratios.

In the outer envelope the gas pressure is low enough that we might expect to see a perturbation of the stellar structure due to the magnetic field pressure itself. We calculate this perturbation under several physical models for intermediate and high mass stars and determine that sufficient magnetic field energy may be available in


the outer envelope to expand a star by about $20 \%$ over its unperturbed radius.

Finally we consider the evidence for the existence of non-magnetic neutron stars, concluding that while no non-magnetic neutron stars have ever been positively identified, we have no evidence that prevents the existence of at least as many non-magnetic as magnetic neutron stars.

We define the appearance of stars by measuring a number of their properties: their position on the H-R diagram, their spectra, their isotope abundance ratios, to name a few. We can describe this appearance remarkably well in terms of standard evolutionary models based only on initial composition and mass, as required by the Russell-Vogt theorem. This is to say that we can do astrophysics. There are some stars however which have observable properties which differ both from those of their siblings and from those of standard evolutionary calculations. The Russell--Vogt theorem predicts that there should be discrete, well defined models for each mass and composition: one red giant, one main sequence star, one white dwarf perhaps. What we actually see are many variations of each type of star, all similar to their standard evolutionary calculation archtype, but differing enough that we cannot explain their variations by simply adjusting the mass and composition in our standard model.

It seems likely that some of this deviate behavior is caused by one of more physical effects which are not included in the standard theory. A few such effects are those of rotation, mass loss, binary evolution, and magnetism. This dissertation describes some perturbations on the standard theory which we might expect to arise from a photospheric magnetic field of < 2000 gauss.

At this point: "Why magnetism and why 2000 gauss?" Magnetic fields on stars are rapidly becoming an observable quantity. Babcock's first (1947) photographic detection of a magnetic field on 78 Vir weighed in at about a kilogauss. Later, two channel photoelectric polarimeters such as the one described by Landstreet et al (1975) were used to measure kilogauss magnetic fields in Ap stars with an accuracy of a couple of hundred gauss. (Borra and Landstreet 1980). These same instruments were used to detect the presence of megagauss fields on a few white dwarfs (Angel, Borra, and Landstreet 1981). The two most recent developments in the measurement of stellar magnetic fields have been the development of the multiple slit photoelectric magnetometer (Borra, Fletcher, and poeckert 1981, and Brown and Landstreet 1981) and the method of determing absolute field strengths through Zeeman broadening of magnetically active lines (Robinson, Worden, and Harvey 1980). The multiple slit magnetometer has made possible the measurement of magnetic fields in some Ap stars with uncertainties of only a few tens of gauss, less than 10 gauss for some bright stars. Fobinson's method, while still having a detection limit of a kilogauss, may be used to detect badly scrambled surface fields whereas polarimetric methods are useful only in detecting residual ordered fields.

From these recent observations, a pattern seems to be emerging that finds that the probability of a main sequence star having an ordered detectable magnetic increases with increasing mass. When cooler stars (G and later) show fields, they seem to nearly always be of the scrambled variety, detectable only with Robinson's technique
(Brown and Landstreet 1981). Since both white dwarfs and main sequence stars seem to show distinct populations of magnetic objects, threaded by about $10^{24}$ maxwells of magnetic flux, and non-magnetic objects without detectable fields. This natural division into magnetic and non-magnetic objects provides a textbook set of test and control samples to test predicted results of stellar magnetic fields. We chose 2000 gauss as a canonical photospheric magnetic field following observations such as those of Borra and Landstreet (1980) for magnetic fields on Ap stars.

Then how might a magnetic field of this type affect the appearence of a star? Does it explain observed phenomona in a natural way, or is it riddled by so many free parameters and ad hoc assumptions as to be useless? In terms of stellar structure a magnetic field has two rather contradictory effects. As a relativistic gas, its effect is to provide a $\mathrm{B}^{2} / 8 \pi$ pressure component and to move the polytropic exponent toward $4 / 3$, destabilizing the star. As a vector field however, its effect is to prevent material from flowing across field lines, providing a source of rigidity in the star's interior. The result may be compared to a pile of soap bubbles which is at the same time more rigid but less stable than its isomer, the puddle of dishwater.

We will investigate scenerios involving both the rigidity and the instability of the soap bubbles. Possibly the most well known destabilizing effect of a magnetic field is demonstrated by sunspots. The additional $\mathrm{B}^{2} / 87$ magnetic pressure term allows material threaded by magnetic flux to rise through surrounding material,
cooling adiabatically until the material surrounding the magnetic element is enough warmer that the extra themal pressure offsets the additional magnetic pressure of the magnetic flux element. At this point the material threaded by the magnetic flux will be significantly cooler than the surrounding material, as in a sunspot. If this magnetic element is buried deep inside the envelope of a star, rather than being on the surface it will be heated by its surroundings and continue to rise. In zones of a composition gradient, this rising flux element will have a composition different from that of the surrounding material and this process will have the effect of mixing the interior of the star at a rate characteristic of the speed with which the flux element rises through its surroundings. This mechanism has been proposed as a source of fuel to prolong the lives of blue stragglers beyond their appointed times (Wheeler 1979). It has also been proposed as a mechanism to account for the anomolous CNO isotope and abundance ratios in some red giants (Dearborn and Eggletion 1977). In Chapter 2, we calculate mixing profiles for a $2.0 \mathrm{M}_{\odot}$ star and use these to predict observable CNO isotope and abundance ratios in red giants under various initial assumptions.

If there is ever a situation where the magnetic field energy is comparable to or exceeds the gas pressure, there is a possibility that the magnetic field will have a major influence on the structure of that region. A kilogauss field at the surface of a main sequence star is such a situation. There are two classes of stars, the Ap's and the $O$ dwarfs, that are often associated with magnetic fields and which show evidence for anomolously large radii. Chapters 3 and 4
describe structural perturbations of a stellar envelope due to a significant magnetic pressure at the photosphere in the context of these anomolous, often magnetic stars.

Whether a magnetic field serves to stabilize a star by adding to its rigidity, or to destabilize it by promoting mixing and lowering the polytropic exponent, is a matter of the detailed geometry of the field. Except for the possibility of scenerios where the magnetic field at the photosphere superstabilizes an already radiative envelope and allows molecular diffusion to generate surface abundance anomalies in Ap stars, the rigidity of a magnetic field would seem to be important only in regions that are unstable to thermal convection. In Chapter 5 we assume that a strong primordial magnetic field can inhibit core convection in intermediate mass stars and ask what effects the radiative core might have on the star's evolution. By computing detailed isochrones of both radiative and convective core stars of the same age, we can compare our results directly to observations of clusters. This will determine whether a few radiative core stars in a cluster help to solve or only augment discrepencies between observations and theoretical models. An example of such a discrepancy is the observed lack of a themal timescale jump between the main sequence and the shell burning branch described by Maeder (1974).

The underlying theme threading this dissertation is the question of what $10^{24}$ maxwells of magnetic flux through a stellar object can do toward explaining deviate behavior of the affected object. In Chapter 6 we take the lead provided by Angel et al (1981)
when they suggest that magnetic white dwarfs and magnetic main sequence stars have a common origin. If magnetic main sequence stars beget magnetic white dwarfs and non-magnetic main sequence stars beget non-magnetic white dwarfs then what of neutron stars? If magnetic main sequence stars beget magnetic neutron stars and so forth then where are the non-magnetic neutron stars? To show the existence of both flux-less and $10^{24}$ gauss versions of all three kinds of stars, main sequence, white dwarfs, and neutron stars would be strong evidence that the surface fields of megnetic stars are coupled to the interior fields and that the fields are primordial, rather than generated at the core convection zone boundary.

The final chapter is an assessment of what we have done and what to do next. Because this dissertation is the result of a preliminary shotgun blast in the direction of magnetic fields in otherwise normal stars, we will consider only the simplest models for these effects which are physically justifiable. All of the calculations are one dimensional and any force exerted by a bundle of magnetic flux lines will be treated as a scalar pressure. Since many magnetism problems are inherently multi-dimensional, this treatment cannot always be expected to yield a definitive answer. However the one dimensional treatment will yield an accurate energy balance for the star and clearly point out those effects which are not worth further consideration.

## CHAPTER 2

## MAGNETIC MIXING

Stars which have evolved from the main sequence to the giant branch are observed to have ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratios from $\sim 7$ to $\sim 25$ (Lambert and Ries 1978). Evolutionary calculations show that the deep convection zone that forms as the star moves off the main sequence is adequate to lower the ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratio from the solar value of $\sim 90$ to the range 20 30, yet for this deep convection to lower the ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratio to the value observed for many stars would require an initial ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratio of about 20, a fact inconsistent with observations of the interstellar medium (Dearborn, Eggleton, and Schramm 1976). Nuclear processing can produce enough additional ${ }^{13} \mathrm{C}$ to lower the ratio if a mechanism can be found that will remove ${ }^{13}$ C from the furnace before it is destroyed, that is, before the CN cycle reaches a steady state (Dearborn and Eggleton 1977).

One such mechanism for mixing material is through magnetic filux tubes, formed at the outer boundary of the core convection zone (Gurm and Wentzel 1967). The magnetic field in these tubes, generated by an $\alpha^{2}$ dynamo and Iimited by energy equipartition with turbulent motion, may be up to $10^{5}$ gauss (Schussler and Pahler 1978) and exerts an additional pressure ( $\mathrm{B}^{2} / 8 \pi$ ) which tends to buoy the flux tubes up. If we consider the plasma that makes up the interior of the star to be of very high electrical conductivity, the material permeated by the
magnetic field is more or less locked onto the field lines and is buoyed up along with the field lines. This mechanism will be active in stars larger than $-1.5 M_{\circ}$ (lowest mass star having a core convection zone) and smaller than $\sim 2.0 \mathrm{M}_{\odot}$ (so that the mixing time scale is less than the main sequence lifetime: Schussler and Pahler 1978).

We shall consider the case in which magnetic mixing provides only a small perturbation on the overall stellar structure, hence magnetic mixing will be active only when the star is stable against convection. We shall also consider that this magnetic mixing begins somewhat outside of the generating convection zone boundary. While it is the convection zone that forms the magnetic field, actual material transport will be inhibited by the molecular weight gradient which exists in the vicinity of the convection zone boundary. We shall, however, test different posible starting points for active magnetic mixing.

To further examine this mechanism, we propose a magnetic mixing model with the following features:

1) $M=2.0 M_{0}$. This will give the star sufficient time to mix on the main sequence and is representative of stars for which the mechanism is active.
2) The "magnetic luminosity" will be constant through the star. We define magnetic Iuminosity as the amount of magnetic field energy ( $B^{2} / 8 \pi$ ) passing through a given closed surface per unit time. Obviously this magnetic luminosity must be much less than the total luminosity. In addition, some models will be considered in
which the magnetic luminosity is allowed to decay with radius due to interference between adiabatically expanding flux tubes.
3) Adopt $B=10^{5}$ gauss at the base of the mixing zone as an equipartition value with convective turbulence (Schussler and Pahler, 1978).

The fraction of mass in the flux tubes will be an input parameter. This is a measure of the number of field lines in the star. As the tube rises, this fraction will change to maintain a constant magnetic luminosity up to the limit that no more than one half of the star can be inside flux tubes. This reflects the fact that matter must sink and that the star cannot be overfilled with flux tubes. We shall consider three cases: First, we will consider the case in which no more than one half of the mass in any zone is contained within flux tubes, the "unsaturated" case. Second, we will consider a star where half of the mass is in flux tubes at the base of the mixing region, and magnetic flux is conserved as the tubes rise by unlocking enough matter from the expanding flux tubes to keep the mass fraction of the tubes at 0.5 ("saturated case"). Finally, we shall consider the case where the material remains locked to the magnetic field lines but enough of the expanding flux tubes are destroyed by conversion into themal energy to keep the total mass fraction of the flux tubes dow to 0.5. Since the magnetic luminosity is far smaller than the total luminosity of the star, the effect of any energy deposited in a zone by a disintegrating flux tube will be small compared to the energy normally passing through that zone. However in this case magnetic flux will not be conserved. In all cases, the
thermal lifetime of the tube (random walk time out for a particle starting at the center) turns out to be comparable to the length of time the flux tube takes to reach the surface.

The depth of magnetic mixing will be an input parameter. The actual value depends in detail on the field production mechanism and is quite uncertain. The molecular weight gradient places some restraints on the depth of mixing however, and some isotope ratios may be insensitive to this choice.

The radius of the flux tubes will be an input parameter. This will determine the rise times of the flux tubes and is, at present, totally unknown. We shall consider a range of values.

We shall exmine the effects of this mixing on the CNO isotope ratios $\left({ }^{12} \mathrm{C} /{ }^{13} \mathrm{C},{ }^{14} \mathrm{~N} /{ }^{15} \mathrm{~N},{ }^{16} \mathrm{O} /{ }^{17} \mathrm{O},{ }^{16} \mathrm{O} /{ }^{18} \mathrm{O}\right.$ ) and abundance ratios ( $\mathrm{C} / \mathrm{N}$ and $C / O$ ) . Sufficient mixing will certainly have the effect of moving the CNO isotope and abundance ratios toward their equilibrium values at the base of mixing, that is, lowering the ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratio toward 3.5 , lowering $\mathrm{C} / \mathrm{N}$ and $\mathrm{C} / \mathrm{O}$, and raising ${ }^{14} \mathrm{~N} /{ }^{15} \mathrm{~N}$. However, if mixing is slow enough that the CN cycle is not allowed to reach equilibrium, it is the manner of approach to equilibuium and not the equilibrium values themselves that matter. In the context of magnetic mixing, each of the isotope and abundance ratios will change in an identifiable way, leaving a magnetic signature.

In the spirit of obtaining a magnetic signature, we shall bury the flux tube radius as a free parameter by varying this parameter to achieve a range of ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratios between 3.5 and 23.5 , leaving the depth of mixing as the only free parameter. Having done this, we can
associate a specific $\mathrm{C} / \mathrm{N}, \mathrm{C} / \mathrm{O},{ }^{14} \mathrm{~N} /{ }^{15} \mathrm{~N},{ }^{16} \mathrm{O} /{ }^{17} \mathrm{O}$, and ${ }^{16} \mathrm{O} /{ }^{18} \mathrm{O}$ ratio with every ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratio for any depth of mixing; more importantly, we can choose a mixing depth that is consistent with any combination of ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ and any other cNO isotope or abundance ratio, for example $\mathrm{C} / \mathrm{N}$. Having done this, we will be able to make predictions of all of the other CNO isotope and abundance ratios given only ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ and $\mathrm{C} / \mathrm{N}$ to determine if magnetic mixing is a viable explanation of the observed anomalies.

## Calculations

Since the magnetic mixing is severely impeded by a molecular weight gradient, the mixing can have little effect on the structure of a star. For this reason we used the technique described by Dearborn and Eggleton (1977) where we first generate an evolutionary series of models following only the hydrogen to helium reaction. These structural models are then used as input for a code which follows the detailed CNO reactions. The magnetic mixing is then treated as a diffusion term in the nucleosysthesis "piggyback" code.

To determine the magnetic diffusion parameter, consider the diffusion to be the amount of mass per init time moving through a mass zone

$$
\begin{equation*}
\sigma=\frac{d M}{d t} \Delta M, \tag{1}
\end{equation*}
$$

where $\mathrm{dM} / \mathrm{dt}=$ mass moving per unit time, $\Delta \mathrm{M}=$ mass through which flux tubes move (mass in zone). We can then write

$$
\begin{equation*}
\frac{d M}{d t}=F\left(4 \pi r^{2} \rho\right) u, \tag{2}
\end{equation*}
$$

where $F=$ fraction of total mass residing in flux tubes, $u=$ velocity of flux tubes.

Parker (1975) derives a rise velocity for magnetic flux tubes in a convectively stable environment by assuming that a magnetic flux element will rise, cooling adiabatically until a temperature differential is established between the flux tube and its environment that will exactly balance the original buoyancy of the flux tube:

$$
\begin{equation*}
d T / T=B^{2} / 8 \pi P \tag{3}
\end{equation*}
$$

After this balance is established, the flux tube can only rise at a rate proportional to the rate at which energy can flow into it from its sourroundings. The result of this calculation is that the rise velocity of the flux tubes may be written as:

$$
\begin{equation*}
u=\frac{\alpha}{\bar{X} G}(\lambda / a)^{2} \frac{B^{2}}{8 \pi P} \frac{I}{P} \tag{4}
\end{equation*}
$$

where $\alpha$ and $\xi$ are factors of order unity, $\lambda=$ temperature scale height, $a=f l u x$ tube radius, and $I=$ intensity of radiation.

When the material making up a flux tube is locked to the flux lines, as in the case of a highly conductive plasma, we can define the run of the field strength and flux tube radius through the star as:

$$
\begin{equation*}
B=B_{0} \rho / \rho_{0} \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
a=a_{0}\left(\rho_{0} / \rho\right)^{1 / 2} \tag{6}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
\sigma=F\left(4 \pi r^{2} \rho\right) \backsim M \tag{7}
\end{equation*}
$$

and we may define the magentic Iuminosity as:

$$
\begin{equation*}
L_{m}=F\left(d E_{m} / d v\right) u\left(4 \pi r^{2}\right) \tag{8}
\end{equation*}
$$

where magnetic energy density $\mathrm{dE}_{\mathrm{m}} / \mathrm{dV}=\mathrm{B}^{2} / 8 \pi$.
Using this model, we may determine a diffusion parameter at every point in a stellar interior either using equation (8) to determine F for the unsaturated case or arbitrarily setting $\mathrm{F}=0.5$ in the saturated case. In the saturated, magnetic flux conserving case, we set $F=0.5$ and use equation (8) rather than equation (5) to solve for the run of magnetic field through the star.

## Results

We ran nucleosynthesis models for the three cases described above: the unsaturated case, the saturated flux-conserving case, and the saturated non-flux-conserving case. We chose first to allow the envelope to mix above a depth of $0.3 \mathrm{M}_{0}$ from the center of the star to simulate the case that mixing is uninhibited by the molecular weight gradient outside of the core discontinuity and then depths of $0.5 M_{\odot} 0.6 M_{\odot}$, and $0.7 M_{\odot}$ from the center to simulate different degrees of inhibition. We also considered models in which we varied the
initial ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratio from 40 to 90 and the initial $\mathrm{C} / \mathrm{N}$ ratio from 3 to 5.8 to test for sensitivity to these parameters.
of these variations, we find the resulting isotope and abundance ratios relatively insensitive to the starting abundance ratios. Also, except in those cases where magnetic mixing extends all of the way down to the convective core, the resulting isotope and abundance ratios are equivalent to within $10 \%$ regardless of the saturation case chosen. In all cases where the final ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratio is between 5 and 20 , the required flux tube radius falls between 30 km and 1000 km . In figures $1-5$ we have plotted all of the CNO isotope and abundance ratios that might be expected given various combinations of ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratios and magnetic mixing depths. The model is absolutely insensitive to variations in the value $B_{0}=1.0 \times 10^{5}$ since we have parameterized our results in terms of the final ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratio. Any variation of the initial B field could be compensated for by a corresponding variation of the initial flux tube radius.

## Discussion

From the models we have run, we see that the post main sequence abundance ratios are consistent with those in stars having a moderate nitrogen enhancement and little or no ${ }^{13} \mathrm{C}$ enhancement. Some of these stars are (Lambert and Ries 1977)

B Gem: $\mathrm{C} / \mathrm{N}=0.9,{ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}=16$;
$\gamma$ Tau: $C / N=0.6,{ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}=19$;
$\delta$ Tau: $\mathrm{C} / \mathrm{N}=0.9,{ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}=23$;


Figure 1. ${ }^{14} \mathrm{~N} /{ }^{15} \mathrm{~N}$ isotope ratios for $2.0 \mathrm{M}_{\mathrm{O}}$ star.


Figure 2. ${ }^{16} 0 /{ }^{17} \mathrm{O}$ iostope ratios for $2.0 \mathrm{M}_{\mathrm{C}}$ star.


Figure 3. ${ }^{16}{ }_{0} /{ }^{18} 0$ isotope ratios for $2.0 M_{0}$ star.


Figure 4. $\mathrm{C} / \mathrm{N}$ abundance ratios for $2.0 \mathrm{M}_{0}$ star.


Figure 5. $\mathrm{C} / 0$ abundance ratios for $2.0 \mathrm{M}_{Q}$ star.
$\varepsilon$ Tau: $\mathrm{C} / \mathrm{N}=0.9,{ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}=22$;
$\theta^{1}$ Tau: $C / N=1.0,{ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}=20$.
The models where mixing does not penetrate $0.5 \mathrm{M}_{0}$ are consistent with E Cyg: $C / N=1.4,{ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}=11.5$,
and approach the value for
$\alpha$ BOO: $C / N=2.2,{ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}=3$.
The question that then comes up is how to justify mixing that begins so far out from the convective core. Hwever, the molecular weight gradient due to the PP I chain extends out to $\sim^{\sim 1.0} \mathrm{M}_{0^{\prime}}$ and the mixing inside the gradient may be very slow because the flux tubes must break free of the gradient.

By equating the fall time of an element in a molecular weight gradient and the rise time of a magnetic flux tube, Kippenhahn (1974) derives the following criterion for a molecular weight gradient to suppress magnetic mixing:

$$
\frac{\mathrm{P}_{\mathrm{m}}}{\mathrm{P}}<\frac{\phi \mathrm{d} \mu}{\mu}
$$

where $P_{m}=$ magnetic pressure $=B^{2} / 8 \pi, \mu=$ molecular weight, and $\phi$ $=$ function of state $=1$ for a perfect gas. For $B=10^{5}$ gauss, $\mathrm{P}=$ $10^{17} \mathrm{dyn} \mathrm{cm}^{-2}$, and $\phi=1$, a molecular weight discontunity of $\mathrm{d} \mu / \mu$ $=4 \times 10^{-9}$ will bottle up the $10^{5}$ gauss field.

At the end of the star's $10^{9}$ year main sequence lifetime, the molecular weight differs by a factor of $2 \times 10^{-3}$ from the surface value as far out in the star as $1.3 \mathrm{M}_{\odot}$, which is clearly too far out for any useful mixing to occur. The next thing to consider is
how quickly the gradient will form to see if the magnetic mixing will inhibit its formation.

With no mixing, a molecular weight gradient forms rather quickly in a stellar envelope. At a mass zone of $0.6 \mathrm{M}_{\ominus}$ from the center of our $2.0 \mathrm{M}_{\mathrm{O}}$ star, the molecular wieght gradient increases at a rate of $\dot{\mu} / \mu \sim 10^{-11}$ per year. Unless a magnetic field can mix this area in less than about a hundred years, magnetic mixing will be snuffed within the first few hundred yers of a star's lifetime.

Although $10^{-9}$ is a very small gradient and a hundred years is a very short timescale this is not an impossible condition. The stellar envelope need only be homogenized over the displacement distance of a flux tube as described by equation (3). As a flux tube rises adiabatically, its temperature drops by an amount:

$$
\begin{equation*}
d T_{B} / T=d z(d \ln P / d z) \nabla_{e} \tag{9}
\end{equation*}
$$

but the temperature of its environment drops by:

$$
\begin{equation*}
\mathrm{dT}_{e} / \mathrm{T}=\mathrm{dz}(\mathrm{~d} \ln P / \mathrm{dz}) \nabla \tag{10}
\end{equation*}
$$

thus the temperature differential between the flux element and its environment is:

$$
\begin{equation*}
\mathrm{d} T / T=\mathrm{dz}(\mathrm{~d} \ln \mathrm{P} / \mathrm{dz})\left(\nabla_{\mathrm{a}}-\nabla\right) \tag{11}
\end{equation*}
$$

Substituting from equation (3) we find the displacement distance to be

$$
\begin{equation*}
\lambda=\frac{H\left(B^{2} / 8 \pi P\right)}{\left(\nabla_{a}-\nabla\right)} \tag{12}
\end{equation*}
$$

where $H$ is the pressure scale height. We can approximate the mixing time by the time required for a flux tube to cross this displacement distance. In Figure 6 we compare the time required to form a molecular weight gradient of $d \mu / \mu=10^{-9}$ with the time required for a flux tube of $10^{5}$ gauss to disperse it. From this consideration, we might expect that a $2.0 \mathrm{M}_{\odot}$ star with flux tubes somewhat smaller than 1000 km would be stable against magnetic mixing below about 0.6 $M_{\odot}$ from the center and would mix above this height.

This sensitivity to the molecular weight gradient is actually a very useful property of the model. In the region in which ${ }^{13} \mathrm{C}$ is enhanced, the CN cycle has produced little or no helium. The major contribution to the $\mu$ gradient is from the helium produced via the PP chain. In deeper regions, both the PP chain and the CNO cycle are active in producing helium, and the increased mean molecular weight provides a natural mechenism for turning off this mixing.

This leads us to propose the following model for magnetic mixing above $0.6 \mathrm{M}_{0}$

1. Field generated at $0.3 \mathrm{M}_{0}$.
2. Field buoyant on a thermal time scale above $0.6 \mathrm{M}_{0}$.
3. There is little mixing below $0.6 \mathrm{M}_{0}$ because the star is 100\% filled with flux tubes that are too heavy to float.
4. The flux can only rise as corresponding material floats off the top. In this case, the time scale for rising may be so long that


Figure 6. Mixing timescales for $2.0 \mathrm{M}_{6}$ star.
the matter is no longer rigidly locked to the flux lines since the plasma is not truly perfectly conducting.

This picture predicts an ${ }^{16} \mathrm{O} /{ }^{17} \mathrm{O}$ ratio of 800 for all stars with low ${ }^{12} \mathrm{C},{ }^{13} \mathrm{C}$ ratios (figure 4) indicating that magnetic mixing has not penetrated to the ${ }^{17} 0$ production region. It also predicts relatively low ${ }^{14} \mathrm{~N} /{ }^{15} \mathrm{~N}$ ratios for all stars with low ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratios (figure 3) because the mixing has not penetrated the ${ }^{15} \mathrm{~N}$ destruction area. Further, for any ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratio and $\mathrm{C} / \mathrm{N}$ ratio, a mixing depth can be detemined from Figure 1 and this used to predict a c/O ratio and an ${ }^{16} 0 /{ }^{18} 0$ ratio from Figures 2 and 5.

Some of our predicted ratios may be affected by the depth of convection on the giant branch. Different numerical treatments can lead to slightly different predicted penetration depths. Convection on the giant branch penetrates well below the regions containing significant amounts of ${ }^{12} \mathrm{C},{ }^{13} \mathrm{C},{ }^{15} \mathrm{~N}$, and ${ }^{18} \mathrm{O}$ (figure 7). Further, the abundance of ${ }^{16} 0$ is not changed significantly outside the convective core. These elements are not then affected by small changes in the penetration depth of the convection zone on the giant branch. The final surface abundance of ${ }^{14} \mathrm{~N}$ and ${ }^{17} \mathrm{O}$ are, however affected by this uncertainty. Since the abundance of ${ }^{14} \mathrm{~N}$ is already relatively high in the outer regions of the star, its abundance is unlikely to change by more than $10 \%$. The initial $\mathrm{C} / \mathrm{N}$ ratio has more effect than this. On the other hand, the abundance of ${ }^{17} \mathrm{O}$ is very 10 w in the original envelope of the star. In models where no magnetic mixing occurs, the surface convection zone eventually penetrates to a region where the abundance of ${ }^{17} 0$ is rapidly increasing. Small


Figure 7. CNO isotope abundance profile for evolved $2.0 \mathrm{M}_{\mathrm{O}}$ star.
differences in the depth of penetration could affect the surface value by as much as $30 \%$.

## Conclusion

We have shown that magnetic bubbles can cause mixing in stars of $2.0 \mathrm{M}_{\odot}$ at reasonable ( $\sim 10^{5}$ gauss) field strengths. If the lower limit of this mixing is less than $0.5 \mathrm{M}_{\odot}$ from the center of - the star, the magnetic mixing strongly depresses the post main sequence $\mathrm{C} / \mathrm{N}$ ratio. However, the magnetic mixing is a very fragile effect which is completely stopped by a mean molecular weight differential of $10^{-9}$. The depth to which mixing can occur is then determined by the timescale of mixing, hence the radius of the flux tubes. For 1000 km flux tubes, mixing can occur starting at about 0.6 $M_{0}$.

We have presented predicted ${ }^{16} \mathrm{O} /{ }^{17} \mathrm{O},{ }^{16} \mathrm{O} /{ }^{18} \mathrm{O},{ }^{14} \mathrm{~N} /{ }^{15} \mathrm{~N}$ and $\mathrm{C} / \mathrm{O}$ ratios for various combinations of $\mathrm{C} / \mathrm{N}$ and ${ }^{12} \mathrm{C} /{ }^{13} \mathrm{C}$ ratios for a 2 $M_{\odot}$ star. We hope that some of these ratios might actually be measured for the stars in question to provide an observational test of this model.

## CHAPTER 3

## MAGNETIC BALLOONS

Another effect of magnetic fields on otherwise normal stars arises from the gradient of the magnetic field pressure in the outer envelope. ©n an Ap star, the magnetic field may be typically 1000 gauss at the photosphere (see for example Landstreet et al 1975). Under LTE conditions, where pressure is a scalar, a star with a global magnetic field of $10^{3}$ gauss will have a magnetic pressure component ( $B^{2} / 88$ ) of about $4 \times 10^{4}$ dynes $/ \mathrm{cm}^{2}$. Stellar atmosphere calculations, such as by kurucz (1979) suggest that this magnetic pressure component is comparable to or larger than the gas pressure at the photosphere of A type main sequence stars. If the gradient of the magnetic field in the stellar envelope is comparable to, or exceeds the gradient of the gas pressure, we may get a feeling for possible effects by modifying the equation of hydrostatic equilibrium to the form:

$$
\begin{equation*}
\frac{d P}{d r}+\frac{d}{d r}\left(B^{2} / 8 \pi\right)=-\rho g(r) . \tag{13}
\end{equation*}
$$

and watching for a change in the structure of the star. In interpreting this analysis, we must ask two questions. First, can we legitimately regard the effects of the magnetic field to be simply an additional scalar pressure term, and second, will the change in the structure have observational consequences, (a change in the effective temperature, for example).

A well defined class of stars, the Ap stars, are to be included in these considerations. They are observed, in many cases, to have strong magnetic fields (Preston 1967) and they appear redward of the main sequence on the H-R diagram (WOIff 1967). This redness is often attributed to heavy line blanketing in the blue which must be fit theoretically to each star individually. Further, as noted by Wolff, whese theoretical considerations do not take the possible effects of the strong field on the structure of the atmosphere into account. There is also some evidence that Ap stars as a group have radii from 1.5 to 2.0 times the expected main sequence radii for their spectral type (Shallis and Blackwell 1979) though this result is not Universal (Baber and Rantela 1978).

## Description of Model

We shall consider the effect of a strong global magnetic field on the envelope of an otherwise normal zero age main sequence star of 2-5 solar masses. To this end, we must first consider the structure of a general magnetic field in any stellar envelope. There are two principle ways in which a generated or fossil magnetic field might be transported from the interior of a star to the surface: It may diffuse through the envelope due to a finite electrical conductivity, or individual flux tubes comprising the general field may be buoyed up, more or less locked to the surrounding material as described by Parker (1975). In any event, we will assume that any flux dissipated in the envelope will be replaced by flux newly transported from the
deed interior. Hence, we are dealing with a steady state, though non-equilibrium condition.

It has been shown (Gilman 1970) that in all cases where a magnetic field increases inwards in a star of infinite electrical conductivity, a slender, horizontal flux tube, an element of a toroidal magnetic field, for example, is unstable to bending into rising and falling flux loops. When the flux is locked onto the material, the total amount of flux threading an element of material will be constant.

$$
\begin{equation*}
\rho a^{2}=\text { ronstant } \tag{14}
\end{equation*}
$$

There are several possibilities for how the total flux threading this element varies with density. In general, for a frozen in field,

$$
\begin{equation*}
B \propto \rho d l \tag{15}
\end{equation*}
$$

where $d l$ is the length of the flux element. If the total length of the flux tube were to remain constant, as it may if one pictures the tube as some sort of garden hose, then.

$$
B \propto \rho
$$

If the length of the tube varies linearly with its radius, as it would if the flux element were a balloon, eqn (15) still holds but with

$$
d 1=\rho^{-1 / 3}
$$

SO:

$$
B \propto \rho^{2 / 3}
$$

There are certainly other possibilities for the run of magnetic field strength with density but it is hard to imagine that the length of a flux tube would increase with increasing density, or that it would decrease faster than $\rho^{1 / 3}$. Hence

$$
\begin{equation*}
B=\rho^{n} \quad 2 / 3<n<1 \tag{16}
\end{equation*}
$$

should cover all cases.
Since our structure calculations are one dimensional, we must define an average magnetic pressure $P_{B}$ at each radius. While very strong fields may dominate the structure and disrupt spherical symmetry, our approximation should provide information on the effect of any given field:

$$
\begin{equation*}
P_{B}=\frac{\oint P_{B}(\theta, \phi) d s}{\oint d s}=\frac{\oint\left[B^{2}(\theta, \phi) / 8 \pi\right] d s}{\oint d s} \tag{17}
\end{equation*}
$$

where $P_{B}(0,0)$ and $B^{\prime}$ refer to the local conditions and ds is a surface element. This average is important because the fraction of surface area occupied by a flux tube changes as the flux tube is raised. We can write:

$$
\begin{equation*}
\left[B^{2}(\theta, \phi) / 8 \pi\right] d s \approx\left(B^{2} / 8 \pi\right) \quad a^{2} N(r) \tag{18}
\end{equation*}
$$

where $N(r)$ is the number of flux tubes piercing the integration
surface, $a$ is the average radius of a flux tube, and $B$ is the magnetic field inside the flux tube. From equation (15), $a \propto B^{-1 / 2} \propto \rho^{-n / 2}$

$$
\begin{align*}
B & =B_{0}\left(\rho / \rho_{0}\right)^{n} \\
a & =a_{0}\left(\rho_{0} / \rho\right)^{n / 2} \\
\oint \frac{B^{2}(\theta, \phi)}{8 \pi} & d s=\frac{B_{0}^{2} \rho}{8 \pi \rho_{0}^{2 n}} \pi a_{0}^{2}\left(\rho_{0} / \rho\right)^{n} N(r)  \tag{19}\\
& =\left(B_{0}^{2} / 8 \pi\right) \pi a_{0}^{2}\left(\rho / \rho_{0}\right)^{n} N(r)
\end{align*}
$$

Then

$$
\begin{equation*}
P_{B}=\frac{\left(B_{0}^{2} / 8 \pi\right) \pi a_{0}^{2}\left(\rho / \rho_{0}\right)^{n} N(r)}{4 \pi r^{2}} \tag{20}
\end{equation*}
$$

The magnetic pressure on the surface is

$$
\begin{equation*}
\left(P_{B}\right)_{0}=\frac{\left(B_{0}^{2} / 8 \pi\right) \pi a_{0}^{2} N_{0}}{4 \pi r^{2}} \tag{21}
\end{equation*}
$$

so the run of magnetic pressure through the star is

$$
\begin{equation*}
P_{B} /\left(P_{B}\right)_{0}=\left(\rho / \rho_{0}\right)^{n}\left(r_{0} / r\right)^{2} N(r) / N_{0} \tag{22}
\end{equation*}
$$

In general, if not all of the flux tubes pierce the surface of the star, $N(r)$ is a non-increasing function of $r$. To define a minimum effect for this model, we will assume that $N(r)=N_{0}$, an assumption
valid in the case of vertical flux tubes which all pierce the surface. Finally, defining an average magnetic field from

$$
\begin{equation*}
P_{B}=\left(B^{2} / 8 \pi\right) \tag{23}
\end{equation*}
$$

We obtain:

$$
\begin{equation*}
B / B_{0}=r_{0} / r\left(\rho / \rho_{0}\right)^{n / 2} \tag{24}
\end{equation*}
$$

However, Gilman's general result for the instability of a flux tube assumes: 1) an infinitely conductive medium, 2) fast themal equilibrium time, 3) isolated flux tubes, and 4) a plane parallel geometry. Moss (1975) suggests that a mixed toroidal poloidal field may inhibit the instability described by Gilman. Certainly a finite conductivity may allow magnetic diffusion through the material on a time scale shorter than the thermal timescale of the flux tubes' buoyancy. In this case, where the material is not forced to move with the flux lines, we have, for a non-constant conductivity

$$
\begin{equation*}
\nabla \times \frac{\nabla \times B}{\sigma}=\left(4 \pi / c^{2}\right) \frac{d B}{d t}=0 \tag{25}
\end{equation*}
$$

for a steady state condition.
Locally, we can assume that a toroidal field.

$$
\begin{equation*}
B=B_{\phi} \hat{\phi} \tag{26}
\end{equation*}
$$

Let:

$$
\begin{equation*}
L=\frac{\nabla \times B}{\sigma}=-\hat{\theta} \frac{1}{\sigma r} \frac{\partial}{\partial r}\left(r B_{\phi}\right) \tag{27}
\end{equation*}
$$

assuming that $B$ and $s$ are functions of $r$ only. Suppressing derivatives with respect to 0 is equivalent to our neglect of the curvature term in the buoyancy case: we will obtain a maximum order of magnitude effect. Then

$$
\begin{equation*}
\nabla \times L=\hat{\phi}\left[\frac{1}{r} \frac{\partial}{\partial r}\left(\frac{\partial / \partial r\left(r B_{\beta}\right)}{\sigma}\right)\right]=0 \tag{28}
\end{equation*}
$$

and integrating

$$
\partial / \partial r .\left(r B_{\phi}\right)=\sigma C
$$

$$
\begin{equation*}
\partial B / \partial r+B / r=\sigma C / r \tag{29}
\end{equation*}
$$

We should expect that any star with a surface magnetic field which originates in in the interior will employ some combination of these two mechanisms in transporting that flux to the surface. Further, it should be reasonable to expect that the parameters of a real magnetic star will be somewhere between those of models using purely diffusive and purely buoyant modes of flux transport. Precisely which flux transport mode dominates in a particular region of a star depends on the relative thermal and magnetic diffusion timescales in that region. Assuming no stabilizing mechanism, a flux tube approaching thermal equilibrium with its surroundings before its magnetic field dissipates will rise as described by Parker (1975). ©n the other hand, if the flux diffuses out of an element faster than its approach to thermal equilibrium, the buoyancy will be inhibited and flux transport will proceed by diffusion alone. Hwever, in view of
the inherent uncertainties of pure timescale arquments, we will consider models where the mode of flux transport is chosen to give the most condensed (maximum density) model. We will also, to establish limits on the effect of magnetic fields on stars, consider both purely diffusive and purely buoyant models without regard to either timescale or energy arguments.

We will complete this model by noting that main sequence stars of $>2.0 \mathrm{M}_{0}$ have essentially radiative envelopes, and that strong magnetic fields may further stabilize the star against convection. Hence, we will consider purely radiative models, realizing that convection may affect our results if the magnetic field alters the star's structure too much. However, as long as the model remains near the main sequence, we may confidently assume a radiative envelope. Finally, we will only consider models in which the magnetic field becomes negligible as the density approaches zero in the atmosphere. This behavior is typical of toroidal fields in stars in that currents cannot pass outside of the region containing material. We will then consider that the measured magnetic fields on stars are of the same order of magnitude as the toroidal field at the photosphere as suggested by Moss (1975). This should be a good assumption if the photospheric field has not relaxed to a force-free configuration.

## Calculations

The effects of magnetic fields that we will consider are important only in the outer envelope of a star where $P_{B} \sim P_{g}$. Realizing this, we will use a modification of the program GOB
described by Paczynski (1969) to calculate stellar envelopes. We will fit the lower boundary conditions of these envelopes to zero age main . sequence (ZAMS) models from 2.0 to 4.2 solar masses, and will adopt $X$ $=0.7, Y=0.28$ and $Z=0.02$ as a typical composition for population $I$ objects.

The magnetic field enters the structure equations through the pressure:

$$
P=P_{g}+P_{B}
$$

where $P_{B}=B^{2} / 8 \pi$. We then re-write Paczynski's equation (28) as follows:

$$
\begin{equation*}
d P=\left(\frac{\partial P}{\partial \rho}\right)_{T B} d \rho+\left(\frac{\partial P}{\partial T}\right)_{\rho B} d T+\left(\frac{\partial P}{\partial B}\right)_{\rho T} d B \tag{30}
\end{equation*}
$$

and following his progression:

$$
\begin{align*}
\frac{d \rho}{d r} & =\left[\frac{d P}{d r}-\left(\frac{\partial P}{\partial T}\right)_{\rho B} \frac{d T}{d r}-\left(\frac{\partial P}{\partial B}\right)_{\rho T} \frac{d B}{d r}\right] /\left(\frac{\partial P}{\partial \rho}\right)_{T B}  \tag{31}\\
\nabla_{\rho} & =\frac{d \ln \rho}{d \ln P} \\
& =\left[P-\left(\frac{\partial P}{\partial T}\right)_{\rho B}^{T V}-\left(\frac{\partial P}{\partial B}\right)_{\rho T} B \nabla_{B}\right] /\left(\frac{\partial P}{\partial \rho}\right)_{T B} / \rho \tag{32}
\end{align*}
$$

where $\left(\frac{\partial P}{\partial B}\right)_{\rho T}=\frac{B}{\Delta \pi}$ and $\nabla_{B}=\frac{d \ln B}{d \ln P}$ is determined by the nature of flux transport.

The selection of flux transport mode may be accomplished either by selecting the mode that will minimize the potential energy of the zone, that is, to maximize $\nabla_{\rho}$, or by the timescale argument
mentioned earlier. The time for magnetic flux to leak out is given by Jackson (1975) as:

$$
\begin{equation*}
\tau_{B}=\frac{4 \pi \sigma L^{2}}{c^{2}} \tag{33}
\end{equation*}
$$

and the time to reach themal equilibrium is

$$
\begin{equation*}
r_{t h}=\frac{c_{v} T}{a T^{4}} \frac{K \rho L^{2}}{c} \tag{34}
\end{equation*}
$$

where $C_{V}$ is the specific heat per unit volume. The ( $L^{2}$ ) factor is just a characteristic size of a flux tube in both cases; thus, the quantities $\frac{\boldsymbol{\tau}_{\mathrm{B}}}{\mathrm{L}^{2}}$ and $\frac{\boldsymbol{\tau}_{\mathrm{th}}}{\mathrm{L}^{2}}$ may be computed directly to determine a dominant method of the flux transport independent of the size of the flux tubes. The use of two independent selection criteria will serve as an internal check on consistency.

We may now generate a family of models by assuming that the magnetic field in the extreme outer envelope is purely diffusive and varying the integration constant $C$ in equation (17) We then use one or another selection criterion to detemine whether the field remains diffusive or becomes buyant at each integration point. We can also generate a family of purely diffusive models by simply suppressing the buoyancy, and a family of purely buoyant models by suppressing diffusion and introdusing a negligible field in the outer atmosphere, allowing it to grow as $B / B_{0}=r_{0} / r\left(\rho / \rho_{0}\right)^{n / 2}$. We will generate models both for the case $n=1$ and $n=2 / 3$.

These calculations assume horizontal, toroidal flux tubes. When we consider vertical tubes, there is a possibility of material
streaming along the flux lines. We suspect however, that the timescale of this streaming will be comparable to the thermal timescale along the length of the tubes; the timescale for buoyancy will be the thermal time across the tube. Thus, for slender tubes, the streaming time may be very long.

Using this physical model, given the magnetic field at the photosphere, we can integrate inward to determine the structure just as we can for the non-magnetic case.

## Results

We find that, except for low (< 100 gauss) photospheric magnetic fields, our maximum effect diffusion case yields no acceptable purely diffusive models. The magnetic pressure increases with depth until the magnetic field itself fully satisfies the hydrostatic equilibrium equation. This results in a density inversion or shell model, clearly not a physical model. The case of low surface fields is simply a model of a normal, non-magnetic star. Aside from this, we can argue against a purely diffusive model by comparing the magnetic and thermal timescales (figure 8) of a ZAMS model and seeing that in all cases the magnetic diffusion timescale is far longer than the thermal timescale. Not only are there no acceptable models where diffusion dominates throughout the envelope, but in the absence of some stabilizing mechanism, a flux tube will always become buoyant on a timescale shorter than the flux dissipation timescale.

Due to the stated uncertainties, we calculated a family of models allowing both diffusion and buoyancy and using the maximum $\nabla_{\rho}$


Figure 8. Relative magnetic and thermal timescales for a $4.2 \mathrm{M}_{0}$ envelope.
criterion to select between the two. These models were constructed by arbitrarily varying both the integration constant in equation (29) and the initial magnetic field to obtain a variety of photospheric magnetic fields. We found from this experiment that the structure does not depend on either the integration constant or the initial field separately, but is well parametrized by the photospheric field alone. This result suggests that the important region of the star is in fact quite small and is likely to be either entirely buoyant or entirely diffusive.

For the purely buoyant models, and $n=1$, we find that, for a $2 \mathrm{M}_{0}$ star, a 1000 gauss photospheric field will result in a 20 per cent change in the star's radius (figure 9). This effect is larger for stars of higher mass because the normal (no magnetic field) surface pressure is lower; the $\mathrm{B}^{2} / 8 \pi$ has a larger effect.

For stars of higher mass ( $>3.0 \mathrm{M}_{6}$ ) we find that there are two distinct models for a range of photospheric magnetic fields as show by the fact that the function in figure 9 is multiple valued in places. One of these models is a collapsed star with a radius of only about $20 \%$ greater than the zero field model. The other one is an extended model with a radius typically more than twice the zero field value. This dramatic incraase in radius is caused by the influence of bound-free and free-free interactions in the outer envelope. In the regime where the expansion occurs, a small expansion of the envelope caused by the magnetic pressure gradient causes a drop in temperatura and an opacity increase. With the higher opacity, the envelope must expand even further to allow the original radiation to escape. Thus,


Figure 9. Effects of a magnetic field on intermediate mass stellar envelopes.
in the outer layers of the envelope, the opacity function amplifies the direct offect of the magnetic pressure gradient by a large factor. For the $n=2 / 3$ case we find the same qualitative result but with the onset of ballooning delayed to 500 gauss for a $3.0 \mathrm{M}_{\circ}$ star (figure 10).

To determine whether convection, in fact would be important in these expanded stars, we considered stars that have evolved to the same point in the H-R diagram as the expanded magnetic stars. We find that the convection in the evolved models is confined to the outer $10^{-8} M_{O}$ of the envelope, making up about 0.18 of the star's radius. This suggests that we were, in fact, justified in assuming radiative envelopes for these models. We have also checked this assumption by comparing the radiative gradient to the adiabatic gradient in our models. In this case, we find that even if convection occurs in all regions where $\nabla_{r}>\nabla_{a}$, our result is qualitatively similar. Thus, even if convection is not rigorously suppressed by the magnetic field, its effect is only to change the details of the model.

It is important to realize that this expansion, either to the extended or to the more condensed model, takes place only in the star's very outer envelope. In figure 11 we show the run of $M(r)$, the mass contained interior to radius $r$ for both the condensed and expanded phases of a $4.2 \mathrm{M}_{\odot} 350$ gauss model. The envelopes differ only in the outer $10^{-4}$ of the star's mass. While it is clear that the envelope is optically thick at this point, it is also certain that such a structure change will have no discernable effects on either the structure or evolution of the core and deeper envelope of such a


Figure 10. Sensitivity of radius to variations in the run of magnetic field with density.


Figure 11. Mass structure of expanded and compact $4.2 M_{G}$ stellar envelopes.
star. These models, despite their large radii and conseguently lower surface temperatures, are still very much ZAMS stars.

## Discussion and Conclusions

We present an H-R diagram of magnetic envelopes in figure 12. We see that the magnetic stars lie redward of the main sequence as would be expected from their larger radii. This redness though, will give magnetic stars the appearance of being more evolved than they actually are, and a modest magnetic field could easily scatter the high end of the main sequence and cause significant systematic error in the assignment of absolute magnitudes based on color affecting cluster membership determinations.

In the region where two distinct, physically acceptable models exist ( $\sim 300$ gauss for $3.0 \mathrm{M}_{0}$ ) the state that a real star assumes should be dependant on the history of the star, in this case, whether the magnetic field existed before the star condensed onto the main sequence or it was generated after the star's collapse.

We would expect that the fossil field of 350 gauss would give rise to the extended $3.0 \mathrm{M}_{0}$ model because magnetic pressure would have stopped the star's contraction before becoming fully condensed. AIternately, if this field had been generated after the star landed on the main sequence, the envelope could not cross the higher field gap to the extended form. This may explain the dispersion of the lower main sequence of the pleiades in that a magnetic field has slowed the collapse of stars to the lower main sequence, making it appear younger than it really is. Also, a magnetic field in the upper main sequence
could make those stars appear older than they really are (figure 12). In stars with $M>3.0 M_{0}$, the field required for the double models is of the order of a few hundred gauss, which is about the current threshold of detectability. It should be possible to make a direct test of whether anomolously red stars in clusters are in fact magnetic using a multiple line magnetometer as described by Borra et al (1981) and Brown and Landstreet (1981).

The primary uncertainty in this discussion is the relationship between the observed magnetic field on a star and the photospheric toroidal field. Further justification of this relationship might come theoretically through a further understanding of the magnetic field structure in stars, or observationally by the verification of these predicted effects of magnetic fields.

Finally, we have neglected the possibility that the magnetic field in the envelope of the star has relaxed to a force free configuration. We do not know how long this relaxation time might be, though from figure 8 we can see that the magnetic relaxation time is about 10 orders of magnitude longer than the thermal time scale throughout the envelope. For a $3.0 \mathrm{M}_{\odot} . s t a r$, this implies that the main sequence lifetime of the star exceeds the relaxation time in the outer $10^{-8}$ of the stellar envelope. This does not necessarily mean that the field will have relaxed outside of the $10^{-8}$ point, since a mechanism to transport flux into this region may result in a steady state condition, but not a force free condition. In any case, only a part of the affected portion of the star lies in the outer $10^{-8}$ of its mass (figure 11).


Figure 12. H-R diagram for magnetic star models.

In conclusion. we have ahown that, for stars of $>2.0 \mathrm{M}_{0} \mathrm{a}$ moderate global magnetic field will have observable effects on the outer envelope, though not on the inner envelope or core. Our models are consistent with the observed redness of Ap stars and explain the larger radii of Ap stars as measured by Shallis and Blackwell (1979). These models also suggest the possibility of errors in the evolutionary state of magnetic stars such as RS CW .

## CHAPTER 4

## MASSIVE STAR ENVELOPES

The extreme upper main sequence of the $H-R$ diagram is observed to be very broad, more so than might be expected from standard .evolutionary considerations. There are a number of explanations for this width, among them high rotation and magnetic fields. Both of these possibilities were discussed by Strothers (1980) with the conclusion that neither magnetic fields nor rotation could adequately explain the breadth of the main sequence from $15 \mathrm{M}_{0}$ to $120 \mathrm{M}_{0}$. We have calculated the effect of a magnetic field in the envelope of a 52 $M_{G}$ ZAMS star using the approach described in chapter 3 with the result that under certain circumstances the observed broadening of the main sequence in this region might be accounted for.

## Model and Calculations

Consistent with the technique described in chapter 3, our model assumes a scalar pressure term associated with the magnetic field strength and the number of flux tubes. At any depth into the envelope we obtain an average magnetic field strength dependant on the local density and zone radius (equation 22, chapter 3).

Our calculations were performed using the same modification of the program GOB (Paczynski 1969) as described in chapter 3 using a composition $\mathrm{X}=0.7, \mathrm{Y}=0.28, \mathrm{Z}=0.02$. The generated envelopes were fit to a ZAMS model at the zone where $\rho=1.0 \mathrm{~g} / \mathrm{cm}^{3}$.

## Results

We find the same multiple models for the magnetic $52 \mathrm{M}_{\odot}$ envelopes as we did in chapter 3 for all masses above $3.0 \mathrm{M}_{\circ}$. In this higher mass star however the effect is far more extreme (figures 13, 14). Condensed models exist for fields $<600$ gauss and expanded models for fields $>40$ gauss at the photosphere. This extreme expansion is confined to the outer part of the envelope (figure 15) in agreement with both'Strothers (1980) and chapter 3. Because the expansion is confined to the outer $10 \%$ or so of the star's mass, we are dealing only with a change in the outer envelope structure and these fields will have little or no effect on the structure and subsequent evolution of the core. This suggests that we were justified in doing only envelope calculations and fitting them to cores, just as we did in chapter 3.

We find that in the expanded phase, the magnetic pressure component is adequate to support an extended (balloon) model when $B_{\text {phot }}>50$ gauss. At the same time a condensed model exists for all photospheric B fields of $<600$ gauss. The actual state of any star probably depends on its evolutionary history; a 50 gauss model in the collapsed state has no way to get puffed up to the expanded state and a 100 gauss star in the expanded state may have trouble deflating to the condensed state (although convective instability might serve to exclude the field). These low field extended models should probably be associated with stars in which the envelope failed to collapse during formation due to the magnetic field pressure.


Figure 13. Effect of magnetic field on radius of $52 \mathrm{M}_{\mathrm{C}}$ stellar envelope


Figure 14. Detail of figure 13 showing extent of convective stability.


Figure 15. Mass structure of $52 M_{\Theta}$ envelope with and without magnetic field.

This brings us to a moment of reckoning. Do we really believe that an envelope magnetic field will balloon a star to more than 50 times its unperturbed radius when hydrostatic equilibrium is being maintained principally through the magnetic field pressure? More specifically, what can we say about the stability of this bloated envelope? We can address three possible modes of instability in our model. Some instabilities which may serve to deflate this model are: 1) the leakage of magnetic flux out of the outer envelope faster than it can be replaced from below, 2) the possible instability of the outer envelope against themal convection, and 3) possible effects of material streaming along magnetic field lines. The first two of these considerations may be properly treated in the context of our one dimensional model.

The functional form of the rum of $B$ with $M$ was developed in chapter 3 assuming that the flux tubes making up the envelope field are unstable to bending as described by Gilman (1970). We expect to see Gilman's instability as long as the timescale for magnetic diffusion in Ionger than the thermal timescale of the zone. We can compare the thermal timescale with the magnetic diffusion timescale at all places in the stellar envelope with the result (figure 16) that in both the condensed and the expanded models, the thermal timescale is always far shorter than the magnetic diffusion timescale.

We can test for instability against thermal convection through Schwarzchild's criterion. Our model was developed under the assumption of radiative equilibrium since the effects of a convection zone on a magnetic field are quite complicated. However, if after


Figure 16. Comparison of magnetic and thermal timescales for $52 \mathrm{M}_{\mathrm{O}}$ star.
calculating this model, it should fail the Schwarzchild stability criterion, the results must be taken with a grain of salt. In figure 14 we have plotted a dashed line at the radius at which the radiative gradient first exceeds the adiabatic gradient in the expanded models. Beyond the left end of the dashed line, the entire model registers as unstable. The physics of these model zones is quite uncertain. Convection may destroy part of the field and deflate the model to this radius. A sufficiently strong magnetic field may however stabilize the forming convection zone. Because the density in this region is very low and the radiative gradient just barely exceeds the adiabatic gradient, the convection will be very weak if it exists at all. The third instability that we must consider is that caused by material streaming along the magnetic field lines in the expanded model. This is an effect that we cannot fully consider in the context of a one dimensional model since the magnetic field does not produce a true scalar pressure. What we can show in terms of a one dimensional model is that sufficient energy may exist in a star with a photospheric magnetic field of only a few hundred gauss to support the outer envelope in a greatly distended configuration. While the magnetic pressure is trying to expand the envelope, the expanded material will be attempting to stream dow the field lines to its original position. If the field is badly convoluted, this streaming may be strongly inhibited and the photosphere may assume some radius not far from its calculated radius.

## Conclusions

We have found that a strong, global magnetic field provides a viable mechanism for broadening the extreme upper main sequence. For photospheric field strengths between 40 and 500 gauss, we find a multiplicity of acceptable models. This arises from the fact that as the photospheric temperatures begin to decline, the opacity rises sharply, amplifying the ballooning effect of the magnetic pressure.

While stellar magnetic fields are inherently multi-dimensional, our calculations provide an accurate account of the energy balance in these bloated models, hence they indicate a range of possible behavior. Streaming along field lines, a complete consideration of convection, and a multi-dimensional treatment of the field structure itself well certainly lead to adjustments in these results but they should not alter the qualitative behavior.

Our results differ from those of chapter 3 for intermediate mass stars in producing a much more drastic effect. The ballooning sets in much more suddenly in the very massive stars than for the intermediate mass stars and expands the bloated model to a far larger radius. On the other hand, wheras chapter 3 admits a collapsed model of about $20 \%$ larger radius than the zero field model, the collapsed model for these massive stars is virtually indistinguishable from the zero field model. This difference probably stems from the fact that for very massive stars, both the temperature and pressure of the photosphere are higher than for stars of $2-5 \mathrm{M}_{0}$. The higher gas pressure requires a larger magnetic pressure perturbation to produce a given effect, and the higher temperature delays the onset of the
opacity amplification effect described earlier. When the magnetic pressure perturbation is large enough to rival the gas pressure and the opacity amplification is effective, the larger magnetic energy density than in the intermediate mass stars causes a much more dramatic expansion.

Our results differ from Strothers' (1980) principally because we calculated models with different ratios of magnetic to gas pressure, including those where the magnetic pressure far exceeds the gas pressure. By doing this we identified a family of highly distended models that Strothers never found. The other differences between Strothers' models and ours, the difference in the functional form of the dependence of the magnetic field with mass and the fact that we only calculated envelopes are probably insignificant in comparison.

We would expect the bloated models to be those assumed by stars whose envelopes are prevented from collapsing all the way to the condensed model by their magnetic fields. If convection sets in, enough field may be expelled from the envelope to allow it to settle to a configuration similar to the one described by the dashed line in figure 14. This scenerio provides, as a natural consequence, the existence of a weak convection zone in the envelopes of 0 type stars. This convection could provide the accoustical energy input necessary to drive the observed 0-star coronas.

A more precise modeling of this magnetic ballooning effect in very massive stars will require a better understanding of the interaction of magnetic fields with regions which are marginally
unstable to convection, and a knowledge of the efficiency of streaming along magnetic field ines. At that point, a more detailed treatment of this problem will be worthwhile. Unitil then, we have established that a photospheric magnetic field of $>50$ gauss on a very massive star may contain enough energy to place that star on a significantly different horizontal position on the H-R diagram than a star with no magnetic field.

## CHAPTER 5

RADIATIVE CORE STARS

Standard evolutionary models of A type stars indicate that they have convective cores, as determined by the Schwarzchild criterion. It has also been shown that convection either on the main sequence or during pre main sequence collapse may be an efficient mechanism for confusing and finally destroying any primordial magnetic field in the star. However, a sufficiently strong magnetic field may stabilize the star's convection and in doing so prevent the destruction of a strong primordial field (Moss and Tayler 1970).

In this chapter we present the results of evolutionary calculations designed to examine the observational aspects of $2-5$ $M_{0}$ stars with their core convection suppressed, as might be expected from a strong primordial magnetic field. Evolutionary tracks for very massive stars ( $5-60 M_{\Theta}$ ) have been published (Strothers and Chin 1973) concluding that the hypothesis that all stars have radiative cores can be ruled out. However, this conclusion is only valid for the extreme upper main sequence and for the proposition that all stars, not just a fraction of the population, have radiative cores. Further, Strothers and Chin present only evolutionary tracks, not detailed isochrones which can be compared directly to observational data. By calculating isochrones of both radiative and convective core
models we can address the question of how to identify stars with radiative cores even if they make up only a few percent of all stars.

## Calculations

We will assume that we are dealing with a magnetic field energy density higher than the convective kinetic energy in to core of the star, yet with the magnetic pressure far smaller than the star's gas pressure. Thus the only major effect that we will see will be the suppression of core convection. To actually carry out the calculations we will use the stellar interiors code described by Eggleton (1971) with $X=0.7, Z=0.02$ and Cox-Stewart opacities as for a normal population I star. We will adopt $\alpha=1.5$ although the exact mixing length should be quite unimportant for these mostly radiative stars. Because we are interested only in gross changes in the star's structure, we will follow only the hydrogen to helium burning. To insure that we are indeed seeing changes brought about by the suppression of core convection, we allow the possibility of the star's development of a convective envelope as it evolves. In both cases, the radiative and the convective core stars, we will generate enough models of various masses to construct fairly detailed isochrones of clusters with turn-off masses of $2 \mathrm{M}_{Q^{\prime}}, 3 \mathrm{M}_{0}$, and 5 $M_{G}$. In this way we will have results that can be compared directly to cluster H-R diagrams.


Figure 17. Evolutionary Track for $3.0 M_{C}$ radiative and convective core models.

## Results

We find that the models with radiative cores have evolutionary tracks that differ substantially from those of standard convective core models (figure 17 for $3.0 \mathrm{M}_{0}$ ). In agreement with Strothers and Chin (1973), zero age main sequence (ZAMS) models with radiative cores are cooler and less luminous than their convective core counterparts (point $A_{C}$ and $A_{r}$, figure 17). As the convective core models burn their core hydrogen they evolve to the right on the H-R diagram until their core hydrogen is depleted (point $B_{C}$ ) then they jump onto a sub-giant branch of thick shell hydrogen burning on a themal time-scale (point $C_{C}$ ). The radiative core models on the other hand, deplete their central hydrogen at a very young age (point $B_{r}$ ) and evolve much more vertically on the H-R diagram. This occurs because an effective thick shell burning stage is established at point $B_{r}$, almost at the ZAMS stage. As the model evolves, this thick burning shell is becoming thinner until point $C_{r}$ where the hydrogen burning shell in the radiative core model is the same size as the hydrogen burning shell in a convective model which has just exhausted its core hydrogen. From this point on, the evolutionary tracks of the two models are almost identical, though the radiative core star is 3.0 x $10^{8}$ years old at point $C_{C}$ and the radiative core star is only 2.5 $\times 10^{8}$ years old at point $C_{r}$.

In contrast to Strothers and Chin (1973), we find that for stars of 2-5 $M_{0}$, the vertical evolution of radiative stars places them at approximately the same point on the H-R diagram as the horizontal evolution of convective stars of higher mass. When we
present the models as isochrones (figures $18,19,20$ ) we never see a difference in $\log T$ of more than 0.03 between the positions of radiative and convective core stars of the same age before the turn-off point of the radiative stars. In fact, the only obvious differences between the two isochrones are the less luminous subgiant granch of the radiative stars (because radiative subgiants of a given age are less massive than their convective counterparts) and the lack of a "hook" in the radiative core models joining the core burning and the thick shell burning branches. This hook is often missing in observed cluster $H-R$ diagrams (Maeder 1974). A few radiative core stars in a population may serve to hide it.

## Conclusions

In contrast to Strothers and Chin's conclusions for the extreme upper main sequence, we have presented evidence for the range of stars from 2-5 $M_{0}$ that suggests that while the supppression of convection in a star's core has a major effect on its evolutionary track, it produces no obvious observational effects. The only observational effects at all of radiative core stars are the luminosity of the subgiant branch and the absence of the hook between the core burning and the thick shell burning branches. The hook has never been clearly seen and the difference of $\log T=0.03$ between the two models could be entirely masked by the effects of line blanketing (Wolff 1967), rotation (Faulkner, Poxburgh, and Strittmatter 1968), the magnetic field on the stellar envelope as discussed in chapters 2 - 4, or perhaps other processes as well. Thus we feel that we cannot


Figure 18. Isochrones of radiative and convective core models of 2.0 Mo turn-off mass.


Figure 19. Isochrones of radiative and convective core models of 3.0 $M_{0}$ turn-off mass.


Figure 20. Isochrones of radiative and convective core models of 5.0 $M_{0}$ turn-off mass.
rule out the possibility of strong central magnetic fields in
intermediate mass stars on evolutionary arguments alone.

## CHAPTER 6

## NEUTRON STARS

Surface magnetic fields are measured in some normal stars and in some collapsed remnants, white dwarfs and neutron stars. Each type . of object has a distinct magnetic class with well ordered fields of approximately $10^{3}, 10^{7}$, and $10^{12}$ gauss respectively (Landstreet et al 1975, Angel et al 1981, and Fuderman 1972, respectively, for example). These magnetic stars are all threaded by about the same flux, $10^{24}$ maxwells, and all have about the same small ratio of magnetic to binding energy ( $10^{-6}$, Woltjer 1975). This fact suggests that the degenerate remnants that are magnetic may simply be those that had magnetic progenators, and that magnetic flux was conserved during the collapse (Ruderman 1972). For a highly conductive plasma conserving flux, the field strength varies as the inverse square of the radius.

The possibility that white dwarfs are formed from magnetic main sequence stars, of type $A$ and earlier, has been explored by Angel et al (1981). The space densities of the two types of object, Ap stars and magnetic white dwarfs, are consistent when lifetimes are taken into account. A recent study of white dwarfs in early clusters with known high mass progenators, did not find any with magnetism (Angel and Stockman 1981). This suggests that factors other than progenator mass determine the magnetism of white dwarfs.

In this chapter we examine the observational evidence which can be used to study the incidence of magnetism in neutron stars to find out how progenitor properties may determine magnetism and to check consistency with fossil field origins.

At the outset, we must realize that there are believed to be at least two possible ways in which neutron stars may be formed (Hoyle and Fowler 1960). The first of these processes involves the core collapse of massive stars and is associated with type II supernovae. Objects of this kind must be associated with very young stars, since only very massive stars can form supercritical iron cores, and should be associated with the galactic plane. The other birth process involves the ignition of a degenerate core. When the core becomes more massive than the Chandrasekhar limit, it will in some way readjust its structure, a type I supernova, and possibly leave behind a neutron star. These neutron stars will not be associated with any age star in particular, and do not require high mass progenitors. In fact they need not even be population I objects since their white dwarf progenitors may have formed billions of years ago.

A second factor that complicates the problem is the possibility that the surface field of magnetic neutron stars may decay rather quickly. White dwarf field decay times are as long as their (rather long) cooling times, so the field will be seen as long as the stars themselves are hot enough to be seen (Fontaine, Thomas, and Van Horn 1973). However, in neutron stars, the crust magnetic field may be effectively decoupled from the interior and decay in only $2 \times 10^{7}$ years (Flowers and Fuderman 1977).

There is yet another difficulty in stuaying the incidence of magnetism in neutron stars, that is in identifying non-magnetic ones. Thermal radiation from the surface of a hot neutron star has not yet been unambiguously identified (Helfand 1980). We therefore must rely on indirect evidence for their existance at all. Neutron stars are found in x-ray binaries, supernova remnants, and as pulsars. Observational evidence from these three areas are reviewed in the next three sections.

## X-Ray Binaries

The emission from $x$-ray binaries is generally believed to be the energy liberated by matter falling into the $\sim 100$ Mev per nucleon potential well of a neutron star component. There are strong arguments that this material comes from the Roche lobe overflow of an evolving companion (Petterson 1978 for example). Since the overflow rate depends only on the evolution of the main sequence or subgiant companion, we should expect that the total luminosity should be independent of the possible existance of a strong magneitc field as long as the particle energy density exceeds the magnetic energy density at the critical point. The only effect of the field may be to redirect the inflow to the polar regions, probably causing the pulsed x-ray emission when the magnetic pole is mis-aligned with the rotation axis.

Using the CGS catalog (Bradt, Droxey, and Jernigan 1979) we can make up a sample of young x-ray objects by selecting those optically associated with 0 and B type stars (table 1) and one of

TABLE 1

## YOUNG X-RAY BINARIES

| Source | Name | Pulse Period | Optical Counterpart |
| :---: | :---: | :---: | :---: |
| 0114+650 |  | non pulsator | B0.5 IIIe |
| 0115-737 | SMC $\mathrm{X}-1$ | 0.75 | BO I |
| 0115+634 | transient | 3.6 s | B -em |
| 0352+309 | $X$ Per | 835 s | Be, 09.5(III-V)e |
| 0532-664 | LMC $\mathrm{X}-4$ | non pulsator | 08 III-V |
| 0535+262 | transient: | 104 s | B0.5 Ve |
| 0900-403 | Vela $\mathrm{X}-1$ | 283 s | B0.5 Ib |
| 1118-615 | transient | 405 s | Be |
| 1119-603 | Cen $\mathrm{X}-3$ | 4.8 s | 06.5 II-III |
| 1145-619 |  | 297 s | Bl Vhe |
| 1223-624 | GX 301-2 | 699 s | B1.5 Ia |
| 1258-613 | GX 304-1 | 272 s | B6-9pe |
| 1538-522 |  | 529 s | BO I |
| 1653-407 | V 851 Sco | non pulsator | BO Iae |
| 1700-377 |  | non pulsator | 05.5 f |
| $1956+350$ | Cyg $x-1$ | non pulsator | 09.7 Iab |

## TABLE 2

## GLOBULAR CLUSTER X-RAY OBJECTS

| Source | Name | Comment |
| :--- | :--- | :--- |
| $0512-401$ | NGC 1851 | Burster |
| $1730-333$ | Liller 1 | Burster |
| $1745-203$ | NGC 6440 |  |
| $1746-370$ | NGC 6441 | Burster? |
| $1820-303$ | NGC 6624 | Burster |
| $1850-087$ | NGC 6712 | Burster |
| $2127+119$ | NGC $7078 / \mathrm{M15}$ |  |

population II objects by selecting objects in globular clusters (table 2). We could obtain a larger sample by using galactic bulge objects as a population II sample but we would have to deal with the possibility of the sample being contaminated with brighter, more easily visable foreground objects.

For the young objects, we will take pulsation as an indicator of a strong magnetic field ( ${ }^{-1} 0^{12}$ gauss, Ruderman 1972), for the population II objects we will take bursting activity as evidence of a weak field ( $<3 \times 10^{12}$ gauss) following Joss (1978). This overlap in the field strength cutoff for the two groups may cause problems if the magnetic field of all neutron stars is in the overlap range. However, since we night expect at least an order of magnitude difference in field strength between the magnetic and non-magnetic varieties of neutron stars (by analogy with Angel et al 1981) this overlap should not affect our results significantly. The immediate conclusion is that about two thirds of the young sample is magnetic by this criterion and two thirds of the old sources are not magnetic. The data are consistent with the idea that massive objects leave magnetic remains more often than population II objects. Either the massive objects start out with fields more often, or old ones lose them on a timescale comparable to the age of the universe. Unfortunately, we cannot rule out the latter possibility since Flowers and Ruderman (1977) find a magnetic field decay time of only $2 \times 10^{7}$ years for a neutron star crust. However, they stress that their decay model only considers the average surface dipole componant of the field. Their decay model does permit magnetic field lines connecting
the neutron star with the interstellar medium to remain. If these open field lines can cause the channeling effect described by Joss and Li (1980) the magnetic field may still supress bursting. Also, the Flowers and Ruderman paper presents a model in which the final result is a neutron star with most of the original flux trapped in the internal superfluid region instead of a truely flux-less object. Any small structural readjustment in the star may cause a fraction of this trapped flux to again permeate the crust for another $2 \times 10^{7}$ years. Thus, while certainly important, flux decay in the crust may not make all old neutron stars apparently non-magnetic.

In any event, the x-ray binaries are consistent with the proposal that the young neutron stars are fundamendally different than the old ones. This difference may come from crust magnetic field decay of the old neutron stars or from a difference in the magnetic field strengths of the progenitors.

## Supernova Remnants

Another source of evidence for non-magnetic neutron stars might come from the thermal detection of single neutron stars in supernova remnants. The Einstein x-ray observatory has been used to search for hot neutron stars in young supernova remnants with four possible detections in nearly fifty observations (Helfand 1980). These data are used to conclude that either the majority of supernovae do not leave neutron stars or that the neutron star cooling rates are much higher than expected. Of the four possible detections, (table 3) two of them are known pulsars, RCW 103 and 3C58 being the only good

## TABLE 3

## POSSIBLE DETECTIONS OF THERMAL EMISSION FROM NEUTRON STARS

| Supernova Remnant | Age | Pulsar? |
| :--- | :---: | :--- |
| 3C 58 | 795 y | No |
| Crab | 926 y | Yes |
| RCW 103 | 2000 y | No |
| Vela | $10,000 \mathrm{y}$ | Yes |

candidates for a neutron star that is not also a pulsar. Taken at face value, these data are hard to reconcile with the premise that there are a large number of non-magnetic neutron stars unless the non-magnetic objects cool much faster than the pulsars, although if appears that just the opposite should be the case, since a large magnetic field has the effect of lowering the opacity (Baym and Pethick 1979).

Nevertheless, as Helfand points out, there exists an indicator of whether a supernova remnant contains a pulsar, independant of any possible direct detection or mis-alignment of beaming. This is the fact that in both cases of a known pulsar associated with a supernova remnant, we see an extended x-ray synchrotron nebula. Five of the seven historical supernova remnants do not show such a spectrum at a radiated power level of $10^{-2}-10^{-3}$ times the energy of the Crab pulsar (Helfand 1980). From this, we can say that of a sample of seven possible neutron stars (the historical supernova remnants) one is a known pulsar and five are known not to be pulsars.

This consideration takes into account all supernovae whereas the binary $x$-ray pulsar route considers.type I and type II supernovae seperately. If type I and type II supernovae have wildly different rates, or if one and not the other leaves remnants, then we are left with the possibility of severe sample contamination. According to Wheeler (1980) the ratio of type II to type I supernavae in our galaxy is likely from 1:1 to 1:4. Therefore we can expect that fom three to five of the historical events were type $I$. Since the type I events are presumed to come from low mass progenitors, we might assume that
they have the same mix of magnetic to non-magnetic objects as the white oivarfs, quoted as $20: I$ in favor of the non-magnetic objects by Angel et al (1981). Thus of the three to five type I remnants in our sample, we might resonably expect them all to be non-magnetic. Therefore, of the two to four type II remnants left, probably one or two are magentic, suggesting again that a large fraction of type II supernovae born neutron stars are threaded by a strong field.

## Pulsars

If we adopt the coustomary evolutionary sequence: main sequence star $\Rightarrow$ supernova $\Rightarrow$ neutron star or pulsar, then we can compare birth and death rates of these objects to get an idea of how many dead main sequence stars are not accounted for as pulsars. These not accounted fo as pulsars may be considered as candidates for neutron stars without magnetic fields. This comparison of birth and death rates has been carried out a number of times (eg. Taylor and Manchester 1977, Endal 1979, Shipman and Green 1980, Hills 1980). Some of the derived galactic birth and death intervals are show in table 4. From these data, it is apparent that there is a great deal of uncertainty in the rates but even so there is not a great deal of room for unseen neutron stars.

Most of this uncertainty centers on the pulsar birthrate. Published galactic pulsar birth intervals range from 6 years (Taylor and Manchester 1977) to 90 years (Hanson 1979) depending on the interstellar electron density $\left\langle n_{e}\right\rangle$ and the statistical treatment of the pulsar data themselves. As Shipman and Green (1980) point out,

## TABLE 4 <br> BIRTH AND DEATH RATES OF SUPERNOVAE, SUPERNOVA PROGENITORS, AND PULSARS

Author Progenitor Death Interval
Supernova Interval
Pulsar Birth Interval
Tayler \&Manchester- (1977)
Hanson (1979)
Endal (1979) $>4 y\left(M>4 M_{0}\right)$
Hills (1979) $\quad>21 y\left(M>4 M_{\complement}\right)$
Clark \& ..... 150y
Caswell (1965)
Tammann ..... 25y
(1974)
Ilovaisky \& ..... 50y
Lequeux (1972)
the expected death rate of pulsar progenitors may be set at practically any desired value by adjusting the mass range of neutron star progenitors. There are practical limits to this freedom however. While we might provide enough progenitors for one pulsar every 6 years by asking that all stars of $M>4 M_{0}$ die and become pulsars, mass loss may prevent such an occurance. Indeed the most compelling evidence in favor or a high lower mass limit on neutron star progenitors may be that of Romanishin and Angel (1980) when they determine that it is nearly certain that some stars of $M>5 M_{0}$ become white dwarfs and that a probable upper limit for white dwarf formation is $7 \mathrm{M}_{0}$. This I imitation can be weakened somewhat however by postulating that type I supemovae with low mass progenitors can form neutron stars. Since type I events have low mass progentiors, the relevant comparison may be with the death rate of binary stars above some critical mass rather than with the death rate of massive single stars.

A more serious problem with a high pulsar birthrate may come from the supernova rate. The highest supernova rate reported (Tamman 1974) for events in other galaxies is one event every 25 years, a rate inconsistent with the very highest pulsar birth rate. It appears Iikely that we must either discount the very highest pulsar birthrates because of their inconsistency with progenitor death rates and the supernova rate or postulate that some other effect is in operation; if pulsars are recycled, the apparent pulsar birthrate is of little consequence.

If we consider lower pulsar formation rates, the observations fit together much more easily. At the extreme lower end of the published birth rates, one every 90 years (Hanson 1979) there are plenty of main sequence progenitors, even if a star must have $\mathrm{M}>10$ $M_{0}$ to become a pulsar (Shipman and Green 1980) and the pulsar birth rate agrees well with the supernova rate. With this low birthrate, there is even room to postulate that there are more supernovae than pulsar births, if there is a supernova every 50 years and a pulsar birth every 90 years, every other supernova may bear a non-magnetic neutron star.

There are a couple of possible conclusions to be drawn from this discussion. The first thought may be that the data themselves may be so uncertain that it would be meaningless to try to extract a conclusion. There is a clear conclusion from this exercise however, that is that a significant fraction of neutron stars are magnetic. it is impossible to say whether this fraction is $50 \%$ or $100 \%$ though. While we cannot show any positive evidence for non-magnetic neutron stars from birth and death rate statistics, the uncertainties will not allow us to exclude the possibility that as many as $50 \%$ of all neutron stars are non-magnetic.

## Conclusions

The data support the possibility that neutron star magnetic fields are fossils that reflect the magnetic field that threaded their progenitors. This will provides the low fields necessary for the bursters, since as population II objects, they may have been born in
type I supernovae or low mass progenators. This provides for the substantial fraction of magneitc pulsators among the $x$-ray binaries containing 0 and $B$ type stars, and it provides for the relatively low ratio of magnetic objects among supernova remnants because a large fraction (50-75\%) are results of Type I events.

If we were to assume that all neutron stars were born magnetic, and that the fields were to decay in $2 \times 10^{7}$ years to poduce bursters, then the non-magnetic SNRs would have to be explained as events in which the progenitor was totally destroyed. This is not very attractive because we would expect to see more of the iron peak elements produced by type I supernovae than are actually observed (Ostriker, Richstone, and Thuan 1974).

The possibility of a link between the magnetic properites of neutron stars and the magnetic properties of their progenitors is always an inportant question involving the fundemental magnetic structure of all stellar objects. A resolution of this question, either yes or no, is essential to the understanding of magneitc fields in all stars.

## CHAPTER 7

## CONCLUSIONS

We have presented a method to combine the phenomona of Ap stars, red giants with anomolous CNO isotope and abundance ratios, the breadth of the extreme upper main sequence, magnetic white dwarfs, and x-ray pulsators and bursters under one roof. This hypothesis, that stellar objects come in two flavors, one without a detectable magnetic field and one threaded by $10^{24}$ maxwells, has come through the five experiments just described without any serious difficulty. This brings us to an additional point that must be raised. Have we defined an effect whose impact on stellar structure can be determined, one which makes definite predictions on observed attributes, or have we merely invented a clever catch-word which can be invoked at will and used to explain anything? What are the crucial experiments and how might their results disprove the dual population hypothesis?

We have at all times considered the magnetic field in a star to be in a non-relaxed, non-force-free configuration. Schussler and Pahler (1978) strongly suggest that this is true at any point that can properly be called the interior of a star of $M>2 M_{O}$. We have assumed throughout a sufficient particle density that forces from magnetic field lines will be thermalized and may reasonably be represented as scalar pressures. This assumption need not always hold, when it doesn't, a multi-dimensional analysis may be indicated.

Most importantly, we have predicted more observable properties than we have inserted free parameters. To the extent that these predictions are born out the hypothesis has strengths, to the extent they fail, the hypothesis has weaknesses.

The other question inevitably raised is what justification, apart from Angel et al's (1981) observation do we have to postulate a dual population of stars rather than a smooth gradient of field strengths. There is certainly no problem in enclosing enough magnetic field lines in a collapsing protostellar gas cloud. Even the most modest magnetic field threading a cloud of one light-year diameter will result in an unacceptably strong field for any star when compressed to a few solar diameters. Thus the problem may not be with having a field in the first place but with getting rid of it, and the efficiency with which it is expelled. The popular wisdom holds that the bulk of this flux slips out of the collapsing cloud before the cloud becomes ionized and locked to the flux lines. Any flux remaining after the cloud does become ionized is assumed to be destroyed during the completely convective Hayashi phase of the star's collapse toward the main sequence.

However some spherical protostar models (e.g. Larson 1972) show that protostars above a certain mass never pass through a phase in which they are fully convective. For Larson's stars this cutoff is at about $1.5 M_{0}$ though it would be easy to believe that rotation and a magnetic field would alter this number. If a star begins core hydrogen burning with its primordial field intact, the core convection would tend to reduce the field to the equipartition value. Its
efficiency in doing so would be the result of a race between the destruction of the field by convection and the creation of a non-convective helium rich core. This mechanism may allow the star to develop a core field strength larger than the equipartition value but smaller than the flux conservation value.

Given this scenerio, the cutoff mass for total convection should certainly depend on the strength of the premordial field because convection in a protostar that is just barely convective should be more easily suppressed than in a strongly convective one. In this case, we should see a gradient of the fraction of magnetic stars as a function of mass. There should be a larger fraction of magnetic $5 M_{0}$ stars than $2 M_{0}$ stars. By using the existence for lack thereof) of Ap characteristics as an indicator of the presence (or lack thereof) of a magnetic field, this study could be done with stars much fainter than for which it is feasible to measure magnetic fields directly.

Another scenerio would have the core magnetic flux be bouyant on a thermal timescale as described by chapter 1 during a radiative protostellar collapse. In this case, the flux that we see in a main sequence star is primordial, but rooted in the inner envelope, above the core convection zone.

Another question to ask may be what initial molecular weight gradient exists in the core at the time of a star's entry onto the main sequence. If the center of the core is somehow stabilized against convection, either by a molecular weight gradient or a rigid magnetic field, the high temperature dependence of the CNO energy
generation rate and the somewhat expanded radiative center may combine to put the entire core into radiative equilibrium.

To conclude, the existance of a strong, possibly primordial magnetic field in some fraction of all intermediate to high mass stars provides an attractive possibility for associating several peculiar objects in an evolutionary sequence. These objects include magnetic white dwarfs and neutron stars, anomolous CNO isotope and abundance ratios in some red giants, the possibly anomolous size of Ap stars, the dispersion of the extreme upper main sequence, and accoustical energy for coronas on 0 type stars. I have presented some possible effects of this dual population and I have proposed experiments to explore other possibilities.

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