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The galactic center stellar population in the near infrared

Haller, Joseph William, Ph.D.

The University of Arizona, 1992

by<br>Joseph William Haller

A Dissertation Submitted to the Faculty of the DEPARTMENT OF ASTRONOMY

In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

THE UNIVERSITY OF ARIZONA GRADUATE COLLEGE

As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Joseph William Waller
entitled THE GALACTIC CENTER STELLAR POPULATION IN THE NEAR INFRARED
and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy


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SIGNED:


## ACKNOWLEDGEMENTS

I would first like to thank Marcia Rieke for introducing me to Galactic Center research and her continuing support through resources, conferences, encouragement, and many opportunities working with the near-infrared cameras. She had already begun the LPV study when I signed on with her. George Rieke provided numerous useful comments and suggestions in the development of this project as well as some timely words of encouragement. The both of them of course must receive a large measure of credit and gratitude for suggesting the CO study, travelling to Chile in June, 1989 and interrupting their excursion to Machu Picchu to obtain the data presented in chapter 5 . Jay Frogel is to be thanked as well for obtaining the $L$ data at CTIO.

The committee composed of Marcia Rieke, Rob Kennicutt, John Black, and Ed Olszewski offered many useful comments and criticisms that sharpened my understanding of the Galactic Center. Any remaining errors or omissions are, of course, entirely my own.

Thanks go to Adam Burrows who taught me a great deal of foundational astrophysics; R. Elston, B. Jannuzi and others for introducing me to IRAF; E. Olszewski and R. Schoemer for help with DAOPHOT and I. Barg for installing and supporting it on the IRCAMERA computer; L. Paley for helping to reduce the $H$ data; many praises to Jim Speer for doing the L photometry and learning database reductions; Robin Catchpole and M. Feast of the SAAO for their expertise in Mira variables, and additional thanks to Robin for many good times during his stay at Steward; Fulvio Melia for offering his comments on the work and giving me more opportunities in Galactic Center research.

My gratitude also goes to Scolt Grossman, Todd Henry, Davy Kirkpatrick, Hans Walter Rix, Demis Zaritsky, and Diana Foss who were the classmates that helped make graduate school enjoyable. Bill and Diane Latter, Doug Kelly, Kim and Brian McLeod, Karen Visnovsky, Jim Bollinger, Barry Meyers-Rice, Tom Green, Rex Safford, Pat Hall, Laird Close, Crystal Martin, and Milagros Ruiz - who have been good friends and/or Steward office mates and have provided uncountable occasions of learring while I have been in Tucson; Jill Kirby for her valuable military insights while my father was on duty in the Gulf War. Thank you James Lowenthal for your friendship and being a groomsman at our wedding. Karen and I enjoyed many barbecues with Joe and Pemny Hora, and Joe planted the seed for the "cell" matching procedure in chapter $2 . .$.

To some special Tucson friends: Mark, Teresa, and Kathleen Aronson, Alexis Stanton and "Little" Alexis; Fr. Michael Sherwin, O.P., Prof. A. Pacholezyk, Prof. M. Hewlett and others at the St. Albert Forum....

To Bishop Joseph Hart, my mentor in life; my lifelong friends Joe Swiatek, Bill O'Donnel, James Schumaker, Sheryl and Aaron Van Pelt; Karen's parents, Spike and Margaret Ann Hladky - for all you have given me and your support...

To Mom, Dad, Jim, Tom, Bob, Karen, and Janet - a part of each of you is in me and this would not have happened without any of you, for countless reasons...
... and most of all to you Karen, for your love, patience, and support through the many ups and downs we have had in the last few years. Your faith in me (and occasional prodding) was indispensible.

## To

## Karen, my wife

## and

## Mark, our son

As the geometer who tries so hard to square the circle, but cannot discoover, think as he may, the principle inooloed,
so did I strive with this new mystery:
I yearned to know how our image fit into that circle, how could it conform;
but my own wings could not take me so high then a great flash of understanding struck my mind, and suddenly its wish was granted.

At this point power failed high fantasy but, like a wheel in perfect balance turning, I felt my will and my desire impelled
by the Love that mooes the sun and the other stars.

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#### Abstract

Near infrared photometry is presented for 659 stars within $2.5^{\prime}$ of the Galactic Center from three epochs of $2.2 \mu \mathrm{~m}$ and one epoch of $1.6 \mu \mathrm{~m}$ observations. The data were taken with a Rockwell $64 \times 64 \mathrm{HgCdTe}$ IR array at the Steward Observatory 1.6 m telescope and are complete to $\mathrm{m}_{\mathrm{k}}=11.0$. Additional photometry at $3.4 \mu \mathrm{~m}$ for a subsample of 258 stars taken with a $58 \times 62$ InSb array at CTIO is also included. The sample has an average ( $\mathrm{H}-\mathrm{K})_{\text {OBS }}$ which is consistent with a late-type population with $(\mathrm{H}-\mathrm{K})_{0}=0.3$ and $\mathrm{A}_{\mathrm{V}}=31.8 \mathrm{mag}$, and luminosities several magnitudes brighter than the Baade's Window AGB. A variability analysis shows 59 stars are long period variable candidates at the $3 \sigma$ level. These stars show correlations in $(H-K)_{0}$ vs. ( K $\mathrm{L})_{0}$ and ( $\left.\mathrm{K}-\mathrm{L}\right)_{0}$ vs. $\Delta \mathrm{m}_{\mathrm{K}}$ consistent with Mira variables on the AGB. The LPV luminosities are consistent with a population $\mathrm{few} \times 10^{8} \mathrm{yr}$ old which is older than the Galactic Center central cluster stars and younger than bulge stars in Baade's Window, implying the Galactic Center has repeating episodes of star formation. Observations of the $C O(v=2 \leftarrow 0) 2.3 \mu \mathrm{~m}$ absorption feature taken at the CTIO 4 m with the IR Spectrometer Array of the unresolved stellar emission at 9 positions within $30^{\prime \prime}$ of $\mathrm{Sgr} \mathrm{A}^{*}$ are presented. It is shown that the CO band strength is highly correlated with $\mathrm{r}_{\mathrm{Sgra}^{\prime}}$ for radii less than $R_{\text {co }} \approx 8.5^{\prime \prime}(0.28 \mathrm{pc})$. The Galactic rotation curve has a radial gradient of $-16 \mathrm{~km} \mathrm{~s}^{-1} \operatorname{arcsec}^{-1}$ inside $\mathrm{r}_{\mathrm{SgTA}^{*}} \approx 10^{\prime \prime}(0.33 \mathrm{pc})$ where it reaches a minimum. Estimates of the enclosed mass vs. radius show there is $(1.7 \pm 0.3) \times 10^{6} \mathrm{M} \odot$ inside $\mathrm{r}_{\text {sgra }}=5.2^{\prime \prime}$ ( 0.17 pc ) with a mass-to-light ratio on the order of 10 . Assuming the CO absorption samples stellar emission at each projected radius, these results are strong evidence for a massive black hole at the Galactic Center.


## CHAPTER 1 :

## INTRODUCTION TO THE GALACTIC CENTER

## Overview

The work presented here focuses on the stellar population within 5 pc of the Galactic Center. The ability to study these stars with greatly improved detail has recently been made possible by advances in near-infrared detector array technology during the 1980's. For the first time, properties of the individual stars can now be studied in this wavelength region using methods that are common in optical astronomy. Instead of laborious scans made from single element detectors with poor spatial resolution, whole arcminute fields can be imaged at subarcsecond pixel scales. Because of the high density of objects in the central arcminutes, small amounts of observing time produce large amounts of photometric information. The large stellar survey presented here, among the first to be done with the new arrays, necessitated the development of numerous data reduction techniques and database procedures that were innovative to near infrared astronomy.

This chapter gives the motivations for the experiments presented in later chapters through a brief overview of the Galactic Center region and the important objects known to reside there. The observations can then be placed within a broader perspective. Chapters 2, 3, and 4 are collectively a detailed photometric investigation of the brightest stars within the central $5^{\prime} \times 5^{\prime}(10 \mathrm{pc} \times 10 \mathrm{pc})$ of the Galactic Center. Data are presented from observations at $1.6 \mu \mathrm{~m}(\mathrm{H}), 2.2 \mu \mathrm{~m}(\mathrm{~K})$, and $3.4 \mu \mathrm{~m}(\mathrm{~L})$ that show the luminosity function, the color characteristics, and variability for a sample of 659 stars which are then compared to a well studied stellar population in the Galactic bulge to test for the presence of late-type supergiants. A small subset are demonstrated to be long period variables on the asymptotic giant branch (AGB) using photometric techniques. Their presence supports the hypothesis that several episodes of star
formation have occurred at the Galactic Center. Chapter 5 presents spectroscopic data of the $2.3 \mu \mathrm{~m}$ CO absorption feature due to the late-type stars within $30^{\prime \prime}$ of $\mathrm{Sgr} \mathrm{A}^{*}$ (a bright radio point source commonly thought to be the dynamical center of the Galaxy), which are among the highest spatial and spectral resolutions attained for this region. A deficit of CO absorption at the center previously observed by other workers is confirmed and is shown to have a highly correlated spatial distribution. An analysis of the enclosed mass vs. radius from $\operatorname{Sgr} \mathrm{A}^{*}$ is then performed using the velocity dispersion and rotational velocities at six positions from $5.2^{\prime \prime}$ to $10.4^{\prime \prime}$ from $\mathrm{Sg} \mathrm{A}^{*}$. The results, which are in agreement with several other determinations, support the hypothesis that a massive compact object resides at the Galactic Center. Chapter 6 offers some concluding remarks and outlines prospects for further research.

## The Galactic Center Environment

Twenty five years ago the study of the Galactic Center was sedate compared to current research. In the late 1960's two important discoveries by Becklin and Neugebauer (1968) and Low et al. (1969) showed that large amounts of radiation, on the order of $10^{\delta}-10^{7} \mathrm{~L} \odot$, were being emitted in the near- and far-infrared from a region that was only a few parsecs in size. These observations established the significant nature of the Galactic Center and laid the foundations for subsequent studies at progressively higher spatial and spectral resolutions which characterize current research. Unusual ejection phenomena observed in HI gas several hundred parsecs from the center characterized Galactic Center research during the 1960's and 1970's (e.g. Oort 1977) and motivated much scrutiny at smaller length scales. The 1970's and early 1980's saw great progress with the discovery of various gaseous and dust components (i.e. the "molecular ring" and the "minispiral") and demonstrated the very high mass density within the central parsec. However, the massive amount of data accumulated on the central few parsecs during the 1980's and early 1990's (e.g. Genzel and Townes 1987; Morris 1989) as yet gives no
clear connection to these larger scale features. Can a $10^{6} \mathrm{M} \odot$ black hole or massive star formation at the very center explain structures at radii of 1 kpc ? A unified theory is currently lacking.

Recent review articles on this subject can be found in Genzel and Townes (1987) and Morris (1989). More synoptic but dated accounts can be found in Oort (1977), Brown and Liszt (1984), and Riegler and Blandford (1982) as well as Backer (1986). The outline of the Galactic Center given below is not comprehensive but does try to emphasize the important properties of those objects generally regarded as significant contributors to the overall structure, dynamics, and energetics as well as the important constraints on models that attempt to explain this highly complex region.

A distance modulus to the Galactic Center of 14.2 will be adopted which corresponds to a true distance, $R_{0}$, of 6.9 kpc . This value will be assumed unless otherwise noted. It was chosen to agree with Frogel and Whitford (1987) in their study of the late-type stellar population in Baade's Window and is based on the distribution of RR Lyrae variables along the line-of-sight to the Galactic Center (Blanco and Blanco 1985), estimates of the distance to the centroid of the Galactic globular cluster distribution (Frenk and White 1982), and $\mathrm{H}_{2} \mathrm{O}$ maser proper motions in newly formed massive stars near the Galactic Center (Reid et al. 1986). It is lower than the values derived in recent reviews of this subject ( $\mathrm{R}_{0}=8.5 \mathrm{kpc}$, Kerr and Lynden-Bell 1986; $\mathrm{R}_{0}=$ 7.7 kpc , Reid 1989). The next three chapters deal with photometry of Galactic Center stars and this value of $\mathrm{R}_{0}$ allows direct comparison with the Frogel and Whitford (1987) data. Since it may be underestimated by 10 to $20 \%$, the corresponding absolute magnitudes will be systematically too faint by $0.2-0.4$ mag.

The first thing to note about the Galactic Center is its close proximity in comparison with other galactic centers. At a distance of $6.9 \mathrm{kpc}(23,000$ light years) 1 parsec has a projected angle of $30^{\prime \prime}$ when viewed from the solar system. The typical seeing limit of ground based nearinfrared work is around $1^{\prime \prime}$ which corresponds to $0.03 \mathrm{pc}(6900$ a.u.) at the Galactic Center. In
contrast, 1 parsec at the distance of M31, the closest spiral galaxy most similar to our own, will have a projected angle of only $0.4^{\prime \prime}$ and the smallest visible structures will be on the order of 2.5 pc in size. Thus, comparable structures seen in the center of our own Galaxy cannot be resolved by ground based instruments in even the closest spiral galaxy. The problem only becomes worse for even more remote galaxies.

The term "Galactic Center" itself has a variety of meanings for observational techniques that consider different concentric solid angles in the constellation of Sagittarius. For gamma and X-ray astronomy the Galactic Center can include a solid angle of width 5 to $10^{\circ}$, with a projected size of 600-1200 pc! Infrared observers tend to concentrate on a solid angle with a projected size of only 5 pc or less. Such vastly different length scales lead to very distinct topics that has occasionally caused confusion from observational limitations. For instance, the object E1740.7-2942 was recently discovered to be the source of e+e-annihilation line radiation at 511 kev but it is significantly displaced from the dynamical center of the Galaxy by a projected angle of $0.5^{\circ}$ (Sunyaev et al. 1991). The annihilation line radiation was viewed as strong evidence for a massive black hole at the Galactic Center (e.g. Brown and Liszt 1984). But such a large separation between E1740.7-2942 and the dynamical center means that the annihilation line source is irrelevant to the energetics within the central $10 \mathrm{pc}\left(5^{\prime \prime}\right)$ of the Galaxy.

The photometric study in chapters 2 through 4 examines the Galactic Center stellar population within a solid angle of approximately $5^{\prime} \times 5^{\prime}$ with a projected radius of 5 pc . The CO absorption study discussed in chapter 5 examines the unresolved stellar emission from within $30^{\prime \prime}(1 \mathrm{pc})$ of the Galactic Center. The projected radii for some significant objects discussed below are listed in table 1-1 for comparison ( $\mathrm{R}_{0}=6.9 \mathrm{kpc}$ ).

The main physical components within the center are observed or inferred from a wide variety of observational techniques spanning nearly the entire electromagnetic spectrum with the exception of the visible and UV bands. These include a stellar population, a multi-phase interstellar medium of gas and dust, a peculiar radio point source commonly presumed to be
a massive black hole, and magnetic fields. Many of the present efforts in Galactic Center research center on the details of these components and their relative importance.

TABLE 1-1 : Projected Radii From Sgr A* For Galactic Center Objects

| Object | Angular Radius (") | Projected Radius (cm) |
| :--- | ---: | ---: |
| Schwartzschild Radius of | $5.8 \times 10^{-6}$ | $5.9 \times 10^{11}$ |
| $2 \times 10^{6} \mathrm{M}_{\odot}$ Black Hole |  |  |
| IRS 16 C | 1.5 | $1.5 \times 10^{17}$ |
| IRS 7 | 6.5 | $6.7 \times 10^{17}$ |
| Mini-spiral | $1-38$ | $1.0-39 . \times 10^{17}$ |
| Molecular Ring | $35-206$ | $3.6-21.2 \times 10^{18}$ |
| Stellar Survey of Chapters 2-4 | $<150$ | $1.5 \times 10^{19}$ |
| E1740.7-2942 | 1800 | $1.85 \times 10^{20}$ |
| Baade's Window | 13700 | $1.41 \times 10^{21}$ |

The Galactic Center Stellar Population - In General

The stars at the Galactic Center seen in the near-infrared (1-3 $\mu \mathrm{m}$ ) are the dominant mass component (Genzel and Townes 1987) in the region for $\mathrm{R}>1 \mathrm{pc}$. When stellar emission was first observed by Becklin and Neugebauer (1968), several fundamental characteristics were established that remained important in subsequent investigations (e.g. Lebofsky et al. 1982; Allen, Hyland and Jones 1983; Rieke and Lebofsky 1987; Sellgren et al. 1987). There is a central peak in the surface brightness of $2.2 \mu \mathrm{~m}$ emission that is proportional to $\theta^{-0.8}$, where $\theta$ is the projected angle away from the central emission peak. This emission was identified with the Galactic Center from its coincidence with the bright radio source Sgr A (Downes and Maxwell 1966) and the dynamical center as determined by observations of HI in the Galactic plane (Oort and Rougoor 1960). For a stellar population at the Galactic Center with a spherically symmetric luminosity density of the form $\rho . \times R_{C C}{ }^{-\alpha}$, no significant occultation effects from crowding, a constant mass-to-light ratio and uniform foreground extinction, the surface brightness $\mu$ at
projected radius $R$ along the $z$-axis line-of-sight will be given by

$$
\mu=\rho_{=} \int_{-\infty}^{\infty} \mathbb{R}_{G C}^{-a} d z \propto R^{-\alpha+1}
$$

This will reproduce the observed surface brightness profile for $\alpha=-1.8$. The assumption of uniform extinction is consistent with the relatively constant ( $\mathrm{H}-\mathrm{K}$ ) color observed with large apertures across the central few arcminutes (Becklin and Neugebauer 1968). If the intrinsic colors of these stars are typical for late-type stars then the foreground visual extinction to the Galactic Center is on the order of $A_{v}=30$. It is also notable that the flux density of $2.2 \mu \mathrm{~m}$ radiation implies a total luminosity of few $\times 10^{6} \mathrm{~L} \odot$ radiating from within the central few parsecs and translates into a stellar density that is $10^{6}$ times greater than in the solar neighborhood if the mass to light ratio is similar to M31 (Becklin and Neugebauer 1968).

There is a dominant central cluster of stars $\left(\theta<30^{\prime \prime}\right)$ which has been intensively studied (e.g. Becklin and Neugebauer 1975; Lebofsky et al. 1982; Allen, Hyland and Jones 1983; Rieke, Rieke and Paul 1989; Depoy and Sharp 1991). Near-infrared spectra show that many of these stars have strong CO absorption at 2.3 $\mathrm{\mu m}$ (Neugebauer et al. 1976; Treffers et al. 1976; Lebofsky et al. 1982; Wollman, Smith and Larson 1982) which is characteristic of high-luminosity late-type stars. It increases in strength with increasing luminosity and decreasing temperature (e.g. Kleinmann and Hall 1986). The strength of the CO feature in the central cluster stars shows they are supergiant stars. The $2.2 \mu \mathrm{~m}$ luminosity of the central cluster is dominated by the point source "IRS 7" which also has the strong CO absorption (Neugebauer et al. 1976; Treffers et al. 1976; Lebofsky et al. 1982; Wollman et al. 1982; Sellgren et al. 1987). Along with IRS 7, Lebofsky et al. (1982) listed 6 other sources (sources 11, 12, 19, 22, 23, and 24) with late-type supergiant spectra. This strong concentration of supergiants within such a small region ( $R<4 \mathrm{pc}$ ) implies considerable recent star formation activity which will be more extensively discussed below.

The enclosed mass versus radius from the Galactic Center has been determined using radial velocities of individual sources (Sellgren et al. 1987; Rieke and Rieke 1988) and the velocity dispersion of the unresolved background stellar emission (McGinn et al. 1989; Sellgren et al. 1990). These efforts will be discussed in more detail in chapter 5 and an extension of the latter method will be made in this work.

## The Galactic Center Stellar Population - Particular Objects

The objects within $30^{\prime \prime}$ of $\mathrm{Sgr} \mathrm{A}^{*}$ have many unusual features and there is no clear consensus on the relative importance for many of them. The more important objects are discussed here.

- IRS 7 - The bright source IRS 7 has an apparent K magnitude of 6.7; its bolometric magnitude is approximately $-8.0\left(1.3 \times 10^{5} \mathrm{~L} \odot\right)$. Near infrared spectra show it has a luminosity class of M2la (Lebofsky et al. 1982) which implies an age $<10^{7} \mathrm{yr}$. The brightness of this supergiant star is so outstanding ( $\sim 1 / 10$ the total $2.2 \mu \mathrm{~m}$ luminosity in the central parsec) that it was detectable in the initial near-infrared aperture scans of Becklin and Neugebauer (1968). Several features have clearly established that IRS 7 is located near the Galactic Center. An excess of Br $\alpha$ emission was interpreted by Rieke and Rieke (1988) as the ionization of a stellar wind from IRS 7 by ambient UV photons. They placed an upper limit on its distance from the center < 0.6 pc . Recently, radio continuum observations (Yusef-Zadeh and Morris 1991) have revealed a cometary "tail" of ionized gas streaming from the location of IRS 7 in a direction away from the IRS 16 complex and the dynamical center. A theoretical analysis of these observations by Yusef-Zadeh and Melia (1991) modeled the tail as the interaction between the stellar wind of IRS 7 and a much more powerful wind from IRS 16 C or Sgr A*.
- The IRS 16 Complex and Evidence for Massive Blue Stars - The resolved sources known as the IRS 16 complex lie within $1-5^{\prime \prime}$ of $\mathrm{Sgr} \mathrm{A}^{*}$. The dereddened colors of these stars
are notable because they are much bluer than those of late-type stars (Becklin and Neugebauer 1975; Allen, Hyland and Jones 1983; Rieke, Rieke and Paul 1989). This compact cluster has four main components (IRS 16 NE, IRS 16 C, IRS 16 NW and IRS 16 SW) with numerous fainter point sources and related unresolved emission. Initially IRS 16 C and $\mathrm{Sgr} \mathrm{A}^{*}$ were not clearly distinguishable, but astrometric comparisons between radio, mid- and near- infrared positions have shown a separation by about 1" (Allen and Sanders 1986; Becklin et al. 1987; Forrest et al. 1987). The vicinity of IRS 16 shows strong Hel ( $2.06 \mu \mathrm{~m}$ ) emission lines (e.g. Hall, Klienmann and Scoville 1982; Geballe et al. 1991; Krabbe et al. 1991) with velocity components that correlate with the $2 \mu \mathrm{~m}$ continuum of the IRS 16 cluster and are very broad with a velocity width - 1500 $\mathrm{km} \mathrm{s}^{-1}$ indicative of high velocity mass-loss winds from early-type stars (Krabbe et al. 1991). The late-type supergiants and the enigmatic nature of the bluer IRS 16 sources suggest that a considerable population of progenitor blue main-sequence stars is present as well. Imaging such sources is impossible however given severe crowding in the central $20^{\prime \prime}$ and the spatial resolutions that are currently available. This is illustrated in table 1-1 that shows the estimated K magnitudes for a progenitor blue main-sequence $O$ star (Maeder 1991) and a red supergiant (Lebofsky et al. 1982). Figure 1-1 shows an $\mathrm{H}-\mathrm{R}$ diagram with sensitivity curves at various $2.2 \mu \mathrm{~m}$ detection thresholds for stars at the Galactic Center assuming they have black body spectra.

TABLE 1-2 : Apparent K Magnitudes of Luminous Stars at the Galactic Center

| Spectral <br> Type | Luminosity | Effective <br> Temperature | $2.2 \mu \mathrm{~m}$ <br> Flux | K |
| :---: | :---: | :---: | :---: | :---: |
| O7V | $2.5 \times 10^{5} \mathrm{~L} \odot$ | $35,000 \mathrm{~K}$ | 2.57 mJy | 13.6 |
| M2I | $1.0 \times 10^{5} \mathrm{~L} \odot$ | $3,000 \mathrm{~K}$ | 500 mJy | 7.8 |

The observations presented in chapters $2-4$ were made with a Rockwell $64 \times 64$ array at


Figure 1-1: The figure shows the region in the $L$ vs. $T_{\text {eff }}$ diagram accessible by the corresponding sensitivity limits at $2.2 \mu \mathrm{~m}$ of $\mathrm{K}=11,12$, and 13 mag for Galactic Center stars, assuming they have black body spectra. Values of $R_{0}=6.9 \mathrm{kpc}$ and foreground extinction $A_{V}=30 \mathrm{mag}$ are assumed and the position of the zero age main sequence (ZAMS) comes from Mihalas and Binney (1981).
$1.2^{\prime \prime} /$ pixel resolution. The images were confusion limited for sources fainter than $\mathrm{K} \approx 12.0 \mathrm{mag}$ but stars could not be reliably detected from overlapping frames for $\mathrm{K}>11.0$ mag. Thus significant improvements in spatial resolution are needed to detect massive blue main-sequence stars. Such stars may perhaps be inferred by effects on surrounding material or by line emission processes that substantially exceed continuum emission.

There are indications that massive blue stars may be present. Within the thermal gas and dust structure known as the "minispiral" (see below) is a collection of point sources visible at $10 \mu \mathrm{~m}$. The interpretation of these sources had been controversial, either attributed to dust density enhancements (Gatley 1982, 1987) illuminated by Ly $\alpha$ photons, or to embedded stars that locally heat the dust (Rieke, Telesco, and Harper 1978). Shorter wavelength observations (Rieke, Rieke, and Paul 1989) show significant J-band emission from at least three of these sources ( 1,9 , and 13) with two-component infrared spectra consistent with hot stars, of spectral type $G$ or earlier (from the lack of $2.3 \mu \mathrm{~m}$ CO absorption), and locally heated dust. These sources also show the strong Hel emission lines (Krabbe et al. 1991).

## Interstellar Dust and Gas Within 10 pc of the Galactic Center

Far infrared observations show that considerable amounts of reprocessed energy (few $\times 10^{6} \mathrm{~L}$ 。) are radiated by dust in the Galactic Center (e.g. Gatley et al. 1977; Becklin, Gatley and Werner 1982). This radiation is comparable to the observed near infrared emission from the late-type stars (Becklin and Neugebauer 1968). The $30 \mu \mathrm{~m}$ dust emission is peaked at the same location as the $2.2 \mu \mathrm{~m}$ radiation as well as the $50-100 \mu \mathrm{~m}$ color temperature. The emission seen at $50 \mu \mathrm{~m}$ and $100 \mu \mathrm{~m}$ is double lobed on either side of the temperature and luminosity peaks indicating a ring structure for the dust which is depleted in the central parsec. The normalized emission radial profiles broaden with increasing wavelength which shows the dust color temperature and luminosity decrease with increasing radius from the Galactic Center. Such a
configuration raises the possibility of a compact central source dominating the energetics of the inner few parsecs. Early models for the dust heating (Gatley et al. 1977) included a hot luminous central source, identified with $\operatorname{Sgr} \mathrm{A}^{*}$, and the local population of late-type stars. An upper limit for the total luminosity of the region assuming thermal equilibrium for the dust grains and a dust absorption coefficient that varies as $\lambda^{-1}$ (e.g. Spitzer 1978) gives $\mathrm{L}_{\text {ToтлL }} \leq 1$ $3 \times 10^{\gamma} \mathrm{L}_{\odot}$ (Becklin et al. 1982). The dust thermal equilibrium relation, which has the form $T_{\text {dua }}{ }^{5}$ $\alpha L R^{-2}$, was calibrated from known Galactic Hn regions and was applied to the Galactic Center using a grain temperature of 70 K from the highest color temperature at $50-100 \mu \mathrm{~m}$ and a characteristic dust radius given by the lobes at $100 \mu \mathrm{~m}$. Producing this $1-3 \times 10^{7} \mathrm{~L} \odot$ of radiation from a volume only several parsecs in size is one of the important constraints to be satisfied by models for the Galactic Center region.

The Circum-Nuclear Disk - The dust seen at mid- and far-infrared wavelengths is associated with a ring of atomic and molecular gas with a sharp inner edge $35^{\prime \prime}$ from the IRS 16 complex. Molecular hydrogen ( $2.12 \mu \mathrm{~m}$ ) at the inner edge of the disk shows evidence of being shocked by an outward flowing wind with an implied mass loss rate of $10^{-3} \mathrm{M} \odot \mathrm{yr}^{-1}$ (e.g. Gatley et al. 1984; Gatley et al. 1986). Atomic lines of hydrogen ( 21 cm ) (Liszt et al. 1983), and oxygen ( $63 \mu \mathrm{~m}$ ) (Lester et al. 1981; Genzel et al. 1985), as well as molecular lines of $\mathrm{CO}(2.6 \mathrm{~mm})$ (Liszt et al. 1983) and HCN (3mm) (Wright et al. 1987) collectively show a disk that extends to at least $3^{\prime}$ from the dynamical center. It rotates in the sense of Galactic Rotation at $-110 \mathrm{~km} \mathrm{~s}^{-1}$ from the inner edge of the disk out to $1.4^{\prime}$ with turbulent velocity widths of $50 \mathrm{~km} \mathrm{~s}^{-1}$. It is inclined with respect to the Galactic Plane by approximately $60^{\circ}$ at the inner edge and shows evidence of being warped (e.g. Güsten 1987). The disk material has a filling factor of only 5$10 \%$ allowing exciting UV radiation from the central 1.5 pc to penetrate nearly 8 pc from the center and thus heat the dust seen in the far-infrared. Comparisons of CO (Lugten et al. 1987) and atomic (Genzel et al. 1985) line ratios with collisional excitation calculations show gas temperatures of few $\times 10^{2} \mathrm{~K}$ and densities of few $\times 10^{4} \mathrm{~cm}^{-3}$ at the inner disk edge and there is
decreasing excitation, energy density, and gas pressure with increasing distance from the center (e.g. Genzel 1989).

Ionized Gas and the "Minispiral" - Within the molecular ring is the ionized gas distribution known as the SGR A West complex, which is confined to several discrete filamentary structures known as the "mini-spiral" (Lo and Claussen 1983). It has been observed both in the radio continuum (e.g. Ekers et al. 1983; Pedlar et al. 1990) and in the [NeII] 12.8 $\mu \mathrm{m}$ line (e.g. Lacy et al. 1980; Lacy et al. 1982; Serabyn et al. 1988; Lacy, Achtermann and Serabyn 1992). Hot dust is intermixed with the ionized gas and is observable at near infrared wavelengths (e.g. Rieke, Telesco and Harper 1978; Gezari and Yusef-Zadeh 1991). The filaments show systematic velocity changes along their lengths that indicate predominantly orbital motion rather than outflow or inflow. The velocities give an inferred enclosed mass of $1.6 \times 10^{6} \mathrm{M} \odot$ inside a radius of 10 " (Serabyn et al. 1988). It is most likely that the "Western Arc" is the inner ionized edge of the molecular ring and the other filaments are probably infalling clumps that have been tidally elongated. Complicating the analysis of the motions of the filaments is the contribution of the local magnetic field. Only a lower limit of 10 mgauss is known for the field strength (Aitken et al. 1986). The excitation of the [NeII] implies a Ly continuum flux of $10^{51}$ photons $\mathrm{s}^{-1}$ emerging from the central parsec. This Ly continuum luminosity is among the largest for Galactic HII regions (e.g. Wynn-Williams and Becklin 1974) yet the implied ionizing spectrum is among the coolest (Lacy et al. 1980). The ratio of $\mathrm{N}\left(\mathrm{Ar}^{+2}\right) / \mathrm{N}(\mathrm{Ar})<0.5$ for the ionized cloud closest to the center implies that $\mathrm{Ar}^{+2}$ photons (and thus $\mathrm{He}^{+}$photons) ionize less than half its material. This places an upper limit to the Hell ionizing flux which in conjunction with the estimated Ly continuum flux gives an upper limit for the temperature of the ionizing UV radiation field. If it arises from distributed sources in the central parsec then the temperature is less than $35,000 \mathrm{~K}$ and less than $31,000 \mathrm{~K}$ if it is from a dominant central source (Lacy et al. 1980).

Unusual line emission centered about $3^{\prime \prime}$ to the south west of $\operatorname{Sgr} \mathrm{A}^{*}$ at $2.217 \mu \mathrm{~m}$ has
recently been seen by several workers (Allen, Hyland and Hillier 1990; Eckart et al. 1992). Eckart et al. (1992) showed the gas is spatially extended with a velocity FWHM of $500 \mathrm{~km} \mathrm{~s}^{-1}$ and offset from Sgr $A^{*}$ forming an arc around a particular feature in the "mini-spiral" known as the "mini-cavity" which is opposite the IRS 16 complex from Sgr A". They suggested it was either an [Fe XII] transition observed in the solar corona or an [Fe III] transition at 2.2178(20) $\mu \mathrm{m}$. Wardle and Yusef-Zadeh (1992) model the emission as the collisional excitation of stellar wind material originating from IRS 16 that is gravitationally focused around $\mathrm{Sgr} \mathrm{A}^{*}$ and then shocked by its impacting on the "mini-spiral" feature. This interpretation favors an [Fe XII] identification for the line emission since it is likely that the post-shock gas temperature would too hot for significant [Fe III] emission.

## Sgr $\mathbf{A}^{*}$ - A. Massive Black Hole at the Galactic Center?

Lynden-Bell and Rees (1971) first postulated that a massive black hole was present at the Galactic Center and a strong nonthermal radio point source was subsequently discovered by Balick and Brown (1974) and became known as Sgr $\mathrm{A}^{*}$. The above discussion has shown that there are several indications from the dust and gas emissions for a centralized energy source with a characteristic temperature $\sim 35,000$ degrees, a total luminosity on the order of $10^{7}$ L. within a region which has a mass density on the order of $10^{6} \mathrm{M} \odot \mathrm{pc}^{-3}$. Much research has attempted to establish the presence of a central engine (see Genzel and Townes 1987; Gatley 1989) and the unique properties of $\mathrm{Sgr} \mathrm{A}^{*}$ have made it a frequent candidate. The motivations for such an hypothesis are obvious. If we can understand the activities at the center of our own Galaxy through processes involving a massive black hole then perhaps it can serve as a scaleddown version of active galactic nuclei in distant galaxies and quasars.

The observational status of $\operatorname{Sgr} \mathrm{A}^{*}$ has been summarized by Lo (1982, 1987, 1989). If any object can be identified as a massive black hole, as suggested by gas dynamics and stellar
velocities, it is likely to be $\operatorname{Sgr} \mathrm{A}^{*}$ because it has a number of features that are unique within the Galaxy. Pulsar interpretations for this object are ruled out because its low variability amplitude, $\sim 20 \%$, and radio spectral index $\left(v^{\alpha}\right)$ of $\alpha=0.2$ are outside typical pulsar values. It is also more radio luminous than the brightest known pulsar by a factor of $10^{3}$. The radio spectrum is similar to objects found in the nuclei of M81 and M101. The angular size of $\mathrm{Sgr} \mathrm{A}^{*}$ varies as $\lambda^{2}$ for wavelengths longer than 1 mm , consistent with thermal scattering by interstellar electrons.

As mentioned above, the early conjectures that $\operatorname{Sgr} \mathrm{A}^{*}$ and IRS 16 were coincident proved incorrect by the demonstration that these objects are separated by at least $1^{\prime \prime}$ (Allen et al. 1986; Becklin et al. 1987; Rieke, Ricke, and Paul 1989). This left a somewhat confusing picture for proponents of a central engine because there was no detectable near-infrared source at the location of $\mathrm{Sgr} \mathrm{A}^{*}$ consistent with the luminosity and temperature constraints for a central source. Recently there have been several efforts to identify $\mathrm{Sgr} \mathrm{A}^{*}$ with high resolution nearinfrared imaging. Depoy and Sharp (1991) reported observations from a $100 \times 1^{5}$ image at $0.3^{\prime \prime} /$ pixel resolution giving a $3 \sigma$ upper limit on Sgr $\mathrm{A}^{*}$ of $\mathrm{K} \geq 13.5$ and Eckart at al. (1992) reported a detection of $\operatorname{Sgr} \mathrm{A}^{*}$ at $\mathrm{K}=13.7 \pm 0.6$ using a $2000 \times\left(0.5^{s}\right.$ or $\left.1^{3}\right)$ image at $0.05^{\prime \prime} /$ pixel resolution. If the Galactic Center is to be a "mini-AGN" then it seems the central engine is in a relatively inactive state and the source of the $10^{7} \mathrm{~L}_{\odot}$ must lie elsewhere, most likely the IRS 16 complex.

The presence of a central black hole in the Galactic Center is also suggested by dynamical evidence that shows central masses on the order of $10^{\gamma} \mathrm{M} \odot$ in the nuclei of M31 and M32 (Tonry 1987; Kormendy 1988; Dressler and Richstone 1988). It is notable that if such an object were within our Galactic Center is would already have been detected by a $250 \mathrm{~km} \mathrm{~s}^{-1}$
velocity dispersion for stars in its proximity. This has been ruled out by near-infrared stellar radial velocities (Sellgren et al. 1987; Rieke and Rieke 1988; McGinn et al. 1989) but a compact object with a mass an order of magnitude less can still be accommodated (see chapter 5).

## Starbursts at the Galactic Center

Lebofsky, Rieke, and Tokunaga (1982) found 7 supergiant stars in the central $1.5^{\prime}$ from near-infrared spectroscopy and the results of Lebofsky, Rieke, Deshpande, and Kemp (1982) suggested five other stars with similar photometric and polarization properties could also be supergiants. The fact that the Galactic Center contained numerous late-type supergiants led to the consideration that starburst-like phenomena in the Galactic Center could explain the overall energetics and characteristics of the ionizing radiation without resorting to explanations involving a central engine. The spectral type of IRS 7 implies a progenitor of $\sim 20 \pm 2.5 \mathrm{M} \odot$ (e.g. Maeder and Maynet 1989) which has formed within the last $10^{7} \mathrm{yr}$.

It is useful to present the calculation of Lebofsky et al. (1982), using revised values of the Galactic Center distance and the stellar evolution tracks of Maeder and Maynet (1989), to demonstrate the significance of massive star formation within the Galactic Center environment. When the spectral types and estimated bolometric luminosities are compared to stellar evolution tracks to find the appropriate progenitor mass, an upper limit to the age of the population is given by the age of the most massive star. These mass estimates show a minimum of $=60-70$ $\mathrm{M}_{\odot}$ was consumed in the formation of this group of stars. The time scale for the starburst can be taken as the age of IRS 7 at $10^{7} \mathrm{yr}$.

TABLE 1-3: Galactic Center Luminous Late-Type Stars

| Source | Spectral Type | $\mathrm{m}^{*}{ }^{\text {a }}$ | (H-K) ${ }^{4}$ | $\mathrm{M}_{\mathrm{K}}{ }^{\text {b }}$ | Progenitor Mass ${ }^{\text {( }}$ (M) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| IRS 7 | M1.3Ia | 6.7 | 0.29 | -11.5 | 20 |
| IRS 12 | M4Ib | 8.1 | 0.35 | -9.5 | 7-9 |
| IRS 19 | M4Ib | 8.1 | 0.49 | -9.7 | 9-12 |
| IRS 11 | M0Ib | 9.5 | 0.22 | -5.0 | 3-4 |
| IRS 22 | M6II | 7.8 | 0.50 | -9.0 | 5-7 |
| IRS 23 | M6II | 7.9 | 0.51 | -9.4 | 7-9 |
| IRS 24 | M6II | 7.9 | 0.49 | -9.8 | 9-12 |

-Lebofsky, Rieke and Tukanaga (1982), ${ }^{\mathrm{b}} \mathrm{R}_{0}=6.9 \mathrm{kpc}, \mathrm{BC}_{\mathrm{K}}=3.0$ Maeder and Maynet (1989)

A lower limit to the star-formation rate can be made by considering an initial mass function (IMF) for star formation of the form $f(m) \propto m^{-2.35}$, which is the Salpeter form. The mass contribution from stars of masses between $m$ and $m+d m$ will be given by $d m=m \times f(m)$ $d m \propto m^{-1} d m$. For a starburst event with an upper cutoff mass of $m_{2}$ and a lower cutoff mass of $m_{1}$, the fractional mass contribution, $F$, for stars with progenitor masses greater than $m_{3}$ will be equal to

$$
F=\frac{\int_{m_{3}}^{m_{2}} m f(m) d m}{\int_{m_{1}}^{m_{2}} m f(m) d m}=\frac{m_{2}^{-0.35}-m_{3}^{-0.35}}{m_{2}^{-0.35}-m_{1}^{-0.35}} .
$$

Using numbers derived from the observed supergiants (and knowing the resulting stars must have $\mathrm{T}_{\text {eff }}<35,000 \mathrm{~K}$ ), we may take $m_{2}=20 \mathrm{M} \odot$, with a minimum amount of $70 \mathrm{M} \odot$ of stars with $m>7 \mathrm{M} \odot$, the resulting lower limits on the star formation rate follow for the assumed lower mass cutoffs listed:

TABLE 1-4: Lower Limits on the Galactic Center Star Formation Rate

| $m_{l}(\mathrm{MO})$ | $F$ | $\begin{gathered} \mathrm{M}_{1} \\ \left(\mathrm{~m}>\mathrm{M}_{\odot}\right) \end{gathered}$ | $\mathrm{M}_{\text {total }}$ (M) | $\begin{gathered} \text { SFR } \\ \left(\mathrm{M} \odot \mathrm{yr}^{-1}\right) \end{gathered}$ | $\begin{gathered} \text { SFR } \\ \left(\mathrm{M} \odot \mathrm{yr}^{-1} \mathrm{pc}^{-2}\right) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 0.08 | 70 | 875 | $8.8 \times 10^{-5}$ | $3.1 \times 10^{-7}$ |
| 1.0 | 0.24 | 70 | 291 | $2.9 \times 10^{-5}$ | $1.0 \times 10^{-7}$ |

So the approximate star formation rate in this burst was on the order of $2 \times 10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1} \mathrm{pc}^{-2}$. This is two orders of magnitude greater than the Galaxy wide average rate of $3-7 \times 10^{-9} \mathrm{M} \odot \mathrm{yr}^{-1}$ $\mathrm{pc}^{-2}$ (Miller and Scalo 1979; $\mathrm{R}_{0}=10 \mathrm{kpc}$ ), but comparable to the rate within the central 800 pc of the bulge of $=4 \times 10^{-7} \mathrm{M}_{\odot} \mathrm{yr}^{-1} \mathrm{pc}^{-2}$ (Smith, Biermann, and Mezger 1978; $\mathrm{R}_{0}=10 \mathrm{kpc}$ ). The amount of mass inferred to be involved in the starburst and the rate of star formation are rather modest compared to the theoretical results of Loose, Krügel and Tutukov (1982) that are discussed below ( $0.01 \%$ ). Nevertheless, the presence of large numbers of late-type supergiants is evidence that starburst episodes might occur and could account for the high luminosity within the region.

There is abundant raw material for starbursts within 1 kpc of the Galactic Center in giant molecular clouds that have been mapped in ${ }^{12} \mathrm{CO},{ }^{13} \mathrm{CO}$, and CS (e.g. Bally et al. 1987, 1988; Stacy, Dame, and Thaddeus 1989). Stark et al. (1991) estimated the masses for the six largest clouds to be on the order of a few $\times 10^{6} \mathrm{M}_{\odot}$ from tidal limitations by the bulge, the Virial theorem, and CO and CS column densities. The perturbed orbits of bulge stars enhance the density of material behind each cloud resulting in dynamical friction that eventually causes the cloud to spiral into the center within only a few hundred million years. Unless the rate of star formation within a cloud is sufficiently high to consume it during its fall, it will cause a starburst upon collapsing to the center. The number of clouds and the time scale for infall suggest a steady state recurrence of starbursts every 100 Myr .

A theoretical study by Loose, Krügel and Tutukov (1982) examined the relevance of massive star formation in the Galactic Center environment. They pointed out that unlike the
disk of the Galaxy, gaseous material collecting at the Galactic Center cannot escape because it is at the bottom of a potential well thus leading to large amounts of raw material to form stars. The mass density distribution of stars, $\rho^{*}$, that determines the gravitational potential varies as $R^{-18}$. For a gaseous cloud's internal gravity to overcome tidal forces its density must be $\rho>$ $\rho^{*} \propto R^{-1.8}$, so clouds must be increasingly dense with decreasing radii from the center to undergo Jeans instability. Because of the increased effects of rotation, gas temperature, turbulence, magnetic fields, and coalescence of fragments after collisions, the stars that form near the center are likely to have higher masses than stars formed in the disk. They described two models that assumed an IMF of the form $\mathrm{dN} \propto M^{-235} \mathrm{~d} M$ with an upper cutoff of $60 \mathrm{M}_{\odot}$. Their "Case A" had a lower IMF cutoff at $1.0 \mathrm{M} \odot$ and exhibited a $10^{8}$ yr periodic cycle of gas accumulation which led to bursts of high mass star formation. Each burst, lasting a few $\times 10^{7}$ yr converted approximately $10^{7} \mathrm{M} \odot$ into $\sim 10^{5} \mathrm{OB}$ stars. These OB stars would each in turn expel $10^{51}$ erg of energy back into the gaseous material via supernova explosions. All of the star formation occurred within 10 pc of the center with a peak luminosity of $10^{9} \mathrm{~L} \circ$. The other model discussed, "Case B", had a lower IMF cutoff at 0.1 M $\odot$ producing a smaller fraction of supernova progenitors. This model showed an initial buildup of gas and a subsequent burst of star formation. However, the amount of energy returned by supernovae was insufficient to disrupt the accumulated gas and led to the formation of a quasi-hydrostatic core in the central few parsecs. Both models show that accumulated material from mass loss by late-type stars and ambient gas at the bottom of the deep gravitational potential of the Galactic Center may lead to unusual star formation conditions unlike anywhere else in the Galaxy.

The claim that star formation is important for understanding the Galactic Center was contested (e.g. Gatley 1987) by the assertion that knot structures observed in the $10 \mu \mathrm{~m}$ emission and associated with thermal emission seen in the radio (which later became known as the "minispiral") should show up in the color temperature distribution of the far-infrared emission if they are internally heated by hot stars. Also it was suggested that a star-formation event should
produce a more chaotic configuration in the gas and dust structures seen at the center. The farinfrared observations indicated that the dust distribution has a central cavity and the symmetric luminosity and temperature profiles about the center meant the energetics could well be dominated by a strong central ionizing source. Subsequent continuum and line measurements of the K sources at the positions of these $10 \mu \mathrm{~m}$ knots show they are likely to be massive blue stars (Rieke, Rieke, and Paul 1989; Allen, Hyland and Hillier 1990; Krabbe et al. 1991). Even so, the picture of star-formation at the Galactic Center does not contend that formation is currently ongoing, merely that it occurred in the recent past and its effects are still felt from the high-mass stars that were produced. The variability study presented here was initiated mainly to find further evidence of recent star-formation (chapter 4).

## The Galactic Center and the Nucleus of M31

Despite being $\sim 10^{2}$ times further away, there are several features in the center of our Galaxy that are also present in the nucleus of M31. The center of the M31 normalized $2.2 \mu \mathrm{~m}$ light profile is similar to the $2.2 \mu \mathrm{~m}$ light distribution of the Galactic Center when the Galactic observations are degraded to comparable resolution (Becklin and Neugebauer 1968; Rieke and Lebofsky 1987). There is a nucleus with a radius on the order of $2^{\prime \prime}(0.6 \mathrm{pc})$ that is distinct from the bulge (Light, Danielson and Schwarzschild 1974; King et al. 1992). King et al. also reported the detection of $\sim 150 \mathrm{UV}$ point sources in the central $44^{\prime \prime}$ which they interpreted as postasymptotic giant branch stars. These PAGB stars only contributed less than $20 \%$ of the total UV light emerging from the M31 nucleus indicating that some fainter and more distributed population is responsible. One possibility they suggested was a high-metallicity AGB population but they add that the characteristic ages and masses of these stars are yet to be determined from stellar evolutionary tracks.

There is the kinematic evidence for a central mass concentration on the order of $10^{7} \mathrm{M}_{\odot}$
from the optical and near-infrared velocity dispersion and rotation curves as mentioned above (Kormendy 1988; Dressler and Richstone 1988) with a mass to light ration in excess of $10^{2}$ Mo/Lo. Recently Crane, Dickel, and Cowan (1992) recently reported the $5 \sigma$ detection of a radio point source at 3.6 cm coincident with the dynamical nucleus with a flux density of $28 \pm$ $5 \mu \mathrm{Jy}$ which is intrinsically $1 / 5$ as bright as Sgr $\mathrm{A}^{*}$ at the same wavelength. Further information as to its nature requires future observations but its proximity to the nucleus and the evidence for a compact mass concentration suggests it may be analogous to Sgr $\mathrm{A}^{*}$.

Unlike the Galactic Center, M31 does not show significant emission in the mid- and farinfrared from dust emission at $10 \mu \mathrm{~m}$ (Rieke and Lebofsky 1978) or $100 \mu \mathrm{~m}$ (Telesco 1977) and there is a low density of gas (Brinks 1984; Soifer et al. 1986) which may account for the low luminosity of the radio point source if it is similar to Sgr A* (Crane et al. 1992). M31 shows many features that are similar to the Galactic Center but there is not a complete one-to-one correspondence. Nevertheless, its is likely that a more comprehensive understanding of either nucleus will help in understanding the other because the larger distance to M31 is compensated to some extent by its lower central extinction (Sandage, Becklin and Neugebauer 1969).

## Summary

The inner 10 pc of the Galactic nucleus is an extremely complex region. There is a central cluster of stars with a density on the order of a few $\times 10^{6}$ stars $\mathrm{pc}^{-3}$ that shows evidence of recent star formation and a hot blue population of unknown extent. There is a ring of molecular and atomic material, in roughly circular orbit about the center, intermixed with dust which absorbs a significant amount of UV radiation and reradiates in the far infrared. Within the inner edge of the molecular ring is a cavity largely devoid of gas except for a few very striking ionized filaments with motions that suggest orbital trajectories. Finally there is the
mysterious Sgr $A^{*}$ which has unique radio properties and is well placed spatially and dynamically to be identified with a massive black hole of approximately $1-2 \times 10^{6} \mathrm{M} \odot$, although energetically it is difficult to view it as an active central engine. Two competing hypotheses to explain the energetics have invoked recent massive star formation or an active central engine analogous to what is thought to be in other galaxies. This is the broad background in which the following observations are to be placed.

## CHAPTER 2 :

## HKL PHOTOMETRY OF GALACTIC CENTER STARS

## Background

The central peak of the Galactic Center $2.2 \mu \mathrm{~m}$ emission was initially attributed to older Population II stars within the core of the Galactic Bulge (Becklin and Neugebauer 1968), similar to the population seen in the center of M31 (Sandage, Becklin and Neugebauer 1969). Subsequent spectroscopic studies showed the brightest point sources to be younger late-type supergiants (Neugebauer et al. 1976, Lebofsky et al. 1982, Wollman, Smith, and Larson 1982). Whatever older bulge-like stars may be present are intermixed with at least a few recently formed massive stars. The first large-scale high resolution imaging of the center (Rieke, 1987) revealed nearly 1000 stars detectable at $K(2.2 \mu \mathrm{~m})$ in a $5^{\prime} \times 5^{\prime}$ area to a limit of $K=12$. The Galactic Center luminosity function suggested there is an excess of high-luminosity stars compared to the population in Baade's Window at Galactic latitude $1=-3.8^{\circ}$ (Frogel and Whitford 1987). Since the only information available came from photometry in two wavebands at a single epoch, remaining questions about the nature of the constituent stars (e.g. their predominant luminosity class and evolutionary state) could not be fully answered using these data. The present chapter describes new observations at $K(2.2 \mu \mathrm{~m}), \mathrm{H}(1.6 \mu \mathrm{~m})$, and $\mathrm{L}(3.4 \mu \mathrm{~m})$ that form an extended investigation of the general traits of the luminous Galactic Center stars seen in the near IR.

## Observations

The K observations were made at three epochs: 1987, May 5; 1987, September 2; 1988 May 26. The first two epochs were taken by M. Rieke and the third by M. Rieke and J. Haller.

Each night was photometric, and the data were acquired close to the local meridian to minimize problems associated with the relatively high air mass of the Galactic Center when observed from Tucson, Arizona ( $z \geq 2.06$ ). The Steward Observatory 1.54 -m telescope was used at $\mathrm{f} / 45$ with a $64 \times 64 \mathrm{HgCdTe}$ infrared detector array with a CCD readout fabricated by Rockwell International. The camera is described in Rieke et al. (1986). The camera's reimaging optics provided a plate scale of $1.2^{\prime \prime} /$ pixel and a field of view of 77 ". The repeated imaging was done at $K(2.2 \mu \mathrm{~m})$ since shorter wavelengths are more severely attenuated by interstellar extinction ( $A_{V}=30$; Becklin and Neugebauer 1968; Rieke et al. 1978; Becklin et al. 1978; Lebofsky et al. 1982; Rieke, Rieke, and Paul 1989), and would have required much longer exposures. Exposure times were limited to 20 seconds because source confusion was becoming apparent. The read noise of 600 electrons was adequate for the images to be background-limited. An additional map was made during the May 1988 observing run at $H(1.6 \mu \mathrm{~m})$ with 90 -second exposures, providing color information.

At each epoch the region was mapped in a series of seven columns of constant right ascension stepped by 4 sec of time, with $4^{\prime \prime}-10^{\prime \prime}$ of overlap between rows to ensure that every star was imaged at least twice. The area of the observed region was approximately $5^{\prime} \times 5^{\prime}$ (the south eastern corner was not mapped because of its high extinction from a foreground molecular cloud) and was centered on IRS $7(\mathrm{~K} \approx 7.0)$. Frames of blank sky were also taken at a position an hour east of the Galactic Center for flat fielding. The flux calibration was based on observations of standards from Elias et al. (1982).

The L data were taken by J. Frogel in May 1988 at the CTIO 1.5 m telescope using a Santa Barbara $58 \times 62 \mathrm{InSb}$ infrared detector array with $60^{3}$ integration times. There were 60 individual readouts of 1 sec each, coadded to produce the total integration time. The angular resolution was $0.9^{\prime \prime} /$ pixel. The central $5^{\prime} \times 5^{\prime}$ of the Galactic Center centered on IRS 7 was mapped as seven rows of constant declination in eighteen steps. Frame overlapping ensured that most of the detected sources were imaged at least twice for confirmation. Since the
background at L due to thermal sky emission is considerable and rapidly changing, flatfields were constructed for each row by median averaging all of the frames within each row.

## Data Reductions : Photometry

The data frames were reduced first by the replacement of data in bad pixels with the average value of surrounding pixels and were then flat-fielded using an averaged sky frame. Because this field is severely crowded, the repeatability of aperture photometry and point spread function fitting (PSF) methods were tested. Photometry was extracted for 40 stars in a region overlapped by two data frames with an aperture diameter ~ FWHM of the PSF and it was found that the dispersion of the magnitudes using PSF fitting was approximately half that for aperture photometry. Consequently, the package DAOPHOT (Stetson, 1987) was used to fit PSFs to measure the stellar fluxes.

The detection threshold for the images was determined from the standard error per pixel that includes contributions from the read noise of the detector, the Poisson noise from the thermal sky background, and the variations of the intrinsic stellar background caused by the unresolved stars in the Galactic Center (see Appendix 1 for a discussion of the likely distribution of these stars along the line-of-sight). Empirical standard error estimates were derived by calculating the dispersion of 50 to 250 pixel values in regions where no resolved sources were apparent. The contributions due to the read noise were negligible while the thermal background accounted for only about $10 \%$ of the total, thus the $3-\sigma$ detection threshold of the images was dominated by the confusion of faint stellar sources. For the K-maps the 3- $\sigma$ detection limit was between $\mathrm{K}=12$ to 12.5 , while for the H map the $3-\sigma$ detection limit was H $=15$. These magnitude limits imply a foreground extinction limit of $A_{v}<43 \mathrm{mag}$ for stars with an intrinsic $\mathrm{H}-\mathrm{K}=0.3$, the observed average in Baade's Window (Frogel and Whitford 1987).

Candidate objects were located by convolving each image with a Gaussian PSF with the
observed FWHM and preliminary magnitudes were derived using aperture photometry. The PSF was constructed empirically using stars selected from a pool of the 20 brightest sources on an image. Groups of stars with overlapping PSFs were iteratively reduced to find the best fitting profile using the DAOPHOT routines GROUP and NSTAR. Because of the severe crowding by faint sources, some of the data frames had groups exceeding 100 stars. These frames had to be regrouped with a smaller fitting radius of the Gaussian core of the PSF until the group sizes were reduced below the maximum of 60 stars per group allowed by DAOPHOT.

The surface density of sources at $L$ was surprisingly lower relative to $K$, even with $60^{s}$ exposures because the high thermal emission still dominated the background. The density of sources was sufficiently low that stellar magnitudes could be measured using aperture photometry. This was also advantageous because poor tracking during some of the exposures caused the images to have very elongated stellar profiles. The IRAF package APPHOT was used instead of DAOPHOT. In the central $20^{\prime \prime}$ the source crowding was considerable so smaller aperture sizes were used with corrections applied from photometry on stars in less crowded areas. These problematic stars are not included in the data presented in this analysis, however.

## Data Reductions : Database Construction

A total of 155 K data frames was processed, which yielded 11,777 stellar measurements. A typical star was likely to have been observed two to three times at each of the three epochs and at each wavelength. Thus a procedure was required to reduce the multiplicity of measurements. An initial database was generated for each epoch from the results of NSTAR. Each record in this database contained a listing of the original data frame, epoch, magnitude, pixel coordinates, and so forth. Each star was measured from two to six times per epoch
because of frame overlapping. All of the frames were placed on a standard coordinate system by measuring relative offsets from brighter stars. The multiple measurements of each star were then combined to produce a final database for each epoch.

The algorithm to produce the compiled databases went through several stages of development to accommodate computational limitations. An initial approach centered on frame-to-frame comparisons that treated each epoch on an equal footing. Records were sorted by data frame and each data frame was was given an index number specifying the epoch and relative position within the map. For instance, the frame labelled " 236 " was taken during the second epoch, September 1987, and was located in the third column, sixth row. To find matches for stars in frame 236, a search would be made in frames $j 25, j 26, j 27, j 35, j 36, j 37, j 45, j 46, j 47$ for $j=1,2,3$ with the provision that frame 236 not be searched against itself. For an average number density of stars per frame, $\rho$, the time for each frame-to-frame search is proportional to $\boldsymbol{\rho}^{\mathbf{2}}$. Restricting the search to particular data frames is a significant improvement over a direct star-to-star search throughout the entire database but with the computer facilities available this was still too unwieldy. It was also more difficult to implement uniqueness criteria that would prevent unwanted duplicate records in the resultant database.

The next phase of development incorporated two important changes. It was realized that significant residual positional errors occurred when comparing maps between two epochs and this caused a significant number of unmatched records. So a fiducial epoch in K was selected (May 1988) in which the frame offsets were measured with great care. Subsequent data frames from other epochs were then to be compared directly to the May 1988 K map to determine their global position rather than to other adjacent frames within the same epoch. The global offsets were then applied to the stars within that frame. This greatly improved the record matching between the epochs without compromising significantly the positional accuracy of the other maps. Also because the crowding in this field was so great it was deemed best not to combine the epochs into a final database until each epoch had its own compiled database.

This again prevented unwanted record duplications.
The problem of time consuming frame-to-frame matching was solved by extending the very same method to smaller length scales. The map was divided up into square cells of uniform size and each cell was given a character string label 8 letters in length such that the cell centered on the coordinate system origin was designated " 00 ". The $80^{\text {th }}$ cell along the $x$-axis and $-167^{\text {th }}$ cell along the $y$-axis would then be designated " $80-167$ ". Each star was then labelled by the character string label appropriate for the cell in which it resided. The cell size is chosen to be slightly larger than the expected positional errors but smaller than the average separation between stars. This procedure is more advantageous because adjacent cells can be addressed easily by computation and are relatively close within the database when it is sorted by cell position. Since it is still possible for a star to reside near the cell edge, it is still necessary for adjacent cells to be searched. However, the average number of stars per cell is on the order of unity and this results in a substantial increase in computational speed. Also there was little computational time spent addressing potentially matching stars because discrete character string cell labels eliminate the need to calculate positional tests of the form $\delta r>\left[\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y_{2}\right)^{2}\right.$ ] in most cases.

When searching adjacent cells for a given test star, two important criteria apply to reject duplications and spurious matches. If two stars happen to lie within the same cell that is tested, then the star to be matched is the one with the minimum spatial separation. Secondly, stars which are within the same data frame as the test star must be rejected since this will produce a spurious match. Using the cell algorithm, stars in each epoch were matched by searching for positional coincidences with cell sizes of $0.75^{\prime \prime}$. The final list of $K$ detections was then collated with the database containing $H$ magnitudes, resulting in a final database containing magnitudes and colors of over 1400 stars. The photometric errors estimated from the repeated measurements of stars were $0.088,0.070$, and 0.095 mag for May 1987, September 1987, and May 1988 respectively.

After the three K observations were compiled into a single list, an additional correction to the photometry was applied because of systematic magnitude differences between epochs. The systematic errors were spatially variable across the map, often correlated with particular data frames or columns of data frames. The fact that the standard stars were observed at slightly different air masses than the Galactic Center, as well as extinction variations, both in time and airmass, contributed to these errors. (The procedure is discussed more in detail in chapter 4 where it is most relevant to the search for variable stars.) All data were normalized to the May, 1988 map because this epoch included the H measurements. The final $\mathrm{H}-\mathrm{K}$ colors for a star were derived from the H data and only the K data from May, 1988. In 15 instances there was no K detection in May 1988 to match with the H measurement because the H map is slightly deeper than the K map relative to typical obscured stellar colors. In these cases the $\mathrm{H}-\mathrm{K}$ color was computed from the May 1988 H magnitude and the average K magnitude from the other epochs in May 1987 and September 1987.

Finally, it was necessary to extract from this compiled database a sample that was complete for each epoch. The 3- 0 detection limits listed above were used by DAOPHOT to locate sources on the data frames, but the data are not complete to those limits. At each epoch, a large percentage of all sources should have been detected at least twice because of the overlapping of the data frames. Figure 2-1 shows the actual fraction of sources detected at least twice for each epoch as a function of K magnitude. The dashed horizontal line on each plot indicates the fraction of the map covered multiple times by the mosaic of frames. To first order, the fraction of the map covered multiple times by image frames is equal to the expected fraction of stars in the database


Figure 2-1: Fraction of multiply detected sources at each $2.2 \mu \mathrm{~m}$ epoch. The dashed lines indicate the expected fraction of multiple detections based on overlapping of images. Every epoch clearly shows a sharp deviation indicating a magnitude limit in multiple detections. The completeness limits of $\mathrm{K}=11.00$ and $\mathrm{H}=13.00$ thus set by the the magnitude limit for multiple detections.
that are detected multiple times, assuming a constant source density in the $5^{\prime} \times 5^{\prime}$ survey field. If the FIND routine in DAOPHOT worked perfectly, and all the data parameters were set correctly, one would expect a higher fraction of multiple detections down to the 3- $\sigma$ detection limit. Based on these diagrams, completeness limits of $K=11.50$ for May, $1987, K=11.25$ for September, $1987, \mathrm{~K}=11.00$ and $\mathrm{H}=13.0$ for May, 1988, were determined by the magnitude where the fraction of multiple detections begins to deviate from the appropriate fraction for each map. The database discussed here is composed of all the sources brighter than these limits and thus preserves much of the measurement redundancy in each epoch. A positional map showing the region surveyed is shown in figure 2-2.

An additional database was constructed for the $L$ photometry that produced approximately 1300 stellar magnitude and position measurements. Global coordinates within the $5^{\prime} \times 5^{\prime}$ field were again measured for each L image to a precision of $\pm 0.5^{\prime \prime}$ against the stellar positions of the previous HK database to minimize residual positional errors between wavelengths. The $L$ frame coordinates were then applied to the individual stars to agree with the global K system. The multiple measurements of each star were then matched using the "cell" procedure using a cell size of $0.75^{\prime \prime}$ and matched records were collated into a final L database. The estimated completeness limit was empirically determined in the same way as the HK database. The fractional distribution of multiple detections vs. $m_{L}$ shows a strong roll over in the data at $m_{L}>8.5$, so only those stars brighter than this limit were compiled with the $H$ and $K$ data. The $L$ magnitudes for sources in regions of low density have an internal error of $\pm 0.25$ mag, based on the repeatability of the measurements from frame overlapping. The final L database was then combined with the previous HK database, again by the "cell" searching technique, to produce a three color sample of 278 stars. The apparent K luminosity function from the HK database as well as the $K$ stars detected at $L$ in the HKL database is shown in figure 2-3.


Figure 2-2: A positional map of the stars used in the HK database complete to $\mathrm{m}_{\mathrm{K}}=11.0$ and $(\mathrm{H}-\mathrm{K})_{0}>1.5$. The bordered outline shows the extent of the overlapping data frames taken at H and K .


Figure 2-3: The apparent K magnitude luminosity function showing the complete sample of H and K stars. The dashed histogram is the subsample comprising the HKL database.

## Summary

The final products of the observations were the sample of 688 H and K stars complete to $m_{K}=11.0$ and another sample of $278 \mathrm{H}, \mathrm{K}$, and L stars complete to $\mathrm{L}=8.5$. Chapter 3 will examine the H and K sample in detail and show that the Galactic Center stellar population is composed of high luminosity stars that are substantially different from stars in the nearby bulge. Chapter 4 undertakes a search for variability using both the HK and HKL databases and shows that luminous late-type AGB stars reside in the central few parsecs of the Galaxy.

## CHAPTER 3 :

## HIGH LUMINOSITY LATE-TYPE STARS IN THE GALACTIC CENTER STELLAR POPULATION


#### Abstract

The basic properties of the Galactic Center stellar population derived from the $\mathrm{H}(1.6 \mu \mathrm{~m})$ and $\mathrm{K}(2.2 \mu \mathrm{~m})$ photometry (Appendix 2 ) will be examined in this chapter by a careful analysis of the foreground extinction, expected intrinsic colors, and bolometric corrections that are needed to compare this population with stars in Baade's Window. A cursory examination of the images reveals an extraordinarily high star density of around 80 stars $\operatorname{arcmin}^{-2}$ which was noted in Rieke (1987). Her older data were taken with a $32 \times 32$ array, with poorer performance, and with barely adequate integration time at H . In spite of these shortcomings, these older data can still be used to compare the newer data of higher quality with the extinction properties derived from the old data being close to the values derived from the newer data.


## Foreground Extinction

The interstellar extinction law has been measured toward the Galactic Center (Rieke and Lebofsky 1985; Rieke, Rieke, and Paul 1989) from 1-13 $\mu \mathrm{m}$ observations which showed no evidence for any significant variations between the A5 star oSco and several stars in the Galactic Center. A known constant extinction law then allows an accurate dereddening of the stars in our images. Figure 3-1 shows the distribution of observed $\mathrm{H}-\mathrm{K}$ colors for the sources in the original complete sample. Because only one color is available it is not possible to assign uniquely the

Observed H-K Colors


Figure 3-1 : The observed H-K color distribution in the final database before removal of suspected foreground stars.
extinction and intrinsic color for each star. However, several pieces of evidence can be used as a guide to estimate the averages of these quantities. First, there is spectroscopy that shows strong $2.3 \mu \mathrm{~m}$ CO absorption for approximately 100 of these stars (Rieke and Rieke unpublished data; Rieke 1988; Rieke and Rieke 1988). Other studies of both resolved sources and unresolved stellar emission confirm that the CO feature is common in this population (McGinn et al. 1989, Sellgren at al. 1990; see also chapter 5). This feature is most pronounced in M giants and supergiants (Klienmann and Hall 1988). Therefore it can be assumed stars are predominately late-type stars and have relatively narrow range of intrinsic $\mathrm{H}-\mathrm{K}$ color (the $\mathrm{H}-\mathrm{K}$ color ranges from 0.0 to 0.5 from spectral type A 0 to M 9 ). Secondly, an empirical estimate of the average H-K color can be derived from the closest comparison population in Baade's Window (Frogel and Whitford 1987) where the average intrinsic $\mathrm{H}-\mathrm{K}=0.3$ and there is a relatively narrow spread in the intrinsic $\mathrm{H}-\mathrm{K}$ colors. The extinction can be separated from the intrinsic color to acceptable accuracy given the limited data. For the complete sample of Galactic Center stars, the average observed H-K of 2.24 is in good agreement with an extinction of about 30 magnitudes (Becklin and Neugebauer 1968) for an assumed average $\mathrm{H}-\mathrm{K}$ color of 0.3 mag.

The relative number of stars with H-K redder than 0.5 in at late-type population is small so the redward spread in Figure 2 is most easily attributed to some stars having higher than average extinction. The relative lack of stars on the southeast side of the Galactic Center as compared to the northwest side seen in large scale images supports the suggestion of some regions having higher extinction than the average (Catchpole, Glass, and Whitelock 1989).

Although no independent distance estimates exist for this group of stars, the amount of reddening corresponds somewhat to the relative locations of these stars. Those stars with the least amount of intervening material are the bluest and therefore the nearest and so are likely to be foreground stars. The foreground star reddening limit was chosen to be an observed $(\mathrm{H}-\mathrm{K})_{\text {obs }}<1.5$. There were 29 such stars, comparable to the expected number of foreground stars observed along this line-of-sight from a calculation that assumes a solar
neighborhood stellar luminosity function and a relatively uniform distribution of foreground dust. Appendix 1 describes this calculation in detail using a simple model of the Galactic stellar disk and dust distributions as well as an estimate of the line-of-sight dispersion for stars in the central bulge. The disk calculations shows that about $20-35$ stars with $\mathrm{H}-\mathrm{K}<=1.5$ would be detected, depending largely on the value of the disk scale length. The values used in the calculation ranged from 3 to 5 kpc (de Vaucouleurs and Pence 1978). The agreement between this simple model and the observed number of blue stars supports the identification of these stars as foreground objects. Sources with H-K significantly less than 1.5 may have a sufficiently low level of extinction that they may be observable at visible wavelengths and ten of the 29 foreground stars have been found on the Palomar red plate. Positions with accuracies of $0.5^{\prime \prime}$ in each coordinate were determined for these stars using the NOAO Grant machine. Table A2-2 in appendix 2 lists the optical and infrared positions and photometry of these stars. The disk calculation also predicts that 3 to 9 foreground stars would have $\mathrm{H}-\mathrm{K}>1.5$. This amount of contamination is at the $1 \%$ level and is certainly acceptable considering the accuracy of the data and cannot affect the conclusions presented later in this chapter. Thus both on the basis of the Galactic structure calculation and optical identifications, sources with ( $\mathrm{H}-\mathrm{K})_{\text {obs }}<1.5$ were removed from the complete sample.

The remaining group of 659 stars lie essentially at the Galactic Center. The distance to the Galactic Center is here taken to be $6.9 \mathrm{kpc}\left((\mathrm{m}-\mathrm{M})_{0}=14.2\right.$; see chapter 1$)$ so that direct comparisons can be made with studies of Baade's Window by Frogel and Whitford (1987). This will be regarded as the distance to the majority of the stars in our sample. The Galactic bulge density calculation described in Appendix 1 shows that $80 \%$ of these stars lie within 20pc of the Galactic Center; $90 \%$ are within 50 pc. The intrinsic spatial depth of this group of stars will produce a photometric dispersion on the order of -0.006 mag is much smaller than the typical photometric error which is on the order of 0.1 mag.

The reddest stars in the sample may lie beyond the Galactic Center, suffer greater
extinction from intervening dust clouds of greater than average density, or have circumstellar dust shells. Background contaminants beyond the Galactic Center would be much less numerous than foreground contaminants. Given that $A_{V}=30 \mathrm{mag}$ has been confirmed for many stars with spectroscopic data, and that the underlying unresolved stellar distribution has a brightness of around 13th magnitude per square arc sec, the probability of detecting many background sources at a limit of $K=11$ is low. Their inferred luminosities would be underestimated and therefore would not change the conclusions of this work. Therefore, with a very high level of confidence, we may treat this sample of stars as representative of the brightest late-type stars at the Galactic Center.

In the final sample of stars, excised of foreground contaminants, the average extinction implied by the observed $\mathrm{H}-\mathrm{K}$ color distribution is $\mathrm{A}_{\mathrm{V}}=31.8 \mathrm{mag}$, for an intrinsic $\langle\mathrm{H}-\mathrm{K}\rangle=0.3$. It would be possible to estimate the individual extinction for every star with the observed $\mathrm{H}-\mathrm{K}$ color and carefully chosen intrinsic color. But the H-K colors in this sample are not accurate enough ( $\approx \pm 0.15 \mathrm{mag}$ ) for this procedure to be implemented effestively. Instead, a uniform extinction estimate based on the above average value was applied to the data to avoid a spurious bright tail in the derived luminosity function. Significant local variations in the extinction exist at the level of $\pm 6 \mathrm{mag}$ in $A_{V}$ from foreground molecular clouds (Glass, Catchpole, and Whitelock 1987) as well as individual clouds in the molecular ring (Genzel 1989) but the resulting average intrinsic $\mathrm{H}-\mathrm{K}$ color is quite sensitive to the choice of the extinction correction. Decreasing the assumed average extinction by 5 magnitudes will shift the average intrinsic $\mathrm{H}-\mathrm{K}$ to 0.6 which would be implausibly large. Increasing the average extinction by 5 magnitudes will shift the intrinsic $\mathrm{H}-\mathrm{K}$ to -0.02 and imply a very hot stellar population, but this is ruled out on spectroscopic grounds by the large number of stars with strong CO absorption. So the dispersion within a given luminosity bin may be increased but there should not be a bias in the average luminosity levels from spuriously bright stars.

## Bolometric Corrections

A bolometric luminosity function is the most useful for direct comparisons with other populations. Thus an appropriate estimate of the bolometric correction must be made to apply to the dereddened K magnitudes. In general these corrections would be derived from spectral types or the J-H and $\mathrm{H}-\mathrm{K}$ colors of each star. Since our data are limited to H and K information, the bolometric corrections were estimated from the expected values for late-type stars. Figure 3-2 shows the K bolometric correction as a function of $\mathrm{H}-\mathrm{K}$ for several different samples of late-type stars. The bolometric correction depends on metallicity and luminosity class, as evidenced by the differences seen between the correction for SMC supergiants, LMC supergiants, Milky Way supergiants (Elias et al. 1985), and Baade's Window giants and long period variables (Frogel and Whitford 1987).

Currently, there are no metallicity determinations for the stars at the Galactic Center. The K giants in Baade's Window, which again are the closest comparison population, appear to be super metal rich (SMR) (Rich 1988) as is the case for the M stars (Terndrup et al. 1991). Blanco (1988) has observed a steep gradient in the relative numbers of early- and late-type M giants with decreasing distance from the center. This was interpreted as evidence for a steep metallicity gradient that drives the observed spectral types to cooler temperatures due to line blanketing effects. Some indirect evidence on the metallicity of the stars at the center (Lester et al. 1981) comes from the argon emission lines of ionized gas in the central $14^{\prime \prime} \times 28^{\prime \prime}$ which shows an abundance 2-3 times solar. Solar abundances at the center cannot be ruled out, and although SMR abundances were not explicitly examined, the Lester et al. (1981) data appear to rule out abundances as high as 10 times solar for the gas. These results, as well as the metallicity gradient in the bulge, suggest stars in the Galactic Center are likely to have higher than solar metallicities but less than 10 times solar.

Since Baade's Window has the most metal rich population with known bolometric
corrections, these are the most suitable to apply to the Galactic Center. A uniform bolometric correction $K_{B C}$ of 3 magnitudes was applied to the stars in the complete sample, corresponding to an intrinsic $\mathrm{H}-\mathrm{K}$ color of 0.3. From the trends in figure 3-2, if the metallicity is higher, $\mathrm{K}_{\mathrm{BC}}$ should be larger. If the stars have higher luminosities than giants, $\mathrm{K}_{\mathrm{BC}}$ should be smaller. Since these two effects change $\mathrm{K}_{\mathrm{BC}}$ in opposite directions, the choice of $\mathrm{K}_{\mathrm{BC}}=3.0$ for the Galactic Center population appears justified.

## Discussion

The result of the above analysis is the derived luminosity function shown in figure 3-3 as well as the luminosity function for Baade's Window (Frogel and Whitford 1987). The assumptions used in converting from K to bolometric magnitudes are a distance modulus of 14.2 ( 6.9 kpc ; see chapter 1) in agreement with the value of Frogel and Whitford (1987), a uniform foreground extinction of $A_{V}=31.8$, and a uniform bolometric correction of 3.0. The most prominent result of Figure 3-3 is the large number of high luminosity Galactic Center stars significantly brighter than the luminosity cutoff in Baade's Window.

The luminosity function for Baade's Window in figure 3-3 has been normalized relative to the Galactic Center following the same procedure as used in Rieke (1988). This normalization is based on the relative surface brightness at $2.4 \mu \mathrm{~m}$ of Baade's Window and the center


Figure 3-2: Bolometric corrections at $\mathrm{K}(2.2 \mu \mathrm{~m})$ as a function of $\mathrm{H}-\mathrm{K}$ color for different stellar populations. The Baade's Window corrections are from Frogel and Whitford (1987) and the others are from Elias et al. 1985.

## Luminosity Functions



- G.C. - B.W.

Figure 3-3: The luminosity function for the Galactic Center compared to Baade's Window (Frogel and Whitford 1987). The normalization between the two regions is discussed in the text.
(Matsumoto et al. 1982). The correction factor from this method is 14; i.e., our survey contains 14 times as many stars as the Frogel and Whitford (1987) survey of Baade's Window. Another method of normalizing the counts is based on normalizing only the actual observed counts in each region at the same luminosity. If the normalization is based on equalizing the number of stars seen at $\mathrm{M}_{\mathrm{bol}}=-4$, a correction of 5 is found.

The Galactic Center luminosity function appears to have an excess of high luminosity stars in comparison to Baade's Window. The important question to ask is whether the high luminosity stars at the Galactic Center are a distinct population from Baade's Window or are they similar stars but the high density of stars has allowed better sampling of the bright end of the luminosity function. If a normalization factor of 14 is applied to the Baade's Window luminosity function then there will be 9 stars with $\mathrm{M}_{\mathrm{bol}} \leq-5$. If the factor is 5 , then 24 such stars should be found. There is only one star brighter than this limit in Baade's Window (Frogel and Whitford 1987). The luminosity functions for fields at $b^{11}$ ranging from $-3^{\circ}$ to $-12^{\circ}$ in Frogel et al. (1990) show a slight trend for the higher latitude (and hence presumably less metal-rich) fields to have luminosity functions extending to brighter magnitudes. The $-12^{\circ}$ field includes stars as bright as $\mathrm{M}_{\mathrm{bot}}=-5.8$. Yet, none of these fields has stars as bright as the bright tail at the Galactic Center. So the high luminosity tail at the Galactic Center cannot easily be discounted as a better statistical sample of a generic bulge luminosity function.

It also does not appear likely that the high luminosity tail is an artifact from the analysis of the data. Since the distance modulus to the center was the same as adopted by Frogel and Whitford (1987), this cannot account for the difference between the populations. The effect on the Galactic Center luminosity function for choices of $A_{V}$ and $K_{B C}$ is shown in Table 3-1. The derived spread in $\mathrm{M}_{\mathrm{BOL}}$ is shown for a range of intrinsic $\mathrm{H}-\mathrm{K}$ between 0.1 to 0.5 . Changes in the extinction drive $\mathrm{M}_{\mathrm{BOL}}$ in an opposite direction from the effects of the $\mathrm{K}_{\mathrm{BC}}$ changes and the maximum change in $\mathrm{M}_{\text {BoL }}$ which could result from changing these assumptions is only 0.11 magnitudes. This is negligible in comparison to the differences between the Galactic Center and

Baade's Window luminosity functions. Given that JHK colors are driven towards bluer values at higher metellicities (eg. Terndrup et al. 1991), it would seem most likely that the average H-K color for the Galactic Center stars might be less than 0.3 rather than larger than 0.3 ; the changes to $\mathrm{M}_{\mathrm{BOL}}$ from this cause would be very small.

TABLE 3-1 : Effect On Derived Bolometric Magnitude From Assumed Color and Extinction

| $(\mathrm{H}-\mathrm{K})_{0}$ | $\mathrm{~A}_{\mathrm{V}}$ | $\Delta \mathrm{BC}_{\mathrm{K}}$ | $\Delta \mathrm{A}_{\mathrm{K}}$ | $\Delta \mathrm{M}_{\mathrm{BOL}}$ |
| ---: | ---: | ---: | ---: | ---: |
| 0.1 | 34.9 | 0.75 | -0.71 | 0.04 |
| 0.2 | 33.3 | 0.20 | -0.17 | 0.03 |
| 0.3 | 31.8 | 0.00 | 0.00 | 0.00 |
| 0.4 | 30.2 | -0.15 | 0.18 | 0.03 |
| 0.5 | 28.6 | -0.25 | 0.36 | 0.11 |

On all accounts the high luminosity tail at the Galactic Center must be regarded as real. It is best explained by a population which is much younger than the typical bulge population in Baade's Window. The supergiants in the central cluster show this is at least partially true. The progenitor mass of IRS 7 is at least $20 \mathrm{M} \odot$ and so has a maximum age of about 10 million years. But aside from the stars in the inner 1.5 ' there are no confirmed supergiants from spectroscopic evidence. It could also be that these stars have a variety of ages. Many of the high luminosity stars would then be on asymptotic giant branch (AGB). For this explanation to be correct, the high luminosity stars would have masses of at least $3 \mathrm{M}_{\odot}$ and an age of no more than about 500 million years. The stars brighter than $M_{B O L}=-6$ require at least $5 \mathrm{M} \odot$ progenitors and ages around 125 million years (Maeder and Meynet 1989). If many of the high luminosity stars are on the AGB, they should be variable. This is the topic of the following chapter. In either case, the Galactic Center stellar population has a younger component than seen in Baade's Window which confirms the earlier suggestions (Lebofsky et al. 1982) of recent star formation at the Galactic Center. Whether this younger population represents a form of the
starburst phenomenon seen in other galaxies awaits further work.
High luminosity stars up to 1 magnitude brighter in $\mathrm{M}_{\mathrm{Box}}$ than the Baade's Window population have also been discovered using near-infrared JHK photometry in the central bulge of M31 (Davies, Frogel and Terndrup 1992; Rich and Mould 1991) and in M32 (Freedman 1992; Elston and Silva 1992). While Davies et al. (1992) contend that the luminous population is due to contamination from the disk of M31, the other workers interpret these populations as luminous AGB stars that have an age less than the Baade's Window population. The Galactic Center population discussed here is yet again another magnitude brighter than the M31 and M32 populations, indicating an even younger age. While the known spatial extent of the luminous Galactic Center population is much smaller by comparison, future survey work will investigate if these populations have further degrees of similarity.

## Conclusions

Two-color near-infrared photometry for a complete sample of 659 stars to an apparent K magnitude of 11.0 from the central $5^{\prime} \times 5^{\prime}$ of the Galactic Center shows that a significant luminous stellar population lies within 6 parsec of the Galactic Center. Self-consistent solutions to the colors of this population lead to an average $\mathrm{A}_{\mathrm{V}}$ of 31.8 magnitudes and an intrinsic $\mathrm{H}-\mathrm{K}$ of 0.3 . The luminosity function of the Galactic Center stars contains a substantial number of stars brighter than the brightest stars at $\mathrm{M}_{\text {bol }}=-4.6$ seen in Baade's Window. The Galactic Center population includes younger components than are present in the Galactic bulge. The inferred bolometric luminosities of these stars indicate an age of $100-500$ million years if they are on the AGB or 10 million years if they are supergiants but from these data alone it is not possible to decide what type of population is predominant.

## CHAPTER 4 :

## A SEARCH FOR LONG PERIOD VARIABLE STARS AT THE GALACTIC CENTER USING HKL PHOTOMETRY

## Background

Near infrared spectra of bright sources within the central parsec of the Galactic Center showed the large concentration of young supergiant stars having an estimated age of $10^{\circ} \mathrm{yr}$ (e.g. Neugebauer et al. 1976; Lebofsky et al., 1982; Wollman et al.. 1982). Lebofsky et al. (1982) suggested these stars were the result of a recent starburst episode in the Galactic Center and could explain the overall energetics of the region in an alternative way to interpretations that invoke a central massive black hole. The data analyzed in chapter 3 showed further evidence of a population that is younger than the AGB stars in Baade's Window from the excess number of high-luminosity stars brighter than the AGB stars in the bulge.

The Galactic Bulge population has been studied extensively in the area known as Baade's Window (Frogel and Whitford 1987) and has an estimated age of 11-14 Gyr (Terndrup 1988). The near-infrared light is dominated by late-type $M$ giant stars on the AGB and a large fraction of the brightest stars are Mira long period variables (LPVs) (Frogel and Whitford, 1987). These stars have been extensively studied using near-infrared techniques and their characteristics are well known. Miras are thought to be a terminal phase of AGB evolution during which the star loses mass at an increasing rate through a wind driven by radial stellar pulsations. The light variations caused by pulsation have a range of periods from 150 to 700 days and amplitudes at $\mathrm{K}(\mathbf{2} .2 \mu \mathrm{~m})$ from 0.1 to 1.0 mag (Feast et al., 1982; Whitelock, Feast and Catchpole 1991). There is also a Period $-\mathrm{M}_{\mathrm{BOL}}$ relation that is similar in many respects to that of Cepheid variables and is insensitive to metallicity differences as shown by a comparison
between the Galactic bulge and the LMC (Feast and Whitelock 1987). There has been a claim for an period-age relation (e.g. Wood and Bessell 1983) but this has been disputed (e.g. Feast 1986) since the P-L relation between the LMC and the bulge are indistinguishable within the errors. While Miras in the bulge are among the brightest members of that population there are also nonvariables at the top of the AGB which are just as bright (e.g. Frogel and Whitford, 1987; figure 17). In contrast, globular cluster Miras typically exceed the luminosity of all the AGB nonvariables (e.g. Frogel and Elias 1988). In LMC clusters, the brightness of stars at the AGB tip (where LPVs are predominantly found) is a decreasing function of time (Mould and Aaronson 1980). Therefore knowledge of the bolometric magnitudes of the LPVs gives an estimate for the age of an AGB population.

In sharp contrast to an AGB population, a stellar population of predominantly younger, high mass, late-type supergiant stars, will not show the Mira-like variability. High mass supergiants can be variable but the amplitudes are typically 0.1 mag or less at $2.2 \mu \mathrm{~m}$ and the luminosity changes are not periodic since they do not have the structural instability to excite the acoustic oscillations that characterize Miras.

This distinction between an older, low-mass population and a younger high-mass population is the basis for the variability experiment presented here. The goal is to determine if either of these two population types can predominantly characterize the high luminosity latetype stars in the central few parsecs of the Galactic Center. This is done using repeated imaging taken during the course of one year to reveal light variations with amplitudes characteristic of Miras. If there are few LPVs above the brightest characteristic luminosity of the bulge luminosity function then the brighter nonvariables are likely to be young supergiants and this would strengthen the starburst hypothesis. If the brightest stars in the luminosity function are predominantly LPVs then this argues for an older population and limits the extent of a recent starburst.

The data used in this experiment were previously introduced in chapter 2. Since these
observations are limited to only three epochs there are no complete light curves. The measurements for variability may also be complicated by the severe crowding. It is desirable to test the reliability of detecting LPVs through light variations and to verify that stars with high amplitude variations can be identified with Miras. The colors derived from the HKL data set, a subset of the complete HK sample, are compared with the properties of Miras in the nearby bulge and Baade's Window.

A possible mimicking of intrinsic variability for a given star at the Galactic Center could perhaps occur because of changes in the foreground extinction from the motions of intervening dust. Given the correlation of foreground extinction with the column density of hydrogen gas (e.g. Spitzer 1978) and the Galactic Center extinction law of Rieke and Lebofsky (1985), then table 4-1 shows the changes in K magnitude and $\mathrm{H}-\mathrm{K}$ color corresponding to the total fractional changes in the column density of foreground material:

TABLE 4-1 : Variability Effects Due To Changing Extinction

| $\Delta \mathrm{K}$ | $\Delta(\mathrm{H}-\mathrm{K})$ | $\Delta \mathrm{N}(\mathrm{H}) / \mathrm{N}(\mathrm{H})$ |
| :---: | :---: | :---: |
| 0.1 | 0.056 | 0.03 |
| 0.2 | 0.112 | 0.06 |
| 0.3 | 0.168 | 0.09 |
| 0.4 | 0.224 | 0.12 |
| 0.5 | 0.280 | 0.15 |

Individual clouds in the molecular ring have sizes on the order of $0.2 \mathrm{pc}\left(6^{\prime \prime}\right)$, rotation speeds of $-110 \mathrm{~km} \mathrm{~s}^{-1}$ and average densities of $-3 \times 10^{4} \mathrm{~cm}^{-3}$ with corresponding extinctions of $A_{V}=$ 30 mag (Genzel 1989). The total fractional column density changes required to show significant effects in the variability analysis discussed below would have to be in excess of $10 \%$. Changes in the fractional column density for a molecular ring cloud would have to be even more extreme, by at least a factor of 2 , since the column density of a cloud is comparable to all other foreground material. The timescale for such an effect can be estimated by the time for a clump
to cross in front of a given star: $-2 \times 10^{3} \mathrm{yr}$. The characteristic length scale that could probed by this experiment is approximately 20 a.u. during one year of observations. Thus the gas would have to be significantly clumped or shocked on length scales $5 \times 10^{-4}$ times smaller than the size of the cloud. It seems unlikely that this would make a significant contribution to the variability of the stars in the HK sample. In any case, a star with time-variable extinction could in principle be identified from a shift parallel to the reddening line which has a slope of 1.78 in the K vs. (H-K) diagram. However, this set of data only has color information from one epoch and would thus not be able to discriminate this effect from intrinsic variability. The extent of its influence on the data will be evaluated by light curves derived from future observations.

## Variable Star Data Set

The details of the of H and K database were discussed in chapter 2 but for completeness some of the important parameters are restated here. The photometric errors estimated from the overlapping of single frames were $\pm 0.088, \pm 0.070$, and $\pm 0.095 \mathrm{mag}$ for May 1987 , September 1987, and May 1988 respectively. The stars have a positional accuracy of $\pm 0.5$ ". The database is a magnitude and color sample of 659 stars empirically determined to be complete to $\mathrm{m}_{\mathrm{K}}=11$. Estimates of the foreground contamination due to disk stars show only $\approx 10$ stars would be in our final sample with an observed $\mathrm{H}-\mathrm{K}>1.5$. Also given the compactness of the Galactic Center stellar density distribution inferred from the $2.2 \mu \mathrm{~m}$ surface brightness profile, $90 \%$ of the observed stars within our field will lie within 50 pc of the Galactic Center and $60 \%$ will lie within $6.4 \mathrm{pc}\left(0.5 \times 5^{\prime} \times 6.9 \mathrm{pc}\right)$.

## Variability Analysis

To determine the intrinsic variability of these stars, residual systematic errors between
the three epochs of data had to be minimized. Our observations were taken at high air mass ( $z>2.00$ ) so there are expected to be significant variations in the sky thermal background. Also the data frames were very crowded and the PSF was nearly critically sampled. This resulted in variations of the PSF across the map on the order of arcminute length scales corresponding to the size of each data frame. This in turn reflects the maximum extent of a group of stars reduced in DAOPHOT. In some cases the group size exceeded the maximum allowed for DAOPHOT ( 60 stars), so the fitting radius of the PSF had to be reduced. To eliminate the systematic errors from the photometry, the average magnitude difference $<\mathrm{m}_{\mathbb{K}}(i)-\mathrm{m}_{\mathbb{K}}(j)>$ between each pair of epochs $i$ and $j$ was calculated for each star within a box of size $20^{\prime \prime}, 40^{\prime \prime}$, or $60^{\prime \prime}$ such that it contained at least 20 stars. If the stellar magnitudes were free from any systematic errors, it would be expected that the average of the distribution of magnitude residuals would be near zero. The observed raw residual magnitudes between the September 1987, $m_{K}(2)$, and May 1988, $m_{K}(3)$, observations are shown in figure 4-1. A noticeable constant offset is present as well as scatter that was most likely introduced from PSF and background variations across the map. The average residual in the immediate vicinity of each star, described above, is shown in figure 4-2. It is clear that systematic errors between epochs are not corrected by a simple constant, but that spatial variations in the systematic errors are present. Visual inspection of the spatial distribution of the average magnitude residual shows correlations with the placement of the data frame columns on the sky. Since the size of the systematic errors is comparable to the size of the amplitude variations to be investigated, they were removed by a simple subtraction. The improvement in the data by subtracting off the local average residual can be seen in figure 4-2. The average residual for each


Figure 4-1. - The observed raw magnitude residuals between the September $1987, \mathrm{~m}_{\mathbb{K}}(2)$, and May 1988, $\mathrm{m}_{\mathrm{K}}(3)$.


Figure 4-2 - The average magnitude residual $<m_{k}(2)-m_{k}(3)>$ between September 1987 and May 1988. The residual included all stars within a box around a given star of size $\operatorname{MIN}\left(20^{\circ}, 40^{\circ}, 60^{\prime \prime}\right)$ such that at least 20 stars were included in the average.


Figure 4-3. - The observed magnitude residual between September 1987 and May 1988 corrected by the average magnitude residual for each star. The average of the residuals is now 0.0 mag and the dispersion is reduced.
star is nearly zero and the dispersion is also reduced.
The data sample the stars at only three epochs and this is inadequate to reliably fit the amplitudes, periods, and phases for potential variable star candidates. Harmon and Gilmore (1988) attempted to estimate statistically the period distribution of Mira stars detected by IRAS using only three epochs of data. Subsequent work on bulge Miras (Whitelock, Feast, and Catchpole 1991) with greatly improved phase sampling showed that the statistical methods overestimated the periods of the Miras and so led to spurious underestimates to their ages. Thus, it was not deemed desirable to proceed with such an analysis on the Galactic Center stellar population. However, the inadequate phase coverage of the data presented here is offset by the ability to detect large amplitude variations that are characteristic of LPVs. The more modest approach applied here uses the amplitude variations to identify potential LPV stars and then evaluates their relative numbers and positions within the bolometric luminosity function.

The RMS magnitude residual between epoch pairs, $\Delta \mathrm{m}_{\mathrm{K}}$ was chosen as the parameter to identify the variable candidates (e.g. Reid, Glass, and Catchpole, 1988). The $\Delta \mathrm{m}_{\mathrm{K}}$ parameter is defined by the following general expression for N discrete observations:

$$
\Delta m_{K}^{2}=\frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N}\left(m_{K}(i)-m_{K}(j)\right)^{2}
$$

For this particular data set, $\mathrm{N}=3$ and the expression reduces simply to:

$$
\Delta m_{K}=\sqrt{\frac{1}{3}\left[\left(m_{K}(1)-m_{k}(2)\right)^{2}+\left(m_{K}(1)-m_{K}(3)\right)^{2}+\left(m_{K}(2)-m_{K}(3)\right)^{2}\right]}
$$

Confirmation of the intrinsic character of $\Delta \mathrm{m}_{\mathrm{K}}$ is shown in figure 4-4 which plots $\Delta \mathrm{m}_{\mathrm{K}}$ vs. $<\mathrm{m}_{\mathrm{K}}$ (3) $-m_{k}(1)>$ and $<m_{k}(3)-m_{k}(2)>$. The amplitude parameter values are not functions of the
corrections to the magnitude residuals, which implies that the effects of variability, apparent by high values of $\Delta \mathrm{m}_{k}$, are intrinsic and not an artifact of corrected systematic errors. Variable candidates were identified by values of $\Delta \mathrm{m}_{\mathrm{K}}$ at least $3 \sigma$ above the expected average for a nonvariable star with the same photometric errors as the stars in this sample. The expected distribution of $\Delta \mathrm{m}_{k}$ for nonvariable stars was estimated using a Monte Carlo calculation with $10^{7}$ trials. The average nonvariable residual, $\left\langle\Delta m_{k}\right\rangle$, was $0.106 \pm 0.056$ mag; thus the imposed minimum $\Delta m_{K}$ for a LPV star to be identified at the $3 \sigma$ level was 0.274 mag. From this calculation we can also reconstruct the nonvariable distribution of $\Delta \mathrm{m}_{\mathrm{K}}$ values expected from the time sampling intervals of our data set, averaged over phase. This distribution is shown in figure 4-5 along with the observed distribution of $\Delta m_{k}$. The estimated nonvariable distribution is fitted to have the same number of stars that occur in the first six bins of the observed distribution ( $78 \%$ of the total sample). The widths of the two distributions match very closely in this part of the diagram, giving confidence that we have reasonably estimated the photometric errors. It is very important to note that, while most of our stars are indistinguishable from nonvariables, there is an excess of stars with high values of $\Delta \mathrm{m}_{\mathrm{k}}$. In figure 4-6 the RMS residuals $\Delta m_{k}$ vs. $m_{k}(3)$ show that there is no significant trend of LPV amplitude with magnitude so we cannot attribute the effect to incompleteness effects. The 59 stars with $\Delta \mathrm{m}_{\mathrm{k}}>0.274$ will be regarded as LPV candidates.

The efficiency of this method in detecting LPVs was estimated by a further extension of the above Monte Carlo calculation. The LPV light curves were modelled as sinusoidal functions of period and amplitude with randomized phase, $\phi$, sampled at times $t_{i}$ of $0^{d}, 120^{d}$, and $387^{\text {d }}$ corresponding to the time intervals of our observations:

$$
m_{\Sigma}(i)=A \sin \left[2 \pi\left(\frac{t_{i}}{P}\right)+\phi\right]+e_{i}
$$



Figure 4-4. - The average magnitude residual correction vs. $\Delta \mathrm{m}_{\mathrm{K}}$. This shows the derived values of $\Delta m_{K}$ are not functions of the corrections for systematic errors.


Figure 4-5 - The observed distribution of $\Delta \mathrm{m}_{\mathrm{K}}$ binned to 0.025 mag. The expected $\Delta \mathrm{m}_{\mathrm{K}}$ distribution for nonvariable stars with the same photometric errors calculated by a Monte Carlo simulation is shown by the dashed line. The simulated nonvariable distribution is normalized to have the same number of stars as the observed distribution for $\Delta \mathrm{m}_{\mathrm{K}}<0.15 \mathrm{mag}$ ( $78 \%$ of our observed sample). The excess number of stars at high values of $\Delta \mathrm{m}_{\mathrm{K}}$ is due to LPVs. Nonvariable stars can be excluded from our LPV sample at the $3 \sigma$ level for $\Delta m_{k}>0.274$.


Figure 4-6-The $\Delta m_{K}$ vs. $m_{k}(3)$ diagram. The stars above the horizontal line at $\Delta m_{K}=0.274$ are the variable candidates; stars below the line cannot be distinguished from nonvariables. The diagram also shows there is no strong correlation of LPV amplitude with magnitude.

At each epoch a random photometric error, $\varepsilon_{j}$ was added that was drawn from a Gaussian distribution with a dispersion corresponding to the photometric error of that epoch. A variety of periods, P, and amplitudes, A, were chosen to encompass the observed characteristics of Galactic Miras with periods ranging from 50 to 800 days and half amplitudes from 0.0 to 0.5 magnitudes (Feast et al., 1982). A large number of trial observations $\left(\approx 10^{6}\right)$ with randomized phase was then made at each period and amplitude and the normalized distribution of $\Delta \mathrm{m}_{\mathrm{K}}$ values was calculated. For nearly the whole range of input amplitude and period values, the $\Delta \mathrm{m}_{\mathrm{K}}$ distributions were approximately Gaussian so the fraction of stars with $\Delta \mathrm{m}_{\mathrm{K}}>0.274$ could be easily estimated.

Thus for each period and amplitude, the average magnitude residual, $\left\langle\Delta \mathrm{m}_{\mathrm{k}}\right\rangle$, and its dispersion were used as the input parameters of a normalized Gaussian distribution. The resulting total, phase-averaged detection probability, for LPVs with $\Delta \mathrm{m}_{\mathrm{K}}>0.274$ mag, as a function of period and amplitude, is shown in figure 4-7. For periods greater than 160 days there is a general trend of increasing detectability with increasing amplitude and a nearly constant level of detectability at a given amplitude with approximately $10 \%$ variations with increasing period. There is rapidly fluctuating sensitivity for periods below 140 days, but in this region of the A-P plane the total detection probability never greatly exceed $50 \%$.

To determine the variable detection efficiency at a given period some form of amplitude distribution must be assumed. From what is known of the Galactic neighborhood Miras (Feast et al., 1982) there is a relatively uniform distribution of amplitudes with $0.2<\mathrm{A}<1.0 \mathrm{~K} \mathrm{mag}$ for the 100 to 300 day periods. The amplitudes at longer periods have large average values because there are few with A $<0.5$ mag. Figure 4-8 shows the average detection probability at a given period for various assumptions of the amplitude distribution. All assume a uniform distribution of variable amplitudes between $\mathrm{A}_{\mathrm{Low}}=0.0,0.1,0.2,0.3$ and $\mathrm{A}_{\mathrm{Hi}}=0.5$ mag. The most conservative case would be for a population containing very low amplitude variables ( $\mathrm{A}_{\text {Low }}=$ 0.0 ). Although


Figure 4-7. - The contours of the total probability that a LPV of a given period and amplitude will have an $\Delta m_{K}>0.274 \mathrm{mag}$. The lowest contour of 0.05 is at the lower part of the figure while the highest contour of 0.95 is in the upper right. Contour intervals are in steps of 0.05 .


Figure 4-8. - The total amplitude-integrated probability that a variable of a given period will have $\Delta m_{K}>0.274$ mag for various amplitude distributions. The solid line assumes amplitudes uniformly distributed between 0.0 and 0.5 mag . Subsequent lines also assume a uniform distribution with the same upper bound but with lower bounds of $0.1,0.2$, and 0.3 mag.


Figure 4-9. - The average $\mathrm{M}_{\mathrm{BOL}}$ vs. (H-K) ${ }_{\text {cbs }}$ for LPVs and the total distribution. The bin size is 0.25 mag. The LPVs on average are systematically redder while the non-LPVs tend to be bluer at the bright end relative to fainter non-LPVs and LPV samples respectively.
the resulting observed amplitude distribution would not well represent the intrinsic distribution, the calculations shows there is a 45\% chance of detecting variables above 200 days, while there is a slightly lower average value for shorter periods. For those distributions that have preferentially higher amplitudes, the detection efficiency increases considerably. If the amplitude distribution of LPVs at the Galactic Center is similar to Miras in the Solar neighborhood, approximately $75 \%$ of the longer period ( $\mathrm{P}>300^{d}$ ) variables in this sample have been distinguished from nonvariables.

## Indications of Variability From HKL Photometry

Having selected LPV candidates from high values of $\Delta \mathrm{m}_{\mathrm{k}}$, they are further distinguishable by higher values of the average dereddened H-K (figure 4-9). The LPV sample is redder in $(\mathrm{H}-\mathrm{K})_{\text {obs }}$ by approximately 0.3 mag from the total sample of stars within the expected range for bulge Miras. This result is consistent with the circumstellar reddening that is common with mass-losing stars on the AGB. The nonvariable stars at the bright end of the distribution are also bluer than both LPVs and non-LPVs below $\mathrm{M}_{\mathrm{BOL}}$ of -4.0.

Figures $4-10$ and $4-11$ show the dereddened ( $\mathrm{H}-\mathrm{K}$ ) vs. ( $\mathrm{K}-\mathrm{L}$ ) diagrams for the nonvariables dereddened by an assumed $A_{v}=31.8 \mathrm{mag}$ (see chapter 3 ). The nonvariable color distribution clusters near $(\mathrm{K}-\mathrm{L})=0.1$ with a dispersion consistent with the photometric errors and with no clear correlation in the two colors. The color-color distribution for the LPV sample is quite different with a strong color correlation present and an average $(\mathrm{K}-\mathrm{L})=0.5$. The range of the LPV (K-L) color distribution is not as great as Baade's Window (Frogel and Whitford 1987) or the sample of Miras at $1= \pm 7$ selected by IRAS colors (Whitelock et al. 1991) because of the completeness criteria that $\mathrm{m}_{\mathrm{k}}<11.0 \mathrm{mag}$ for the complete K sample. In any case, the correlation in the LPV sample is clearly consistent with the expected effects of varying degrees of reddening caused by circumstellar dust shells that condense out of the stellar winds
associated with Mira pulsation (e.g. Bowen and Willson 1991).
A more interesting result comes from the comparison of the ( $\mathrm{K}-\mathrm{L}$ ) and ( $\mathrm{H}-\mathrm{K}$ ) colors with $\Delta m_{k}$, the amplitude parameter used to select LPVs. Figure 4-12 shows dereddened (H-K) vs. $\Delta \mathrm{m}_{\mathrm{K}}$, with a vertical line at $\Delta \mathrm{m}_{\mathrm{K}}=0.274$ that separates the nonvariable stars and LPV candidates. It is apparent in the diagram that LPV candidates are redder on average than the nonvariables in (H-K) by 0.3 mag. But they never dominate any portion of the diagram at constant ( $\mathrm{H}-\mathrm{K}$ ). They are only distinguishable from the nonvariable because there are relatively few at bluer ( $\mathrm{H}-\mathrm{K}$ ) colors, but overall the color distributions in ( $\mathrm{H}-\mathrm{K}$ ) overlap to such an extent that the LPV sample cannot be clearly separated. However, the corresponding diagram of dereddened (K-L) vs. $\Delta \mathrm{m}_{\mathrm{K}}$ shows a detectable correlation as illustrated in figure 4-13; (K-L) increases as $\Delta m_{k}$ increases. Figure 14 from Whitelock et al. (1991) shows a similar behavior for IRAS-selected Miras in the bulge at $1= \pm 7^{\circ}$. This effect can again be understood as an increase in mass loss with increasing pulsational amplitude. It is important to note that this effect is peculiar to Miras. Cepheid variables, for instance, are not subject to substantial mass-loss effects while for Miras the size of the circumstellar dust shell (and hence the amount of increased reddening) is directly related to the pulsational amplitude.

Even in the absence of complete light curves, it is clear the HKL color differences between the LPV and nonvariable samples show that the $\Delta \mathrm{m}_{\mathrm{K}}$ parameter is a robust indicator of Mira variability despite the small number of observations. The Mira sample is certainly incomplete and poor phase coverage means the reddest stars in the nonvariable sample are likely to be undetected LPVs. Further confirmation comes by comparing these stars with known $\mathrm{OH} /$ IR stars that lie in this field. There are 4 out of $11 \mathrm{OH} /$ IR stars of the Lindquist et al. (1992a) sample within the central $5^{\prime} \times 5^{\prime}$ that match with objects in the HK sample (table 4-2). Each has at least one color and/or amplitude consistent with the properties of $\mathrm{OH} / \mathrm{IR}$ stars. The fact that all were not identified is not surprising considering the large circumstellar extinction caused by dust shells around $\mathrm{OH} / \mathrm{IR}$ stars.


Figure 4-10. The dereddened ( $\mathrm{K}-\mathrm{L}$ ) vs. ( $\mathrm{H}-\mathrm{K}$ ) diagram for the "nonvariable" sample of stars detected at $\mathrm{H}, \mathrm{K}$, and L . The dispersion in the diagram is consistent with the photometric errors.


Figure 4-11. The dereddened (K-L) vs. (H-K) diagram for the LPV sample of stars detected at $\mathrm{H}, \mathrm{K}$, and L . There is a strong color-color correlation for this sample of stars selected on the basis of $\Delta \mathrm{m}_{\mathrm{K}}$.


Figure 4-12 The $(\mathrm{H}-\mathrm{K})_{0}$ vs. $\Delta \mathrm{m}_{\mathrm{K}}$ diagram for the HKL sample. The vertical line shows the $3 \sigma$ cutoff for nonvariables with $\Delta m_{K}>0.274$. The LPV candidates are shown with filled circles and nonvariables with open circles. The horizontal line is the average ( $\mathrm{H}-\mathrm{K})_{0}$ for the nonvariable sample and emphasizes the systematically redder ( $\mathrm{H}-\mathrm{K})_{0}$ color of the LPVs, but a clear color correlation is not so apparent.


Figure 4-13: The (K-L) ${ }_{0}$ vs. $\Delta m_{\mathrm{K}}$ diagram for the HKL sample. The vertical line shows the $3 \pi$ cutoff that excludes nonvariables for $\Delta m_{K}>0.274$. The LPV candidates are shown in filled circles and nonvariables are shown with open circles. The nonvariables have an average ( $\mathrm{K}-\mathrm{L})_{0}$ near zero while the LPVs show a correlation of color with amplitude parameter. This is characteristic of Mira variables studies in the bulge (Whitelock et al. 1991, fig. 14).

TABLE 4-2: OH/R Star Identifications

| Source $^{\mathbf{a}}$ | $\mathrm{ID}^{\mathrm{b}}$ | $\mathrm{m}_{\mathrm{k}}$ | $\Delta \mathrm{m}_{\mathrm{K}}$ | $(\mathrm{H}-\mathrm{K})_{0}$ | $(\mathrm{~K}-\mathrm{L})_{0}$ | $\mathrm{M}_{\mathrm{BOL}}$ |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: |
| OH359.946-0.047 | 455 | 9.17 | 0.63 | 0.58 | - | -5.44 |
| OH359.954-0.041 | 436 | 10.73 | 0.25 | 0.73 | 0.82 | -3.88 |
| OH359.970-0.047 | 562 | 8.22 | 0.15 | 0.61 | -0.27 | -6.38 |
| OH349.986-0.061 | 647 | 10.15 | 0.64 | 2.50 | 2.03 | -4.45 |

"Lindquist et al. (1992a), ${ }^{\text {b }}$ Appendex 2

## Discussion

The bolometric luminosity function derived from the data set in chapter 2 is shown in figure 4-14 along with the distribution of the LPV candidates. The expected periods from the P- $\mathrm{M}_{\mathrm{BOL}}$ relation for Miras are shown at the top of the diagram for comparison (Feast et al., 1982). Several features are to be noted in interpreting these results. First, these stars are well above the He flash luminosity of the RGB extrapolated to $[\mathrm{Fe} / \mathrm{H}]=1.0$ by the He flash - metallicity relation of Sweigart and Gross (1978) and so they are too bright to be first-ascent giant stars. Secondly, they lie below the maximum luminosity of an $A G B$ star of $M_{B O L}=-7.1$ implied by the core mass luminosity relationship (Paczynski, 1971) and so on the basis of their luminosities they are either bright AGB stars or lower luminosity supergiants. Studies in the LMC show that AGB stars characteristically have K amplitudes greater than 0.25 mag while supergiant K amplitudes are less than 0.25 amplitudes (Wood, Bessel, and Fox, 1983). The imposed cutoff of $\Delta m_{K}$ of 0.274 mag in the Galactic Center sample selects against supergiants and favors AGB stars. For these reasons and the fact that stars in our sample with high values of $\Delta \mathrm{m}_{\mathrm{K}}$ are systematically redder in $(\mathrm{H}-\mathrm{K})_{\text {obs }}$ and $(\mathrm{K}-\mathrm{L})_{\text {obs }}$ and the correlation between $(\mathrm{K}-\mathrm{L})_{0}$ and $\Delta \mathrm{m}_{\mathrm{K}}$, we infer that these stars are Mira LPVs on the AGB. Applying the $M_{B O L}$ vs. age calibration of the peak AGB luminosity and turnoff age (Mould and Aaronson 1980; Aaronson and Mould 1982) with the consideration that these stars are likely to be super-metal-rich (Blanco 1988), they have


Figure 4-14. - The derived bolometric luminosity function for the 659 stars in our complete sample with $\mathrm{m}_{\mathrm{K}}<11.0$ and $\mathrm{H}-\mathrm{K}$ color $>1.50$. The solid histogram shows the total distribution while the dashed histogram is for the LPV candidates with $\Delta m_{K}>0.27$ mag. The expected periods for LPVs from the $\mathrm{M}_{\mathrm{BCL}}$ vs. Period relation of LMC and bulge Miras is shown for comparison (Feast et al. 1982).
ages on the order of $2-5 \times 10^{8} \mathrm{yr}$. This is an order of magnitude younger than stars in Baade's Window but also an order of magnitude older than the young $10^{7} \mathrm{yr}$ supergiant population in the central cluster. Blöcker and Schönberner (1991) recently reported there may be a breakdown in the core mass-luminosity relation for AGB stars with masses of the order of 7 M©. The brightest stars in our sample have bolometric luminosities at approximately the same level, and if this result holds true it would substantially reduce the age estimates below the values estimated by the Aaronson and Mould calibration (1982). Unfortunately, more detailed interpretations of this population are problematic because stellar evolution models are not generally available for intermediate- and high-mass stars with high metallicities and young ages. Such models would be highly desirable for interpreting the Galactic Center and other highmetallicity star-forming regions, such as starburst galaxies.

There are several other features to note when comparing the LPV distribution at the Galactic Center to the bulge population in Baade's Window at $-3.8^{\circ}$ and other outlying fields (Frogel and Whitford, 1987; Frogel et al., 1990). The greatest frequency of Galactic Center LPVs occurs in the magnitude range of the bulge LPVs $\left(-4.5 \leq \mathrm{M}_{\mathrm{BOL}} \leq-3.2\right)$. This result should be compared to the fractional distribution of bulge LPVs shown in figure 4-15 (Frogel and Whitford 1987; Frogel et al. 1990). The greatest fractional contribution of LPVs in the bulge luminosity function occurs at the tip of the AGB near $M_{B O L}=-4.5$ above which there is only one LPV. Since the highest frequency of LPV candidates in our sample occurs near this bulge cutoff it is conceivable that this is due to a bulge population component. If this is true, then the bulge population has been traced in the near infrared from the halo at $-12^{\circ}$ (Frogel and Whitford, 1987; Frogel et al., 1990) all the way into the center of our Galaxy. But significantly unlike the bulge population, the Galactic Center LPV population has no cutoff above $\mathrm{M}_{\mathrm{BOL}}=-4.5$ as would be expected for an older AGB sample. There are significant numbers of LPVs extending to higher luminosities, by as much as 1.5 mag. Even so, the LPVs never dominate the luminosity function within any bin. Given the estimated LPV detection frequencies, this implies that most of the


Figure 4-15. - The fractional distribution of LPVs in the bolometric luminosity function. The fractional contribution of Bulge LPVs to each bin of the Baade's Window luminosity function is represented by the dashed line (Frogel and Whitford 1987 and Frogel et al. 1990). The fractional contribution of LPVs to each bin in the observed Galactic Center luminosity distribution is shown by the solid line while the dotted line shows the same distribution corrected for $=75 \%$ detection efficiency expected for an LPV popuiation with preferentially high amplitudes.
stars at the bright end of the bolometric luminosity function are nonvariable.
The remaining bright nonvariables with $\mathrm{M}_{\mathrm{BO}}<-4.5$ are either older AGB stars or younger supergiants. Supergiants would be distinguished by bluer (H-K) colors than AGB stars and this agrees with the observed color distribution. The presence of stars like IRS 7 in the central parsec and the low frequency of LPVs in the luminosity distribution suggest that many of these remaining stars could be supergiants as well. If this is so, then massive star formation associated with the central cluster happened over a much larger volume than the central few parsecs. The best way to confirm supergiants would be to examine the $2.3 \mu \mathrm{~m}$ CO absorption feature between the LPV and nonvariable samples. If the nonvariables are supergiants they should show systematically deeper CO .

In globular clusters, LPVs derive their luminosity from H and He shell burning and are always brighter than the H shell burning stars on the first ascent of the RGB (e.g. Frogel and Elias 1988). This feature has been incorporated in semi-empirical theoretical studies of stellar evolution (e.g. Charlot and Bruzual 1991) based on stellar evolutionary tracks (Maeder and Maynet 1989). One important exception is the SMR population observed in Baade's Window (Frogel and Whitford 1987) where not all of the brightest stars on the AGB are LPVs. As mentioned in chapter 2, counts of M giants in the bulge at different Galactic latitudes (Blanco 1988) show the frequency of $M$ giants decreases faster with increasing distance away from the Galactic Center than would be expected from an $\mathrm{R}^{1 / 4}$ law. This was attributed to the effect of increasing metallicity toward the Galactic Center. Frogel and Whitford (1987) asserted that stars on the giant branch have enhanced luminosities because higher metallicity translates into a longer time spent on the main sequence. So the strong possibility of high metallicity in the Galactic Center would mean that at least some of the bright nonvariables in the luminosity function will be nonvariable giants of the same population as the Miras. The contribution of these nonvariables is presently unknown. Even so, they are still be far too bright for an old bulge population and are more similar to the young clusters in the LMC (Mould and Aaronson
1980). Those Miras with $M_{B O L}=-6$ (there are 4 such stars in our sample) imply a main-sequence turnoff mass of $=5 \mathrm{M} \odot$ indicating a prior episode of star formation occurring in the same region approximately $10^{8} \mathrm{yr}$ ago (Charlot and Bruzual A., 1991). The calculation in Appendix 1 estimating the proximity of these stars to the Galactic Center shows that $90 \%$ of the observed population is within 50 pc of $\mathrm{R}_{0}$ because of the sharply peaked stellar density distribution that is implied by the $2.2 \mu \mathrm{~m}$ surface brightness profile so explanations resorting to foreground contamination are untenable.

It is significant that either choice for the nonvariables is consistent with theoretical models of periodic star formation at the Galactic Center discussed previously (e.g. Loose, Krügel, and Tutukov, 1982), where one would expect to find different temporal "strata" of stars within the region. Whether or not a central engine drives the overall dynamics or energetics of this region, star formation has occurred at least several times, within intervals much less than the age of the Galaxy.

Such a picture of continual star formation at the Galactic Center rests on the identification of the LPVs as young AGB stars. In the absence of complete light curves, one may question the strength of these conclusions. The obvious way to confirm this hypothesis would be to obtain accurate light curves by follow up observations over several years. Such an experiment is now underway but data have not been reduced to a sufficient extent to derive periods.

It is then clear that star formation has occurred in the Galactic Center at different times. An important question to ask is what is its spatial extent? It would be useful to have some way to sample easily these populations through future near-infrared surveys. This may indeed be possible for a young AGB population. Because of the strong circumstellar reddening by dust shells it is likely that those stars with a (K-L) $>1.0$ are LPVs. Comparing the fractional contribution of stars with red K-L to the total luminosity function may indicate whether the remaining bright nonvariables are supergiants. If the supergiant population is confined to the
central few arcminutes of the Galactic Center, then with increasing projected distance from the center, a larger fraction of luminous stars will have $(\mathrm{K}-\mathrm{L})_{0}>1.00$. This can then also be used to find LPV candidates for further study. There were approximately $2 \mathrm{LPVs}_{\mathrm{arcmin}}{ }^{-2}$ in the K photometry sample. With the extremely high source density ( $\approx 25$ stars $\operatorname{arcmin}^{-2}$ ) in this region the opportunities for acquiring large samples of complete LPV light curves would be promising because variable candidates could be identified by a photometric index that avoids the tedious procedure of acquiring complete light curves for a large sample of stars to identify the variables.

Lindqvist, Habing, and Winnberg (1992b) showed their sample of OH/IR stars divided into two distinct spatial groups by the expansion velocity of the OH emission line. Those with $v_{\text {exp }}>18 \mathrm{~km} \mathrm{~s}^{-1}$ were spatially concentrated to within $16^{\prime}(32 \mathrm{pc})$ of the center and they interpreted them as a population of relatively younger AGB stars (Baud et al. 1981). This may be an indication of how far from the center the high luminosity AGB population extends. All of the Lindqvist et al. (1992a) OH/IR stars that were identified with the HK sample stars in table $4-2$ belonged to this younger population.

## Conclusions

We have searched for LPVs within the central $5^{\prime} \times 5^{\prime}$ of the Galactic Center from a data set of three $2.2 \mu \mathrm{~m}$ epochs and one $1.6 \mu \mathrm{~m}$ epoch using the RMS residual magnitude differences between epochs. Monte Carlo calculations show that, with our photometric errors, nonvariable stars will have $\left\langle\Delta m_{K}\right\rangle=0.106 \pm 0.056$ mag. This was used to determine a $3 \sigma$ cutoff in $\Delta m_{K}$ to exclude nonvariable candidates from our LPV sample. If the amplitude distribution of the Galactic Center Miras is similar to that in the solar neighborhood, then a majority of the brightest LPV's have been detected. The number of detected variables is greatest in the range characteristic of bulge LPVs but does not show the expected cutoff at $\mathrm{M}_{\mathrm{BOL}}<-4.5$. Galactic Center LPVs have an average ( $\mathrm{H}-\mathrm{K})_{\text {obs }}$ color $0.2-0.3$ mag greater than the sample without LPVs in nearly every magnitude bin of the luminosity distribution. From this we conclude the LPVs in our sample are Miras on the AGB.

The results of HKL photometry for 264 stars taken from the complete sample of HK stars show that stars selected with as LPV candidates have strongly correlated $(\mathrm{H}-\mathrm{K})_{0}$ and $(\mathrm{K}-\mathrm{L})_{0}$ as well as (K-L) $)_{0}$ vs. $\Delta \mathrm{m}_{\mathrm{K}}$. Both correlations support the procedure used to identify the variables and their identification as Miras.

The luminosities of the LPVs and presumed high metallicity indicate the ages for these stars is on the order of $5 \times 10^{8}$ years. This shows that star formation has occurred over at least 6pc length scales within the Galactic Center and the importance of this process to the energetics to the region merits consideration. Given the presence of the $10^{\circ}$-year-old supergiants in the central cluster, it would also appear that star formation is also recurrent. The stars at the Galactic Center region are not a simple bulge population but are a composite of at least three distinct stellar populations characterized by different periods of star formation. First, there is the expected older bulge-like population identified by the peak in the number distribution of
the Mira LPVs at the characteristic cutoff luminosity of the bulge. Secondly, there are the extremely young massive stars which reside within the central $1.5^{\prime}$ identified by such members as IRS 7 and the IRS 16 complex. Thirdly, there is an intermediately bright distribution of bright Mira LPVs, that are brighter than the typical bulge AGB. The nature of the intermediately bright nonvariable population remains to be determined.

## CHAPTER 5 :

## THE UNRESOLVED LATE-TYPE STELLAR POPULATION WITHIN 30" OF SGR A* AND KINEMATIC EVIDENCE FOR A $1.7 \times 10^{6} \mathrm{M} \odot$ BLACK HOLE

## Background

Previous chapters examined the late-type stars within 6 pc of the Galactic Center showing that a composite population was likely formed by several episodes of star formation. This chapter will focus on the unresolved stars within 1 pc of the dynamical center, provisionally identified with Sgr A*. Previous spectral studies of this central stellar cluster have clarified the general nature of the brightest resolved sources. Several attempts have been made to traced the enclosed mass distribution about the center, using the motions of the ionized and molecular gas as well as the resolved and unresolved stars, to place upper limits on the mass of a central black hole. The surest way to probe for a large compact mass would be the surrounding stars because they are not dynamically affected by ambient magnetic fields and interact only through gravitation. If such a compact object exists it could ultimately serve as the central engine driving the energetics and of surrounding phenomena. This task has been complicated because of ambigous dynamical evidence and the presence of other known sources (i.e. IRS 16) that could alternatively explain the overall features through something resembling a starburst.

## Previous Studies of the CO $2.3 \mu \mathrm{~m}$ Absorption Feature

The only near infrared spectral feature that has been used to measure the enclosed mass distribution by stars is the $2.3 \mu \mathrm{~m} C O(v=2 \leftarrow 0)$ transition because the extinction in the K band (approximately $0.1 \times A_{V}$ ) is rather modest, it is a strong feature in the spectra of late-type stars, and the fact that no other line can be reliably used at shorter wavelengths. Neugebauer et al.
(1976) first classified the bright sources of the central cluster as supergiants because of strong CO absorption and the blue continuum of IRS 16 implied it was either a cluster of M dwarfs or luminous early-type stars. These results were confirmed by many subsequent workers (Hall, Kleinmann, and Scoville 1982; Wollman, Smith, and Larson 1982; Lebofsky, Rieke, and Tokunaga 1982). Lebofsky et al. (1982) extended the list of supergiant stars and proposed the starburst hypothesis. Wollman et al. (1982) measured radial velocities for sources 7, 12, and 19 and found they were all blue shifted to the local standard of rest, with a slight trend of increasing velocity with decreasing distance to IRS 16.

Four extensive CO velocity studies have been made in recent years (Sellgren et al. 1987; Rieke and Rieke 1988; McGinn et al. 1989; Sellgren et al. 1990), attempting to measure the enclosed mass versus radius in the vicinity of IRS 16 and $\operatorname{Sgr} \mathrm{A}^{*}$. The first two works measured the radial velocities of resolved sources while the second pair used the velocity dispersion of the fainter unresolved emission. Because of the common theoretical basis of these studies as well as the work presented here, it is appropriate to outline the three methods used to measure the enclosed mass vs. radius using velocity dispersions.

Theoretical Relations for the Enclosed Mass vs. Radius of Spherically Symmetric Systems

For radial velocity measurements of individual sources, a minimum estimate of the enclosed mass is made by assuming a gravitational potential dominated by a single object resulting in stellar orbits that approach Keplerian (Sellgren et al. 1987). From conservation of energy, the minimum enclosed mass within a projected radius $r$ will be given by

$$
M(r) \geq \frac{v_{\operatorname{los}}^{2} r}{G}
$$

where $v_{\text {bs }}$ is the line-of-sight radial velocity.
A more precise estimate can be made using the "projected mass" method of Bachall and Tremaine (1981). The mass estimate derived from each star is of the same form but is suitably weighted for the predominant type of stellar orbit (i.e. circular or radial) and overall mass distribution (dominant central mass or distributed mass). The general form of this mass estimator is given by

$$
M(R)=\frac{C}{\pi} \frac{1}{N} \sum_{i=1}^{N} \frac{v_{i}^{2} r_{i}}{G} \quad r_{i} \leq R
$$

where the normalization constant for a dominant central mass, $C$, is $32 / 3$ for circular orbits, 16 for isotropic orbits, and 32 for radial orbits. These normalizations are doubled for a self gravitating stellar system (Heisler et al. 1985). The fractional error of the estimated mass is proportional $N^{-0.5}$ and depends slightly on the mass distribution and orbit type. The virial theorem is not preferred for these estimates because it does not always converge on the correct value of the mass, or does so only very slowly as $N \rightarrow \infty$. The projected mass method will give similar answers as the virial theorem (when the virial theorem works correctly) and its variance is also well behaved (unlike the variance of the virial theorem mass estimator).

A third method derives the enclosed mass from the observed velocity dispersion, $\sigma(r)$, and rotational velocity, $\mathrm{V}_{R O T}(\mathrm{r})$, as well as information from the total light distribution by utilizing the first moment of the equation of stellar hydrodynamics (Hartwick and Sargent 1978):

$$
M(r)=\frac{r \sigma(r)^{2}}{G} \times\left[-\frac{d \ln n(r)}{d \ln r}-\frac{d \ln \sigma(r)^{2}}{d \ln r}+(\lambda-2)+\frac{V_{R O T}(r)^{2}}{\sigma(r)^{2}}\right]
$$

Here $n(r)$ is the number density of test particles, and $\lambda$ is the ratio of the tangential to radial
velocity dispersions. This method requires more information about the distribution of sources and their orbits but, generally, reasonable assumptions can be made to reduce the complexity of its application. For instance, the run of $n(r)$ is generally derived from the $2.2 \mu \mathrm{~m}$ surface brightness distribution which implies a density law of $n \propto r^{-1.8}$ for a constant mass to light ratio (Neugebauer et al. 1968; Allen et al. 1983). There is also evidence that the stellar orbits are isotropic and so $\lambda=2$ (McGinn et al. 1989).

The Galactic Center Stellar Kinematics

A study of $\mathrm{OH} / \mathrm{IR}$ stars by Winnberg et al. (1985) measuring the 1612 MHz OH maser line radial velocities for 33 stars in the central $34^{\prime} \times 34^{\prime}$ was able to determine a rotational velocity gradient of $270 \pm 80 \mathrm{~km} \mathrm{~s}^{-1}$ per degree in this region. Several of these stars have also been used in the following studies of the enclosed mass vs. radius near the Galactic Center.

The first extensive velocity study in the central few parsecs by Sellgren et al. (1987) used $120 \mathrm{~km} \mathrm{~s}^{-1}$ resolution data on sources $7,11,12,19,22$, and 23 with a $3.8^{\prime \prime}$ aperture. They applied each of the above methods to their data, augmented by seven $\mathrm{OH} / \mathrm{IR}$ star velocities (Winnberg et al. 1985). Their data showed that the motions of the gas and the stars are distinct and are not likely to be coupled. Assuming no velocity dispersion gradient, no bulk stellar rotation, and isotropic orbits they were able to apply the stellar hydrodynamic equation, which reduced to a simple linear function of radius. All three enclosed mass estimators indicated approximately $1 \times 10^{6} \mathrm{M} \odot$ of mass was concentrated within 0.3 pc of IRS 16 and $2-10 \times 10^{6} \mathrm{M} \odot$ within $1 \mathrm{pc}\left(\mathrm{R}_{0}=8.5 \mathrm{kpc}\right)$. In general the enclosed mass vs. radius was proportional to radius, unlike the results derived from gas velocities which show a constant mass distribution inside of 10 pc (Gatley et al. 1984; Ekers et al. 1983; Geballe et al. 1984).

The list of radial velocities was greatly extended by Rieke and Rieke (1988) who measured CO velocities for 43 stars within $6.5 \mathrm{pc}\left(\mathrm{R}_{0}=8.5 \mathrm{kpc}\right)$ of the Galactic Center at 277 km
$\mathrm{s}^{-1}$ resolution. They also included eight sources from Winnberg et al. (1985) and the IRS 19 and IRS 23 measurements of Sellgren et al. (1985) in their analysis. Significantly, the velocity dispersion of their sample was constant at $72 \mathrm{~km} \mathrm{~s}^{-1}$ out to a distance of 2 pc . This was much smaller than the velocities of the gas which are typically $\sim 300 \mathrm{~km} \mathrm{~s}^{-1}$ within the same region which again showed that the stars and the gas have very different kinematic properties. The projected mass method also excluded a dominant central mass model because the enclosed mass was an increasing function of distance. Whatever compact object may reside at the center, it does not dominate the total mass beyond 0.5 pc , since the stellar cluster contributes a nonnegligible fraction as evidenced by the rising $\mathrm{M}(\mathrm{r})$ curve. Further, if the core radius of the stellar distribution is 0.1 pc (Allen, Hyland, and Jones 1983) then no anomalous M/L ratio results but a core radius of 0.6 pc (Rieke and Lebofsky 1987) would make their data compatible with a central compact object of $2 \times 10^{6} \mathrm{M} \odot$.

McGinn et al. (1989) and Sellgren et al. (1990) examined the CO band velocities of the diffuse stellar emission within $\pm 90^{\prime \prime}$ of $\mathrm{Sgr} \mathrm{A}^{*}$. Using $120 \mathrm{~km} \mathrm{~s}^{-1}$ velocity resolution observations and a $20^{\prime \prime}$ aperture that sampled positions every $10^{\prime \prime}$ along the Galactic equator and meridian passing through IRS 16, they were able to measure both the rotation curve and the gradient in the velocity dispersion. The velocity dispersion peaked at $=125 \mathrm{~km} \mathrm{~s}^{-1}$ within $20^{\prime \prime}$ of IRS 16 and exceeded the rotational velocity by at least a factor of four, indicating the stellar distribution is pressure supported and the stars rotate only half as fast as the gas at a distance of 1.6 pc . Unlike the constant value of $\sigma_{\mathrm{v}}$ measured by Rieke and Rieke (1988), there was a monotonic decline in velocity dispersion of $-0.79 \pm 0.14 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{arcsec}^{-1}$ with increasing distance as well as large fluctuations. They estimated that brighter resolved sources contributed $25 \%$ of the total average flux in their apertures, yet simulations indicated no significant effects on their results. Possibilities for the differing results from Rieke and Rieke (1988) included small number statistical problems with the earlier sample, dynamical differences between the bright and faint populations due to differing ages (a $10^{\gamma}$ yr population would not have time to become
dynamically relaxed with the central cluster), or perhaps significant foreground contamination in the bright star sample. They derived the $M(r)$ vs. $r$ relation via the stellar hydrodynamic method. It was a significant improvement over Sellgren et al. (1987) because the velocity dispersion gradient and the rotational velocity could be meaningfully evaluated, and assumption of an isotropic velocity dispersion could be further justified. They measured $2.5 \times 10^{6}$ $\mathrm{M}_{\odot}$ of material within 0.6 pc of IRS $16\left(\mathrm{R}_{0}=8.5 \mathrm{kpc}\right)$, in close agreement with Sellgren et al. (1987) and Rieke and Rieke (1988).

Sellgren et al. (1990) extended the McGinn et al. (1989) study to the central 1 pc , near IRS 16. They obtained two measurements with a $5.0^{\prime \prime}$ aperture and five with a $7.8^{\prime \prime}$ aperture at 120 $\mathrm{km} \mathrm{s}^{-1}$ spectral resolution and another eight measurements using a $2.7^{\prime \prime}$ aperture using the cooled grating array spectrometer on the NASA IRTF at $250 \mathrm{~km} \mathrm{~s}^{-1}$ resolution. Their goal was velocity dispersion measurements at locations selected to avoid the bright resolved sources to determine the enclosed mass to within 0.05 pc of IRS 16. Their data suffered from significant contamination by the bright $2.2 \mu \mathrm{~m}$ sources of the central cluster and the larger apertures. They were able to confirm a rising velocity dispersion (McGinn et al. 1989) in the vicinity of IRS 16 to approximately $125 \mathrm{~km} \mathrm{~s}^{-1}$. However, they also detected a significant decrease in the depth of the CO absorption feature within the central $20^{\prime \prime}$. This prevented velocity dispersion measurements within 0.4 pc . Even with drift errors up to half the aperture size, they claimed that four of their $2.7^{\prime \prime}$ aperture measurements within $15^{\prime \prime}$ of IRS 16 had no contamination and showed the average CO depth was $21 \%$, much smaller than the average value of $35 \%$ outside of $15^{\prime \prime}$. Their interpretation was that stars within $15^{\prime \prime}$ have diminished CO strength and hence the measured velocity dispersions are indicative of foreground and background stars. Thus the velocity dispersion does not reflect the stars at the projected distance from IRS 16 but instead this technique is limited to extracting information outside the central cluster where CO is depleted. This interpretation will be further discussed below.

Lindqvist, Habing and Winnberg (1992) recently used a sample of $\mathrm{OH} / \mathrm{IR}$ stars to
measure the enclosed mass from 4-80 pc by both of the above methods. They found good agreement using the projected mass method with the enclosed mass vs. radius derived by Sellgren et al. (1987) and McGinn et al. (1989). Their OH/IR star sample also extended the enclosed mass curve, as measured by stars, to a considerably larger projected radius.

## Observations and Data Reductions

The new CO spectra presented here were acquired at the CTIO 4 m telescope at $\mathrm{f} / 33$ using the Infrared Spectrometer on the nights of June 24, 26, and 28, 1989. The detector was a SBRC $58 \times 62 \mathrm{InSb}$ array with a read noise of 380 electrons. The spatial scale along the slit was $0.81^{\prime \prime}$ /pixel and the grating was set for a pixel resolution of -3000 which is half the spectral resolution, with a corresponding velocity resolution of $99.6 \mathrm{~km} \mathrm{~s}^{-1}$. The slit was $15^{\prime \prime} \times 1.3^{\prime \prime}$ in projected size on the sky, oriented in the north-south direction. Table 5-1 lists the nominal slit coordinates offset from IRS 7 and the cumulative exposure times. At each slit position spectra were taken at the nominal wavelength of $2.3 \mu \mathrm{~m}$ (the " A " set of spectra) as well as another set with the grating "jogged" by $1 / 2$ of a pixel element (the " $B$ " set of spectra) so the light was sampled at twice the pixel resolution. The selection of the slit positions (figure 5-1) were determined from a high resolution $K$ map (Rieke and Rieke, unpublished) to minimize the contamination from bright point sources that would diminish the CO absorption feature. The signal was peaked on IRS 7 with short test exposures and then the telescope was wobbled to the offset position. Sky frames at the same exposure were obtained from a position $\sim 90^{\prime \prime}$ south of the nominal slit positions in a region of very high extinction ( $A_{v} \approx 60 \mathrm{mag}$ ) due to a foreground molecular cloud (Glass, Catchpole, and Whitelock 1987). Velocity reference spectra were made from IRS 7 with 60 exposures and sky transmission frames were made from 30 sec exposures of a G dwarf star, all of which bracketed the data exposures in time. Bias frames, dome flats, and dark current frames at each exposure time were made at the end of the night.


Figure 5-1: The placement of the nine slit positions with respect to a high resolution $2.2 \mu \mathrm{~m}$ image (Rieke and Rieke, unpublished). The nominal positions of IRS 7 and Sgr A* are shown in boxes and the IRS 16 complex is directly to the left of $\operatorname{Sgr} A^{*}$. The diagonal line is the Galactic Equator. The slit "Position 1" is in the "valley" of emission immediately to the west of IRS 16. The slit designated "wi16" is listed in the log book with the same offset position, however the intensity profiles do not agree, indicating an error in the R.A. coordinate.

TABLE 5-1
LIST OF $2.3 \mu \mathrm{~m}$ CO SPECTRA SLIT POSITIONS

| Night of <br> Observations | Slit Position | $\alpha_{\text {IRS }}$ <br> $(\eta)$ | $\delta_{\text {IRST }}$ <br> $(\eta)$ | Total Exposure <br> Time (Sec) |
| :---: | :---: | :---: | :---: | :---: |
| June 24/25, 1989 | 1 | -1.5 | -8.0 | 1200 |
|  | 2 | -3.5 | -7.0 | 480 |
|  | 3 | +3.5 | -7.0 | 480 |
|  | 5 | -4.0 | +3.0 | 480 |
| June 26/27, 1989 | 6 | -9.0 | -18.0 | 480 |
|  | 7 | +11.0 | +0.0 | 240 |
|  | 8 | +5.5 | -4.5 | 480 |
| June 28/29, 1989 | 9 | +0.0 | -14.5 | 480 |
|  | WI16 | -1.5 | -8.0 | 240 |

Only a small $62 \times 19$ pix $^{2}$ region of the array contained usable data. This was extracted and a small number ( -10 ) of dead pixels were masked. After correction for dark current and bias levels, a flat field was made from the dome exposures and applied to the data and sky transmission frames. The sky transmission spectrum was replicated in the spatial direction of the array and divided into the flat fielded data frames to remove telluric features. Finally, those spectra with multiple exposures were coadded. Eighteen spectra were extracted from each slit position, two pixels wide in the spatial direction with an effective beam size of $\Delta \alpha \times \Delta \delta=$ $1.3^{\prime \prime} \times 1.6^{\prime \prime}$ and sequentially spaced by one pixel from north to south . The extracted spectra were normalized to unity in the continuum blueward of the $2.29 \mu \mathrm{~m}$ CO absorption feature. The "A" spectra and the " B " spectra data were then combined by interleaving the two sets. There were 162 final spectra, which after trimming of the edges had a velocity range of $4700 \mathrm{~km} \mathrm{~s}^{-1}$ with an inverse dispersion of $49 \mathrm{~km} \mathrm{~s}^{-1}$ pixel ${ }^{-1}$ (figure 5-2).

An estimate of the signal-to-noise in each spectrum was made from the ratio of the average blueward continuum to the scatter of the "A" spectra interpolated to the shifted "B"


Figure 5-2: The extracted long slit spectra of the CO absorption feature at $2.3 \mu \mathrm{~m}$. North is at the top and each spectrum has been normalized to unity in the continuum level blueward of the CO feature as shown by the dotted lines. Each spectrum is coadded by 2 pixels in the spatial direction and sequentially spaced across the slit by one pixel. The effective beam size is $1.6^{\prime \prime} \times 1.3^{\prime \prime}$. The spectra are separated by $1 / 2$ of the continuum level. Shorter wavelengths are to the right.


Figure 5-2: Continued


Figure 5-2: Continued.


Figure 5-3: Estimated Signal-to-Noise in the CO spectra. The dotted line is the theoretical $\mathrm{S} / \mathrm{N}$ relation for a slit spectrum with a peak relative flux of 1.0 at $\mathrm{S} / \mathrm{N}=100$.


Figure 5-3: Continued.


Figure 5-3: Continued.


Figure 5-4: The CO Band Strength vs. Projected Angle from Sgr A*. The absorption strength has a high spatial correlation with separation from Sgr $A^{*}$. The legend shows the correspondence between the markers and the slits. Obvious sources of contamination have been removed from the sample. The correlation is insensitive to recentering to within $\pm 1^{\prime \prime}$ of $\mathrm{Sgr} \mathrm{A}^{*}$.
spectra positions. Figure 5-3 shows the estimated signal-to-noise vs. the relative continuum intensity along the spatial direction of the slit. In general, there is good agreement with the theoretical expectation that the signal to noise is proportional to the square root of the accumulated counts.

The CO Absorption Feature

The CO band strength is here defined as the depth of the absorption feature relative to the blueward continuum and this was measured for each spectrum. The intensity profile along each slit was examined for contamination from known point sources on the high resolution map and these were excluded from the CO sample. To determine the spatial position of each spectrum, a fiducial pixel position was identified by the average location of IRS 7 in the velocity standard exposures prior and subsequent to each data frame exposure. This nominal pixel position was assigned the positional offset from IRS 7 recorded in the logbook. Subsequent pixel positions were calculated in the north-south direction by the pixel angular scale.

The CO depth vs. angular distance from Sgr A* is shown in figure 5-4. We confirm the result of earlier studies (Allen et al. 1989; Sellgren et al. 1990) that CO absorption is diminished within the central 0.35 pc . The most significant result is that the CO depletion is highly correlated with projected distance from $\mathrm{Sgr} \mathrm{A}^{*}$. The dispersion of the CO spatial correlation is insensitive to recentering within $\pm 1.0^{\prime \prime}$ of $\mathrm{Sgr} \mathrm{A}^{*}$, beyond which it becomes considerable. Sellgren et al. (1990) showed that the CO strength is constant at a level of $-35 \%$, as expected for a late-type stellar population, from $20^{\prime \prime}$ out to at least $90^{\prime \prime}$. Between $10-20^{\prime \prime}$ their few data points had an average CO strength of $21 \%$. These data in the same region have an average CO strength of $27 \%$, but there is noticeable scatter caused by contaminating blue sources. But it is important to note that at this point the line depths have been uncorrected for the effects of the line-of-sight velocity dispersion. That will be addressed below.


Figure 5-5: The $\chi^{2}(r)$ vs. $V_{\text {OFFSET }}$ curves at velocity dispersions of $0,25,50,75,100,125,150,175$, 200,250 , and $300 \mathrm{~km} \mathrm{~s}^{-1}$ for the six spectra listed in Table 5-2. In each diagram the top curve corresponds to a velocity dispersion of $0 \mathrm{~km} \mathrm{~s}^{-1}$.


Figure 5-6: Examples of spectra that could not be cross correlated. Examples shown are $\chi^{2}$ curves that are flat, discontinuous, non-monotonic, and inverted.

Velocity standard exposures were taken from IRS 7 before and after every program spectrum as templates for measuring the radial velocity and velocity dispersion. The templates were made by convolving each spectrum of IRS 7 with a Gaussian function with a width corresponding to velocity dispersions of $0,25,50,75,100,125,150,200,250$, and $300 \mathrm{~km} \mathrm{~s}^{-1}$. The spectra were then cross correlated at each velocity dispersion in the following way. The algorithm linearly interpolated the spectra to four times the pixel resolution and renormalized to the CO band depth of the velocity standard at the position of the maximum slope in the CO profile. A series of discrete offset velocities $v_{1}, v_{2}, \ldots, v_{N}$ were used to compute a correlation coefficient, $R\left(v_{j}\right)$, and its square, $R^{2}\left(v_{j}\right)$, about the velocity reference spectrum at $v_{\mathrm{K}}=0.0(\mathrm{k}=$ $1 / 2 \mathrm{~N}$ ).

$$
\begin{aligned}
R\left(v_{j}\right) & =\frac{1}{N} \sum_{i=1}^{N} \frac{F_{r o f}\left(v_{j}\right)}{F_{t a s}\left(v_{i}-v_{j}\right)} \\
R^{2}\left(v_{j}\right) & =\frac{1}{N} \sum_{i=1}^{N}\left(\frac{F_{r e f}\left(v_{j}\right)}{F_{\text {rest }}\left(v_{i}-v_{j}\right)}\right)^{2}
\end{aligned}
$$

From the correlation coefficient curves, curves of $\chi^{2}\left(v_{j}\right)$ vs. $v_{l}$ were computed at each velocity dispersion, $D$, following the usual definition of $\chi^{2}$ :

$$
\chi_{D}^{2}\left(v_{j}\right)=\frac{N R_{D}^{2}\left(v_{j}\right)-\left(R_{D}\left(v_{j}\right)\right)^{2}}{N-1}
$$

Unfortunately the yield of successful cross correlations was not very high. Out of 162 spectra, only six from slit Position 1 gave satisfactory results (Figure 5-5). Types of failure included flat $\chi^{2}$ curves, discontinuous curves, non-monotonic curves with discontinuities, and inverted curves
(Figure 5-6). One possible cause that may be hampering the procedure is the method used to renormalize the spectra. At each position of the spectrum, the slope is computed from the average of the slopes at positions within $\pm 5$ pixels. This is perhaps too sensitive to the effects of decreased signal-to-noise as well as the diminished CO strength near the center. The velocity dispersion, $\sigma_{R A W}$, and offset velocity from IRS 7, $V_{I R S S}$, were determined by finding the minimum of the $\chi^{2}(v)$ curves in the $\sigma_{R A W}-V_{\mathrm{RS}}$ plane by parabolic interpolation. The slit positions and measured values of $\sigma_{\text {RAW }}$, and $V_{\text {IRS }}$ values are shown in Table 5-2.

Error analysis was done by a Monte Carlo simulation procedure. A template spectrum of IRS 7 was convolved with a Gaussian velocity dispersion, $\sigma_{v}(0)$, and duplicated 100 times at signal-to-noise ratios of $10,20,30, \ldots, 90,100$. The synthetic spectra were cross correlated and the resulting $\sigma_{R A W}$ and $V_{\text {IRS7 }}$ values were measured from the $\chi^{2}$ curves in the same way as the

TABLE 5-2 :
Raw Velocity Dispersions and Relative Velocities From IRS 7

| Slit <br> Position | $\mathrm{S} / \mathrm{N}$ | $\theta_{\text {SGRA }}$ <br> $\left({ }^{(n)}\right.$ | $\sigma_{\text {RAW }}$ <br> $\left(\mathrm{km}^{-1}\right)$ <br> $\left.\mathrm{s}^{-1}\right)$ | $\mathrm{V}_{\text {IRS }}$ <br> $\left(\mathrm{km} \mathrm{s}^{-1}\right)$ |
| :---: | :---: | ---: | ---: | ---: |
| P1-11 | 48 | 5.2 | 142.5 | +41.0 |
| P1-12 | 61 | 6.4 | 98.5 | +29.8 |
| P1-13 | 51 | 6.9 | 101.2 | +60.7 |
| P1-14 | 44 | 7.7 | 111.3 | +77.7 |
| P1-15 | 43 | 8.6 | 112.3 | +80.3 |
| P1-17 | 22 | 10.3 | 108.6 | +127.8 |

data. For those trials that produced reliable $\chi^{2}$ curves, the average and dispersion of the measured $\left\langle\sigma_{V}\right\rangle$ and $\left\langle V_{\mathrm{IRS}}\right\rangle$ were calculated. In general it was found that successful cross correlation systematically overestimated the velocity dipsersion and systematic velocity with errors that monotonically decreased with increasing signal-to-noise (S/N) (Figure 5-7). Below a S/N of 10 , the measured velocity dispersion was $-190 \mathrm{~km} \mathrm{~s}^{-1}$ independent of the true


Figure 5-7: Figure 5-7a shows the mean measured velocity dispersion vs. $\mathrm{S} / \mathrm{N}$ for synthetic input spectra of IRS 7 with intrinsic velocity dispersions of $75,100,125$, and $150 \mathrm{~km} \mathrm{~s}^{-1}$. In all instances the cross correlation technique overestimates the dispersion. Figure $5-7 \mathrm{~b}$ is the mean measured systematic velocity vs. $\mathrm{S} / \mathrm{N}$ for the same spectra. Generally the systematic error and its standard deviation (Figure 5-7c) depend only on the signal-to-noise ratio for $\mathrm{S} / \mathrm{N}$ $\mathbf{> 2 0}$. Figure 5-7d is the fractional error of the velocity dispersion vs. signal to noise and Figure 5 -7e shows its standard deviation. Estimates of the velocity dispersion systematic errors can be reliably made for $\mathrm{S} / \mathrm{N}>40$. The percentage yield of successful cross correlations vs. $\mathrm{S} / \mathrm{N}$ is shown in 5-7f.


Figure 5-7: Continued.


Figure 5-7: Continued.
velocity dispersion while for $\mathrm{S} / \mathrm{N}>40$ the fractional error of the measured velocity dispersion was independent of the true velocity dispersion. Thus for sufficiently high quality spectra, a reliable correction for systematic errors could be made to the measured results. The dispersion of the fractional systematic error was a function of $\mathrm{S} / \mathrm{N}$ and $\sigma_{\mathrm{v}}(0)$, varying from $0.1-0.2$ for $\sigma_{v}(0)=75-150 \mathrm{~km} \mathrm{~s}^{-1}$. The yield rate of the cross correlation program was also an increasing function of $S / N$. At fixed $S / N$, the yield rate decreased significantly for $\sigma_{v}(0)$ greater than 100 $\mathrm{km} \mathrm{s}^{-1}$. Thus the failure of the program to measure the velocity dispersion can be at least partially attributed to insufficient $\mathrm{S} / \mathrm{N}$ in the spectra. There is also the consideration that the intrinsic CO depth is decreasing monotonically inside $10^{\prime \prime}$. The fractional error in determining the strength of the feature, $\varepsilon$, as a function of $S / N$ in the continuum will go as

$$
\frac{\Delta e}{\varepsilon}=\frac{\sqrt{(1-e)(2-e)}}{e} \frac{1}{(S / N)}
$$

The decreased $\mathrm{S} / \mathrm{N}$ in the continua of those spectra with $r \geq 10^{\prime \prime}$ from the center, as well as the diminished intrinsic strength of the CO feature for those close to the center both have the effect of minimizing the chances for the cross correlation program to renormalize the CO band correctly to that of the template spectrum. The measured systematic velocity with respect to the template spectrum was always on the order of a few $\mathrm{km} \mathrm{s}^{-1}$ with an error on the order of $5 \mathrm{~km} \mathrm{~s}^{-1}$ for $\mathrm{S} / \mathrm{N}>20$ and was not a strong function of intrinsic velocity dispersion.

Table 5-3 lists the corrected velocity dispersions, $\mathrm{V}_{\text {LSR }}$ velocities and estimated errors for the measurements listed in Table 5-2. The corrected velocity dispersion is $\sigma_{R A W} /(1+\beta)$. The column listing $\Delta \mathrm{V}_{\text {LSR }}$ gives the systematic error correction and dispersion respectively. The $\mathrm{V}_{\mathrm{LSR}}$ of IRS 7 was taken to be $-130 \mathrm{~km} \mathrm{~s}^{-1}$ (McGinn et al. 1989).

TABLE 5-3
Velocity Dispersions and Line-of Sight Velocities Corrected For Systematic Errors

| Slit <br> Position | $S / N$ | $\beta$ | $\Delta \beta$ | $\Delta \sigma_{R A W}$ <br> $\left(\mathrm{~km} \mathrm{~s}^{-1}\right)$ | $\Delta V L S R$ <br> $(\mathrm{~km} \mathrm{s-1)}$ | $\sigma_{v}$ <br> $(\mathrm{~km}$ <br> $\left.\mathrm{s}^{-1}\right)$ | $V_{L S R}$ <br> $\left(\mathrm{~km} \mathrm{~s}^{-1}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P1-11 | 48 | 0.13 | 0.05 | 21 | $-4.5 / 7$ | $126 \pm 19$ | $-93 \pm 7$ |
| $\mathrm{P} 1-12$ | 62 | 0.08 | 0.02 | 15 | $-2.8 / 6$ | $91 \pm 15$ | $-103 \pm 6$ |
| $\mathrm{P} 1-13$ | 52 | 0.11 | 0.03 | 17 | $-3.7 / 7$ | $91 \pm 17$ | $-66 \pm 7$ |
| $\mathrm{P} 1-14$ | 45 | 0.12 | 0.06 | 19 | $-4.3 / 8$ | $99 \pm 18$ | $-56 \pm 8$ |
| $\mathrm{P} 1-15$ | 44 | 0.12 | 0.06 | 19 | $-4.3 / 8$ | $100 \pm 18$ | $-54 \pm 8$ |
| $\mathrm{P} 1-17$ | 22 | 0.45 | 0.50 | 38 | $-14.4 / 7$ | $75 \pm 37$ | $-16 \pm 17$ |

Given the relatively poor $\mathrm{S} / \mathrm{N}$ of the last measurement, the data will be examined in two groups: set \#1 containing all six data points in Table 5-3 and set \#2 containing only the first five. The $\sigma_{v}$ vs. $\mathrm{r}_{\mathrm{SgrA}^{\prime}}$ and $\mathrm{V}_{\mathrm{LSR}}$ vs. $\mathrm{r}_{\mathrm{S}_{\mathrm{grA}}}$ diagrams are shown in Figure 5-8. The average velocity dispersion of the sample shows a slight gradient but the average value, $98.8 \pm 2.4 \mathrm{~km} \mathrm{~s}^{-1}$ is intermediate between the results of Rieke and Rieke (1988) at $75 \mathrm{~km} \mathrm{~s}^{-1}$ and McGinn et al. (1989) at $125 \mathrm{~km} \mathrm{~s}^{-1}$ for this same region. The most striking result is the $\mathrm{V}_{\mathrm{LSR}} \mathrm{vs} . \mathrm{r}_{\mathrm{SgrA}}$. diagram. The positions of these velocities have small angular inclinations from the Galactic equator and agree in sign with the results of McGinn et al. (1989). It is reasonable to interpret them as a measurement of the Galactic rotation curve over the same range of radius. The data show the Galactic Center stellar rotation curve inside $10^{\prime \prime}$ from $\mathrm{Sgr} \mathrm{A}^{*}$ has a gradient of $\boldsymbol{\sim} 16 \mathrm{~km} \mathrm{~s}^{-1}$ $\operatorname{arcsec}^{-1}$. Figure 5-9 shows the velocity data plotted with the $V_{L S R}$ values measured by McGinn et al. (1989) (their positive velocity data have been included by reflecting them onto the approaching side) and there is good agreement with the velocity measurements at $10.3^{\prime \prime}$. Starting with a minimum velocity near $0 \mathrm{~km} \mathrm{~s}^{-1}$ at $10^{\prime \prime}$, the curve begins to rise to at least 100 $\mathrm{km} \mathrm{s}^{-1}$ at the $5^{\prime \prime}$ position. Although it has an intrinsic characteristic length scale 100 times smaller, this rotation curve has similar behavior to the rotation curve of the central few


Figure 5-8: The top figure is the velocity dispersion vs. projected angle from Sgr $\mathrm{A}^{*}$ for Position 1 data points. The north and south ends of the slit are indicated. The bottom figure shows the $V_{\text {LSR }}$ VS. $\mathrm{r}_{\mathrm{S}_{8-A}}$ curve which indicates rapidly increasing Galactic rotational velocities inside of $10^{\circ}$ from Sgr $A^{*}$.


Figure 5-9: The figure shows the Galactic Center rotation curve of McGinn et al. (1989) from the CO absorption feature using $20^{\prime \prime}$ apertures. Data points marked by ( $\square$ ) have approaching LSR velocities while points marked by ( $\nabla$ ) have receeding velocites and have been reflected onto the approaching side. The filled circles are the data points from the present study. The rotation curve reaches a minimum of $-0 \mathrm{~km} \mathrm{~s}^{-1}$ near $10^{\prime \prime}$ and then begins an abrupt rise with decreasing radius.


Figure 5-10: The fractional change in the intrinsic CO band depth due to velocity broadening as measured from the velocity template spectra of IRS 7. The fractional change for the measured data points at slit Position 1 is of the order of $\mathbf{1 5 - 2 0 \%}$
arcseconds of M31 at optical wavelengths (Kormendy 1988; see Figure 2).
The CO band strength is at least partially depleted by the effects of velocity broadening. The fractional change in CO strength vs. velocity dispersion was computed from the IRS 7 template spectra (Figure 5-10). In the range of the measured velocity dispersions of the data, the fractional change in the CO strength is approximately $\mathbf{1 5 - 2 0 \%}$. A nominal correction of $\mathbf{2 0 \%}$ due to velocity broadening was applied to the CO band strengths that are treated below in the discussion section.

The Enclosed Mass vs. Radius

By assuming the velocity dispersion and bulk motion of the CO emission primarily originates from a region along the line-of-sight at each projected radius, the enclosed mass vs. radius may be derived by application of the first moment of the Boltzmann equation as discussed above. Here it was assumed that the velocity ellipsoid was isotropic $(\lambda=2)$ and this is partially supported by the fact that $\mathrm{V}_{\mathrm{LSR}} /<\sigma_{\vee}>\sim 1$ (Kormendy and Illingworth 1982; McGinn et al. 1989) but complete justification can only come with isophotal information of the unresolved emission for this region which is presently not available. Since the CO absorption profile cannot be fitted properly unless $R_{\text {CORE }}$ of the $2.2 \mu \mathrm{~m}$ light distribution is smaller than $\sim 2.5^{\prime \prime}$ (see below) we will use $n(r) \propto \mathrm{r}^{-1.8 \pm 0.1}$ (Becklin and Neugebauer 1968; Allen, Hyland, and Jones 1983) for the number density of $2.2 \mu \mathrm{~m}$ sources, in agreement with McGinn et al. (1989). Two equivalent forms of the moment equation were applied to the two data sets:

$$
\begin{aligned}
& M(r)=\frac{r \sigma_{v}^{2}}{G}\left(-\frac{d \ln n(r)}{d \ln r}-\frac{d \ln \sigma_{v}^{2}}{d \ln r}+\frac{V_{L S R}}{\sigma_{v}^{2}}\right) \\
& M(r)=\frac{r \sigma_{v}^{2}}{G}\left(-\frac{d \ln n(r)}{d \ln r}-\frac{2 r}{\sigma_{v}} \frac{d \sigma_{v}}{d r}+\frac{V_{L S R}}{\sigma_{v}^{2}}\right)
\end{aligned}
$$

with the additional three case assumptions

| CASE I | DATA SET \#1 | $\mathrm{d} \ln \sigma_{\mathrm{v}}{ }^{2} / \mathrm{d} \ln \mathrm{r}=-0.99 \pm 0.80$ |
| :---: | :---: | :---: |
|  | DATA SET \#2 | $\mathrm{d} \ln \sigma_{\mathrm{v}}{ }^{2} / \mathrm{d} \ln \mathrm{r}=-0.93 \pm 0.86$ |
| CASE II | DATA SET \#1 | $\mathrm{d} \sigma_{\mathrm{v}} / \mathrm{dr}=-5.8 \pm 6.0 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{arcsec}^{-1}$ |
|  | DATA SET \#2 | $\mathrm{d} \sigma_{\mathrm{v}} / \mathrm{dr}=-5.2 \pm 7.1 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{arcsec}^{-1}$ |
| CASE III | DATA SET \#1/\#2 | $\mathrm{d} \sigma_{\mathrm{v}} / \mathrm{dr}=0$ |

The velocity dispersion gradients were determined by a linear least squares fit to the data. (Case III is for an isothermal sphere with no velocity dispersion gradient.) Given these assumptions, and applying the above equations to both data sets the following $M(r)$ vs, $r$ values are derived.

Including the velocity dispersion of the P1-17 spectrum into the estimates of the velocity dispersion gradient does not greatly affect the average enclosed mass, but it does tend to produce a more negative gradient in the $M(r)$ values. The logarithmic velocity gradient in Case I produces higher mass estimates than the simple gradient of Case II. Assuming positional errors of $\pm 0.5^{\prime \prime}$, the major contributors to the errors of the above mass estimates are from the velocity dispersion and its gradient. Case II produced larger formal errors because the error term for the dispersion gradient depends on $r^{2}$ while in Case I this error only depends on $r$.

TABLE 5-4 : ENCLOSED MASS VS. RADIUS ESTIMATES

| Slit Position | $\begin{gathered} \begin{array}{c} r_{\text {sgr^ }} \\ \left({ }^{(1 / p c)}\right. \end{array} \\ \mathrm{R}_{0}=6.9 \mathrm{kpc} \end{gathered}$ | ENCLOSED MASS ( $\left.10^{6} \mathrm{M} \odot\right)$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | CASE I |  | CASE II |  | CASE IIIData Set <br> $\# 1 / \# 2$ |
|  |  | Data Set \#1 | Data Set ${ }^{\text {\# }}$ 2 | Data Set \#1 | Data Set \#2 |  |
| P1-11 | $5.2 \mid 0.17$ | $2.14 \pm 0.78$ | $2.11 \pm 0.80$ | $1.82 \pm 0.55$ | $1.78 \pm 0.58$ | $1.51 \pm 0.39$ |
| P1-12 | $6.4 \mid 0.22$ | $1.68 \pm 0.53$ | $1.66 \pm 0.53$ | $1.61 \pm 0.49$ | $1.57 \pm 0.53$ | $1.27 \pm 0.27$ |
| P1-13 | $6.9 \mid 0.23$ | $1.48 \pm 0.60$ | $1.45 \pm 0.61$ | $1.43 \pm 0.57$ | $1.39 \pm 0.62$ | $1.03 \pm 0.32$ |
| P1-14 | $7.7 \mid 0.26$ | $1.83 \pm 0.77$ | $1.79 \pm 0.78$ | $1.78 \pm 0.75$ | $1.72 \pm 0.82$ | $1.24 \pm 0.40$ |
| P1-15 | $8.6 \mid 0.30$ | $2.06 \pm 0.87$ | $2.02 \pm 0.89$ | $2.07 \pm 0.90$ | $2.00 \pm 0.99$ | $1.40 \pm 0.45$ |
| P1-17 | 10.3\| 0.35 | $1.28 \pm 1.30$ | - | $1.56 \pm 1.38$ | - | $0.83 \pm 0.80$ |
| $<\mathrm{M}(\mathrm{r})>$ |  | $1.75 \pm 0.14$ | $1.80 \pm 0.12$ | $1.71 \pm 0.09$ | $1.69 \pm 0.10$ | $1.21 \pm 0.10$ |

## Discussion

Interpretations of the CO Absorption Feature

Sellgren et al. (1990) (SMBH) were the first to examine what changes in the unresolved population would be needed to produce the observed CO depletion. The two models they discussed with their observations will be reintroduced here and compared to the present results. Additionally, more generalized models will also be compared.

We first consider that the $2.2 \mu \mathrm{~m}$ light distribution is known to depend on radius as $\mathrm{R}^{-0.8}$, which implies a stellar number density that goes as $\mathrm{R}^{-18}$. The core radius of this distribution has been the subject of some controversy with Allen et al. (1983) reporting a core radius of $\approx 1.0^{\prime \prime}$ while Rieke and Lebofsky (1987) favored a value of $\approx 20^{\circ}$. In any case, the luminosity density of $2.2 \mu \mathrm{~m}$ sources can be represented by:

$$
\begin{array}{ll}
F_{\Sigma}(r)=F_{\Sigma}(0) & ; r \leq R_{C O R E} \\
F_{\Sigma}(r)=F_{\Sigma}(0) \times\left(\frac{r}{R_{\text {CORS }}}\right)^{-1.8} ; r>R_{C O R E}
\end{array}
$$

The luminosity density of CO absorbing sources in the models presented by SMBH were:

## MODEL SMBH-A

$$
\begin{array}{ll}
F_{C O}(r)=0 & ; r \leq R_{C O} \\
F_{C O}(r)=F_{E}(r) & ; r>R_{C O}
\end{array}
$$

## MODEL SMBH-B

$$
\begin{aligned}
& F_{C O}(r)=F_{K}\left(R_{C O}\right) ; r \leq R_{C O} \\
& F_{C O}(r)=F_{K}(r) \quad ; r>R_{C O}
\end{aligned}
$$

Model A represents a situation where CO absorbing stars are completely absent inside a distance of $\mathrm{R}_{\text {co }}$. This could be due to an interface between two populations (a very unlikely situation) or the CO in the stellar photospheres is somehow being removed (by star-star collisions; photoionizations; ablation by outflowing material; etc.). Model B has a cool population extending into the core, but the number density of CO absorbing stars is constant for $r<R_{\mathrm{CO}}$. The remaining $2.2 \mu \mathrm{~m}$ emission is augmented by a hotter population, but in such a way that the total light profile still obeys the $\mathrm{R}^{-0.8}$ law.

For clarity, the following definitions will be made:
$F_{K}(i) \equiv$ the continuum luminosity density at $K$.
$F_{C O}(i) \equiv$ the $2.3 \mu \mathrm{~m}$ luminosity density.
$\mu_{\mathrm{K}}(i) \equiv$ the continuum surface brightness at $K$.
$\mu_{c o}(i) \equiv$ the surface brightness at $2.3 \mu \mathrm{~m}$.
$i=1$ will represent the stellar component with CO absorption.
$i=2$ will represent the stellar component without CO absorption.
$\varepsilon \equiv$ the average CO depth/star for the $i=1$ stars.
$r=r(x, y, z)=$ the radius from the Galactic Center.

Placing the origin of the coordinates at the Galactic Center and orienting the $z$-axis along the line-of-sight toward the center, the surface brightness $\mu$ at projected radius $\left(x^{2}+y^{2}\right)^{1 / 3}$ and the luminosity density $F(r)$ are related by

$$
\mu(x, y) \propto \int_{-R_{0}}^{+\infty} F(r(x, y, z)) d z
$$

For stars with $C O$ absorption we take $F_{C O}(1)=(1-\varepsilon) \times F_{k}(1)$, and for stars without $C O$ absorption $F_{c o}(2)=F_{k}(2)$. Using these definitions, the observed $C O$ band depth at a each position is given by

$$
\begin{aligned}
e_{\alpha f f} & =\frac{\mu_{K}-\mu_{C O}}{\mu_{K}} \\
& =\frac{\left(\mu_{K}(1)-\mu_{C O}(1)\right)+\left(\mu_{K}(2)-\mu_{C O}(2)\right)}{\mu_{K}} \\
& =e \frac{\mu_{K}(1)}{\mu_{K}}
\end{aligned}
$$

Thus for the SMBH models, where $\varepsilon$ is regarded as a constant for all CO absorbing stars, the observed band depth is a measure of the fractional contribution of CO absorbers to the total $2.2 \mu \mathrm{~m}$ surface brightness. Neither of the SMBH models with the parameter values stated by them compare well with these observations but SMBH-A can be matched with $R_{\text {Co }}=8.5^{\prime \prime}$ (Figure 5-11). Their preferred model, SMBH-A, matches $\mathrm{R}_{\mathrm{CO}}=10^{\prime \prime}$ adequately but it falls away too steeply at smaller projected radii. Model SMBH-B with $\mathrm{R}_{\mathrm{Co}}=15^{\prime \prime}$ does not fit the shoulder of the CO absorption curve very well, but it does make a better fit inside of $10^{\prime \prime}$. SMBH were


Figure 5-11: The top figure shows the observed CO absorption profile (corrected by $20 \%$ for velocity broadening effects) compared to the Sellgren et al. (1990) models SMBH-A ( $F_{c o}(r)=0$ for $r<10^{\prime \prime}$ ) and SMBH-B $\left(F_{c o}(r)=F_{K}\left(15^{\prime \prime}\right)\right.$ for $\left.r<15^{\prime \prime}\right)$. Neither of the models give a satisfactory fit over the range of the observations. The bottom figure shows model SMBH-A with $R_{C O}=7^{\prime \prime}$, $8^{\prime \prime}, 9^{\prime \prime}$, and $10^{\prime \prime}$ from which an approximate fit of $8.5^{\prime \prime}$ is indicated.


Figure 5-12: The top figure shows the distribution of the total number of $2.2 \mu \mathrm{~m}$ sources and the number of CO absorbers for various values of the parameter $\alpha$ for $R_{c o}=10$ " and $R_{\text {cors }}=1.0$ ". The Sellgren et al. (1990) models "A" and "B" correspond to parameter values of $\alpha=0$ and $\alpha=\infty$. The CO absorption will only be sampled at the projected radius if $\alpha>0$, otherwise the observed emission will be due to material at a true radius of $R_{c o}$ The bottom figure is the resulting CO absorption strength vs. rspA. . curves with a velocity broadening correction of $+20 \%$ applied. The assumed intrinsic CO band strength for the population is $35 \%$ (Sellgren et al. 1990).
not satisfied with the interpretation of model B because of the contrived situation that would have to exist for a bluer population component to be added in such a particular way that the $2.2 \mu \mathrm{~m}$ light profile would retain its $\mathrm{R}^{-0.8}$ form into the core radius. They preferred model A , which, again, can be interpreted as the total destruction of CO in the photospheres of those stars with $\mathbf{r}<\mathrm{R}_{\mathrm{CO}}$. They estimated that the ambient UV radiation field emerging from the central parsec ( $\approx 10^{51} \sec ^{-1} ;$ Becklin, Gatley, and Werner 1982) was insufficient to cause the depletion out to such a large distance but stellar collisions (Phinney 1989; see below) were not ruled out.

These models can be further extended by considering the CO luminosity density to be varying as

$$
\begin{array}{ll}
F_{C O}(r)=F_{\Sigma}\left(R_{C O}\right) \times\left(\frac{r}{R_{C O}}\right)^{\alpha} & ; r \leq R_{C O} \\
F_{C O}(r)=F_{\Sigma}(r) & ; r>R_{C O}
\end{array}
$$

where $F_{K}(r)$ has the same form given above. The parameter values $\alpha=\infty, \mathrm{R}_{\mathrm{Co}}=10^{\prime \prime}$ and $\alpha=$ $0, \mathrm{R}_{\mathrm{CO}}=15^{\prime \prime}$ reproduce models $\mathrm{SMBH}-\mathrm{A}$ and $\mathrm{SMBH}-\mathrm{B}$ respectively. Figure $5-12$ shows the luminosity density for both the total $2.2 \mu \mathrm{~m}$ and CO absorbing distributions for various values of $\alpha$ and $\mathrm{R}_{\mathrm{CO}}=10^{\prime \prime}$ and the resulting CO band strength profiles compared to the observations. (For models with $\alpha<0$ an additional constraint for $\mathrm{r}<\mathrm{R}_{\mathrm{CORE}}$ is applied such that $F_{\mathrm{co}}(r)=$ $F_{\mathrm{CO}}\left(R_{\text {CORE }}\right)$ so that the number density of CO absorbers will remain bounded.) The model fits are sensitive to the ratio $R_{\text {CO }} / R_{\text {Cores }}$ with values less than $\approx 5$ giving excessive amounts of $C O$ absorption inside of $\mathrm{R}_{\mathrm{co}}$. One of the important results of these data is that they favors values of the $2.2 \mu \mathrm{~m}$ core radius that are no greater than $\approx 2.0^{\prime \prime}$. If a two component stellar population is present, there must be a very large fraction of stars too hot for significant CO absorption near the center of the stellar distribution to adequately diminish the CO strength.

Yet another situation can be considered instead of a two-component population. The

CO band strength is a function of both luminosity class and temperature (Kleinmann and Hall 1986) so we consider the effect of gradual changes in the feature. We allow the $2.2 \mu \mathrm{~m}$ light distribution to be due to a single component of stars, but instead let the parameter $\varepsilon$ be a function of radius inside the projected distance $\mathrm{R}_{\mathrm{co}}$.

$$
\begin{array}{ll}
e(r)=e\left(R_{c O}\right) \times\left(\frac{r}{R_{c o}}\right)^{\beta} & ; r \leq R_{c o} \\
e(r)=e\left(R_{c O}\right) & ; r>R_{c o}
\end{array}
$$

The observed CO band strength will then be given by

$$
e_{a f f}=\frac{\mu_{E}-\mu_{C O}}{\mu_{K}}=\frac{\int_{-\infty}^{+\infty} e(r) \times F_{K}(r) d z}{\int_{-\infty}^{+\infty} F_{K}(r) d z}
$$

which is the average band depth along the line-of-sight, weighted by the luminosity density. If we now examine the integrand of the numerator in the range $R_{\text {CORE }}<r<R_{\text {CO }}$ we find

$$
\begin{aligned}
\mathrm{e}(r) \times F_{K}(r) & =\mathrm{e}\left(R_{C O}\right)\left(\frac{r}{R_{C O}}\right)^{\beta} \times F_{\Sigma}\left(R_{C O}\right)\left(\frac{r}{R_{C O}}\right)^{-1.8} \\
& =\mathrm{e}\left(R_{C O}\right) \times F_{K}\left(R_{C O}\right)\left(\frac{r}{R_{C O}}\right)^{\beta-1.8}
\end{aligned}
$$

Thus for models with a single population component and $\varepsilon=\varepsilon(r)$, a given value of $\beta$ is equivalent to a SMBH model with a value of $\alpha+1.8$.

This result has very important consequences for interpretations of the line-of-sight
velocity dispersions used in measuring the enclosed $\mathrm{M}(\mathrm{r})$. If the stellar population inside $R_{\text {co }}$ is consistent with model SMBH-A with of $\alpha>0.5-1.0$, as favored by the observations, then the observed CO band depth is dominated by stars at a true radius of $R_{\mathrm{co}}$ and efforts to measure the enclosed mass vs. radius cannot reliably proceed to smaller projected radii. But if the CO depletion is caused by a process that gradually affects a single population of late-type stars where $\varepsilon \propto r^{\beta}$, and $\beta<1.8$, the effective value of $\alpha$ will be less than 0 and the observed line-ofsight velocity dispersion will be dominated by stars at the projected radius, even for $r<R_{\text {co }}$ The model fits seem to favor parameter values of $\alpha \gg 1$ and $R_{c O} \approx 8.5^{\prime \prime}$. By just considering the observed CO absorption profile alone, it appears that the CO absorption feature observed at projected radii less than $R_{\text {CO }}$ is dominated by stars with a true radius of $R_{C O}$. If this is the true situation then it would mean the enclosed mass vs. radius measurements have not been consistently applied.

The Galactic Center Rotation Curve

The models above that attribute the observed CO feature to stars near a true radius of $R_{C O}$ are incompatible with the turnover in the $V_{\text {LSR }}$ vs. $r_{S_{g r A}}$ curve near $10^{\prime \prime}$. Figure 5-13 shows that the CO strength is correlated with both $\sigma_{v}$ and $V_{\text {LSR }}$. The rotational velocity gradient cannot be due to a corrective $1^{\circ}$ rotation that was applied to the data because of a tilt in the array. Even if systematic velocity errors are present, the change in the velocity gradient across 6 pixels (corresponding to the spacing of the data points examined here) would only be on the order of $1 \mathrm{~km} \mathrm{~s}^{-1} \operatorname{arcsec}^{-1}$.

The measurements from P1-14 and P1-15 are nearly identical in CO depth, $\mathrm{V}_{\text {LSR }}$ and $\sigma_{\mathrm{v}}$. The intensity profile at this position shows a slight enhancement in the emission and the CO absorption profile shows a plateau superimposed on a nearly parabolic profile (Figure 5-14). There is a faint resolved source near this position that is apparent on the high resolution K map


Figure 5-13: The top figure is the $\sigma_{v}$ vs. $\mathrm{r}_{\mathrm{Sg} \mathrm{H}^{\prime}}$. for the measured points at from slit P1. The bottom figure is the corresponding $V_{L S R}$ vs. $\mathrm{r}_{\mathrm{SgrA}^{\prime}}$. diagram. Both plots show the CO band strength is anticorrelated with these kinematic quantities.


Figure 5-14: Relative Intensity and CO band strength vs. Pixel Position for slit P1. The plateau in the CO absorption for pixels 14 and 15 , as well as the enhanced emission suggest these pixels are contaminated by a resolved source.


Figure 5-15: The effect of the intrinsic CO band strength on the systematic errors of the cross correlation algorithm at $\mathrm{S} / \mathrm{N}=60$ calculated from synthetic spectra made from IRS 7. The variations in the error of $\sigma_{v}$ is from $5-10 \%$ over the range of CO band strength measured in the data. More important for the stellar rotation curve are the $\mathrm{V}_{\text {LSR }}$ errors which only vary by - 2 $\mathrm{km} \mathrm{s}^{-1}$ over the range of CO depths.


Figure 5-16: The endosed mass vs. $r_{5_{p x a}}$. for Case I, Data Set \#1 (top) and Case II, Data Set \#1 (bottom). The dotted line is the predicted enclosed mass model of McGinn et al. (1989) (see their fig. 10) derived from the $2.2 \mu \mathrm{~m}$ light distribution (Becklin and Neugebauer 1968; Matsumoto et al. 1982), assuming a M/L ratio of $1 \mathrm{Mo} / \mathrm{L}_{0}$ and calibrated by the enciosed mass estimates of HI gas at 300 pc (Listz and Burton 1978) and then corrected for a value of $\mathrm{R}_{0}=6.9 \mathrm{kpc}$. The open squares are the enclosed mass estimates by MCGinn et al. (1989) made by the CO absorption velocity dispersion of the unresolved stellar emission with $r>9^{\wedge}$. The results of the present study are consistent with a constant enclosed mass with a high M/L ratio.
(Rieke and Rieke, unpublished) that would then also imply a small positional error in the placement of the slit by about $0.5^{\prime \prime}$. These facts would all indicate contamination by a point source. But this is perhaps not the whole explanation because the CO velocity dispersion of a single source would clearly be separable from the measured velocity dispersions of $\sim 100 \mathrm{~km}$ $s^{-1}$. So either there are still significant contributions in the emission from the unresolved background or this is perhaps a binary or small cluster of late-type stars. Slit positions P1-12 and P1-13 have similar velocity dispersions but they have very different values of $\mathrm{V}_{\text {LSR }}$. The remaining two positions show no indication of any contamination. Even with the rejection of the measurements that have possible resolved source contamination, the gradients in $V_{\text {LSR }}$ and $\sigma_{v}$ are still present and are perhaps better determined.

These velocity results are not due to spurious results of the cross correlation technique due to decreasine CO band strength. This is shown by simulating different CO band strengths from the template IRS 7 spectrum that are then cross correlated with the original. Figure 5-15 shows that for a test at a $S / \mathrm{N}=60$ there are no significant variations in the systematic errors due to decreasing $C O$ strength.

If the interpretation of the SMBH models discussed above is correct, CO absorption at a true radius of $R_{\text {Co }}$ would be expected to have a measured $V_{L S R}(\theta)$ vs. $r_{S_{g 7 A}}$ curve that would decrease with decreasing separation from $\operatorname{Sgr} \mathrm{A}^{*}$ as $\mathrm{V}_{\mathrm{LSR}}(r)=\mathrm{V}_{\mathrm{LSR}}\left(R_{\mathrm{CO}}\right) \times \operatorname{SIN}\left(\pi / 2 \times \mathrm{r}_{\mathrm{SgFA}^{*}} / R_{\mathrm{CO}}\right)$ due to projection effects. If this unlikely situation were the case, then there are large azimuthal changes in the rotation speed of the stars and the true rotation speed is in fact much larger. Thus the velocity observations indicate that CO absorption lies mostly at the projected radius.

The velocity curve implies that CO depletion in the central 0.5 pc is due to dynamical effects on the stars orbiting inside $10^{\prime \prime}$. Phinney (1989) estimated that probability for a giantdwarf collision in the central cluster to be $-0.02 \sigma_{80} \mathrm{rpc}^{-2}$ based on the expected rate of stellar collisions and the time spent in the red giant phase. Giant stars at a distance of 0.2 pc would have a $97 \%$ chance for their envelopes to be stripped through a collision. The fact that CO is
still seen means that at least some remnants are able to reestablish a giant star atmosphere after collision. The rising velocity dispersion gradient with decreasing distance means that velocity broadening effects are more important at smaller radii. Incorporating this effect into the CO profile models would adjust the model predictions in the right direction to make them consistent, but this has yet to be demonstrated.

The Central Mass Distribution

The CO absorption models and the rotational velocity curves are inconsistent. The models suggest that the absorption is dominated by emission at $R_{\mathrm{co}}$ which would mean that the enclosed mass estimates are inappropriate. Yet, the Galactic rotation curve clearly shows a strong gradient inconsistent with this interpretation thus the weight of evidence falls toward failure of the CO models. Given the rising velocity dispersion and rotational velocity toward the center it would seem the enclosed mass vs. radius is being measured. The enclosed mass departs very strongly from predictions made by the $\mathbf{2 . 2 \mu m}$ surface brightness distribution at a radius of 0.17 pc by $6-7$ standard deviations in dex (Figure $5-18$ ) and the $\mathrm{M} / \mathrm{L}$ ratio within $R_{\text {co }}$ is rapidly increasing. Table $5-5$ lists the enclosed mass estimated from the $2.2 \mu \mathrm{~m}$ light distribution for a model that assumes a $\mathrm{M} / \mathrm{L}=1 \mathrm{M}_{\odot} / \mathrm{L}_{\odot}$ (McGinn et al. 1989). Table $5-6$ lists the corresponding $\mathrm{M} / \mathrm{L}$ ratios for the enclosed mass estimates listed in Table 5-3.

For all three case assumptions, the enclosed mass measurement at $8.6^{\prime \prime}$ agrees with the result of McGinn et al. (1989) and the measurement at $5.2^{\prime \prime}$ departs significantly from the predictions of the $2.2 \mu \mathrm{~m}$ light distribution. It also agrees with the Rieke and Rieke (1988) radial velocity study using the projected mass method and the assumption of isotropic velocities and a dominant central mass. Lacy, Actermann, and Serabyn (1992) showed the morphology of the ionized gas structures extending to within $2^{\prime \prime}$ of $\mathrm{Sgr} \mathrm{A}^{*}$ can be modelled as a linear spiral structure on circular orbits around a central mass of $(1.7 \pm 0.4) \times 10^{6} \mathrm{M} \odot$. The model parameters
they used in their fit indicate that $10 \%$ of the mass distribution inside of 0.17 pc is due to stars and this also agrees well with the present enclosed mass measurements and the estimated $2.2 \mu \mathrm{~m}$ light distribution. Melia (1991) has modeled the total emission from $\operatorname{Sgr} \mathrm{A}^{*}$, from the radio to gamma rays, as the result of accreting material from a $10^{-3} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ stellar wind originating from close to IRS 16 C. The preferred range for the mass of Sgr $A^{*}$ from the model fits was 0.7 $\times 10^{6} \mathrm{M}_{\odot} \leq \mathrm{M}_{\mathrm{SgrA}^{*}} \leq 1.7 \times 10^{6} \mathrm{M} \odot$. The results of this experiment lead to the conclusion that the stellar kinematics between $0.17-0.35 \mathrm{pc}$ from $\mathrm{Sgr} \mathrm{A}^{*}$ show a very compact mass distribution of $1.7 \times 10^{6} \mathrm{M} \odot$ inside a radius of 0.17 pc with a $\mathrm{M} / \mathrm{L}$ ratio $>10$. This is strong evidence for the presence of a massive black hole at the Galactic Center.

TABLE 5-5
$2.2 \mu \mathrm{~m}$ ENCLOSED MASS MODEL

| Slit Position |  | $2.2 \mu \mathrm{~m}$ ENCLOSED MASS ${ }^{\circ}$ ( $10^{\circ} \mathrm{Mo}$ ) |
| :---: | :---: | :---: |
| P1-11 | $5.2 \mid 0.17$ | 0.200 |
| P1-12 | $6.4 \mid 0.22$ | 0.264 |
| P1-13 | $6.9 \mid 0.23$ | 0.293 |
| P1-14 | 7.710 .26 | 0.341 |
| P1-15 | $8.6 \mid 0.28$ | 0.397 |
| P1-17 | 10.3\| 0.35 | 0.509 |

TABLE 5-6
ENCLOSED MASS TO LIGHT RATIO VS. RADIUS

| Slit Position | $\begin{gathered} r_{\text {sgra* }} \\ \left({ }^{\prime \prime} \mid \mathrm{pc}\right) \\ \mathrm{R}_{0}=6.9 \mathrm{kpc} \end{gathered}$ | ENCLOSED MASS TO LIGHT RATIO ( $\mathrm{M}_{\odot} / \mathrm{L}_{\odot}$ ) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | CASE I |  | CASE II |  | CASE III |
|  |  | Data Set \#1 | Data Set \#2 | Data Set \#1 | Data Set \#2 | Data Set \#1/\#2 |
| P1-11 | $5.2 \mid 0.17$ | $10.84 \pm 3.90$ | $10.56 \pm 4.02$ | $9.18 \pm 2.80$ | $9.02 \pm 2.95$ | $7.61 \pm 1.95$ |
| P1-12 | $6.4 \mid 0.22$ | $6.37 \pm 2.00$ | $6.27 \pm 2.02$ | $6.10 \pm 1.86$ | $5.97 \pm 2.01$ | $4.82 \pm 1.04$ |
| P1-13 | $6.9 \mid 0.23$ | $5.04 \pm 2.05$ | $4.95 \pm 2.07$ | $4.88 \pm 1.94$ | $4.74 \pm 2.10$ | $3.53 \pm 1.08$ |
| P1-14 | $7.7 \mid 0.26$ | $5.36 \pm 2.27$ | $5.26 \pm 2.30$ | $5.22 \pm 2.20$ | $5.06 \pm 2.40$ | $3.65 \pm 1.19$ |
| P1-15 | 8.6 \| 0.28 | $5.18 \pm 2.20$ | $5.10 \pm 2.23$ | $5.22 \pm 2.28$ | $5.04 \pm 2.50$ | $3.52 \pm 1.13$ |
| P1-17 | 10.3\| 0.35 | $2.51 \pm 2.55$ | - | $3.06 \pm 2.71$ | - | $1.63 \pm 1.58$ |

## Conclusions

New measurements of $2.29 \mu \mathrm{~m}$ CO absorption from the unresolved stellar emission within $30^{\prime \prime}$ of $\mathrm{Sgr} \mathrm{A}^{*}$ confirms there is strong CO depletion within a distance $R_{\mathrm{CO}} \approx 8.5^{\prime \prime}$ and in addition shows that it is highly correlated with radius. The stellar velocity dispersion begins to rise inside this distance as $\sim^{-4 /}$ and the Galactic stellar rotation curve begins to rise rapidly from a minimum near $0 \mathrm{~km} \mathrm{~s}^{-1}$ at $10^{\prime \prime}$ to $100 \mathrm{~km} \mathrm{~s}^{-1}$ at $5.2^{\prime \prime}$ with a gradient of $16 \mathrm{~km} \mathrm{~s}^{-1}$ $\operatorname{arcsec}^{-1}$.

The CO absorption distribution has been sufficiently determined from model comparisons to contrain the core radius of the unresolved $2.2 \mu \mathrm{~m}$ light distribution to $<-0.2 \times$ $R_{\mathrm{co}}$, and if the average CO band strength varies as $r^{\beta}$, then $\beta$ must be no larger than 1.8 if the stellar rotation and velccity dispersion are to be sampled at each projected radius. Given the velocity curve observations, this is a strong prediction on future models using stellar collision theory and/or population synthesis that attempt to explain the CO depletion. The near coincidence of $R_{\mathrm{CO}}$ and the radius of minimum stellar rotation suggest dynamical processes may be responsible.

The enclosed mass vs. radius derived from these measurements within 5-10" (0.17-0.35 pc ) of $\mathrm{Sgr} \mathrm{A}^{*}$ is relatively constant at $-(1.7 \pm 0.35) \times 10^{6} \mathrm{M} \odot$ departing from mass models based on the $2.2 \mu \mathrm{~m}$ light at $5.2^{\prime \prime}$ by $6 \sigma$ in dex. The M/L ratio of the enclosed mass increases with decreasing radius with $M / L=-9-10 \mathrm{M} \odot / \mathrm{L}_{\odot}$ at $5.2^{\prime \prime}$. This result agrees with the enclosed mass determinations by [NeII] ionized gas in the "mini-spiral" (Lacy et al. 1991), radial velocities of resolved sources with isotropic orbits (Rieke and Rieke 1988), unresolved stellar velocity dispersions with $r_{s_{\text {grA }}}>9^{\prime \prime}$ (McGinn et al. 1989) and accretion models for Sgr A* (Melia 1991). Given the fact that the stars are overwhelmingly likely to be dynamically influenced only through gravity, this result is strong evidence for a massive black hole at the Galactic Center.

## CHAPTER 6:

## SUMMARY AND PROSPECTS FOR FUTURE RESEARCH

In the last decade, comprehensive explanations of the phenomena of the Galactic Center has been proposed by the "Central Engine" and "Starburst" models. Many of the results discussed in chapter 1 suggest that the energetics of this region are strongly influenced by the presence of massive stars in the central cluster. The studies presented in chapters 3 and 4 show that star formation has been recurrent in the Galactic Center and it is highly possible that the population of bright nonvariables are an extension of the late-type massive star population in the central cluster, further supporting a starburst model. However, the results of chapter 5 show that the dynamics of the central parsec are governed by a dark matter concentration that can most easily be interpreted as a massive black hole. Perhaps the presence of the massive stars is only a proximate explanation to the energetics and that ultimately the reason we see them is because their formation, in addition to accretion processes, is one way that energy is dissipated by infalling material. It is interesting that the results presented in this work, and by others, are the fruits of the earlier discoveries of huge structures of gas with large amounts of kinetic energy and the $10^{7} L_{\odot}$ of radiation from the central parsec that initiated a search for a $10^{8} \mathrm{M} \odot$ black hole in the nucleus. The search seems to have found a rather "puny" massive black hole only about $1 \%$ of the originally expected mass.

## Future Research

Additional $2.2 \mu \mathrm{~m}$ photometry of the central few parsecs is already available to establish the light curves of the Galactic Center LPVs and place further limits on the amplitudes of the nonvariables. The photometric results should be improved because of the better performance
characteristics of the NICMOS $256 \times 256$ arrays with lower read noise, larger formats, better sampling of the PSF, and more experience with performing photometric measurements in crowded fields. A map corresponding to the central region discussed here can be acquired in only 8 data frames with the NICMOS camera as opposed to 50 frame with the older $64 \times 64$ camera. If the insensitivity of the $\mathrm{M}_{\mathrm{BoL}}$ vs. Period relation to metallicity holds true in the Galactic Center, then a new way of determining $\mathrm{R}_{0}$ will be available that makes a minimum of geometric assumptions regarding the objects of interest because of their highly compact spatial distribution, although the foreground and intrinsic reddening effects will have to be carefully considered. The new $256 \times 256$ NICMOS array spectrometer will also be useful for a detailed study of metallicity and age for these AGB stars to test repeating star formation theories. Knowing the mass loss rates of these stars would contribute to understanding the formation and dynamics of the molecular ring.

The CO absorption spectra may have been limited in their results more by the cross correlation algorithm than by intrinsic noise. If a better procedure for renormalization can be found it would be possible to obtain an enclosed mass estimate to within $1.5^{\prime \prime}$ ( $0.05 \mathrm{pc}=0.17 \mathrm{Lt}$ yr). If the enclosed mass is constant to within that distance, any hypothesis other than a massive black hole would be difficult to support. Incorporating velocity dispersion corrections to the CO band strength may make the models consistent with the fact that a rising rotation curve is observed. High resolution imaging with the NICMOS $256 \times 256$ IR camera should clear up the issue of the discrepant value of $R_{\text {CORE }}$ for the $2.2 \mu \mathrm{~m}$ light distribution. This quantity has become crucial for understanding the CO absorption strength as a function of distance. If $R_{\text {CORE }}$ turns out to be comparable to $R_{\text {CO }}$ then new models must be developed with quite different assumptions to reproduce the observed profiles.

The NICMOS $256 \times 256$ IR spectrometer could also be used for a more detailed study of the additional CO absorption features out to the atmospheric cutoff at $2.5 \mu \mathrm{~m}$ to confirm the rotation curve results presented here. Changes in the continuum as a function of $\mathrm{r}_{\mathrm{S}_{87 A}}$ and the
strength of the near IR Mg and Si lines would help diagnose a changing stellar population. The onset of CO depletion occurs near the minimum velocity of the rotation curve. It would be interesting to sample the region within $2^{\prime \prime}$ of the dynamical center of M31 to see if there is a corresponding effect. This would strengthen the hypothesis that the CO depletion is due to dynamical effects on the stellar photospheres.

A better understanding of the details of star-star collisions inside of $R_{c o}$ is also needed. What fraction of the envelope remains after the collision? How important is heat dissipation subsequent to collision and how rapidly does a giant-like atmosphere become reestablished? The results of this work predict that whatever processes are at involved, the average CO bandstrength of the stellar population as a function of $\mathrm{r}_{\mathrm{S}_{\mathrm{gA}}}$. must go approximately as $\mathrm{r}^{\beta}$ where $\beta \leq 1.8$ in order for material to be observed predominantly at the projected radius.

## APPENDIX 1 :

## ESTIMATION OF STELLAR FOREGROUND CONTAMINATION

The calculations of the estimated foreground star contamination within the observed Galactic Center field are described. Simple models of the stellar density and extinction distributions similar to the Bahcall-Soneira Galaxy model (Bahcall and Soneira 1980) are used with luminosity functions appropriate to the near infrared (e.g. Garwood and Jones 1987; Ruelas-Mayorga 1991). Small scale variations of the stellar density and extinction distributions along the line of sight (e.g. spiral arms, star clusters, molecular clouds) are neglected. The bulge and disk contributions are considered separately.

## Estimate of Disk Contamination

Based on the formalism in Mihalas and Binney (1981), the space density of stars of absolute K magnitude $M_{K}$ and spectral class $i$ at a distance $r$ from the Sun is given by $v\left(r, M_{K}, i\right)$. This function is assumed to be the product of a density function $p_{i}(r)$ and a luminosity function $\mu_{i}\left(M_{k}\right):$

$$
\begin{equation*}
v\left(r, M_{k} d\right)=\rho_{i} \times \mu_{i}=\rho_{t}(r) \times \mu_{i}\left(M_{k}\right) \tag{6}
\end{equation*}
$$

The luminosity function $\mu_{i}\left(M_{k}\right)$ for each spectral class, parameterized by the average absolute $K$ magnitude $M_{\kappa}(0)_{i}$ and dispersion $\sigma_{i}$, is given the exponential form:

$$
\begin{equation*}
\mu_{i}\left(M_{K}\right)=\frac{1}{\sqrt{2 \pi \sigma_{l}^{2}}} \times \exp \left[-\frac{1}{2}\left(\frac{M_{K}-M_{K}(0)_{i}}{\sigma_{i}}\right)^{2}\right] . \tag{2}
\end{equation*}
$$

The space density of each spectral class $p_{i}(r)$ was assumed to have the form

$$
\begin{align*}
& \rho_{t}(r)=\rho_{\infty} \exp \left(\frac{r}{\alpha_{\mathrm{p}}}\right) ; r<R_{0}  \tag{8}\\
& \rho_{t}(r)=\rho_{\odot} \exp \left(\frac{R_{0}}{\alpha_{\mathrm{p}}}\right) \exp \left(\frac{-\left(r-R_{0}\right)}{\alpha_{\mathrm{p}}}\right) ; r \geq R_{0}
\end{align*}
$$

where $R_{\mathrm{o}}$ is the Galactic Center distance, $\alpha_{\mathrm{p}}$ is the characteristic length scale of the stellar density distribution, and $\rho_{\infty}$ is the space density of spectral class $i$ in the solar neighborhood. The assumed parameter values for each spectral class were taken from Garwood and Jones (1987).

A similar form is used for the distribution of the foreground extinction where $A_{\lambda}(r)$ is the extinction at wavelength $\lambda$ for stars at a distance $r$ from the sun:

$$
\begin{align*}
& A_{\lambda}(r)=\int_{0}^{r} a_{\lambda}(r) d r \\
& a_{\lambda}(r)=a_{\lambda \odot} \exp \left(\frac{r}{\alpha_{A}}\right) ; r<R_{0}  \tag{9}\\
& a_{\lambda}(r)=a_{\lambda \odot} \exp \left(\frac{R_{0}}{\alpha_{A}}\right) \exp \left(\frac{-\left(r-R_{0}\right)}{\alpha_{A}}\right) ; r \geq R_{0}
\end{align*}
$$

The extinction function was normalized so that $\mathrm{A}_{\mathrm{v}}\left(\mathrm{R}_{\mathrm{D}}\right)=30 \mathrm{mag}$ and with $\mathrm{a}_{\mathrm{v}_{0}}=1.000 \mathrm{mag}$ $K p c^{-1}$. For a given value of $R_{o}$ the value of $\alpha_{p}$ is then fixed. We use the Galactic Center extinction curve of Rieke and Lebofsky (1985) which gives $A_{K}=0.112 \times A_{V}$ and $A_{H}=0.175 \times A_{v}$.

For a field of solid angle $\omega$, the observed number of star counts $A\left(m_{k}\right)$ in a magnitude
bin of width $\Delta \mathrm{m}$ with apparent magnitude $\mathrm{m}_{\mathbf{k}}$ is given by (Mihalas and Binney 1981):

$$
A\left(m_{K}\right)=\omega \sum_{k=1}^{s} \int_{0}^{0} \rho_{\gamma}(r)\left[\int_{m_{k}-\frac{1}{2} \Delta m}^{m_{k}+\frac{1}{2} \Delta_{m}} \mu_{i}\left(m_{k}-5 \log \left(\frac{r}{10, ~}\right)-A_{\Sigma}(r)\right) d m_{\Sigma}\right] r^{2} d r \quad(10)
$$

and the total number of foreground stars detected $N_{\text {Tor }}\left(m_{\text {lim }}\right)$ to a limiting apparent magnitude $m_{l i m}$ is

$$
\begin{equation*}
N_{10 x}\left(m_{\mu_{k \infty}}\right)=\omega \sum_{i=1}^{S} \int_{0}^{\infty} \rho_{i}(r)\left[\int_{-\infty}^{m_{m_{1}}} \mu_{1}\left(m_{k}-5 \log \left(\frac{r}{10 p c}\right)-A_{\Sigma}(r)\right) d m_{K}\right] r^{2} d r \tag{11}
\end{equation*}
$$

The color distribution of foreground stars is, however, most instructive. For convenience let $y$ be the observed (H-K) color, then given a constant ( $\mathrm{H}-\mathrm{K})_{0}$ for all stars in each spectral class the contribution of the foreground stars to the observed ( $\mathrm{H}-\mathrm{K}$ ) color distribution of bin width $\Delta y$ is given by:

$$
\begin{align*}
N(y)=\omega \sum_{i=1}^{s} \int_{0}^{\infty} \rho_{\Delta}(r) & \left.\int_{y-\frac{1}{2} \Delta y}^{y+\frac{1}{2} \Delta y} 8\left(y^{\prime}-(H-K)_{\alpha}-\left(\frac{a_{H}}{a_{K}}-1\right) A_{K}(r)\right) d y^{\prime}\right]  \tag{12}\\
& \times\left[\int_{-\infty}^{m m_{m_{m}}} \mu_{i}\left(m_{k}-5 \log \left(\frac{r}{10 p c}\right)-A_{K}(r)\right) d m_{K}\right] r^{2} d r
\end{align*}
$$

The first factor in brackets represents the color distribution for the particular spectral class (approximated here as a Dirac delta function ) while the second factor in brackets represents the fraction of the luminosity function at distance $r$ above the threshold detection limit $m_{K}$. The H-K colors for each spectral class were taken from Frogel et al. 1978. The resulting color distribution
of foreground stars (figure A1-1) shows that most contaminants in the observed distribution are likely to have $(\mathrm{H}-\mathrm{K}$ ) $<1.5$.

The distribution of stars along the line-of-sight to the Galactic Center is shown in figure A1-2. Nearby main sequence stars make a very small contribution at small distances. The most likely contaminants are giant branch stars at distances a few kiloparsecs out from the sun where the extinction is still minimal. Those with $(\mathrm{H}-\mathrm{K})_{\mathrm{OBS}}>1.5$ are within 1 kpc of the Galactic Center and so would most likely be indistinguishable from our HK sample. The curve begins to rise again from the combined effects of the sampled volume along the line-of-sight increasing as $\mathrm{r}^{2}$ and the exponentially increasing stellar density of supergiants near the Galactic Center. The luminosity function model parameters are probably being extrapolated beyond valid limits. Nevertheless supergiants within 1 kpc of the Galactic Center would be indistinguishable from our HK sample, but even disk stars would produce very little contamination.

## Estimate of Bulge Contamination

Since the stellar density distribution is so sharply peaked near the nucleus, it is easier to consider the fractional distribution of stars within distance $\left|x-R_{0}\right|$ from the Galactic Center. In other words, we want to calculate the dispersal of stars along the line of sight using several simplifying assumptions. The surface brightness profile of $2.2 \mu \mathrm{~m}$ emission implies a density distribution of $\rho \propto R^{-1.8}$ (Sanders and Lowinger 1968). We assume the extinction and the luminosity function are constant to first order and then calculate the fraction of stars, $F$, within the distance along the line of sight $\left|x-R_{0}\right|$ about the center for a stellar density profile of $\rho$ $\propto \mathrm{R}^{-18}, \mathrm{R}_{\mathrm{o}}=8 \mathrm{kpc}$, along a square solid angle of side angle $\theta$, and core radii $\mathrm{R}_{\text {core }}=0.1,0.6,1.0$ pc.

The results (figure A1-3) show that approximately $60 \%$ of the stars in our luminosity

$$
\begin{aligned}
& \rho(\vec{R})=P_{\text {cors }},\left|\vec{R}-\vec{R}_{0}\right|<R_{\text {conss }} \\
& \rho(\vec{R})=\rho_{\text {coss }}\left(\frac{\left|\vec{R}-\vec{R}_{0}\right|}{R_{\text {coss }}}\right)^{(-1.8)},\left|\vec{R}-\vec{R}_{0}\right| \geq \vec{R}_{\text {coss }}
\end{aligned}
$$

function are within the projected distance corresponding to the field of view ( 6.4 pc ); $90 \%$ are within $50 \mathrm{pc} ; 95 \%$ within 104 pc , independent of the core radius. The dispersion in magnitude caused by the depth effects corresponding to these distances from the center is much smaller than the photometric errors of our sample. Thus we can be confident that the luminosity function derived from the complete sample principally resides within 20 pc of the Galactic Center.


Figure A1-1 The estimated ( $\mathrm{H}-\mathrm{K})_{\text {obs }}$ color distribution of foreground disk stars along the line-ofsight to the Galactic Center. The parameter $\alpha_{p}$ is the scale length for the exponential stellar density distribution. A vertical line at $(\mathrm{H}-\mathrm{K})_{\text {oss }}=1.5$ shows the cutoff used to select the complete HK sample.


Figure A1-2 The distribution of stars along the line-of-sight to the Galactic Center. The small spike near 200 pc is the small contribution make by nearby main sequence stars. The peak near 2000 pc is from giant branch stars and the rise near 6000 pc is from the increasing contribution of supergiants near the nucleus. The color cutoff of $(\mathrm{H}-\mathrm{K})_{\text {oBS }}>1.5$ is reflected in the sharp vertical drop near 7000 pc , eliminating nearly all the stars along the line of sight.


Figure A1-3 The percentage of stars within a $5^{\prime} \times 5^{\prime}$ field-of-view along the line-of-sight to the Galactic Center and inside a distance of $\left|x-R_{0}\right|$. The Galactic Center is located at a distance $R_{0}$ from the observer and it is assumed that the extinction is uniform across the FOV. The stellar density distribution is given by $\rho \propto \mathrm{R}^{-18}$ outside a core radius $\mathrm{R}_{\text {core. }}$. The three curves represent core radii of $0.1,0.6$, and 1.0 pc but this has negligible effect on the fractional line-of-sight dispersion.

## APPENDIX 2 :

## GALACTIC CENTER HK PHOTOMETRY

The following table is a listing of the HK database of 638 stars used in the discussions of chapters 3 and 4. The column names are designated by:

## ID : Star Identification Number

$\alpha_{\text {RS }}$ : Right ascension offset position from IRS 7, $17^{\mathrm{h}} 42^{\mathrm{m}} 29.2^{\mathrm{s}}$ (1950), in arcseconds.
$\delta_{\text {IRs }}$ : Declination offset position from IRS 7, $-28^{\circ} 59^{\prime} 12.9^{\prime \prime}$ (1950), in arcseconds.
$n_{\mathrm{K} 1}$ : Number of K detections in May 1987
$\mathrm{n}_{\mathrm{K}_{2}}$ : Number of K detections in September 1987
$n_{K 3}$ : Number of $K$ detections in May 1988
$\mathrm{n}_{\mathrm{H} 3}$ : Number of H detections in May 1988
$<m_{K}>$ : Average $K$ magnitude. The photometric errors are $0.09,0.07$, 0.10 , and $0.15(\mathrm{H})$ mag respectively
( $\mathrm{H}-\mathrm{K})_{\text {OBS }}$ : Observed H-K color
( $\mathrm{H}-\mathrm{K})_{0}$ : Intrinsic $\mathrm{H}-\mathrm{K}$ color, dereddened with the extinction curve of Rieke and Lebofsky (1985) ( $\mathrm{A}_{\mathrm{H}}=0.175 \times \mathrm{A}_{\mathrm{V}} ; \mathrm{A}_{\mathrm{K}}=0.112 \times \mathrm{A}_{\mathrm{V}}$ ) and assuming a forground visual extinction of 31.8.
$\Delta \mathrm{m}_{\mathrm{K}}$ : The RMS magnitude residual between epochs. Nonvariables will have $\Delta \mathrm{m}_{\mathrm{K}}<0.274$ at the $3 \sigma$ level.
$\mathrm{M}_{\mathrm{BOL}}$ : The inferred bolometric magnitude assuming a constant foreground visual extinction of 31.8 mag a distance modulus to the Galactic Center of 14.2, and bolometric correction $\mathrm{BC}_{\mathrm{K}}=$ 3.0.

TABLE A2-1

| ID | $\alpha_{\text {IRS }}$ | $\delta_{\text {ITST }}$ | $\mathrm{n}_{\mathrm{K} 1}$ | $\mathrm{n}_{\mathrm{K} 2}$ | $n_{\mathrm{ks}}$ | $\mathrm{n}_{\mathrm{Hs}}$ | < $\mathrm{m}_{\mathrm{k}}>$ | $(\mathrm{H}-\mathrm{K})_{\text {de }}$ | (H-K) ${ }_{0}$ | $\Delta \mathrm{m}_{\mathrm{k}}$ | $\mathrm{M}_{\mathrm{BOL}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -187.6 | -103.8 | 0 | 2 | 2 | 2 | 10.383 | 2.283 | 0.280 | 0.157 | -4.222 |
| 2 | -184.5 | -72.8 | 0 | 2 | 2 | 2 | 10.617 | 2.753 | 0.750 | 0.000 | -3.988 |
| 3 | -183.6 | 13.5 | 0 | 2 | 2 | 1 | 9.929 | 2.508 | 0.505 | 0.000 | -4.676 |
| 4 | -183.0 | -24.3 | 0 | 2 | 2 | 2 | 9.884 | 2.062 | 0.059 | 0.000 | -4.721 |
| 5 | -182.7 | -89.9 | 1 | 2 | 2 | 2 | 10.746 | 2.163 | 0.160 | 0.079 | -3.859 |
| 6 | -181.7 | -106.7 | 3 | 2 | 2 | 2 | 10.088 | 2.014 | 0.011 | 0.152 | -4.517 |
| 7 | -180.7 | -27.4 | 1 | 2 | 2 | 2 | 9.766 | 1.982 | -0.021 | 0.214 | -4.839 |
| 8 | -180.3 | -118.5 | 2 | 2 | 2 | 2 | 9.750 | 1.664 | -0.339 | 0.107 | -4.855 |
| 9 | -180.2 | -123.8 | 2 | 2 | 2 | 2 | 10.829 | 2.302 | 0.299 | 0.156 | -3.776 |
| 10 | -178.6 | -20.1 | 2 | 2 | 2 | 2 | 10.156 | 1.898 | -0.105 | 0.080 | -4.449 |
| 11 | -178.0 | 96.6 | 0 | 1 | 1 | 1 | 10.887 | 2.205 | 0.202 | 0.060 | -3.718 |
| 12 | -177.2 | -6.7 | 3 | 2 | 2 | 2 | 10.883 | 2389 | 0.386 | 0.099 | -3.722 |
| 13 | -176.9 | 22.0 | 2 | 2 | 2 | 2 | 9.245 | 2.847 | 0.844 | 0.395 | -5.360 |
| 14 | -176.8 | -37.1 | 2 | 2 | 2 | 2 | 10.853 | 1.940 | -0.063 | 0.063 | -3.752 |
| 15 | -176.5 | 34.1 | 2 | 3 | 3 | 3 | 10.599 | 3.006 | 1.003 | 0.108 | -4.006 |


| ID | $\alpha_{\text {U88 }}$ | $\delta_{\text {per }}$ | $n_{k 1}$ | $n_{\text {K2 }}$ | $\mathrm{n}_{\mathrm{xS}}$ | $\mathrm{n}_{\mathrm{HS}}$ | $\left\langle m_{k}\right\rangle$ | (H-K) ${ }_{\text {d }}$ | ( $\mathrm{H}-\mathrm{K})_{0}$ | $\Delta \mathrm{m}_{\mathrm{k}}$ | $\mathrm{M}_{\mathrm{sox}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16 | -175.6 | 41.6 | 2 | 2 | 2 | 2 | 10.352 | 2.328 | 0.325 | 0.102 | -4.253 |
| 17 | -174.9 | -163.6 | 1 | 1 | 1 | 1 | 10.705 | 1.764 | -0.239 | 0.189 | -3.900 |
| 18 | -174.7 | 4.6 | 2 | 2 | 1 | 2 | 10.883 | 2.209 | 0.206 | 0.061 | -3.722 |
| 19 | -174.4 | -107.3 | 3 | 2 | 2 | 2 | 10.468 | 2.201 | 0.198 | 0.105 | -4.137 |
| 20 | -174.3 | 162.9 | 1 | 1 | 1 | 1 | 9.412 | 2.028 | 0.025 | 0.135 | -5.193 |
| 21 | -173.7 | 8.0 | 2 | 2 | 2 | 2 | 10.686 | 1.725 | -0.278 | 0.042 | -3.919 |
| 22 | -173.1 | -167.4 | 1 | 0 | 1 | 1 | 10.129 | 2.465 | 0.462 | 0.030 | -4.476 |
| 23 | -173.0 | 131.2 | 2 | 2 | 2 | 2 | 10.763 | 2.238 | 0.235 | 0.036 | -3.842 |
| 24 | -172.3 | 46.6 | 2 | 2 | 2 | 2 | 10.956 | 2.330 | 0.327 | 0.190 | -3.649 |
| 25 | -171.9 | 72.0 | 2 | 1 | 2 | 2 | 10.375 | 1.786 | -0.217 | 0.165 | -4.230 |
| 26 | -168.2 | 95.8 | 3 | 2 | 2 | 2 | 10.825 | 2.199 | 0.196 | 0.070 | -3.780 |
| 27 | -167.7 | -158.0 | 1 | 1 | 1 | 1 | 10.529 | 2.867 | 0.864 | 0.100 | -4.076 |
| 28 | -166.1 | 17.9 | 2 | 1 | 2 | 2 | 10.766 | 1.850 | -0.153 | 0.221 | -3.839 |
| 29 | -165.8 | -91.2 | 2 | 2 | 2 | 2 | 10.701 | 1.984 | -0.019 | 0.082 | 3.904 |
| 30 | -165.1 | -125.2 | 2 | 2 | 2 | 2 | 10.435 | 1.509 | -0.494 | 0.117 | -4.170 |
| 31 | -164.9 | 112.6 | 2 | 2 | 2 | 2 | 10.559 | 2.126 | 0.123 | 0.059 | -4.046 |
| 32 | -164.7 | 21.6 | 2 | 2 | 2 | 1 | 10.772 | 2.075 | 0.072 | 0.193 | -3.833 |
| 33 | -164.5 | 65.8 | 2 | 2 | 3 | 1 | 9.888 | 2.398 | 0.395 | 0.157 | -4.717 |
| 34 | -164.4 | 71.4 | 2 | 1 | 2 | 2 | 10.118 | 1.532 | -0.471 | 0.093 | -4.487 |
| 35 | -164.1 | 53.5 | 2 | 2 | 2 | 2 | 10.566 | 2.334 | 0.331 | 0.209 | -4.039 |
| 36 | -161.0 | 109.3 | 2 | 2 | 2 | 2 | 10.129 | 1.528 | -0.475 | 0.065 | -4.476 |
| 37 | -160.8 | -26.8 | 2 | 2 | 2 | 2 | 10.771 | 2.293 | 0.290 | 0.100 | -3.834 |
| 38 | -159.9 | 57.8 | 2 | 2 | 2 | 2 | 10.622 | 1.721 | -0.282 | 0.068 | -3.983 |
| 39 | -159.8 | -128.8 | 2 | 1 | 2 | 1 | 9.642 | 1.973 | -0.030 | 0.491 | -4.963 |
| 40 | -159.7 | -88.5 | 2 | 2 | 2 | 2 | 10.616 | 1.706 | -0.297 | 0.086 | -3.989 |
| 41 | -158.6 | -18.6 | 2 | 2 | 2 | 2 | 10.837 | 2.198 | 0.195 | 0.061 | -3.768 |
| 42 | -158.5 | -56.9 | 2 | 2 | 2 | 2 | 10.346 | 2.031 | 0.028 | 0.145 | -4.259 |
| 43 | -158.2 | -168.9 | 1 | 0 | 0 | 1 | 11.252 | 1.703 | -0.300 | -1.00 | -3.353 |
| 44 | -157.4 | 23.7 | 2 | 2 | 2 | 2 | 10.388 | 2.050 | 0.047 | 0.035 | -4.217 |
| 45 | -157.4 | -44.8 | 2 | 2 | 2 | 2 | 9.302 | 2.086 | 0.083 | 0.074 | -5.303 |
| 46 | -157.3 | 90.4 | 2 | 2 | 2 | 2 | 9.329 | 2.672 | 0.669 | 0.127 | -5.276 |
| 47 | -156.5 | -50.3 | 2 | 2 | 2 | 2 | 10.315 | 2.197 | 0.194 | 0.064 | -4.290 |
| 48 | -156.5 | 107.8 | 2 | 2 | 2 | 2 | 10.769 | 2.008 | 0.005 | 0.131 | -3.836 |
| 49 | -155.4 | -23.0 | 2 | 2 | 2 | 2 | 10.810 | 2.357 | 0.354 | 0.111 | -3.795 |
| 50 | -154.4 | 8.2 | 2 | 2 | 2 | 2 | 10.051 | 2.993 | 0.990 | 0.250 | -4.554 |
| 51 | -154.0 | 130.7 | 2 | 2 | 2 | 2 | 8.239 | 2.151 | 0.148 | 0.080 | -6.366 |
| 52 | -153.8 | -71.4 | 2 | 2 | 2 | 2 | 10.705 | 1.609 | -0.394 | 0.051 | -3.900 |
| 53 | -152.9 | -120.5 | 2 | 2 | 2 | 2 | 10.853 | 1.503 | -0.500 | 0.026 | -3.752 |
| 54 | -152.4 | -132.2 | 2 | 2 | 2 | 2 | 10.744 | 1.998 | -0.005 | 0.196 | -3.861 |
| 55 | -152.3 | -88.2 | 2 | 2 | 2 | 2 | 10.242 | 1.585 | -0.418 | 0.066 | -4.363 |
| 56 | -152.3 | 79.0 | 2 | 2 | 2 | 2 | 10.687 | 3.098 | 1.095 | 0.069 | -3.918 |
| 57 | -151.9 | 89.0 | 2 | 2 | 2 | 2 | 10.313 | 2.926 | 0.923 | 0.105 | -4.292 |
| 58 | -151.8 | -63.9 | 2 | 3 | 2 | 2 | 9.456 | 1.883 | -0.120 | 0.047 | -5.149 |
| 59 | -151.4 | -78.2 | 1 | 2 | 2 | 1 | 10.546 | 2.536 | 0.533 | 0.199 | -4.059 |
| 60 | -148.1 | 60.0 | 2 | 2 | 2 | 2 | 10.087 | 2.022 | 0.019 | 0.060 | -4.518 |


| ID | $\alpha_{\text {ITS }}$ | $\delta_{\text {ITST }}$ | $\mathrm{n}_{\mathrm{Kl}}$ | $\mathrm{n}_{\mathrm{K} 2}$ | $\mathrm{n}_{\mathrm{xS}}$ | $\mathrm{n}_{43}$ | $\left\langle\mathrm{m}_{\mathrm{x}}>\right.$ | ( $\mathrm{H}-\mathrm{K})_{\text {des }}$ | ( $\mathrm{H}-\mathrm{K})_{0}$ | $\Delta m_{k}$ | $\mathrm{M}_{\mathrm{pox}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 61 | -147.8 | -81.0 | 2 | 2 | 2 | 2 | 8.671 | 2.452 | 0.449 | 0.489 | -5.934 |
| 62 | -147.4 | -27.6 | 2 | 2 | 2 | 2 | 10.608 | 2.286 | 0.283 | 0.083 | -4.003 |
| 63 | -147.3 | -96.3 | 2 | 2 | 2 | 2 | 10.219 | 2.408 | 0.405 | 0.095 | -4.386 |
| 64 | -147.0 | 24.5 | 2 | 1 | 2 | 2 | 9.939 | 2.500 | 0.497 | 0.247 | -4.666 |
| 65 | -146.7 | -78.2 | 1 | 2 | 0 | 1 | 10.593 | 2.580 | 0.577 | 0.155 | -4.012 |
| 66 | -146.7 | -122.8 | 2 | 2 | 2 | 2 | 10.075 | 1.910 | -0.093 | 0.079 | -4.530 |
| 67 | -146.6 | 111.4 | 2 | 2 | 2 | 2 | 9.988 | 2.267 | 0.264 | 0.058 | -4.617 |
| 68 | -146.5 | -4.7 | 3 | 2 | 2 | 2 | 10.382 | 3.638 | 1.635 | 0.119 | -4.223 |
| 69 | -146.1 | 31.4 | 0 | 0 | 1 | 1 | 10.858 | 2.589 | 0.586 | -1.00 | 3.747 |
| 70 | -146.0 | -154.6 | 1 | 1 | 1 | 1 | 10.548 | 2.353 | 0.350 | 0.147 | -4.057 |
| 71 | -145.5 | -39.6 | 3 | 2 | 2 | 2 | 9.851 | 1.787 | -0.216 | 0.229 | -4.744 |
| 72 | -144.8 | -106.3 | 3 | 2 | 2 | 2 | 10.290 | 2.787 | 0.784 | 0.207 | -4.315 |
| 73 | -144.3 | 15.5 | 2 | 2 | 2 | 2 | 10.564 | 1.824 | -0.179 | 0.046 | -4.041 |
| 74 | -144.2 | 140.1 | 1 | 1 | 1 | 1 | 10.196 | 3.295 | 1.292 | 0.127 | -4.409 |
| 75 | -143.0 | -109.5 | 2 | 2 | 2 | 2 | 10.302 | 2.919 | 0.916 | 0.100 | -4.303 |
| 76 | -142.9 | -25.7 | 2 | 2 | 2 | 2 | 10.799 | 2.505 | 0.502 | 0.105 | -3.806 |
| 7 | -142.9 | -45.6 | 2 | 3 | 2 | 1 | 10.911 | 2.538 | 0.535 | 0.161 | -3.694 |
| 78 | -142.2 | -67.4 | 2 | 3 | 2 | 3 | 9.195 | 2.031 | 0.028 | 0.123 | -5.410 |
| 79 | -141.4 | -30.5 | 2 | 3 | 2 | 2 | 10.082 | 2.241 | 0.238 | 0.168 | -4.523 |
| 80 | -141.2 | 133.3 | 1 | 2 | 2 | 1 | 10.863 | 2.154 | 0.151 | 0.112 | -3.742 |
| 81 | -139.5 | 21.4 | 2 | 2 | 2 | 2 | 10.561 | 2.394 | 0.391 | 0.106 | -4.044 |
| 82 | -138.8 | 40.1 | 2 | 2 | 2 | 2 | 10.726 | 2.449 | 0.446 | 0.126 | -3.879 |
| 83 | -138.7 | -24.9 | 2 | 2 | 2 | 2 | 10.546 | 1.726 | -0.277 | 0.103 | -4.059 |
| 84 | -138.3 | -134.3 | 2 | 2 | 4 | 4 | 10.418 | 2.565 | 0.562 | 0.101 | -4.187 |
| 85 | -138.2 | -167.5 | 1 | 2 | 2 | 2 | 8.911 | 2.709 | 0.706 | 0.242 | -5.694 |
| 86 | -138.0 | -114.6 | 2 | 2 | 2 | 4 | 10.011 | 2.443 | 0.440 | 0.067 | -4.594 |
| 87 | -137.8 | 112.6 | 2 | 2 | 2 | 2 | 10.985 | 2.268 | 0.265 | 0.091 | -3.620 |
| 88 | -137.6 | 8.6 | 2 | 2 | 2 | 2 | 10.363 | 2.268 | 0.265 | 0.051 | -4.242 |
| 89 | -137.5 | 71.3 | 2 | 1 | 2 | 2 | 10.769 | 2.163 | 0.160 | 0.293 | -3.836 |
| 90 | -137.4 | -138.6 | 2 | 2 | 2 | 3 | 9.595 | 1.931 | -0.072 | 0.163 | -5.010 |
| 91 | -137.2 | -127.7 | 2 | 4 | 4 | 4 | 10.597 | 2.206 | 0.203 | 0.062 | -4.008 |
| 92 | -136.8 | 28.9 | 3 | 2 | 2 | 2 | 9.588 | 1.973 | -0.030 | 0.166 | -5.017 |
| 93 | -136.7 | -109.1 | 2 | 4 | 4 | 4 | 10.842 | 1.705 | -0.298 | 0.083 | -3.763 |
| 94 | -136.1 | -159.9 | 1 | 2 | 2 | 2 | 10.812 | 1.621 | -0.382 | 0.088 | -3.793 |
| 95 | -136.0 | -121.8 | 2 | 4 | 4 | 4 | 10.970 | 2.415 | 0.412 | 0.103 | -3.635 |
| 96 | -136.0 | -78.3 | 2 | 4 | 2 | 4 | 10.909 | 2.656 | 0.653 | 0.066 | -3.696 |
| 97 | -135.9 | 50.6 | 2 | 2 | 2 | 2 | 10.219 | 1.787 | -0.216 | 0.074 | -4.386 |
| 98 | -134.8 | -54.9 | 2 | 4 | 4 | 4 | 9.734 | 1.713 | -0.290 | 0.043 | -4.871 |
| 99 | -134.6 | -83.8 | 2 | 4 | 4 | 4 | 10.619 | 2.226 | 0.223 | 0.017 | -3.986 |
| 100 | -134.6 | 7.7 | 0 | 1 | 0 | 2 | 10.514 | 1.975 | -0.028 | -1.00 | -4.091 |
| 101 | -133.7 | 56.7 | 2 | 2 | 2 | 2 | 10.770 | 2.043 | 0.040 | 0.094 | -3.835 |
| 102 | -132.4 | 38.3 | 2 | 4 | 4 | 4 | 10.989 | 2.190 | 0.187 | 0.113 | 3.616 |
| 103 | -132.0 | -48.1 | 2 | 3 | 4 | 4 | 10.636 | 1.890 | -0.113 | 0.286 | -3.969 |
| 104 | -131.4 | 112.0 | 2 | 3 | 3 | 2 | 9.356 | 1.810 | -0.193 | 0.031 | -5.249 |
| 105 | -131.1 | 139.4 | 1 | 1 | 1 | 1 | 8.953 | 1.826 | -0.177 | 0.057 | -5.652 |


| ID | $\alpha_{\text {TSG }}$ | $\delta_{\text {IJS }}$ | $\mathrm{n}_{\mathrm{K} 1}$ | $n_{k 2}$ | $\mathrm{n}_{\mathrm{kS}}$ | $\mathrm{n}_{\mathrm{HS}}$ | $\left\langle m_{x}>\right.$ | ( $\mathrm{H}-\mathrm{K})_{\text {d }}$ | ( $\mathrm{H}-\mathrm{K})_{0}$ | $\Delta \mathrm{m}_{k}$ | $\mathrm{M}_{\text {DOL }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 106 | -131.0 | 20.5 | 3 | 4 | 4 | 4 | 10.847 | 2.007 | 0.004 | 0.058 | -3.758 |
| 107 | -130.3 | -149.5 | 2 | 2 | 2 | 2 | 10.478 | 2.462 | 0.459 | 0.027 | -4.127 |
| 108 | -130.3 | -75.5 | 4 | 4 | 4 | 4 | 10.883 | 2.008 | 0.005 | 0.164 | -3.722 |
| 109 | -129.8 | 78.3 | 2 | 4 | 4 | 4 | 9.924 | 2.284 | 0.281 | 0.086 | -4.681 |
| 110 | -129.1 | -164.5 | 2 | 2 | 2 | 2 | 10.740 | 2.361 | 0.358 | 0.118 | -3.865 |
| 111 | -129.0 | -16.0 | 4 | 4 | 4 | 4 | 10.659 | 2.473 | 0.470 | 0.023 | -3.946 |
| 112 | -128.7 | -88.0 | 4 | 4 | 4 | 4 | 10.951 | 2.127 | 0.124 | 0.076 | -3.654 |
| 113 | -128.7 | -169.1 | 2 | 1 | 2 | 1 | 10.605 | 2.348 | 0.345 | 0.197 | -4.000 |
| 114 | -128.0 | -67.4 | 4 | 6 | 4 | 6 | 10.296 | 2.035 | 0.032 | 0.067 | -4.309 |
| 115 | -127.9 | 136.4 | 2 | 1 | 3 | 3 | 10.763 | 2.260 | 0.257 | 0.060 | -3.842 |
| 116 | -126.1 | 51.5 | 4 | 4 | 4 | 4 | 10.636 | 1.936 | -0.067 | 0.072 | -3.969 |
| 117 | -125.5 | -111.6 | 4 | 4 | 4 | 4 | 10.322 | 2.024 | 0.021 | 0.068 | -4.283 |
| 118 | -124.2 | 10.0 | 4 | 4 | 4 | 4 | 10.325 | 2.884 | 0.881 | 0.242 | -4.280 |
| 119 | -122.2 | 93.1 | 4 | 4 | 4 | 4 | 10.252 | 1.890 | -0.113 | 0.014 | -4.353 |
| 120 | -121.5 | -165.0 | 2 | 2 | 2 | 2 | 10.585 | 2.199 | 0.196 | 0.178 | -4.020 |
| 121 | -121.4 | -80.7 | 3 | 4 | 4 | 4 | 10.032 | 1.899 | -0.104 | 0.108 | -4.573 |
| 122 | -121.3 | 44.3 | 4 | 4 | 4 | 1 | 9.945 | 3.449 | 1.446 | 0.615 | -4.660 |
| 123 | -121.1 | -159.1 | 2 | 2 | 2 | 2 | 9.218 | 2.164 | 0.161 | 0.113 | -5.387 |
| 124 | -120.4 | 152.5 | 2 | 1 | 2 | 2 | 8.418 | 2.029 | 0.026 | 0.186 | -6.187 |
| 125 | -119.8 | -75.7 | 4 | 4 | 4 | 4 | 9.383 | 3.491 | 1.488 | 0.477 | -5.222 |
| 126 | -119.4 | 75.0 | 4 | 4 | 4 | 4 | 10.459 | 2.668 | 0.665 | 0.045 | -4.146 |
| 127 | -117.8 | 93.9 | 4 | 3 | 4 | 4 | 10.212 | 1.663 | -0.340 | 0.095 | -4.393 |
| 128 | -116.7 | -72.5 | 1 | 2 | 0 | 4 | 11.002 | 1.921 | -0.082 | 0.206 | -3.603 |
| 129 | -116.6 | -53.7 | 4 | 2 | 2 | 3 | 9.618 | 1.911 | -0.092 | 0.217 | -4.987 |
| 130 | -116.5 | 146.7 | 2 | 1 | 2 | 2 | 10.449 | 1.823 | -0.180 | 0.201 | -4.156 |
| 131 | -115.3 | 31.0 | 5 | 1 | 5 | 5 | 10.094 | 1.518 | -0.485 | 0.129 | -4.511 |
| 132 | -115.2 | -6.1 | 5 | 2 | 2 | 2 | 10.593 | 3.528 | 1.525 | 0.118 | -4.012 |
| 133 | -114.8 | 93.6 | 3 | 1 | 4 | 1 | 10.852 | 1.632 | -0.371 | 0.169 | -3.753 |
| 134 | -114.7 | 104.6 | 4 | 3 | 4 | 5 | 9.981 | 1.559 | -0.444 | 0.045 | -4.624 |
| 135 | -114.5 | 85.7 | 4 | 2 | 3 | 4 | 9.623 | 2.043 | 0.040 | 0.237 | -4.982 |
| 136 | -114.3 | -36.9 | 5 | 2 | 2 | 2 | 10.794 | 1.563 | -0.440 | 0.005 | -3.811 |
| 137 | -114.2 | -60.5 | 4 | 2 | 2 | 4 | 10.760 | 2.587 | 0.584 | 0.252 | -3.845 |
| 138 | -113.4 | -88.0 | 3 | 2 | 2 | 2 | 10.028 | 2.562 | 0.559 | 0.108 | -4.577 |
| 139 | -113.0 | -72.4 | 4 | 2 | 2 | 2 | 10.858 | 1.975 | -0.028 | 0.077 | -3.747 |
| 140 | -112.4 | 113.1 | 4 | 1 | 2 | 4 | 10.500 | 1.937 | -0.066 | 0.105 | -4.105 |
| 141 | -111.8 | 160.4 | 2 | 0 | 2 | 1 | 10.989 | 3.390 | 1.387 | 0.173 | -3.616 |
| 142 | -111.6 | 10.9 | 4 | 2 | 2 | 4 | 10.697 | 2.057 | 0.054 | 0.194 | -3.908 |
| 143 | -111.0 | -13.4 | 3 | 2 | 2 | 2 | 10.931 | 2.021 | 0.018 | 0.155 | -3.674 |
| 144 | -110.5 | 20.6 | 2 | 2 | 2 | 4 | 10.780 | 2.016 | 0.013 | 0.075 | -3.825 |
| 145 | -110.3 | -53.0 | 2 | 2 | 2 | 2 | 10.399 | 2.077 | 0.074 | 0.046 | -4.206 |
| 146 | -109.9 | -108.0 | 2 | 2 | 2 | 2 | 10.817 | 2.098 | 0.095 | 0.062 | -3.788 |
| 147 | -109.6 | -98.0 | 2 | 1 | 2 | 3 | 9.858 | 2.005 | 0.002 | 0.345 | -4.747 |
| 148 | -108.8 | 8.3 | 2 | 2 | 2 | 2 | 10.107 | 2.506 | 0.503 | 0.159 | -4.498 |
| 149 | -108.2 | -153.8 | 1 | 1 | 1 | 1 | 8.832 | 1.776 | -0.227 | 0.069 | -5.773 |
| 150 | -107.3 | -134.6 | 2 | 2 | 2 | 2 | 9.282 | 1.568 | -0.435 | 0.068 | -5.323 |


| ID | $\alpha_{\text {TST }}$ | $\delta_{\text {TST }}$ | $\mathrm{n}_{\mathrm{k} 1}$ | $\mathrm{n}_{\mathrm{K} 2}$ | $\mathrm{n}_{\mathrm{ks}}$ | $\mathrm{n}_{\mathrm{HJ}}$ | $\left\langle m_{k}>\right.$ | ( $\mathrm{H}-\mathrm{K})_{\text {dx }}$ | ( $\mathrm{H}-\mathrm{K}$ ) ${ }_{0}$ | $\Delta \mathrm{m}_{\mathrm{K}}$ | $\mathrm{M}_{\text {BOL }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 151 | -107.2 | 117.6 | 4 | 1 | 2 | 4 | 10.129 | 1.583 | -0.420 | 0.161 | -4.476 |
| 152 | -107.0 | -24.2 | 2 | 2 | 2 | 2 | 10.487 | 1.939 | -0.064 | 0.198 | -4.118 |
| 153 | -105.7 | -165.9 | 1 | 1 | 1 | 1 | 9.417 | 1.660 | -0.343 | 0.142 | -5.188 |
| 154 | -105.5 | 65.0 | 3 | 2 | 2 | 2 | 10.487 | 1.728 | -0.275 | 0.042 | -4.118 |
| 155 | -105.4 | 56.6 | 2 | 2 | 2 | 2 | 10.701 | 2.224 | 0.221 | 0.045 | -3.904 |
| 156 | -105.4 | 120.6 | 1 | 1 | 2 | 2 | 10.582 | 1.640 | -0.363 | 0.008 | -4.023 |
| 157 | -105.3 | 41.3 | 2 | 2 | 2 | 2 | 10.636 | 2.716 | 0.713 | 0.055 | -3.969 |
| 158 | -105.0 | -21.9 | 0 | 1 | 1 | 1 | 10.739 | 1.914 | -0.089 | 0.127 | -3.866 |
| 159 | -104.0 | 110.9 | 2 | 1 | 2 | 2 | 10.001 | 2.132 | 0.129 | 0.035 | -4.604 |
| 160 | -108.6 | -104.0 | 1 | 2 | 3 | 2 | 9.410 | 2.357 | 0.354 | 0.502 | -5.195 |
| 161 | -103.5 | -96.2 | 2 | 2 | 2 | 2 | 10.854 | 2.437 | 0.434 | 0.125 | -3.751 |
| 162 | -103.3 | -150.5 | 1 | 1 | 1 | 1 | 10.023 | 1.686 | -0.317 | 0.114 | -4.582 |
| 163 | -103.2 | 52.8 | 2 | 2 | 2 | 2 | 10.726 | 2.427 | 0.424 | 0.133 | -3.879 |
| 164 | -102.9 | -122.4 | 1 | 2 | 2 | 2 | 10.822 | 1.598 | -0.405 | 0.105 | -3.783 |
| 165 | -102.7 | -1.6 | 2 | 3 | 3 | 3 | 9.114 | 2.074 | 0.071 | 0.139 | -5.491 |
| 166 | -102.2 | -138.3 | 2 | 1 | 2 | 1 | 10.233 | 2.181 | 0.178 | 0.077 | -4.372 |
| 167 | -102.1 | 72.1 | 2 | 2 | 2 | 2 | 10.682 | 1.910 | -0.093 | 0.083 | -3.923 |
| 168 | -102.0 | -125.0 | 2 | 2 | 2 | 2 | 10.784 | 2.047 | 0.044 | 0.031 | -3.821 |
| 169 | -101.4 | -100.8 | 2 | 3 | 2 | 2 | 10.418 | 3.093 | 1.090 | 0.458 | -4.187 |
| 170 | -101.3 | 55.9 | 2 | 2 | 2 | 2 | 10.415 | 2.313 | 0.310 | 0.088 | -4.190 |
| 171 | -100.3 | 154.9 | 1 | 0 | 1 | 1 | 9.488 | 3.505 | 1.502 | 0.303 | -5.117 |
| 172 | -99.5 | 44.0 | 2 | 2 | 2 | 2 | 10.426 | 1.946 | -0.057 | 0.141 | -4.179 |
| 173 | -99.2 | -94.8 | 2 | 2 | 2 | 2 | 9.926 | 3.497 | 1.494 | 0.100 | -4.679 |
| 174 | -98.9 | -159.0 | 1 | 1 | 1 | 1 | 10.842 | 1.917 | -0.086 | 0.199 | -3.763 |
| 175 | -98.4 | -165.9 | 1 | 1 | 1 | 1 | 10.282 | 2.429 | 0.426 | 0.152 | -4.323 |
| 176 | -98.0 | -23.8 | 2 | 1 | 2 | 2 | 10.275 | 1.929 | -0.074 | 0.147 | -4.330 |
| 177 | -97.6 | 95.1 | 2 | 2 | 2 | 2 | 10.529 | 3.138 | 1.135 | 0.076 | -4.076 |
| 178 | -97.2 | -126.4 | 2 | 2 | 2 | 2 | 10.179 | 1.891 | -0.112 | 0.027 | -4.426 |
| 179 | -97.0 | 14.4 | 1 | 1 | 2 | 1 | 10.186 | 3.101 | 1.098 | 0.117 | -4.419 |
| 180 | -96.6 | -62.6 | 2 | 2 | 2 | 1 | 10.720 | 2.609 | 0.606 | 0.215 | -3.885 |
| 181 | -95.5 | -35.5 | 2 | 3 | 3 | 3 | 10.201 | 2.121 | 0.118 | 0.149 | -4.404 |
| 182 | -95.2 | -153.0 | 1 | 1 | 1 | 1 | 10.880 | 2.479 | 0.476 | 0.053 | -3.725 |
| 183 | -95.2 | -159.1 | 1 | 1 | 1 | 1 | 10.589 | 2.409 | 0.406 | 0.298 | -4.016 |
| 184 | -94.9 | -41.7 | 2 | 2 | 2 | 2 | 10.243 | 2.491 | 0.488 | 0.080 | -4.362 |
| 185 | -94.8 | 67.8 | 2 | 3 | 2 | 3 | 10.853 | 2.566 | 0.563 | 0.232 | -3.752 |
| 186 | -94.1 | 31.9 | 2 | 3 | 3 | 3 | 10.752 | 2.369 | 0.366 | 0.249 | -3.853 |
| 187 | -93.9 | -32.0 | 2 | 2 | 2 | 2 | 10.818 | 2.793 | 0.790 | 0.081 | -3.787 |
| 188 | -93.8 | -89.9 | 2 | 2 | 2 | 2 | 10.000 | 2.311 | 0.308 | 0.050 | -4.605 |
| 189 | -93.4 | 83.8 | 2 | 2 | 2 | 2 | 9.673 | 1.649 | -0.354 | 0.134 | -4.932 |
| 190 | -93.3 | -19.6 | 2 | 2 | 2 | 2 | 10.722 | 2.104 | 0.101 | 0.168 | -3.883 |
| 191 | -92.8 | 127.8 | 2 | 1 | 2 | 2 | 10.441 | 2.250 | 0.247 | 0.068 | -4.164 |
| 192 | -92.1 | -81.0 | 2 | 2 | 2 | 2 | 8.907 | 2.190 | 0.187 | 0.026 | -5.698 |
| 193 | -91.9 | 161.6 | 1 | 0 | 1 | 1 | 9.755 | 2.144 | 0.141 | 0.091 | -4.850 |
| 194 | -91.6 | 3.8 | 2 | 2 | 2 | 2 | 9.129 | 2.414 | 0.411 | 0.133 | -5.476 |
| 195 | -91.0 | 60.5 | 1 | 2 | 0 | 2 | 10.561 | 2.739 | 0.736 | 0.362 | -4.044 |


| ID | $\alpha_{\text {LRT }}$ | $\delta_{\text {ISS }}$ | $\mathrm{n}_{\mathrm{Kl}}$ | $\mathrm{n}_{\mathrm{k}}$ | $\mathrm{n}_{\mathrm{K}}$ | $\mathrm{n}_{\text {Hs }}$ | $\left\langle m_{k}>\right.$ | ( $\mathrm{H}-\mathrm{K})_{\text {dxa }}$ | ( $\mathrm{H}-\mathrm{K}$ ) | $\Delta m_{k}$ | $\mathrm{M}_{\text {boh }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 196 | -91.0 | -113.5 | 2 | 2 | 2 | 2 | 8.668 | 1.887 | -0.116 | 0.103 | -5.937 |
| 197 | -91.0 | 147.9 | 1 | 0 | 1 | 1 | 10.113 | 1.913 | -0.090 | 0.028 | -4.492 |
| 198 | -90.6 | 135.7 | 1 | 0 | 2 | 2 | 9.980 | 2.122 | 0.119 | 0.017 | -4.625 |
| 199 | -90.1 | -45.4 | 2 | 2 | 2 | 2 | 9.920 | 2.547 | 0.544 | 0.423 | -4.685 |
| 200 | -89.8 | -49.9 | 2 | 2 | 2 | 2 | 10.639 | 1.641 | -0.362 | 0.159 | -3.966 |
| 201 | -89.7 | 31.2 | 3 | 2 | 3 | 2 | 11.042 | 2.033 | 0.030 | 0.142 | -3.563 |
| 202 | -89.5 | 46.4 | 2 | 2 | 2 | 1 | 10.851 | 2.499 | 0.496 | 0.132 | -3.754 |
| 203 | -89.5 | -89.7 | 2 | 2 | 2 | 2 | 10.843 | 2.043 | 0.040 | 0.104 | -3.762 |
| 204 | -87.7 | 159.5 | 1 | 0 | 1 | 1 | 10.636 | 1.961 | -0.042 | 0.059 | -3.969 |
| 205 | -87.7 | -11.9 | 2 | 2 | 2 | 2 | 10.866 | 2.362 | 0.359 | 0.152 | -3.739 |
| 206 | -87.6 | 61.6 | 2 | 2 | 2 | 2 | 9.588 | 2.768 | 0.765 | 0.590 | -5.017 |
| 207 | -87.5 | 5.2 | 1 | 2 | 2 | 2 | 10.919 | 1.765 | -0.238 | 0.051 | -3.686 |
| 208 | -87.3 | -53.4 | 2 | 2 | 2 | 2 | 10.346 | 2.005 | 0.002 | 0.009 | -4.259 |
| 209 | -87.2 | 55.5 | 2 | 2 | 2 | 2 | 10.659 | 2.121 | 0.118 | 0.049 | -3.946 |
| 210 | -86.1 | 105.8 | 2 | 1 | 2 | 2 | 9.924 | 1.875 | -0.128 | 0.093 | -4.681 |
| 211 | -85.4 | 122.4 | 2 | 1 | 2 | 1 | 10.257 | 1.851 | -0.152 | 0.085 | -4.348 |
| 212 | -84.6 | -131.8 | 0 | 1 | 0 | 1 | 10.985 | 2.904 | 0.901 | -1.00 | -3.620 |
| 213 | -84.5 | 9.5 | 2 | 2 | 4 | 2 | 10.939 | 2.117 | 0.114 | 0.083 | -3.666 |
| 214 | -84.3 | 87.1 | 2 | 2 | 2 | 2 | 8.284 | 2.012 | 0.009 | 0.167 | -6.321 |
| 215 | -83.6 | -74.5 | 4 | 4 | 4 | 4 | 10.868 | 1.983 | -0.020 | 0.069 | -3.737 |
| 216 | -83.0 | 80.1 | 2 | 2 | 3 | 2 | 10.803 | 1.680 | -0.323 | 0.212 | -3.802 |
| 217 | -83.0 | 141.9 | 1 | 0 | 1 | 1 | 9.713 | 1.916 | -0.087 | 0.070 | -4.892 |
| 218 | -82.7 | -130.1 | 4 | 4 | 4 | 4 | 10.209 | 3.140 | 1.137 | 0.362 | -4.396 |
| 219 | -82.6 | 68.6 | 2 | 1 | 4 | 3 | 10.336 | 1.685 | -0.318 | 0.199 | -4.269 |
| 220 | -82.3 | 24.1 | 2 | 2 | 4 | 2 | 10.133 | 2.522 | 0.519 | 0.055 | -4.472 |
| 221 | -82.2 | 13.1 | 2 | 3 | 4 | 3 | 10.747 | 2.776 | 0.773 | 0.081 | -3.858 |
| 222 | -82.0 | -137.7 | 4 | 2 | 3 | 3 | 10.567 | 3.315 | 1.312 | 0.092 | -4.038 |
| 223 | -81.8 | -141.5 | 3 | 2 | 2 | 2 | 10.560 | 2.289 | 0.286 | 0.111 | -4.045 |
| 224 | -81.5 | 108.2 | 2 | 1 | 4 | 2 | 10.831 | 1.946 | -0.057 | 0.048 | -3.774 |
| 225 | -81.3 | -44.9 | 3 | 4 | 4 | 4 | 10.103 | 2.059 | 0.056 | 0.093 | -4.502 |
| 226 | -80.5 | -109.4 | 4 | 4 | 4 | 4 | 10.257 | 3.071 | 1.068 | 0.164 | -4.348 |
| 227 | -79.8 | 71.3 | 2 | 4 | 5 | 3 | 9.565 | 2.046 | 0.043 | 0.092 | -5.040 |
| 228 | -79.7 | -51.7 | 4 | 4 | 4 | 4 | 10.745 | 2.330 | 0.327 | 0.103 | -3.860 |
| 229 | -79.2 | -36.3 | 4 | 5 | 5 | 5 | 9.026 | 2.186 | 0.183 | 0.061 | -5.579 |
| 230 | -78.9 | 35.8 | 4 | 5 | 5 | 6 | 10.097 | 2.238 | 0.235 | 0.145 | -4.508 |
| 231 | -78.6 | 50.3 | 4 | 4 | 4 | 4 | 10.869 | 1.704 | -0.299 | 0.157 | -3.736 |
| 232 | -78.5 | 21.7 | 4 | 4 | 4 | 4 | 10.369 | 2.664 | 0.661 | 0.226 | -4.236 |
| 233 | -78.5 | -129.9 | 1 | 4 | 4 | 3 | 10.927 | 2.408 | 0.405 | 0.210 | -3.678 |
| 234 | -78.4 | 3.6 | 4 | 4 | 5 | 4 | 10.223 | 2.328 | 0.325 | 0.126 | -4.382 |
| 235 | -77.1 | 89.5 | 4 | 4 | 4 | 4 | 10.833 | 2.142 | 0.139 | 0.170 | -3.772 |
| 236 | -77.1 | -40.2 | 4 | 1 | 2 | 2 | 9.821 | 2.138 | 0.135 | 0.090 | -4.784 |
| 237 | -77.0 | -66.9 | 4 | 5 | 3 | 4 | 10.842 | 2.657 | 0.654 | 0.117 | -3.763 |
| 238 | -76.8 | -120.1 | 4 | 4 | 4 | 4 | 10.660 | 1.755 | -0.248 | 0.066 | -3.945 |
| 239 | -76.8 | -45.5 | 4 | 4 | 4 | 4 | 10.015 | 2.114 | 0.111 | 0.053 | -4.590 |
| 240 | -76.4 | -160.1 | 2 | 2 | 2 | 2 | 10.768 | 2.779 | 0.776 | 0.016 | -3.837 |


| ID | $\alpha_{\text {DS }}$ | $\delta_{\text {DET }}$ | $\mathrm{n}_{\mathrm{k} 1}$ | $\mathrm{n}_{10}$ | $\mathrm{n}_{\mathrm{ks}}$ | $\mathrm{n}_{\text {Hs }}$ | $\left\langle m_{k}\right\rangle$ | ( $\mathrm{H}-\mathrm{K})_{\text {dut }}$ | (H-K) | $\Delta \mathrm{m}_{\mathrm{k}}$ | $\mathrm{M}_{\mathrm{DCO}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 241 | -75.9 | 154.6 | 2 | 1 | 2 | 2 | 9.382 | 2.048 | 0.045 | 0.075 | -5.223 |
| 242 | -75.7 | -95.9 | 4 | 4 | 4 | 4 | 10.424 | 2.554 | 0.551 | 0.085 | -4.181 |
| 243 | -75.6 | -108.5 | 2 | 4 | 3 | 5 | 10.210 | 2.468 | 0.465 | 0.556 | -4.395 |
| 244 | -75.4 | 140.0 | 2 | 1 | 2 | 2 | 10.494 | 2.288 | 0.285 | 0.078 | -4.111 |
| 245 | -75.4 | 68.5 | 4 | 4 | 4 | 6 | 10.160 | 1.911 | -0.092 | 0.035 | -4.445 |
| 246 | -75.0 | 73.6 | 4 | 4 | 4 | 4 | 10.595 | 1.947 | -0.056 | 0.105 | -4.010 |
| 247 | -74.5 | 13.9 | 4 | 4 | 4 | 4 | 9.819 | 2.584 | 0.581 | 0.263 | -4.786 |
| 248 | -74.0 | 80.7 | 4 | 4 | 4 | 4 | 10.937 | 1.834 | -0.169 | 0.071 | -3.668 |
| 249 | -73.7 | -33.4 | 4 | 6 | 4 | 6 | 10.619 | 1.707 | -0.296 | 0.026 | 3.986 |
| 250 | -73.6 | -69.7 | 3 | 5 | 4 | 6 | 9.669 | 1.607 | -0.396 | 0.083 | -4.936 |
| 251 | -73.0 | -131.4 | 4 | 4 | 4 | 4 | 9.797 | 2.642 | 0.639 | 0.089 | -4.808 |
| 252 | -73.0 | 7.2 | 4 | 4 | 4 | 4 | 10.792 | 2366 | 0.363 | 0.057 | -3.813 |
| 253 | -72.6 | 105.8 | 1 | 3 | 3 | 4 | 10.959 | 1.549 | -0.454 | 0.224 | -3.646 |
| 254 | -72.3 | 98.0 | 4 | 4 | 4 | 4 | 10.882 | 2.131 | 0.128 | 0.019 | -3.723 |
| 255 | -72.0 | 53.8 | 4 | 4 | 4 | 4 | 10.656 | 2309 | 0.306 | 0.089 | -3.949 |
| 256 | -72.0 | -110.8 | 4 | 4 | 3 | 4 | 10.705 | 3.056 | 1.053 | 0.149 | -3.900 |
| 257 | -71.8 | 108.1 | 4 | 3 | 4 | 2 | 10.748 | 1.986 | -0.017 | 0.171 | -3.857 |
| 258 | -71.8 | 143.9 | 2 | 1 | 2 | 2 | 10.517 | 2.254 | 0.251 | 0.186 | -4.088 |
| 259 | -70.5 | 19.0 | 4 | 4 | 4 | 3 | 10.515 | 2.342 | 0.339 | 0.179 | -4.090 |
| 260 | -70.0 | 126.2 | 4 | 3 | 4 | 4 | 10.748 | 2.413 | 0.410 | 0.086 | -3.857 |
| 261 | -69.9 | 26.8 | 4 | 3 | 3 | 1 | 10.566 | 2.092 | 0.089 | 0.196 | -4.039 |
| 262 | -69.8 | -90.1 | 4 | 4 | 4 | 4 | 9.498 | 2.318 | 0.315 | 0.386 | -5.107 |
| 263 | -69.2 | -55.0 | 4 | 4 | 4 | 4 | 10.624 | 2.441 | 0.438 | 0.009 | -3.981 |
| 264 | -69.2 | -122.2 | 4 | 4 | 4 | 4 | 9.305 | 2.393 | 0.390 | 0.076 | -5.300 |
| 265 | -68.3 | -84.6 | 3 | 1 | 4 | 3 | 10.591 | 2.276 | 0.273 | 0.119 | -4.014 |
| 266 | -67.4 | -81.9 | 3 | 4 | 4 | 3 | 10.574 | 2.204 | 0.201 | 0.169 | -4.031 |
| 267 | -66.8 | 85.8 | 4 | 4 | 4 | 4 | 10.356 | 1.921 | -0.082 | 0.175 | -4.249 |
| 268 | -66.6 | -131.4 | 4 | 2 | 2 | 3 | 10.126 | 2.212 | 0.209 | 0.150 | -4.479 |
| 269 | -66.6 | 7.8 | 4 | 4 | 4 | 3 | 10.697 | 1.731 | -0.272 | 0.159 | -3.908 |
| 270 | -66.4 | -102.6 | 4 | 4 | 1 | 5 | 10.849 | 2.043 | 0.040 | 0.160 | -3.756 |
| 271 | -66.0 | -108.1 | 4 | 0 | 2 | 3 | 10.649 | 1.583 | -0.420 | 0.234 | -3.956 |
| 272 | -64.5 | 27.8 | 5 | 1 | 2 | 2 | 10.454 | 2.042 | 0.039 | 0.197 | -4.151 |
| 273 | -64.4 | 80.5 | 4 | 4 | 4 | 3 | 10.652 | 2.058 | 0.055 | 0.139 | -3.953 |
| 274 | -64.1 | -115.4 | 4 | 2 | 2 | 2 | 10.552 | 2.440 | 0.437 | 0.138 | -4.053 |
| 275 | -62.8 | -25.3 | 3 | 1 | 1 | 2 | 10.255 | 1.834 | -0.169 | 0.130 | -4.350 |
| 276 | -62.5 | -79.8 | 2 | 2 | 2 | 2 | 10.288 | 1.726 | -0.277 | 0.005 | -4.317 |
| 277 | -61.8 | 95.5 | 4 | 4 | 4 | 4 | 10.528 | 2.198 | 0.195 | 0.061 | -4.077 |
| 278 | -60.5 | 44.3 | 4 | 2 | 2 | 3 | 10.679 | 2.335 | 0.332 | 0.108 | -3.926 |
| 279 | -60.4 | 99.2 | 5 | 1 | 2 | 5 | 10.662 | 1.913 | -0.090 | 0.116 | -3.943 |
| 280 | -59.0 | -160.0 | 1 | 1 | 1 | 1 | 10.828 | 3.413 | 1.410 | 0.250 | -3.777 |
| 281 | -58.9 | -43.1 | 2 | 2 | 2 | 2 | 10.860 | 2.452 | 0.449 | 0.133 | -3.745 |
| 282 | -57.9 | -90.1 | 2 | 2 | 2 | 2 | 10.176 | 2.217 | 0.214 | 0.070 | -4.429 |
| 283 | -57.5 | -102.2 | 2 | 1 | 2 | 3 | 10.857 | 2.103 | 0.100 | 0.245 | -3.748 |
| 284 | -56.7 | -59.3 | 2 | 2 | 2 | 2 | 10.825 | 2.209 | 0.206 | 0.248 | -3.780 |
| 285 | -56.7 | -23.7 | 2 | 2 | 2 | 2 | 10.921 | 2.325 | 0.322 | 0.052 | -3.684 |


| ID | $\alpha_{\text {ITST }}$ | $\delta_{\text {ITS } 7}$ | $\mathrm{n}_{\mathrm{K} 1}$ | $\mathrm{n}_{\mathrm{k} 2}$ | $\mathrm{n}_{\mathrm{xs}}$ | $\mathrm{n}_{4}$ | $\left\langle\mathrm{m}_{\mathrm{k}}>\right.$ | (H-K) ${ }_{\text {dobe }}$ | ( $\mathrm{H}-\mathrm{K})_{0}$ | $\Delta m_{k}$ | M ${ }_{\text {box }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 286 | -56.6 | -94.0 | 0 | 2 | 2 | 2 | 10.717 | 2.389 | 0.386 | 0.000 | 3.888 |
| 287 | -56.4 | 65.9 | 2 | 3 | 2 | 2 | 10.863 | 2.560 | 0.557 | 0.090 | -3.742 |
| 288 | -56.0 | -75.1 | 2 | 2 | 2 | 2 | 10.658 | 2.935 | 0.932 | 0.090 | 3.947 |
| 289 | -55.5 | 40.4 | 2 | 2 | 2 | 2 | 10.077 | 2.409 | 0.406 | 0.020 | -4.578 |
| 290 | -55.1 | -114.0 | 2 | 2 | 2 | 2 | 10.554 | 1.945 | -0.058 | 0.156 | -4.051 |
| 291 | -54.6 | -146.1 | 1 | 1 | 1 | 1 | 9.388 | 2.897 | 0.894 | 0.028 | -5.217 |
| 292 | -54.2 | 3.2 | 0 | 2 | 1 | 1 | 11.051 | 2.091 | 0.088 | 0.108 | -3.554 |
| 293 | -54.1 | -1.4 | 2 | 3 | 2 | 3 | 10.256 | 1.875 | -0.128 | 0.195 | -4.349 |
| 294 | -54.1 | -50.5 | 2 | 2 | 2 | 2 | 10.981 | 2.566 | 0.563 | 0.065 | 3.624 |
| 295 | -53.7 | -29.1 | 2 | 2 | 2 | 2 | 9.735 | 2.235 | 0.232 | 0.120 | -4.870 |
| 296 | -53.5 | 168.8 | 0 | 1 | 1 | 1 | 10.366 | 2.037 | 0.034 | 0.026 | -4.239 |
| 297 | -53.3 | -3.8 | 0 | 1 | 2 | 1 | 10.778 | 2.015 | 0.012 | 0.128 | -3.827 |
| 298 | -53.2 | 6.4 | 2 | 2 | 2 | 2 | 10.529 | 2.262 | 0.259 | 0.154 | -4.076 |
| 299 | -52.9 | -117.4 | 2 | 2 | 2 | 2 | 10.906 | 2.911 | 0.908 | 0.255 | -3.699 |
| 300 | -52.8 | -12.3 | 2 | 2 | 2 | 2 | 8.529 | 2.112 | 0.109 | 0.026 | -6.076 |
| 301 | -51.9 | 11.5 | 2 | 2 | 2 | 1 | 10.260 | 2.457 | 0.454 | 0.071 | -4.345 |
| 302 | -51.9 | 97.8 | 1 | 1 | 2 | 2 | 10.784 | 1.998 | -0.005 | 0.084 | -3.821 |
| 303 | -51.2 | 146.5 | 1 | 1 | 1 | 1 | 10.423 | 2.046 | 0.043 | 0.086 | -4.182 |
| 304 | -51.0 | -30.8 | 1 | 2 | 0 | 2 | 10.739 | 1.868 | -0.135 | 0.292 | -3.866 |
| 305 | -51.0 | 161.7 | 1 | 1 | 1 | 1 | 10.063 | 3.174 | 1.171 | 0.441 | -4.542 |
| 306 | -50.0 | -24.2 | 2 | 2 | 2 | 2 | 10.465 | 2.979 | 0.976 | 0.221 | -4.140 |
| 307 | -49.3 | -17.5 | 0 | 1 | 1 | 2 | 10.367 | 2.204 | 0.201 | 0.119 | -4.238 |
| 308 | -49.3 | -123.5 | 2 | 2 | 2 | 2 | 9.817 | 2.808 | 0.805 | 0.277 | -4.788 |
| 309 | -49.0 | 135.4 | 1 | 2 | 2 | 2 | 10.585 | 2.736 | 0.733 | 0.185 | -4.020 |
| 310 | -48.9 | -164.1 | 1 | 1 | 1 | 1 | 10.669 | 2.871 | 0.868 | 0.112 | -3.936 |
| 311 | -47.4 | 5.6 | 1 | 2 | 2 | 2 | 9.683 | 2.013 | 0.010 | 0.315 | -4.922 |
| 312 | -47.4 | 59.4 | 2 | 2 | 2 | 2 | 10.979 | 2.468 | 0.465 | 0.134 | -3.626 |
| 313 | -47.3 | 81.4 | 2 | 2 | 2 | 2 | 10.984 | 2.336 | 0.333 | 0.101 | -3.621 |
| 314 | -46.2 | -62.2 | 2 | 2 | 2 | 2 | 10.069 | 2.543 | 0.540 | 0.192 | -4.536 |
| 315 | -45.4 | -1.7 | 2 | 2 | 2 | 3 | 10.336 | 2.180 | 0.177 | 0.400 | -4.269 |
| 316 | -45.0 | -57.6 | 2 | 0 | 1 | 1 | 10.939 | 2.182 | 0.179 | 0.014 | -3.666 |
| 317 | -43.7 | 7.4 | 2 | 2 | 2 | 2 | 9.152 | 1.990 | -0.013 | 0.153 | -5.453 |
| 318 | -43.4 | $-66.2$ | 2 | 1 | 3 | 3 | 9.350 | 2.094 | 0.091 | 0.131 | -5.255 |
| 319 | -42.7 | 33.3 | 2 | 3 | 3 | 2 | 10.702 | 2.163 | 0.160 | 0.118 | -3.903 |
| 320 | -42.4 | -142.9 | 1 | 1 | 1 | 1 | 9.960 | 2.596 | 0.593 | 0.076 | -4.645 |
| 321 | -42.4 | -59.6 | 2 | 2 | 2 | 2 | 10.512 | 2.176 | 0.173 | 0.203 | -4.093 |
| 322 | -42.3 | 72.4 | 2 | 2 | 2 | 2 | 10.610 | 2.636 | 0.633 | 0.055 | -3.995 |
| 323 | -42.2 | 163.5 | 0 | 1 | 1 | 1 | 10.413 | 1.785 | -0.218 | 0.026 | -4.192 |
| 324 | -41.9 | 25.4 | 2 | 2 | 2 | 2 | 10.743 | 2.305 | 0.302 | 0.065 | -3.862 |
| 325 | -41.8 | -101.1 | 2 | 3 | 3 | 3 | 10.092 | 1.670 | -0.333 | 0.255 | -4.513 |
| 326 | -41.6 | -51.0 | 2 | 2 | 2 | 2 | 10.313 | 2.702 | 0.699 | 0.091 | -4.292 |
| 327 | -41.4 | -2.0 | 2 | 2 | 2 | 3 | 10.015 | 2.193 | 0.190 | 0.176 | -4.590 |
| 328 | -41.3 | 92.2 | 2 | 2 | 2 | 2 | 9.019 | 1.975 | -0.028 | 0.026 | -5.586 |
| 329 | -41.2 | 160.7 | 1 | 1 | 1 | 1 | 9.331 | 2.841 | 0.838 | 0.022 | -5.274 |
| 330 | -40.0 | -34.1 | 2 | 3 | 3 | 1 | 10.249 | 2.879 | 0.876 | 0.094 | -4.356 |


| ID | $\alpha_{\text {DST }}$ | $\delta_{\text {IDS }}$ | $\mathrm{n}_{\mathrm{k} 1}$ | $\mathrm{n}_{\mathrm{k} 2}$ | $\mathrm{n}_{5}$ | $\mathrm{n}_{\text {HS }}$ | $\left\langle m_{k}\right\rangle$ | ( $\mathrm{H}-\mathrm{K})_{\text {da }}$ | ( $\mathrm{H}-\mathrm{K}$ ) | $\Delta \mathrm{m}_{k}$ | $\mathrm{M}_{\mathrm{DOL}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 331 | -39.7 | -9.8 | 0 | 1 | 0 | 2 | 11.141 | 2.088 | 0.085 | -1.00 | 3.464 |
| 332 | -39.7 | 1.2 | 1 | 3 | 0 | 1 | 10.765 | 2.252 | 0.249 | 0.416 | -3.840 |
| 333 | -39.3 | -41.6 | 2 | 2 | 2 | 2 | 10.160 | 3.061 | 1.058 | 0.209 | -4.445 |
| 334 | -38.9 | -107.0 | 3 | 2 | 2 | 2 | 9.925 | 1.874 | -0.129 | 0.341 | -4.680 |
| 335 | -38.1 | 112.5 | 2 | 2 | 2 | 2 | 10.442 | 1.812 | -0.191 | 0.109 | -4.163 |
| 336 | -38.0 | 155.5 | 1 | 1 | 1 | 1 | 10.505 | 2.719 | 0.716 | 0.121 | -4.100 |
| 337 | -37.9 | -8.1 | 2 | 2 | 2 | 2 | 10.212 | 2.335 | 0.332 | 0.447 | -4.393 |
| 338 | -37.6 | 144.8 | 1 | 1 | 1 | 1 | 10.433 | 2.483 | 0.480 | 0.170 | -4.172 |
| 339 | -36.5 | 58.7 | 0 | 1 | 1 | 2 | 10.963 | 2.456 | 0.453 | 0.000 | -3.642 |
| 340 | -36.2 | 6.4 | 0 | 2 | 2 | 2 | 10.433 | 1.756 | -0.247 | 0.087 | -4.172 |
| 341 | -36.2 | -49.5 | 1 | 2 | 2 | 2 | 10.404 | 1.962 | -0.041 | 0.160 | -4.201 |
| 342 | -34.9 | 74.3 | 2 | 2 | 2 | 2 | 9.862 | 1.876 | -0.127 | 0.158 | -4.743 |
| 343 | -34.6 | 90.3 | 2 | 2 | 2 | 2 | 10.499 | 2.239 | 0.236 | 0.026 | -4.106 |
| 344 | -33.9 | -146.4 | 1 | 3 | 2 | 1 | 10.462 | 3.620 | 1.617 | 0.050 | -4.143 |
| 345 | -33.8 | -43.5 | 0 | 1 | 2 | 2 | 10.941 | 2.100 | 0.097 | 0.203 | -3.664 |
| 346 | -33.7 | 14.6 | 1 | 2 | 1 | 3 | 10.534 | 2.331 | 0.328 | 0.231 | -4.071 |
| 347 | -33.6 | 155.9 | 1 | 1 | 1 | 1 | 10.870 | 2.391 | 0.388 | 0.072 | -3.735 |
| 348 | -33.6 | 59.9 | 2 | 2 | 2 | 2 | 10.393 | 2.849 | 0.846 | 0.030 | -4.212 |
| 349 | -33.1 | 79.7 | 2 | 2 | 2 | 2 | 9.288 | 2.004 | 0.001 | 0.017 | -5.317 |
| 350 | -32.9 | -18.4 | 4 | 2 | 4 | 3 | 10.228 | 2.849 | 0.846 | 0.217 | -4.377 |
| 351 | -32.4 | -165.7 | 1 | 3 | 2 | 2 | 10.564 | 2.623 | 0.620 | 0.089 | -4.041 |
| 352 | -31.6 | 120.6 | 2 | 2 | 2 | 2 | 10.335 | 2.278 | 0.275 | 0.148 | -4.270 |
| 353 | -30.4 | 114.9 | 3 | 2 | 3 | 2 | 9.073 | 2.192 | 0.189 | 0.101 | -5.532 |
| 354 | -30.3 | -51.8 | 3 | 3 | 3 | 3 | 10.681 | 2.556 | 0.553 | 0.157 | -3.924 |
| 355 | -29.4 | 2.6 | 4 | 3 | 4 | 4 | 10.952 | 2.430 | 0.427 | 0.129 | -3.653 |
| 356 | -28.0 | -22.2 | 4 | 2 | 4 | 3 | 10.433 | 2.439 | 0.436 | 0.259 | -4.172 |
| 357 | -28.0 | -111.6 | 3 | 4 | 4 | 4 | 10.746 | 3.350 | 1.347 | 0.094 | -3.859 |
| 358 | -27.5 | -34.5 | 4 | 2 | 4 | 6 | 10.081 | 2.103 | 0.100 | 0.144 | -4.524 |
| 359 | -26.8 | -161.6 | 1 | 3 | 2 | 2 | 10.934 | 2.120 | 0.117 | 0.051 | -3.671 |
| 360 | -26.7 | -39.1 | 5 | 3 | 4 | 4 | 9.804 | 3.032 | 1.029 | 0.183 | -4.801 |
| 361 | -25.7 | 18.1 | 2 | 3 | 2 | 3 | 10.225 | 3.023 | 1.020 | 0.171 | -4.380 |
| 362 | -25.3 | 129.5 | 5 | 4 | 4 | 4 | 10.518 | 2.604 | 0.601 | 0.158 | -4.087 |
| 363 | -25.1 | 165.7 | 2 | 1 | 2 | 2 | 10.999 | 2.751 | 0.748 | 0.080 | -3.606 |
| 364 | -24.7 | 36.4 | 2 | 4 | 3 | 4 | 10.888 | 2.397 | 0.394 | 0.194 | -3.717 |
| 365 | -24.5 | 71.6 | 4 | 4 | 3 | 4 | 9.681 | 1.906 | -0.097 | 0.035 | -4.924 |
| 366 | -24.0 | 75.9 | 4 | 4 | 3 | 4 | 10.288 | 2.188 | 0.185 | 0.057 | -4.317 |
| 367 | -23.7 | 68.1 | 3 | 5 | 4 | 4 | 10.145 | 2.032 | 0.029 | 0.103 | -4.460 |
| 368 | -22.6 | 96.8 | 6 | 4 | 3 | 4 | 9.807 | 2.407 | 0.404 | 0.115 | -4.798 |
| 369 | -22.6 | -100.0 | 4 | 6 | 5 | 5 | 8.506 | 2.517 | 0.514 | 0.556 | -6.099 |
| 370 | -22.5 | 16.2 | 4 | 3 | 4 | 4 | 9.619 | 2.385 | 0.382 | 0.074 | -4.986 |
| 371 | -22.3 | 11.6 | 2 | 3 | 3 | 1 | 10.614 | 2.777 | 0.774 | 0.156 | -3.991 |
| 372 | -21.6 | 40.7 | 4 | 4 | 3 | 4 | 9.233 | 2.383 | 0.380 | 0.051 | -5.372 |
| 373 | -21.4 | 115.9 | 4 | 4 | 4 | 4 | 10.030 | 2.175 | 0.172 | 0.039 | -4.575 |
| 374 | -20.8 | -143.7 | 1 | 3 | 2 | 2 | 10.892 | 3.249 | 1.246 | 0.115 | -3.713 |
| 375 | -20.0 | -91.3 | 4 | 4 | 4 | 4 | 10.642 | 2.430 | 0.427 | 0.167 | -3.963 |


| ID | $\alpha_{\text {LST }}$ | $\delta_{\text {tus }}$ | $n_{\text {K1 }}$ | $n_{k 2}$ | $n_{k S}$ | $\mathrm{n}_{4 \mathrm{ss}}$ | $\left\langle m_{k}\right\rangle$ | (H-K) | (H-K) | $\Delta \mathrm{m}_{\mathbf{x}}$ | $\mathrm{M}_{\mathrm{BOL}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 376 | -19.8 | -161.9 | 1 | 3 | 1 | 2 | 10.607 | 2.234 | 0.231 | 0.143 | 3.998 |
| 377 | -19.1 | 76.8 | 3 | 4 | 3 | 4 | 10.153 | 2.102 | 0.099 | 0.049 | -4.452 |
| 378 | -18.9 | -30.7 | 3 | 2 | 4 | 3 | 10.422 | 3.214 | 1.211 | 0.355 | -4.183 |
| 379 | -18.7 | -22.9 | 2 | 2 | 3 | 3 | 10.816 | 2.592 | 0.589 | 0.267 | -3.789 |
| 380 | -18.2 | 24.6 | 4 | 3 | 4 | 3 | 10.457 | 2.175 | 0.172 | 0.136 | -4.148 |
| 381 | -17.9 | 73.1 | 4 | 4 | 3 | 4 | 9.632 | 1.939 | -0.064 | 0.040 | -4.973 |
| 382 | -17.3 | 130.6 | 5 | 4 | 4 | 4 | 10.202 | 1.988 | -0.015 | 0.080 | -4.403 |
| 383 | -17.0 | 27.6 | 3 | 2 | 4 | 3 | 10.673 | 2.424 | 0.421 | 0.287 | -3.932 |
| 384 | -16.4 | 10.1 | 4 | 3 | 4 | 4 | 10.023 | 1.869 | -0.134 | 0.280 | -4.582 |
| 385 | -15.7 | -76.1 | 2 | 4 | 2 | 4 | 9.896 | 2.990 | 0.987 | 0.049 | -4.709 |
| 386 | -15.4 | 18.0 | 4 | 3 | 4 | 4 | 10.047 | 2.258 | 0.255 | 0.033 | -4.558 |
| 387 | -15.0 | 158.5 | 3 | 2 | 2 | 2 | 10.226 | 2.273 | 0.270 | 0.002 | -4.379 |
| 388 | -14.9 | 23.0 | 2 | 1 | 2 | 1 | 11.041 | 2.311 | 0.308 | 0.188 | -3.564 |
| 389 | -14.3 | 121.9 | 4 | 4 | 4 | 4 | 10.600 | 1.998 | -0.005 | 0.293 | -4.005 |
| 390 | -12.5 | -6.7 | 0 | 1 | 0 | 3 | 10.376 | 2.479 | 0.476 | -1.00 | -4.229 |
| 391 | -12.4 | 84.5 | 4 | 4 | 2 | 4 | 10.158 | 1.894 | -0.109 | 0.299 | -4.447 |
| 392 | -12.4 | 105.2 | 4 | 4 | 3 | 3 | 10.592 | 1.978 | -0.025 | 0.204 | -4.013 |
| 393 | -12.2 | -16.2 | 0 | 2 | 2 | 2 | 9.943 | 2.388 | 0.385 | 0.558 | -4.662 |
| 394 | -12.0 | -23.8 | 0 | 1 | 1 | 1 | 10.799 | 2.076 | 0.073 | 0.295 | -3.806 |
| 395 | -11.0 | 128.9 | 4 | 4 | 2 | 4 | 10.290 | 1.836 | -0.167 | 0.041 | -4.315 |
| 396 | -10.9 | 77.8 | 4 | 4 | 0 | 4 | 10.280 | 1.892 | -0.111 | 0.136 | -4.325 |
| 397 | -10.8 | -86.4 | 2 | 2 | 2 | 2 | 10.533 | 2.736 | 0.733 | 0.190 | -4.072 |
| 398 | -10.4 | 158.4 | 3 | 2 | 0 | 2 | 10.774 | 1.602 | -0.401 | 0.298 | -3.831 |
| 399 | -10.3 | 45.4 | 4 | 1 | 1 | 4 | 10.395 | 1.904 | -0.099 | 0.138 | -4.210 |
| 400 | -10.1 | 19.8 | 3 | 1 | 2 | 4 | 9.847 | 2.064 | 0.061 | 0.280 | -4.758 |
| 401 | -9.8 | 93.2 | 4 | 0 | 1 | 4 | 10.960 | 2.302 | 0.299 | 0.046 | -3.645 |
| 402 | -9.4 | 113.2 | 4 | 2 | 2 | 4 | 8.747 | 2.212 | 0.209 | 0.131 | -5.858 |
| 403 | -8.9 | -19.8 | 0 | 0 | 2 | 1 | 10.498 | 1.500 | -0.503 | -1.00 | -4.107 |
| 404 | -8.5 | -33.2 | 2 | 0 | 2 | 3 | 9.154 | 2.250 | 0.247 | 0.110 | -5.451 |
| 405 | -8.4 | -7.7 | 0 | 0 | 1 | 2 | 10.114 | 2.434 | 0.431 | -1.00 | -4.491 |
| 406 | -8.3 | -16.2 | 2 | 0 | 2 | 3 | 9.687 | 1.717 | -0.286 | 0.208 | -4.918 |
| 407 | -8.2 | 17.9 | 0 | 1 | 2 | 2 | 10.230 | 2.050 | 0.047 | 0.026 | -4.375 |
| 408 | -8.1 | 7.8 | 2 | 1 | 2 | 3 | 8.868 | 2.172 | 0.169 | 0.058 | -5.737 |
| 409 | -8.0 | 162.9 | 2 | 1 | 1 | 2 | 10.769 | 1.896 | -0.107 | 0.103 | -3.836 |
| 410 | -7.0 | 89.1 | 1 | 2 | 1 | 4 | 10.637 | 2.085 | 0.082 | 0.210 | -3.968 |
| 411 | -6.8 | 128.7 | 2 | 2 | 2 | 4 | 10.837 | 1.844 | -0.159 | 0.091 | -3.768 |
| 412 | -6.2 | -26.3 | 1 | 0 | 2 | 2 | 9.898 | 2.315 | 0.312 | 0.173 | -4.707 |
| 413 | -6.0 | 104.1 | 2 | 2 | 2 | 3 | 8.743 | 1.939 | -0.064 | 0.176 | -5.862 |
| 414 | -6.0 | 0.2 | 1 | 1 | 2 | 3 | 9.998 | 2.637 | 0.634 | 0.521 | -4.607 |
| 415 | -5.1 | -21.3 | 2 | 0 | 2 | 2 | 9.935 | 1.658 | -0.345 | 0.112 | -4.670 |
| 416 | -4.9 | 47.0 | 2 | 2 | 1 | 2 | 9.309 | 2.081 | 0.078 | 0.085 | -5.296 |
| 417 | -4.8 | 35.6 | 1 | 2 | 1 | 2 | 10.955 | 2.861 | 0.858 | 0.233 | -3.650 |
| 418 | -3.4 | -12.4 | 2 | 0 | 2 | 2 | 8.503 | 2.157 | 0.154 | 0.047 | -6.102 |
| 419 | -3.4 | -31.0 | 2 | 0 | 2 | 2 | 8.166 | 1.986 | -0.017 | 0.017 | -6.439 |
| 420 | -2.9 | 95.0 | 2 | 2 | 1 | 2 | 10.713 | 2.445 | 0.442 | 0.229 | -3.892 |


| ID | $\alpha_{\text {BTS }}$ | $\delta_{\text {TBS }}$ | $\mathrm{n}_{\mathrm{K} 1}$ | $\mathrm{n}_{\mathrm{k} 2}$ | $\mathrm{n}_{\mathrm{kJ}}$ | $\mathrm{n}_{\text {H3 }}$ | $\left\langle\mathrm{m}_{k}\right\rangle$ | ( $\mathrm{H}-\mathrm{K})_{\text {dut }}$ | (H-K) | $\Delta \mathrm{m}_{k}$ | $\mathrm{M}_{\text {dod }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 421 | -1.2 | -23.9 | 0 | 0 | 2 | 2 | 10.952 | 1.715 | $-0.288$ | -1.00 | 3.653 |
| 422 | -1.0 | -48.6 | 2 | 1 | 2 | 2 | 10.161 | 2.178 | 0.175 | 0.173 | -4.444 |
| 423 | -0.8 | 12.1 | 2 | 1 | 2 | 1 | 9.764 | 2.311 | 0.308 | 0.276 | -4.841 |
| 424 | -0.3 | 30.8 | 2 | 2 | 1 | 3 | 10.547 | 1.940 | -0.063 | 0.189 | -4.058 |
| 425 | 0.0 | 0.0 | 2 | 1 | 2 | 3 | 6.901 | 2.693 | 0.690 | 0.148 | -7.704 |
| 426 | 0.3 | -13.4 | 2 | 0 | 2 | 2 | 8.718 | 2.198 | 0.195 | 0.057 | -5.887 |
| 427 | 0.5 | 94.2 | 2 | 2 | 1 | 2 | 10.590 | 1.668 | -0.335 | 0.048 | -4.015 |
| 428 | 0.5 | 154.6 | 2 | 1 | 1 | 1 | 9.269 | 2.184 | 0.181 | 0.112 | -5.336 |
| 429 | 0.7 | 111.8 | 2 | 2 | 2 | 2 | 10.588 | 2.676 | 0.673 | 0.122 | -4.017 |
| 430 | 1.0 | 5.4 | 1 | 1 | 2 | 2 | 8.904 | 2.325 | 0.322 | 0.133 | -5.701 |
| 431 | 1.0 | -27.1 | 2 | 0 | 2 | 2 | 9.931 | 2.550 | 0.547 | 0.068 | -4.674 |
| 432 | 1.5 | 72.9 | 0 | 2 | 1 | 2 | 10.243 | 2.481 | 0.478 | 0.300 | -4.362 |
| 433 | 1.7 | 42.0 | 2 | 2 | 1 | 2 | 9.419 | 2.370 | 0.367 | 0.080 | -5.186 |
| 434 | 1.9 | 120.0 | 2 | 2 | 2 | 2 | 9.613 | 2.318 | 0.315 | 0.088 | -4.992 |
| 435 | 1.9 | 23.4 | 2 | 1 | 2 | 2 | 9.759 | 3.502 | 1.499 | 0.083 | -4.846 |
| 436 | 2.1 | 34.4 | 2 | 2 | 1 | 2 | 10.729 | 2.729 | 0.726 | 0.247 | -3.876 |
| 437 | 2.7 | -5.2 | 1 | 0 | 1 | 2 | 7.938 | 2.027 | 0.024 | 0.095 | -6.667 |
| 438 | 3.0 | 75.7 | 2 | 2 | 1 | 1 | 10.022 | 2.402 | 0.399 | 0.122 | -4.583 |
| 439 | 3.3 | -16.4 | 1 | 0 | 2 | 1 | 9.930 | 2.554 | 0.551 | 0.248 | -4.675 |
| 440 | 4.4 | -23.4 | 2 | 0 | 2 | 2 | 10.709 | 2.156 | 0.153 | 0.153 | -3.896 |
| 441 | 4.4 | 80.8 | 2 | 2 | 1 | 2 | 10.241 | 2.123 | 0.120 | 0.129 | -4.364 |
| 442 | 4.6 | 28.1 | 2 | 1 | 2 | 2 | 9.630 | 2.259 | 0.256 | 0.156 | -4.975 |
| 443 | 5.0 | 144.1 | 2 | 1 | 1 | 1 | 10.561 | 1.653 | -0.350 | 0.047 | -4.044 |
| 444 | 5.1 | -11.7 | 2 | 0 | 2 | 2 | 8.550 | 2.291 | 0.288 | 0.033 | -6.055 |
| 445 | 5.5 | 138.5 | 2 | 1 | 1 | 1 | 9.697 | 1.854 | -0.149 | 0.117 | -4.908 |
| 446 | 5.7 | -4.5 | 2 | 0 | 1 | 1 | 8.171 | 2.493 | 0.490 | 0.098 | -6.434 |
| 447 | 5.8 | 165.1 | 2 | 1 | 1 | 1 | 9.956 | 1.891 | -0.112 | 0.098 | -4.649 |
| 448 | 5.8 | 110.4 | 2 | 2 | 2 | 2 | 11.006 | 2.242 | 0.239 | 0.260 | -3.599 |
| 449 | 6.1 | 121.8 | 2 | 2 | 2 | 1 | 10.618 | 1.831 | -0.172 | 0.087 | -3.987 |
| 450 | 6.1 | 160.4 | 2 | 1 | 1 | 1 | 10.826 | 3.034 | 1.031 | 0.077 | -3.779 |
| 451 | 6.3 | -58.8 | 2 | 1 | 2 | 2 | 9.656 | 1.930 | -0.073 | 0.110 | -4.949 |
| 452 | 6.5 | 98.5 | 3 | 2 | 1 | 3 | 10.433 | 2.363 | 0.360 | 0.471 | -4.172 |
| 453 | 6.8 | 102.6 | 2 | 2 | 2 | 2 | 10.314 | 1.946 | -0.057 | 0.129 | -4.291 |
| 454 | 7.2 | 154.6 | 2 | 1 | 1 | 1 | 10.919 | 1.697 | -0.306 | 0.038 | -3.686 |
| 455 | 7.6 | -1.4 | 0 | 1 | 2 | 2 | 9.166 | 2.578 | 0.575 | 0.625 | -5.439 |
| 456 | 8.5 | -23.5 | 2 | 0 | 2 | 2 | 10.155 | 2.560 | 0.557 | 0.254 | -4.450 |
| 457 | 8.6 | 66.6 | 2 | 2 | 1 | 2 | 10.085 | 2.619 | 0.616 | 0.236 | -4.520 |
| 458 | 8.7 | 136.6 | 2 | 1 | 1 | 1 | 10.426 | 1.773 | -0.230 | 0.125 | -4.179 |
| 459 | 9.4 | -83.7 | 2 | 3 | 3 | 2 | 10.954 | 1.535 | -0.468 | 0.330 | -3.651 |
| 460 | 10.1 | 104.6 | 2 | 2 | 2 | 2 | 10.116 | 2.325 | 0.322 | 0.151 | -4.489 |
| 461 | 10.2 | -149.7 | 0 | 2 | 1 | 1 | 10.934 | 2.028 | 0.025 | 0.221 | -3.671 |
| 462 | 10.2 | -11.2 | 2 | 0 | 2 | 2 | 9.534 | 2.831 | 0.828 | 0.053 | -5.071 |
| 463 | 10.6 | 71.0 | 2 | 2 | 1 | 1 | 10.410 | 2.847 | 0.844 | 0.143 | -4.195 |
| 464 | 11.2 | 78.2 | 2 | 2 | 1 | 2 | 10.181 | 2.364 | 0.361 | 0.215 | -4.424 |
| 465 | 12.5 | -120.7 | 1 | 2 | 2 | 4 | 10.738 | 2.190 | 0.187 | 0.049 | -3.867 |


| ID | $\alpha_{\text {TST }}$ | $\delta_{\text {TIS } 7}$ | $\mathrm{n}_{\mathrm{K} 1}$ | $\mathrm{n}_{\mathrm{k} 2}$ | $\mathrm{n}_{\mathbf{x S}}$ | $\mathrm{n}_{\mathrm{HS}}$ | $\left\langle m_{x}\right\rangle$ | ( $\mathrm{H}-\mathrm{K})_{\text {data }}$ | (H-K) | $\Delta m_{k}$ | $\mathrm{M}_{\text {DOW }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 466 | 12.6 | -0.2 | 2 | 1 | 2 | 3 | 8.911 | 2.064 | 0.061 | 0.042 | -5.694 |
| 467 | 13.6 | 17.8 | 2 | 1 | 2 | 3 | 10.238 | 2.713 | 0.710 | 0.163 | -4.367 |
| 468 | 14.2 | -24.1 | 2 | 0 | 2 | 3 | 8.468 | 2.852 | 0.849 | 0.101 | -6.137 |
| 469 | 14.9 | -16.2 | 2 | 0 | 2 | 3 | 9.721 | 2.493 | 0.490 | 0.145 | -4.884 |
| 470 | 15.4 | 3.9 | 1 | 1 | 2 | 4 | 9.784 | 2.029 | 0.026 | 0.060 | -4.821 |
| 471 | 15.7 | -10.1 | 1 | 0 | 2 | 3 | 10.390 | 2.505 | 0.502 | 0.144 | -4.215 |
| 472 | 15.8 | 97.3 | 3 | 2 | 1 | 2 | 10.081 | 1.675 | -0.328 | 0.101 | -4.524 |
| 473 | 15.8 | 24.6 | 1 | 1 | 2 | 1 | 10.539 | 2.891 | 0.888 | 0.538 | -4.066 |
| 474 | 16.1 | 89.0 | 2 | 1 | 1 | 2 | 10.195 | 2.322 | 0.319 | 0.302 | -4.410 |
| 475 | 16.2 | -63.7 | 2 | 1 | 2 | 3 | 10.762 | 2.765 | 0.762 | 0.385 | -3.843 |
| 476 | 16.8 | 13.6 | 2 | 1 | 2 | 3 | 10.650 | 2.122 | 0.119 | 0.354 | -3.955 |
| 477 | 16.9 | -0.5 | 2 | 2 | 2 | 5 | 9.687 | 2.025 | 0.022 | 0.082 | -4.918 |
| 478 | 17.2 | 44.0 | 2 | 2 | 1 | 4 | 8.562 | 2.391 | 0.388 | 0.086 | -6.043 |
| 479 | 17.8 | 8.2 | 0 | 1 | 1 | 4 | 10.360 | 2.168 | 0.165 | 0.110 | -4.245 |
| 480 | 17.9 | 90.3 | 0 | 2 | 1 | 3 | 10.404 | 1.819 | -0.184 | 0.300 | -4.201 |
| 481 | 19.2 | 23.3 | 2 | 3 | 2 | 4 | 9.880 | 2.230 | 0.227 | 0.152 | -4.725 |
| 482 | 19.5 | 58.4 | 2 | 1 | 1 | 1 | 9.477 | 2.596 | 0.593 | 0.205 | -5.128 |
| 483 | 20.1 | 14.6 | 1 | 3 | 2 | 4 | 10.406 | 2.527 | 0.524 | 0.067 | -4.199 |
| 484 | 20.2 | -7.5 | 2 | 1 | 2 | 4 | 9.984 | 2.551 | 0.548 | 0.036 | -4.621 |
| 485 | 20.3 | 32.3 | 2 | 4 | 1 | 1 | 10.568 | 2.522 | 0.519 | 0.038 | -4.037 |
| 486 | 20.8 | -98.5 | 2 | 2 | 4 | 4 | 10.931 | 2.252 | 0.249 | 0.126 | -3.674 |
| 487 | 22.0 | 9.3 | 2 | 3 | 3 | 4 | 10.021 | 2.981 | 0.978 | 0.078 | -4.584 |
| 488 | 22.3 | 17.6 | 2 | 3 | 3 | 4 | 9.911 | 3.216 | 1.213 | 0.169 | -4.694 |
| 489 | 23.5 | 118.9 | 2 | 4 | 2 | 4 | 9.539 | 2.420 | 0.417 | 0.007 | -5.066 |
| 490 | 23.7 | 166.5 | 2 | 1 | 1 | 2 | 10.998 | 2.017 | 0.014 | 0.142 | -3.607 |
| 491 | 24.2 | -16.0 | 2 | 0 | 3 | 1 | 10.934 | 2.358 | 0.355 | 0.352 | -3.671 |
| 492 | 24.5 | -2.4 | 1 | 3 | 3 | 2 | 10.322 | 3.206 | 1.203 | 0.200 | -4.283 |
| 493 | 24.7 | -21.5 | 0 | 1 | 0 | 3 | 10.960 | 2.792 | 0.789 | -1.00 | -3.645 |
| 494 | 25.1 | 38.5 | 2 | 4 | 3 | 4 | 10.297 | 2.155 | 0.152 | 0.129 | -4.308 |
| 495 | 25.3 | 58.1 | 2 | 4 | 3 | 3 | 10.632 | 2.358 | 0.355 | 0.192 | -3.973 |
| 496 | 25.6 | 85.0 | 2 | 4 | 3 | 4 | 10.265 | 1.905 | -0.098 | 0.084 | -4.340 |
| 497 | 26.0 | -13.0 | 1 | 0 | 0 | 1 | 10.995 | 2.297 | 0.294 | -1.00 | -3.610 |
| 498 | 26.0 | 98.7 | 3 | 4 | 4 | 6 | 10.746 | 2.254 | 0.251 | 0.040 | -3.859 |
| 499 | 26.4 | -22.7 | 2 | 1 | 3 | 1 | 10.909 | 3.176 | 1.173 | 0.303 | -3.696 |
| 500 | 26.9 | 46.8 | 2 | 4 | 3 | 4 | 9.624 | 2.250 | 0.247 | 0.157 | -4.981 |
| 501 | 26.9 | 4.1 | 2 | 3 | 3 | 3 | 10.555 | 3.253 | 1.250 | 0.250 | -4.050 |
| 502 | 27.5 | 129.7 | 1 | 4 | 4 | 4 | 9.136 | 2.105 | 0.102 | 0.007 | -5.469 |
| 503 | 28.3 | 42.9 | 2 | 3 | 1 | 4 | 10.692 | 2.352 | 0.349 | 0.195 | -3.913 |
| 504 | 28.4 | 21.2 | 4 | 3 | 3 | 4 | 10.697 | 2.336 | 0.333 | 0.133 | -3.908 |
| 505 | 29.7 | 131.4 | 0 | 1 | 1 | 2 | 10.130 | 1.910 | -0.093 | 0.000 | -4.475 |
| 506 | 30.2 | 61.4 | 0 | 1 | 0 | 1 | 10.953 | 2.962 | 0.959 | -1.00 | -3.652 |
| 507 | 30.3 | -85.1 | 2 | 1 | 4 | 3 | 10.895 | 2.154 | 0.151 | 0.111 | -3.710 |
| 508 | 30.9 | 30.9 | 4 | 3 | 3 | 6 | 9.554 | 2.110 | 0.107 | 0.087 | -5.051 |
| 509 | 31.0 | 63.9 | 4 | 5 | 5 | 4 | 10.401 | 2.819 | 0.816 | 0.246 | -4.204 |
| 510 | 31.2 | 168.0 | 1 | 0 | 0 | 2 | 10.911 | 2.525 | 0.522 | -1.00 | -3.694 |


| ID | $\alpha_{\text {IRT }}$ | $\bar{o}_{\text {ITST }}$ | $\mathrm{n}_{\mathrm{k} 1}$ | $\mathrm{n}_{\mathrm{K} 2}$ | $\mathrm{n}_{\mathrm{ks}}$ | $\mathrm{n}_{\mathrm{Hs}}$ | $\left\langle m_{k}\right\rangle$ | ( $\mathrm{H}-\mathrm{K})^{+\infty}$ | ( $\mathrm{H}-\mathrm{K}$ ) | $\Delta \mathrm{m}_{\mathrm{k}}$ | $\mathrm{M}_{\mathrm{BOL}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 511 | 31.4 | 92.5 | 5 | 4 | 3 | 4 | 10.848 | 2.497 | 0.494 | 0.058 | -3.757 |
| 512 | 31.7 | 43.5 | 4 | 4 | 2 | 1 | 10.735 | 2.956 | 0.953 | 0.074 | -3.870 |
| 513 | 32.6 | 48.3 | 4 | 4 | 3 | 4 | 10.792 | 1.870 | -0.133 | 0.168 | -3.813 |
| 514 | 32.7 | 66.7 | 4 | 4 | 3 | 4 | 10.627 | 2.615 | 0.612 | 0.038 | -3.978 |
| 515 | 33.7 | -20.0 | 1 | 0 | 0 | 1 | 11.006 | 2.832 | 0.829 | -1.00 | -3.599 |
| 516 | 33.9 | -62.8 | 2 | 1 | 4 | 4 | 9.380 | 1.961 | -0.042 | 0.315 | -5.225 |
| 517 | 34.0 | -13.8 | 1 | 0 | 0 | 1 | 10.509 | 3.106 | 1.103 | -1.00 | -4.096 |
| 518 | 34.2 | 75.5 | 3 | 4 | 3 | 4 | 10.449 | 2.492 | 0.489 | 0.080 | -4.156 |
| 519 | 34.5 | 128.6 | 4 | 4 | 4 | 4 | 9.694 | 1.880 | -0.123 | 0.131 | -4.911 |
| 520 | 34.8 | 24.4 | 4 | 3 | 3 | 4 | 8.411 | 2.557 | 0.554 | 0.372 | -6.194 |
| 521 | 35.0 | -121.6 | 0 | 1 | 2 | 3 | 9.578 | 2.461 | 0.458 | 0.246 | -5.027 |
| 522 | 35.1 | 121.7 | 4 | 4 | 4 | 4 | 10.054 | 1.898 | -0.105 | 0.052 | -4.551 |
| 523 | 36.0 | -68.5 | 0 | 1 | 1 | 5 | 10.568 | 1.983 | -0.020 | 0.090 | -4.087 |
| 524 | 36.7 | 141.7 | 3 | 2 | 2 | 2 | 10.757 | 2.915 | 0.912 | 0.014 | -3.848 |
| 525 | 36.8 | 38.9 | 3 | 4 | 3 | 4 | 9.515 | 2.096 | 0.093 | 0.062 | -5.090 |
| 526 | 37.1 | 30.5 | 1 | 4 | 3 | 4 | 10.766 | 2.439 | 0.436 | 0.161 | -3.839 |
| 527 | 38.3 | 79.1 | 3 | 4 | 3 | 4 | 9.197 | 3.191 | 1.188 | 0.491 | -5.408 |
| 528 | 39.3 | -39.5 | 1 | 1 | 2 | 2 | 10.808 | 2.160 | 0.157 | 0.116 | -3.797 |
| 529 | 39.4 | -27.8 | 1 | 1 | 1 | 2 | 9.297 | 2.793 | 0.790 | 0.437 | -5.308 |
| 530 | 39.8 | 9.3 | 2 | 3 | 1 | 3 | 8.579 | 2.375 | 0.372 | 0.214 | -6.026 |
| 531 | 41.4 | 40.3 | 2 | 2 | 2 | 2 | 10.490 | 2.532 | 0.529 | 0.126 | -4.115 |
| 532 | 41.5 | 120.8 | 2 | 4 | 0 | 4 | 10.982 | 1.644 | -0.359 | 0.062 | -3.623 |
| 533 | 41.6 | -22.0 | 1 | 1 | 1 | 2 | 10.559 | 2.836 | 0.833 | 0.087 | -4.046 |
| 534 | 43.5 | 12.1 | 0 | 2 | 0 | 1 | 10.602 | 2.446 | 0.443 | -1.00 | -4.003 |
| 535 | 44.0 | 1.6 | 2 | 2 | 1 | 2 | 10.384 | 2.013 | 0.010 | 0.127 | -4.221 |
| 536 | 44.3 | 152.4 | 1 | 1 | 1 | 1 | 8.961 | 1.991 | -0.012 | 0.219 | -5.644 |
| 537 | 44.6 | 16.6 | 2 | 2 | 1 | 2 | 9.719 | 2.316 | 0.313 | 0.090 | -4.886 |
| 538 | 44.9 | 37.7 | 2 | 2 | 2 | 2 | 10.971 | 2.802 | 0.799 | 0.125 | -3.634 |
| 539 | 45.3 | 93.1 | 3 | 2 | 2 | 2 | 10.047 | 2.218 | 0.215 | 0.145 | -4.558 |
| 540 | 45.6 | 115.8 | 2 | 2 | 1 | 2 | 9.636 | 2.906 | 0.903 | 0.106 | -4.969 |
| 541 | 46.1 | 23.5 | 2 | 2 | 1 | 2 | 10.525 | 2.087 | 0.084 | 0.129 | -4.080 |
| 542 | 47.1 | 73.7 | 2 | 2 | 2 | 2 | 10.551 | 2.420 | 0.417 | 0.066 | -4.054 |
| 543 | 47.5 | 161.4 | 1 | 1 | 1 | 1 | 10.521 | 1.910 | -0.093 | 0.098 | -4.084 |
| 544 | 48.7 | 20.6 | 0 | 1 | 0 | 1 | 10.937 | 2.082 | 0.079 | -1.00 | -3.668 |
| 545 | 49.2 | 90.1 | 2 | 2 | 2 | 2 | 9.754 | 1.954 | -0.049 | 0.068 | -4.851 |
| 546 | 49.8 | 84.6 | 2 | 2 | 2 | 1 | 10.536 | 1.872 | -0.131 | 0.168 | -4.069 |
| 547 | 50.3 | -10.3 | 1 | 1 | 1 | 2 | 10.821 | 2.759 | 0.756 | 0.191 | -3.784 |
| 548 | 50.5 | 69.6 | 0 | 0 | 1 | 1 | 10.949 | 2.029 | 0.026 | $-1.00$ | -3.656 |
| 549 | 50.8 | 58.2 | 2 | 2 | 2 | 2 | 9.910 | 1.989 | -0.014 | 0.095 | -4.695 |
| 550 | 50.8 | 26.6 | 3 | 2 | 1 | 2 | 10.115 | 1.957 | -0.046 | 0.151 | -4.490 |
| 551 | 50.9 | -68.9 | 0 | 0 | 2 | 3 | 10.487 | 2.469 | 0.466 | -1.00 | -4.118 |
| 552 | 51.8 | 22.8 | 1 | 2 | 1 | 1 | 10.701 | 2.215 | 0.212 | 0.161 | -3.904 |
| 553 | 52.9 | 72.6 | 2 | 2 | 2 | 2 | 10.156 | 1.941 | -0.062 | 0.155 | -4.449 |
| 554 | 54.1 | 107.7 | 2 | 2 | 2 | 2 | 10.645 | 1.951 | -0.052 | 0.178 | -3.960 |
| 555 | 54.4 | 28.6 | 3 | 3 | 2 | 3 | 10.014 | 2.027 | 0.024 | 0.029 | -4.591 |


| ID | $\alpha_{\text {bST }}$ | $\delta_{\text {LTST }}$ | $\mathrm{n}_{\mathrm{K} 1}$ | $n_{x 2}$ | $\mathrm{n}_{\mathrm{ks}}$ | $\mathrm{n}_{4 \times 1}$ | < $\mathrm{m}_{k}>$ | ( $\mathrm{H}-\mathrm{K})_{\infty}$ | (H-K) | $\Delta m_{k}$ | M ${ }_{\text {POL }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 556 | 54.6 | 19.5 | 2 | 2 | 1 | 2 | 10.305 | 2.132 | 0.129 | 0.078 | -4.300 |
| 557 | 54.8 | 58.8 | 2 | 2 | 2 | 1 | 10.538 | 3.089 | 1.086 | 0.131 | -4.067 |
| 558 | 55.3 | 125.0 | 2 | 2 | 2 | 2 | 9.024 | 1.719 | -0.284 | 0.055 | - -5.581 |
| 559 | 55.9 | 111.2 | 2 | 2 | 2 | 1 | 10.852 | 2.282 | 0.279 | 0.145 | -3.753 |
| 560 | 55.9 | 143.8 | 1 | 1 | 1 | 1 | 10.389 | 2.280 | 0.277 | 0.065 | -4.216 |
| 561 | 56.1 | -34.4 | 1 | 1 | 1 | 2 | 10.520 | 3.812 | 1.809 | 0.165 | -4.085 |
| 562 | 56.3 | 67.8 | 2 | 2 | 2 | 2 | 8.223 | 2.613 | 0.610 | 0.149 | -6.382 |
| 563 | 57.6 | 75.6 | 2 | 2 | 2 | 2 | 9.461 | 2.258 | 0.255 | 0.104 | -5.144 |
| 564 | 57.8 | 87.1 | 2 | 2 | 2 | 2 | 10.377 | 2.339 | 0.336 | 0.097 | -4.228 |
| 565 | 57.9 | 71.3 | 1 | 0 | 2 | 1 | 10.009 | 2.083 | 0.080 | 0.151 | -4.596 |
| 566 | 59.1 | 54.4 | 2 | 1 | 2 | 2 | 10.434 | 2.154 | 0.151 | 0.133 | -4.171 |
| 567 | 59.7 | 110.9 | 1 | 2 | 2 | 2 | 10.748 | 2.731 | 0.728 | 0.110 | -3.857 |
| 568 | 60.0 | 22.8 | 2 | 2 | 1 | 2 | 9.897 | 3.130 | 1.127 | 0.354 | -4.708 |
| 569 | 60.1 | 78.1 | 2 | 2 | 2 | 1 | 9.826 | 2.433 | 0.430 | 0.109 | -4.779 |
| 570 | 60.4 | 126.6 | 1 | 1 | 2 | 2 | 9.082 | 1.798 | -0.205 | 0.170 | -5.523 |
| 571 | 60.4 | 57.4 | 2 | 2 | 2 | 2 | 10.217 | 2.427 | 0.424 | 0.195 | -4.388 |
| 572 | 60.4 | 13.4 | 1 | 2 | 0 | 1 | 10.966 | 3.311 | 1.308 | 0.623 | -3.639 |
| 573 | 61.7 | 119.4 | 2 | 2 | 2 | 2 | 10.353 | 1.860 | -0.143 | 0.043 | 4.252 |
| 574 | 61.9 | -40.7 | 0 | 0 | 2 | 2 | 10.825 | 2.272 | 0.269 | -1.00 | -3.780 |
| 575 | 61.9 | 51.8 | 2 | 2 | 2 | 2 | 9.923 | 2.271 | 0.268 | 0.073 | -4.682 |
| 576 | 62.4 | 45.8 | 2 | 2 | 2 | 2 | 10.649 | 2.407 | 0.404 | 0.078 | -3.956 |
| 577 | 62.6 | 71.5 | 2 | 2 | 2 | 1 | 10.159 | 2.290 | 0.287 | 0.157 | -4.446 |
| 578 | 62.9 | 35.6 | 2 | 2 | 2 | 2 | 10.668 | 2.356 | 0.353 | 0.116 | -3.937 |
| 579 | 63.3 | 126.1 | 2 | 2 | 2 | 2 | 8.598 | 2.943 | 0.940 | 0.335 | -6.007 |
| 580 | 63.5 | -110.4 | 0 | 0 | 2 | 2 | 10.637 | 2.348 | 0.345 | -1.00 | -3.968 |
| 581 | 63.9 | 66.3 | 2 | 2 | 2 | 2 | 10.570 | 2.459 | 0.456 | 0.158 | -4.035 |
| 582 | 64.3 | 62.4 | 2 | 3 | 3 | 3 | 9.431 | 2.498 | 0.495 | 0.106 | -5.174 |
| 583 | 64.5 | 143.1 | 1 | 1 | 1 | 1 | 10.411 | 2.987 | 0.984 | 0.110 | -4.194 |
| 584 | 64.6 | 16.6 | 2 | 2 | 1 | 2 | 10.166 | 1.798 | -0.205 | 0.124 | -4.439 |
| 585 | 65.7 | 138.6 | 1 | 1 | 1 | 1 | 10.891 | 2.403 | 0.400 | 0.061 | -3.714 |
| 586 | 66.2 | 76.4 | 2 | 2 | 2 | 2 | 9.622 | 2.417 | 0.414 | 0.042 | -4.983 |
| 587 | 66.6 | -47.9 | 0 | 0 | 2 | 2 | 9.892 | 2.403 | 0.400 | -1.00 | -4.713 |
| 588 | 67.0 | -79.1 | 0 | 0 | 2 | 2 | 10.322 | 2.566 | 0.563 | $-1.00$ | -4.283 |
| 589 | 67.4 | 26.4 | 1 | 1 | 0 | 1 | 11.157 | 2.011 | 0.008 | 0.028 | -3.448 |
| 590 | 69.1 | 159.9 | 1 | 1 | 1 | 1 | 8.839 | 2.153 | 0.150 | 0.194 | -5.766 |
| 591 | 69.2 | 37.6 | 2 | 4 | 2 | 4 | 9.233 | 2.869 | 0.866 | 0.161 | -5.372 |
| 592 | 69.6 | 21.2 | 2 | 4 | 1 | 4 | 9.300 | 2.750 | 0.747 | 0.068 | -5.305 |
| 593 | 69.8 | -80.4 | 0 | 0 | 2 | 2 | 10.215 | 2.447 | 0.444 | -1.00 | -4.390 |
| 594 | 70.3 | 50.8 | 2 | 3 | 2 | 3 | 10.502 | 2.067 | 0.064 | 0.090 | -4.103 |
| 595 | 70.5 | 65.7 | 2 | 5 | 2 | 6 | 9.916 | 3.308 | 1.305 | 0.121 | -4.689 |
| 596 | 70.5 | 46.7 | 2 | 4 | 2 | 4 | 9.334 | 1.918 | -0.085 | 0.142 | -5.271 |
| 597 | 71.0 | -168.8 | 0 | 0 | 1 | 1 | 10.367 | 2.350 | 0.347 | -1.00 | -4.238 |
| 598 | 72.4 | 123.0 | 2 | 4 | 2 | 4 | 10.482 | 3.413 | 1.410 | 0.231 | -4.123 |
| 599 | 72.7 | 76.5 | 2 | 4 | 3 | 4 | 10.337 | 2.185 | 0.182 | 0.102 | -4.268 |
| 600 | 73.0 | 85.7 | 2 | 4 | 3 | 4 | 10.846 | 2.565 | 0.562 | 0.186 | -3.759 |


| ID | $\alpha_{\text {TS } 7}$ | $\delta_{\text {TST }}$ | $\mathrm{n}_{\mathrm{K} 1}$ | $\mathrm{n}_{\mathrm{K} 2}$ | $\mathrm{n}_{\mathrm{ks}}$. | $\mathrm{n}_{\text {Hs }}$ | $<m_{k}>$ | ( $\mathrm{H}-\mathrm{K})_{\infty}$ | ( $\mathrm{H}-\mathrm{K}$ ) | $\Delta m_{k}$ | $\mathrm{MaO}_{\text {col }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 601 | 74.3 | 51.9 | 2 | 3 | 3 | 4 | 10.226 | 2.596 | 0.593 | 0.317 | -4.379 |
| 602 | 74.4 | 117.9 | 2 | 4 | 3 | 4 | 10.398 | 1.874 | -0.129 | 0.052 | -4.207 |
| 603 | 74.9 | -114.6 | 0 | 0 | 2 | 2 | 10.362 | 3.505 | 1.502 | -1.00 | -4.243 |
| 604 | 75.2 | 81.0 | 2 | 4 | 4 | 4 | 10.389 | 2.246 | 0.243 | 0.089 | -4.216 |
| 605 | 75.3 | 41.0 | 2 | 4 | 4 | 4 | 10.127 | 3.170 | 1.167 | 0.133 | -4.478 |
| 606 | 76.1 | 88.9 | 2 | 4 | 4 | 4 | 10.439 | 2.394 | 0.391 | 0.068 | -4.166 |
| 607 | 76.2 | 14.7 | 2 | 4 | 3 | 4 | 10.614 | 2.028 | 0.025 | 0.171 | -3.991 |
| 608 | 76.3 | -172.6 | 0 | 0 | 1 | 1 | 10.436 | 3.454 | 1.451 | -1.00 | -4.169 |
| 609 | 76.6 | 702 | 2 | 4 | 4 | 5 | 9.432 | 2.322 | 0.319 | 0.087 | -5.173 |
| 610 | 76.7 | 23.1 | 2 | 4 | 3 | 2 | 10.100 | 2.814 | 0.811 | 0.174 | -4.505 |
| 611 | 77.2 | 49.5 | 0 | 0 | 1 | 3 | 10.828 | 3.028 | 1.025 | -1.00 | -3.777 |
| 612 | 79.3 | 147.1 | 1 | 2 | 2 | 2 | 9.203 | 2.949 | 0.946 | 0.067 | -5.402 |
| 613 | 81.1 | -22.9 | 2 | 2 | 2 | 2 | 10.555 | 2.150 | 0.147 | 0.090 | -4.050 |
| 614 | 81.4 | -34.6 | 2 | 2 | 2 | 3 | 10.058 | 2.396 | 0.393 | 0.051 | -4.547 |
| 615 | 81.7 | 108.1 | 2 | 4 | 4 | 4 | 10.942 | 2.925 | 0.922 | 0.120 | -3.663 |
| 616 | 82.3 | 44.3 | 1 | 4 | 3 | 2 | 10.575 | 2.481 | 0.478 | 0.106 | -4.030 |
| 617 | 82.4 | -26.0 | 2 | 2 | 2 | 1 | 10.909 | 2.217 | 0.214 | 0.045 | -3.696 |
| 618 | 82.5 | 73.0 | 4 | 4 | 4 | 1 | 10.737 | 3.374 | 1.371 | 0.206 | -3.868 |
| 619 | 84.1 | 55.6 | 4 | 4 | 4 | 2 | 10.595 | 2.612 | 0.609 | 0.058 | -4.010 |
| 620 | 84.9 | -22.9 | 2 | 2 | 2 | 3 | 10.699 | 1.712 | -0.291 | 0.098 | -3.906 |
| 621 | 85.5 | 44.4 | 1 | 4 | 3 | 3 | 10.615 | 2.505 | 0.502 | 0.349 | -3.990 |
| 622 | 87.1 | 99.5 | 4 | 4 | 5 | 5 | 10.179 | 1.993 | -0.010 | 0.069 | -4.426 |
| 623 | 87.6 | 130.8 | 2 | 4 | 2 | 4 | 10.723 | 2.773 | 0.770 | 0.124 | -3.882 |
| 624 | 87.6 | 16.8 | 4 | 4 | 3 | 3 | 10.865 | 3.132 | 1.129 | 0.185 | -3.740 |
| 625 | 88.4 | 53.8 | 4 | 4 | 4 | 4 | 10.841 | 2.692 | 0.689 | 0.167 | -3.764 |
| 626 | 88.4 | 153.5 | 2 | 2 | 2 | 2 | 11.001 | 1.899 | -0.104 | 0.028 | -3.604 |
| 627 | 89.9 | 111.0 | 4 | 4 | 4 | 4 | 10.471 | 3.216 | 1.213 | 0.095 | -4.134 |
| 628 | 90.0 | 46.8 | 4 | 1 | 4 | 2 | 10.165 | 2.606 | 0.603 | 0.124 | -4.440 |
| 629 | 90.4 | 90.1 | 4 | 2 | 4 | 3 | 10.697 | 1.927 | -0.076 | 0.163 | -3.908 |
| 630 | 91.6 | 40.1 | 4 | 2 | 4 | 2 | 9.362 | 2.408 | 0.405 | 0.092 | -5.243 |
| 631 | 93.4 | 45.3 | 4 | 2 | 3 | 2 | 10.252 | 2.099 | 0.096 | 0.159 | -4.353 |
| 632 | 98.2 | 129.6 | 1 | 2 | 2 | 2 | 10.556 | 1.831 | -0.172 | 0.170 | -4.049 |
| 633 | 100.8 | 1.0 | 2 | 2 | 2 | 2 | 10.571 | 2.160 | 0.157 | 0.124 | -4.034 |
| 634 | 102.6 | 76.9 | 2 | 2 | 2 | 2 | 9.170 | 1.897 | -0.106 | 0.062 | -5.435 |
| 635 | 103.1 | -20.6 | 1 | 1 | 1 | 1 | 9.767 | 1.825 | -0.178 | 0.059 | -4.838 |
| 636 | 104.0 | -6.7 | 2 | 1 | 1 | 1 | 10.973 | 2.133 | 0.130 | 0.087 | -3.632 |
| 637 | 104.2 | 118.4 | 2 | 2 | 2 | 2 | 10.097 | 2.321 | 0.318 | 0.251 | -4.508 |
| 638 | 105.0 | 1.9 | 2 | 2 | 2 | 2 | 10.156 | 2.406 | 0.403 | 0.057 | -4.449 |
| 639 | 106.1 | 21.2 | 2 | 2 | 2 | 2 | 10.368 | 2.699 | 0.696 | 0.101 | -4.237 |
| 640 | 108.3 | 143.2 | 1 | 1 | 1 | 1 | 10.879 | 2.012 | 0.009 | 0.109 | -3.726 |
| 641 | 109.6 | 93.9 | 3 | 2 | 2 | 2 | 9.409 | 2.005 | 0.002 | 0.094 | -5.196 |
| 642 | 111.6 | 159.7 | 1 | 1 | 1 | 1 | 9.429 | 2.626 | 0.623 | 0.096 | -5.176 |
| 643 | 111.8 | 147.1 | 1 | 1 | 1 | 1 | 9.987 | 1.904 | -0.099 | 0.060 | -4.618 |
| 644 | 112.7 | 89.0 | 2 | 2 | 2 | 2 | 10.714 | 2.162 | 0.159 | 0.188 | -3.891 |
| 645 | 120.4 | 153.4 | 1 | 1 | 1 | 1 | 10.902 | 2.045 | 0.042 | 0.037 | -3.703 |


| ID | $\alpha_{\text {DES }}$ | $\delta_{\text {trs }}$ | $\mathrm{n}_{\mathrm{K} 1}$ | $n_{K 2}$ | nxs | $\mathrm{n}_{4 \mathrm{H}}$ | $\left\langle m_{x}>\right.$ | ( $\mathrm{H}-\mathrm{K})_{\text {d }}$ | (H-K) | $\Delta \mathrm{m}_{k}$ | $\mathrm{M}_{\text {DOL }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 646 | 120.7 | -11.3 | 1 | 2 | 2 | 1 | 10.792 | 2.221 | 0.218 | 0.214 | -3.813 |
| 647 | 122.6 | 90.9 | 2 | 3 | 3 | 1 | 10.154 | 4.498 | 2.495 | 0.637 | -4.451 |
| 648 | 124.2 | 137.1 | 1 | 2 | 1 | 1 | 10.828 | 3.097 | 1.094 | 0.050 | -3.777 |
| 649 | 125.8 | 139.2 | 1 | 2 | 2 | 1 | 10.753 | 2.687 | 0.684 | 0.074 | -3.852 |
| 650 | 126.0 | 120.9 | 2 | 4 | 4 | 2 | 10.683 | 2.951 | 0.948 | 0.106 | -3.922 |
| 651 | 127.0 | 62.4 | 1 | 6 | 5 | 2 | 10.050 | 2.552 | 0.549 | 0.039 | -4.555 |
| 652 | 128.4 | 159.2 | 1 | 2 | 2 | 1 | 10.824 | 2.281 | 0.278 | 0.054 | -3.781 |
| 653 | 129.0 | -20.5 | 1 | 2 | 2 | 1 | 9.079 | 2.280 | 0.277 | 0.198 | -5.526 |
| 654 | 131.6 | 108.8 | 2 | 4 | 4 | 2 | 9.094 | 1.996 | -0.007 | 0.068 | -5.511 |
| 655 | 131.8 | 150.1 | 1 | 2 | 2 | 1 | 10.190 | 3.064 | 1.061 | 0.201 | -4.415 |
| 656 | 133.6 | 161.3 | 2 | 2 | 2 | 1 | 9.427 | 2.161 | 0.158 | 0.245 | -5.178 |
| 657 | 133.7 | 46.4 | 3 | 4 | 4 | 2 | 10.584 | 2.612 | 0.609 | 0.091 | -4.021 |
| 658 | 137.2 | 60.9 | 5 | 4 | 5 | 2 | 10.928 | 2.160 | 0.157 | 0.169 | -3.677 |
| 659 | 1372 | 124.0 | 4 | 4 | 4 | 2 | 10.227 | 1.934 | -0,069 | 0.079 | -4.378 |

TABLE A2-2
GALACTIC CENTER STARS WITH OPTICALLY DETERMINED POSITIONS

| No. | Optical (1950) |  | Infrared (1950) |  | Optical-Infrared |  | $\begin{gathered} K \\ (2.2 \mu \mathrm{~m}) \end{gathered}$ | ( $\mathrm{H}-\mathrm{K}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | RA | DEC | RA | DEC | $\begin{gathered} \text { ARA } \\ (7) \end{gathered}$ | $\triangle D E C$ (") |  |  |
| 1 | 174232.70 | -28 5917.3 | 174232.5 | -28 5917.3 | +2.6 | 0.0 | 10.83 | 1.04 |
| 2 | 174232.19 | -28 5828.4 | 174232.1 | -28 5827.9 | +1.2 | -0.5 | 10.53 | 0.76 |
| 3 | 174230.20 | -29 0019.3 | 174230.0 | -29 0019.0 | +2.6 | -0.3 | 9.44 | 1.10 |
| 4 | 174235.72 | -28 5837.6 | 174235.5 | -28 5838.8 | -2.9 | +0.8 | 10.33 | 0.61 |
| 5 | 174222.37 | -29 0011.8 | 174222.3 | -290009.3 | +1.0 | -2.5 | 10.66 | 1.48 |
| 6 | 174220.95 | -28 5947.7 | 174221.1 | -295947.2 | -2.0 | +0.5 | 7.29 | 0.54 |
| 7 | 174228.89 | -29 0111.7 | 174228.7 | -28 0110.8 | +2.5 | -0.9 | 8.56 | 0.55 |
| 8 | 174237.94 | -28 5728.1 | 174237.8 | -29 5730.0 | -1.8 | +1.9 | 9.29 | 0.90 |
| 9 | 174229.39 | -290149.4 | 174229.1 | -29 0147.8 | -3.8 | -1.6 | 9.91 | 0.71 |
| 10 | 174219.74 | -2901 16.6 | 174219.8 | -29 0115.8 | -0.8 | -0.8 | 9.59 | 0.37 |

## APPENDIX 3 :

## GALACTIC CENTER HKL PHOTOMETRY

The following table is a listing of the HKL database of 257 stars used in the discussions of chapter 4. The column are designated by:

## ID : Star Identification Number

$\alpha_{\text {RSS }}:$ Right Ascension position offset from IRS 7; $17^{\text {h }} 42^{\mathrm{m}} 29.2^{\text {² }}$ (1950), in arcseconds.
$\delta_{\text {IRS7 }}$ : Declination offset position from IRS 7; $-29^{\circ} 59^{\prime} 12.9^{\prime \prime}$ (1950), in arcseconds.
$\mathrm{n}_{\mathrm{k} 3}$ : Number of K detections in May 1988
$\mathrm{n}_{\mathrm{H} 3}$ : Number of H detections in May 1988
$\mathrm{n}_{\mathrm{L} 3}$ : Number of L detections in June 1988
$<\mathrm{m}_{\mathrm{K}}>$ : Average K magnitude
$\Delta \mathrm{m}_{\mathrm{K}}$ : The RMS residual K magnitude between epochs
$(\mathrm{H}-\mathrm{K})_{\mathrm{OBS}}$ : Observed $\mathrm{H}-\mathrm{K}$ color, $\mathrm{m}_{\mathrm{H}}-\mathrm{m}_{\mathrm{K}}(3)$
( $\mathrm{K}-\mathrm{L}$ ) OBs : Observed K-L color, $\mathrm{m}_{\mathrm{K}}(3)-\mathrm{m}_{\mathrm{L}}$
$(\mathrm{H}-\mathrm{K})_{0}$ : Intrinsic $\mathrm{H}-\mathrm{K}$
(K-L) ${ }_{0}$ : Intrinsic K-L
All colors were dereddened by $A_{v}=31.8$ mag using the extinction curve of Rieke and Lebofsky (1985).

TABLE A3-1

| ID | $\alpha_{\text {IRS }}$ | $\delta_{\text {IRS }}$ | $n_{K}$ | $n_{H}$ | $n_{L}$ | $<m_{K}$ | $\Delta m_{K}$ | $(H-K)_{\text {OBS }}$ | $(\mathrm{K}-\mathrm{L})_{\text {ORS }}$ | $(\mathrm{H}-\mathrm{K})_{0}$ | $(\mathrm{~K}-\mathrm{L})_{0}$ |
| ---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -184.5 | -72.8 | 2 | 2 | 1 | 10.62 | 0.00 | 2.75 | 2.13 | 0.75 | 0.38 |
| 2 | -183.6 | 13.5 | 2 | 1 | 1 | 9.93 | 0.00 | 2.51 | 2.03 | 0.51 | 0.28 |
| 3 | -178.6 | -20.1 | 2 | 2 | 1 | 10.16 | 0.08 | 1.90 | 1.60 | -0.10 | -0.14 |
| 4 | -176.9 | 22.0 | 2 | 2 | 1 | 9.24 | 0.40 | 2.85 | 2.06 | 0.84 | 0.31 |
| 5 | -176.5 | 34.1 | 3 | 3 | 1 | 10.60 | 0.11 | 3.01 | 2.31 | 1.00 | 0.56 |
| 6 | -175.6 | 41.6 | 2 | 2 | 1 | 10.35 | 0.10 | 2.33 | 2.08 | 0.33 | 0.33 |
| 7 | -174.7 | 4.6 | 1 | 2 | 1 | 10.88 | 0.06 | 2.21 | 2.32 | 0.21 | 0.57 |
| 8 | -174.3 | 162.9 | 1 | 1 | 1 | 9.41 | 0.14 | 2.03 | 1.81 | 0.03 | 0.07 |
| 9 | -167.7 | -158.0 | 1 | 1 | 1 | 10.53 | 0.10 | 2.87 | 2.02 | 0.86 | 0.27 |
| 10 | -165.8 | -91.2 | 2 | 2 | 1 | 10.70 | 0.08 | 1.98 | 2.00 | -0.02 | 0.25 |
| 11 | -164.5 | 65.8 | 3 | 1 | 1 | 9.89 | 0.16 | 2.40 | 2.01 | 0.40 | 0.26 |
| 12 | -160.8 | -26.8 | 2 | 2 | 1 | 10.77 | 0.10 | 2.29 | 2.38 | 0.29 | 0.63 |
| 13 | -159.8 | -128.8 | 2 | 1 | 1 | 9.64 | 0.49 | 1.97 | 2.23 | -0.03 | 0.48 |
| 14 | -158.5 | -56.9 | 2 | 2 | 1 | 10.35 | 0.14 | 2.03 | 1.78 | 0.03 | 0.03 |
| 15 | -157.4 | -44.8 | 2 | 2 | 2 | 9.30 | 0.07 | 2.09 | 2.07 | 0.08 | 0.32 |
| 16 | -157.3 | 90.4 | 2 | 2 | 2 | 9.33 | 0.13 | 2.67 | 1.51 | 0.67 | -0.23 |
| 17 | -156.5 | -50.3 | 2 | 2 | 2 | 10.31 | 0.06 | 2.20 | 2.00 | 0.19 | 0.25 |
| 18 | -154.4 | 8.2 | 2 | 2 | 2 | 10.05 | 0.25 | 2.99 | 2.52 | 0.99 | 0.77 |
| 19 | -154.0 | 130.7 | 2 | 2 | 2 | 8.24 | 0.08 | 2.15 | 2.05 | 0.15 | 0.30 |
| 20 | -151.9 | 89.0 | 2 | 2 | 1 | 10.31 | 0.10 | 2.93 | 1.80 | 0.92 | 0.05 |
| 21 | -151.8 | -63.9 | 2 | 2 | 1 | 9.46 | 0.05 | 1.88 | 1.67 | -0.12 | -0.08 |


| ID | $\alpha_{\text {Prs }}$ | $\delta_{\text {TST }}$ | $\mathrm{n}_{\mathrm{K}}$ | $\mathrm{n}_{\mathrm{H}}$ | $\mathrm{n}_{\mathrm{L}}$ | $<m_{k}$ | $\Delta \mathrm{m}_{\mathrm{k}}$ | (H-K) oss | (K-L) ${ }_{\text {O8S }}$ | (H-K) | (K-L) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 22 | -148.1 | 60.0 | 2 | 2 | 2 | 10.09 | 0.06 | 2.02 | 1.95 | 0.02 | 0.20 |
| 23 | -147.0 | 24.5 | 2 | 2 | 2 | 9.94 | 0.25 | 2.50 | 2.34 | 0.50 | 0.59 |
| 24 | -146.6 | 111.4 | 2 | 2 | 1 | 9.99 | 0.06 | 2.27 | 1.56 | 0.26 | -0.19 |
| 25 | -146.5 | -4.7 | 2 | 2 | 2 | 10.38 | 0.12 | 3.64 | 2.39 | 1.64 | 0.64 |
| 26 | -145.5 | -39.6 | 2 | 2 | 2 | 9.86 | 0.23 | 1.79 | 1.85 | -0.22 | 0.10 |
| 27 | -144.8 | -106.3 | 2 | 2 | 1 | 10.29 | 0.21 | 2.79 | 2.03 | 0.78 | 0.28 |
| 28 | -144.2 | 140.1 | 1 | 1 | 3 | 10.20 | 0.13 | 3.29 | 1.62 | 1.29 | -0.13 |
| 29 | -142.2 | -67.4 | 2 | 3 | 3 | 9.20 | 0.12 | 2.03 | 1.65 | 0.03 | -0.10 |
| 30 | -141.4 | -30.5 | 2 | 2 | 3 | 10.08 | 0.17 | 2.24 | 1.44 | 0.24 | -0.31 |
| 31 | -138.7 | -24.9 | 2 | 2 | 1 | 10.55 | 0.10 | 1.73 | 2.01 | -0.28 | 0.26 |
| 32 | -138.3 | -134.3 | 4 | 4 | 1 | 10.42 | 0.10 | 2.56 | 1.70 | 0.56 | $-0.05$ |
| 33 | -138.2 | -167.5 | 2 | 2 | 2 | 8.91 | 0.24 | 2.71 | 1.80 | 0.71 | 0.05 |
| 34 | -138.0 | -114.6 | 2 | 4 | 2 | 10.01 | 0.07 | 2.44 | 1.52 | 0.44 | -0.22 |
| 35 | -137.8 | 112.6 | 2 | 2 | 1 | 10.98 | 0.09 | 2.27 | 3.01 | 0.27 | 1.26 |
| 36 | -137.6 | 8.6 | 2 | 2 | 1 | 10.36 | 0.05 | 2.27 | 1.77 | 0.27 | 0.02 |
| 37 | -137.4 | -138.6 | 2 | 3 | 1 | 9.60 | 0.16 | 1.93 | 1.21 | -0.07 | -0.54 |
| 38 | -136.8 | 28.9 | 2 | 2 | 2 | 9.59 | 0.17 | 1.97 | 1.58 | -0.03 | -0.17 |
| 39 | -135.9 | 50.6 | 2 | 2 | 3 | 10.22 | 0.07 | 1.79 | 1.59 | -0.22 | -0.16 |
| 40 | -134.8 | -54.9 | 4 | 4 | 2 | 9.73 | 0.04 | 1.71 | 1.26 | -0.29 | -0.49 |
| 41 | -133.7 | 56.7 | 2 | 2 | 2 | 10.77 | 0.09 | 2.04 | 2.24 | 0.04 | 0.49 |
| 42 | -131.4 | 112.0 | 3 | 2 | 2 | 9.36 | 0.03 | 1.81 | 1.46 | -0.19 | -0.29 |
| 43 | -131.1 | 139.4 | 1 | 1 | 2 | 8.95 | 0.06 | 1.83 | 1.53 | -0.18 | -0.22 |
| 44 | -129.8 | 78.3 | 4 | 4 | 1 | 9.92 | 0.09 | 2.28 | 2.06 | 0.28 | 0.32 |
| 45 | -128.0 | -67.4 | 4 | 6 | 3 | 10.30 | 0.07 | 2.04 | 1.76 | 0.03 | 0.01 |
| 46 | -124.2 | 10.0 | 4 | 4 | 2 | 10.32 | 0.24 | 2.88 | 2.28 | 0.88 | 0.53 |
| 47 | -121.4 | -80.7 | 4 | 4 | 3 | 10.03 | 0.11 | 1.90 | 1.60 | -0.10 | -0.15 |
| 48 | -121.3 | 44.3 | 4 | 1 | 2 | 9.95 | 0.61 | 3.45 | 3.32 | 1.45 | 1.57 |
| 49 | -121.1 | -159.1 | 2 | 2 | 3 | 9.22 | 0.11 | 2.16 | 1.79 | 0.16 | 0.04 |
| 50 | -120.4 | 152.5 | 2 | 2 | 3 | 8.42 | 0.19 | 2.03 | 1.94 | 0.03 | 0.19 |
| 51 | -119.8 | -75.7 | 4 | 4 | 3 | 9.38 | 0.48 | 3.49 | 2.61 | 1.49 | 0.86 |
| 52 | -117.8 | 93.9 | 4 | 4 | 3 | 10.21 | 0.10 | 1.66 | 1.98 | -0.34 | 0.23 |
| 53 | -116.6 | -53.7 | 2 | 3 | 1 | 9.62 | 0.22 | 1.91 | 1.67 | -0.09 | -0.08 |
| 54 | -115.3 | 31.0 | 5 | 5 | 2 | 10.09 | 0.13 | 1.52 | 1.79 | -0.48 | 0.04 |
| 55 | -115.2 | -6.1 | 2 | 2 | 1 | 10.59 | 0.12 | 3.53 | 2.65 | 1.52 | 0.91 |
| 56 | -114.8 | 93.6 | 4 | 1 | 3 | 10.85 | 0.17 | 1.63 | 2.34 | -0.37 | 0.59 |
| 57 | -114.7 | 104.6 | 4 | 5 | 3 | 9.98 | 0.04 | 1.56 | 2.01 | -0.44 | 0.27 |
| 58 | -114.5 | 85.7 | 3 | 4 | 1 | 9.62 | 0.24 | 2.04 | 2.15 | 0.04 | 0.40 |
| 59 | -114.2 | -60.5 | 2 | 4 | 1 | 10.76 | 0.25 | 2.59 | 2.14 | 0.58 | 0.39 |
| 60 | -113.4 | -88.0 | 2 | 2 | 3 | 10.03 | 0.11 | 2.56 | 1.93 | 0.56 | 0.18 |
| 61 | -111.8 | 160.4 | 2 | 1 | 2 | 10.99 | 0.17 | 3.39 | 2.48 | 1.39 | 0.73 |
| 62 | -110.3 | -53.0 | 2 | 2 | 2 | 10.40 | 0.05 | 2.08 | 2.11 | 0.07 | 0.36 |
| 63 | -108.2 | -153.8 | 1 | 1 | 3 | 8.83 | 0.07 | 1.78 | 1.64 | -0.23 | -0.11 |
| 64 | -107.3 | -134.6 | 2 | 2 | 3 | 9.28 | 0.07 | 1.57 | 1.36 | -0.43 | -0.39 |
| 65 | -107.2 | 117.6 | 2 | 4 | 2 | 10.13 | 0.16 | 1.58 | 1.65 | -0.42 | -0.10 |
| 66 | -105.7 | -165.9 | 1 | 1 | 2 | 9.42 | 0.14 | 1.66 | 1.20 | -0.34 | -0.55 |


| ID | $\alpha_{\text {DST }}$ | $\delta_{\text {TSS }}$ | $\mathrm{n}_{\mathrm{K}}$ | $\mathrm{n}_{\mathrm{H}}$ | $\mathrm{n}_{\mathrm{L}}$ | $<m_{x}$ | $\Delta m_{k}$ | ( $\mathrm{H}-\mathrm{K})_{\text {OSS }}$ | (K-L) ${ }_{\text {oss }}$ | ( $\mathrm{H}-\mathrm{K})_{0}$ | (K-L) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 67 | -104.0 | 110.9 | 2 | 2 | 3 | 10.00 | 0.04 | 2.13 | 1.91 | 0.13 | 0.16 |
| 68 | -103.2 | 52.8 | 2 | 2 | 1 | 10.73 | 0.13 | 2.43 | 2.15 | 0.42 | 0.40 |
| 69 | -102.7 | -1.6 | 3 | 3 | 2 | 9.11 | 0.14 | 2.07 | 1.86 | 0.07 | 0.11 |
| 70 | -100.3 | 154.9 | 1 | 1 | 3 | 9.49 | 0.30 | 3.50 | 3.00 | 1.50 | 1.25 |
| 71 | -99.2 | -94.8 | 2 | 2 | 2 | 9.93 | 0.10 | 3.50 | 2.02 | 1.49 | 0.27 |
| 72 | -97.6 | 95.1 | 2 | 2 | 3 | 10.53 | 0.08 | 3.14 | 1.99 | 1.14 | 0.24 |
| 73 | -93.8 | -89.9 | 2 | 2 | 2 | 10.00 | 0.05 | 2.31 | 1.68 | 0.31 | -0.07 |
| 74 | -93.4 | 83.8 | 2 | 2 | 2 | 9.67 | 0.13 | 1.65 | 1.28 | -0.35 | -0.47 |
| 75 | -92.1 | -81.0 | 2 | 2 | 3 | 8.91 | 0.08 | 2.19 | 1.61 | 0.19 | -0.14 |
| 76 | -91.9 | 161.6 | 1 | 1 | 2 | 9.76 | 0.09 | 2.14 | 1.37 | 0.14 | -0.38 |
| 77 | -91.6 | 3.8 | 2 | 2 | 2 | 9.13 | 0.13 | 2.41 | 1.69 | 0.41 | -0.06 |
| 78 | -91.0 | 60.5 | 3 | 2 | 2 | 10.56 | 0.36 | 2.74 | 1.99 | 0.74 | 0.24 |
| 79 | -91.0 | 147.9 | 1 | 1 | 2 | 10.11 | 0.03 | 1.91 | 1.64 | -0.09 | -0.11 |
| 80 | -90.6 | 135.7 | 2 | 2 | 3 | 9.98 | 0.02 | 2.12 | 1.70 | 0.12 | -0.05 |
| 81 | -90.1 | -45.4 | 2 | 2 | 3 | 9.92 | 0.42 | 2.55 | 2.42 | 0.54 | 0.67 |
| 82 | -87.6 | 61.6 | 2 | 2 | 2 | 9.59 | 0.59 | 2.77 | 2.08 | 0.77 | 0.33 |
| 83 | -86.1 | 105.8 | 2 | 2 | 3 | 9.92 | 0.09 | 1.88 | 1.66 | -0.13 | -0.09 |
| 84 | -85.4 | 122.4 | 2 | 1 | 3 | 10.26 | 0.09 | 1.85 | 1.85 | -0.15 | 0.10 |
| 85 | -84.3 | 87.1 | 2 | 2 | 3 | 8.28 | 0.17 | 2.01 | 1.80 | 0.01 | 0.06 |
| 86 | -83.0 | 141.9 | 1 | 1 | 3 | 9.71 | 0.07 | 1.92 | 1.59 | -0.09 | -0.15 |
| 87 | -82.7 | -130.1 | 4 | 4 | 2 | 10.21 | 0.36 | 3.14 | 2.38 | 1.14 | 0.63 |
| 88 | -82.6 | 68.6 | 4 | 3 | 2 | 10.34 | 0.20 | 1.69 | 1.92 | -0.32 | 0.17 |
| 89 | -82.3 | 24.1 | 4 | 2 | 2 | 10.13 | 0.06 | 2.52 | 1.75 | 0.52 | 0.00 |
| 90 | -81.3 | -44.9 | 4 | 4 | 1 | 10.10 | 0.09 | 2.06 | 1.56 | 0.06 | -0.19 |
| 91 | -80.5 | -109.4 | 4 | 4 | 1 | 10.26 | 0.16 | 3.07 | 1.70 | 1.07 | -0.05 |
| 92 | -79.8 | 71.3 | 5 | 3 | 2 | 9.56 | 0.09 | 2.05 | 1.96 | 0.04 | 0.21 |
| 93 | -79.2 | -36.3 | 5 | 5 | 2 | 9.03 | 0.06 | 2.19 | 1.80 | 0.18 | 0.05 |
| 94 | -78.9 | 35.8 | 5 | 6 | 3 | 10.10 | 0.14 | 2.24 | 1.87 | 0.23 | 0.12 |
| 95 | -78.4 | 3.6 | 5 | 4 | 2 | 10.22 | 0.13 | 2.33 | 1.65 | 0.33 | -0.10 |
| 96 | -77.1 | -40.2 | 2 | 2 | 2 | 9.82 | 0.09 | 2.14 | 1.65 | 0.14 | -0.10 |
| 97 | -76.8 | -45.5 | 4 | 4 | 2 | 10.02 | 0.05 | 2.11 | 1.53 | 0.11 | -0.21 |
| 98 | -75.9 | 154.6 | 2 | 2 | 1 | 9.38 | 0.07 | 2.05 | 1.70 | 0.04 | -0.05 |
| 99 | -75.6 | -103.5 | 3 | 5 | 2 | 10.21 | 0.56 | 2.47 | 2.27 | 0.47 | 0.52 |
| 100 | -75.4 | 68.5 | 4 | 6 | 1 | 10.16 | 0.04 | 1.91 | 1.70 | -0.09 | -0.05 |
| 101 | -74.5 | 13.9 | 4 | 4 | 2 | 9.82 | 0.26 | 2.58 | 1.85 | 0.58 | 0.10 |
| 102 | -73.6 | -69.7 | 4 | 6 | 3 | 9.67 | 0.08 | 1.61 | 2.16 | -0.40 | 0.41 |
| 103 | -73.0 | -131.4 | 4 | 4 | 3 | 9.80 | 0.09 | 2.64 | 1.55 | 0.64 | -0.20 |
| 104 | -71.8 | 143.9 | 2 | 2 | 2 | 10.52 | 0.19 | 2.25 | 2.22 | 0.25 | 0.47 |
| 105 | -70.0 | 126.2 | 4 | 4 | 2 | 10.75 | 0.09 | 2.41 | 2.25 | 0.41 | 0.51 |
| 106 | -69.8 | -90.1 | 4 | 4 | 2 | 9.50 | 0.39 | 2.32 | 2.01 | 0.32 | 0.26 |
| 107 | -69.2 | -55.0 | 4 | 4 | 1 | 10.62 | 0.01 | 2.44 | 1.99 | 0.44 | 0.24 |
| 108 | -69.2 | -122.2 | 4 | 4 | 2 | 9.30 | 0.08 | 2.39 | 1.58 | 0.39 | -0.17 |
| 109 | -66.8 | 85.8 | 4 | 4 | 3 | 10.36 | 0.17 | 1.92 | 2.07 | -0.08 | 0.32 |
| 110 | -66.6 | -131.4 | 2 | 3 | 1 | 10.13 | 0.15 | 2.21 | 1.73 | 0.21 | -0.01 |
| 111 | -60.5 | 44.3 | 2 | 3 | 3 | 10.68 | 0.11 | 2.33 | 2.05 | 0.33 | 0.30 |


| ID | $\alpha_{\text {LrS }}$ | $\delta_{\text {TRS }}$ | $\mathrm{n}_{\mathrm{K}}$ | $\mathrm{n}_{\mathrm{H}}$ | $\mathrm{n}_{\mathrm{L}}$ | $\left\langle\mathrm{m}_{\mathrm{K}}\right.$ | $\Delta \mathrm{m}_{\mathrm{k}}$ | (H-K) ${ }_{\text {cos }}$ | (K-L) ${ }_{\text {ose }}$ | ( $\mathrm{H}-\mathrm{K})_{0}$ | (K-L) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 112 | -57.9 | -90.1 | 2 | 2 | 1 | 10.18 | 0.07 | 2.22 | 1.57 | 0.21 | -0.18 |
| 113 | -55.5 | 40.4 | 2 | 2 | 3 | 10.03 | 0.02 | 2.41 | 2.06 | 0.41 | 0.31 |
| 114 | -54.6 | -146.1 | 1 | 1 | 3 | 9.39 | 0.03 | 2.90 | 1.84 | 0.89 | 0.09 |
| 115 | -54.1 | -1.4 | 2 | 3 | 2 | 10.26 | 0.20 | 1.88 | 1.96 | -0.13 | 0.21 |
| 116 | -53.7 | -29.1 | 2 | 2 | 3 | 9.73 | 0.12 | 2.23 | 1.51 | 0.23 | -0.24 |
| 117 | -52.8 | -12.3 | 2 | 2 | 1 | 8.53 | 0.03 | 2.11 | 1.64 | 0.11 | -0.11 |
| 118 | -51.9 | 11.5 | 2 | 1 | 2 | 10.26 | 0.07 | 2.46 | 2.03 | 0.45 | 0.28 |
| 119 | -51.2 | 146.5 | 1 | 1 | 1 | 10.42 | 0.09 | 2.05 | 2.48 | 0.04 | 0.73 |
| 120 | -51.0 | -30.8 | 3 | 2 | 2 | 10.74 | 0.29 | 1.87 | 2.43 | -0.14 | 0.68 |
| 121 | -51.0 | 161.7 | 1 | 1 | 3 | 10.06 | 0.44 | 3.17 | 2.76 | 1.17 | 1.01 |
| 122 | -49.3 | -17.5 | 1 | 2 | 1 | 10.37 | 0.12 | 2.20 | 2.26 | 0.20 | 0.51 |
| 123 | -49.0 | 135.4 | 2 | 2 | 3 | 10.59 | 0.18 | 2.74 | 1.83 | 0.73 | 0.08 |
| 124 | -43.7 | 7.4 | 2 | 2 | 2 | 9.15 | 0.15 | 1.99 | 1.33 | -0.01 | -0.42 |
| 125 | -42.4 | -142.9 | 1 | 1 | 1 | 9.96 | 0.08 | 2.60 | 1.32 | 0.59 | -0.43 |
| 126 | -42.3 | 72.4 | 2 | 2 | 2 | 10.61 | 0.06 | 2.64 | 1.88 | 0.63 | 0.13 |
| 127 | -41.6 | -51.0 | 2 | 2 | 1 | 10.31 | 0.09 | 2.70 | 1.95 | 0.70 | 0.20 |
| 128 | -41.4 | -2.0 | 2 | 3 | 2 | 10.02 | 0.18 | 2.19 | 1.82 | 0.19 | 0.07 |
| 129 | -41.3 | 92.2 | 2 | 2 | 2 | 9.02 | 0.03 | 1.98 | 1.44 | -0.03 | -0.31 |
| 130 | -41.2 | 160.7 | 1 | 1 | 2 | 9.33 | 0.02 | 2.84 | 2.25 | 0.84 | 0.50 |
| 131 | -40.0 | -34.1 | 3 | 1 | 2 | 10.25 | 0.09 | 2.88 | 2.04 | 0.88 | 0.29 |
| 132 | -38.9 | -107.0 | 2 | 2 | 2 | 9.93 | 0.34 | 1.87 | 1.47 | -0.13 | -0.28 |
| 133 | -38.0 | 155.5 | 1 | 1 | - 1 | 10.51 | 0.12 | 2.72 | 2.11 | 0.72 | 0.36 |
| 134 | -37.6 | 144.8 | 1 | 1 | 1 | 10.43 | 0.17 | 2.48 | 2.33 | 0.48 | 0.58 |
| 135 | -33.6 | 155.9 | 1 | 1 | 1 | 10.87 | 0.07 | 2.39 | 2.17 | 0.39 | 0.42 |
| 136 | -33.1 | 79.7 | 2 | 2 | 1 | 9.29 | 0.02 | 2.00 | 1.78 | 0.00 | 0.03 |
| 137 | -32.9 | -18.4 | 4 | 3 | 1 | 10.23 | 0.22 | 2.85 | 2.08 | 0.85 | 0.33 |
| 138 | -31.6 | 120.6 | 2 | 2 | 3 | 10.34 | 0.15 | 2.28 | 1.65 | 0.28 | -0.10 |
| 139 | -30.4 | 114.9 | 3 | 2 | 3 | 9.07 | 0.10 | 2.19 | 1.75 | 0.19 | 0.01 |
| 140 | -27.5 | -34.5 | 4 | 6 | 2 | 10.08 | 0.14 | 2.10 | 1.83 | 0.10 | 0.08 |
| 141 | -26.7 | -39.1 | 4 | 4 | 2 | 9.80 | 0.18 | 3.03 | 1.95 | 1.03 | 0.20 |
| 142 | -25.7 | 18.1 | 2 | 3 | 2 | 10.22 | 0.17 | 3.02 | 1.82 | 1.02 | 0.08 |
| 143 | -25.3 | 129.5 | 4 | 4 | 1 | 10.52 | 0.16 | 2.60 | 1.87 | 0.60 | 0.12 |
| 144 | -24.7 | 36.4 | 3 | 4 | 2 | 10.89 | 0.19 | 2.40 | 2.50 | 0.39 | 0.75 |
| 145 | -24.5 | 71.6 | 3 | 4 | 2 | 9.68 | 0.04 | 1.91 | 1.66 | -0.10 | -0.09 |
| 146 | -24.0 | 75.9 | 3 | 4 | 2 | 10.29 | 0.06 | 2.19 | 1.59 | 0.18 | -0.15 |
| 147 | -23.7 | 68.1 | 4 | 4 | 2 | 10.14 | 0.10 | 2.03 | 1.95 | 0.03 | 0.20 |
| 148 | -22.6 | 96.8 | 3 | 4 | 2 | 9.81 | 0.12 | 2.41 | 1.63 | 0.40 | -0.12 |
| 149 | -22.6 | -100.0 | 5 | 5 | 3 | 8.51 | 0.56 | 2.52 | 2.52 | 0.51 | 0.77 |
| 150 | -22.5 | 16.2 | 4 | 4 | 2 | 9.62 | 0.07 | 2.38 | 2.20 | 0.38 | 0.45 |
| 151 | -21.6 | 40.7 | 3 | 4 | 3 | 9.23 | 0.05 | 2.38 | 1.83 | 0.38 | 0.08 |
| 152 | -21.4 | 115.9 | 4 | 4 | 2 | 10.03 | 0.04 | 2.17 | 1.41 | 0.17 | -0.34 |
| 153 | -19.1 | 76.8 | 3 | 4 | 3 | 10.15 | 0.05 | 2.10 | 1.65 | 0.10 | -0.10 |
| 154 | -18.9 | -30.7 | 4 | 3 | 3 | 10.42 | 0.35 | 3.21 | 2.84 | 1.21 | 1.09 |
| 155 | -17.9 | 73.1 | 3 | 4 | 3 | 9.63 | 0.04 | 1.94 | 1.16 | -0.06 | -0.59 |
| 156 | -173 | 130.6 | 4 | 4 | 3 | 10.20 | 0.08 | 1.99 | 2.22 | -0.01 | 0.47 |


| ID | $\alpha_{\text {PRT }}$ | $\delta_{\text {trs7 }}$ | $\mathrm{n}_{k}$ | $\mathrm{n}_{\mathrm{H}}$ | $\mathrm{n}_{\mathrm{L}}$ | $<m_{10}$ | $\Delta m_{k}$ | ( $\mathrm{H}-\mathrm{K})_{\text {oes }}$ | (K-L) oss | (H-K) | (K-L) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 157 | -17.0 | 27.6 | 4 | 3 | 3 | 10.67 | 0.29 | 2.42 | 2.50 | 0.42 | 0.75 |
| 158 | -15.7 | -76.1 | 2 | 4 | 3 | 9.90 | 0.05 | 2.99 | 1.87 | 0.99 | 0.12 |
| 159 | -15.4 | 18.0 | 4 | 4 | 2 | 10.05 | 0.08 | 2.26 | 1.75 | 0.26 | 0.00 |
| 160 | -15.0 | 158.5 | 2 | 2 | 1 | 10.23 | 0.00 | 2.27 | 1.64 | 0.27 | -0.11 |
| 161 | -12.4 | 84.5 | 2 | 4 | 2 | 10.16 | 0.30 | 1.89 | 1.80 | -0.11 | 0.05 |
| 162 | -12.4 | 105.2 | 3 | 3 | 2 | 10.59 | 0.20 | 1.98 | 2.25 | -0.03 | 0.50 |
| 163 | -12.2 | -16.2 | 2 | 2 | 2 | 9.94 | 0.56 | 2.39 | 2.04 | 0.39 | 0.29 |
| 164 | -10.9 | 77.8 | 8 | 4 | 3 | 10.28 | 0.14 | 1.89 | 2.44 | -0.11 | 0.69 |
| 165 | -10.3 | 45.4 | 1 | 4 | 2 | 10.39 | 0.14 | 1.90 | 1.79 | -0.10 | 0.05 |
| 166 | -10.1 | 19.8 | 2 | 4 | 1 | 9.85 | 0.28 | 2.06 | 1.75 | 0.06 | 0.00 |
| 167 | -9.8 | 93.2 | 1 | 4 | 1 | 10.96 | 0.05 | 2.30 | 2.34 | 0.30 | 0.59 |
| 168 | -9.4 | 113.2 | 2 | 4 | 3 | 8.75 | 0.13 | 2.21 | 1.92 | 0.21 | 0.17 |
| 169 | -8.3 | -16.2 | 2 | 3 | 1 | 9.69 | 0.21 | 1.72 | 1.49 | -0.29 | -0.26 |
| 170 | -8.1 | 7.8 | 2 | 3 | 2 | 8.87 | 0.06 | 2.17 | 2.06 | 0.17 | 0.31 |
| 171 | -6.8 | 128.7 | 2 | 4 | 2 | 10.84 | 0.09 | 1.84 | 2.22 | -0.16 | 0.47 |
| 172 | -6.0 | 104.1 | 2 | 3 | 2 | 8.74 | 0.18 | 1.94 | 1.83 | -0.06 | 0.09 |
| 173 | -5.1 | -21.3 | 2 | 2 | 2 | 9.94 | 0.11 | 1.66 | 1.52 | -0.34 | -0.23 |
| 174 | -4.9 | 47.0 | 1 | 2 | 2 | 9.31 | 0.09 | 2.08 | 1.66 | 0.08 | -0.09 |
| 175 | -2.9 | 95.0 | 1 | 2 | 3 | 10.71 | 0.23 | 2.44 | 2.22 | 0.44 | 0.47 |
| 176 | -1.0 | -48.6 | 2 | 2 | 2 | 10.16 | 0.17 | 2.18 | 1.56 | 0.17 | -0.19 |
| 177 | -0.8 | 12.1 | 2 | 1 | 1 | 9.76 | 0.28 | 2.31 | 1.83 | 0.31 | 0.08 |
| 178 | 0.5 | 94.2 | 1 | 2 | 1 | 10.59 | 0.05 | 1.67 | 1.92 | -0.34 | 0.17 |
| 179 | 0.5 | 154.6 | 1 | 1 | 3 | 9.27 | 0.11 | 2.18 | 1.62 | 0.18 | -0.13 |
| 180 | 1.0 | 5.4 | 2 | 2 | 3 | 8.90 | 0.13 | 2.33 | 1.45 | 0.32 | -0.30 |
| 181 | 1.7 | 42.0 | 1 | 2 | 3 | 9.42 | 0.08 | 2.37 | 1.64 | 0.37 | -0.11 |
| 182 | 1.9 | 120.0 | 2 | 2 | 3 | 9.61 | 0.09 | 2.32 | 1.98 | 0.32 | 0.23 |
| 183 | 1.9 | 23.4 | 2 | 2 | 3 | 9.76 | 0.08 | 3.50 | 3.81 | 1.50 | 2.06 |
| 184 | 2.1 | 34.4 | 1 | 2 | 2 | 10.73 | 0.25 | 2.73 | 2.57 | 0.73 | 0.82 |
| 185 | 3.0 | 75.7 | 1 | 1 | 3 | 10.02 | 0.12 | 2.40 | 1.79 | 0.40 | 0.04 |
| 186 | 3.3 | -16.4 | 2 | 1 | 2 | 9.93 | 0.25 | 2.55 | 2.18 | 0.55 | 0.43 |
| 187 | 4.6 | 28.1 | 2 | 2 | 1 | 9.63 | 0.16 | 2.26 | 1.97 | 0.26 | 0.22 |
| 188 | 5.5 | 138.5 | 1 | 1 | 2 | 9.70 | 0.12 | 1.85 | 1.46 | -0.15 | -0.29 |
| 189 | 6.3 | -58.8 | 2 | 2 | 3 | 9.66 | 0.11 | 1.93 | 1.13 | -0.07 | -0.62 |
| 190 | 6.5 | 98.5 | 1 | 3 | 1 | 10.43 | 0.47 | 2.36 | 1.36 | 0.36 | -0.39 |
| 191 | 6.8 | 102.6 | 2 | 2 | 2 | 10.31 | 0.13 | 1.95 | 1.57 | -0.06 | -0.18 |
| 192 | 8.6 | 66.6 | 1 | 2 | 3 | 10.09 | 0.24 | 2.62 | 1.45 | 0.62 | -0.30 |
| 193 | 10.1 | 104.6 | 2 | 2 | 2 | 10.12 | 0.15 | 2.33 | 1.60 | 0.32 | -0.15 |
| 194 | 10.2 | -11.2 | 2 | 2 | 3 | 9.53 | 0.05 | 2.83 | 1.87 | 0.83 | 0.12 |
| 195 | 11.2 | 78.2 | 1 | 2 | 2 | 10.18 | 0.21 | 2.36 | 1.45 | 0.36 | -0.30 |
| 196 | 12.6 | -0.2 | 2 | 3 | 3 | 8.91 | 0.04 | 2.06 | 1.43 | 0.06 | -0.32 |
| 197 | 13.6 | 17.8 | 2 | 3 | 1 | 10.24 | 0.16 | 2.71 | 2.09 | 0.71 | 0.34 |
| 198 | 14.9 | -16.2 | 2 | 3 | 3 | 9.72 | 0.14 | 2.49 | 1.28 | 0.49 | -0.47 |
| 199 | 15.8 | 97.3 | 1 | 2 | 2 | 10.08 | 0.10 | 1.68 | 1.76 | -0.33 | 0.01 |
| 200 | 16.1 | 89.0 | 1 | 2 | 2 | 10.20 | 0.30 | 2.32 | 1.98 | 0.32 | 0.23 |
| 201 | 16.9 | -0.5 | 2 | 5 | 1 | 9.69 | 0.08 | 2.02 | 1.62 | 0.02 | -0.13 |


| ID | $\alpha_{\text {ITST }}$ | $\delta_{\text {TSE7 }}$ | $\mathrm{n}_{\mathrm{k}}$ | $\mathrm{n}_{\mathrm{H}}$ | $\mathrm{n}_{\mathrm{L}}$ | $\left\langle m_{k}\right.$ | $\Delta \mathrm{m}_{\mathrm{k}}$ | ( $\mathrm{H}-\mathrm{K})_{\text {ons }}$ | (K-L) ${ }_{\text {ors }}$ | ( $\mathrm{H}-\mathrm{K})_{0}$ | (K-L) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 202 | 17.2 | 44.0 | 1 | 4 | 3 | 8.56 | 0.09 | 2.39 | 1.68 | 0.39 | -0.07 |
| 203 | 20.2 | -7.5 | 2 | 4 | 1 | 9.98 | 0.04 | 2.55 | 1.63 | 0.55 | -0.12 |
| 204 | 22.3 | 17.6 | 3 | 4 | 3 | 9.91 | 0.17 | 3.22 | 1.93 | 1.21 | 0.18 |
| 205 | 23.5 | 118.9 | 2 | 4 | 3 | 9.54 | 0.01 | 2.42 | 1.91 | 0.42 | 0.16 |
| 206 | 24.2 | -16.0 | 3 | 1 | 1 | 10.93 | 0.35 | 2.36 | 2.13 | 0.35 | 0.38 |
| 207 | 25.1 | 38.5 | 3 | 4 | 2 | 10.30 | 0.13 | 2.15 | 1.52 | 0.15 | -0.23 |
| 208 | 25.6 | 85.0 | 3 | 4 | 2 | 10.27 | 0.08 | 1.91 | 1.79 | -0.10 | 0.04 |
| 209 | 26.9 | 46.8 | 3 | 4 | 3 | 9.62 | 0.16 | 2.25 | 1.30 | 0.25 | -0.45 |
| 210 | 27.5 | 129.7 | 4 | 4 | 1 | 9.14 | 0.01 | 2.10 | 1.44 | 0.10 | -0.31 |
| 211 | 30.9 | 30.9 | 3 | 6 | 2 | 9.55 | 0.09 | 2.11 | 3.10 | 0.11 | 1.35 |
| 212 | 31.0 | 63.9 | 5 | 4 | 3 | 10.40 | 0.25 | 2.82 | 2.92 | 0.82 | 1.17 |
| 213 | 31.2 | 168.0 | 1 | 2 | 1 | 10.91 | -1.00 | 2.52 | 2.62 | 0.52 | 0.87 |
| 214 | 32.7 | 66.7 | 3 | 4 | 1 | 10.63 | 0.04 | 2.62 | 2.22 | 0.61 | 0.47 |
| 215 | 33.9 | -62.8 | 4 | 4 | 2 | 9.38 | 0.32 | 1.96 | 1.52 | -0.04 | -0.23 |
| 216 | 34.5 | 128.6 | 4 | 4 | 2 | 9.69 | 0.13 | 1.88 | 1.23 | -0.12 | -0.52 |
| 217 | 36.8 | 38.9 | 3 | 4 | 3 | 9.52 | 0.06 | 2.10 | 1.33 | 0.09 | -0.42 |
| 218 | 38.3 | 79.1 | 3 | 4 | 3 | 9.20 | 0.49 | 3.19 | 2.99 | 1.19 | 1.24 |
| 219 | 39.3 | -39.5 | 2 | 2 | 1 | 10.81 | 0.12 | 2.16 | 2.35 | 0.16 | 0.60 |
| 220 | 39.4 | -27.8 | 1 | 2 | 2 | 9.30 | 0.44 | 2.79 | 2.04 | 0.79 | 0.29 |
| 221 | 39.8 | 9.3 | 1 | 3 | 2 | 8.58 | 0.21 | 2.38 | 2.01 | 0.37 | 0.26 |
| 222 | 44.3 | 152.4 | 1 | 1 | 3 | 8.96 | 0.22 | 1.99 | 1.82 | -0.01 | 0.07 |
| 223 | 44.6 | 16.6 | 1 | 2 | 2 | 9.72 | 0.09 | 2.32 | 1.63 | 0.31 | -0.12 |
| 224 | 45.3 | 93.1 | 2 | 2 | 3 | 10.05 | 0.14 | 2.22 | 1.50 | 0.21 | -0.24 |
| 225 | 45.6 | 115.8 | 1 | 2 | 2 | 9.64 | 0.11 | 2.91 | 1.90 | 0.90 | 0.15 |
| 226 | 49.2 | 90.1 | 2 | 2 | 3 | 9.75 | 0.07 | 1.95 | 1.21 | -0.05 | -0.54 |
| 227 | 54.4 | 28.6 | 2 | 3 | 2 | 10.01 | 0.03 | 2.03 | 1.64 | 0.02 | -0.11 |
| 228 | 55.3 | 125.0 | 2 | 2 | 1 | 9.02 | 0.06 | 1.72 | 1.66 | -0.28 | -0.09 |
| 229 | 56.3 | 67.8 | 2 | 2 | 2 | 8.22 | 0.15 | 2.61 | 1.48 | 0.61 | -0.27 |
| 230 | 57.6 | 75.6 | 2 | 2 | 3 | 9.46 | 0.10 | 2.26 | 1.41 | 0.26 | -0.34 |
| 231 | 57.8 | 87.1 | 2 | 2 | 1 | 10.38 | 0.10 | 2.34 | 1.67 | 0.34 | -0.07 |
| 232 | 60.0 | 22.8 | 1 | 2 | 3 | 9.90 | 0.35 | 3.13 | 2.23 | 1.13 | 0.48 |
| 233 | 60.1 | 78.1 | 2 | 1 | 3 | 9.83 | 0.11 | 2.43 | 1.32 | 0.43 | -0.43 |
| 234 | 61.9 | 51.8 | 2 | 2 | 2 | 9.92 | 0.07 | 2.27 | 1.24 | 0.27 | -0.51 |
| 235 | 63.3 | 126.1 | 2 | 2 | 1 | 8.60 | 0.34 | 2.94 | 2.30 | 0.94 | 0.55 |
| 236 | 64.3 | 62.4 | 3 | 3 | 2 | 9.43 | 0.11 | 2.50 | 1.54 | 0.49 | -0.21 |
| 237 | 69.1 | 159.9 | 1 | 1 | 3 | 8.84 | 0.19 | 2.15 | 1.71 | 0.15 | -0.04 |
| 238 | 69.2 | 37.6 | 2 | 4 | 3 | 9.23 | 0.16 | 2.87 | 1.16 | 0.87 | -0.59 |
| 239 | 69.6 | 21.2 | 1 | 4 | 3 | 9.30 | 0.07 | 2.75 | 1.57 | 0.75 | -0.18 |
| 240 | 70.5 | 65.7 | 2 | 6 | 3 | 9.92 | 0.12 | 3.31 | 1.28 | 1.30 | -0.47 |
| 241 | 74.3 | 51.9 | 3 | 4 | 1 | 10.23 | 0.32 | 2.60 | 1.51 | 0.59 | -0.24 |
| 242 | 75.3 | 41.0 | 4 | 4 | 3 | 10.13 | 0.13 | 3.17 | 1.54 | 1.17 | -0.21 |
| 243 | 76.6 | 70.2 | 4 | 5 | 3 | 9.43 | 0.09 | 2.32 | 1.23 | 0.32 | -0.52 |
| 244 | 76.7 | 23.1 | 3 | 2 | 2 | 10.10 | 0.17 | 2.81 | 1.53 | 0.81 | -0.22 |
| 245 | 79.3 | 147.1 | 2 | 2 | 3 | 9.20 | 0.07 | 2.95 | 1.44 | 0.95 | -0.31 |
| 246 | 81.1 | -22.9 | 2 | 2 | 2 | 10.55 | 0.09 | 2.15 | 2.12 | 0.15 | 0.37 |


| ID | $\alpha_{\text {CBS }}$ | $\delta_{\text {IRS }}$ | $n_{K}$ | $n_{H}$ | $n_{L}$ | $\left\langle m_{K \mathcal{S}}\right.$ | $\Delta m_{K}$ | $(H-K)_{\text {OES }}$ | $(K-L)_{\text {OBS }}$ | $(H-K)_{0}$ | $(K-L)_{0}$ |
| :--- | ---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 247 | 81.4 | -34.6 | 2 | 3 | 3 | 10.06 | 0.05 | 2.40 | 1.70 | 0.39 | -0.05 |
| 248 | 84.1 | 55.6 | 4 | 2 | 2 | 10.60 | 0.06 | 2.61 | 1.91 | 0.61 | 0.16 |
| 249 | 90.0 | 46.8 | 4 | 2 | 3 | 10.16 | 0.12 | 2.61 | 1.74 | 0.60 | -0.01 |
| 250 | 91.6 | 40.1 | 4 | 2 | 3 | 9.36 | 0.09 | 2.41 | 1.71 | 0.41 | -0.04 |
| 251 | 93.4 | 45.3 | 3 | 2 | 3 | 10.25 | 0.16 | 2.10 | 2.03 | 0.10 | 0.28 |
| 252 | 102.6 | 76.9 | 2 | 2 | 3 | 9.17 | 0.06 | 1.90 | 1.44 | -0.11 | -0.31 |
| 253 | 103.1 | -20.6 | 1 | 1 | 2 | 9.77 | 0.06 | 1.82 | 1.46 | -0.18 | -0.29 |
| 254 | 104.2 | 118.4 | 2 | 2 | 1 | 10.10 | 0.25 | 232 | 1.19 | 0.32 | -0.56 |
| 255 | 105.0 | 1.9 | 2 | 2 | 2 | 10.16 | 0.06 | 2.41 | 1.53 | 0.40 | -0.22 |
| 256 | 109.6 | 93.9 | 2 | 2 | 3 | 9.41 | 0.09 | 2.00 | 1.19 | 0.00 | -0.56 |
| 257 | 111.6 | 159.7 | 1 | 1 | 3 | 9.43 | 0.10 | 2.63 | 1.56 | 0.62 | -0.19 |
| 258 | 122.6 | 90.9 | 3 | 1 | 3 | 10.15 | 0.64 | 4.50 | 3.78 | 2.50 | 2.03 |

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