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# Disk components in early type galaxies 

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# DISK COMPONENTS IN EARLY TYPE GALAXIES 

by<br>Hans-Walter Reinhard Rix

A Dissertation Submitted to the Faculty of the DEPARTMENT OF ASTRONOMY

In Partial Fulfillment of the Requirements For the Degree of DOCTOR OF PHILOSOPHY In the Graduate College THE UNIVERSITY OF ARIZONA

## THE UNIVERSITY OF ARIZONA GRADUATE COLLEGE

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#### Abstract

This thesis clarifies the role of disk components embedded in the spheroids of early type galaxies, with particular focus on the frequency and structure of disks in galaxies conventionally classified as "ellipticals". We discuss both photometric and spectroscopic means of assessing disks.

Using simple photometric models, we explore what physical disk parameters result in detectable photometric signatures. We discuss in particular the deviations of the projected isophotes from perfect ellipses in disk/spheroid systems. We show that a wide range of intrinsic disk-to-spheroid ratios ( $\mathrm{D} / \mathrm{S}$ ) can produce very similar photometric signatures, depending on viewing angle. We find the distribution of observed isophote distortions in a sample of ellipticals with published surface photometry to be consistent with the $\mathrm{D} / \mathrm{S}$ hypothesis, implying that about half of the sample members could contain disks with $D / S \sim 0.25$.

To confront our models with a more suitable set of data, we obtained surface photometry at $0.4 \mu$ and $1.6 \mu$ for a statistical sample of about 80 galaxies, comprised of both E's and S0's. Analyzing this data set we find that in any given luminosity bin of early type galaxies, one third of the objects contain disks whose detectability depends on a favourably high inclination. This fraction was estimated independently from isophote distortions and from radial luminosity profiles. The apparent smooth transition between disk galaxies and purely spheroidal objects can be explained exclusively by changes in the viewing angle, even assuming two discrete classes of early type galaxies (either having substantial disks or none at all). There is no need to invoke continuity along the Hubble sequence from E's to S 0 's.

For the members of this sample we find a considerable range in D/S, $0.15<$ $D / S<5$. However, most of that variation is caused by changes in the relative scale lengths rather than by changes in disk surface brightness.

To analyze kinematic signatures of disk components we develop an optimal algorithm to extract the line-of-sight velocity distribution (LOSVD) from the broadening of absorption line spectra. Analyzing the LOSVD's in two kinematically distinct cores of elliptical galaxies, we find that they can be modelled dynamically as small disks embedded in the large spheroid. The range in rotational support, $1.3<v / \sigma<4$, of these disks suggests that some of them have formed dissipatively and others through a merger event.


## CHAPTER 1

## INTRODUCTION

### 1.1 The Hubble Sequence

More than half a century ago Hubble set out to bring order to the "Realm of the Nebulae". He devised a classification sequence for the different morphological types of galaxies, ranging from "early type" elliptical to "late type" spiral galaxies (1923,1936). In this scheme, early types (E,S0) are galaxies with an overall smooth luminosity distribution, while late types ( $\mathrm{Sa}-\mathrm{Sc}, \mathrm{Irr}$ ) are galaxies with spiral arms, bright knots, dust lanes and other features. This sequence was devised on a purely phenomenological basis and implied in its terminology, if anything, a temporal evolution (see also Sandage 1961), which failed to hold after closer scrutiny. Nonetheless, it has over the decades proven very useful in understanding the global properties of intrinsically bright galaxies. This usefulness results from a number of physical characteristics, relevant both for the final structure and for the formation history, that vary systematically along this essentially one dimensional sequence. These characteristics include the current star formation rate, the present gas content, the overall age of the stellar population, the mass density of the environment and the relative importance of the spheroidal component of the system to the flat disk component. In this thesis we will concern ourselves mostly the last of these
aspects.
Since most galaxies show a certain amount of symmetry, their overall shapes can often be described by simple geometric figures. A very successful way of describing the three dimensional shapes of most intrinsically bright galaxies has been to separate their luminous parts into a spheroidal component and a highly flattened disk component. Although there is a great deal of scatter, the disk to spheroid luminosity ratio, the $D / S$ ratio, increases strongly from early to late types along the Hubble sequence: while most of the light in many late type spirals comes from the disk, the spheroid dominates in most early type galaxies (e.g. Simien and de Vaucouleurs, 1986). According to the conventional definition, S0s are early type galaxies with disk components, while ellipticals are pure spheroids. However, it has never been determined whether there is a continuous transition between the two types, i.e. whether disk components can constitute an arbitrarily small fraction of the total light in ellipticals. In the present work we want to take several steps to clarify this issue and to explore the range of properties of disk components in early type galaxies. Before posing specific questions, we will illustrate in the subsequent sections, first, why it is important to distinguish between spheroid and disk components and second, how to untangle the two components in practice, given the observable information.

### 1.2 Why Distinguish Spheroids and Disks?

The distinction between spheroids and disks is important in two respects: First, to analyze the dynamics of a stellar system from kinematic data, we need to know the underlying geometry. Second, disks and spheroids are likely to have had very different formation processes. The current stucture of these two components still bears imprints of the formation processes.

### 1.2.1 Dynamics

Spheroids and disks differ in the extent to which they are sustained against selfgravity and external (halo) forces by streaming motions of the constituent stars rather than by randomly oriented motions: while disks are predominantly rotationally supported, spheroids are "pressure" supported. However, streaming motions do play a role in spheroids and, conversely, there is a random component to the motions of disk stars. The relative importance of rotational support compared to pressure support can be conveniently expressed by the ratio $v / \sigma$, where $v$ is the luminosity weighted mean streaming velocity and $\sigma$ is the velocity dispersion. For spheroidal systems $v / \sigma$ ranges from zero (non-rotating system) to about unity (Davies et al. 1983, Kormendy and Illingworth 1982, Dressler and Sandage 1983). In contrast, disks in late type galaxies have $v / \sigma$ of about 5 to 8 (van der Kruit and Freeman 1986, Gilmore et al. 1989). Disks in S0's may have random motions twice as large as the later types (Kormendy 1984a, 1984b). Since the ratio of rotational energy to the energy in random kinematic motions is proportional to $(v / \sigma)^{2}$, the observed $v / \sigma$ differences imply a one or two order of magnitude difference in the rotational support between the two components. This difference in rotational support also leads to the difference in intrinsic shapes: while the ratio of longest to shortest axis of the isodensity contours in spheroids ranges from one to two (Mihalas and Binney 1981), disks can have axis ratios of up to 10 (e.g. Sandage 1961).

The primary goals of kinematic studies of galaxies are to infer their mass distribution, to estimate their mass-to-light ratio ( $\mathrm{M} / \mathrm{L}$ ) and investigate whether this ratio changes with radius and to constrain the shapes of the stellar orbits. It is crucial in these studies to understand the underlying geometry of the light, for example whether the mass is in a flattened disk or in a spheroid. This may be illustrated by the following, admittedly exaggerated, example: suppose kinematic measurements
$v(r)$ and $\sigma(r)$ were taken of a face-on disk. Since the object appears to be circular in projection, the data could be interpreted with a spherical mass model. With this geometry a mass distribution could be found in which a luminous tracer population would produce the observed kinematic data, but this analysis would yield an extremely low $M / L$ estimate, together with an outward drop in the $M / L$ ratio, due to the outward drop of the disk dispersion. In this case the error in geometry leads to such an absurd mass distribution that in practice the spherical geometry would be rejected and the presence of a face-on disk would be inferred.

For more realistic cases of ambiguous geometry, for example where spheroids and disks co-exist, a faulty geometry will lead to less dramatic inferences about the mass distribution. Yet, since detailed modelling of the mass distributions, constraining velocity anisotropies and searching for $M / L$ changes often is very dependent on the assumed underlying geometry (e.g. Binney and Tremaine 1987, Binney, Illingworth and Davies 1990), just these errors may be more "dangerous", because they are harder to recognize. The modelling of NGC4697 by Binney et al. (1990) may serve as a specific example of the practical implications of an ambiguous geometry. Binney et al. found it impossible to model the kinematics of this object by an isotropic, axisymmetric model: the rotation velocity $v$ decreased too rapidly away from the major axis ${ }^{1}$. One possible explanation is to invoke a slight bias towards more circular orbits, compared to an isotropic model. An alternative, and, as we will show in Chapter 2, for this particular case a more likely possibility is that NGC4697 contains in addition to its main spheroid a quite highly inclined disk whose light contributes to the measured rotation along the major axis. Although the two scenarios produce similar kinematic signatures, their underlying structure and formation history would be quite different. In Chapter 4 we will discuss ways

[^0]to distinguish these alternatives observationally.

### 1.2.2 Dynamical Interactions of Spheroids and Disks

The existence and longevity of S 0 galaxies shows that spheroids and disks can coexist in equilibrium; nonetheless, they will affect each other dynamically.

Disk components, as dynamically cold as they are observed to be, would be grossly bar unstable unless they are embedded in a spheroid, which may be luminous or not (Hohl 1971, Ostriker and Peebles 1973); cold disks need some spheroidal mass distribution to exist. Qualitatively speaking, the more the potential is dominated by a spheroidal component and the less the disk is self-gravitating, the less fragile it is with respect to global instabilities.

The way in which spheroidal components are affected by the presence or absence of a disk is much less clear: on the one hand, observations show that virtually all bulges (i.e. spheroids that have disks associated with them) rotate rapidly enough to explain their considerable flattening (Kormendy and Illingworth 1982, Dressler and Sandage 1983). On the other hand, theoretical arguments suggest that bulges should be flatter than expected from their rotation: Barnes and White (1984) showed through N -body experiments that the adiabatic addition (through gas infall and star formation) of a disk component will make the bulges flatter and increase their velocity dispersion, without speeding them up. This conflict between observations and N -body experiment implies that the scenario assumed for the numerical simulations, namely the slow growth of the disk after the spheroid formation, is incorrect; the majority of the disk material must have been in place at the epoch of the spheroid formation.

This indication that the same galaxy formation mechanism which results in disks also results in rapidly rotating spheroids, has led to the conjecture (e.g. Nieto 1988,

Capaccioli et al. 1990) that in turn rotationally flattened "ellipticals" contain disk components. The presence of a disk components could then explain the difference in rotational support among elliptical galaxies: while the very brightest ellipticals exhibit very small $v / \sigma$, many of the other apparently spheroidal galaxies are consistent with being rotationally flattened (e.g. Davies et al. 1983).

### 1.2.3 Formation

The formation of galaxies, including the formation of their components, is a field that is currently very actively pursued. For an outline of current thought on the topic we refer to review papers (e.g. Carlberg 1987 and White 1990). Here we shall briefly mention some of the reasons why spheroids and disks are thought to have had rather different formation histories. The small net angular momentum and large random motions in spheroids can be explained if their formation history has been violent. Merging of either pre-existing galaxies (Toomre 1977, White 1978, Barnes 1988) or protogalactic clumps (van Albada 1982, Aguilar and Merritt 1989) can produce remnants that resemble observed spheroids in shape and specific angular momentum. While in these calculations the rotation of the final spheroid strongly reflects the initial conditions, the radial profile is almost always found to resemble an $R^{1 / 4}$ law (de Vaucouleurs 1948) over a large range of $R$ (White 1978). However, purely dissipationless collapse or merger models have difficulties explaining the high phase space densities observed at the cores of many spheroids (Carlberg 1986): no progenitor objects of high enough phase space density are known. An additional constraint on spheroid formation is that most of the luminous material must already have been converted from gas into stars by the time the system reached its present equilibrium configuration; otherwise, dissipation of the progenitor gas would have led to disk formation.

The existence of disks provides immediate evidence for the importance of dissipational processes in galaxy formation. If gas can cool efficiently, it will lose radiatively as much energy as it can, given the constraints of angular momentum conservation. This process will force the gas into a rotationally supported disk. Since dissipationless stellar material does not have a similar a cooling mechanism, it follows that most of the material in disks has reached its current location in gaseous form. At present it is unclear whether any disks formed at about the same epoch as the spheroids (e.g. Katz 1990) or whether all disks formed later and over an extended period of time (Binney and May 1986).

The fragility of a disk component can give clues to the dynamical history of their host galaxies: tidal interactions can heat disks, i.e. increase the random motions; violent tidal interactions will destroy disks. Barnes (1988) showed that a merger of two disk galaxies of comparable size will lead to a diskless remnant and Quinn (1987) demonstrated that an infalling satellite with as little mass as $4 \%$ of the primary, can heat the primary disk by a factor of two. The abundance of dynamically cold disks can therefore be used to constrain the number of past interactions (Ostriker 1990). Similarly, the existence of any disk component residing in a spheroid excludes a merger origin of the system from equal size progenitors, or from dissipationless formation. However, there are merger processes that can lead to the formation of disk-like structures comprising a small fraction of the total material (Balcells and Quinn 1990): a satellite system which is disrupted as it spirals into the main body of the galaxy will deposit material preferentially in its orbital plane, with large streaming velocities reflecting the orbital motion, and relatively small random motions. The deposited material will then have kinematic properties typical of a disk.

Finally, disks and spheroids have different typical luminosity profiles: the out-
ward fall-off of the light in disks can very often be fit by the empirical law:

$$
I(R)=I_{0} \cdot \exp \left(-\frac{R}{R_{\operatorname{cxp}}}\right)
$$

(Freeman 1970), while in spheroids it can be described as

$$
I(R)=I_{e f f} \cdot \exp (\gamma) \cdot \exp \left(-\gamma\left(\frac{R}{R_{e f f}}\right)^{1 / 4}\right)
$$

with $\gamma=6.67$ (de Vaucouleurs 1948). Each profile is characterized by an intensity scale, $I_{0}$ or $I_{\text {eff }}$, and a radial scale length, $R_{\text {exp }}$ or $R_{\text {eff }}$. These particular fitting formulae are purely empirical, yet it appears that the stereotypical profiles are directly related to the formation process: mergers and violent relaxation lead to an $R^{1 / 4}$ law for a wide range of initial profiles (White 1978, van Albada 1982), while recent numerical simulations of dissipational proto-galaxy collapse (Katz and Gunn 1991) suggest that angular momentum transfer in gas will establish an exponential profile. Although in most cases the empirical fitting functions will only hold approximately, and in some cases will fail miserably, the overall pattern in the luminosity profile differences is highly significant.

### 1.3 How to Tell a Disk from a Spheroid

Although a conceptual separation of spheroids and disks is usually straightforward, the observational assessment of these two components is not. The basic observational tools that can be employed to untangle the two are surface photometry and absorption line spectroscopy. The first technique utilizes the fact that disks are much flatter than spheroids and usually show a different radial luminosity profile. The second technique uses the kinematic "coldness" of the disks, which is reflected in rapid rotation and a small velocity dispersion.

### 1.3.1 Photometric Decompositions

The fundamental problem with photometric decompositions of spheroids and disks is that they cannot be done unambiguously without making stringent a priori assumptions about the properties of each component. Rybicki (1987) showed formally that, even for an axisymmetric system, the projected light distribution could not provide enough information to reconstruct the 3-D luminosity, unless the system is seen edge-on ${ }^{2}$.

The standard way to escape this dilemma is to assume that the disk and the spheroid are axisymmetric and obey their stereotypical radial profiles exactly (an exponential profile for the disk and an $R^{1 / 4}$ law for the spheroid). Futhermore, the spheroid is conventionally assumed to be of constant ellipticity. With these assumptions the problem is reduced to determining six parameters ( $I(0), I_{\text {eff }}, R_{\text {exp }}$, $R_{e f f}, \cos (i)$ and $\left.\epsilon_{S}\right)$, which can be done by fitting the major and minor-axis profiles (e.g. Kent 1985). For spiral galaxies such a procedure appears to give robust decompositions, because in many objects the light at small radii is dominated by the spheroid, while at large radii it is dominated by the disk. Thus the parameters of the two components can in essence be fixed independently.

If the radial scales and luminosities of the two components are very similar, such a decomposition becomes much more difficult, since the two profiles can add up to a composite profile which may still be well fit by an $R^{1 / 4}$ law. Further complications can arise when the assumption of axisymmetry is relaxed: For triaxial ellipsoids, rather than spheroids, the projected major axes of spheroid and disk will not coincide, making major-minor axis fitting an ill-defined procedure.

For substantially inclined systems an additional constraint could be imposed:

[^1]suppose the light of each component projects into perfect ellipses. For any inclined system the light from the disk component will be projected into more flattened isophotes than the spheroid, causing the shapes of the resulting isophotes of the total light to deviate from ellipses: the isophotes become extended along the major axis or "lemon-shaped" (see Figure 1.1). Even if a disk component is too faint to contribute significantly to the overall luminosity profile, it can still reflect itself in the isophote shapes. In general it holds true that disks are most easily detected by photometric means if they are seen nearly edge-on. It is then that their geometric differences from the spheroids become most evident.

### 1.3.2 Spectroscopic Detection of Disk Components

An alternative, or complementary, way to distinguish disks from spheroids, in particular in early type galaxies, is by their kinematic characteristics. These are reflected in the Doppler shift and the apparent line broadening of absorption lines in the observed spectra.

As mentioned in Section 1.2.2., the presence of a disk can be inferred by interpreting the kinematic data in the context of a particular geometry, say a spheroidal model. Then one can check how plausible the deduced mass distribution is: For example a rapid, substantial outward drop in the measured velocity dispersion may mean that the light has become dominated by a disk component. Inclined objects, that exhibit rotation too fast to be consistent with the oblate rotator hypothesis (Binney 1978), have been argued to have disks (e.g. Jedrzejewski and Schechter 1989).

A more direct method of inferring the presence of a dynamically cold component can be applied when disk and spheroid contribute comparable amounts to the total light. In this case each component will leave its characteristic imprint on the
resulting line-of-sight velocity distribution. This distribution will then be composed of a broad component with small velocity off-set, stemming from the spheroid, and a narrower component with larger off-set from the disk. This idea is illustrated in Figure 1.2. A careful reconstruction of this velocity distribution can then recover the contributions from each component, which we will discuss extensively in Chapter 4.

### 1.4 Disks in Early Type Galaxies: Scope of this Thesis

So far we have discussed, in quite general terms, the importance of a spheroid/disk assessment for our understanding of the dynamics and the formation history of a galaxy and we have pointed out some of the practical difficulties in making this distinction. We now focus on the particular questions that will be addressed in the course of this work.

Until very recently the term elliptical galaxy implied a monolithic object whose structure could be described by a few global quantities, and which consisted only of a spheroid, or ellipsoid, without a disk. However, closer observational scrutiny through photometry and spectroscopy revealed substructure over a wide range of radial scales, which in many cases could be explained by a spheroid/disk (S/D) geometry: some "ellipticals" have distinctly lemon shaped isophotes, some show "humps" in the ellipticity profiles, some contain small radial regions of rapid rotation. Such galaxies are usually referred to as "disky" ellipticals. Most authors presumed that these objects do contain very weak (a few percent of the total light) stellar disk component and interpreted them as a "transition objects" from S0s to Es (e.g. Carter 1987, Jedrzejewski 1987, Capaccioli 1987, Bender 1988a, Nieto 1988). Unfortunately, in all cases the presence or absence, and properties of such disk components were argued only anecdotally for individual objects, rather than systematically for statistical sets of galaxies. Since for each individual case the ex-


Figure 1.1: Isophote Shapes of S/D Sytems
This Figure illustrates qualitatively how changes in viewing angle can mimic dramatic changes in the strength of a disk component embedded in the spheroid. The three panels in the left column show isophotes for intrinsically identical D/S systerns ( $D / S=0.3, R_{\text {exp }}=R_{\text {eff }}$ ). The apparent fading of the disk component from the top to the bottom is merely caused by tilting the system towards face-on: $\cos (i)=0.2, \cos (i)=0.3$ and $\cos (i)=0.5$, respectively. In the right column we show the isophotes of three systems, all viewed from $\cos (i)=0.3$, but with a wide range of disk-to-spheroid ratios: $D / S=1.6, D / S=0.3$ and $D / S=0.07$.
planation of "diskiness" by means of a spheroid/disk model is not unique, these studies remained inconclusive.

One of the primary goals of this thesis is a systematic, observational test of this S/D hypothesis; in Chapters 2 and 3 we will use photometric data and modelling to investigate the nature of "ellipticals" with disky or "pointed" isophotes. We would like to know:

- Is it sensible to attribute the isophote shapes to the presence of a disk? If yes, what range of disk and spheroid parameters can produce the observed isphote shapes? For any given S/D model, how do the isophote shapes depend on the inclination of the system?
- We will show in Chapter 2 that changing the strength of a disk component at a given inclination and changing the inclination for a given disk, can have very similar effects on the isophote shapes (as illustrated qualitatively in Figure 1.1). Thus one hypothesis about the nature of disky ellipticals is that they contain very faint, edge-on disks (a few percent of the total light), as suggested by Jedrzejewski (1987), Carter (1987) and Capaccioli et al. (1990). Alternatively they could be physically identical objects to well known edgeon disk galaxies, such as NGC3115 or the Sombrero galaxy NGC4594, which have disks that constitute $15 \%$ to $40 \%$ of the total light, just seen from a less favorable viewing angle. We will design and employ statistical tests to untangle these alternatives. The answer to this question will also tell us whether these potential disk components are expected to have a significant impact on the kinematics and dynamics of the galaxy, or whether they just a very small deviation from an overall spheroidal, or ellipsoidal, mass distribution.
- We will use these results to address the question of continuity along the Hubble
sequence from S0's to $E$ 's: how much of the apparent continuity in photometric properties can be explained by assuming a physical discreteness of E's and S0's and merely varying viewing angles? Is there a wide and continuous range of disk-spheroid luminosity ratios, D/S? If so, do disks just become smaller compared to the spheroid or do they become dimmer? Is there a wide range in disk surface brightnesses or does Freeman's law (1970) hold also for these disks.
- We will estimate what fraction of disks is photometrically unrecognizable (in any individual case), and how likely it is to make a significant error by assuming a purely spheroidal geometry, when modelling "spheroidal" galaxies, which contain disks. Finally, we can make a revised estimate of how many galaxies are presumably disk-less, or have negligible disks, and are thus "available" as merger products.

In Chapter 4 we concentrate on a particular class of potential disks in elliptical galaxies: kinematically distinct cores. Over the last few years, an increasing number of ellipticals have been found in which small nuclear regions, $<5^{\prime \prime}$, have very different angular momentum than the main body of the galaxy (e.g. Franx and Illingworth 1988, Bender 1988b, Schechter and Jedrzejewski 1989). This is usually manifested in rapid rotation, either parallel, in the opposite sense, or perpendicular to the rotation of the outer parts, which could be naturally understood through the presence of a disk. We develop and apply tools to test whether the paradigm of a small nuclear disk embedded in and coexisting with the spheroid is viable. As mentioned earlier, the simultaneous presence of a hot spheroid and a cold disk leads to a characteristic, composite velocity distribution along the line of sight (see Figure 1.2). We develop and discuss an algorithm to extract optimally such a velocity distribution from absorption line spectra. Combining this information with photometry we


Figure 1.2: Kinematic Detection of Disks
This figure illustrates qualitatively how disks imprint themselves into kinematic data. The top panel shows the (major axis) luminosity profile of an D/S system. The two bottom panels show the expected velocity distribution at the two opposite major axis points where the components contribute equally to the total light. The narrow component, offset in velocity space, arises from the kinematically "cold", rapidly rotating disk material. The broad component stems from the "hot" spheroid. The resulting total velocity distribution is clearly asymmetric, and is expected to be antisymmetric at opposing positions. This distribution can in principle be recovered from the line broadening in spectroscopic data (Chapter 4).
can construct very well constrained disk/spheroid models for these kinematically distinct cores. These models allow us to estimate the rotational support of these nuclear disks and thus enable us to infer plausible formation mechanisms for these objects.

To conclude, we briefly summarize in Chapter 5 the main results of this thesis and draw attention to some of the remaining issues.

## CHAPTER 2

## PHOROMETRIC SIGNATURES OF DISKS ${ }^{1}$

### 2.1 Non-Elliptical Isophotes: A Brief Overview

For a long time theoretrical interest in elliptical galaxies concentrated on their high degree of symmetry and their apparently simple structure. These properties allowed a rigorous and even largely analytic treatment of their dynamics. Surprising observational findings, such as the low rotation speed of the brightest objects (Bertola and Capaccioli, 1975) and the presence of isophote twists (e.g. Leach, 1981), were explained elegantly in terms of velocity anisotropies and triaxial models (e.g. Binney, 1978).

## CCD Photometry and Isophote Shapes

With the advent of CCDs, the observed isophotes in elliptical galaxies were found to deviate systematically from ellipses at the percent level (Lauer, 1985; Jedrzejewski, 1987; Franx, 1988; Bender, Döbereiner and Möllenhoff, 1988, hereafter BDM). One particular form of distortion is found frequently in otherwise regular galaxies; if the residual intensity variations along a best fitting ellipse are expanded as a Fourier

[^2]series in the azimuthal angle $\theta$, the coefficient " $A_{4}$ ", associated with the $\cos (4 \theta)$ term, is found to have a relatively large amplitude. If $A_{4}$ is negative, the isophotes are boxy; if $A_{4}$ is positive the isophotes are pointed ${ }^{2}$, as illustrated in Figure 2.1. In their "almost statistical" sample of ellipticals, BDM found such distortions to be very common; roughly one third of the objects showed pointed isophotes, and a comparable fraction showed boxy isophotes. The amplitudes of these distortions are typically about $1 \%$.

## Interpreting Non-Elliptical Isophote-Shapes

In this chapter we will discuss galaxies which have pointed isophotes, i.e. show positive $A_{4}$. There is no good physical reason why "elliptical" galaxies should have perfectly elliptical isophotes. One can view the isophotes as the result of a time- , or sample-, average over the orbits of the constituent stars in the galaxy. No individual orbit will result in elliptical isophotes by itself ${ }^{3}$ (e.g. Binney and Tremaine 1987). It is only the superposition of many orbits which will yield the observed smooth isophotes. A strong population of particular orbital families could lead to nonelliptical isophotes: Binney and Petrou (1985) show orbits whose superposition can produce a very boxy apperance and Contopulos and Grosbol (1989), in an analysis of triaxial bar potentials, found an orbital family which can yield pointed isophotes.

Even a mass distribution (or light distribution) which is constant on spheroids at every given radius may not project into ellipses. Merritt (1991) has considered the projection of spheroidal models becoming much rounder towards the outside. However, he found that for the actually observed ellipticity gradients in early type

[^3]

Figure 2.1: Pointed and Boxy Isophote Distortions This figure illustrates qualitatively the shape of isophotes which deviate from perfect ellipses by a $\cos (4 \theta)$ perturbation. The solid lines represent ellipses. The dashed line at the top shows "pointed" isophotes (with positive $A_{4}$ ), at the bottom "boxy" isophotes. For illustrative purposes the amplitude of $A_{4}$ was chosen to be $5 \%$, larger than the distortions found in most ellipticals.
galaxies the resulting $A_{4}$ is less than $0.5 \%$. Thus ellipticity changes are not a viable explanation for the observed pointed isophotes.

As indicated in the introductory chapter, there is a simple, natural, but, as just mentioned, by no means exclusive, interpretation of ellipticals with pointed isophotes, which we will investigate here: we assume that these galaxies are composed of a flat disk component superimposed on a truly spheroidal bulge, a morphology similar to that of S0s, but possibly with a more dominant spheroid.

If such a disk is assumed to be flat and nearly edge-on, a disk with about one percent of the total light can produce the observed amplitudes for $A_{4}$. For example, Carter (1987) modelled NGC4697, which has an $A_{4}$-parameter of +0.02 , with a highly inclined disk which contributes only $2 \%$ of the total light. This assumption of a weak, nearly edge-on disk has been adopted by all authors who have tried to estimate such a disk contribution quantitatively (Carter 1987, Jedrzejewski et al. 1987, Capaccioli et al. 1988), and has led them all to conclude that disk components are very weak and dynamically unimportant.

## Outline of this Chapters

The main goal of this chapter is to develop tools for assessing the importance of disks from their photometric signatures, with emphasis on the isophote shapes. Specifically, we set out to investigate how observables such as the luminosity profile, $I(r)$, the ellipticity profile, $\epsilon(r)$, and $A_{4}(r)$, relate to physical parameters such as spheroid-to-disk ratio, disk inclination, and the scale lengths of spheroid and disk. We choose a very simple model for the light distribution: an oblate spheroid of constant ellipticity and an exponential disk. With this model we explore the space of physical parameters, and "project" our models into the plane of observable parameters by creating pseudo-data and performing surface photometry on them.

Theses models will allow us to address some of the questions raised in the introducory chapter:

- For which regions of parameter space do disks result in measurable isophote distortions?
- Are the observed isophote distortions predominantly caused by very weak, highly inclined disks?
- Can we estimate detection probabilities for such disks, in particular as a function of their contribution to the total light?
- What is the expected distribution of $A_{4}$ for a population of galaxies with disks seen from random viewing angles?
- What fraction of "ellipticals" might have disks which are strong enough to be dynamically and kinematically important?

We will focus on the more technical aspects of the problem here and refer some of the implications of the results, such as the question of continuity along the Hubble sequence to Chapter 3. The remainder of the chapter is organized as follows: In Section 2.2 we characterize our photometric models. Section 2.3 describes how disks of various strengths and inclinations are reflected in the isophote shapes. Section 2.4 addresses the detection probability of disks. Alternative photometric means of detecting such disks are discussed in Section 2.5, where we also model the disk in NGC4660. Section 2.6 gives a summary and conclusions.

### 2.2 Photometric Modelling

We analyze the photometric effects of an exponential disk superimposed on a spheroid by creating a sequence of "pseudo-CCD" frames and analyzing them with surface
photometry software, kindly provided by Marijn Franx (Franx et al., 1989). In this section we give details of our photometric models and our "observing" procedure.

The spheroidal components of our galaxies are represented by an oblate spheroid with a constant intrinsic ellipticity, $\epsilon_{0}$, and an $R^{\frac{1}{4}}$-law luminosity profile ${ }^{4}$ with unit effective radius and unit total luminosity. The disks are taken to be exponential, with scale length $R_{\text {exp }} / R_{\text {eff }}$, and total light contribution $\mathrm{D} / \mathrm{S}$. The disk lies in the equatorial plane of the spheroid, and the whole galaxy is inclined with respect to the line of sight at an angle $i$. Thus the disk isophotes are ellipses with ellipticity,

$$
\begin{equation*}
\epsilon_{d i a k}=1-\cos (i), \tag{2.1}
\end{equation*}
$$

and the spheroid isophotes are ellipses with ellipticity

$$
\begin{equation*}
\epsilon_{\text {spheroid }}=1-\sqrt{\cos ^{2}(i)+\left(1-\epsilon_{0}\right)^{2} \sin ^{2}(i)} \tag{2.2}
\end{equation*}
$$

In addition to the overall scale parameters $L_{S p h e r o i d}$ and $R_{\text {eff }}$, this parametrization gives us the following four "shape" parameters: $\cos (i), R_{\text {exp }} / R_{\text {eff }}, \mathrm{D} / \mathrm{S}$ and $\epsilon_{0}$. For comparison with observations we need to simulate the photometry over an appropriate radial range. We want our models to represent nearby bright ellipticals, as in the observational samples of Djorgovski (1985), Jedrzejewski (1987), BDM and Franx (1988). BDM find $\left\langle R_{\text {eff }}\right\rangle \approx 20^{\prime \prime}$ for their sample, with a scatter of less than a factor of two. Assuming that CCD-photometry is conventionally done in the range of 2 " to 80 ", we perform "photometry" on our models over the range, $0.1 \cdot R_{\text {eff }}$ to $4 \cdot R_{\text {eff }}$. In practice, our data frame consists of a $512 \times 512$ array on which we placed a (in most cases noiseless) superposition of the spheroid and the disk. The photometric parameters we want to extract from these models are the luminosity profile, $I(r)$, the ellipticity profile, $\epsilon(r)$, and the profile of residual Fourier coeffi-

[^4]cients (e.g. $\left.A_{4}(r)\right)$. These residual coefficients will be defined and thei: significance will be discussed in the next section.

Before proceeding, we should again state quite clearly the limitations of this model. Although we assume that the spheroid is oblate and has constant ellipticity, we know from measured isophote twists and other photometric evidence that neither assumption holds strictly in most real ellipticals and S0s. Furthermore, an $R^{\frac{1}{4}-}$ law luminosity profile has little physical justification and is known to hold only approximately (within 0.1 magnitudes) in most cases. However, we wanted to keep our modelling as straightforward as possible, because any additional freedom would cause a proliferation of free parameters and thus obfuscate the reader who has to wade through the results.

### 2.3 Isophote Distortions from Disk Components

### 2.3.1 The Significance of Fitting Residuals

The two dimensional image is mapped into the photometric profiles, $I(r), \epsilon(r)$, etc., by fitting (in a least squares sense) a sequence of concentric ellipses to the image pixel values. The deviations of the isophotes from ellipses are characterized by the residual terms of the ellipse fitting procedure. The essential steps of this algorithm are as follows: the image intensity is first sampled along a trial ellipse. This intensity string, $I(\theta)$, is subsequently analyzed in a Fourier series:

$$
\begin{equation*}
I(\theta)=I_{0}+\sum_{i=1}^{N} a_{\mathbf{i}} \cdot \cos (i \theta)+b_{\mathbf{i}} \cdot \sin (i \theta) \tag{2.3}
\end{equation*}
$$

where $\theta$ is the azimuthal angle as measured from the major axis and $N$ determines the truncation of the Fourier expansion; here we used $N=6$. The first and second order expansion coefficients are used to improve the fitting parameters, the
center of the ellipse, the ellipticity, and the position angle. Starting from these new parameters, the procedure is repeated until some convergence criterion is satisfied. For perfectly elliptical isophotes the intensity should then be constant along the sampled annulus, and all Fourier coefficients except $I_{0}$ should vanish. Since the fitting forces the first and second order coefficients to be very small, significant residuals can only be of order three or higher.

A disk component will always have a higher apparent ellipticity than the spheroid, and will lead to excess intensity near the major axis. Thus it is expected to give residual Fourier components that have even symmetry and a maximum on the major axis $(\theta=0)$, i.e. $\cos (2 n \theta)$, with $n \geq 2$. Furthermore, the dominant coefficient, usually $a_{4}$, must be positive. Indeed, most of the "power" of the disk signature in such an expansion is concentrated in $a_{4}$. Any non-vanishing residuals with other symmetry cannot be explained by this model, and must be attributed to tidal distortions, dust obscuration or an intrinsically complex (e.g. triaxial) structure (Franx, 1988).

A positive $a_{4}$-parameter should only be taken as indicating a disk if it is the dominant residual; for the majority of galaxies with significant $a_{4}$ this appears to be the case (BDM), although there are exceptions. Franx et al.(1989) have shown that $b_{4}$ is of comparable amplitude in some cases and have argued that this phenomenon could be explained by the superposition of a triaxial bulge and a disk.

To give the residual terms a geometric meaning, $a_{4}$ is usually divided by the slope of the intensity profile $\partial \ln (I) / \partial \ln (R)$, to yield $A_{4}$ (e.g. Jedrzejewski 1987). This residual $A_{4}$ can be viewed to lowest order as the fraction by which an isophote is longer than the semi-major axis of the best fitting ellipse. Throughout the remainder of this chapter we will use this geometrical $A_{4}$, because it is the quantity customarily published (Jedrzejewski 1987, BDM, Franx 1988).

Before we discuss our simulations, we should address qualitatively one further
point that has plagued previous work: the light attributable to disk-like components is usually assessed by subtracting the best fitting model of elliptical isophotes from the data and comparing the integrated $r m s$ residual light to the total light of the fitted model (see e.g. Jedrzejewski 1987). This procedure results in disk-to-spheroid ratio estimates of at most a few percent. However, as we show below, it systematically and substantially underestimates the disk contribution because much of the disk light signature can be "absorbed" by ellipticity changes.

### 2.3.2 The $A_{4}$ Signature of Disks

We now proceed to investigate how much $A_{4}(r)$ can tell us quantitatively about the presence of disks. Before carrying out a systematic search of the parameter space, let us illustrate our method by analyzing the isophotes of a sequence of four models. The data we extract from these models are: the ellipticity, $\epsilon(r)$, the isophote fitting residual, $A_{4}(r)$, and $\delta I(r)$, the residual from the best fitting $R^{\frac{1}{4}}$-law over the range $0.1 R_{\text {eff }}<r<4 r_{\text {eff }}$. Here $r$ is the mean radius of the fitting ellipse, defined as the geometric mean of major and minor axis, $r=\sqrt{a b}$. The spheroid ellipticity, $\epsilon_{0}$, was set to be 0.35 , as a fiducial value to represent a range of bulge ellipticities from 0.15 to 0.55 . We varied the disk inclination, $\cos (i)$, from 0.1 to 0.4 , and chose (by experiment) $\mathrm{D} / \mathrm{S}$ to yield a maximum $A_{4}$ value, $A_{4, m}$, of about $2 \%$ in each case. The results are shown in Figure 2.2.

This sequence illustrates a number of points. Firstly, even with our simple model the $A_{4}$-parameter does not yield unique information about the disk component. Even for fixed $\epsilon_{0}$ and $R_{\text {exp }} / R_{\text {eff }}$, there is a trade-off between disk mass and disk inclination that can result in very similar $A_{4}$ profiles. Secondly, the above mentioned notion (e.g. Jedrzejewski, 1987) that " $n \%$ " of $A_{4}$ translates into roughly an " $n \%$ " disk light contribution, could lead to an underestimate of the disk fraction by an


Figure 2.2: Non-Uniqueness of $A_{4}$ Profiles
The top panel shows the $A_{4}$-profile for three models. Although the disk light fraction $\mathrm{D} / \mathrm{S}$ varies by a factor of 20 between these models, the $A_{4}$-profiles are very similar. This illustrates that the effects of disk strength and disk inclination on $a_{4}(r)$ are indistinguishable. The two bottom panels show the run of the ellipticity and the deviations of the radial profile from an $R^{\frac{1}{4}-l a w, ~ r e s p e c t i v e l y, ~ f o r ~ t h e ~ s a m e ~ t h r e e ~}$ models.
order of magnitude! Thirdly, we find that (unless the disk dominates the light over a wide range of radii) the maximum of $A_{4}$ always occurs near the maximum disk light contribution along the major axis,

$$
\begin{equation*}
r_{m d} \approx 2.4 \cdot\left(R_{e x p}^{4} / R_{e f f}\right)^{1 / 3} . \tag{2.4}
\end{equation*}
$$

This formula can be derived by simply comparing the relative contributions of an $R^{\frac{1}{4}}$-law and an exponential profile of given scale lengths. For the above sequence $R_{\text {exp }} / R_{\text {eff }}$ was chosen so as to keep the radial coordinate of $A_{4, m}$ constant, taking into account that this radial coordinate is the geometric mean of the apparent major and minor axes and thus depends on the apparent ellipticity of the system at this radius. Finally, we note that the profiles have qualitatively similar shapes, so that we can use $A_{4, m}$ to characterize the $A_{4}$ profile.

We now turn to a more systematic investigation of the dependence of $A_{4}$ on our model parameters, $\epsilon_{0}, \cos (i), \mathrm{D} / \mathrm{S}$, and $R_{\text {exp }} / R_{\text {eff }}$. An exploration of parameter space for these four quantities shows that the dependence of $A_{4}$ on $R_{\text {exp }} / R_{\text {eff }}$ and $\epsilon_{0}$ is substantially weaker than its dependence on $\mathrm{D} / \mathrm{S}$ and $\cos (i)$. We considered parameters in the ranges $0.15<\epsilon_{0}<0.55$ and $0.25<R_{\text {exp }} / R_{\text {eff }}<2$. The range for $R_{\text {exp }} / R_{\text {eff }}$ was chosen to allow comparison with the existing data samples. For the galaxies of interest these limits imply $4^{\prime \prime}<R_{\text {exp }}<35^{\prime \prime}$, which spans the range in which isophote shapes can be well measured without being affected by seeing. The variation in $A_{4}$ for both parameters is only about $50 \%$ and we decided to use fiducial values of $\epsilon_{0}=0.35$ and $R_{\text {exp }} / R_{\text {eff }}=0.75$.

We consider values for $\mathrm{D} / \mathrm{S}$ ranging from $2 \%$ to about unity. For $\cos (i)$ we choose limits of 0.05 and 0.95 . We do not consider more edge-on disks because our models do not include any vertical disk structure. For random viewing angles, $\cos (i)$ is uniformly distributed between 0 and 1 . We therefore exclude only a small


Figure 2.3: Inclination Dependence of $A_{4}$
The four lines illustrate the dependence of $a_{4, m}$ on $\cos (i)$ for four different disk strengths, $\mathrm{D} / \mathrm{S}$. For any given $\mathrm{D} / \mathrm{S}$, the maximal $A_{4}$ declines exponentially towards more face-on viewing angles. Since for random viewing angles $\cos (i)$ is evenly distributed, any observed cumulative distribution of $A_{4}$ is expected to have this same functional form. The open circles indicate the cumulative distribution of $A_{4}$ measured by BDM, which, in its shape, is consistent with the model prediction. Note however, that if any edge-on objects are missing in the BDM sample (because they were classified as S 0 s ), the data points will shift to the right, towards higher disk fractions.
fraction of cases by setting $\cos (i)_{\min }=0.05$. Figure 2.3 shows the dependence of $A_{4}$ on $\cos (i)$ for several values of $\mathrm{D} / \mathrm{S}$. We have only shown the range $0.5 \%<A_{4}<$ $10 \%$, since for bigger $A_{4}$ the disk is obvious to the eye, and smaller $A_{4}$ values are hard to establish as disk signatures. We see that, for given $\mathrm{D} / \mathrm{S}, A_{4}(\cos (i))$ is well approximated over this range by an exponential decline. In particular, we note that $A_{4, m}$ is small $(<1 \%)$ for $\cos (i)>0.6$ for all D/S. In addition, Figure 2.3 shows that, for suitable inclination, $A_{4}$ is of the order of a few percent for a wide range of D/S. Finally we note that $A_{4, m}$ varies by large factors over our chosen parameter
range for $\mathrm{D} / \mathrm{S}$ and $\cos (i)$. This is an a posteriori justification for ignoring variations in $R_{\text {exp }} / R_{\text {eff }}$ and $\epsilon_{0}$; these changed $A_{4, m}$ by factors of 1.5 or less over the range of interest.

### 2.4 Detectability of Disks

### 2.4.1 Detection Probabilities

We mentioned in the previous section that for random viewing angles $\cos (i)$ is distributed uniformly. Therefore any graph showing some quantity, say $A_{4}$, as a function of $\cos (i)$ can be re-interpreted as a cumulative probability distribution of this quantity. Exploiting this fact we can make a-connection between our models and the observable distribution of $A_{4}$ in a sample of galaxies; such a comparison will allow us to estimate the possible abundance and strength of disks in observed "ellipticals". From Figure 2.3 we see that for given $\mathrm{D} / \mathrm{S}, \log \left(A_{4, m}\right)$ is roughly a linear function of $\cos (i)$ over the range plotted. For random viewing angles there is an equal probability of measuring $A_{4, m}$ in each interval, $\Delta \log \left(A_{4, m}\right)$, for $0.5 \%<$ $A_{4, m}<A_{4, m, 0}$, where $A_{4, m, 0}$ is the value of $A_{4, m}$ for edge-on disks. Since this holds true for each $\mathrm{D} / \mathrm{S}$, it also holds approximately for any distribution of $\mathrm{D} / \mathrm{S}$ as long as the measured values of $A_{4, m}$ are less than $A_{4, m, 0}$ for most of the population. Thus, treating the $\cos (i)$-axis as a probability axis, the cumulative distribution of observed $A_{4}$ values is expected to have the same functional form in this diagram as our theoretical lines. This then allows us to estimate a "typical" value for $\mathrm{D} / \mathrm{S}$ by comparing, say, the number of objects with $1 \%<A_{4, m}<5 \%$ to the number of objects with $A_{4, m}<1 \%$ (including all objects without detectable $A_{4}$ ).

At present, only BDM's "nearly statistical" sample contains a large enough number of galaxies for such statistical tests. We extracted $A_{4, m}$ values (from BDM's
figures) for all objects in which $A_{4}$ was the dominant (positive) residual. Assuming that these $A_{4}$ indeed do arise from a disk component, we sorted the $A_{4}$, normalized their abscissa values by the total number of sample members, and plotted them as open circles in Figure 2.3. The distribution of observed $A_{4}$ is, at least qualitatively, consistent with the hypothesis that they arise from disk components seen at random angles. Note, that this construction assumes that no edge-on galaxies are missing because they were classified as S 0 's; a situation which is quite improbable if our basic hypothesis about disky galaxies is correct. Also, van den Bergh's (1990) result shows that the Hubble type classification of early type galaxies is probably viewing angle dependent. Unfortunately, there is also some ambiguity as to which of the elliptical galaxies should be included in the total sample. All galaxies were included in Figure 2.3, but Bender et al.(1989) argue that the radio-loud ellipticals should be considered as qualitatively different objects from the other galaxies. Their argument is based on the very good correlation between radio luminosity and isophote shape the majority of the boxy ellipticals have $L_{R}>10^{21} \mathrm{WHz}^{-1}$ (at 1.4 GHz ), while none of the significantly disky galaxies have $L_{R}>10^{21} W H z^{-1}$. It is thus plausible that strong radio emission indicates a structurally distinct population of galaxies. Note that if we were to exclude all galaxies in BDM's sample which show significant boxiness at all radii ( 8 objects), we would obtain virtually the same results from a statistical analysis as we obtain by excluding all objects with strong radio emission. Thus it does not matter in this context which criterion is chosen to distinguish the potential two classes of ellipticals.

If we include all of Bender's objects in our disk fraction estimate outlined above (i.e. 12 objects with $1 \%<A_{4, m}<5 \%$ out of a total of 47 objects) we obtain $\mathrm{D} / \mathrm{S} \approx 0.05$; if we exclude the nine radio-loud objects ( $L_{R}>10^{21} \mathrm{~W} / \mathrm{Hz}$ ) then 12 out of 38 objects have large $A_{4}$ and we obtain $D / S \approx 0.30$. Purely statistical errors
in these fractions also produce large uncertainties in $\mathrm{D} / \mathrm{S}$, reflecting the relatively weak sensitivity of $A_{4}$ to $\mathrm{D} / \mathrm{S}$. The main conclusion of this exercise would seem to be that disks containing of the order $20 \%$ of the light could be present in most ellipticals without contradicting the $A_{4}$ data. Furthermore, "typical" disks of this order are actually suggested by the rather substantial fraction of radio weak ellipticals which have $A_{4, m}>1 \%$.

### 2.4.2 Mapping Parameter Space

Given these uncertainties, let us address a more general problens: For any given parameter set $[D / S, \cos (i)]$, what is the probability that such a disk can be detected photometrically? As we have seen, for $\cos (i)>0.5$ the isophote shapes yield hardly any information even for strong disks. In such cases the major axis profile might be used as an alternative indicator of a disk. Although we have no secure a priori knowledge of what the underlying luminosity profile (without disk) might be, it appears that an $R^{\frac{1}{4}-l a w}$ approximates $I(r)$ within 0.1 mag for the majority of giant ellipticals (Burstein et al. 1987), when fitted over a factor of twenty or so in radius.

For our models we evaluate the rms deviation, $\overline{\delta I}$, of the major axis profile from an $R^{\frac{1}{4}-l a w ~(w h e r e ~ t h e ~ i n f l u e n c e ~ o f ~ a ~ d i s k ~ c o m p o n e n t ~ i s ~ m a x i m i z e d), ~ f i t t e d ~}$ over a factor of 40 in radius, and we use it as a second observable signature of the presence of disks. Although in real galaxies there is no rigorous way to decide which deviations are significant and which ones are not, it seems reasonable to use the luminosity profile to put upper limits on $D / S$, because it is unlikely that an underlying spheroid profile will conspire with an exponential disk to mimic an $R^{\frac{1}{4}}$ law. With these two disk detection criteria, we can estimate the fraction of disk systems which still eludes recognition. Figure 2.4 shows contours in the [D/S, $\cos (i)$ ] plane both for $A_{4, m}$ and for $\overline{\delta I}$. The solid contours present essentially the same
information as Figure 2.3. They again demonstrate that $A_{4}$ detects disks very efficiently when $\cos (i)<0.4$, but not at all when $\cos (i)>0.6$. The dashed lines are contours of $\overline{\delta I}$. Requiring $\overline{\delta I}>0.1$ mag to detect a disk, we see that, for $\mathrm{D} / \mathrm{S}<0.25$, $A_{4}$ is a more sensitive indicator of disks than the luminosity profile. The shaded area indicates the region of parameter space in which disks cannot be detected photometrically, adopting $A_{4, m}>1 \%$ and $\overline{\delta I}>0.1 \mathrm{mag}$ as detection criteria.

Again we recall that, for a sample without viewing angle bias, the $\cos (i)$ axis can be regarded as a probability axis. Figure 2.4 then shows that, for random viewing angles and for disk-to-spheroid ratios of less than a quarter, fewer than $50 \%$ of the disks can be detected photometrically. Consequently, there should be many undetected systems with disks as strong as those found so far. Using BDM's sample, and tentatively adopting the Bender et al.(1989) view that the ellipticals, which have strong radio emission (or are significantly boxy at all radii) are fundamentally different objects from those which do not, we find that somewhat less than half of these "quiet" objects show a significantly positive $A_{4}$ parameter. If all these pointed isophotes are due to the presence of a disk, this again implies that the majority of the other "quiet" ellipticals should also have disky components. If we include all ellipticals this means that still about $40 \%$ to $60 \%$ of all galaxies classified as ellipticals have disk components. Figure 2.4 also shows that there is no reason to suppose that most of these disks are extremely weak.

It seems that disky components may not be small "flaws" of a perfectly ellipsoidal model, but rather might turn out to be substantial components, which have important consequences for the kinematics and dynamics.


Figure 2.4: Disk Detectability
This figure shows the impact of a disk component on the isophote shapes and the radial profile in the $[\mathrm{D} / \mathrm{S}, \cos (i)]$ parameter plane. Loci of constant $A_{4, m}$ (solid lines) and constant $r m s$ deviation from an $R^{\frac{1}{4}}$-law, $\overline{\delta I}$ (dashed lines), are indicated. It is apparent that for highly inclined disks, $\cos (i)<0.40, A_{4}$ is a sensitive diagnostic, while for low inclinations, $\cos (i)>0.60$, there is virtually no $A_{4}$ signature, independent of $D / S$. For near face-on disks the radial profile must be employed to detect any disk component. If we adopt $A_{4}>1 \%$ and $\overline{\delta I}>0.1 \mathrm{mag}$ as our detection criteria, then disks in the shaded area of the parameter plane are undetectable by photometric means. Since the $\cos (i)$-axis can be interpreted as a probability axis, this implies that $50 \%$ of all disks with $D / S<0.25$ cannot be detected by photometric means.

### 2.5 Other Photometric Signatures of Disks

In the last section we were forced to retreat to the somewhat swampy ground of statistics because the knowledge of $A_{4}$ alone was insufficient to determine both $\mathrm{D} / \mathrm{S}$ and $\cos (i)$. It is clear that additional photometric information can overcome this ambiguity. Yet, there is a good number of problems associated with doing so in practice, which we will discuss now.

### 2.5.1 Luminosity and Ellipticity Profiles

As seen in the bottom panels of Figure 2.2, for a given $A_{4}$ both $\epsilon(r)$ and $\delta I(r)$ are, in principle, capable of differentiating between various $[\mathrm{D} / \mathrm{S}, \cos (i)]$ combinations. The obvious problem is that this is only true if the underlying structure of the subcomponents is well described by our model. Nevertheless, there are two regimes in which $\epsilon(r)$ and $\delta I(r)$ can provide useful constraints. Firstly, they can yield upper limits on D/S by assuming that the observed profile should not be flatter or smoother than predicted by the model. Secondly, our models predict both $\epsilon(r)$ and $\delta I(r)$ to have a very characteristic shape in the presence of a disk. For example, the ellipticity shows a maximum at the radius of the maximum disk light contribution, as originally pointed out by Michard (1984). Therefore, if the observed variations in, say, $\epsilon(r)$ match the predictions of the models, then the fit can be attributed more significance. We will illustrate this point in section 2.6.3.

### 2.5.2 Higher Order Residuals

As mentioned before, $A_{4}$ is not the only Fourier residual with the appropriate symmetry to show the presence of a flat disk; the next higher order one is $A_{6}$. We implemented the calculation of $A_{6}$ in the isophote analysis software to test its usefulness in untangling $\cos (i)$ and $\mathrm{D} / \mathrm{S}$. We found in all cases that $A_{6}$ is smaller than


Figure 2.5: Higher Order Residuals
This figure illustrates the importance of higher order residuals as a function of disk inclination. While for near edge-on disks the higher order coefficients (here $A_{6}$ compared to $A_{4}$ ) are of comparable importance, they are small compared to $A_{4}$ for less highly inclined disks. This should allow an observational test of the hypothesis that most observed disk components are very weak and nearly edge-on.
$A_{4}$, but that, for high inclinations ( $\cos (i)<0.2$ ), both coefficients are of comparable size. The relation of $A_{6}$ to $A_{4}$ as a function of $\cos (i)$ is shown in Figure 2.5, for several values of $\mathrm{D} / \mathrm{S}$. For inclinations lower than $\cos (i)=0.40, A_{6}$ is almost negligible.

How feasible is it to employ $A_{6}$ in practice? We performed Monte-Carlo simulations, adding various amounts of Poisson noise to the "data" frames before analyzing the isophotes. We found, both from the "smoothness" of $A_{4}(r)$ and $A_{6}(r)$ and from the formal error calculation, that $A_{6}$ is as robust as $A_{4}$ with respect to Poisson noise. Therefore $A_{6}$ should be measurable to an accuracy level comparable to $A_{4}$. However, the only published data on $A_{6}$ to date (Michard and Simien, 1988) are not of sufficient quality to test this idea.

Unfortunately, the interpretation of $A_{6}$ is very sensitive to the breakdown of
the model assumptions. We can illustrate the consequences of an inappropriate model by two examples: Suppose the bulge is triaxial, resulting in a non-zero angle between the projected bulge and disk major axis. For an edge-on disk it takes only a twist (and resulting phase shift in $A_{6}$ ) of $360^{\circ} / 6 * 0.25=15^{\circ}$ to change the $A_{6}$ amplitude from maximum to zero. Thus, the higher the order of the Fourier residuals, the more sensitive they are to non-axisymmetries. Secondly, $A_{6}$ is only significant for nearly edge-on disks; for these orientations the finite thickness of the disks is most noticeable. The few existing measurements of vertical scale heights in S0 disks (e.g. Silva et al.1990) suggest 5:1 as a reasonable "ball park" number for the edge-on axis ratio of the disk isophotes. A comparison with Figure 2.5 shows that therefore the measured $A_{6}$ may reflect more the disk thickness rather than the disk inclination.

### 2.5.3 The Disk in NGC4660

As an example, we apply the techniques discussed above to determine the disk parameters of NGC4660 (classified as E5 in the RC2). This galaxy is one of the best examples of pointed isophotes in BDM's sample. The data for the ellipticity and the $A_{4}$ profile were taken from BDM; the luminosity profile was taken from Djorgovski's thesis (1985). The open squares in Figure 2.6 show the data. Superimposed are the predictions of two models that roughly approximate the observed $A_{4}$ profile. The dashed lines are for a model with $\mathrm{D} / \mathrm{S}=3 \%$ and $\cos (i)=0.1$ and the solid lines are for a model with $\mathrm{D} / \mathrm{S}=48 \%$ and $\cos (i)=0.40$. The model with the more luminous disk is clearly a better fit to the data. The observed stronger "curvature" in both the ellipticity and the luminosity residual profile are probably due to a disk that cuts off more steeply than an exponential disk. To avoid the introduction of additional parameters, we did not try to model this.


Figure 2.6: Disk-Spheroid Model for NGC 4660
The squares represent photometric parameters for NGC4660, taken from BDM and Djorgovski (1985). We try to determine the strength and inclination of the disk component in this galaxy, using the profiles of $A_{4}$, the ellipticity, and the deviations of the radial profile from an $R^{\frac{1}{4}-l a w . ~ T h e ~ l i n e s ~ i n d i c a t e ~ t h e ~ p r e d i c t i o n s ~ o f ~ t w o ~ o f ~}$ our models: the dashed line represents a model with $\mathrm{D} / \mathrm{S}=0.03$ and $\cos (i)=0.10$, while the solid line shows a model with $\mathrm{D} / \mathrm{S}=0.48$ and $\cos (i)=0.40$. It is obvious that the latter model provides an acceptable fit, while the model with the weak disk must be rejected. Despite its classification as an elliptical galaxy this object has a bulge-to-disk ratio of only $2: 1$. As a future check of this interpretation we show the predicted $A_{6}$-profiles.

So far we have used $\epsilon(r)$ and $\delta I(r)$ to untangle $\mathrm{D} / \mathrm{S}$ and $\cos (i)$. The bottom right panel of Figure 2.6 shows the $A_{6}$ profiles predicted by our two models. Although we do not have data on $A_{6}$, we note that the models make very different predictions for $\mathrm{it}^{5}$. Based on the other profiles, we would predict $A_{6}$ to be undetectable over the main body of this galaxy.

NGC4660 is a rapidly rotating elliptical: $v_{\max }=150 \mathrm{~km} / \mathrm{s}$ (Bender, 1988a). Bender linked this to the isophote shapes, showing that ellipticals with pointed isophotes are consistent with oblate rotators. In light of our modelling, another explanation seems viable: between $4^{\prime \prime}$ and $50^{\prime \prime}$ the disk in our model contributes more than $50 \%$ of the light on the major axis. Assuming that the disk is kinematically "colder" than the spheroid, it is likely that the rotation curve measurement is dominated by the disk rotation. The deprojected disk velocity (assuming $\cos (i)=0.4$ ) would be at least $200 \mathrm{~km} / \mathrm{s}$. The observed velocity dispersion of $150 \mathrm{~km} / \mathrm{s} \mathrm{im}-$ plies, assuming an isotropic velocity distribution, an expected rotation velocity of $\approx \sqrt{3} \sigma=250 \mathrm{~km} / \mathrm{s}$. Taking into account that in both the velocity measurement and the dispersion measurement a possible multi-component structure of the velocity distribution has been neglected, the two estimates for the rotation velocity appear consistent.

The simultaneous presence of these two, kinematically distinct, components is expected to cause characteristic, asymmetric absorption line profiles, as illustrated in the introductory Chapter. Detailed interpretation of such line profiles will require careful modelling both of the structure of the galaxy and of the observational procedures employed to obtain the data; we will illustrate such modelling in detail in Chapter 4.

[^5]
### 2.6 Summary and Conclusion

We have set out to investigate the photometric signatures of disk components in early type galaxies, concentrating on their isophote shapes and modelling them by an $R^{\frac{1}{4}}$-law spheroid of constant ellipticity and a flat exponential disk. For these simple photometric models we have found the following results:

- Deviations of the isophotes from perfect ellipses, as characterized by the parameter $A_{4}$, depend predominantly on the the spheroid-to-disk ratio and the galaxy's inclination. Dependences on the scale lengths of the luminosity profiles and on the intrinsic spheroid ellipticity are substantially weaker for the parameter range of interest.
- Similar $A_{4}$ profiles can be produced by a wide range of combinations of spheroid-to-disk ratio and inclination. In particular, there is little reason to assume that most observed "disky" elliptical galaxies have weak, nearly edge-on disks.
- The expected shape of the probability distribution of $A_{4}$ parameters is consistent with the observed distribution of Bender et al. (1988). Surveying a wide range of disk-to-spheroid ratios and inclinations we found that for many interesting parameter combinations the disks are practically undetectable. In particular we find that for $\cos (i)>0.6$, there is no detectable $A_{4}$ signature in the isophote shapes for any $\mathrm{D} / \mathrm{S}$. Deviations from a perfect $R^{\frac{1}{4} \text {-law are also }}$ difficult to detect for most galaxies with $\mathrm{D} / \mathrm{S}<0.25$.
- The detection probability for disks in current observational samples is less than $50 \%$. Applying this correction to $\mathrm{BDM}^{\prime}$ s sample, we find that, if all
the isophote distortions are indeed caused by disk components, most of the radio weak (or non-boxy) "ellipticals" have disks, many of which may well be substantial, and thus dynamically important. Statistical estimates of "typical" values for $\mathrm{D} / \mathrm{S}$ are of the order of $25 \%$ for radio weak galaxies. The presence of substantial disks might also explain Bender's (1988a) observation that almost all pointed ellipticals show strong rotation.
- We applied our modelling to NGC4660 and find that it has a disk-to-spheroid ratio of about $1: 2$ and an inclination of $\cos (i)=0.4$. A disk of this strength could dominate the measured rotation speed of this object over a wide range of radii, independent of the rotation of the spheroid. NGC4660 should "properly" be classified as an SO . Such classification issues seriously confuse the problem of estimating the frequency and strength of disks in early type galaxies, because visual classifications have been used to define the samples of "elliptical" and "disk" galaxies without a full understanding of their limitations. We will try to remedy this problem by obtaining photometry of a statistical sample consisting of both Es and S0s.

We stress that our analysis depends critically, at least in its quantitative aspects, on the assumption that most galaxies are well modelled as a superposition of an $R^{\frac{1}{4} \text {-law spheroid and an exponential disk. While there is no known a priori reason }}$ for this to be true, and there is considerable evidence that at least some bulges are triaxial (e.g. Gerhard et al., 1989), there is corroborating evidence that our interpretation may hold approximately in the majority of cases: We know that "S0s" with small values of D/S exist (e.g. NGC3115: $\mathrm{D} / \mathrm{S}=0.14$ in the $r$-band, Silva et al. 1990) and we know that such disks are only readily detectable for nearly edge-on viewing angles. As stressed by Capaccioli et al.(1990) and van den Bergh
(1990), analogous objects seen more face-on would reveal only much more subtle signs of a disk and would probably be termed "ellipticals". Furthermore our model implies a correlation between the apparent ellipticity of the galaxy and the measured value of $A_{4}$, in the sense that flatter objects should have a larger (positive) $A_{4}$. Such a correlation is indeed found for pointed ellipticals (Bender et al., 1989).

Analysis of the observed absorption line profiles, which should clearly reflect the two component nature of the velocity distribution, would appear to offer a powerful test for the presence of disks which is quite independent of the assumptions of our photometric models. For two galaxies from the BDM sample (NGC 5322 and NGC 3610) we will present such an analysis in Chapter 4.

The photometric modelling we have presented supports the view that many "ellipticals" are two component systems, with disks somewhat fainter than the canonical $D / S \approx 1$ found for classical S0s (Burstein 1979). Two somewhat different formation scenarios appear possible. The spheroid could have formed from the merger of two bona fide galaxies, with the disk condensing from residual gas either immediately or during a later episode of accretion. Alternatively, a substantial amount of subclumping during the formation of the galaxy could have converted most of the gas into stars before the final collapse (Katz,1989). Only relatively little gas would have been left to settle into a disk. Both these pictures can be tested by suitable simulations, once more information is available on the scale-lengths and surface densities of the disks.

## CHAPTER 3

## A DISK CENSUS OF EARLY TYPE GALAXIES

### 3.1 Introduction

In the previous chapter we argued that the frequency and strength of disk components, in particular weak disk components, in early type galaxies is best constrained by a statistical approach, assuming the members of the sample are seen from random viewing angles. However, if a sizeable fraction of galaxies have inconspicuous disk components, their Hubble classification (E or S0) will depend on the viewing angle. Therefore, no sample that was selected on a purely morphological basis will satisfy this requirement of random viewing angles. At present, all published samples of detailed CCD surface photometry (Lauer 1985a,b, Jedrzejewski 1987, Franx et al. 1989, Bender et al. 1988, Peletier et al. 1990) are afflicted with this problem. Other samples (e.g. Michard and Simien 1988, Capaccioli et al. 1990), that include both E's and S0's, were not selected on a well defined basis.

To improve on this situation we need to obtain surface photometry on a statistically well defined sample of early type galaxies, comprised both of E's and SOs. Suppose that early type galaxies were objects with smooth, symmetric light distributions, no spiral structure, negligible dust content and small current star formation rate. Then their classification could change from E to S 0 and vice versa, but not to any other Hubble type; thus, taken together, they should comprise a sample whose
selection did not depend on inclination. In reality, early type galaxies do contain small amounts of dust and gas, which may cause classification as Sa or such, if seen edge-on. Nevertheless, a magnitude limited sample of E's and S0's can certainly be expected to be much less sensitive to inclination selection effects than any sample of either subclass.

In addition to the advantages for studying weak disks, such a sample can also provide additional data on the general spheroid and disk properties of early type galaxies. Although several large CCD surface photometry studies of elliptical galaxies have been conducted over the last decade, not much new high quality surface photometry on S0s has become available since the study of Burstein (1979, hereafter B79). Only a few cases (e.g. NGC 3115, Capaccioli et al. 1987, Silva et al. 1990) have been studied in detail and a few disk-spheroid decompositions for S0s are included in the studies by Boroson et al. (1983) and Kent (1985, hereafter K85). For his sample of S 0 s , Burstein found the spheroids to be somewhat more luminous than their disks, and the disk light's characteristic radial scale to be somewhat larger than the spheroid's. This is in contrast to the properties of NGC3115, which is seen nearly edge-on: it has a disk comprising only $20 \%$ of the total light (Silva et al. 1990) and a disk scale $40 \%$ smaller than the spheroid's scale. Since at present it is not clear how representative each of these results is, a statistical study of disk properties would be useful to settle this question.

From the discussion in the previous chapter and from the available studies on S0 galaxies there appears to be tentative evidence for disk components in early type galaxies over a wide range in disk-to-spheroid ratios, $\mathrm{D} / \mathrm{S}$, and relative radial scales, $R_{\text {exp }} / R_{\text {eff }}$. Yet, as we discussed previously, changes in viewing angle can mimic closely a vast change in intrinsic disk properties (see Figure 1.1). Consequently, when it comes to quantifying the relative frequency and strengths of these disk
components, the present state is confusing at best.
A further point which needs clarification is the relative age of the two subcomponents in early type disk galaxies. While it is well established for later type spiral galaxies that the mean age of their disks is substantially less than that of their spheroids ${ }^{1}$, the situation for earlier types is unclear at present: there is no significant, present epoch star formation in S0 disks and there is no evidence for young stars ( $<10^{9}$ years) (Kennicutt 1991, in prep.). Recently, Gregg (1989) and Bothun and Gregg (1990) have claimed evidence that the disks in S0's are bluer and thus presumably younger. Since in many S 0 's, however, the light is not dominated by the disk at any radius, the measurement of disk colors is difficult.

To clarify and answer the various points raised in the previous paragraphs, we will present and analyze surface photometry of a statistical sample of early type galaxies. Specifically, we want to address the following questions: What fraction of all early type galaxies are truly spheroidal or ellipsoidal, i.e. are disk-less or have negligible disks ${ }^{2}$. What fraction of all objects has disks, which can only be seen from favorable vantage points? How many of those disks will elude detection in any given sample and thus constitute seemingly spheroidal galaxies with disks? For the cases where a $\mathrm{D} / \mathrm{S}$ decomposition is possible, what are the ranges and distributions of $R_{\text {exp }} / R_{\text {eff }}, D / S$ and of the central disk surface brightness $I_{0}$ ? Finally, we will employ two-color photometry to test recent claims that S 0 disks are substantially younger than their spheroids.

The remainder of this chapter is organized as follows: in Section 2 we characterize the sample. Section 3 describes the observations and Section 4 discusses the data reduction. The data analysis and photometric modelling are discussed in Section 5.

[^6]In Section 6, finally, we use this information to address the questions just raised.

### 3.2 The Sample

To assemble a sample whose selection is as far as possible inclination independent, we selected its members from the Uppsala Galaxy Catalog (UGC, Nielson 1973), revised and complemented by D. Burstein (unpublished), according to the following criteria:

1. The objects must have Zwicky magnitudes between 12.5 and 13.3.
2. They must be classified as $\mathrm{E}, \mathrm{E} / \mathrm{S} 0, \mathrm{E}-\mathrm{S} 0$ or S 0 .
3. The Galactic absorption (in the B-band) along their line of sight (Burstein and Heiles, 1984) must be less than 0.2 mag .
4. A lower limit on the "optical diameter" of 1 ' is implicitly set by the UGC selection criteria (Nielson 1973); the small fields of the available detectors forced us to impose an upper limit on the optical diameter of $4^{\prime}$. This size cut-off also disposed of a number of diffuse, nearby dwarf ellipticals.

The magnitude range was chosen to yield a sample size of nearly 100 , which is required for robust viewing angle statistics. We decided against the largest and brightest early type galaxies ( $<12.5 \mathrm{mag}$ ) for two reasons: first, surface photometry in some form exists for many of these brightest objects, although not in a homogeneous fashion for both Es and S0s. To avoid duplication of existing information we chose the set of the next brightest objects on whose isophote shapes no information is available. Second, the "optical" diameters (Nielson 1973) of our objects are typically $1.5^{\prime}$ to $2.5^{\prime}$. Therefore a larger portion of each object can be fit within the limited fields of views ( $\approx 2^{\prime} \times 2^{\prime}$ ), set by the array detectors.

These criteria are satisfied by 93 objects, 82 of which were actually observed in at least one wavelength band (Table 1). Of the observed galaxies 41 were classified as ellipticals, 33 as S 0 's and 10 as intermediate types (E/SO, E-S0). Finally, it is worth noting that this sample, similar to other samples of bright early type galaxies, is skewed toward members of the Virgo and Coma clusters.

### 3.3 Observations

Although the gas and dust content of early type galaxies is usually low (e.g. Knapp 1990), it is possible that dust extinction can mimic subtle features in the underlying light distribution (Möllenhof and Bender 1987). To avoid misinterpretation of such features as bars, disks, boxiness, etc., we decided to obtain photometry in two color bands spanning a wide range in wavelength. Specifically, we observed the objects in the Mould B Band and the Johnson H-band. For standard extinction curves (Rieke and Lebofsky 1985) the dust absorption, $A_{H}$, at $1.6 \mu$ is expected to be a factor of 7.5 lower than $A_{B}$ in the blue. Thus, the impact of dust absorption can be neglected at $H$, unless there is a very high optical depth at $B$.

Observations in both bands were gathered for 49 objects, while 18 galaxies each were observed only in either $B$ or $H$. In the following we shall describe the instrumental set-ups, the data acquisition procedures and the basic data reduction steps separately for each color band.

The B images were obtained in two observing runs (April 1990 and September 1990) at the Steward 2.3 m telescope in direct imaging mode. The detector was a TI CCD, binned to $400 \times 400$ pixels, resulting in a pixel scale of $0.3^{\prime \prime}$ and a field of view of 120 " on a side. A combination of medianized sky images and dome flats was used to correct for the differing sensitivities of the individual pixels, resulting in flat fielding errors of $<0.5 \%$ on all scales. The exposure times ranged from 5 to 15

Table 3.1: Sample Members and Log of Observations

| UGC | NGC | Cl. | $m_{B}$ | $V_{\text {rad }}$ | Date(B) | Date(H) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 80 | 16 | E | ??.? | 3041 | 9/16/90 | 10/7/90 |
| 292 | 128 | S0 | 13.2 | 4227 | 9/16/90 | 10/7/90 |
| 597 | 315 | ?? | ??.? | 5048 | 9/16/90 | - |
| 735 | 410 | E | 12.6 | 5294 | 9/16/90 | - |
| 801 | 448 | S0 | 13.2 | 1917 | 9/16/90 | - |
| 848 | 467 | S0 | 13.3 | 5467 | 9/16/90 | - |
| 859 | 473 | S0 | 13.2 | 2137 | - | 10/6/90 |
| 926 | 499 | S0 | 13.0 | 4375 | - | 10/6/90 |
| 938 | 507 | E | 13.0 | 4915 | - | 10/6/90 |
| 992 | 533 | E | 13.1 | 5544 | 9/16/90 | - |
| 995 | 529 | E-S0 | 13.1 | 4862 | 9/16/90 | 10/6/90 |
| 1250 | 670 | S0 | 13.1 | 3692 | - | 10/6/90 |
| 1283 | 679 | E-S0 | 13.1 | 5045 | 9/16/90 | - |
| 1475 | - | ?? | ??.? | 4209 | - | 10/7/90 |
| 1476 | 777 | E | 12.7 | 5040 | 9/16/90 | 10/7/90 |
| 1631 | 821 | E | 12.7 | 1716 | - | 10/7/90 |
| 2128 | 1016 | E | 13.3 | 6585 | - | 10/7/90 |
| 3063 | 1587 | E | 13.3 | - | - | 10/7/90 |
| 3153 | 1653 | E | 12.9 | - | - | 10/7/90 |
| 4551 | - | S0? | 13.1 | 1745 | - | 4/10/90 |
| 4674 | 2693 | E | 13.1 | 4865 | 4/26/90 | 4/15/90 |
| 4763 | 2749 | E | 13.3 | 4180 | 4/25/90 | - |
| 4791 | 2765 | S0 | 13.3 | 4064 | 4/26/90 | 4/11/90 |
| 4840 | 2778 | E | 13.1 | 2016 | 4/25/90 | 4/10/90 |
| 5018 | 2872 | E | 13.0 | 3226 | 4/25/90 | - |
| 5292 | 3032 | S0 | 13.0 | 1561 | 4/27/90 | 4/11/90 |
| 5350 | 3070 | E | 13.2 | 5391 | 4/26/90 | 4/11/90 |
| 5503 | 3156 | So | 12.8 | 1296 | 4/27/90 | 4/11/90 |
| 5617 | 3226 | E | 13.3 | 1275 | 4/25/90 | - |
| 6037 | 3458 | S0 | 13.2 | 1898 | 4/27/90 | 4/10/90 |
| 6281 | 3599 | E/S0 | 13.0 | 850 | 4/26/90 | 4/14/90 |
| 6295 | 3605 | E-S0 | 12.7 | 1991 | 4/25/90 | 4/10/90 |


| UGC | NGC | Cl. | $m_{B}$ | $V_{\text {rad }}$ | Date(B) | Date(H) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6409 | 3658 | E-S0 | 13.3 | 2044 | 4/25/90 | 4/13/90 |
| 6444 | 3674 | S0 | 13.1 | 1885 | 4/27/90 | 4/11/90 |
| 6605 | 3773 | S0 | 13.1 | 973 | - | 4/12/90 |
| 6704 | 3842 | E | 13.3 | 6237 | 4/26/90 | 4/15/90 |
| 6738 | 3872 | E | 12.9 | 3210 | 4/26/90 | 4/15/90 |
| 6779 | 3894 | S0 | 12.9 | 3275 | 4/27/90 | 4/12/90 |
| 6953 | 4008 | E-S0 | 13.1 | 3680 | 4/26/90 | 4/12/90 |
| 7117 | 4124 | S0 | 12.7 | 1674 | - | 4/13/90 |
| 7165 | 4150 | S0 | 12.6 | 244 | 4/27/90 | 4/12/90 |
| 7202 | 4169 | S0 | 12.9 | 3784 | 4/27/90 | 4/13/90 |
| 7203 | 4168 | E | 12.7 | 2307 | 4/26/90 | 4/15/90 |
| 7214 | 4179 | S0 | 12.8 | 1228 | - | 4/15/90 |
| 7311 | 4233 | S0 | 13.2 | 2371 | 4/26/90 | 4/13/90 |
| 7376 | 4270 | S0 | 13.3 | 2347 | 4/27/90 | 4/10/90 |
| 7390 | 4283 | E | 13.1 | 1078 | 4/25/90 | - |
| 7461 | 4339 | E | 13.1 | 1298 | 4/25/90 | - |
| 7502 | 4379 | S0 | 12.6 | 1071 | - | 4/14/90 |
| 7517 | 4387 | E | 13.2 | 584 | 4/26/90 | 4/12/90 |
| 7571 | 4434 | E | 13.2 | 1068 | 4/25/90 | - |
| 7610 | 4458 | E | 13.3 | 684 | 4/25/90 | - |
| 7634 | 4474 | S0 | 12.6 | 1624 | 4/27/90 | 4/11/90 |
| 7637 | 4476 | S0 | 13.3 | 1955 | 4/27/90 | 4/13/90 |
| 7655 | 4489 | E | 13.2 | 960 | 4/26/90 | 4/15/90 |
| 7701 | 4515 | E-S0 | 13.3 | 940 | 4/25/90 | 4/14/90 |
| 7722 | 4528 | S0 | 12.9 | 1347 | - | 4/12/90 |
| 7759 | 4551 | E | 13.1 | 1198 | 4/26/90 | 4/15/90 |
| 7793 | 4578 | S0 | 12.9 | - | - | 4/15/90 |
| 7850 | 4612 | S0 | 12.9 | 1884 | - | 4/14/90 |
| 7860 | 4627 | E ? | 13.3 | 828 | 4/25/90 | - |
| 8028 | 4789 | E-S0 | 13.3 | 8372 | 4/26/90 | 4/12/90 |


| UGC | NGC | Cl. | $m_{B}$ | $V_{\text {rad }}$ | Date(B) | Date(H) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8866 M | 5389 | S0 | 13.2 | 1832 | - | $4 / 10 / 90$ |
| 8974 | 5444 | E | 12.8 | 3994 | $4 / 25 / 90$ | $4 / 10 / 90$ |
| 9137 | 5532 | S0 | 13.3 | 7410 | $4 / 27 / 90$ | $4 / 14 / 90$ |
| 9188 | 5582 | E | 13.0 | 1435 | $4 / 26 / 90$ | $4 / 11 / 90$ |
| 9395 | 5687 | SO | 13.3 | - | $4 / 27 / 90$ | $4 / 11 / 90$ |
|  |  |  |  |  |  |  |
| 9642 | 5820 | $\mathrm{E}-\mathrm{S} 0$ | 13.0 | 3235 | $4 / 25 / 90$ | $4 / 14 / 90$ |
| 9678 | 5831 | E | 13.1 | 1683 | $4 / 27 / 90$ | $4 / 12 / 90$ |
| 9726 | 5854 | SO | 13.1 | 1669 | $4 / 27 / 90$ | $4 / 13 / 90$ |
| 9851 | 5929 | $\mathrm{E}-\mathrm{S} 0$ | 13.0 | 2550 | $4 / 26 / 90$ | $4 / 14 / 90$ |
| 9903 | 5953 | SO | 13.3 | 1983 | $4 / 26 / 90$ | $4 / 13 / 90$ |
|  |  |  |  |  |  |  |
| 10345 | 6127 | E | 13.0 | 4609 | $4 / 25 / 90$ | $4 / 12 / 90$ |
|  |  |  |  |  | $9 / 16 / 90$ | - |
| 10916 | 6411 | E | 13.2 | 3690 | $4 / 25 / 90$ | $4 / 14 / 90$ |
|  |  |  |  |  | $9 / 16 / 90$ |  |
| 12523 | 7619 | E | 12.7 | 3758 | $9 / 16 / 90$ | - |
| 12528 | $? ? ? ?$ | $? ?$ | $? ? . ?$ | - | $9 / 16 / 90$ | - |
| 12531 | 7626 | E | 12.8 | 3416 | $9 / 16 / 90$ | $10 / 6 / 90$ |
| 12760 | 7742 | $? ?$ | $? ? . ?$ | 1655 | $9 / 16 / 90$ | - |
| 12841 | 7785 | E | 13.0 | 3824 | $9 / 16 / 90$ | $10 / 6 / 90$ |
|  |  |  |  |  |  |  |
| 8110 | 4889 | E | 13.3 | 6497 | $4 / 25 / 90$ | $4 / 11 / 90$ |
| 8125 | 4914 | E | 12.7 | 4663 | $4 / 25 / 90$ | $4 / 10 / 90$ |
| 8423 | 5129 | E | 13.3 | 6908 | $4 / 26 / 90$ | $4 / 13 / 90$ |
| 8499 | 5198 | E | 13.2 | 2569 | $4 / 26 / 90$ | $4 / 12 / 90$ |
| 8675 | 5273 | S 0 | 12.7 | 1098 | $4 / 27 / 90$ | $4 / 14 / 90$ |

This table lists the observed sample members and gives the UGC and NGC catalog numbers, Hubble type classification and estimated total blue magnitudes from Nielson (1973). The radial velocities were taken from the CfA redshift catalog (Huchra et al. 1990 and references therein). The last two columns list the date of the observations in the two color bands. All B-band data as well as the H -band data from October 1990 were taken at the Steward $90^{\prime \prime}$ telescope on Kitt Peak. The H -band data from April were taken at the Steward 61" telescope on Mount Bigelow.
minutes in seeing from 1.2" to 1.9". Landolt (1973), and Christian et al (1985) flux calibration standards were observed throughout the night. Unfortunately, slight cirrus during most of the observations prevented a photometric calibration of the data. From the scatter of the standards, we estimate the uncertainty in the absolute magnitude scale to be $\lesssim 0.25$ mag. Note, however, that the relative photometry, i.e. the shape of all photometric profiles, is unaffected.

For infrared surface photometry, the H -band provides the best trade-off between galaxy brightness and atmospheric background signal. The $1.6 \mu$ images were obtained on the Steward 1.5m (April 1990) and the Steward 2.3m (October 1990) telescopes. The detector was a NICMOS $128 \times 128 \mathrm{HgCdTe}$ array. Its quantum efficiency in the H band is about $55 \%$; the dark current is about $1.5 e^{-} / s$ (Rieke $e t$ al. 1989). Since at $1.6 \mu$ the sky emission dominates the galaxy signal at virtually all radii except the core and the sky brightness varies on short time scales (<1min), the IR observing technique is drastically different from the optical CCD work. Every $30-60$ seconds the telescope is "wobbled" back and forth between the objects and an adjacent piece of "blank" sky. The whole observation of a galaxy then results in a stack of alternating object and background frames. Typical total exposure times for each galaxy were 20 minutes, with an equal amount of time spent on the sky; thus 40 minutes were required per object. The basic data reduction steps are as follows: the average of adjacent sky images was subtracted from each object frame and all sky frames were renormalized and medianized to map the detector response to a uniform illumination. Subsequently, each object image was divided by this flat-field and shifted to a common reference position. This procedure results in flat-fielding to $\lesssim 5 \cdot 10^{-4}$. Finally, all object frames were averaged to yield the final image.

### 3.4 Data Reduction

### 3.4.1 Ellipse Fitting

To analyze the projected luminosity distributions of the objects in our sample, the two dimensional images were collapsed into a series of photometric profiles with the ellipse fitting procedure (Franx et al. 1989) described in Chapter 2. Here, we will only give a brief description of the additional procedures required when analyzing "real" data. Before fitting the ellipses to the image, all regions containing chip defects, cosmic rays, the images of stars or other background and foreground objects were masked. This procedure excluded in most cases only a small fraction of the total pixels ${ }^{3}$, leaving most of the data available for the fit. By means of Monte-Carlo simulations we checked that the masking was efficient in removing all objects that could affect the photometric profiles. In particular we found that the $A_{4}$ profile was hardly ever perturbed by more than $0.2 \%$ by (stellar) images missed in the masking routine.

For the intensity profiles, the ellipse fitting routine returns the quantity $I_{t}(r)$, which represents the mean count rate, or the $O^{t h}$ order Fourier term, within an elliptical annulus. Here, $r$ stands for the mean radius of the ellipse, defined as $\sqrt{a b}$. This is the quantity most commonly used to describe the radial profiles (e.g. Jedrzejewski 1987, Bender et al. 1988, Franx et al. 1989, Peletier et al. 1990). Some authors (e.g. Lauer 1935, Djorgovski 1985, Kent 1985) use this same quantity as a function of the semi-major axis, $I_{t}(a)$, as the "major axis profile". Note that, for objects with isophote twists and non-elliptical isophotes, this is not equivalent to the straight major axis cuts used e.g. by B79. However, for most objects these differences are negligible. To obtain the luminosity profile of any galaxy we must convert the directly measurable quantity $I_{t}(r)$, to the galaxy's surface brightness

[^7]$\mu(r)$; the two quantities are related by
\[

$$
\begin{equation*}
\mu(r)=\mu_{0}-2.5 \log \left[I_{t}(r)-I_{s k y}\right], \tag{3.1}
\end{equation*}
$$

\]

where $\mu_{0}$ represents the zero point of the magnitude scale, as determined from the standard stars, and $I_{s k y}$ is the rate of background counts. For observations where the object fills, or overfills, the data frame this conversion is difficult, because $I_{\text {sky }}$ cannot be measured directly ${ }^{4}$.

With our instrumental setup ( $2^{\prime} \times 2^{\prime}$ field of view) most objects contribute in excess of $1 \%$ to the total counts at the edge of the frame. To estimate the true sky level we must fit it simultaneously with a model of the luminosity profile such as an $R^{1 / 4}$ law. Therefore our sky level estimate, and consequently our luminosity profile (Eq. 3.1), $\mu(r)$, is model dependent. To explore the sensitivity of the sky level estimate on the particular luminosity model chosen, we fitted each profile, $I_{t}(r)$, with an $R^{1 / 4}$ law and with a composite disk-spheroid model. Typically, the change in sky level resulted in an 0.1 magnitude uncertainty of the profile at $30^{\prime \prime}$. We will discuss this model fitting and the resulting sky estimates in more detail below.

Appendix A shows the resulting photometric profiles for all our objects in the B-band and Appendix B shows the data for the H-band. The top panel of each column shows the major axis profile, $\mu(a)$, of each object, as reconstructed from the ellipse fitting. The two panels below show the run of the ellipticity, $\epsilon$, and the major axis position angle, P.A., for the best fitting ellipses as a function of the major axis coordinate, $a$. The bottom panel shows the $A_{4}$ Fourier component of deviations of the isophotes from perfect ellipses. The error bars in the geometric profiles (P.A., $\epsilon$ and $A_{4}$ ) represent the formal statistical errors from the fitting routine. The errors in the intensity profile include estimates of the just mentioned uncertainties that

[^8]arise from our ignorance of the true sky level. We have included the position angle profile, even though we will not use it in this present context, because it is useful for kinematic studies through long-slit spectroscopy.

### 3.4.2 External Comparisons

There are several ways to check the accuracy and reproducibility of these photometric profiles: comparison with published surface photometry, internal comparison of different observations of the same object within the same color band and a comparison between the H -Band and the B -Band data.

Since rather little published B or H band photometry exists for the objects in our sample, only limited external comparisons can be made: we have about 35 objects in common with Djorgovski (1985), eight objects with K85 and three objects with B79.

Comparison with Djorgovski (1985)

For the objects in common with Djorgovski's (1985) R-band data, we compared the isophote geometry (the ellipticity $\epsilon$ and the position angle, P.A.) at a fiducial radius ( $10^{\prime \prime}$ ), assuming that the ellipticity and the position angle of the isophotes are independent of color. We found a median agreement of 0.03 in the ellipticity and $3.5^{\circ}$ in the position angle. A detailed comparison of the luminosity profiles did not appear sensible since several authors (e.g. Djorgovski 1985, Franx et al. 1989, Peletier et al. 1990) noted strong disagreement between Djorgovski's $\mu(r)$ and other, internally consistent, sets of data; these discrepancies appear to be caused in part by errors in the sky level setting.

## Comparison with Kent (1985)

We can estimate the accuracy of our luminosity profiles in comparison with the $r$-band data from K85. Kent's images are much larger ( $5^{\prime} \times 5^{\prime}$ ) than ours and yield a model independent estimate of $I_{s k y}$. The expected color gradients in B-r are expected to be 0.05 or less per radial decade (e.g. Franx et al. 1989), and are thus to lowest order negligible. Figure 3.1 shows the comparison of the profiles, together with the luminosity models fitted to the profiles which will be discussed in Section 3.7. With the exception of UGC3872 the profiles agree - after a color offset - to about 0.1 magnitudes for $r<35^{\prime \prime}$, usually covering a range of 5 magnitudes. Beyond $35^{\prime \prime}$ the errors are dominated by the uncertainties in the sky level. If we chose an appropriate model for the luminosity profile, the agreement is quite good at large radii (U80, U2778, U3894, U4169 and U7626). If our assumed model was wrong, as apparently is the case for U7785, the disagreement at large radii can be vast. The error bars in the bottom panel for each object indicate the typical errors expected to arise from our ignorance of the sky level; they represent the sample mean of the difference between the inferred $\mu(r)$, assuming an $R^{1 / 4}$ law and a composite S/D model for the sky level determination. For the majority of objects this error estimate appears reasonable. For UGC 3872 the discrepancies are larger than could be explained by errors in the sky; however, a misidentification of this object appears unlikely since the P.A. measurements agree within a few degrees.

## Comparison with Burstein (1979)

Our data can be most directly compared with the photographic B-band data from B79. Figure 3.2 illustrates the comparison, with the same notation as Figure 3.1. Since Burstein takes a straight, one pixel wide, luminosity cut along the major axis, instead of fitting ellipses, we reconstructed the major axis profile from our photo-


Figure 3.1: Comparison with Kent's (1985) R-Band Photometry This figure compares the eight objects in common with Kent (1985). In the top panel for each object the open circles represent K85's measurements and the solid squares show our data. A color off-set was added to the data to illustrate the extent of the agreement. The solid lines indicate K85's models for the luminosity profiles in comparison with ours (dash-dotted lines); the models are offset by a magnitude with respect to the data. The bottom panels show the difference between the two data sets, without magnitude off-set. In most cases the agreement is good to $\approx 0.1 \mathrm{mag}$ in the range from 3 to $30^{\prime \prime}$. (Contd.)


Comparison with K85 (Contd.)
The error bars in the small panels indicate the uncertainties in the profile arising from the model dependent setting of the sky level (see text). UGC 3872 may have a large disk which becomes apparent only at radii $>40^{\prime \prime}$ and thus remains undetected by our data. UGC3894 is classified as an elliptical and thus K85 only fits a simple $R^{1 / 4}$ law; however, the fit can be significantly improved by an S/D model. The decompositions for the two objects with prominent disks (UGC 80 and UGC 2778) are quite similar.
metric profiles including the Fourier residuals up to $6^{\text {th }}$ order. The reconstruction appears to produce consistency with B79, except for UGC7634: our major axis profile of this edge-on S0 changes less abruptly than B79's at $10^{\prime \prime}$, possibly due to the effective smoothing in the profile reconstruction.

### 3.4.3 Internal Comparison of the Optical and IR Data

Before comparing the B and H band data, we demonstrate the reproducability of the photometry by a comparison of B data on the same objects obtained during different observing runs. Figure 3.3 compares the photometric profiles for UGC 10916, obtained in the Spring and Fall of 1990, respectively. Aside from a 0.25 mag offset in the brightness calibration due to non-photometric observing conditions, the agreement is excellent. Note, however, that the systematic errors in the two profiles, arising e.g. from a faulty sky level setting or unremoved background sources are expected to be the same in both data sets.

There is no external comparison possible for the H-Band data, since the only published IR surface photometry (Peletier 1989) was obtained at J and K. However, we can check the quality of the IR data through a comparison with the optical data, for the objects which were observed at both wavelengths. A look at Appendices A and B reveals immediately that the B -band observations are generally of much higher signal-to-noise; thus at least the geometrical comparisons can serve as a check on the accuracy of the H-band data. The agreement between the two bands is reasonable for most cases; a "typical" example is shown shown in Figure 3.4. The presence of dust can lead to gross discrepancies between the optical and IR photometric profiles. These differences are almost exclusively due to extinction of the blue image. The most drastic example for the presence of dust in the sample is UGC7637, shown in Figure 3.5.


Figure 3.2: Comparison with Burstein's (1979) B-Band Photometry This figure compares the major axis profiles and the $S / D$ decompositions for the three objects in common between our sample and B79. The symbols are the same as in the previous figure; again a small offset was added to the profiles. The bottom panels show the difference between the two data sets, without magnitude off-set. In most cases the agreement is good to $\approx 0.1 \mathrm{mag}$ in the range from 3 to $30^{\prime \prime}$. The discrepancies in UGC7634 at $15^{\prime \prime}$, may arise from differences in the way the major axis profile was determined: B79 used a straight cut, only one resolution element wide, while we reconstructed $I(a)$ from the photometric profiles, resulting in some smoothing. No model comparison was possible for UGC 7634, because B79 did not fit a two component model to his data.


Figure 3.3: Reproducability of the B-band Photometry Comparison of the surface photometry for UGC 10916 between data obtained during two independent observing runs. The upper left panel shows the "color" profile, i.e. the magnitude difference between the two observations: aside from a 0.25 mag offset in the magnitude calibration, due to non-photometric conditions, the rms agreement is 0.02 mag between $2^{\prime \prime}$ and $40^{\prime \prime}$. The discrepancies in the ellipticity and the position angle inside $2^{\prime \prime}$ are due to differences in the seeing and slight guiding errors. The discrepancies in the ellipticity profile outside of $50^{\prime \prime}$ are caused by convergence problems of the ellipse fitting algorithm. Note that this is the radius beyond which data cannot be sampled along the entire ellipse, because of the limited data frame size. The $A_{4}$ residuals are reproduced within $0.2 \% \mathrm{rms}$.


Figure 3.4: Comparison of B and H Band Photometry for UGC 10916 The color gradient between $3^{\prime \prime}$ and $35^{\prime \prime}$ reflects the radial population change in the galaxy. The sharp turn in the color gradient beyond $35^{\prime \prime}$ is due to differing estimates of the sky level in the two bands. In the other three panels the triangles represent the optical data and the crosses represent the IR data. Between $3^{\prime \prime}$ and $40^{\prime \prime}$ there is good agreement in ellipticity and position angle. The deviations in the H-band data inside $3^{\prime \prime}$ are caused by coarse sampling ( $0.9^{\prime \prime} / p i x$ ) and by poor guiding. The comparison of the Fourier residuals shows that, even between $3^{\prime \prime}$ and $30^{\prime \prime}, A_{4}$ can only be determined to an accuracy of $0.5 \%$ to $1 \%$.


Figure 3.5: Photometric Evidence for Dust
This object contains a dust lane at a radius of about $10^{\prime \prime}$. The H-band image (crosses) appears unaffected by the dust: the ellipticity and P.A. profiles are smooth. However, the dust has imprinted itself clearly onto the optical profiles as well as onto the $\mathrm{B}-\mathrm{H}$ color profile. Note that the $A_{4}$ amplitude in the blue is twice as large as in the IR, reflecting the dust distribution rather than the projected shape of the density distribution. Yet there is an $A_{4}$ signature at H , indicating the presence of an underlying small disk, which is also reflected in the luminosity profile (see text).


Figure 3.6: Normalized B-H Color Profile
The four panels show the B-H color gradient for the 44 galaxies for which both acceptable optical and IR data were obtained. The color is defined as the magnitude difference between the radial profiles after model fitting and sky level fitting (see Section 3.5) Only the radial range from $3^{\prime \prime}$ to $20^{\prime \prime}$ is shown, because coarse spatial sampling and vast sky background make the H-Band data unreliable outside this interval. Since the observations in neither band were obtained under photometric conditions, all color profles were normalized to their mean value of $\mathrm{B}-\mathrm{H}=3.5$ at $3^{\prime \prime}$. Each object is offset from its neighbors by 0.25 mag in color, each small tick mark represents a change $\Delta \mathrm{B}-\mathrm{H}=0.1 \mathrm{mag}$ (not the true $\mathrm{B}-\mathrm{H}$ color). For most galaxies the color profiles are smooth and exhibit outward blueing. The few irregular color profiles (e.g. U7202, U7637, U9903) are caused by dust that affects the B band image.


Figure 3.7: B-H Comparison of the Ellipticity Profiles
The difference between the ellipticity profiles derived at B and at H between $3^{\prime \prime}$ and $20^{\prime \prime}$ is shown for all objects with photometry in two colors. The solid line represents $\epsilon_{B}-\epsilon_{H}=0$; each tickmark represents an ellipticity difference of 0.1. The agreement is typically within $\Delta \epsilon \approx 0.03$. There are no gross systematic differences between the profiles derived at the two wavelengths. There are a few objects that show humps in $\epsilon_{B}-\epsilon_{H}$, such as U4840, U6779, U7165 and U8675. As we will show in Section 3.5 all these objects have strong disk components. If these inclined disks are bluer than the spheroids, then they will cause the B band image to flatten more strongly than the IR image.


Figure 3.8: B-H Comparison of the Isophote Shapes
The four panels compare the $A_{4}$ residuals derived from the optical and IR data. Each tickmark corresponds to a difference of $2 \%$ in the isophote shapes. For most objects there is reasonable agreement ( $<0.5 \%$ ); note in particular U292 and U7634, which have $A_{4}(\max )>10 \%$. Strong disagreement ( $>2 \%$ ), as in U7202, U7637, U9642 and U9903, can be attributed to dust, because the IR residual profile is smoother than that in the visible.

Figures 3.6 through 3.8 present a systematic comparison of the photometry at the two colors. We have limited our comparison to the radial range between $3.5^{\prime \prime}$ and $20^{\prime \prime}$ because the IR photometry becomes increasingly unreliable outside this range: at small radii this is due to seeing ( $1^{\prime \prime}-2^{\prime \prime}$ ), coarse sampling ( $\left.0.9^{\prime \prime} / p i x\right)$ and poor guiding at the $61^{\prime \prime}$ telescope; at large radii it is due to the dominant contribution of the sky emission to the total signal. Note, however, that the uncertainties in $\mu(r)$, arising from the ignorance of the sky level, are comparable in H and in B , since they mostly depend on the relative apparent size of the galaxy and the size of the detector array.

In the first of these figures we present the color profiles, normalized ${ }^{5}$ to $\mathrm{B}-\mathrm{H}=$ 3.5 at $3.5^{\prime \prime}$. Most objects exhibit a smooth outward decrease in B-H; this color gradient is intrinsic to the stellar population in the galaxies and will be discussed in Section 3.7 (There we will also discuss the errors in these gradients.) The few irregular profiles can be attributed to dust. Figure 3.7 compares the ellipticities derived from the B and H data. Typical mean differences $\left\langle\epsilon_{B}-\epsilon_{H}\right\rangle$ are only 0.02 , with a comparable dispersion. This agreement is somewhat better than the one found by Peletier (1989), for a B-K comparison, presumably because of the increased IR detector quality. Finally, Figure 3.8 shows a comparison of the $A_{4}$ residuals at B and H . The profiles generally agree within $0.5 \%-1.0 \%$, as expected from the formal errors at H. Again there is a number of discrepant objects. In these cases the IR residual profile is smoother than the optical, again pointing to dust extinction as the prime source of the disagreement.

There are no gross systematic discrepancies between the $B$ and $H$ data, supporting the reliability of the IR surface photometry. Nevertheless, in the absence of dust, the optical data provide more accurate information on the luminosity pro-

[^9]file and the isophote shapes. Therefore the IR data are most useful in indicating the presence of an obscuring medium, which resulted in "spurious" features in the photometric profiles at B. Furthermore the IR data allow us the assessment of color gradients, which we will discuss below.

### 3.5 Data Analysis

We now proceed to characterize and interpret this host of photometric data (Appendices $A$ and $B$ ), in terms of simple photometric models, either spheroid/disk (S/D) models or pure spheroid models. We divide the process into several steps:

1. We fit an $R^{1 / 4}$ law to the major axis profile to see whether there is any need to invoke an S/D model. If a disk is present, it will imprint itself most strongly onto the luminosity profile along the major axis.
2. If the $R^{1 / 4}$ law is a "poor" fit to the major axis profile, we fit an S/D model to the data along the two principal axes.
3. We reject or accept the S/D fit, depending on the formally derived disk properties.
4. Once an S/D fit is accepted, we have a formal value for the inclination and we can rotate the model to a face-on orientation. Then we can calculate how much the radial profile would have deviated from an $R^{1 / 4}$ law had we seen the galaxy face-on. This allows us to estimate whether we could have detected its disk component from that angle.
5. We tabulate the resulting photometric parameters and superimpose the fit and the data in the appendices.

### 3.5.1 Fitting Photometric Models

## a) Choice of Method

In Chapter 1 we mentioned that a wide variety of S/D decomposition methods is used throughout the literature. The main ones are:

1. Fitting an azimuthally averaged, one-dimensional profile (e.g. Kodaira et al., Schombert and Bothun 1987)
2. Fitting the major axis profile in conjunction with an inclination estimate (B79, Bothun and Gregg 1990)
3. Fitting major and minor axis profiles simultaneously (e.g. K85)
4. Fitting the two dimensional image (Boroson et al. 1983, Silva et al. 1990)

Even though there is no optimal fitting method (see Section 1.3.1), some methods clearly have disadvantages that others do not have, in particular when the sky level must be considered a free parameter. These differences can be seen most clearly when considering the recovery of the photometric scale parameters for idealized models ${ }^{6}$ : 1) Fitting only the angle-averaged luminosity profile, will not recover correctly the S/D characteristics, except if the system is seen face-on. This is simply because in a 1-D profile the signature of an inclined disk is virtually indistinguishable from a smaller face-on disk of higher surface brightness, unless additional information is used. 2) The second method improves on that situation, by including an inclination estimate. However, an estimate of the disk inclination from the maximum ellipticity of the total light distribution will be in error unless the disk dominates at some radius. 3) The third method is capable of recovering the input

[^10]parameters correctly, even if the sky level must be fit, because the inclination is well determined through the difference between major and minor axis profiles. 4) Using the full 2-D data can clearly recover the correct parameters for idealized models. It has been implemented in various ways: Boroson et al. (1983) fitted simultaneously a number of one dimensional radial cuts at different position angles. This amounts to fitting the image, rebinned to a cylindrical grid. Silva et al. (1990) fitted a a 2-D model on a Cartesian grid to the image of NGC 3115, which requires much more computing time.

For the present data set, we employed simultaneous fitting of the major and minor axis profiles. For idealized models this procedure is equivalent to the twodimensional fitting. In realistic situations, the dominant uncertainties in fitting a model arise from our ignorance of the sky level. This uncertainty is equal for the two methods, since it depends only on the relative size of the galaxy compared to the data frame. Furthermore, for most of our objects the isophotes can be described as ellipses to within a few percent, less than typical discrepancies between the image and the models described below. In that case only two independent functions of the radius, $I_{t}(r)$ and $\epsilon(r)$, are necessary to describe the image ${ }^{7}$. Thus the two principal axis profiles contain virtually all the available information.

## b) Executing the Fit

Each of our photometric models is characterized by seven parameters, the two length scales, $R_{\text {eff }}$ and $R_{\text {exp }}$, the two intensity scales, $I_{\text {eff }}$ and $I_{0}$, the spheroid ellipticity, $\epsilon$, the inclination of the system $\cos (i)$ and the level of sky counts, $I_{s k y}$. Each model is fitted in a least squares sense to the count profile along the two principal axes.

[^11]In this model the galaxy brightness at major axis points, $a_{j}$, is given by

$$
\begin{equation*}
I_{\text {major }}\left(a_{j}\right)=I_{0} / \cos (i) \cdot \exp \left(-a_{j} / r_{e x p}\right)+e^{\gamma} I_{e f f} \cdot \exp \left[-\gamma\left(a_{j} / r_{e f f}\right)^{1 / 4}\right], \tag{3.2}
\end{equation*}
$$

where $a_{j}$ is the projected major axis distance from the center, and $\gamma=7.67$. Similarly, the minor axis intensity profile is :

$$
\begin{equation*}
I_{\text {minor }}\left(b_{j}\right)=I_{0} / \cos (i) \cdot \exp \left(-b_{j}^{D} / r_{e x p}\right)+e^{\gamma} I_{e f f} \cdot \exp \left[-\gamma\left(b_{j}^{S} / r_{e f f}\right)^{1 / 4}\right] \tag{3.3}
\end{equation*}
$$

with the de-projected minor axis distances (see Chapter 2)

$$
\begin{equation*}
b_{j}^{D}=b_{j} / \cos (i) \tag{3.4}
\end{equation*}
$$

and

$$
\begin{equation*}
b_{j}^{S}=b_{j} / \sqrt{\cos ^{2}(i)+\sin ^{2}(i) \cdot(1-\epsilon)^{2}} \tag{3.5}
\end{equation*}
$$

For any set of data, consisting of M data triplets $\left[a(j), I_{a}(j), \delta I_{a}(j)\right]$ along the major axis and $N$ data triplets $\left[b(j), I_{b}(j), \delta I_{b}(j)\right]$ along the minor axis, the best fitting model is defined as the one which minimizes

$$
\begin{equation*}
\chi^{2} \equiv \sum_{j=1}^{M} \frac{\left[I_{a}(j)-I_{\text {major }}(a(j))-I_{s k y}\right]^{2}}{\delta I_{a}^{2}(j)}+\sum_{j=1}^{N} \frac{\left[I_{b}(j)-I_{\operatorname{minor}}(b(j))-I_{s k y}\right]^{2}}{\delta I_{b}^{2}(j)} \tag{3.6}
\end{equation*}
$$

For this maximum likelihood fit the data points are only required to be mutually independent; their (radial) distribution is unspecified. There are two obvious choices for the spatial binning: either linear or logarithmic in radius. We decided to use even spacing in $\log (R)$, since it reduces the number of necessary radial points without loss of information, given the exponential nature of the luminosity profiles. Ideally, the weights, $\delta I$, should represent only the errors expected from Poisson statistics. However, the relative errors, $\delta I / I$, as inferred from counting statistics, can be as small as $10^{-3}$. Yet, in practice the errors near the galaxy centers are dominated by other sources such as flat fielding and guiding. Through comparison of the two sides
of the major axis, we found that the innermost parts ( $1^{\prime \prime}$ to $5^{\prime \prime}$ ) generally agree within $1 \%$ to $3 \%$. We therefore assumed a minimum relative error of $3 \%$. Since these errors are no longer Poisson distributed and independent (there is some dependence on the binning in radius), a translation of a measured $\chi^{2}$ into likelihood estimates of the fit is not possible. Therefore, the quantities $\chi^{2}$ quoted in the next sections should only be understood as relative measures of goodness of fit. To characterize discrepancies between the data and the photometric models we will use three quantities:

1. $\chi_{\text {model }}^{2}$, characterizing the deviation of a composite major axis fit from the best $R^{1 / 4}$ law. It is used to decide whether there is any reason to invoke a composite model.
2. $\chi_{\text {noisc }}^{2}$, characterizing the small scale deviations of the angle averaged profile from a composite model fit (which in all but a few cases describes well the overall shape of the profile). It is used to characterize the typical deviations expected even for a "good" model.
3. $\chi_{\text {face-on }}^{2}$, characterizing the deviations of the best fitting composite S/D model, turned face-on, from an $R^{1 / 4}$ law, using the same radial points and errors as the data. It describes how much the overall shape of the luminosity profile would have deviated from an $R^{1 / 4}$ law had we seen the galaxy face on. We will use this quantity to estimate the face-on detectability of any given disk.

We decided to restrict ourselves to axisymmetric systems to avoid a further increase in fitting parameters. Even with this minimum of seven parameters, MonteCarlo simulations showed that nearly degenerate solutions are possible. As we will illustrate with specific examples below, the meaningfulness of most decompositions is limited not by the fitting technique, but by deviations of the galaxies from the
idealized model, by the interpretation of the fitted components as dynamically hot or cold systems, and, most of all, the finite size of the detectors.

In order to obtain the data points for the model fitting, we reconstructed the major and minor axis profiles from the photometric profiles, partly shown in Appendices $A$ and $B$, including the residuals up to $6^{\text {th }}$ order. This reconstruction has the advantage that we can easily obtain slightly smoothed profiles, without worrying about foreground objects, chip defects, etc. Since the highest order Fourier residuals are small ( $<1 \%$ ) in all but very few cases, we can reconstruct the principal axis profiles to a high degree of accuracy. We defined the major axis direction as the luminosity weighted mean of the position angle.

Since we want to fit a non-linear model with many parameters it is important to have a robust algorithm which finds the best fitting solution in a fail-proof manner, independent of the initial parameter guesses. The conventional gradient-search methods (e.g. Press et al. 1986) are insufficient for this. Instead we employed a "biased random walk" technique, which is described in the Section 4.7. MonteCarlo simulations showed that this method finds the correct solution for a wide range of initial parameter guesses in the vast majority of cases. It still can fail in a small fraction of cases ( $\approx 5 \%$ ), simply because it is extremely difficult to survey the seven-dimensional parameter space for $\chi^{2}$ minima ${ }^{8}$.

## c) Judging the Goodness of the Fit

In principle there is a straightforward way to decide a) whether a model fit to a data set is "good" or "bad" and b) whether a particular model fits significantly better than another: the first question can be decided by comparing the $\chi^{2}$ of the model fit to the $\chi^{2}$ expected from the number of degrees of freedorn; the second question

[^12]Table 3.2: B Band Photometric Parameters

| UGC | $\chi_{n}^{2}$ | $\chi^{2}$ | $M_{S}$ | $M_{D}$ | $I_{\text {aff }}$ | $I_{0}$ | $R_{\text {eff }}$ | $R_{\text {ex }}$ | (c) | Cman | $\Delta P . A$. | R/R | D/S | $\cos (\mathrm{i})$ | $A_{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 80 | 0.44 | 0.04 | -20.5 | -18.5 | 21.9 | 22.5 | 3.2 | 3.1 | 0.35 | 0.50 | 35.00 | 0.96 | 0.16 | 0.36 | 3.0 |
| 292 | 0.89 | 0.30 | -20.3 | -20.2 | 21.6 | 22.0 | 2.5 | 5.2 | 0.53 | 0.67 | 0.90 | 2.05 | 0.87 | 0.23 | 12.0 |
| 597 | 0.07 | - | -22.6 | - | 23.8 | - | 19.0 | - | 0.27 | 0.29 | 1.50 | - | - | - | -0.8 |
| 735 | 0.03 | 0.89 | -20.7 | . 20.6 | 21.6 | 22.1 | 2.9 | 6.8 | 0.24 | 0.31 | 1.10 | 2.36 | 0.97 | 0.63 | 1.9 |
| 801 | 0.22 | 0.13 | -18.9 | -17.2 | 21.8 | 21.4 | 1.4 | 1.0 | 0.58 | 0.63 | 1.60 | 0.71 | 0.21 | 0.23 | 3.8 |
| 848 | 0.03 | 0.10 | -21.6 | -20.8 | 23.7 | 24.0 | 11.7 | 17.7 | 0.07 | 0.12 | 12.10 | 1.51 | 0.50 | 0.91 | 0.0 |
| 992 | 0.06 | - | -22.1 | - | 24.0 | - | 16.7 | - | 0.24 | 0.25 | 2.20 | - | - | - | 0.0 |
| 995 | 0.10 | - | -21.0 | - | 22.4 | - | 5.0 | - | 0.18 | 0.22 | 13.00 | - | - | - | 2.0 |
| 1283 | 0.10 | 2.70 | -21.0 | -19.5 | 25.0 | 19.1 | 16.3 | 1.0 | 0.09 | 0.11 | 29.00 | 0.06 | 0.26 | 0.96 | 1.0 |
| 1476 | 0.05 | - | -20.4 | - | 23.8 | - | 6.9 | - | 0.17 | 0.18 | 0.90 | - | - | - | -0.2 |
| 4674 | 0.19 | - | -21.5 | - | 22.5 | - | 6.6 | - | 0.25 | 0.29 | 3.80 | - | - | - | 1.2 |
| 4763 | 0.04 | 0.22 | -20.5 | -19.7 | 21.8 | 23.2 | 3.0 | 7.5 | 0.22 | 0.28 | 2.30 | 2.49 | 0.48 | 0.73 | -0.5 |
| 4791 | 0.11 | 0.05 | -19.7 | -19.5 | 22.8 | 21.9 | 3.2 | 3.7 | 0.56 | 0.67 | 1.20 | 1.15 | 0.87 | 0.23 | 5.7 |
| 4840 | 0.05 | 1.12 | -18.0 | -18.1 | 20.1 | 21.5 | 0.4 | 1.6 | 0.21 | 0.24 | 1.90 | 3.77 | 1.17 | 0.75 | -1.0 |
| 5018 | 0.08 | - | -20.6 | - | 22.4 | - | 4.1 | - | 0.22 | 0.23 | 1.40 | - | - | - | -0.4 |
| 5292 | 2.01 | 7.82 | -18.2 | -18.3 | 17.5 | 21.9 | 0.1 | 2.1 | 0.16 | 0.22 | 18.00 | 15.58 | 1.12 | 0.84 | 0.0 |
| 5350 | 0.12 | - | -21.3 | - | 22.5 | - | 5.8 | - | 0.12 | 0.13 | 2.10 | - | -- | - | 0.4 |
| 5617 | 0.10 | - | -19,3 | - | 24.0 | - | 4.6 | - | 0.15 | 0.19 | 8.30 | - | - | - | 2.1 |
| 6037 | 0.54 | 1.36 | -18.8 | -18.2 | 19.8 | 21.6 | 0.5 | 1.7 | 0.23 | 0.31 | 18.70 | 3.10 | 0.54 | 0.74 | -10.0 |
| 6281 | 0.50 | 3.37 | -16.8 | -17.0 | 17.4 | 20.6 | 0.1 | 0.6 | 0.16 | 0.24 | 8.50 | 9.22 | 1.24 | 0.83 | 2.4 |
| 6295 | 0.16 | 0.01 | -18.9 | -16.6 | 22.2 | 22.7 | 1.6 | 1.4 | 0.36 | 0.41 | 0.80 | 0.88 | 0.13 | 0.28 | -1.5 |
| 6409 | 0.12 | 7.40 | -18.8 | -18.9 | 18.7 | 21.7 | 0.3 | 2.6 | 0.19 | 0.25 | 13.80 | 8.18 | 1.14 | 0.83 | 1.5 |
| 6444 | 0.08 | 0.60 | -17.9 | -17.7 | 19.6 | 21.6 | 0.3 | 1.4 | 0.52 | 0.65 | 1.60 | 4.54 | 0.89 | 0.31 | 7.0 |
| 6704 | 0.10 | 0.44 | -21.0 | -20.6 | 22.3 | 22.6 | 4.7 | 8.7 | 0.19 | 0.23 | 3.10 | 1.84 | 0.71 | 0.72 | 0.0 |
| 6704 | 0.05 | - | -20.0 | - | 22.4 | - | 3.0 | - | 0.18 | 0.20 | 1.70 | - | - | - | 0.0 |
| 6738 | 0.02 | - | -20.1 | - | 22.5 | - | 3.4 | - | 0.26 | 0.33 | 8.00 | - | - | - | 0.6 |
| 6779 | 0.01 | 0.01 | -20.4 | -17.9 | 22.8 | 23.1 | 4.5 | 3.0 | 0.37 | 0.42 | 1.50 | 0.68 | 0.10 | 0.19 | 1.3 |
| 6953 | 0.10 | - | -21.3 | - | 23.2 | - | 7.8 | - | 0.33 | 0.44 | 1.50 | - | - | - | 1.1 |
| 7165 | 0.43 | 7.57 | -17.8 | . 17.9 | 17.4 | 21.1 | 0.1 | 1.2 | 0.31 | 0.36 | 4.40 | 11.07 | 1.07 | 0.66 | 0.4 |
| 7202 | 0.79 | 0.04 | -20.8 | -18.5 | 20.8 | 22.8 | 2.1 | 3.4 | 0.44 | 0.47 | 9.60 | 1.65 | 0.12 | 0.27 | 1.0 |
| 7203 | 0.02 | - | -20.5 | - | 24.0 | - | 8.0 | - | 0.13 | 0.16 | 1.80 | - | - | - | 0.9 |
| 7311 | 1.54 | - | -19.7 | - | 22.2 | - | 2.4 | - | 0.34 | 0.55 | 8.40 | - | - | - | 3.2 |
| 7376 | 0.05 | 0.59 | -18.7 | -19.0 | 20.9 | 21.6 | 0.8 | 2.5 | 0.49 | 0.62 | 1.20 | 2.99 | 1.31 | 0.36 | 1.5 |


| UGC | $\chi_{n}^{2}$ | $\chi^{2}$ | $M_{S}$ | $M_{D}$ | $I_{\text {aff }}$ | $I_{0}$ | $R_{\text {eff }}$ | $R_{\text {app }}$ | (c) | eman | $\Delta P \cdot A$. | R/R | D/S | cos(i) | $A_{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7390 | 0.18 | - | -17.9 | - | 21.2 | - | 0.7 | - | 0.04 | 0.06 | 10.00 | - | - | - | 0.0 |
| 7461 | 0.24 | 0.56 | -18.5 | -18.7 | 22.3 | 22.9 | 1.4 | 4.0 | 0.06 | 0.07 | 6.70 | 2.78 | 1.27 | 0.89 | 0.0 |
| 7517 | 1.26 | 0.25 | -17.1 | -16.3 | 22.6 | 21.1 | 0.9 | 0.6 | 0.35 | 0.40 | 1.20 | 0.64 | 0.45 | 0.46 | -1.2 |
| 7571 | 0.06 | 4.11 | -17.3 | -17.3 | 18.8 | 21.1 | 0.2 | 0.9 | 0.05 | 0.08 | 9.80 | 5.47 | 0.98 | 0.95 | 0.0 |
| 7610 | 0.05 | 0.07 | -17.7 | -16.4 | 23.2 | 23.7 | 1.5 | 2.1 | 0.10 | 0.15 | 2.30 | 1.33 | 0.30 | 0.82 | 0.6 |
| 7634 | 0.17 | 0.07 | -18.7 | -17.6 | 22.0 | 22.2 | 1.4 | 1.8 | 0.45 | 0.58 | 0.50 | 1.29 | 0.38 | 0.19 | 13.0 |
| 7637 | 0.21 | - | -19.6 | - | 22.3 | - | 2.4 | - | 0.35 | 0.45 | 2.10 | - | - | - | 2.5 |
| 7655 | 0.13 | 1.41 | -16.9 | -17.1 | 21.5 | 21.9 | 0.5 | 1.2 | 0.08 | 0.09 | 25.60 | 2.36 | 1.15 | 0.94 | 0.3 |
| 7701 | 0.03 | - | -17.4 | - | 21.6 | - | 0.6 | - | 0.18 | 0.31 | 2.40 | - | - | - | 0.8 |
| 7759 | 0.01 | 0.09 | -18.2 | -16.2 | 23.2 | 21.8 | 1.9 | 0.8 | 0.26 | 0.30 | 1.10 | 0.41 | 0.17 | 0.45 | -1.0 |
| 7860 | 0.05 | - | -18.4 | - | 24.8 | - | 4.5 | - | 0.35 | 0.38 | 12.20 | - | - | - | 0.1 |
| 8028 | 0.11 | - | -22.2 | - | 23.4 | - | 13.5 | - | 0.26 | 0.30 | 0.70 | - | - | - | 0.5 |
| 8110 | 0.13 | - | -22.5 | - | 23.6 | - | 16.8 | - | 0.34 | 0.37 | 1.30 | - | -- | - | -0.9 |
| 8125 | 0.08 | 0.67 | -21.0 | -20.4 | 20.8 | 22.1 | 2.4 | 6.3 | 0.37 | 0.41 | 1.50 | 2.67 | 0.59 | 0.53 | 0.5 |
| 8423 | 0.16 | - | . 21.9 | - | 23.5 | - | 12.5 | - | 0.33 | 0.36 | 1.10 | - | - | - | -1.5 |
| 8499 | 0.06 | 2.07 | .18.3 | -18.9 | 19.7 | 21.1 | 0.4 | 1.9 | 0.12 | 0.15 | 5.00 | 4.83 | 1.74 | 0.87 | 0.2 |
| 8675 | 0.15 | 10.41 | -16.9 | -18.7 | 19.7 | 21.5 | 0.2 | 2.1 | 0.14 | 0.17 | 9.30 | 10.17 | 5.39 | 0.86 | 0.6 |
| 8974 | 0.05 | - | -20.9 | - | 22.7 | - | 5.4 | - | 0.19 | 0.21 | 2.40 | - | - | - | 0.6 |
| 9137 | 0.03 | 0.20 | -22.0 | -21.8 | 22.2 | 23.7 | 7.0 | 23.9 | 0.19 | 0.25 | 3.00 | 3.39 | 0.84 | 0.79 | 0.0 |
| 9188 | 0.15 | 0.70 | -18.3 | -18.0 | 21.3 | 23.0 | 0.8 | 3.0 | 0.30 | 0.34 | 0.90 | 3.56 | 0.78 | 0.59 | 1.5 |
| 9395 | 0.08 | 0.17 | -17.6 | -16.5 | 21.7 | 23.5 | 0.8 | 1.9 | 0.35 | 0.40 | 1.10 | 2.52 | 0.37 | 0.45 | 0.5 |
| 9642 | 3.28 | 0.36 | -18.9 | -18.8 | 22.3 | 21.0 | 1.8 | 1.8 | 0.46 | 0.56 | 2.70 | 1.00 | 0.94 | 0.34 | 3.6 |
| 9678 | 0.08 | - | -19.6 | - | 22.7 | - | 2.9 | - | 0.18 | 0.29 | 8.40 | - | - | - | 0.8 |
| 9726 | 0.41 | 0.26 | -18.6 | -18.2 | 21.6 | 22.1 | 1.1 | 2.2 | 0.52 | 0.68 | 3.00 | 1.96 | 0.72 | 0.23 | 5.0 |
| 9903 | 1.15 | - | -19.0 | - | 20.4 | - | 0.8 | - | 0.17 | 0.35 | 27.20 | - | - | - | 0.4 |
| 10345 | 0.10 | 0.74 | -20.4 | -19.8 | 21.0 | 22.3 | 1.9 | 5.1 | 0.04 | 0.06 | 4.10 | 2.63 | 0.59 | 0.93 | 0.0 |
| 10528 | 0.11 | 0.19 | -20.1 | -20.4 | 19.9 | 23.7 | 1.0 | 12.6 | 0.38 | 0.42 | 3.70 | 12.07 | 1.28 | 0.54 | -10.0 |
| 10916 | 0.07 | - | -21.2 | - | 23.6 | - | 9.0 | - | 0.31 | 0.34 | 2.80 | - | - | - | -1.0 |
| 12523 | 0.02 | - | -21.3 | - | 22.4 | - | 5.5 | - | 0.22 | 0.27 | 1.40 | - | - | - | 0.4 |
| 12531 | 0.11 | 3.37 | -20.1 | -20.5 | 20.6 | 21.7 | 1.4 | 5.3 | 0.11 | 0.14 | 7.90 | 3.72 | 1.44 | 0.90 | 0.3 |
| 12760 | 3.10 | 0.54 | -19.9 | -18.0 | 23.7 | 21.0 | 5.3 | 1.2 | 0.12 | 0.44 | 30.40 | 0.22 | 0.18 | 0.71 | 1.5 |
| 12841 | 0.16 | 0.01 | -21.1 | -18.5 | 22.7 | 23.3 | 5.7 | 4.5 | 0.42 | 0.48 | 1.60 | 0.79 | 0.09 | 0.24 | -2.5 |

## Notes on the Table:

All disk quantities are only listed for the objects with accepted S/D decompositions. In the other cases the spheroid quantities refer to the single component fit of an $R^{1 / 4}$ law.
Col. 1: UGC designation
Col. 2: $\chi_{\text {noise }}^{2}$, characterizing the smoothness of the luminosity profile, as defined in Section 3.6.1. Col. 3: $\chi_{\text {face-on }}^{2}$, characterizing the deviations of the best fitting two component model from an $R^{1 / 4}$ law, if seen face on.
Cols. 4 and 5: Absolute magnitude of the spheroid and the disk, assuming $H_{0}=75 \mathrm{~km} / \mathrm{s}$.
Col. 6: Spheroid surface brightness in $\left[\mathrm{mag} / \square^{2}\right]$ at the effective radius.
Col. 7: Disk central surface brightness in $\left[\mathrm{mag} / \square^{2}\right]$, corrected to face-on.
Col. 8: Effective radius of the spheroid in $\left[\mathrm{mag} / \square^{2}\right]$, as defined in Section 3.5.
Col. 9: Exponential radius of the disk in $\left[\mathrm{mag} / \square^{2}\right]$, as defined in Section 3.5.
Col. 10: Luminosity weighted (from $2 \square^{2}$ to $60 \square^{2}$ ) mean ellipticity.
Col. 11: Maximum ellipticity between $2 \square^{2}$ and $60 \square^{2}$.
Col. 12: Position angle variation, defined as the root of the luminosity weighted second moment of the P.A. distribution.
Col. 13: Ratio of the spheroid scale length, $R_{\text {eff }}$, to the disk scale length, $R_{\text {exp }}$.
Col. 14: Luminosity ratio of the disk to the spheroid.
Col. 15: Formally derived disk inclination estimate, uncorrected for finite disk thickness.
Col. 16: Estimated extremum (in $\%$ ) for the amplitude $A_{4}$ of the $\cos (4 \theta)$ residual in the isophote fitting procedure. For most objects the maximum value is listed. For objects which are predominantly "boxy", the minimum is listed. We assigned the value $-10 \%$ to objects with irregular isophotes.
can be decided by comparing the $\chi^{2}$ of a simple $R^{1 / 4}$ fit to the $\chi^{2}$ of the composite S/D fit. Unfortunately, the statistically well defined answers to these questions are not helpful: for the majority of the objects neither model is statistically "good", i.e. fits within the errors. Second, for virtually all objects the two-component S/D model fits statistically better than a single component model. A similar, formal result was found by Kodaira et al. (1986), but was accepted outright by them as real. The problem with this finding is that an excessive number of formally derived disks appear to be face-on. It appears more likely that most "pure" spheroids do not follow an $R^{1 / 4}$ law - and why should they, after all - and that the twocomponent model just accommodates the particular shape of the spheroid profile. Consequently, following statistics strictly will yield an unphysical answer.

Thus we are forced to employ empirical methods to decide what is a good fit and what is not. We will use a combination of statistical criteria and physical constraints to decide whether to accept a disk fit. We can introduce a typical scale for the extent of deviations of a model from the data: For most objects the overall shape of the radial profile, can be approximated well by a two component model over the available radial range. However, significant deviations between the model and the data remain on small scales, due to dust, ripples, the end of bars, etc. We can characterize their typical size by the quantity $\chi_{n o s e}^{2}$, which is defined as the deviation per degree of freedom of the best fitting S/D model of the angle averaged profile. This quantity can be compared to the deviation of the smooth, best fitting two component model from the best fitting $R^{1 / 4}$ law, expressed as $\chi_{\text {model }}^{2}$, assuming the same errors as before. This allows us to asses whether a two component model deviates in its overall shape more from a simple model, than the typical small scale noise in the profile. Although not a rigorous or unique criterion, this comparison allows us to retain some statistical objectivity, while avoiding the all-disk paradox
mentioned above.

## d) Other Constraints

One could consider imposing additional constraints to avoid the fitting of "spurious" disks and to improve the limits on the derived photometric parameters. In particular, we have not yet mentioned in this chapter the role of $A_{4}$ as an indicator of the inclination. One could envision using $A_{4, \max }$ as a measure of $\cos (i)$ and then use the principal axis profiles to determine the best decomposition for this inclination. Although this would lead to formally better constrained fits, it has several disadvantages: First, there is no straightforward way of combining goodness of fit measures for deviations from the radial profiles with the ones of the $A_{4}$ profile ${ }^{9}$. Second, by employing $A_{4}$ as a constraint we must rely on the assumption that the observed $A_{4}$ in all objects is due to an S/D geometry. However, we would like to test this very hypothesis.

### 3.5.2 Results of the Model Fits

We applied this model fitting procedure to all B and H images. The resulting major axis fits, for either the $R^{1 / 4}$ law or the composite $S / D$ model, are superimposed onto the radial profiles in Appendices A and B. An example of major and minor axis fits is given in Figure 3.9. We accepted the S/D decomposition if the following criteria were satisfied:

- Any disk will imprint itself most strongly onto the major axis profile. If the major axis data can be well fit by an $R^{1 / 4}$ law, as quantified by $\chi_{\text {model }}^{2}$, then the luminosity profiles are consistent with a single component model and there is

[^13]no justification for invoking a two component model. We will quantify below what we mean by "good" fit, in terms of $\chi_{\text {model }}^{2}$.

- The face-on central surface brightness of the disk, $I_{0}$, exceeded $23.5 m_{B} / \square^{2}$ or $20.0 m_{H} / \square^{2}$, respectively. This limit was set to eliminate fits that showed large, low surface brightness disks that can easily be traded off against a change in the sky level ${ }^{10}$. We did not choose a cut-off in the directly measured quantity $I_{0} / \cos (i)$, because such a choice would result in many formal fits of nearly edge-on disks of very low face-on surface brightnesses, accommodating small profile bumps along the major axis. Most objects that yielded such fits did not show any diskiness in the isophotes, thus making the presence of an edge-on disk unlikely.
- The fit must yield a disk scale length of less than $50^{\prime \prime 11}$ to avoid near-degeneracy with changes in the sky level.
- The disk must contribute at least $30 \%$ of the total light at some measured major axis points.

For all the objects whose decompositions were rejected we fit an $R^{1 / 4}$ law to the major axis profile, again leaving the sky level as a free variable. Tables 3.2 and 3.3 present the results of this analysis for the B and H band data. The absolute magnitudes have been calculated from the velocities in Table 1, assuming $H_{0}=$ $75 \mathrm{~km} / \mathrm{s}$. Since the two data sets are of discrepant quality, we give the results independently and present a comparison below. Acceptable S/D decompositions have been found for about $40 \%$ of the total sample. The de-projected spheroid ellipticity is very poorly determined in almost all cases and is not of particular

[^14]interest for our present study; thus it is not listed in the tables. Instead we list the mean and maximum projected ellipticity of the total light.

### 3.5.3 Checking the Decompositions

## External Comparisons

To caution the reader before the interpretation of these decompositions, we illustrate their limitations. The main uncertainties arise from the limited radial range for which data are available and from our ignorance of the sky level. As mentioned before, our estimate of the sky level is only well defined in the context of a luminosity model and it will differ for any given object, depending on whether we fit an $R^{1 / 4}$ law or an S/D model. Obviously this uncertainty in our data points, $\mu(r)$, will be reflected in the model fits to those data. The quality of our decompositions can be best tested by an external comparison with the results of Burstein (1979) and Kent ${ }^{12}$ (1985) shown in Figures 3.1 and 3.2.

Two decompositions can be compared with B79: there is excellent agreement for UGC 7202 and there is reasonable agreement for UGC 7165. Our model for UGC 7165 actually provides a better fit (in a $\chi^{2}$ sense) to Burstein's data. B79 did not attempt a decomposition for UGC 7634, claiming evidence for an exponential spheroid profile. Yet, we find a very sensible fit for this galaxy, as shown in Figure 3.9 .

For four of the eight objects in common with K85 (UGC 80, UGC 2778, UGC 4169 and UGC 7619) our model fits are in reasonable agreement (within a factor of 1.5). The four examples of blatant disagreement, not necessarily in the luminosity profiles but in the physical implications of the fitted models, illustrate the problems

[^15]Table 3.3: H Band Photometric Parameters

| UGC | $\chi_{n}^{2}$ | $\chi_{\text {m }}^{2}$ | $M_{S}$ | $M_{D}$ | $I_{\text {aff }}$ | $r_{0}$ | $R_{\text {eff }}$ | $\boldsymbol{R}_{\text {emp }}$ | (c) | Cmae | $\triangle P . A$. | R/R | D/S | $\cos (\mathrm{i})$ | $A_{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 80 | 0.43 | 0.13 | . 23.6 | -22.4 | 17.9 | 18.5 | 2.1 | 2.9 | 0.34 | 0.47 | 30.70 | 1.37 | 0.31 | 0.48 | 3.0 |
| 292 | 1.13 | 0.76 | .23.1 | -23.7 | 18.6 | 16.8 | 2.2 | 2.4 | 0.43 | 0.44 | 0.50 | 1.10 | 1.63 | 0.34 | 12.0 |
| 859 | 0.24 | - | . 23.0 | - | 20.6 | - | 5.3 | - | 0.31 | 0.33 | 6.20 | - | - | - | -1.0 |
| 926 | 0.30 | 0.40 | -24.4 | -23.7 | 18.3 | 20.0 | 3.5 | 10.8 | 0.35 | 0.38 | 0.60 | 3.05 | 0.54 | 0.56 | -0.5 |
| 938 | 0.69 | 4.41 | -24.3 | -26.0 | 17.0 | 18.6 | 1.8 | 15.8 | 0.14 | 0.25 | 17.70 | 8.57 | 4.68 | 0.90 | 0.5 |
| 995 | 0.10 | - | -24.6 | - | 18.9 | - | 5.0 | - | 0.16 | 0.21 | 7.60 | - | - | - | 2.0 |
| 1250 | 0.67 | 0.01 | -23.7 | -21.3 | 18.9 | 19.4 | 3.4 | 2.7 | 0.51 | 0.52 | 0.30 | 0.79 | 0.11 | 0.33 | 5.2 |
| 1475 | 0.35 | - | -23.5 | - | 18.5 | - | 2.5 | - | 0.15 | 0.18 | 4.40 | - | - | - | 0.0 |
| 1476 | 1.79 | 1.32 | -24.7 | -24.7 | 17.5 | 18.4 | 2.7 | 8.1 | 0.16 | 0.18 | 2.10 | 2.95 | 1.06 | 0.83 | -0.2 |
| 1631 | 1.08 | - | -23.8 | - | 19.0 | - | 3.7 | - | 0.36 | 0.39 | 0.90 | - | - | - | 3.6 |
| 2128 | 0.32 | 4.22 | -24.7 | -26.7 | 18.0 | 18.4 | 3.6 | 20.4 | 0.05 | 0.07 | 8.00 | 5.69 | 5.98 | 0.98 | 0.3 |
| 4561 | 0.92 | 0.50 | -22.3 | -20.8 | 14.6 | 19.9 | 0.2 | 2.7 | 0.27 | 0.54 | 4.40 | 11.45 | 0.27 | 0.36 | 10.0 |
| 4674 | 0.07 | - | -25.1 | - | 18.7 | - | 6.0 | - | 0.24 | 0.26 | 2.80 | - | - | - | 1.2 |
| 4791 | 0.09 | 0.06 | -23.0 | . 23.0 | 18.7 | 18.3 | 2.3 | 3.6 | 0.58 | 0.66 | 0.80 | 1.61 | 1.03 | 0.30 | 5.7 |
| 4840 | 0.40 | - | -22.5 | - | 19.4 | - | 2.5 | - | 0.21 | 0.24 | 5.70 | - | - | - | -1.0 |
| 5503 | 0.08 | 0.05 | -20.9 | -20.2 | 19.8 | 19.0 | 1.4 | 1.3 | 0.47 | 0.50 | 1.00 | 0.97 | 0.57 | 0.40 | -10.0 |
| 6037 | 0.90 | 0.59 | -21.9 | -21.6 | 16.1 | 17.6 | 0.4 | 1.3 | 0.24 | 0.33 | 17.80 | 3.12 | 0.72 | 0.81 | -10.0 |
| 6281 | 1.32 | 3.64 | -20.3 | -19.8 | 14.3 | 18.1 | 0.1 | 0.7 | 0.20 | 0.26 | 8.30 | 8.46 | 0.61 | 0.83 | 2.4 |
| 6295 | 0.18 | - | -22.6 | - | 19.2 | - | 2.4 | - | 0.38 | 0.42 | 1.20 | - | - | - | -1.5 |
| 6409 | 4.44 | 4.70 | -22.2 | -22.2 | 15.1 | 18.1 | 0.3 | 2.2 | 0.19 | 0.24 | 14.10 | 7.53 | 0.97 | 0.87 | 1.5 |
| 6444 | 0.33 | 0.28 | -21.6 | -21.0 | 16.0 | 18.2 | 0.3 | 1.4 | 0.52 | 0.67 | 1.70 | 4.05 | 0.60 | 0.29 | 7.0 |
| 6605 | 0.33 | - | -20.3 | - | 22.2 | - | 3.2 | - | 0.10 | 0.14 | 93.40 | - | - | - | 0.0 |
| 6704 | 0.42 | - | -25.5 | - | 20.0 | - | 12.9 | - | 0.19 | 0.23 | 5.30 | - | - | - | 0.0 |
| 6738 | 0.75 | - | -24.3 | - | 18.0 | - | 2.9 | - | 0.30 | 0.35 | 4.20 | - | - | - | 0.6 |
| 6779 | 0.06 | - | -25.0 | - | 19.5 | - | 7.9 | - 0 | 0.36 | 0.42 | 0.70 | - | - | - | 1.3 |
| 6953 | 0.01 | - | -24.7 | - | 19.6 | - | 7.4 | - 0 | 0.26 | 0.36 | 1.30 | - | - | - | 1.1 |
| 7117 | 0.23 | 0.15 | -22.0 | -22.2 | 17.1 | 18.9 | 0.7 | 3.2 | 0.52 | 0.56 | 0.70 | 4.76 | 1.28 | 0.46 | 3.2 |
| 7165 | 0.08 | 4.42 | -21.0 | -20.8 | 13.9 | 17.9 | 0.1 | 1.1 | 0.26 | 0.31 | 1.30 | 10.54 | 0.81 | 0.71 | 0.4 |
| 7202 | 0.09 | - | -24.6 | - | 18.9 | - | 5.2 | - 0 | 0.42 | 0.46 | 5.60 | - | - | - | 1.0 |
| 7203 | 0.05 | - | -23.9 | - | 19.8 | - | 5.6 | - | 0.14 | 0.18 | 2.00 | - | - | - | 0.9 |
| 7214 | 0.08 | - | -23.9 | - | 19.1 | - | 4.2 | - | 0.51 | 0.62 | 0.70 | - | - | - | 9.0 |


| UGC | $\chi_{n}^{2}$ | $\chi_{m}^{2}$ | $M_{S}$ | $M_{D}$ | Iaff | $I_{0}$ | $R_{\text {eff }}$ | $\boldsymbol{R}_{\text {eap }}$ | (c) | eman | $\Delta P . A$. | R/R | D/S | $\cos (\mathrm{i})$ | $A_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7311 | 0.24 | 0.31 | . 23.1 | -22.0 | 17.2 | 19.3 | 1.2 | 3.5 | 0.21 | 0.38 | 9.70 | 2.92 | 0.36 | 0.48 | 3.2 |
| 7376 | 0.27 | - | -24.4 | - | 20.7 | - | 10.6 | - | 0.45 | 0.53 | 0.50 | - | - | -. | 1.5 |
| 7502 | 0.34 | - | -21.9 | - | 19.5 | - | 1.9 | - | 0.04 | 0.05 | 11.90 | - | - | - | 0.2 |
| 7517 | 0.09 | 0.08 | . 21.0 | -19.4 | 18.7 | 18.2 | 0.9 | 0.6 | 0.36 | 0.42 | 1.00 | 0.70 | 0.22 | 0.48 | -1.2 |
| 7634 | 0.16 | 0.10 | -22.3 | -21.3 | 18.2 | 18.8 | 1.3 | 2.0 | 0.47 | 0.61 | 0.50 | 1.56 | 0.39 | 0.31 | 13.0 |
| 7637 | 1.27 | 0.27 | -22.4 | -20.7 | 20.7 | 16.3 | 4.3 | 0.5 | 0.38 | 0.45 | 1.10 | 0.11 | 0.20 | 0.51 | 2.5 |
| 7701 | 0.11 | - | -20.5 | - | 18.2 | - | 0.6 | - | 0.17 | 0.26 | 4.70 | - | - | - | 0.8 |
| 7722 | 0.09 | 0.14 | -22.0 | -21.0 | 17.3 | 17.6 | 0.8 | 1.0 | 0.29 | 0.43 | 23.30 | 1.34 | 0.38 | 0.67 | 3.0 |
| 7759 | 0.11 | - | -22.4 | - | 19.6 | - | 2.6 | -- | 0.29 | 0.31 | 0.70 | - | - | - | -1.0 |
| 7850 | 0.35 | 3.58 | -22.2 | -22.4 | 15.8 | 17.7 | 0.4 | 2.0 | 0.19 | 0.21 | 10.90 | 5.08 | 1.22 | 0.79 | 0.3 |
| 8028 | 0.10 | - | -25.8 | - | 19.8 | - | 13.2 | - | 0.25 | 0.29 | 2.00 | - | - | - | 0.5 |
| 8110 | 0.18 | - | -26.1 | - | 19.7 | - | 15.0 | - | 0.33 | 0.37 | 1.00 | - | - | - | -0.9 |
| 8125 | 0.16 | - | -25.1 | - | 19.3 | - | 7.7 | - | 0.36 | 0.39 | 0.70 | - | - | - | 0.5 |
| 8423 | 0.10 | - | -25.5 | - | 19.4 | - | 10.0 | - | 0.32 | 0.34 | 0.50 | - | - | - | -1.5 |
| 8499 | 0.22 | 1.63 | -22.3 | -22.7 | 15.5 | 17.3 | 0.4 | 1.9 | 0.12 | 0.15 | 3.90 | 5.12 | 1.38 | 0.88 | 0.2 |
| 8675 | 0.14 | 6.94 | -20.2 | -21.0 | 15.1 | 18.4 | 0.1 | 1.4 | 0.11 | 0.16 | 7.50 | 11.98 | 2.03 | 0.86 | 0.6 |
| 8866 | 0.62 | 0.41 | -22.5 | -22.3 | 17.0 | 18.8 | 0.8 | 3.2 | 0.52 | 0.59 | 2.50 | 3.90 | 0.86 | 0.42 | 6.0 |
| 8974 | 0.23 | - | -24.4 | - | 19.0 | - | 5.0 | - | 0.18 | 0.19 | 2.20 | - | - | - | 0.6 |
| 9137 | 0.04 | - | -25.8 | - | 19.0 | - | 9.6 | - | 0.21 | 0.24 | 1.80 | - | - | - | 0.0 |
| 9188 | 0.05 | - | -22.1 | - | 18.9 | - | 1.6 | - | 0.31 | 0.34 | 1.50 | - | - | - | 1.5 |
| 9395 | 0.21 | - | -21.5 | - | 19.4 | - | 1.6 | - | 0.33 | 0.39 | 1.50 | - | - | - | 0.5 |
| 9642 | 0.14 | 0.20 | -22.4 | -22.7 | 16.8 | 17.1 | 0.7 | 1.8 | 0.52 | 0.58 | 0.50 | 2.54 | 1.32 | 0.38 | 3.6 |
| 9678 | 0.05 | - | -22.9 | - | 18.5 | - | 2.0 | - | 0.22 | 0.30 | 4.50 | - | - | - | 0.8 |
| 9726 | 0.09 | - | -23.9 | - | 20.5 | - | 7.5 | - | 0.47 | 0.62 | 2.10 | - | - | - | 5.0 |
| 9851 | 1.26 | - | -23.3 | - | 20.0 | - | 4.6 | - | 0.20 | 0.24 | 36.70 | - | - | - | . 10.0 |
| 9903 | 0.18 | - | -22.4 | - | 16.7 | - | 0.7 | - | 0.17 | 0.22 | 6.80 | - | - | - | 0.4 |
| 10345 | 0.44 | - | -24.4 | - | 18.6 | - | 4.1 | - | 0.05 | 0.07 | 7.30 | - | - | - | 0.0 |
| 10916 | 0.15 | - | -24.4 | - | 19.7 | - | 6.7 | - | 0.29 | 0.30 | 1.40 | - | - | - | -1.0 |
| 12531 | 0.76 | 0.16 | -23.7 | -22.7 | 18.2 | 18.1 | 2.4 | 2.9 | 0.07 | 0.07 | 1.40 | 1.19 | 0.42 | 0.90 | 0.3 |
| 12841 | 0.13 | - | -25.3 | - | 19.9 | - | 11.0 | - | 0.40 | 0.42 | 1.30 | - | - | - | -2.5 |



Figure 3.9: S/D Decomposition of UGC 7634
To illustrate our S/D decomposition procedure, we show the result of fitting a composite model to the major and minor axis profiles of UGC 7634. For most of the data points the error bars are smaller than the symbols. The fit results in a highly inclined disk, which comprises about $25 \%$ of the total light. Even though the disk is very prominent from our particular viewing angle ( $A_{4}=13 \%!!$ ), the system could not have been distinguished from a pure spheroid if seen face-on.
inherent to these decompositions. For UGC 3872 Kent finds a large, low surface brightness disk which dominates the light between $50^{\prime \prime}$ and $150^{\prime \prime}$, while our data cannot provide support for this hypothesis. Again, this shows that the present data are only sensitive to disks within the parameter brackets specified in Section 3.5.2. Conversely, the luminosity profile of UGC 3894 is much better fit by an S/D model than a simple $R^{1 / 4}$ law. However, K85 does not even attempt a decomposition (due to the galaxy's a priori classification as an elliptical?). A similar argument applies to UGC 7626. UGC 7785 provides a clear example for a discrepancy due to problems in setting the sky level. Apparently, the sky level in our data frames can be lowered sufficiently to result in an acceptable $R^{1 / 4}$ fit, rather than the S/D fit that Kent finds.

## Internal Comparison of $B$ and $H$

As an internal check of our S/D parameter estimates we can compare the photometric parameters at B and H for the objects with decompositions in both colors. Figure 3.10 shows a comparison of the spheroid-to-disk ratios, scale lengths and inclinations derived from the two data sets. For the galaxies that had acceptable decompositions in both colors, there is reasonable agreement in derived parameters; nonetheless, the scatter is clearly much larger than expected from intrinsic color differences. The outliers in these comparisons are again an indicator of the inherent instability of these decompositions, which can result in vastly discrepant parameters.

## Disk Inclination and Isophote Shapes

Since we did not use information from the isophote shapes directly in fitting our models, we can now employ a comparison of the formally derived values of $\cos (i)$ with $A_{4}$, as an additional consistency check both on the quality of our decompositions and on our simple photometric models from Chapter 2: the inclinations were formally derived from the minor and major axis fitting procedures and there is no guarantee that they will not be in conflict with the inclination constraints derived from $A_{4}$. The values listed in Tables 3.2 and 3.3 were not corrected for the finite thickness of the disk; therefore there are no values below $\cos (i) \approx 0.15$. Figure 3.11 compares the isophote shapes to the formally derived inclinations $\cos (i)$. There are two things to note about this figure: first, in agreement with our preconceptions, all objects with large $A_{4}$ are found to be near edge-on, and no nearly face-on objects show large $A_{4}$. Second, there is very substantial scatter in the relation: on the one hand, there are objects with small values of $\cos (i)(<0.5)$ but no diskiness at all. Some of these discrepancies may arise from problems with the fitting: despite


Figure 3.10: Comparison of $B$ and $H$ Band Decompositions
The top panel compares the derived magnitudes for all objects in observed in both bands. Crosses represent spheroids (for all objects) and triangles represent disks. We plotted the absolute magnitudes to avoid overcrowding of the plot. The solid line indicates the loci of $M_{B}-M_{H}=3.7$. The second panel compares $R_{\text {exp }} / R_{\text {eff }}$. In most cases there is reasonable agreement, although differences as large as a factor of three exist. Again, most of the scatter must be attributed to the H-band data. The bottom panel compares the derived inclinations, uncorrected for disk thickness.
our S/D fit rejection criteria, some of the objects without disks appear to be well fit by an $R^{1 / 4}$ law along the minor axis, but are much better fit by a composite profile along the major axis: this yields a formal fit of a highly inclined disk. On the other hand, the observed positive $A_{4}$ in some face-on systems may be due to some non-axisymmetric structure, such as a favorably oriented bar.

All these checks illustrate that the photometric decompositions are on the one hand feasible and without obvious bias towards overemphasizing either the spheroid or the disk component, yet on the other hand are very "risky" and can be completely spurious in some case. Nonetheless, they may be used for statistical studies, even if some of the individual decompositions are very arguable.

### 3.6 Discussion

After having discussed the characteristics and limitations of both the data and their modelling, we will now try to answer the initially posed questions about the properties and frequency of disks in early type galaxies. While we concentrated on purely geometric techniques in Chapter 2 (i.e. analyzing the isophote shapes), we have focussed so far on the luminosity profiles in this chapter. We will continue to keep the two methods of disk estimation separate, to employ their comparison as a consistency check.

To discuss the overall physical properties of the sample we should strive for statistical completeness, which would require us to combine the B and H data. Yet, in many respects the inclusion of the H band data is problematic: the decompositions are much less secure due to the larger contribution of the sky to the total counts. Since the H data were taken under non-photometric conditions, the combination of intensity scales ( $I_{0}$ and $I_{\text {eff }}$ ) from the two samples is difficult, when comparing


Figure 3.11: $A_{4}(\max )$ vs. $\cos (i)$
This figure compares Fourier residuals $A_{4}$ to the uncorrected values for the disk inclinations, for all B-band decompositions. All objects with large $A_{4}(>3 \%)$ are found to be highly inclined, as expected for S/D systems. However, there are highly inclined systems $\cos (i)<0.4$ without significant $A_{4}$, as well as apparently face-on systems with considerable Fourier residuals. The cause for the former could be either spurious fits of a disk component or errors in the inclination estimates as large as 0.2 . The latter effect just emphasizes again that inclined S/D systems are not the only possibility to yield disky isophotes. These Fourier residuals could e.g. be caused by a favorably oriented bar.
spheroid and disk luminosities. However, for isophote shape statistics the completeness of the sample is more important than the details of the photometric calibration. In the following discussion we will rely mostly on the $B$ band data, and only complement them by the IR data for the objects not observed in the blue. Furthermore, in the cases where the $A_{4}$ residuals appeared much more regular in H than in B, we used the IR data to assign $A_{4}$ rather than the optical data. Obviously, we will use all the data when discussing color gradients.


Figure 3.12: Inclination Bias in the Disk Detection This figure shows the formally derived inclination (corrected for an intrinsic disc flattening of $c / a=0.15$ ) against the scale length ratio of the two components. While systems with disk scales substantially larger than the spheroids are detected both when highly inclined and when face-on, systems with $R_{\text {exp }} \approx R_{\text {eff }}$ are only found if considerably inclined ( $\cos (i)<0.55$ ). Note the lack of highly inclined systems with very large $R_{\text {exp }} / R_{\text {eff }}$. They are presumably missing from the sample (see text).

### 3.6.1 Which Disks Would Have Been Seen From All Angles?

We have mentioned repeatedly that disk components are easier to detect, the more inclined they are, because their surface brightness is then larger compared to the spheroid's due to projection effects. In light of that we might ask how many of the S/D systems we found from our modelling are "conspicuous" in the sense that we would we have seen them even if they had been face-on? These are the objects which can be recognized as " SO s" independent of viewing angle. Figure 3.12 is the key to understanding which disks are conspicuous: it shows the derived inclinations, $\cos (i)$, as a function of $R_{\text {exp }} / R_{\text {eff }}$ and reveals the viewing angle bias in the disk detection.

While S/D systems with $R_{\text {exp }} / R_{\text {eff }} \gtrsim 2$ are found both highly inclined and faceon, objects with $R_{\text {exp }} / R_{\text {eff }} \approx 1$ are only detected for $\cos (i)<0.55^{13}$. The cause of this bias becomes apparent from the bottom panel of Figure 3.13, which shows for all objects with composite models how much their radial (model) profile would deviate from an $R^{1 / 4}$ law if they were seen face-on. The quantity $\chi_{\text {face-on }}^{2}$, characterizing these deviations, is small ${ }^{14}$ for almost all systems with $R_{\text {exp }} / R_{\text {eff }} \approx 1$.

It follows that S/D systems with $R_{\text {exp }} / R_{\text {eff }}$ substantially different from unity can be detected from any vantage point (even from face-on), while objects where the spheroid and the disk have comparable scale lengths are virtually indistinguishable from pure spheroids unless they are substantially inclined. We will refer to the latter class as objects with "inconspicuous" disks, a more appropriate term than "weak" disks. This difference arises because S/D systems with discrepant component scale lengths show a characteristic "inflection" (see e.g. UGC 7165 in Figure 3.2) in their total luminosity profile, which can be recognized from all viewing angles.

Closer examination of Figure 3.12 shows that the majority of large disks have large values of $\cos (i)$, i.e. appear preferentially face-on ${ }^{15}$. This deficit of edge-on galaxies "worsens" for objects with $R_{\text {exp }} / R_{\text {eff }}$ of about 10, where all objects, but one, have $\cos (i)>0.6$. This deficiency is not caused by any flaw or bias in our fitting procedure, because in that case we should expect (truly inclined) objects with formal low inclinations $(\cos (i)>0.5)$, yet large $A_{4}$; according to Figure 3.11, this does not occur.

There are several possible explanations for this lack of objects. Among the ran-

[^16]

Figure 3.13: Face-On Detectability of Disk Components
The top panel compares the quantity $\chi_{\text {noise }}^{2}$ to the quantity $\chi_{\text {face-on }}^{2}$ for all objects, regardless of the acceptance of their $\mathrm{S} / \mathrm{D}$ decomposition. Circles represent the B band data and tripods represent the H -band data. The quantity $\chi_{\text {noise }}^{2}$ characterizes the small scale deviations of the angle averaged profile from a composite model fit, i.e. it quantifies the smoothness of the profile. The quantity $\chi_{\text {face-on }}^{2}$ quantifies the residuals of an $R^{1 / 4}$ law fit to the smooth profile of the two component model fit to the object and rotated face-on; it characterizes the overall deviations of the profile from an $R^{1 / 4}$ law. We use this comparison to decide when a disk component is detectable through the luminosity profile. Most galaxies have $\chi_{\text {noise }}^{2}<0.3$, as indicated by the vertical line. We accept a disk fit if the $\chi^{2}$ from the major axis fit exceeds this value of 0.3. Similarly, we presume that we could have detected a disk if a fit to the model still yields $\chi_{\text {face-on }}^{2}>0.3$, after having rotated the model to a face-on position.
The bottom panel shows $\chi_{\text {face-on }}^{2}$ as a function of $R_{\text {exp }} / R_{\text {eff }}$. The two component nature of galaxies with $R_{\text {exp }} / R_{\text {eff }}>2$ can be and is detected from a face-on vantage point. The disks in all objects with $R_{\text {exp }} / R_{\text {eff }} \approx 1$, however, were only detected because of their favorable orientation.
dom $10 \%$ of galaxies which satisfied our sample selection criteria, but were not observed, three have overall ellipticities of 0.65 or higher and thus presumably are highly inclined objects with large $R_{\text {exp }} / R_{\text {eff }}$. Second, the smallness of these spheroids only becomes apparent for inclined objects. Since the S/D ratio is one of the classification criteria for the Hubble type, the highly inclined counterparts of these sample members may be classified as " $\mathrm{S} 0 / \mathrm{Sa}$ " and therefore be missing from the sample. Furthermore, large disks with small spheroids usually contain considerable amounts of gas and dust ${ }^{16}$; this dust content may only become apparent for inclined systems, again leading to re-classification of these objects as "S0/Sa."

### 3.6.2 How Many Disks Did Elude Detection?

Using the radial luminosity profiles and the isophote shapes, we can estimate the number of photometrically unrecognizeable or unrecognized disks in the sample in two distinct and nearly independent ways.

## Estimate from Principal Axis Fitting

The first estimate is based on the principal axis fitting just described. As shown in Figure 3.12, we found acceptable S/D decompositions for fourteen objects with $R_{\text {exp }} \approx R_{\text {eff }}$, all of which are considerably inclined ( $\cos (i)<0.55$ ). A comparable number of less inclined counterparts with undetectable disks must be contained in the sample. Thus about 26 objects, or $30 \%$ of all galaxies in the sample, contain disks that can only be detected from a favorable perspective. This estimate would be a lower limit if the sample contained objects with flat disks of very low surface brightness, which would not significantly impact the radial luminosity profile, and thus remain undetected by this method, regardless of their orientation.

[^17]
## Estimate from Isophote Shape Statistics

Alternatively, we can estimate the frequency of inconspicuous disks by means of the viewing angle statistics described in Chapter 2. Suppose we took the total sample, and made a small correction for the missing edge-on objects with large $R_{\text {exp }} / R_{\text {eff }}$, then we would find that just under half of the objects show pointed isophotes ( $>1 \%$ ) at some radius (see Tables 3.2 and 3.3). From the arguments made in Chapter 2 this implies that the majority of the sample members have significant disks.

More stringent statistical constraints on the number of diskless objects and objects with inconspicuous disks can be obtained if we manage to dispose of all galaxies with conspicuous disks without introducing a viewing angle bias. This can be achieved by discarding all objects whose disks could have been detected regardless of inclination, i.e. objects for which $\chi_{\text {face-on }}^{2}$ exceeds a certain limit. We chose twice the median value of $\chi_{\text {noise }}^{2}$ as this cut-off value (Figure 3.12), that is we eliminated S/D systems for which the overall deviation from an $R^{1 / 4}$ law was larger than typical deviations from the fit due to small scale variations in the profile. As Figure 3.14 will show, the subsequent results are not very sensitive to the exact choice of this cut-off. This process eliminates 23 sample members with conspicuous disks.

We can now reexamine the $A_{4}$ statistics from Chapter 2 for this remainder of the sample. Before doing so, we should stress again that we are now in a much stronger position to interpret these statistics, because the members of our present sample are, by construction, seen at random inclinations. When we applied these statistics previously to a sample of elliptical galaxies (Chapter 2), edge-on galaxies with large $A_{4}$ "ran the risk" of being classified as $S 0 \mathrm{~s}$ and thus be excluded from the sample.

Figure 3.14 shows the distribution of diskiness, $A_{4}$, in this trimmed sample: $30 \%$ of the objects show $A_{4}>1 \%$. The three symbols in Figure 3.14 represent the
distributions for different values of the $\chi_{\text {face-on }}^{2}$ cut-off, and for a sample in which the B band data were complemented by the H band data. They demonstrate that the fraction of objects with $A_{4}>1 \%$ is insensitive to the details of the sample selection and the process of eliminating conspicuous disks. There are no objects with $A_{4}>15 \%$, which would be expected from the simple models of Chapter 2 if the sample contained edge-on objects with significant disks. This can be attributed to the finite thickness of realistic disks: since the parameter $\cos (i)$ in our models refers to the axis ratio of the disk component isophotes, disks of a finite thickness will exhibit a minimum formal $\cos (i)$ greater than zero, even if seen perfectly edge-on. Specifically, if we denote the axis ratio of the disk isodensity surfaces (the intrinsic disk thickness) as $(c / a)_{\text {disk }}$, then the fitted and the true disk inclination are related by

$$
\begin{equation*}
\cos (i)_{o b s}=\sqrt{(c / a)_{d i s k}^{2}+\left(1-(c / a)_{d i s k}^{2}\right) \cos ^{2}(i)_{t r u c}} . \tag{3.7}
\end{equation*}
$$

As the next step, we must decide on a statistic to compare the $A_{4}$ distribution to the models of the previous chapter. From a statistical point of view, we would like to compare the measured $A_{4}$ with the model expectations for as large a fraction of the sample as possible, i.e. the whole sample. However, since virtually all objects show isophote deviations from perfect ellipses at some level, a comparison of very small $A_{4}$ becomes conceptually meaningless ${ }^{17}$. On the other hand, if we only use the fraction of objects for which the interpretation of $A_{4}$ is absolutely safe, such as $f\left(A_{4}>5 \%\right)$, then we are left with only a handful of objects and our test loses any statistical power. Thus we must compromise between the two extremes, and choose some limit above which we consider isophote deviations significant, in the sense of our modelling. Here, we have chosen to compare the fraction $f\left(A_{4}>1 \%\right)$ between

[^18]

Figure 3.14: $A_{4}$ Distribution for Objects Without Conspicuous Disks This figure shows the cumulative distribution of $A_{4}$ for subsamples from which objects with conspicuous disks have been removed. Only $A_{4}>0.5 \%$ is shown, to allow logarithmic scaling of the $A_{4}$ axis. The open triangles represent the distribution for the $B$-band sample, with all objects removed which had $\chi_{\text {face-on }}^{2}>0.3$; the open squares show the distribution for a cut-off at 0.15 . The open pentagons, represent the distribution for the total sample ( B and H combined) and a cut-off of $\chi_{\text {face-on }}^{2}>0.3$. The fraction of objects with $A_{4}>1 \%$ is found to be about $25 \%$ in all cases and thus not to be sensitive to the details of the subsample selection. The solid line indicates the predicted $A_{4}$ distribution from the modelling of Chapter 2, assuming that $50 \%$ of the sample has $S / D \approx 0.3$ and $R_{\text {exp }} \approx R_{\text {eff }}$, while the other half is disk less. No attempt to model the finite thickness of the disk components (and their impact on $A_{4}$ ) has been made.
the data and the models. An additional advantage to not restricting ourselves to only the most inclined objects is that our analysis is less sensitive to the exact thickness of the disk components. The disk thickness enters into this comparison because the measured $A_{4}$ relates to $\cos (i)_{\text {observed }}$, while it is the quantity $\cos (i)_{\text {true }}$ which is uniformly distributed. The difference between the two is largest for edge-on objects.

If we were to assume that all galaxies in the sample are identical, the simple model from Chapter 2 would yield a D/S ratio estimate of 0.05 , given the $30 \%$ fraction of objects with $A_{4}>1 \%$ (see Figure 2.2). However, from the parameters derived for edge-on sample members with secure S/D decompositions (e.g. UGC 292 and UGC 7634) and from the modelling of NGC 4660 (Chapter 2), it appears more likely that the two components in these objects have $\mathrm{D} / \mathrm{S}$ ratios around 0.5 . If we now suppose, that a certain fraction, $f_{D / S}$, of the sample had disk components contributing a sizeable fraction, say between $15 \%$ and $50 \%$, to the total light, while the remainder were diskless objects, we can estimate how large $f_{D / S}$ would need be to yield a sufficient number of objects with significantly positive $A_{4}$. Figure 2.2 of Chapter 2 shows that in this regime of disk light fraction such an estimate is relatively insensitive to the exact value of $\mathrm{D} / \mathrm{S}$ : about $50 \%$ of all spheroid-disk systems would show $A_{4}>1 \%$. Since the present sample contains $25 \%-30 \%$ of objects with such pointed isophotes, we infer $f_{D / S} \approx 0.5$. Just on the basis of small number statistics this value has an uncertainty of

$$
\begin{equation*}
\Delta f_{D / S} \approx 1 / \sqrt{N_{A_{4}>1 \%}} \tag{3.8}
\end{equation*}
$$

with $N_{A_{4}>1 \%} \approx 14$ (see Figure 3.15), and thus could range from 0.3 to 0.65 . It would be futile to be more formal about these statistical estimates, since substantial systematic uncertainties are connected to the reliability of attributing the diskiness of the isophotes to a $S / D$ superposition.

It is reassuring that these two different estimates, one based on the radial profiles the other on the isophote shapes, yield comparable results: about 25 to 30 objects in the total sample contain inconspicuous disks. Admittedly, each method is based on idealized assumptions about the light distributions, namely the stereotypical radial light profiles in the one case and the projection of each component into perfect ellipses in the other. Therefore their agreement in the results is non-trivial, and it implies that even though in individual cases these idealized assumptions can fail, they appear to give a consistent picture in a statistical sense.

### 3.6.3. Is There Continuity From SO's to E's?

Do our data provide corroborating evidence for the hypothesis that there is physical continuity along the Hubble sequence from Es to S0s, rather than only an observational grey-zone of disk detectability? Does the original suggestion by Carter (1987), Jedrejewski (1987), Capaccioli (1987) and Capaccioli et al. (1990) hold that for $R_{\text {exp }} \approx R_{\text {eff }}$ there exists a wide range of spheroid-to-disk ratios, with the disks just fading away compared to the spheroid?

We were able to demonstrate that all the photometric signatures are consistent with a simple model for the S/D population of early type galaxies: about one third of all objects have disks with scale lengths larger than the spheroids and $D / S$ ratios near unity. The second, physically not distinct, third consists of objects of somewhat lower D/S ( $\sim 0.5$ ), for which the disks are not much larger than their spheroids; their two-component nature can only become apparent for favorable orientations. The last third of objects are diskless spheroids ${ }^{18}$. Such a population is consistent with all the observational evidence presented here and there is certainly no need to invoke continuity. Beyond that, the highly inclined objects from the second

[^19]group which permit S/D decompositions indicate substantial disks of relatively high surface brightness $\left(I_{0}(B) \approx 22 \mathrm{mag} / \square^{2}\right)$.

Their less inclined counterparts must be present in the sample and will mimic the presence of a faint disk. Identifying the sample members with subtle disk signatures with these less inclined counterparts does not leave much "room" for an additional population of galaxies with flat, lower surface brightness disks: most instances of observed diskiness already have been accounted for by the second group of objects.

There are, however, systems with very small values of $D / S$, which we will discuss in the next chapter. Yet, they must be termed more appropriately "small" disks rather than "faint" or "weak" disks. Their small D/S is exclusively caused by a small $R_{\text {exp }} / R_{\text {eff }}$ rather than by low surface brightness ${ }^{19}$.

Of course we cannot exclude the existence of $D / S$ systems, in which the disk does not contribute substantially to the total light at any observed radius, even for highly inclined galaxies. But from all the observable signs of disks in early type galaxies we conclude that whenever a flat disk is present it is of a face-on surface brightness comparable to spiral galaxies and it constitutes a significant fraction ( $15 \%$ to $70 \%$ ) of the total luminous matter.

Finally, let us return for a moment to Hubble classifications: about $60 \%$ of our sample were classified as E or $\mathrm{E} / \mathrm{S} 0$, the rest as S 0 . The discussion above indicates only that $30 \%$ to $40 \%$ of our sample are diskless objects. Thus one third to one half of all galaxies classified as ellipticals have significant disks. This statement refers directly only to the particular way that the UGC galaxies were classified. Yet, since the number ratio of E's to SO's per luminosity bin in the UGC is comparable to the one in the RC2 (De Vaucouleurs et al. 1976) and the Shapley-Ames Catalog (Sandage and Tammann 1981), this result should be typical of other comparable

[^20]sets of galaxies and classifications.

### 3.6.4 What Do Disks and Spheroids Know About Each Other?

Given the photometric scale parameters from our model fits, $I_{\text {eff }}, I_{0}, R_{\text {eff }}$ and $R_{\text {exp }}$, we could ask to what extent these parameters are correlated, and if there are correlations, how many independent parameter combinations exist. The standard way of addressing this question is by Principal Component Analysis (e.g. Kormendy and Djorgovski 1990). However, there are several problems with our data set, preventing the application of this algorithm: first, we have accepted decompositions for only $40 \%$ of the sample members. Second, to increase the reliability of the disk detection, we have severely limited the parameter subspace of "acceptable" D/S combinations. Thus apparent correlations may very well reflect only our selection criteria. One particular spurious correlation can be introduced through the conservation of total luminosity: e.g. large disks are found to have low surface brightnesses and vice versa (see also K85 and Kodaira et al., 1986). Therefore we will restrict ourselves to a few conservative comments on some of the parameter correlations found in our data:

Figure 3.15 compares the absolute magnitudes of disks and their associated spheroids. A clear correlation is seen, implying that large disks and spheroids go together. Nonetheless, for each spheroid luminosity the disk luminosity can vary by an order of magnitude. Thus the apparent correlation just means that the spread in absolute magnitudes in the sample is larger than the range of $\mathrm{D} / \mathrm{S}$ ratios at any given luminosity. Is this merely a result of our inability to carry out decompositions for sub-components of very unequal luminosity? For very large values of $D / S$ this is certainly not the case; even very small bulges will dominate the light at the innermost radii. Rather, it seems likely that the Hubble type based selection of


Figure 3.15: Absolute Spheroid and Disk Magnitudes
This figure compares the (absolute) disk and spheroid brightnesses in the B band for the composite systems in the sample. It shows that in general more luminous disks are associated with more luminous spheroids; however, for any given spheroid there is an order of magnitude spread in disk luminosities. The correlation of the two parameters stems from the two order of magnitude spread in total absolute magnitudes among the sample members. Phrased differently, at each given luminosity there is a factor of ten spread in the $\mathrm{D} / \mathrm{S}$ ratio.
our sample is responsible for the upper limit on D/S: as indicated in Appendix C, most of the objects with $D / S>5$ contain conspicuous dust and hints of spiral structure. Similarly, Kent (1985) and Boroson et al. (1983) find that all galaxies with $D / S>10$ are late type spirals. For very small D/S the detectability of disks is an obvious limit. Nevertheless, we can exclude a large population of flat disks of lower surface brightness, because its highly inclined members would still yield an $A_{4}$ signature without being detectable through principal axes fitting. As discussed above this appears not to be the case. However, very small, nuclear disks with scale lengths of only a few arcsecond may be missed in our analysis.

In Figure 3.16 we show how the characteristic surface brightnesses of the disks
(top panel) and the spheroids (bottom panel) depend on their physical scale length (in kpc ). The solid line in both panels of Figure 3.16 connects loci of constant luminosity. The top panel deserves several comments: It shows that most S0 disks have surface brightnesses that are comparable to later type disk galaxies, confirming the findings by Kent (1985). Second, most of the spread in disk luminosities,

$$
\begin{equation*}
L_{D i a k}=2 \pi I_{0} R_{e x p}^{2} \tag{3.9}
\end{equation*}
$$

can be traced to the range of disk scale lengths, rather than surface brightness changes: the disk scale lengths vary by more than an order of magnitude, resulting in two orders of magnitude change in disk luminosity, while the central surface brightnesses only span a smaller range. In particular, for the disks there is an anticorrelation between their size and their surface brightness, causing small disks to be brighter than expected from scaling larger disks down in size. This is not, at least not exclusively, due to problems of detectability, because this correlation is stronger when comparing $I_{0}$ with $R_{\text {exp }}$, rather than with $R_{\text {exp }} / R_{\text {eff }}$. A similar result was found by Kent (1985) for later type galaxies, where the decompositions are more robust. Kent attributed this correlation to his sample selection; he chose a volume limited sample within a given absolute luminosity bin. Since the present sample has a spread of more than five in distance (mostly from the relative distances of Virgo and Coma), it samples a range of at least 4-5 magnitudes in absolute luminosity. It is therefore unlikely that this correlation is entirely a selection effect.

### 3.6.5 Are Disks Younger Than Their Spheroids?

Age differences between various sub-components of a galaxy, will be reflected in differences of the stellar population, which in turn can result in color differences. Even though we can not measure an absolute color from our data, we can still assess color gradients. In this section we will briefly describe the measurement and


Figure 3.16: Intensity vs. Radial Scale Parameters
This figure shows the B band intensity scales ( $I_{0}$ and $I_{\text {eff }}$ ) versus the radial scales ( $R_{\text {exp }}$ and $R_{\text {eff }}$ ) for the disks (top panel) and the spheroids (bottom panel). In the bottom panes the open triangle refer to the spheroids with detected disk components, while all the others are shown as solid dots. The lines in the diagrams connect loci of constant luminosity.
significance both of the overall color gradients and of the limits that these gradients impose on the presence of color differences between the spheroids and the disks.

## Color Gradients

Gradients are frequently observed in the optical colors of early type galaxies (for a recent review, see Kormendy and Djorgovski 1990), with most galaxies exhibiting outward blueing. As discussed e.g. in Peletier (1989), these changes in color can be attributed to gradients in two basic physical characteristics of the stellar population, its mean age and its mean metallicity: either the stellar population becomes younger at larger radii, or it becomes more metal-poor, or both. At present most authors favor metallicity changes as the prime cause of the optical color gradients in most ellipticals (e.g. Kormendy and Djorgovski 1990, Gorgas and Efstathiou 1987, Peletier 1989), for several reasons. First, a vast range in ages is required (4 Gyrs to 15 Gyrs) to explain the color changes purely by age changes; however, we know from observations of high redshift ellipticals (e.g. Eisenhardt and Lebofsky 1987), that the temporal evolution of the optical and near-IR colors has been very modest since $z=1$. Second, most early type galaxies exhibit an outward drop in the mean absorption line strength of their stellar population (e.g. Gorgas and Efstathiou 1987), implying an outward decrease of the metallicity. Finally, all dissipative formation scenarios of galaxies predict a metallicity gradient (Larson 1976, Carlberg 1986). All these results suggest that a metallicity gradient of $\partial([\mathrm{Fe} / \mathrm{H}]) / \partial(\log R)$ of about 0.15 dex is typical for intrinsically bright ellipticals (Kormendy and Djorgovski 1990, Peletier 1989).

In Figure 3.3 we showed the color profiles for the galaxies observed both at $B$ and at H , as derived from the angle averaged run of the luminosity. All profiles had to be normalized to the sample mean of about $\mathrm{B}-\mathrm{H}=3.7$ at $3^{\prime \prime}$, because the data are
not photometric. Except for a few profiles which are irregular due to dust extinction in the B band, most profiles are smooth and generally indicate an outward blueing of the galaxy. We can sensibly characterize the radial dependence of $B-H$ for most galaxies with a simple gradient $\partial(\mathrm{B}-\mathrm{H}) / \partial(\log R)$, by fitting a linear function to the profiles shown in Figure 3.3. The distribution of the resulting gradients is presented in Figure 3.11, separately for S/D systems and for "pure" spheroids (according to Table 3.2). In both cases it is clustered around $\partial(\mathrm{B}-\mathrm{H}) / \partial(\log R)=-0.2$, with a dispersion of about 0.15 for the S/D systems and a somewhat smaller dispersion for the spheroids. Some of the outliers in the distribution are caused by irregular profiles; only UGC7637 and UGC9903 were excluded.

The errors for these gradients are difficult to quantify since they depend on the setting of the sky level, the seeing, etc., which we tried to minimize by restricting the radial fitting range. Monte-Carlo simulations showed that different ways of setting the sky level (e.g. fitting the profiles with an $R^{1 / 4}$ law and, alternatively, with a composite S/D model lead to a scatter of about 0.07 in $\partial(\mathrm{B}-\mathrm{H}) / \partial(\log R)$. An error of this size is indicated by the horizontal bar in the bottom panel of Fig 3.11. Furthermore, the constancy of the slope throughout the entire radial range for many objects and the small scatter for the spheroids, can be used to argue the reality of the gradients.

For one of our objects we can check the derived color gradient against external measurements. Peletier (1989) found that the optical and IR color gradients in ellipticals are well correlated; in particular objects with radially independent optical colors also do not show optical-IR color gradients. Peletier (1989) found no significant gradients for UGC 7517; this galaxy is among the few for which we also find constant colors over the observed radial range (see Figure 3.5).

With only one color available there is no hope of untangling age and metallicity
effects. Following the arguments above, we will for now assume that the gradients in the spheroids are predominantly caused by changes in metallicity and will estimate the required $\partial([\mathrm{Fe} / \mathrm{H}]) / \partial(\log R)$ to produce the observed outward blueing. Peletier (1989) has used the revised Yale isochrones (Green et al. 1987) to calculate opticalIR colors for an old stellar population for a range of metallicities ( 0.2 to 2 of solar metallicity). For an age of about 12 Gyrs he finds that V-K changes by -0.65 mag , and $\mathrm{B}-\mathrm{V}$ by -0.08 mag , when decreasing the metallicity by a factor of four ( 0.6 dex ). Neglecting $\mathrm{H}-\mathrm{K}$ gradients, this implies a B-H gradient of -0.17 mag for a $40 \%$, or 0.15 dex, drop in metallicity. Applying this to the distribution shown in Figure 3.13 we find that the mean color gradient derived from our B-H data implies a mean metallicity gradient which is in excellent agreement with the one derived from the optical colors, confirming Peletier's (1989) results.

## Color Differences Between Spheroids and Disks?

In a recent paper, Bothun and Gregg (1990) claimed to have measured B-H color differences between the spheroids and disks in 35 S 0 galaxies: they found from their aperture measurements that disks were on average half a magnitude bluer than their associated spheroids. Arguing from the similarity of the J, H and K colors of both components ${ }^{20}$ that this difference should be caused by age differences, they find a much younger age for the disks components ( $\sim 5$ Gyrs) .

We can use the color gradients derived above to check this claim. As seen from Appendix A, the light at small radii ( $<5^{\prime \prime}$ ) in virtually all S/D models is dominated by the spheroid, while the disk contributes significantly to the light at $20^{\prime \prime}$. Suppose each component had radially constant colors, but there was a color offset between them. In the resulting composite luminosity profile this will

[^21]

Figure 3.17: Distribution of $\mathrm{B}-\mathrm{H}$ Color Gradients
This figure shows histogram of the global $\mathrm{B}-\mathrm{H}$ color gradients, derived by fitting a linear function to the color profiles (Fig. 3.3) over the range of $3.5^{\prime \prime}$ to $20^{\prime \prime}$. The top panel shows the distribution for S/D systems (according to Table 3.2) and the bottom panes shows the distribution for "pure" spheroids. The error bar in the bottom panel indicates the uncertainty in the gradient, estimated from changes in the gradient due to different ways of assessing the sky level. Virtually all objects have negative gradients, i.e. become bluer towards larger radii. The S/D systems have a larger spread in color gradients and in particular have more frequently steep $\mathrm{B}-\mathrm{H}$ gradients (statistical significance $\approx 90 \%$ ). The expected $\mathrm{B}-\mathrm{H}$ gradient for a metallicity change of $0.15 d e x$ per radial decade is indicated (from Peletier 1989). Thus the metallicity gradients inferred from optical colors and from optical-IR colors are in good agreement. The arrows in the top panel indicate the expected increase in color gradient for two typical S/D systems (see text), assuming the disk is 0.5 mags. bluer than the spheroid.
result in a color gradient: B-H decreases towards large radii, where the bluer disk contributes more to the total light. Figure 3.18 quantifies this statement. We show the resulting color gradients for two representative S/D systems; the first has $R_{\text {exp }} / R_{\text {eff }}=0.75$ and $D / S \approx 0.35$, the second has $R_{\text {exp }} / R_{\text {eff }}=3.0$ and $D / S \approx 1.0$, both in the B band. The panels show the resulting differences between the B and H profiles, assuming the disk is fainter in H by 0.2 mag and 0.5 mag , respectively. The spheroid-disk transition causes significant color gradients: for the small disk model the resulting color gradient, $\partial(\mathrm{B}-\mathrm{H}) / \partial(\log R)$, is -0.11 and -0.24 , respectively. For the larger disk the gradients are even stronger, -0.19 and -0.45 . If each component by itself had an intrinsic color gradient, these S/D gradients would have to be added to them.

Do the measured color gradients place limits on the possible color differences between spheroids and disks? Figure 3.17 shows that the mean color gradient for S/D systems and "pure" spheroids is comparable, with the S/D systems showing a larger spread. If most of these disk galaxies had color differences between their sub-components as large as 0.5 magnitudes, the distribution in the top panel should be shifted to the left by about 0.2 to 0.4 (as indicated by the arrows), which is clearly inconsistent with the measurements. The formal limit on the mean gradient difference between the two subsets is quite small ( $<0.1$ ). This argument assumes that disk galaxies typically have the same color gradient as the spheroids in the bottom panel, if they have no color offset in color between the sub-component. It may very well be that the neglect of intrinsic color gradients has led Bothun and Gregg (1990) to attribute the outward blueing of S0s to a much bluer and thus younger disk. As a check on these results one could use the difference of the mean ellipticity between B and H (see middle panel of Figure 3.18). For a disk differing by 0.5 mag in color $\langle\epsilon\rangle$ should be about +0.02 to +0.05 . Although such


Figure 3.18: Photometric Signatures of a Blue Disk
This figure illustrates the differences between the B and H photometric profiles of an S/D galaxy assuming the disk is bluer than the spheroid. We have picked two "representative" S/D models; the first is characterized by $R_{\text {exp }} / R_{\text {eff }}=0.75$ and $\mathrm{D} / \mathrm{S}=0.35$, the second by $R_{\text {exp }} / R_{e f f}=3.0$ and $\mathrm{D} / \mathrm{S}=1.0$. The top panel shows the resulting color gradient, the center panel shows the differences in the ellipticity profile and the bottom panel the $A_{4}$ differences. In each panel the results of the first model are shown at the top. The solid symbols refer to a spheroid-disk color difference of half a magnitude, the open symbols to a difference of 0.2 mags.
a color dependent flattening cannot be ruled out by the present data (we have no independent check on the errors in $\langle\epsilon\rangle$ ), it does not appear to be present in the data.

Before we discuss the astrophysical implications of this lack of color difference we should note two points. First, the top panel includes all objects that had accepted S/D decompositions in B. From the earlier discussion about decompositions, this does not mean that every single object contains a disk. Also there are S/D systems for which the disk does not contribute strongly to the major axis light between $3^{\prime \prime}$ and $20^{\prime \prime}$. Nevertheless, this should affect only a minority of S/D sample members. Second, the S/D systems show a larger dispersion in gradients than the spheroids. This may, in parts, be attributed to the increased uncertainty in measuring the color gradient, because we have more freedom to fit the sky level. Monte-Carlo simulations of this error source, however, indicate that this is insufficient to explain the spread, which therefore must be real. Finally, one should note that there is a good number of S/D systems which exhibits stronger outward blueing than any of the spheroids, consistent with the presence of a bluer disk.

What does this comparison of color gradients imply, for the "typical" (i.e. sample mean) difference in population between the disk and the spheroids? Let us suppose first that the two components have comparable metallicity, at any given radius, as argued from the JHK colors by Bothun and Gregg (1989) and that any difference is attributed to age. The limit on the age difference between these components depends on the overall age of the population. Bothun et al. (1984) have modelled the $\mathrm{B}-\mathrm{H}$ color evolution for populations between 3 and 10 Gyrs, while Peletier (1989) has calculated colors for old populations ( $10-20 \mathrm{Gyrs}$ ). A color difference of 0.1 magnitudes corresponds to an age difference of only 1.5 Gyrs for a 7 Gyrs old population, but it corresponds to 6 Gyrs for a mean age of 13 Gyrs. Thus, at least if the whole population is relatively young, the disk and spheroid must be
nearly coeval. Alternatively, we could suppose that the populations are coeval and constrain the metallicity differences between the two components. In that case, we find that the metallicities should be very similar, $\Delta[F e / H]<0.1 d e x$, (thus the similar color gradients for the subsets), with a possible spread in metallicity differences of about 0.15dex.

With either of these two assumptions we find that the populations of spheroids and disks in the majority of observed $\mathrm{S0s}$ are quite similar. The only alternative is to presume that the similarity in $\mathrm{B}-\mathrm{H}$ is due to compensating effects despite the presence of differing stellar populations. Without further color information this cannot be straightforwardly disproved. Yet, some arguments suggest that this is unlikely: First, since disks are dynamically cold they must either be coeval with or younger than the spheroid. Second, in the cases where we can separate the spheroid and the disk clearly, as in the case of the Milky Way, the disk may be more metal rich than the spheroid at a given radius; however, to reach significantly higher metallicities it had to form much later. As a result it is nonetheless bluer than the spheroid at the same radius. Thus, age and metallicity differences are not likely to trade off yielding a vanishing $\mathrm{B}-\mathrm{H}$ color difference. Is it possible that dust makes the disk components appear redder? For some objects this may be the case, and could explain the shallow gradients for some S/D systems. However, for the majority of the objects this effect should be negligible, judging from the obscuration in late type spirals and scaling the dust content down to the smaller HI content of S0s (e.g. Knapp 1990).

### 3.7 Summary

We have obtained and analyzed surface photometry at $0.4 \mu$ and $1.6 \mu$ for a magnitude limited sample of eighty early type ( E and S 0 ) galaxies. We have used these data
to study the frequency and characteristics of disk components in these objects, employing both the fitting of composite spheroid/disk models to the principal axis luminosity profiles, as well as the isophote shape statistics developed in Chapter 2. The main results are the following:

- The overall disk census showed that one third of the objects have "conspicuous" disks. These are disks that have a substantially larger scale length than their associated spheroid ( $R_{\text {exp }} / R_{\text {eff }}>2$ ) and which would have been detected from all viewing angles. A second third has "inconspicuous" disks, in the sense that a favorably high inclination is required for these disks to show clear photometric signatures. This category consists of S/D systems in which the two sub-components have similar scale lengths ( $0.3<R_{\text {exp }} / R_{\text {eff }}<2$ ); thus their composite light profile can be well approximated by a simple luminosity model such as an $R^{1 / 4}$ law. Only if these systems are substantially inclined does the disk become apparent through its flatness. The final third of the sample are "diskless" objects. Diskless here means that any present disk is too weak to cause pointed isophotes even if seen edge-on.
- We have estimated the frequency of inconspicuous disks independently by means of luminosity profile fitting and isophote shape statistics. The two methods yield consistent estimates. Thus all of the observed "diskiness" in the isophotes can be explained by disks which are strong enough to imprint themselves in the luminosity profiles. Therefore, disks in "elliptical" galaxies do not appear to be very weak as originally proposed by Carter (1987), Jedrzejewski (1987) and Capaccioli et al. (1990).
- In particular we find that these inconspicuous disks do not have abnormally low surface brightnesses ( $\approx 22 \mathrm{mag} / \square^{2}$ in $B$ ) and thus they may have formed
in the same fashion as their more prominent counterparts. The considerable spread in disk-to-spheroid ratios observed among the sample members ( $0.15 \lesssim D / S \lesssim 5$ ) is predominantly caused by changes in the scale length ratios, $R_{\text {exp }} / R_{\text {eff }}$, rather than a wide spread in disk surface brightnesses.
- These results do not support the hypothesis of continuity along the Hubble sequence from S0's to E's in the sense of a progression in which disks fade away compared to the spheroid. All the apparent continuity can be explained through changes in viewing angles, assuming a physically discrete model, in which galaxies either have substantial disks (typically $30 \%$ of the total light) or none at all.
- Comparing the luminosity profiles in two colors ( B and H ) we were able to derive radial color gradients for a subset of forty objects in our sample; most objects show outward blueing with a mean $\partial(\mathrm{B}-\mathrm{H}) / \partial(\log R)$ of about 0.15 . Using these gradients we were able to put limits on how much bluer the disks are in the S/D systems than the spheroids. We find $\Delta(B-H)<0.15$, in conflict with an earlier claim of 0.5 by Bothun and Gregg (1990). Although on the basis of the available information a trade-off between lower age of the disk (making it bluer) and higher metallicity (making it redder) cannot be ruled out, this lack of color difference suggests that the mean ages of disks and spheroids in these galaxies are not very different.
3.8 Appendix A: B-Band Photometric Profiles


















### 3.9 Appendix B: H-Band Photometric Profiles


















### 3.10 Appendix C: Notes on Individual Objects

In this appendix we present brief comments on the objects whose luminosity distributions show features which are not clearly, or uniquely, reflected in the photometric profiles.

UGC 80 Small bar perpendicular to the apparent disk major axis
UGC 292 The famous "peanut" of " $X$ " shaped bulge, edge-on disk two companions warping the disk in the outer parts?

UGC 734 Disk appears too face-on to explain diskiness with simple model

UGC 848 Dust lane at $5^{\prime \prime}-10^{\prime \prime}$
UGC 938 Face-on disk with clearly non-axisymmetric bulge, bar?
UGC 995 central bar, isophotes appear to be diamond shaped, but not an edge-on disk

UGC 1250 Possibly very small nucleus $<2^{\prime \prime}$, wrong model fit?
UGC 1283 Barred S0
UGC 1631 Highly inclined, faint (?) disk
UGC 2128 Face-on Disk
UGC 4551 Highly inclined S/D system, weak spiral structure
UGC 4674 Companion at 70 "
UGC 4763 Well established $4^{\text {th }}$ order isophote distortion which is skewed with respect to the major axis

UGC 4791 Nearly edge-on, warped disk
UGC 4840 Face-on disk

UGC 5018 Small radial data range due to reflection of bright star in one corner of the CCD frame

UGC 5292 Very dusty, spiral structure (?)
UGC 5503 Very small spheroid, marginally resolved in the $H$ data
UGC 5617 Very dusty
UGC 6037 Small bar (?)
UGC 6295 Small radial data range due to companion galaxy
UGC 6444 Highly inclined, large disk
UGC 6605 Asymmetric nucleus, low surface brightness
UGC 6704 Two companions
UGC 6779 S/D decomposition in good agreement with K85
UGC 6953 Large disk
UGC 7117 Asymmetric nucleus, appears smaller than fit
UGC 7165 Some dust in the disk
UGC 7202 Central dust lane at $5^{\prime \prime}$
UGC 7214 Edge-on disk
UGC 7311 Polar dust ring at $8^{\prime \prime}$
UGC 7376 Very flat but with distinctly boxy isohpotes, "fluffed up" disk (?)

UGC 7461 Very large, large uncertainties in the sky level estimate
UGC 7610 Several small companions
UGC 7634 Edge-on disk
UGC 7637 Irregular dust at $\approx 5^{\prime \prime}$
UGC 7701 Flattened core, not a bar!

UGC 7722 Bar perpendicular to the projected major axis
UGC 7850 Companion, irregular isophotes at $15^{\prime \prime}$
UGC 7860 Clumpy inside $10^{\prime \prime}$
UGC 8028 Isophote distortion possibly due to refraction spikes from a bright star

UGC 8110 Two companions
UGC 8499 Face-on disk
UGC 8675 Dust, weak spiral structure
UGC 9137 Companion, flat core, possible nuclear disk
UGC 9642 Isophote distortions outside of $30^{\prime \prime}$ possibly due to light from companion galaxy

UGC 9678 Flat core
UGC 9726 Dust lane at $7^{\prime \prime}$ causing boxiness; spiral arms(?)
UGC 9851 Interacting, barred, some spiral structure
UGC 9903 Clumpy, star formation, companion
UGC 12760 Flocculent spiral, luminosity ring at $6^{\prime \prime}$

## CHAPTER 4

## NUCLEAR DISKS IN ELLIPTICAL GALAXIES ${ }^{1}$

### 4.1 Introduction

Stellar dynamical models of early-type galaxies are constrained observationally by two types of data: the projected surface density (see Chapter 3) and the line-ofsight velocity distribution (LOSVD) of the luminous material. The LOSVD as a function of position on the image carries all the accessible information about the kinematics of the galaxy. The LOSVD is conventionally characterised by its first two moments, the bulk motion and the velocity dispersion. This simple description is often sufficient for a comparison with stellar dynamical models, particularly those based on the Jeans equations, since these are usually formulated in terms of loworder moments of the velocity distribution.

The LOSVD "broadens" features in a galaxy spectrum. Other broadening effects arise in the atmospheres of individual stars and from the finite wavelength resolution of the spectrograph. The instrumental broadening can usually be approximated by a

[^22]gaussian, and is in typical observing configurations (spectral resolution of $\sim 3000$ ) a significant fraction of the velocity broadening. It can thus lead to substantial smoothing of line profiles, and so of any derived LOSVD. In many circumstances both the integration of velocity distributions along the line-of-sight, and the spatial averaging caused by seeing and by the finite width of the spectrograph slit can drive the observed LOSVD towards a gaussian shape. Thus there is some justification for the standard assumption that the observed absorption line profiles in early-type galaxies are gaussian. This assumption is, of course, mainly one of convenience. Except for a normalisation, the line profile is then determined by only two parameters, and can be estimated reliably in data of relatively low signal-to-noise. Gaussian parametrisations of the instrumental broadening, of the LOSVD, and so of the observed line profiles have been almost universally employed in the literature throughout the last two decades. Only with very high signal-to-noise data can a more detailed description be justified.

However, there are several situations where the LOSVD of a galaxy is not expected to be gaussian, either as a result of observational limitations, or because of the intrinsic structure of the galaxy.

The nuclei of M31 and M32 provide well known examples where the observed LOSVD is asymmetric at small radii because of seeing and instrumental smoothing effects. In both objects the rotation velocity rises extremely rapidly near the center (Tonry 1987, Dressler and Richstone 1988, Kormendy 1988). Within the inner few arc seconds, light scattered across the nucleus causes extended low velocity tails on the LOSVD, and so leads to biased measurements both of the rotation velocity (too small) and of the velocity dispersion (too large).

Even away from the center of rapidly rotating systems, the line-of-sight integration can lead to an asymmetrical observed velocity distribution. This is because
the streaming of stars well in front of or behind the tangent point is predominantly across the line-of- sight; such stars therefore produce a low velocity "tail" in the distribution.

In a non-rotating galaxy, local velocity anisotropies, such as those postulated for M87 by Binney and Mamon (1982), can lead to LOSVDs which, although symmetric, are significantly more - or less - peaked than a gaussian. Specific examples have been calculated by Dejonghe (1987) and Gerhard (1991).

Finally, whenever a dynamically "hot", slowly rotating stellar component coexists with a dynamically "cold", rapidly rotating component (as in an S0 galaxy) the resulting LOSVD is expected to be bimodal, or at least asymmetric.

Since the modelling of early-type galaxies is always underconstrained by observation, it is crucial to extract as much kinematic information from the data as possible. Recently, Franx and Illingworth (1988) and Bender (1990) have made significant progress in going beyond the gaussian hypothesis to extract more information about the LOSVD from absorption line spectra. The technique used by both groups was, in essence, a "cleaning" scheme, in which the auto-correlation of a stellar template is deconvolved from the cross-correlation of the galaxy with the template. These authors were able to demonstrate the presence of bimodal or at least asymmetric velocity distributions in a few ellipticals with kinematically distinct cores.

Here we present an alternative and more direct approach to extracting LOSVDs from spectra. Our method preserves the maximum information content of the spectrum, allows rigorous treatment of statistical errors, is simple in concept, and provides several ways to suppress potential systematic errors from template mismatch, for example, by including a spectral synthesis which finds the best possible match of the galaxy spectrum to a mixture of the available templates. We discuss two
slightly different approaches. In the first the LOSVD is to be estimated in the absence of any a priori assumptions about its shape. This is a relatively standard optimal filtering problem. In the second the LOSVD is assumed to belong to a parametrized family of models (e.g. single or multiple gaussians) and we set up optimal techniques for estimating both the parameters and their uncertainties. As an illustration of the power of these schemes, we apply them to a few elliptical galaxies with kinematically distinct cores.

The remainder of this chapter is organized as follows. In Section 4.2 we review standard methods for extracting kinematic information from observations of elliptical galaxies. Section 4.3 describes our methods, while Section 4.4 illustrates their application to the counter-rotating core of NGC 5322. The final section gives our conclusions.

### 4.2 Standard Techniques for Extracting Line Profiles

As a starting point we discuss some frequently used algorithms for extracting kinematic information from the spectra of early-type galaxies. These algorithms were originally developed to obtain only streaming velocities and velocity dispersions, but some of them can be, and have been, extended to obtain more detailed information about LOSVDs.

### 4.2.1 The Fourier Quotient Method

Over the last decade one of most frequently used algorithms has been the Fourier quotient technique, originally devized by Schechter (see Sargent et al. 1978). If the galaxy spectrum is assumed to be the convolution of a template with a broadening function, then the latter can be retrieved by dividing the Fourier transform of the
galaxy spectrum by that of the template, and back-transforming the result. In realistic situations the ratio of the transforms is quite noisy, and must be fit to a simple model for the broadening function (i.e. a gaussian). Although this method is quite fast and yields reliable mean velocities and velocity dispersions, it has two serious disadvantages when used for the more general problem. Firstly, the errors in the quotient are strongly correlated, significantly complicating a rigorous error analysis. More importantly, since the broadening function is fitted to the data in Fourier space, absorption features from all parts of the spectrum interact with each other. This makes the fitted broadening function very sensitive to a wide variety of spectral mismatches between template and object. This sensitivity is often too great to allow detailed analysis of the shape of the line profile. This problem is discussed in some detail by Bender (1990).

### 4.2.2 Cross Correlation Methods

The cross-correlation method was originally suggested by Simkin (1974), and was later developed by Tonry and Davis (1979). The galaxy and template spectra are correlated directly as a function of relative shift in wavelength, and the location of the highest correlation peak is identified with the mean velocity. The velocity dispersion is then estimated by subtracting the width of the template autocorrelation peak in quadrature from the width of the cross-correlation peak, thus correcting for instrumental broadening. If there is no mismatch between template and object, formal errors in these estimates can be calculated (see Tonry and Davis 1979). However, as these authors note, for high signal-to-noise data the real uncertainties are dominated by template mismatch. For this scheme, in contrast to the Fourier quotient method, only mismatch between "overlapping" spectral features contributes to the uncertainties. We address the problem of spectral mismatch in some detail
below.
The use of the autocorrelation peak to correct for intrinsic and instrumental broadening can be generalized to obtain a detailed line profile. By hypothesis, the only difference between the template autocorrelation and the object-template cross-correlation (apart from a shift of origin due to differing redshifts) is that one of the two spectra in the latter case has been convolved with the LOSVD. Thus the LOSVD can be derived by deconvolving the template autocorrelation from the cross-correlation. This idea was first employed by Franx and Illingworth (1988). Their spectra of the counter-rotating core in IC 1459 showed evidence for an asymmetric broadening function: the Fourier quotient and cross-correlation methods gave substantially different results for the rotation curve. They removed instrumental broadening, intrinsic stellar line widths and negative side-lobes from the cross-correlation peak using a modified version of the CLEAN (Högbom, 1974) algorithm with the template autocorrelation as the "antenna pattern". The LOSVD retrieved in this fashion showed a kinematically "hot", slowly rotating component superimposed on a cooler, rapidly rotating component. As a check on the reality of these structures, they compared the LOSVDs of two points on the major axis on opposite sides of the center; these turned out to be roughly antisymmetric, as expected for a stellar system in equilibrium. However, the nonlinear character of the CLEAN algorithm precluded a rigorous mathematical assessment of the errors. Such an analysis would be essential to study less symmetric objects, or more subtle asymmetries in the line profile.

Bender (1990) has chosen an alternative but closely related approach in which he explicitly deconvolves the template autocorrelation peak from the cross-correlation using Fourier techniques. A Wiener filter must be employed in this deconvolution in order to get smooth solutions without undue noise. With this technique Bender
is able to give a convincing demonstration of asymmetry in the line profiles of NGC 4621. However, as in Franx and Illingworth's approach, the filtering seriously complicates a rigorous calculation of the errors and may introduce systematic errors. Questions such as "Is the observed asymmetry in the profile significant?" are then difficult to address. Furthermore, if the LOSVD is to be interpreted in terms of a particular model, for example a superposition of gaussians, it is unclear how to fit the model to the filtered line profile in order to obtain estimates and confidence intervals for the model parameters.

### 4.2.3 Direct Fitting Methods

Before the invention of the FFT algorithm allowed the efficient computation of Fourier transforms, kinematic information was often extracted from spectra by direct comparison of a broadened template to the object spectrum (e.g. Burbidge et al. 1962). This approach is very straightforward. For a family of broadening models, characterised by a parameter set $\underline{\alpha}$, the best parameter values are simply those which minimise $\chi^{2}$ for the difference between the broadened template and the object. Twenty years ago, the disadvantage of this method was that the computation of $\chi^{2}$ for a given set of trial parameters was quite costly, and a full exploration of parameter space was therefore difficult. In the original applications, only a few trial parameter sets were compared with the data (e.g. Richstone and Sargent, 1972). This is no longer a serious problem, and a variation of this method was developed recently by Franx, Illingworth and Heckman (1989) ${ }^{2}$. Their scheme, which they call the Fourier Fitting Method, is similar to the direct fitting method in that it minimises the $\chi^{2}$ of the template-object match; it differs in that the $\chi^{2}$ sum is evaluated in Fourier space in order to buy computational speed.

[^23]There are, nevertheless, several advantages to treating the problem entirely in pixel space as we describe below. This approach allows a clear-cut separation between fitting the continuum and fitting the line profile, thereby facilitating a rigorous treatment of the noise in the result. In addition, there is no need to "bell down" (i.e. taper with a bell-shaped curve) the ends of the spectrum, which can significantly reduce its information content. Such belling is necessary in Fourier schemes in order to eliminate spurious high frequency signal from edge effects. Finally, crosstalk between features in different parts of the spectrum is absent in our scheme, and the effects of template mismatch are significantly reduced in consequence. These considerations have rather little impact on the determination of a mean velocity and a velocity dispersion, and can usually be safely ignored if these are the only data required for kinematic modelling of a galaxy. However, as we illustrate below, the reliable extraction of more detailed information about the LOSVD requires the greatest possible care, both in optimizing the use of the available information in the spectrum, and in assessing the uncertainties in the result. We believe that the direct fitting approach we describe below fulfills both these criteria.

### 4.3 Direct Line Profile Fitting

In this section we outline two different approaches to the estimation of velocity distributions from absorption line spectra. The first makes no assumptions about the velocity distribution other than taking it to be bandwidth-limited in velocity space. Optimal filtering techniques are applied to estimate a line profile which is as close as possible (in an rms sense) to the true profile, given the known properties of the photon noise. We show how systematic errors from spectral mismatch can be minimized by a proper choice of template, and by using a priori knowledge of
the symmetry properties of the LOSVD. This approach is suitable for preliminary exploration of complex situations where the assumption of any specific model is unwarranted. On the other hand, a simple family of models, for example, single or double gaussians, or symmetric profiles with varying kurtosis, can often provide an appropriate description of the expected LOSVDs. Analysis of the line profile then reduces to obtaining the best possible estimates of the parameters, together with their associated uncertainties. The mathematical background we require for both these approaches is the same. Throughout this section we illustrate our methods by applying them to artificial spectra generated by Monte Carlo simulation, and to spectra of NGC 3610 and NGC 5322.

### 4.3.1 General Considerations

As a starting point we make the conventional assumption that the spectrum of a galaxy can be modelled as a superposition of continuum terms and of a broadened template spectrum. The line widths in the template are assumed to reflect effects from stellar atmospheres and from instrumental resolution; it is thus important that the template should be obtained with exactly the same instrumental setup as the galaxy to be analysed. However, the template can be a spectrum of composite stellar type, for example the integrated spectrum of a globular cluster, or the superposition of a number of stellar spectra. We represent the broadened template as a linear combination of identical spectra shifted by integer pixel amounts with respect to each other; the coefficients of this linear combination are the broadening coefficients and carry all the accessible information about the line-of-sight velocity distribution (see Figure 4.1). Clearly, spectral bins must be equally spaced in the logarithm of the wavelength for this formulation to work. Since this does not normally correspond to the detector pixels, rebinning is necessary to get the data into the correct format,
resulting in a slight loss of information. We have elected to live with this smoothing in the present work, and have rebinned our data logarithmically, choosing a width equal to that of our CCD pixels at the center of our spectral range. This also introduces a correlation between the noise in adjacent bins, which we ignore in the analysis below. In practice it does not seem to cause problerns, probably because our detector pixels are significantly smaller than our effective spectral resolution, and because the velocity information arises from many uncorrelated parts of the spectrum. A formulation of the problem without rebinning is possible, but is more complex than the one we give below.

We assume that the galaxy spectrum, $\underline{\hat{o}}$, consists of $N$ logarithmic bins, that the total bandwidth of the broadening function is $M$ bins, and that we include $L$ continuum components. Furthermore, we assume that each of the shifted template spectra is given on bins identical to the galaxy spectrum. Initial estimates of the redshift and velocity dispersion of the galaxy are thus required in order to construct an appropriate set of shifted templates. The ends of all the spectra must then be trimmed to the maximum spectral range over which data are available both for the galaxy and for all the shifted templates. Provided $M$ is kept as small as possible, no useful information is lost through this trimming. After this initial preparation, the spectral model, $\widehat{\underline{m}}$, for the galaxy can be written as

$$
\begin{equation*}
\underline{\widehat{m}}=\underline{\widehat{c}} \cdot \underline{\delta}+\widehat{\underline{T}} \cdot \underline{b}, \tag{4.1}
\end{equation*}
$$

where $\underline{\underline{\hat{C}}}$ is a $N \times L$ matrix, whose columns represent a basis of possible continuum shapes (we generally use a set of $\sim 10$ polynomials) and $\widehat{\underline{T}}$ is a $N \times M$ matrix whose columns $\widehat{\hat{t}_{i}}(i=1, \ldots, M)$ are the template spectra shifted with respect to each other by one pixel per column. We define the best fitting model to be that specified by the values of $\underline{\delta}$ and $\underline{b}$ which minimise


Figure 4.1: Illustration of the Basic Algorithm
Schematic illustration of our method for extracting the line-of-sight velocity distribution from the absorption line spectrum of a galaxy. The galaxy spectrum is modelled as a superposition of the following: a) a set of template spectra shifted by single pixel increments to represent the various velocity populations contributing to the integrated light; b) a set of continuum components to represent low frequency differences between the template and galaxy spectra; c) noise (predominantly photon noise) in the galaxy spectrum. The template spectra are assumed to be of sufficiently high $\mathrm{S} / \mathrm{N}$ that their noise contribution is negligible. The best estimate for the velocity distribution is taken to be the set of template amplitudes, $\underline{b}$, which best fits (in a $\chi^{2}$ sense) the observed spectrum.

$$
\begin{equation*}
\chi^{2}(\underline{\delta}, \underline{b})=\left\|\underline{\underline{\sigma}}^{-1} \cdot(\underline{\hat{c}} \cdot \underline{\delta}+\underline{\underline{\hat{T}}} \cdot \underline{b}-\underline{\hat{o}})\right\|^{2} . \tag{4.2}
\end{equation*}
$$

Here $\underline{\underline{\sigma}}^{-1}$ is the diagonal noise-weighting matrix whose elements are given by the inverse of the noise in the galaxy spectrum at the $i^{\text {th }}$ pixel. In practice the noise in the spectrum is well approximated by a Poisson distribution, and $\sigma_{i}$ can be calculated straightforwardly, taking into account both photon noise and readout noise from the detector; we assume that noise in the template observations can be neglected.

The solution to the least squares problem posed in Equation 4.2 is straightforward. Nevertheless we will outline our particular way of solving it in considerable detail, because we will need to refer to many of the intermediate steps later when we discuss the noise filtering. Readers not interested in the mathematical details of the technique may skip to Section 4.3.3 for an illustration of our optimal filtering scheme, and to Section 4.3.4 for applications of our model fitting procedure.

We proceed by multiplying $\underline{\underline{\sigma}}^{-1}$ with the terms following it in Equation 4.2, leading to

$$
\begin{equation*}
\chi^{2}(\underline{\delta}, \underline{b})=\|\underline{\tilde{\tilde{C}}} \cdot \underline{\delta}+\underline{\underline{\tilde{T}}} \cdot \underline{b}-\underline{\tilde{\sigma}}\|^{2}, \tag{4.3}
\end{equation*}
$$

where $\underline{\underline{\tilde{C}}}=\underline{\underline{\sigma}}^{-1} \cdot \underline{\underline{\hat{C}}}$, etc. This operation divides the count in each pixel of the continuum, template, and object spectra by the noise in the corresponding pixel of the object spectrum. We can now eliminate all continuum terms from the problem without additional low pass filtering and without tapering the ends of the spectra, therefore without any loss of information. We subtract from each noise-weighted spectrum (i.e. from $\underline{\underline{\tilde{o}}}$ and from each column, $\underline{\tilde{t}_{\text {i }}}$, of $\underline{\underline{T}}$ ) a best fit continuum, defined as its projection onto the vector space spanned by the columns of $\underline{\underline{\tilde{C}}}$. It is easy to show that this operation is equivalent to minimizing Equation 4.3 over $\underline{\delta}$. Denoting
the continuum subtracted quantities by $\underline{\underline{T}}$ and $\underline{o}$, we then have,

$$
\begin{equation*}
\chi^{2}(\underline{b})=\|\underline{\underline{T}} \cdot \underline{b}-\underline{o}\|^{2}, \tag{4.4}
\end{equation*}
$$

where $\underline{\underline{\tilde{T}}}^{T} \cdot \underline{\underline{T}}$ and $\underline{\underline{\tilde{T}}}^{T} \cdot \underline{o}$ both vanish. In this expression, $\chi^{2}$, can be minimized in the same way by subtracting from $\underline{o}$ its projection onto the space spanned by the columns of $\underline{\underline{T}}$. The minimum $\chi^{2}$ is then just the square of the norm of the residual, and the corresponding value of $\underline{b}$ is the desired broadening function. This whole procedure comes down to choosing $\underline{\delta}$ and $\underline{b}$ in the model of Equation 4.1 so that the residual, $\widehat{\hat{r}}=\underline{o}-\underline{\widehat{\widehat{p}}}$, is orthogonal, with appropriate noise-weighting, to the space spanned by the continuum and template spectra, i.e. so that $\underline{\underline{\widehat{C}}}^{T} \cdot \underline{\underline{\sigma}}^{-2} \cdot \hat{\underline{r}}$ and $\underline{\underline{T}}^{T} \cdot \underline{\underline{\sigma}}^{-2} \cdot \hat{\underline{r}}$ both vanish, where $\underline{\underline{\sigma}}^{-2}$ is the square of $\underline{\underline{\sigma}}^{-1}$.

One particular, and in this context very practical, way to find the broadening $\underline{b}$ which minimises $\chi^{2}$ in Equation 4.4 is through singular value decomposition (see e.g. Press et al. 1986). A standard theorem of linear algebra allows us to rewrite $\underline{\underline{T}}$ as

$$
\begin{equation*}
\underline{\underline{T}}=\underline{\underline{U}} \cdot \underline{\underline{W}} \cdot \underline{\underline{V}}^{T} \tag{4.5}
\end{equation*}
$$

where $\underline{\underline{U}}$ is a $N \times M$ matrix, whose columns form an orthonormal basis for the space spanned by the noise-weighted, continuum-subtracted template vectors $\underline{t_{i}} ; \underline{\underline{V}}$ is an $M \times M$ matrix, whose columns form an orthonormal basis for the space of all broadening functions, $\underline{b}$, and $\underline{\underline{W}}$ is a diagonal matrix with elements, $w_{i}$. Note that these matrices depend on the galaxy spectrum only through the noise weighting ${ }^{3}$. The solution for $\underline{b}$ which minimises $\chi^{2}$ in Equation 4.1 can then be written as

$$
\begin{equation*}
\underline{b}=\underline{\underline{V}} \cdot \underline{\underline{W}}^{-1} \cdot \underline{\underline{U}}^{T} \cdot \underline{o} . \tag{4.6}
\end{equation*}
$$

[^24]An exactly similar procedure can be used to subtract the continuum from the object and template spectra in order to obtain Equation 4.4 from Equation 4.3.

The procedure we have just described allows us to obtain a best fit model (in a $\chi^{2}$ sense) to the galaxy spectrum. However, the broadening functions it produces are usually rapidly varying and often contain large negative values. (All elements of the broadening $\underline{b}$ should, of course, be positive.) This breakdown occurs because high frequency components of the broadening function are only weakly constrained by the signal in the data (this shows up as small values of the corresponding $w_{i}$ ) and the algorithm adjusts their coefficients in order to follow noise spikes. This is a well known problem in signal processing, and we investigate two ways of circumventing it. One is to weight components of the broadening function according to the expected signal-to-noise of their determination. This produces the equivalent of a Wiener filter in the present context. The second procedure is to assume that the true broadening function is a member of some parametrized family of models. Equation 4.4 must then be minimized over this family, rather than over all possible $\underline{b}$.

### 4.3.2 Noise Filtering

We now set out to find the estimate of the broadening, $\underline{b}$, for which the mean square error,

$$
\left\langle E^{2}\right\rangle=\left\langle\left\|\underline{b}-\underline{b}_{\text {true }}\right\|^{2}\right\rangle,
$$

is minimised, given the known noise properties of the data. (Here $\underline{b}_{\text {true }}$ denotes the solution of Equation 4.6 for an idealised, noiseless signal, $\underline{s}$, from the galaxy.) This can be achieved by modifying the elements of the diagonal matrix, $\underline{\underline{W}}^{-1}$, thereby suppressing the coefficients of those broadening basis vectors, $\underline{v}_{i}$, of the broadening function which are most heavily affected by noise. The modified version of the matrix, $\underline{\underline{W}}^{-1}$, will be denoted $\underline{\underline{\Omega}}^{-1}$, with diagonal elements $\omega_{i}{ }^{-1}$.

Let us assume that the measured spectrum, $\underline{\underline{g}}$, differs from the signal, $\underline{s}$, by the noise, $\underline{n}$ : then $\underline{o}=\underline{s}+\underline{n}$. Because of the noise weighting applied in going from 4.2 to 4.3 we have $\left\langle n_{i}\right\rangle=0$ and $\left\langle n_{i} n_{j}\right\rangle=\delta_{i j}$. As noted above, this second equation is, in fact, an approximation, since rebinning of the galaxy spectrum leads to a correlation between the noise in adjacent bins. Using 4.6, the squared error in the broadening estimate is:

$$
\begin{equation*}
E^{2}=\left\|\underline{\underline{V}} \cdot\left(\underline{\underline{\Omega}}^{-1}-\underline{\underline{W}}^{-1}\right) \cdot \underline{\underline{U}}^{T} \cdot \underline{s}+\underline{\underline{V}} \cdot \underline{\underline{\Omega}}^{-1} \cdot \underline{\underline{U}}^{T} \cdot \underline{n}\right\|^{2} \tag{4.7}
\end{equation*}
$$

Using the orthonormal properties of $\underline{\underline{V}}$ and $\underline{\underline{U}}$, one can take the expectation value of this error over the noise distribution and obtain:

$$
\begin{equation*}
\left\langle E^{2}\right\rangle=\sum_{i=1}^{M}\left[\left(\omega_{i}^{-1}-w_{i}^{-1}\right)^{2} \cdot\left(\underline{u_{i}^{T}} \cdot \underline{s}\right)^{2}+\left(\omega_{i}^{-1}\right)^{2}\right] \tag{4.8}
\end{equation*}
$$

where $\underline{u}_{i}^{T}$ denotes the $i^{t h}$ line of the matrix $\underline{\underline{U}}^{T}$. This expression is easily minimized with respect to the $\omega_{i}$. Using the relation, $\underline{u}_{i}^{T} \cdot \underline{s}=w_{i} \underline{v}_{i}^{T} \cdot \underline{b}_{\text {true }}$, (see Equation 4.6) we find the minimizing values of $\omega_{i}^{-1}$ to be,

$$
\begin{equation*}
\omega_{i}^{-1}=w_{i}^{-1} \cdot \frac{\left(w_{i} \underline{v}_{i}^{T} \cdot \underline{b}_{t r u e}\right)^{2}}{1+\left(w_{i} \underline{v}_{i}^{T} \cdot \underline{b}_{t r u c}\right)^{2}} \tag{4.9}
\end{equation*}
$$

Since we do not know $\underline{b}_{\text {true }}$, Equation 4.9 cannot be solved without further assumption. This reflects the fact that an optimal filtering cannot be defined without some knowledge of the true line profile. To make progress, $\left(\underline{v_{i}^{T}} \cdot \underline{b}_{\text {true }}\right)^{2}$ must be replaced by its average over a suitable set of broadening functions. It is important to avoid being overly restrictive in defining this set. For example, if one chose a set of symmetric profiles, the reconstructed broadening function would always be symmetric, because Equation 4.9 would then give zero weight to all the antisymmetric components of $\underline{b}$. We have chosen to evaluate $\left\langle\left(\underline{v_{i}^{T}} \cdot \underline{b}_{\text {model }}\right)^{2}\right\rangle$ over an ensemble
of broadening functions defined in the following way: each element of $\underline{b}$ is given a random value distributed uniformly between 0 and 1 , and the resulting vector is scaled to unit norm. The members of this ensemble are bandwidth-limited, positive definite, and normalised, but no other a priori assumption is made about their shape, location, and frequency content. By refusing to be more specific, we hope to avoid bias towards any particular model. Finally, the desired broadening, $\underline{b}$, is calculated as

$$
\begin{equation*}
\underline{b}=\underline{\underline{V}} \cdot \underline{\underline{\Omega}}^{-1} \cdot \underline{\underline{U}}^{T} \cdot \underline{o} \tag{4.10}
\end{equation*}
$$

where the elements of $\underline{\underline{\Omega}}^{-1}$ are given by 4.9 once the above substitution has been made for the terms involving $\underline{b}_{\text {true }}$.

Although the broadening estimate of Equation 4.10 has, by construction, the smallest possible sampling variance, it is biased, and it can differ substantially from the true broadening for data of marginal signal-to-noise. The bias is easily determined from Equations 4.6 and 4.10 to be,

$$
\begin{equation*}
\left\langle\underline{b}-\underline{b}_{\text {true }}\right\rangle=\underline{\underline{V}} \cdot\left(\underline{\underline{\Omega}}^{-1}-\underline{\underline{W}}^{-1}\right) \cdot \underline{\underline{U}}^{T} \cdot \underline{\underline{s}}, \tag{4.11}
\end{equation*}
$$

and is small provided the signal, $\underline{s}$, contains little of the high frequency power filtered out by Equation 4.9. Unfortunately, Equation 4.11 cannot be used to correct for any bias in practice, since this would require an a priori knowledge of $\underline{b}_{t r u e}$. Formal errors for the estimate of $\underline{b}$ can be obtained from the covariance matrix,

$$
\begin{equation*}
\underline{\underline{\Sigma}}=\left\langle\left(\underline{b}^{-}-\underline{b}_{\text {true }}\right) \cdot\left(\underline{b}^{T}-\underline{b}_{\text {true }}^{T}\right)\right\rangle-\left\langle\left(\underline{b}-\underline{b}_{\text {true }}\right)\right\rangle \cdot\left\langle\left(\underline{b}^{T}-\underline{b}_{\text {true }}^{T}\right)\right\rangle \tag{4.12}
\end{equation*}
$$

The averages over the noise distribution are easily carried out leaving:

$$
\begin{equation*}
\underline{\underline{\Sigma}}=\underline{\underline{V}} \cdot \underline{\underline{\Omega}}^{-2} \cdot \underline{\underline{V}}^{T}, \tag{4.13}
\end{equation*}
$$

where $\underline{\underline{\Omega}}^{-2}$ is the square of $\underline{\underline{\Omega}}^{-1}$. Thus, in the orthonormal basis, $\underline{v}_{i}$, the errors on the different components of $\underline{b}$ are independent. However, the same is not true in the original velocity space representation. Notice that although the bias in our estimate of $\underline{b}$ depends on the form of the signal, the scatter in the estimate does not. We use the diagonal elements of Equation 4.13 to assign error bars to the LOSVDs we plot in the next section. Examination of the full covariance matrix shows that the errors on different elements of $\underline{b}$ are strongly correlated, with the correlation extending across a good fraction of the bandwidth.

### 4.3.3 Testing the Filtering Scheme

We now use spectra obtained at the MMT with the instrumental set-up described in section 4.4.1 to carry out a series of Monte-Carlo simulations. A stellar spectrum is convolved with a known broadening function, and perturbed with Poisson noise; it then serves as the "galaxy" spectrum. Adopting the original stellar spectrum as a template, we then use the above techniques to recover the broadening function. (The systematic errors caused by mismatch between the broadened stellar spectrum and the template are discussed in section 4.3 .5 below.) We adopt a broadening function which is the superposition of two gaussians of equal amplitude. One is centered at the origin and has a variance of 4 pixels, while the other is offset by 4 pixels and has a variance of two pixels. With our set-up one pixel corresponds to $45 \mathrm{~km} / \mathrm{s}$. We find that even small amounts of noise prevent recovery of a realistic line profile unless filtering is employed. Figure 4.2 compares the input line profile to those retrieved when optimal filtering is applied to individual realisations with various noise amplitudes. The error bars on each recovered profile were calculated from the diagonal elements of Equation 4.13; they appear to give a reasonable characterisation of the deviation from the true profile. Note that for decreasing

S/N the filtering introduces increasingly severe systematic errors. For a $\mathrm{S} / \mathrm{N}$ of 10 the derived profile is virtually symmetric and has substantial negative wings.

As an application of the algorithm to "real" data, we reconstruct the broadening function of a major axis spectrum of NGC 5322 at $2^{\prime \prime}$ from the center. The signal-to-noise per pixel in this spectrum averages about 40. The counter-rotating core of this galaxy will be discussed in detail in Section 4.4. As Figure 4.3 shows, the velocity distribution is clearly asymmetric and is similar to the model of Figure 4.2. This lends weight to the interpretation that the inner regions of this galaxy are a superposition of a hot, slowly rotating "bulge", and a cooler, rapidly rotating "disk".

Although retrieving line profiles in this fashion is very valuable for a preliminary, qualitative investigation of their shape, the result is not ideally suited either to hypothesis testing (e.g. asking "Is the profile significantly asymmetric?" or "Does it deviate significantly from a simple gaussian?") or to parameter estimation (e.g. "What are the parameters of the best double gaussian fit with their associated errors?"). This is because the bias in Equation 4.11 depends on the intrinsic profile, and because the weighting of Equations 4.9 and 4.10 is not optimal for these more detailed questions. A direct insertion of model broadening functions in Equation 4.4 provides a much more straightforward approach to such problems.

### 4.3.4 Fitting Models to Line Profiles

In this section we will consider the case where the $M$ components of the broadening vector, $\underline{b}$, (typically $M \sim 20-30$ ) are viewed as functions of the $K$ parameters of some model line profile. We denote these parameters by the vector, $\underline{\alpha}$. Normally $K$ will be much less than $M$. The archetype of such a model is the ubiquitous gaussian, characterized by three parameters, a location, a width, and a strength. A logical


Figure 4.2: MC Simulations of the LOSVD Reconstruction
Monte-Carlo simulations of line profile recovery using the algorithm described in Section 4.3.2. No particular assumptions about the shape of the LOSVD are made, but optimal filtering is employed to suppress noise. The simulated galaxy spectrum was created by broadening a stellar spectrum with the LOSVD given by the dashed line. This input LOSVD is composed of two gaussians of equal amplitude and dispersions of 2 and 4 pixels, respectively. The "cold" component is offset by four pixels. Subsequently, various amounts of Poisson noise were added independently to each pixel to give an "observed galaxy spectrum" from which the LOSVD could be recovered. The four panels show the recovered LOSVD (indicated by the triangles) for four $\mathrm{S} / \mathrm{N}$ ratios. The errors were estimated from the diagonal elements of the covariance matrix defined in Equation 4.13. Note the increasing systematic deviations of the reconstructed profile from the input profile, due to increased filtering and the consequent smoothing. For $S / N=10$ the reconstructed profile is almost symmetric.


Figure 4.3: Reconstructed LOSVD of NGC 5322
This figure shows the LOSVD on the major axis of NGC 5322 at $2^{\prime \prime}$. It was obtained using the method of Section 4.3.2. The velocity distribution is clearly asymmetric and could sensibly be decomposed into two components.
extension is a six parameter, double gaussian; such a model seems a reasonable first description of the LOSVD in a galaxy where bulge and disk components are both significant, and we focus on it in some detail below. One should keep in mind, however, that the techniques we develop are not specific to this model, and could be used for any other parametrised family of LOSVDs. For example, one might generalise the single gaussian model by allowing the power in the exponential to differ from 2. This would add a fourth parameter and might make a suitable family of models to search for the kurtosis expected in the LOSVDs of nonrotating, near-spherical galaxies with anisotropic velocity distributions (see, for example, the models of Dejonghe 1987). Model fitting methods of this kind are routinely used to determine atmospheric parameters of stars, where in contrast to our application, the model is usually fit to a single, well resolved line of extremely high signal-to-noise.

The best-fitting parameter set is easily obtained from Equation 4.4, where $\chi^{2}$ is
now viewed as a function of $\underline{\alpha}$, through our assumption that $\underline{b}=\underline{b}(\underline{\alpha})$. Since this latter functional dependence is, in general, nonlinear, the value of $\underline{\alpha}$ which minimises $\chi^{2}$ must be found by some numerical search scheme. For a simple gaussian model, the standard gradient search method (e.g. Press et al. 1986) works both efficiently and reliably. However, for more complex models, such as the double gaussian, tradeoffs between various parameter combinations are often possible, and the search then tends to get "stuck" in a local minimum of the $\chi^{2}$-surface. For the models we have tested so far, we have got much more reliable results from a biased random-walk scheme which finds the global minimum robustly with only a moderate amount of computing time. (In the case of a double gaussian there are of course always two equivalent global minima.) This scheme has the added advantage that constraints on the parameters, for example to ensure a positive line profile, are easily implemented. We describe it in more detail in the Appendix.

In addition to determining a best fit parameter set, it is critical that we should be able to assign realistic uncertainties to the result. For example, when fitting double gaussian models, we may wish to assess whether the data really require the presence of a second component. If such a second component is required, we usually want confidence limits for the parameters of both gaussians. The principal sources of uncertainty in our derived LOSVD's are usually the Poisson noise in the data, and the spectral mismatch between the galaxy and the template. For the observational set-up we have used, photon noise is usually dominant for $\mathrm{S} / \mathrm{N}<40$ while systematic errors due to template mismatch can predominate for $S / N>40$. However, as we discuss in the next section, there are several methods available which reduce such systematic errors.

In realistic situations the values of $\chi^{2}$ for the best single gaussian, $\chi_{\text {min }}^{2}(s g)$, and for the best double gaussian, $\chi_{\min }^{2}(d g)$, will indicate either that both models should
be rejected $\left(\chi^{2} \gg N\right)$ or that both models are acceptable $\left(\chi^{2} \approx N\right)$. To determine whether the double gaussian fit ( $N-6$ degrees of freedom) is significantly better than a single gaussian ( $N-3$ degrees of freedom) we must examine the reduction in $\chi^{2}, \Delta \chi^{2} \equiv \chi_{\min }^{2}(s g)-\chi_{\min }^{2}(d g)$. If the true broadening were indeed a single gaussian, $\Delta \chi^{2}$ would be distributed as $\chi^{2}$ with $(N-3)-(N-6)=3$ degrees of freedom (see e.g. Hoel, 1971). A value of $\Delta \chi^{2}$ much larger than expected from this distribution then indicates that a second component has been detected significantly. For example, $\Delta \chi^{2}>7.8$ implies that a double gaussian is preferred over a single gaussian at the $95 \%$ confidence level. Once a model has been chosen (e.g. single or double gaussian), the $\chi^{2}$ surface can be used to place confidence limits on its parameters in the standard way (see, for example, Avni 1976).

To illustrate this procedure, and to estimate the $\mathrm{S} / \mathrm{N}$ required to detect a second component, we now show results from some Monte Carlo simulations. These simulations use the same input LOSVD as those of Figure 4.2. Each artificial spectrum was fit both by single and by double gaussian models, using the original stellar spectrum as template. The distribution of the reduction in $\chi^{2}$ for each ensemble is shown in Figure 4.4a, and is compared with the theoretical distribution for $\chi^{2}$ with 3 degrees of freedom. At a $\mathrm{S} / \mathrm{N}$ of 10 , the single gaussian model is rejected with $95 \%$ confidence only about a fifth of the time. For a $\mathrm{S} / \mathrm{N}$ of about 30 , rejection at this level is possible about $95 \%$ of the time. For our chosen model profile, the galaxy spectrum must therefore contain $\sim 10^{6}$ photons for a reliable detection of non-gaussian LOSVDs ${ }^{4}$ It may seem surprising that this is only larger by a factor of three than the number of photons needed to determine a reliable ( $<10 \%$ accuracy) dispersion for a single gaussian (Franx, Illingworth and Heckman 1989); yet, it must be borne in mind that $\sim 10^{6}$ photons are only enough to reject a sin-

[^25]gle gaussian model, not necessarily enough to determine well the parameters of a complex LOSVD. If we had chosen gaussians with unequal normalisation or with less disparate location, discrimination could have been more difficult. For one hour exposures with an efficient spectrograph ( $>10 \%$ ) at a 4 -meter class telescope, $10^{6}$ counts correspond to a $V$ magnitude of 18.5. By averaging along the slit, it should be possible to detect multiple components in nearby (large) galaxies out to about the point where their surface brightness approaches that of the sky.

### 4.3.5 Systematic Errors

To argue that a line profile, extracted from the data, is a true reflection of the line-of-sight velocity distribution, we must exclude all alternative explanations for any apparent "peculiarity". In the following, we concentrate on two problems that can substantially modify a derived LOSVD, but are entirely unrelated to the galactic velocity distribution. One is an instrumental broadening which could differ between object and template, and the other a mismatch between the intrinsic shape of spectral features in object and template. If any single star is used as a template, such a mismatch is to be expected, since galaxies clearly have composite spectra.

The first source of error can be largely eliminated if template and galaxy spectra are obtained with the same instrumental set-up. This means using the same grating, slit, and spectrograph focus, and paying considerable attention to trail stars during template observations so that the slit is uniformly illuminated. The spectral range of the template and the object spectra also need to be close to avoid effects from the variation of spectrograph resolution with wavelength. Care will be needed to eliminate such errors in work on galaxies at high redshift. In addition, template and galaxy spectra must be reduced in the same way. In particular, all rebinning from the detector pixels must be done in the same way for both, otherwise the difference


Figure 4.4: Reduction of $\chi^{2}$ due to a Complex LOSVD
This figure shows the distributions of the reductions in $\chi^{2}$ when the LOSVD is fit with a double, rather than a single gaussian. Each distribution is calculated from a set of 30 Monte-Carlo realisations. The input profile was the same double gaussian used in Figure 4.2. The input spectrum and the spectrum used for profile recovery were identical. Poisson noise of the indicated amount was added to each broadened input spectrum. The dashed line indicates the expected distribution of $\Delta \chi^{2}$ had the input profile been a single gaussian. A signal-to-noise ratio of about 30 is necessary to reject this null hypothesis safely for the particular profile shape considered here.


Figure 4.5: Reduction of $\chi^{2}$ due to Template Mismatch This figure shows how template mismatch can mimic a non-gaussian LOSVD. A series of Monte Carlo simulations were carried out in which a K5III spectrum was broadened with a single gaussian and then perturbed with Poisson noise. A K0III template was subsequently used to recover the LOSVD. The dashed line gives the distribution of the reduction in $\chi^{2}$ expected in the absence of mismatch when comparing single and double gaussian fits. This is just a $\chi^{2}$ distribution with three degrees of freedom. At high $\mathrm{S} / \mathrm{N}$, considerably larger values of $\Delta \chi^{2}$ (apparently indicating the presence of a spurious second component) are often found. The solid lines show the strength of this effect for three $\mathrm{S} / \mathrm{N}$ values.
in the effective smoothing will systematically affect the derived LOSVDs.
The second source of error is more difficult to control; we begin by illustrating its effects. We took the spectrum of a K5III star, broadened it with the double gaussian of Figure 4.2, and applied the model fitting techniques of Section 4.3.4 to reconstruct the LOSVD using now a KOIII star as template. The noise was taken as the sum in quadrature of that in the two stellar spectra. This experiment was then repeated, exchanging the roles of the stars. In Figure 4.5a the reconstructed profiles are compared to the "true" profile. Mismatch causes clear systematic errors which are complementary in the two cases. Since our fitting procedure is constrained to a certain velocity range, typically $1000-1500 \mathrm{~km} / \mathrm{s}$, only mismatch between crowded and thus overlapping spectral features will afflict the reconstruction of the LOSVD. Thus the algorithms proposed by Franx and Illingworth (1988) and Bender (1990) are expected to suffer from similar systematic effects.

It is important in this context to distinguish between the systematic errors in the line profile, i.e. in the coefficients $b_{i}$, and the corresponding errors in the parameters which characterise a particular broadening model. The relative size of these latter errors can be substantially larger than the systematic deviations in the line profile, if the particular model chosen is sensitive to small changes in the line profile. The case of Figure 4.5 a may serve as an illustration; the template mismatch results in errors for the gaussian parameters of up to $40 \%$ (The mismatch here is admittedly a very severe one). For generality's sake we will refer to the errors in the line profile in the following. Their translation into errors of a broadening model must be evaluated individually for each case at hand.

To simulate template mismatch in a more controlled fashion we started out again with two identical spectra as the "galaxy" and the "template." Subsequently, we added random perturbations to the unbroadened "galaxy" spectrum, as mismatch,


Figure 4.6: Systematic Errors in the LOSVD due to Template Mismatch This figure shows the systematic errors in the reconstructed (double gaussian) LOSVD if a mismatched template is used. The left two panels show tests with a K5III "galaxy" spectrum and a K0III template. The roles of the two spectra were switched for the right two panels. The top panels show the true LOSVD and its mean reconstruction. The bottom panels highlight the deviations from the true profile. The error bars indicate the scatter in an ensemble of 50 reconstructions due to Poisson noise in the "galaxy" spectrum (we took $S / N \approx 50$ ). The mean reconstructed parameters ( $V$ and $\sigma$ ) for a double gaussian model are given in the top panel. The input parameters were $v_{1}=0, v_{2}=4, \sigma_{1}=4$ and $\sigma_{2}=2$.
and broadened it with a known LOSVD, added noise again, simulating photon noise, and tried to recover the LOSVD with the "template" spectrum. Although the impact of mismatch on the recovered LOSVD will depend on the particular form of the mismatch, the following results may serve as a guide line: for an RMS difference between the two spectra of $3 \%$ (of the mean counts in the original spectrum), added as random perturbations to the "galaxy" spectrum before broadening, the resulting systematic deviations were found to be $2 \%$ to $3 \%$ of the peak value of the LOSVD. In general, the input difference and the resulting LOSVD error appear to scale linearly for differences of less than $10 \%$.

As we now show, a further consequence of template mismatch is the mimicking of the presence of a spurious, "second component" in truly gaussian line profiles, as inferred from a $\Delta \chi^{2}$ test. We construct 3 ensembles of 30 artificial spectra by broadening the spectrum of a K5III star with a single gaussian of dispersion 3 pixels and adding noise. The mean $\mathrm{S} / \mathrm{N}$ per pixel in the three ensembles is 20 , 50 , and 100 . We then fit single and double gaussians to each spectrum using a KOIII star as the template. The distributions of $\Delta \chi^{2}$ (defined above) are compared with that expected for insignificant mismatch ( $\chi^{2}$ with 3 degrees of freedom) in Figure 4.5. Clearly mismatch is not detectable above the noise for $\mathrm{S} / \mathrm{N}=20$, but is dominant for $S / N=100$. In the latter case, the dispersion of the primary component is typically underestimated by $25 \%$, while a second, spurious component with a relative contribution of order $30 \%$ is added to fit the mismatch.

The most obvious way to minimise systematics due to mismatch is to use the best possible template. In practice this means creating a synthetic template from a library of unbroadened stellar spectra spanning a suitable range in spectral type. Because of the narrow slits used for kinematic studies, spectrophotometry is impossible, and we must restrict ourselves to matching the strength of spectral features.


Figure 4.7: Two Sided Fitting of the LOSVD
Systematic errors in the profile reconstruction can be suppressed if a single LOSVD model is fitted simultaneously to two spectra (assumed to be taken at diametrically opposed points) while enforcing reflection symmetry in the line profile. The left panels are identical to those in Figure 4.6. The right panels show the improvement when such two-sided fitting is employed.


Figure 4.8: Matching Composite Templates
The three panels illustrate how the match of spectral features can be improved by using composite templates. The spectrum is taken from NGC 5322 at $4^{\prime \prime}$. The top panel shows the best fit with a K0 III template, the center panel shows a fit with a K5 III template and the bottom panel shows the best fit with a composite template, as described in Section 4.3.4. The difference between the galaxy spectrum and the fit is shown in each panel, offset by -20 . The spectra are normalized so that the expectation value of the noise is unity at each pixel (see Section 4.3.2).

This is, in any case, all that is required - our continuum fit takes out much of the effect of large-scale variations. This procedure might still give spurious results if there are multiple kinematic components with very different stellar populations. At present, there is no observational evidence for major population differences between the various kinematic subsystems in the kind of galaxy we are studying. A generalisation of the technique we now discuss could be used to search for such differences.

We have investigated optimizing our template for the spectral region from $4900 \AA$ to $5600 \AA$, which is frequently used for kinematic studies, and in which we have taken most of our own data. Standard spectral synthesis models (e.g. Pickles 1985) show that in this spectral range the light in most ellipticals is dominated by giants between late $G$ and early $M$. We therefore assembled a set of six template spectra ranging from G6III to M1III, and employed an iterative technique to find the best composite template for each galaxy. We determine a first estimate of the broadening function using a single stellar template. We then shift all the templates to a common reference velocity and broaden them with this function. The biased random walk technique described in the Appendix can now be used to find the set of non-negative coefficients which minimise $\chi^{2}$ for the fit of a superposition of these broadened templates to the galaxy spectrum. With these coefficients we construct a composite unbroadened template which leads to an improved estimate of the broadening function. The whole procedure can then be repeated; however, we have found that, in practice, only little additional reduction in $\chi^{2}$ results from further iteration.

Figure 4.8 shows that the overall match of spectral features can be improved dramatically by this technique. The top panel shows the best fit to a major axis spectrum of NGC 5322 at $3^{\prime \prime}(S / N \approx 60)$ when a KOIII star is used as template; the second panel shows results for a K5III star; and the bottom panel shows the
greatly improved fit when a composite template is used. The $\chi^{2}$ values for the fits shown in these three plots are 1120, 1717, and 840 respectively (for 756 degrees of freedom). The improvement is thus highly significant. The best fit template is $32 \%$ $\mathrm{K} 0,66 \% \mathrm{~K} 5,2 \% \mathrm{M} 1$. (We used several different templates of type K0 and K5). This mix is somewhat "later" than usually inferred for the nuclei of ellipticals from long baseline spectral synthesis or broad-band colours, but is not an unreasonable result.

As inferred from the $\chi^{2}$ value, even our best match corresponds to an $r m s$ difference of about $3 \%$ mismatch between the galaxy and the template spectrum. Thus, even after the template matching we must assume that systematic errors in the gaussian model parameters at the $10 \%$ to $15 \%$ level could be present. Since this problem is caused by the intrinsic sensitivity of a double gaussian decomposition to small changes in the line profile, there is little hope to substantially improve on that. In the next section, however, we describe another technique which can often be used to further reduce systematic errors in the reconstructed LOSVD.

### 4.3.6 Additional Checks

As mentioned earlier, in equilibrium stellar systems the velocity distributions at diametrically opposed points are expected to show reflection symmetry. This expectation holds on or off the principal axes, and for triaxial ellipsoids with any regular streaming motion. It has been used by Franx and Illingworth (1988) and Bender (1990) as a direct check on the shape of their derived line profiles. Figure 4.9 illustrates the situation: the two dashed lines represent the LOSVDs of NGC 3610 at $\pm 3^{\prime \prime}$ along the major axis; the velocities of one of them have been reflected about $0 \mathrm{~km} / \mathrm{s}$. The agreement is excellent, and provides convincing evidence for a two component structure. Since the line profile is so strongly asymmetric the system-
atic errors are likely to be considerably smaller than in the examples of the last section. Notice that in this galaxy, unlike NGC 5322, the "disk" component is quite cold, with $V_{\text {rot }} / \sigma \approx 4.5$.

This symmetry constraint provides an additional "trick" which can improve our results. While the LOSVDs are expected to show reflection symmetry, systematic errors due to mismatch have, at least to lowest order, the same shape on each side. It is therefore advantageous to fit spectra on the two sides simultaneously with a single set of broadening coefficients, $\underline{\boldsymbol{b}}$, while enforcing reflection symmetry. This can be done by doubling the length of the $\underline{t}_{i}$ and of $\underline{o}$ in Equation 4.4. The observational vector is simply the two continuum-subtracted, noise-weighted spectra laid end to end; the $t_{i}$ are pairs of templates corresponding to the same broadening coefficient given the assumed reflection symmetry. The result of such a simultaneous fit for NGC 3610 is shown as the solid line in Figure 4.9. Not surprisingly, it is a "compromise" between the individual fits on either side. This procedure substantially reduces systematic deviations in the reconstructed LOSVD. We illustrate this in Figure 4.7 which extends the experiments of Figure 4.6 to this two-sided fitting. Deviations from the true profile are reduced by a factor of 2 or 3 . Again, the translation of this improvement into gaussian parameter space depends sensitively on the specific line profile. This scheme also improves the $\mathrm{S} / \mathrm{N}$ for the fit at a given radius by $\sqrt{2}$. Finally, we find that it lowers "spurious" $\chi^{2}$-reductions of the kind shown in Figure 4.5 by more than a factor of two. These advantages make two-sided fitting the method of choice whenever the symmetry assumption is justified.

A second, independent check which does not depend on assumptions of symmetry is provided by a comparison of results from different spectral regions. For example, Sargent et al. (1978) carried out such a test to establish the validity of their Fourier quotient method. We will compare spectra of NGC 3610 in the $5300 \AA$ re-


Figure 4.9: Disk Signature in NGC 3610
Best double gaussian fit to the LOSVD of NGC 3610 at $3^{\prime \prime}$ on either side of the center along the major axis. The two dashed lines show individual fits to the two spectra, while the solid line represents the simultaneous fit to both sides, enforcing reflection symmetry. The presence of a cold, rapidly rotating component is evident, with $v / \sigma \approx 4.5$.


Figure 4.10: Asymmetric Ca II Triplet Lines in NGC 3610
Near IR spectra, around a line of the Ca II triplet at $8500 \AA$ can be used to check the LOSVD for NGC 3610 which was derived from a spectrum centered at $5200 \AA$. A K5 template spectrum was broadened by the LOSVD shown in Figure 4.9 and is here compared to the observed near-IR spectra of NGC 3610 at $\pm 3^{\prime \prime}$. Note the good agreement while only the line strength was adjusted.
gion, obtained with the instrumental set-up of section 4, to spectra of a line in the Ca triplet region near $8500 \AA$. Due to low spectrograph efficiency, we were unable to obtain near-IR spectra with very high signal-to-noise. We therefore convolved the near-IR spectrum of a template star with the broadening found at $5300 \AA$ and compared the result directly to the spectrum of NGC 3610 . We found excellent agreement, as shown in Figure 4.10. Note that the only free parameter in the modelling of Figure 4.10 was the line strength. While infrared spectra of higher $\mathrm{S} / \mathrm{N}$ are clearly necessary for a definitive test, this comparison further strengthens our faith in the reliability of the methods outlined above.

### 4.3.7 Summary of the Method

In summary it seems worthwhile to stress once more that the main advantage of the methods outlined here, over the ones published previously (Franx and Illingworth 1988, Bender 1990), is the ability to quantify the uncertainties and to reduce systematic errors, rather than any dramatically improved exploitation of the available signal. Some progress, however, has been made in the latter respect by avoiding any tapering of the spectra and by allowing for two-sided fitting (if the stellar system is assumed to be symmetrical). In the presented method no usable information is discarded during the analysis process. For gaussian broadening, we have compared the recovery of $V$ and $\sigma$ through direct fitting and through standard cross-correlation (CC) methods in Monte-Carlo simulations of spectra with $\mathrm{S} / \mathrm{N}$ of 30 and 10 ; for a dispersion of $230 \mathrm{~km} / \mathrm{s}$, the direct fitting recovers $V$ and $\sigma$ unbiasedly, with a scatter of $8 \mathrm{~km} / \mathrm{s}(\mathrm{S} / \mathrm{N}=30)$ and $30 \mathrm{~km} / \mathrm{s}(\mathrm{S} / \mathrm{N}=10)$ in each parameter. The reconstruction of $V$ from the CC is equally good. However, the recovered values of $\sigma$ show both an increased scatter, by a factor of 1.5 to 2 , and systematic errors due to the fact that the CC peak is not gaussian (which leads to a faulty correction for the instrumental
width).
As illustrated in the previous sections, any filtering process will introduce substantial, irreversible bias in the reconstructed LOSVD. In many applications it may thus be more beneficial to formulate the bias explicitly in terms of a parametrized model, and then obtain unbiased estimates for the model parameters. Even in this approach biased estimates can result from template mismatch. From the discussion in 3.5 and 3.6 it follows that such biases can be substantially reduced but not completely eliminated, and therefore may limit the accuracy of the LOSVD reconstruction in many circumstances.

### 4.4 The Core Dynamics of NGC 5322: A Worked Example

We now apply the above techniques to an analysis of the dynamical state of NGC 5322. Bender (1988b) found the core of this galaxy to be counter-rotating at $80 \mathrm{~km} / \mathrm{s}$ with respect to the outer parts ( $V_{\text {rot }}=-30 \mathrm{~km} / \mathrm{s}$ ), and also published detailed photometry indicating the presence of a flattened central component of small scale length (Bender et al. 1988). Far from the center, the mean ellipticity is 0.3, the isophotes are significantly "boxy", and $V_{\text {rot }} / \sigma \approx 0.15$. In the core the isophotes alter their shape and become noticeably "disky". There is thus a clear suggestion that the kinematic peculiarities may be due to a counter-rotating nuclear disk.

### 4.4.1 Observations and Data Reduction

The subsequent modelling of NGC 5322 is based on major axis spectra obtained at the MMT $^{5}$ on May 5, 1989 under good seeing conditions (FWHM $\approx 1.2^{\prime \prime}$ ). We used

[^26]the Red Channel Spectrograph (Schmidt et al. 1989) with a 1200 lines/mm grating and a $1.25^{\prime \prime} \times 180^{\prime \prime}$ slit to obtain a spectrum that covers the region from $4900 \AA$ to $5500 \AA$ at $45 \mathrm{~km} / \mathrm{s} /$ pixel. The total integration time is 60 min , the FWHM of an unresolved line is 2.6 pixels, and the spatial sampling is $0.6^{\prime \prime} /$ pixel; this gives a $\mathrm{S} / \mathrm{N}$ of 35 per pixel at a $V$ surface brightness of $18.3 \mathrm{mag} /\left[^{\prime \prime}\right]^{2}$.

Spectra of our template stars were taken with the same instrumental set-up and at many positions along the slit in order to minimise any difference in instrumental broadening between template and galaxy. The $\mathrm{S} / \mathrm{N}$ was always much higher for our templates than for the galaxy. All spectra were reduced with standard IRAF tasks. This software requires only a single interpolation which we used to derive spectra in $45 \mathrm{~km} / \mathrm{s}$ logarithmic bins. This resulted in minimal degradation of the instrumental resolution. Finally, whenever necessary, we summed galaxy spectra along the slit to obtain at least $8 \cdot 10^{5}$ photons/spectrum.

For each spectrum we found a best composite template and a best double gaussian line profile using the techniques of section 3.5. In addition, we assumed the galaxy to be in equilibrium so that we could fit both sides of the center simultaneously, enforcing reflection symmetry in the way described in section 3.6. The mix of spectral types we found was reasonable (see section 3.5) and was similar at all radii ${ }^{6}$. This confirms earlier findings that kinematically distinct cores do not have dramatically different stellar populations from the rest of the galaxy (Franx and Illingworth 1988, Bender 1990). The results of the double gaussian fitting are shown in Figures 4.11. At 4" a comparison with the decomposition of Bender (1990) is possible: the results agree within $10 \%$ to $20 \%$ (in the gaussian parameters). The discrepancy could quite easily be explained by systematic errors (see Section 4.3.5).

[^27]Parameter sets at each radius refer to independent spectra, independently reduced, so that the point-to-point scatter should give a reliable measure of the random uncertainties. These random errors are comparable to the systematic uncertainties discussed in section 3.5. We clearly detect both "bulge" and "disk" components out to $10^{\prime \prime}$ ( $\approx 0.4 R_{\text {eff }}$ ) (see Fig. 9 a and 9 b ). The rapidly rotating "disk" contributes a maximum of $40 \%$ to the major axis line strength (at $2.5^{\prime \prime}$ ) but its relative strength has dropped by a factor of $5-10$ at $10^{\prime \prime}$. It is this drop, rather than the overall drop in surface brightness, which prevents us from seeing both components at larger radii. The inferred profile of "disk" surface brightness is roughly consistent with an exponential of scale-length $2.1^{\prime \prime}$ (see Fig. 9b). The rotation and velocity dispersion we find for the "bulge" agree roughly with an inward extension of the values found from a single gaussian fit to our data at larger radii, which are in turn are consistent with those obtained earlier by Bender (1988).

### 4.4.2 A Dynamical Model

We can further interpret the results of Figures 4.11 and 4.12 by making a simple dynamical model for NGC 5322. We assume that the potential well is determined by the bulge component and that the disk can be viewed as a population of test particles in this potential. This approximation is a little rough since the secondary component constitutes about $13 \%$ of the light inside $12^{\prime \prime}$. To simplify further we model the main body of the galaxy (which is actually E3, with $V_{\text {rot }} / \sigma=0.15$ ) by a spherical Hernquist (1990) model with an isotropic velocity dispersion. We believe that such a crude approximation to the structure of the primary is justified in light of the fact that the dynamics of the embedded secondary component depend only on the radial run of the circular velocity. Scaling this model to match the observed effective radius, $25^{\prime \prime}$, and the observed line-of-sight velocity dispersion of our primary
component, the rotation curve becomes

$$
\begin{equation*}
V_{c}(x)=C \cdot \frac{\sqrt{x}}{x+1}, \tag{4.14}
\end{equation*}
$$

where $C=720 \mathrm{~km} / \mathrm{s}$ and $x \equiv R / 14^{\prime \prime}$. In this model the circular velocity has a maximum value of $360 \mathrm{~km} / \mathrm{s}$ at about half the effective radius. The extrapolated run of the primary dispersion agrees reasonably with the dispersion measured at larger radii where the secondary component is unimportant.

The dynamics of the secondary component can be described by the Jeans equations in cylindrical coordinates (e.g. Binney and Tremaine 1986). In the equatorial plane, the equation of radial equilibrium is:

$$
\begin{equation*}
\frac{R}{\rho} \frac{\partial}{\partial R}\left(\rho \sigma_{R}^{2}\right)+R \frac{\partial}{\partial z}\left(\overline{v_{R} v_{x}}\right)+\sigma_{R}^{2}-\overline{v_{\phi}^{2}}+R \frac{\partial \Phi}{\partial R}=0 \tag{4.15}
\end{equation*}
$$

To parametrise the observed slight outward decrease in the line-of-sight dispersion of the secondary component, we assume that $\sigma_{l o s}=\sigma_{l o s}^{0} \exp (-x / M)$, with $\sigma_{l o s}^{0}=$ $154 \mathrm{~km} / \mathrm{s}$ and $M=1.65$ corresponding to $23^{\prime \prime}$, to match the observations. The resulting run of $\sigma_{l o s}(x)$ is shown in the right panel of Figure 4.11. This observed velocity dispersion is related to $\sigma_{\phi}$ through

$$
\begin{equation*}
\sigma_{l o s}^{2}=\sin ^{2}(i) \sigma_{\phi}^{2}+\cos ^{2}(i) \sigma_{z}^{2}=\left(\sin ^{2}(i)+\gamma^{2} \cos ^{2}(i)\right) \sigma_{\phi}^{2} \tag{4.16}
\end{equation*}
$$

where we parametrized $\sigma_{z}=\gamma \sigma_{\phi}$. The overall density profile can be approximated by $\rho=\rho_{0} \exp (-x / L)$, with $L=0.15$ (see Figure 4.12). 'This allows us to write $x / \rho \partial \rho / \partial x=-x / L$ and $\partial \sigma_{\phi}^{2} / \partial x=-2 \sigma_{\phi}^{2} / M$. Finally, we can express $\sigma_{R}^{2}$ in terms of $\sigma_{\phi}^{2}$ through the standard relation from epicyclic theory $\sigma_{\phi}^{2}=\left(1+d \ln V_{c} / d \ln x\right) \sigma_{R}^{2} / 2$. Given the large velocity dispersion in the disk, the epicyclic approximation is admittedly poor, but provides at least a well defined relation between $\sigma_{R}$ and $\sigma_{\phi}$. For the circular velocity specified in Equation 4.14 this relation yields

$$
\begin{equation*}
\sigma_{\phi}^{2} / \sigma_{R}^{2} \equiv \beta(x)=\frac{3}{4}-\frac{x}{2 x+2} \tag{4.17}
\end{equation*}
$$

With these relations we obtain for the first term in Equation 4.15

$$
\begin{equation*}
\frac{x}{\rho} \frac{\partial}{\partial x}\left(\rho \sigma_{R}^{2}\right)=-\frac{\sigma_{\phi}^{2}}{\beta} \times\left(\frac{x}{L}+\frac{2 x}{M}-\frac{d \ln \beta}{d \ln x}\right) \tag{4.18}
\end{equation*}
$$

The second term in Equation 4.15, is normally expected to be small in the vicinity of the disk plane and we therefore neglect it. Furthermore, we can write $\overline{v_{\phi}}{ }^{2}=\sigma_{\phi}^{2}+\overline{v_{\phi}^{2}}$. This leads to our final model for the observable mean streaming velocity of the disk

$$
\begin{equation*}
V_{L o s}(x)=\overline{v_{\phi}}(x) \sin (i)=\sqrt{V_{C}^{2}-\sigma_{\phi}^{2} \cdot\left[\beta^{-1}\left(\frac{x}{L}+\frac{2 x}{M}-\frac{d \ln \beta}{d \ln x}-1\right)+1\right]} \times \sin (i) . \tag{4.19}
\end{equation*}
$$

where $i$ is the inclination of the disk to the plane of the sky $(\cos (\mathrm{i})(i)=1$ for face-on). The parameters that are free in fitting this model to the data are the disk dispersions, $\sigma_{\phi}$ and $\sigma_{z}$, and the inclination. The first two can be chosen to match the observed disk dispersion, while the last, in the absence of an independent inclination estimate, can be adjusted to give the disk rotation rate. It is thus not surprising that quite a good fit to the data is possible. The curves in Figure 4.11 assume $\sigma_{\phi}^{0}=148 \mathrm{~km} / \mathrm{s}, \gamma=0.7$, and $i=60^{\circ}$. For this viewing angle the $z$-dispersion $\sigma_{z}$ contributes relatively little to $\sigma_{L O S}$ and is thus only poorly constrained. As a result we are unable to constrain the thickness of the disk, although its inclination appears reasonably well determined

By viewing the inclination as an independent parameter we may have taken too many liberties: an independent estimate of the disk inclination can be derived from the isophote shapes. Bender et al. (1988) find that the isophotes are "pointed" with $a_{4} / a \approx 0.012$ at about 5 arcsec, where we estimate that the secondary component
contributes about $40 \%$ of the major-axis light. Assuming the same disk had been superposed on an exactly spheroidal bulge, we can estimate the inclination, with a slight extension of the modelling described in Chapter 2, to be $i \gtrsim 60^{\circ}$. More detailed modelling by Bender (pers. comm.) yields $i=68^{\circ} \pm 3^{\circ}$. Thus the inclination implied by the photometry is reassuringly close to that we have estimated independently from the dynamics.

As outlined above, it is therefore possible to draw a picture of the core dynamics of NGC 5322, which is consistent with all the available kinematic and photometric data, and is based on a two quite distinct components. It is remarkable that the secondary component rotates at only $170 \mathrm{~km} / \mathrm{s}$, despite the fact that the circular velocity of the galaxy peaks at $360 \mathrm{~km} / \mathrm{s}$. The "disk" is thus apparently supported primarily by pressure rather than by rotation; the first term in equation (15) dominates over the fourth. This term is so important here because the scale length of the secondary component is so small, resulting in a large $\partial \rho / \partial x$. Finally, it is worth noting that in order to understand the dynamics of NGC 5322, it is not crucial that the two components be assumed completely distinct. It is clear that stars with a wide range of orbital properties are present, and can undoubtedly be modelled as a continuum of subcomponents with differing $v / \sigma$. Here we have shown that a decomposition into just two subcomponents can explain all the available data. The "disky" isophotes of the inner regions of the galaxy, and the likely external origin of the counter-rotating material they contain, seem to favour a model with two such discrete components ${ }^{7}$.

[^28]NGC 5322 (Major Axis)


Figure 4.11: Kinematic Model for NGC 5322
Kinematic data for the two-component model for NGC 5322 described in Section 4.4. The left panel shows the streaming velocities, while the right panel shows the line-of-sight dispersions. Triangles represent the primary, and circles the secondary component. The lines represent the results of the dynamical modelling described in 4.4. The line superimposed to the velocities of the primary merely represents an intrapolation of the velocity field in the outer parts of NGC 5322. Note that adjacent points in this plot (and in Fig. 4.12) are obtained from independent data sets. The scatter is thus a good indication of the random errors.


Figure 4.12: Spheroid and Disk Luminosity Profiles (NGC5322)
Major axis intensity profiles of the primary and secondary components of NGC 5322 (weighted by relative line strength) as inferred from the line profile decompositions. The disk, whose peak contribution to the total light is $40 \%$, can be approximated by an exponential profile with a scale length of $2.1^{\prime \prime}$. Note that the relative intensity of the two components is determined by the line profile shapes and is independent of any photometric data.

### 4.5 Conclusions

We have outlined two new techniques for extracting line-of-sight velocity distributions (LOSVDs) from absorption line spectra. These methods operate exclusively in pixel (or velocity) space, they are straightforward in concept, they preserve all the information originally contained in the spectrum, they allow rigorous error analysis, they are easily generalized to include a simultaneous synthesis of the stellar population mix, and they are flexible enough to test a wide range of models for the form or spatial symmetry of the LOSVDs. One method is a form of optimal, or Wiener filtering, and requires no assumption about the form of the LOSVD other than that it be bandwidth limited. The second is appropriate whenever the data are to be fit with parametrized models, for example, a gaussian, a superposition of two gaussians, or a symmetrical profile with variable wing-shape. We have demonstrated how standard $\chi^{2}$ techniques can then be used for hypothesis testing, parameter estimation, and confidence interval construction. An analysis of random and systematic errors gives an estimate of the signal-to-noise required for reliable observational detection of a complex LOSVD. For a typical set-up at a 4 -meter telescope, this can be done at a $V$ surface brightness of $18.5 \mathrm{mag} . /\left[{ }^{[\prime \prime}\right]^{2}$ in a 1 hour exposure. For typical giant ellipticals this implies that complex line profiles can be analysed out to $r_{e f f}$, with suitable spatial averaging. Mismatch between spectral features in the templates and the galaxies can introduce systematic bias in the derived LOSVDs. This bias has particularly severe consequences if the velocity distribution is to be interpreted in the context of a double gaussian model, where the parameters react very sensitively to small changes in the line profile. We have found that the construction of suitable
enclosed mass $M_{\text {enclosed }}=\frac{v^{2} \text { airc } r}{G}$ to be $8 \cdot 10^{9} M_{\text {Sun }}$. The light enclosed by the $3^{\prime \prime}$ isophote is $m_{R} \approx 12.3$. Assuming a distance modulus of 32 , we find $L_{\text {enclosed }} \approx 3.5 \cdot 10^{9} L_{S u n}$, implying a mass-to-light ratio of a few
composite templates reduces this problem significantly.
We have applied our techniques to two ellipticals which were already known to have kinematically distinct cores. We have found significantly asymmetric LOSVDs in both NGC 3610 and NGC 5322. In particular, we have shown that the kinematic data on NGC 5322 agree with a simple dynamical model for a small, counterrotating disk embedded in a slowly rotating bulge. This model agrees well with the photometric data of Bender et al. (1988). The two components are clearly detected in the observed LOSVD out to $10^{\prime \prime}$, and the inclination required for the dynamical model to fit is close to that estimated from photometric determinations of isophote shape. In this system the "disk" component has $V_{\text {rot }} / \sigma \sim 1.3$. Such a small value may make formation by a dissipationless merger, as suggested by Balcells and Quinn (1990), a plausible possibility. However, $V_{\text {rot }} / \sigma$ is about 4 for the disk in NGC 3610 (at $2^{\prime \prime}$ ) which would seem to require formation of the stellar material in situ from a gaseous disk. Such a formation scenario may be further supported by the extremely close alignment, better than a few degrees, of the disk in NGC 3610 with the main body of the galaxy (Scorza and Bender, 1990); this might arise naturally if the disk material had originally been dissipative. Since the "disk" and the "bulge" corotate in this galaxy, there is no particularly strong indication that the disk material was acquired in a merger event.

Finally, it is worth noting that these nuclear disks may be a significant source of scatter in the central $M / L$ ratios estimated for ellipticals. The naively measured velocity dispersion may not reflect the $\sigma$ of the spheroidal component. In addition, the presence of nuclear disks may be a significant source of the scatter in the $D_{n}-\sigma$ relation (Dressler et al. , 1987).

Our results so far in these nearby galaxies, together with similar results obtained by Franx and Illingworth (1988) and by Bender (1990), show that line profile fitting
provides a new and powerful way to study the structure of elliptical and S 0 galaxies.

### 4.6 Appendix: $\chi^{2}$ - Minimisation by a Biased Random Walk

The idea behind this technique is extremely simple: suppose we want to minimize

$$
\begin{equation*}
\chi^{2} \equiv \chi^{2}(\underline{\alpha}) \tag{4.20}
\end{equation*}
$$

where $\chi^{2}$ in a non-linear function of the $K$ model parameters, $\alpha$. For each parameter $i$, define a "box", $\left[\alpha_{i}^{\min }, \alpha_{i}^{\max }\right]$, a maximum step size, $\Delta \alpha_{i}^{\max }$, and an initial guess $\alpha_{i, 0}$. Now start a random walk in parameter space by drawing a change in each parameter at random from the interval, $\pm \Delta \alpha_{i}^{\max }$ and rederiving $\chi^{2}$ for the new parameters. Any step which decreases $\chi^{2}$ is accepted; Steps which increase $\chi^{2}$ by $\Delta \chi^{2}$ are accepted with probability $\exp \left(-\Delta \chi^{2} / \Delta \chi_{\text {acale }}^{2}\right)$, where $\Delta \chi_{\text {scale }}^{2}$ can be viewed as an adjustable "uphill penalty". Any step that would leave the parameter box is rejected. Initially the step size is chosen generously, to allow an "exploration" of the $\chi^{2}$-surface. After a sufficient number of steps (a few hundred to a few thousand) the step size is gradually decreased and the penalty is stiffened to force the parameter values to a $\chi^{2}$ minimum.

This method, despite its brute force character, has two important advantages over gradient search techniques: first, it is very robust in finding the global minimum in the box, independent of the initial guess. This is achieved by allowing uphill steps (to overcome local minima) and by avoiding linearization of $\chi^{2}(\underline{\alpha})$ with respect to the model parameters is required, required in the gradient search method. Secondly, it is trivial to enforce boundaries on the parameters (such as non-negativity of the broadening amplitudes, etc...) by rejecting steps that would cross those boundaries.

However, two conditions must be satisfied to make this method practical. Firstly, we must be able to make sensible choices for the parameter box and the step sizes
a priori. Secondly, we must devise a fast way to calculate $\chi^{2}$ for each set of parameters, since several thousand steps are usually required to find the global minimum reliably and accurately. The first problem is rarely serious. For example, if the broadening model is a single gaussian, the parameter limits for the amplitude are given by zero and the relative line strengths of template and object, the range of possible dispersions and velocities can be estimated from the total luminosity of the object (allowing a safe margin), the initial step size can be chosen as some fraction ( $5 \%$ to $10 \%$ ) of the allowed parameter range, and the final step size will be a similar fraction of the expected accuracy of the parameter determination. Monte Carlo simulations show that, for single and double gaussian broadening models, an initial uphill penalty of $\Delta \chi^{2} \approx 10$, works well. The uphill penalty is subsequently tightened by at least a factor of ten.

A first glance at Equation 4.4 suggests that the number of operations required to calculate $\chi^{2}$ is similar to that needed to calculate $\underline{\underline{T}} \cdot \underline{b}$, namely $M \times N$. However if we rewrite this equation as

$$
\begin{equation*}
\chi^{2}(\underline{b})=\underline{b}^{T} \cdot\left(\underline{T}^{T} \cdot \underline{\underline{T}}\right) \cdot \underline{b}-\underline{b}^{T} \cdot\left(\underline{\underline{T}}^{T} \cdot \underline{o}\right)-\left(\underline{o}^{T} \cdot \underline{\underline{T}}\right) \cdot \underline{b}+\left(\underline{o}^{T} \cdot \underline{o}\right), \tag{4.21}
\end{equation*}
$$

we see that all the quantities in parentheses need to be calculated only once, at the beginning of the random walk. Thereafter, the calculation of $\chi^{2}$ is dominated by that of the first term which takes of order $M \times M$ operations. Since the broadening bandwidth, $M$, is $\sim 30$, while the number of pixels in the spectrum, $N$, is $\sim 1000$, this implies a speed-up of a factor of 30 to 50 .

## CHAPTER 5

## SUMMARY

The goal of this thesis was to help clarifying the role of disk components in early type galaxies. In this last chapter we will briefly summarize what we were able to answer in the course of this work, discuss remaining questions and give an outlook on avenues for future work.

## Main Results

Before discussing some of the broader and more speculative implications of the results, we will restate the main conclusions of the individual chapters:

- The pointed isophotes, seen in many "elliptical" galaxies, can be naturally explained by a spheroid-disk (S/D) geometry. The observed deviations in the isophotes from perfect ellipses (as characterized by the parameter $A_{4}$ ) can be produced by a wide range of intrinsic disk and spheroid parameters. This range is a result of the similarity in photometric signatures between weak, nearly edge-on disk components, and their more luminous, but less inclined counterparts.
- In particular, the original conjecture that diskiness is caused by very weak ( $D / S \sim 0.02$ ) disk in the majority of the cases is unlikely on statistical grounds.
- The spheroid/disk hypothesis is supported by a comparison of the $A_{4}$ distribution observed in a statistically complete sample of ellipticals (BDM89) with the expected distribution of S/D models seen from random viewing angles. If all pointed isophotes are caused by disks, this comparison implies that a large fraction of "ellipticals" has disk components constituting a substantial fraction of their total light ( $\sim 25 \%$ ).
- To obtain unbiased statistical estimates of disk frequency and strength in early type galaxies, we obtained surface photometry (in B and H ) of a magnitude limited sample of 80 objects, encompassing both E's and S0's.
- From fitting S/D models to the principal axis luminosity profiles in these galaxies we find that roughly one third of all early type galaxies has conspicuous disks, one third has inconspicuous disks and one third is diskless. Here, conspicuous means that the disk could have been detected photometrically from all viewing angles, while inconspicuous means that a favorably high inclination with respect to the line-of-sight is required to detect the disk. By the term diskless we imply the absence of any flat components that contributes strongly (at least if edge-on) to the major axis light at any observed radius.
- Whether a disk is conspicuous or not is not so much a function of its contribution to the total light as a function of its scale length compared to that of the spheroid. Face-on disks with $R_{\text {exp }} \sim R_{\text {eff }}$ will go unnoticed even if they contribute nearly as much to the total light as the spheroid.
- Disk frequency estimates derived from isophote statistics and luminosity profile fitting are mutually consistent. This implies that most disks are strong enough to imprint themselves on the luminosity profile and therefore make up a sizeable fraction of the light. The central surface brightnesses we infer
for inconspicuous disks are comparable to the ones of bona fide disk galaxies ( $\sim 22 \mathrm{mag} / \mathrm{l}$ in B ).
- Comparing the derived disk frequencies to the distribution of Hubble types in the sample suggests that one third to one half of all galaxies classified as ellipticals have substantial disks.
- The available photometric information is consistent with the assumption of discrete categories of early type galaxies: objects either have disks with $D / S \gtrsim$ 0.25 or none at all. The apparent fading of disks (from S0's to E 's) can be fully explained by viewing angle effects. There is no need to invoke continuity along the Hubble sequence
- We calculated B-H color gradients for a subset of our sample and found that for the "pure" spheroid systems these gradients are in excellent agreement with the expectations from optical colors, and other optical-IR colors (assuming metallicity changes as the dominant underlying physical gradient). For S/D systems we find similar color gradients to the spheroids', implying that in the majority of early type galaxies disks and spheroids have similar colors, and presumably similar stellar populations.
- To analyze the kinematic signatures of disks, in particular nuclear disks, we have developed an algorithm to extract the maximum information about the line-of-sight velocity distribution from absorption line data.
- Applying this algorithm to spectroscopic data of kinematically distinct cores in elliptical galaxies shows that a nuclear disk embedded in the much larger spheroid is a viable paradigm for these phenomena.
- We constructed a stellar dynamical equilibrium model for the counter-rotating core in NGC 5322, showing that such cores can be dynamically long-lived. We found that the extent to which these cores are supported by rotation varies substantially from object to object ( $1.5<v / \sigma<4$ ); some of the nuclear disks are kinematically as cold as "regular" S0 disks. This spread in rotational support suggest that not all nuclear disks have had the same origin: the dynamically hot ones may have formed in a dissipationless merger, while the cold ones formed from a gas disk.

All in all, we found disks in early type galaxies to be more frequent and dynamically more important than previously thought. The abundance of undetected disks, in particular nuclear disks, needs not be devastating for studies of the global properties, such as the $D_{n}-\sigma$ relation, the fundamental plane, etc. because there orders of magnitude are spanned in luminosity and kinetic energy, while a confusion between disk and spheroid geometry will lead to errors not too different from unity ${ }^{1}$. Yet, if nuclear disks are abundant, their presence can certainly explain most of the apparent nuclear "mass-to-light ratio variation". Detailed dynamical studies of individual objects (search for orbital anisotropy, changes in $M / L$, etc.) based only on measurements of $v$ and $\sigma$, however, can be severely afflicted by errors in geometry. In that case the measured effects and the possible errors due to a faulty geometry are of the same order. There are two ways to overcome these ambiguities: First, using the algorithm from Chapter 4 one can check for the presence of a disk. Second, since we have found that a good fraction of "ellipticals" are truly spheroidal (or ellipsoidal), statistical studies of the kinematics (e.g. Franx, de Zeeuw and Illingworth 1991 and van der Marel (1991) can draw a global picture of the "typical"

[^29]behavior.

We have shown that nuclear disks can contribute to the scatter in global relations. However, depending on the orientation of the system, a nuclear disk can increase or decrease the measured velocity dispersion compared to the structure of the underlying potential well; therefore nuclear disks may not introduce systematic errors.

Another implication of our result is that disks in early type galaxies are more "normal" than they appear, i.e. they occupy a smaller region in parameter space than would be naively inferred from the data. In particular they have surface brightnesses nearly as high as spiral galaxies along with comparable scale lengths. Therefore the main difference between the disks in early and late type galaxies appears to be the recent star formation history (or lack thereof). All the properties of the disks we found can be explained by the same disk formation mechanism as in spirals.

It is somewhat surprising that the disks in early type galaxies are as bright as they are found to be. As we mentioned in the introductory chapter, there is dynamical evidence that the disk material was brought to its present location at the epoch of spheroid formation. Furthermore, the similarity in colors between disks and bulges in many $S 0$ argues that most of the material was converted into stars soon thereafter; thus disks in early type galaxies are old, or at least substantially older than in spirals. Consequently, they must have had a very intense epoch of disk star formation early on to reach these high surface brightnesses.

The only category of disk-like structures in ellipticals which may have had a very different history than just outlined, are the nuclear disks which may have formed by a merger (e.g. NGC 5322).

## Remaining Questions

- We have been able to take a disk census for an (almost) complete set of early type galaxies. However, to make statistically sound statements about disk frequencies we had to consider the sample as a whole. Thus we had to average over five magnitudes in absolute luminosity and over various environments. Since many structural properties of galaxies depend on both the luminosity (e.g. Davies et al. 1983) and environment (Dressler 1980), it would be interesting to study disks statistically as a function of these two parameters. However, since the $\sim 80$ objects were just enough to bring the statistical uncertainties down to the level of systematic errors, such an analysis would require a sample larger by at least a factor of five. The ESO key project on surface photometry may eventually provide such a database.
- Multicolor optical and IR surface photometry is a quite powerful method to estimate similarity of stellar populations (even if the interpretation of color differences may be ambiguous). Such analyses have just become possible (with the advent of IR array detectors) and could be employed to study the similarity of disks and spheroids in SOs. We have collected data at only two wavelengths and could only place very conservative limits on possible differences in the S/D populations.
- In this thesis we have focussed on the mathematical tools for assessing the structure of kinematically distinct cores. Since there is evidence that these cores are common, it would be desirable to overcome the stage of modelling a few individual cores and proceed to answer what the "typical" structure of these cores is. It would be interesting to know how well these cores are aligned with the outer parts of the host galaxy. Furthermore, we would like to know
whether counter-rotating cores (which are most likely of external origin) are systematically different, e.g. with respect to $v / \sigma$, than the nuclear disks that rotate prograde.

But, as they say, "One thesis at a time..."

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[^0]:    ${ }^{1}$ Binney et al. experimented with a disk component, yet restricted themselves to disk models contributing at most $10 \%$ to the total light at any point.

[^1]:    ${ }^{2}$ A system, however, whose emissivity is constant on concentric spheroids, can be deprojected for any known inclination (e.g. Kent 1988)

[^2]:    ${ }^{1}$ This Chapter has already been published in the Astrophysical Journal (Rix, H.-W. and White S. D. M., 1990, Ap.J.,362, 52). We have made only a few minor changes, mostly to avoid overlap with the other chapters. We also changed the notation to achive consistency throughout this thesis.

[^3]:    ${ }^{2}$ Note that the term "pointed", used throughout the chapter, is equivalent to the terms "disky" or "lemon-shaped" used by some other authors
    ${ }^{3}$ Orbital families are usually named after the geometrical figure their time averaged orbits resemble in projection: e.g. box, tube, pretzel, fish and banana orbits

[^4]:    ${ }^{4}$ See the introductory Chapter (Section 1.2.3) for a definition of the luminosity profiles

[^5]:    ${ }^{5}$ It should be borne in mind that the $A_{6}$ prediction for the faint edge-on disk is based on a perfectly flat (2-D) disk model (see 2.6.2)

[^6]:    ${ }^{1}$ Virtually all disks in spiral galaxies are forming stars (Caldwell et al. 1990) even now
    ${ }^{2}$ In the sense that they do not show detectable photometric signatures, even if seen edge-on.

[^7]:    ${ }^{3}$ If large areas had to be excluded, e.g. due to companion galaxies, this is noted in Appendix $C$

[^8]:    ${ }^{4}$ For the H-band observations we do monitor a blank patch of sky during the observations. Yet, the large and rapid variations in the IR background do not allow a sufficiently accurate determination of the sky level.

[^9]:    ${ }^{5}$ Neither B nor H band observations were taken under photometric conditions. For old populations, $\mathrm{B}-\mathrm{H}$ is expected to be in the range 3.5-4.0 (e.g. Impey et al. 1986 and Peletier 1989)

[^10]:    ${ }^{6}$ Axisymmetric systems in which spheroid and disk follow the stereotypical luminosity profiles and in which the spheroid is of constant ellipticity

[^11]:    ${ }^{7}$ Aside from P.A. twists, which cannot be explained with axisymmetric models, anyway.

[^12]:    ${ }^{8}$ We fitted each data set repeatedly, starting from different initial parameter guesses, and "fixed" by hand the few cases that yielded differing values of $\chi_{\min }^{2}$

[^13]:    ${ }^{9}$ Note that the isophote deviations from perfect ellipses are included in the principal axis profiles.

[^14]:    ${ }^{10}$ After all, the sky counts and a disk of infinite scale length are completely indistinguishable.
    ${ }^{11}$ The radius of a circle enscribed to the data frames is $60{ }^{\prime \prime}$

[^15]:    ${ }^{12}$ Even though Kent's data were obtained in the $r$-band, the color gradients, B-r, are negligible compared to the other uncertainties involved.

[^16]:    ${ }^{13}$ The slight face-on bias of objects with very large $R_{e x p} / R_{\text {eff }}$ will be discussed at the end of this section.
    ${ }^{14}$ Note, however, that all objects shown in Figure 3.12 have acceptable S/D decompositions. Thus their major axis profile deviates more from an $R^{1 / 4}$ law than the limiting $\chi_{f a c e-o n}^{2}$, given the actual inclination at which these galaxies are viewed
    ${ }^{15}$ For a population seen from random viewing angles, $\cos (i)$ is uniformly distributed (Chapter 2).

[^17]:    ${ }^{16}$ There are no "pure" disk galaxies with only an old population (Sandage 1961)

[^18]:    ${ }^{17}$ E.g. the simple models predict that $25 \%$ of the sample show deviations from ellipses of less $0.1 \%$. Yet, at look at Appendices $A$ and $B$ shows that virtually all objects exhibit (statistically significant) deviations in excess of that.

[^19]:    ${ }^{18}$ We only consider data at radii larger than $2.5^{\prime \prime}$ and are thus not very sensitive to the detection of nuclear disks (see Chapter 4).

[^20]:    ${ }^{19}$ For example, the nuclear disk in NGC $3610(\mathrm{D} / \mathrm{S} \approx 0.08$ ) has a surface brightness of $19.6 \mathrm{mag} / \square^{2}$ (see Scorzca and Bender 1990).

[^21]:    ${ }^{20}$ Bothun et al. (1984) have promoted the IR colors as "metallicity indicators"

[^22]:    ${ }^{1}$ The contents of this chapter are to be published in a paper by H.-W. Rix and S. D. M. White, which is in press at the Monthly Notices of the Royal Astronomical Society

[^23]:    ${ }^{2}$ Franx and Illingworth (1988) note that a generalization of this method to complex line profiles could yield more reliable results for the LOSVD in IC 1459 than the CLEAN algorithm.

[^24]:    ${ }^{3}$ This decomposition is unique up to corresponding permutations of column and rows in the matrices.

[^25]:    ${ }^{4}$ This assumes that the galaxy surface brightness substantially exceeds that of the sky.

[^26]:    ${ }^{5}$ The Multiple Mirror Telescope is a joint facility of the Smithonian Institution and the University of Arizona

[^27]:    ${ }^{6}$ To test for the presence of a younger stellar population, we even included an early $G$ main sequence star in our template library, which was rejected in the fit, i.e. it did not contribute to the fit. However, the wavelength region chosen for the kinematic observations is not very sensitive to population changes

[^28]:    ${ }^{7}$ A similar dynamical model for NGC 3610 will be published in a forthcoming paper (Rix and Peterson, in prep.); here we note just briefly that our decomposition from Figure 4.9 has sensible physical implications. Compared to NGC 5322 this object has two properties that simplify its modelling: first, its disk is highly inclined and therefore $v_{\text {circ }} \approx v_{\text {projected }}$ and, second, the disk is "cold" and therefore the asymmetric drift is small. At a distance of $25 \mathrm{Mpc}\left(v=1873 \mathrm{~km} / \mathrm{s}, H_{0}=\right.$ 75 ). three arcseconds correspond to 120 pc . Making a small asymmetric drift correction of $30 \mathrm{~km} / \mathrm{s}$ to the measured rotation of the $\operatorname{disk}(260 \mathrm{~km} / \mathrm{s})$ and assuming spherical symmetry, we find the

[^29]:    ${ }^{1}$ The difference between an assumed isotropic velocity dispersion and one contaminated by a disk will typically be only of order $10 \%$. Note, however, that this error will endter the luminesity estimate in the fourth power and the distance estimate quadratically.

