Davila, Joseph Michael

THE PROPAGATION OF ENERGETIC PARTICLES IN FINITE TEMPERATURE ASTROPHYSICAL PLASMAS

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THE PROPAGATION OF ENERGETIC PARTICLES IN FINITE TEMPERATURE ASTROPHYSICAL PLASMAS

by

Joseph Michael Davila

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As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Joseph Michael Davila

entitled The Propagation of Energetic Particles in Finite Temperature Astrophysical Plasmas

and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

W. J. Cocke — Date April 26, 1982

[signature] — Date 4/26/82

[signature] — Date "

J. R. Jokipii — Date "

John T. Emery — Date April 26, 1982

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So, first, I would like to express thanks to my advisor, John Scott. Early in the preparation of this work, our lively discussions of wave polarization, dispersion relations and resonant scattering brought us very near to total neuron failure. Complete brain death was avoided only through a rigorous program of therapy consisting primarily of stochastic golf and warm friendship. John has been a continuous source of optimism and enthusiasm throughout the preparation of this dissertation, and for that I am truly grateful.

I would also like to thank Jim Ionson. Discussions of this work with Jim and John were both enlightening and enjoyable. However, many people on Steward's second floor have complained that at times these discussions were too loud. Several claim to have suffered temporary hearing loss and to them I apologize.

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ABSTRACT

Solutions to the dispersion relation for waves propagating parallel to the static magnetic field in a plasma of arbitrary β are obtained. (β is the ratio of thermal to magnetic pressure.) Resonant scattering by these waves is evaluated. It is found that the magnetostatic approximation, used extensively in the past, breaks down for particles with pitch angles near 90°, and one must consider the more complicated process of particle scattering in electromagnetic turbulence. Many aspects of particle propagation in a finite temperature plasma can be discussed without assuming magnetostatic turbulence. This is accomplished by using a graphical method to obtain the solutions of the resonance condition.

Results show that in a high β plasma, wave damping causes a gap, or hole, in μ-space where the resonant particle scattering rate is severely depressed. It is found that only high energy ($\gamma \gtrsim 10^5$) electrons can be trapped within a typical supernova remnant.

When the notion of electromagnetic resonance is applied to particle propagation in the interplanetary ($\beta \lesssim 1$) plasma, it is found that significant modifications to the conventional scattering picture must be made. It is found that a resonance gap exists which is similar to the one in a high β plasma. For electrons, this gap provides a natural explanation for scatter-free events. Theory predicts that these events should occur for kinetic energies $T \lesssim 300$

keV while observations indicate that the majority have $T \lesssim 500$ keV. For protons and energetic electrons, the scattering mean free path is critically dependent on the non-resonant scattering rate for particles within the gap. This fact provides a way to resolve the well known discrepancy between the theoretical and observational values for the mean free path, λ.

CHAPTER 1

INTRODUCTION AND SUMMARY

Particle scattering in electromagnetic turbulence is a process which appears to operate in many astrophysical environments. For example, cosmic ray and solar flare particles propagating through the interplanetary medium are known to exhibit the diffusive behavior characteristic of strong scattering, and the observed isotropy of cosmic rays seems to imply the existence of an efficient scattering mechanism in the interstellar medium. In addition, scattering is an essential ingredient in physical theories concerning particle driven winds around QSO's or clusters of galaxies, and in some theories of particle acceleration by interstellar shocks.

With few exceptions, previous analyses of energetic particle propagation in plasma turbulence have assumed a cold background plasma. However, x-ray observations of many of the plasmas discussed above have demonstrated that temperatures of 10^6-10^7 °K are not uncommon. The thermal speed of the particles in these high temperature plasmas is typically much greater than the Alfven speed. For these cases, the cold plasma approximation cannot be justified. Finite temperature effects, such as collisionless damping, must be incorporated into particle propagation theories before reliable predictions regarding particle scattering in these hot plasmas can be made.

The most important exception to the picture described above is the interplanetary medium. In the solar wind the Alfven speed is typically slightly greater than the thermal speed of the protons. Nevertheless, it is demonstrated below that even under these conditions thermal effects can affect particle propagation. Since particle propagation in the interplanetary medium can be observed directly, it also provides an important check for many of the predictions of particle scattering theory.

This work is begun by considering wave propagation in a finite temperature plasma. It is found that the cyclotron turnovers, which occur at $\Omega_e(\Omega_i)$ for the RH(LH) waves in a cold plasma, occur at significantly lower frequencies in a finite β plasma. (β is the ratio of thermal to magnetic pressure and Ω_α is the non-relativistic cyclotron frequency for particles of type α.) In fact, lightly damped wave modes of a high β ($\beta \gtrsim 10$) plasma are confined to frequencies $\omega \lesssim \Omega_i$. In addition, these waves are significantly damped whenever $k \gtrsim \frac{1}{3}\Omega_i/v_{ti}$ where v_{ti} is the thermal speed of the ions.

Waves with values of k this large, resonantly interact with particles having very small values of μ. Therefore, to evaluate the effect of this damping on particle propagation, it is necessary to consider particle scattering at small values of μ. But in this region of phase space the magnetostatic approximation, used extensively in the past, begins to break down, and one must consider the more complicated process of particle scattering in electromagnetic turbulence.

Many aspects of particle propagation in a finite temperature plasma can be discussed without assuming magnetostatic turbulence. This is accomplished by using a graphical method to obtain the solutions of the resonance condition. Results show that in a high β plasma, wave damping causes a gap, or hole, in μ-space where the resonant particle scattering rate is severely depressed. Details of the actual size of the gap depend on the particle energy and the wave excitation mechanism.

It is found that the damping in a high β plasma is strong enough to suppress the self-generated turbulence of cosmic ray particles streaming from a supernova remnant. The result is that the streaming particles are probably not isotropized within the remnant, and hence are not subject to the large adiabatic expansion losses previously predicted. Use of the full electromagnetic resonance condition allows a more accurate estimate of the size of the resonance gap for these particles. However, it has no significant impact on the qualitative picture of particle propagation as presented initially by Holman, Ionson and Scott (1979).

The exception to this general picture is high energy electrons. Calculations show that there is no resonance gap for electrons having $\nu \gtrsim 10^5$. It is probable that these particles are isotropized within the remnant.

When the notion of electromagnetic resonance is applied to particle propagation in the interplanetary ($\beta \lesssim 1$) plasma, it is found that significant modifications to the conventional scattering picture must be made. With the usual assumptions regarding interplanetary turbulence, it is found that a resonance gap exists which is similar to the one in a high β plasma.

For electrons, this gap provides a natural explanation for scatter-free events. Theory predicts that these events should occur for kinetic energies $T \lesssim 300$ keV while observations indicate that the majority have $T \lesssim 500$ keV.

For protons and energetic electrons, the scattering mean free path is critically dependent on the non-resonant scattering rate for particles within the gap. This fact provides a way to resolve the well known discrepancy between the theoretical and observational values for the mean free path, λ.

An estimate of the mean free path due to particle mirroring on compressive fluctuations within the gap seems to give values of λ in agreement with experimental values. However, additional work must be done before this estimate can be confirmed.

CHAPTER 2

WAVES IN FINITE β PLASMAS

The normal modes of an infinite, homogeneous, magnetized, cold plasma are well known and can be found in any standard plasma textbook. These include, of course, Alfven and ion-cyclotron waves in the low-frequency region, whistlers and electron-cyclotron waves in the intermediate frequency range and high-frequency electromagnetic modes (light) in the high-frequency region. It has been demonstrated in the past that cold plasma modes are excellent approximations to the normal modes of a warm plasma as long as $\beta \ll 1$, where β is the ratio of thermal pressure to magnetic pressure. In most terrestrial applications, where the plasma is confined by a large magnetic field, the magnetic pressure exceeds the thermal pressure by several orders of magnitude and the assumption of $\beta \ll 1$ is well satisfied. The wave modes of these plasmas are then very well approximated by the usual cold plasma modes.

However, in many astrophysical environments, thermal plasmas are confined by a combination of magnetic and gravitational forces. It is reasonable to expect that for many cases of interest the $\beta \ll 1$ approximation will not be valid. Observational estimates of β in several situations have been summarized by Achterberg (1981). These results, presented in Table 1, clearly show that for many cases of astronomical interest $\beta \ll 1$. Therefore, for astrophysical applications, analysis of the effects of high β on plasma wave modes is desirable.

Table 1. Estimates of Various Physical Parameters in Several Astrophysical Environments.

The parameters shown are the thermal electron density n_e, the temperature T, the magnetic field strength B, and the plasma for supernova remnants (S.N.R.), the hot interstellar medium (H.I.S.M.), extra-galactic radio sources (E.G.R.) and the intra cluster gas (I.C.G.). From Achterberg (1981).

	S.N.R.	H.I.S.M.	E.G.R.	I.C.G.
n_e (cm^{-3})	0.1 - 1	10^{-3} - 0.1	10^{-5} - 10^{-3} (in clusters)	10^{-3}
T (K)	1 - 5 x 10^8	3 x 10^5-10^6	2 x 10^6 - 10^8	10^8
B (G)	10^{-6} - 10^{-5}	10^{-6}	10^{-6} - 10^{-5}	10^{-6}
β	10^2 - 10^5	1 - 350	0.1 - 10^3	350

These waves have been investigated before. Foote and Kulsrud (1979) obtained numerical solutions to the dispersion relation for arbitrary propagation direction. They demonstrated that only waves which propagate nearly parallel to the ambient magnetic field are lightly damped. Achterberg (1981) continued this work by obtaining analytic solutions to the dispersion relation for the case of $\vec{k}$ parallel to $\vec{B}_o$, the ambient field. He obtained expressions for the wave frequency $\omega = \omega(k)$ and the damping rate $\Gamma = \Gamma(k)$ for arbitrary β. Both of these analyses considered only waves in the range $\omega << \Omega_i$ and $k << \Omega_i / v_{ti}$. In a cold plasma this is the region where Alfven waves are found.

Holman, Ionson and Scott (1979) had previously suggested that wave damping in a $\beta >> 1$ plasma can prevent energetic particles from scattering through pitch angles of 90°. Achterberg (1981) confirmed these suggestions. However, it was unknown whether scattering by waves at other frequencies or larger values of k could fill in this resonance gap.

In this chapter, waves of frequency $\omega \lesssim \Omega_e$ are considered for all values of k. The specific goals of this portion of the investigation are to obtain analytic expressions for the wave frequency and damping rate, and to develop an overall picture of plasma wave modes in a finite β plasma.

In this chapter, it is demonstrated that the normal modes of high β plasma are significantly different from those of the more familiar cold plasma. As β is increased, the cyclotron turnovers which normally occur at Ω_i and Ω_e, occur at significantly lower frequencies. For $\beta > 10$ the electron cyclotron resonance is so severely depressed, that there are no lightly damped waves having $\omega > \Omega_i$. In the low frequency range $\omega < \Omega_i$

both the RH and LH wave dispersion relations exhibit significant differences from their cold plasma counterparts. The LH wave is essentially confined to frequencies $\omega<(2/\beta)\Omega_i$ in a high β plasma, and the RH wave has a dispersion relation $\omega \propto k^2$ for $\omega > (2/\beta)\Omega_i$. In addition, waves having large values of k ($kv_{ti}/\Omega_i \gtrsim 0.3$) are strongly damped.

These results have significant implications for the evaluation of collisionless transport phenomena in astrophysical plasmas, e.g. the heat flux instability, as well as for particle pitch angle scattering. In later chapters, only pitch angle scattering will be considered. The results developed in this chapter will be used to examine scattering near supernova remnants and in the interplanetary medium.

The dielectric for plane waves propagating in a homogeneous, magnetized plasma has been obtained from linearized Vlasov theory by many previous authors (e.g. Stix 1962). It has been shown that two linearly independent, circularly polarized transverse wave modes exist with dielectric functions given by

$$\varepsilon(k,\omega) = 1 - \frac{\omega_e^{\,2}}{\omega k v_{te}} f_o(Z^e_{\mp}) - \frac{\omega_i^{\,2}}{\omega k v_{ti}} f_o\,(Z^i_{\pm}) \tag{2.1}$$

where the upper (lower) sign refers to right (left) hand circularly polarized waves. In this relation $v_{tj} = (KT/m_j)^{1/2}$ is the thermal speed of particles of type j with mass m_j and $\omega_j = (4\pi n_j q_j/m_j)$ is the plasma frequency of component j. The argument Z is given by $Z^j_m = (\omega + m\Omega j/ kv_{tj}$. The wave frequency ω and wave vector k are related through the dispersion relation, given as usual, by:

$$\frac{k^2c^2}{\omega^2} - \varepsilon(k,\omega) = 0 \tag{2.2}$$

Analytic expressions are not available which describe the function f_o for all values of Z, however, series expansions can be obtained which are applicable in the following ranges of Z.

For $Z_m^j >> \sqrt{2}$

$$f_o(Z_m^j) = \frac{1}{Z_m^j}\left(1 + \frac{1}{Z_m^{j2}}\right) - i\sqrt{\frac{\pi}{2}}\ \frac{k}{|k|}\ \exp - \frac{1}{2} Z_m^{j^2} \qquad (2.3)$$

and for $Z_m^j << \sqrt{2}$

$$f_o(Z_m^j) = Z_m^j - i\sqrt{\frac{\pi}{2}}\ \frac{k}{|k|} \qquad (2.4)$$

To use these approximations, we must divide the ω,k plane into four distinct regions, (see Figures 1 and 2).

Region I is defined by the inequalities $Z_{\pm}^i >> \sqrt{2}$ and $Z_{\mp}^e >> \sqrt{2}$. In this region waves are least affected by thermal properties of the plasma and damping is usually small. The low frequency portion has been studied previously.

Region II is defined by the inequalities $Z_{\pm}^i << \sqrt{2}$, $Z_{\mp}^e >> \sqrt{2}$. In this region the wavelength of the plasma oscillation is small compared to the gyroradius of a typical thermal particle and significant wave damping results.

Region III is defined by the opposite inequalities $Z_{\pm}^i >> \sqrt{2}$, $Z_{\mp}^e << \sqrt{2}$. In this region significant damping is caused by thermal electrons in the plasma. In addition the inequalities defining this region can only be satisfied for waves having $\omega > \Omega_i$ which means that this region need be considered only when the higher frequency waves are being investigated.

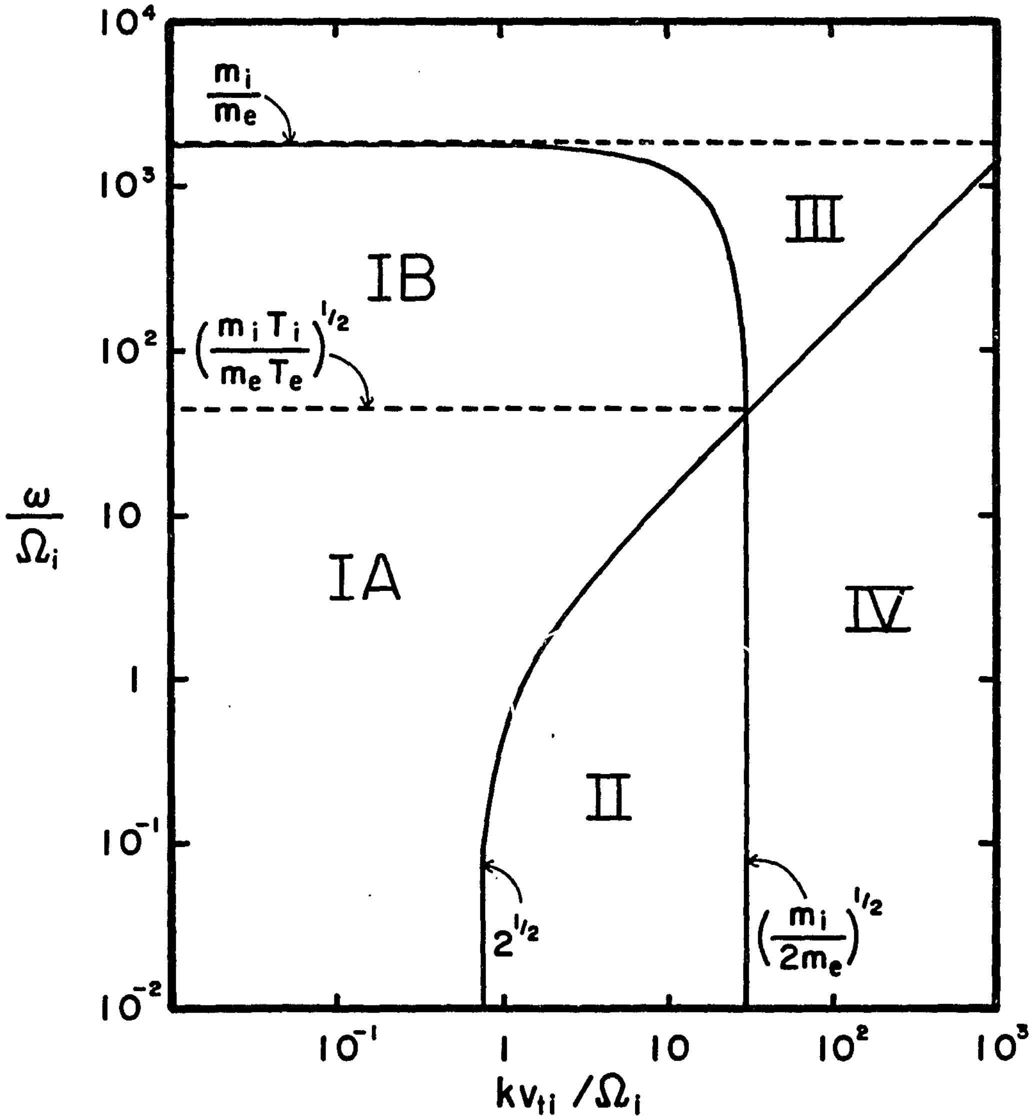


Figure 1. Regions of Dispersion Relation Approximation for RH Wave.

The numerical values presented assume an electron-proton plasma.

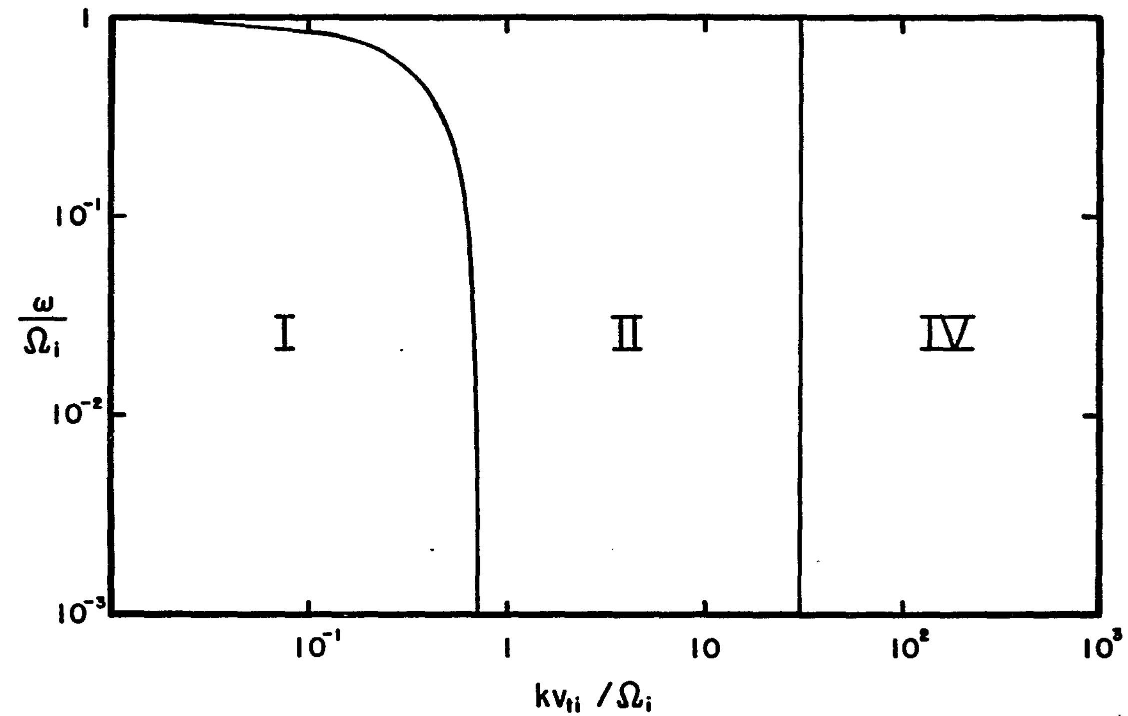


Figure 2. Regions of Dispersion Relation Approximation for LH Wave.

The numerical values presented assume an electron-proton plasma.

Finally, in region IV ($Z^i_\pm \ll \sqrt{2}$, $Z^e_\pm \ll \sqrt{2}$), the wave motion is almost completely dominated by thermal motions in the plasma. All waves are heavily damped. This region is discussed only in a qualitative way.

A. Solution of the Dispersion Relation in Region I

Over most of region I, the normal modes of the finite β plasma are lightly damped ($\Gamma \ll \omega$). In areas where this assumption is satisfied an approximate solution to the dispersion region given in equation 2.2 can be obtained by first dividing the dielectric into real and imaginary parts,

$$\varepsilon(k,\omega) = \varepsilon_1(k,\omega) + i\,\varepsilon_2(k,\omega) \qquad (2.5)$$

The solution of the real part gives the wave frequency using

$$\frac{k^2c^2}{\omega_r^2} - \varepsilon_1(k,\omega_r) = 0 \qquad (2.6)$$

and the damping rate is obtained by Taylor expansion in the usual way as:

$$\Gamma(k) = - \left. \frac{\varepsilon_2(k,\omega)}{\frac{1}{\omega^2}\frac{\partial}{\partial\omega}\omega^2\,\varepsilon_1(k,\omega)} \right|_{\omega=\omega_r} \qquad (2.7)$$

Low Frequency Waves in Region I

For frequencies $\omega < \Omega_i (m_i T_i / m_e T_e)^{1/2} \ll \Omega_e$ the dominant wave damping mechanism is ion cyclotron damping. In this frequency range the dielectric can be approximated as

$$\varepsilon_1(k,\omega) = \frac{c^2}{v_a^{\,2}}\left[\frac{1}{(1 \pm \tilde{\omega}_i)} \pm \frac{\tilde{k}_i^{\,2}}{\omega_i(1 \pm \tilde{\omega}_i)^3}\right] \tag{2.8}$$

$$\varepsilon_2(k,\omega) = \frac{c^2}{v_a^{\,2}}\sqrt{\frac{\pi}{2}}\;\frac{1}{\tilde{\omega}_i\,|\tilde{k}_i|}\;\exp - \frac{1}{2}\left(\frac{1 \pm \tilde{\omega}_i}{\tilde{k}_i}\right)^2 \tag{2.9}$$

It was also assumed that $v_a^{\,2}/c^2 << 1$ to obtain this result. The dimensionless variables used in the dielectric are defined by $\tilde{\omega}_i = \omega/\Omega_i$ and $\tilde{k}_i = kv_{ti}/\Omega_i$.

Using these relations the dispersion relation can be solved for $\tilde{k}_i\ (\tilde{\omega}_i)$ and Γ/ω.

$$\tilde{k}_i^{\,2} = \frac{\beta}{2}\;\frac{\tilde{\omega}_i^{\,2}\,(1 \pm \tilde{\omega}_i)^2}{(1 \pm \tilde{\omega}_i)^3 \pm 1/2\,\beta\,\tilde{\omega}_i} \tag{2.10}$$

$$\frac{\Gamma}{\omega} = \frac{-\sqrt{\frac{\pi}{8}}\;\exp - 1/2\left(\frac{1 \pm \tilde{\omega}_i}{\tilde{k}_i}\right)^2}{\tilde{\omega}_i\,|\tilde{k}_i|\left[\frac{1}{(1 \pm \tilde{\omega}_i)} \pm \frac{\tilde{\omega}_i}{2(1 \pm \tilde{\omega}_i)^2} \pm \frac{\tilde{k}_i^{\,2}}{2\tilde{\omega}_i(1 \pm \tilde{\omega}_i)^3} + \frac{3\tilde{k}_i^{\,2}}{2(1 \pm \tilde{\omega}_i)^4}\right]} \tag{2.11}$$

Previous authors have considered only waves having $\tilde{\omega}_i << 1$. They have demonstrated that finite β has a profound effect on the dispersion relation of the waves at these frequencies. However, these analyses give very little insight into the effects of finite β for waves having $\tilde{\omega}_i \gtrsim 1$, i.e. in the transition region between low frequency and high frequency waves. In the expressions above only terms of order ω/Ω_e have been neglected, so they correctly describe the transition between the low and high frequency wave regimes. The results of previous authors can be obtained by taking the limit $\tilde{\omega}_i << 1$.

Graphs of these functions are presented in Figures 3 through 8. The solution for the RH wave is plotted at two different scales to specifically exhibit the dispersion relation and damping rate for a $\beta \gtrsim 10$ plasma.

This dispersion relation exhibits several interesting differences from the cold plasma result. These have been discussed previously, so they are only summarized briefly here. Consider the denominator of equation 2.10, then

(1) For $\beta\tilde{\omega}_i << 2(1 \pm \tilde{\omega}_i)^3$ one obtains the usual cold plasma result $\omega = k v_a$.

(2) For $\beta\tilde{\omega}_i \sim 2(1 - \tilde{\omega}_i)^3$ the LH wave dispersion relation turns over so that the frequency is no longer a function of k. This is analogous to the ion-cyclotron turnover in a cold plasma, but it occurs at a significantly lower frequency, i.e. <u>finite β tends to depress the cyclotron turnover frequency</u>.

(3) For $\beta\tilde{\omega}_i > 2(1 + \tilde{\omega}_i)^3$ the RH wave has a whistler-like dispersion relation $\omega \propto k^2$. <u>If β is large enough this transition can occur far below the ion cyclotron frequency</u>. These high β whistlers are analogous to the whistlers of cold plasma theory.

Finally, for $\beta > 10$ the frequency of the RH wave remains below $(m_i T_i / m_e T_e)^{1/2} \Omega_i$ until k becomes large enough so that the assumption $\omega + \Omega_i >> \sqrt{2}\, k v_{ti}$ is violated. This conclusion was not apparent from previous analyses which neglected terms of order ω/Ω_i.

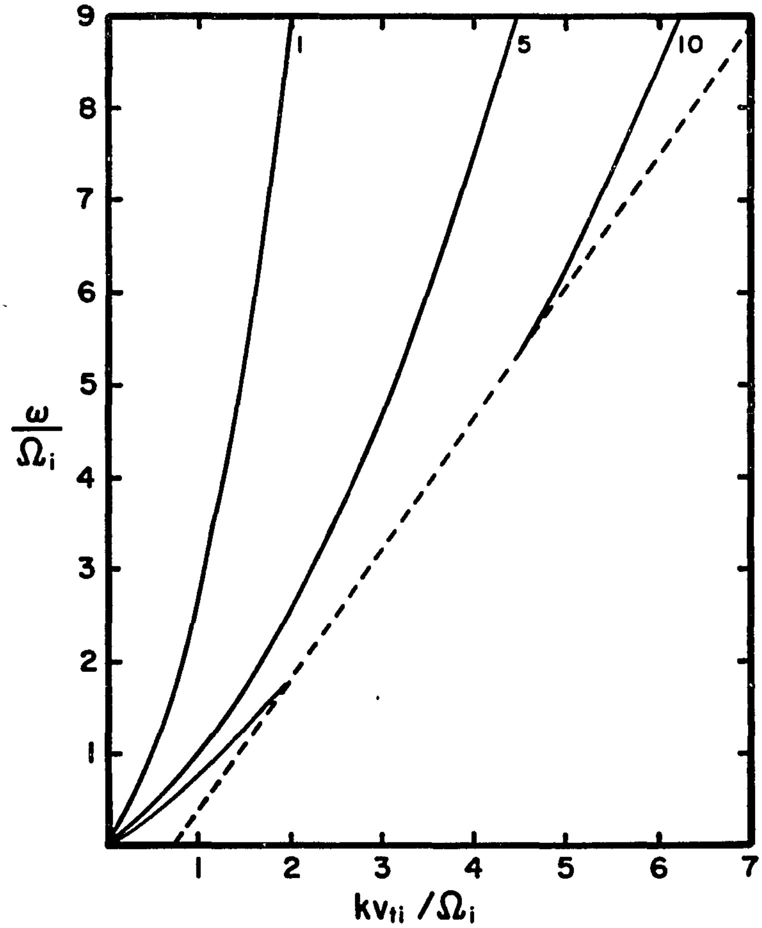


Figure 3. Dispersion Relation for RH Wave in Region IA.

The dispersion relation is shown for three values of β. The damping rate for these waves is shown in Figure 4. Dashed lines indicate boundary of the region where approximation for f(Z) is valid.

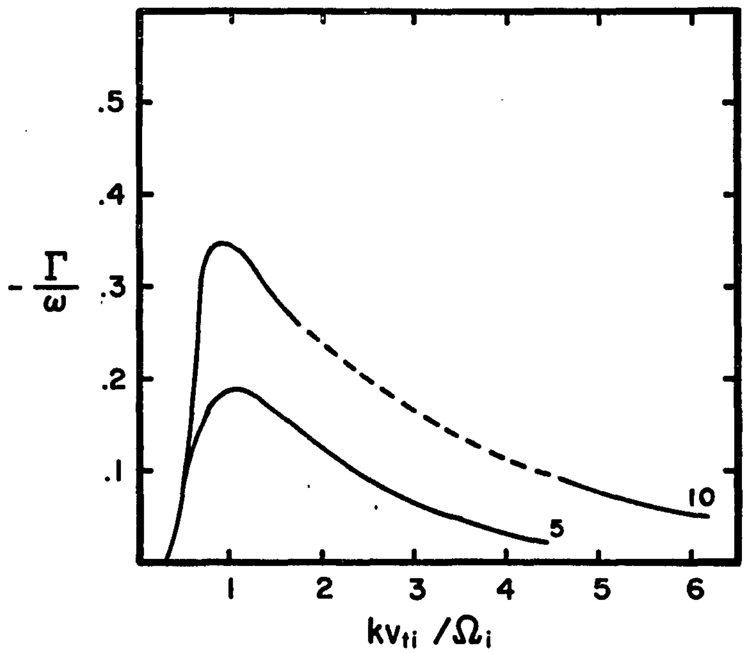


Figure 4. Damping Rate for RH Wave in Region IA.

The values of β shown correspond to those of Figure 3. Damping for $\beta=1$ is too small to be apparent at this scale.

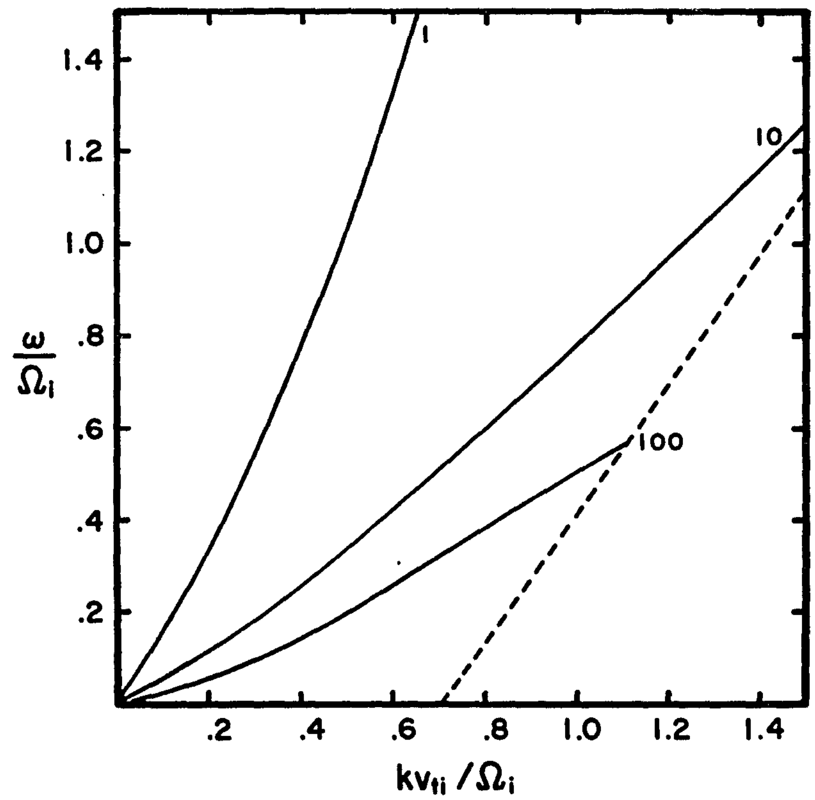


Figure 5. Dispersion Relation for RH Wave and Large β in Region IA.

The corresponding damping rates are shown in Figure 6. Dashed line indicates boundary of approximation region.

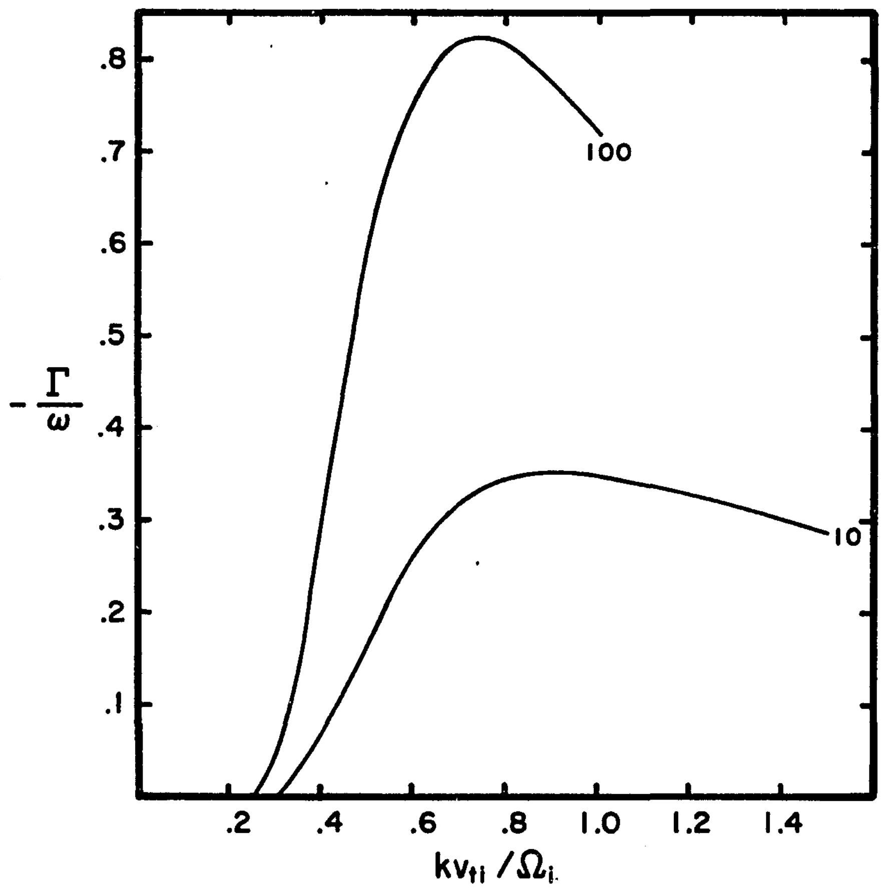


Figure 6. Damping Rate for RH Wave and Large β in Region IA.

The corresponding dispersion relations are shown in Figure 5.

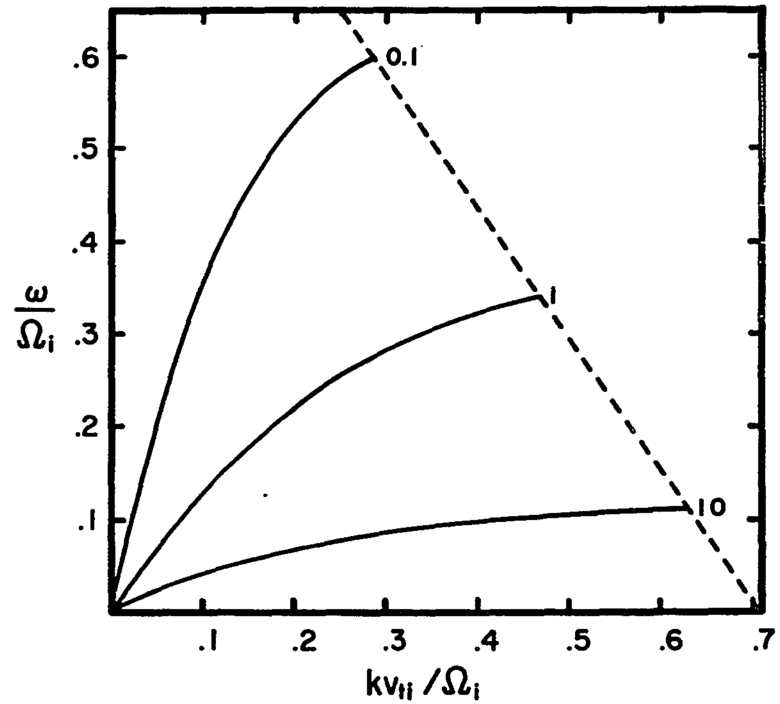


Figure 7. Dispersion Relation for LH Wave in Region IA.

The dispersion relation is shown for three values of β . The corresponding damping rates are shown in Figure 8. Dashed lines again show the boundary of the region where approximation for f(Z) is valid.

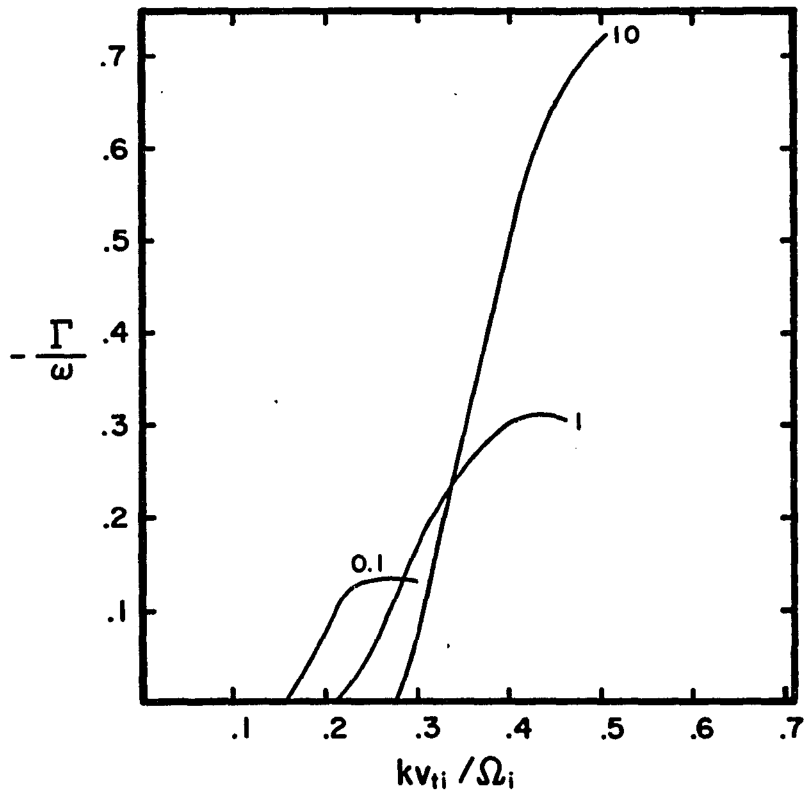


Figure 8. Damping Rate for LH Wave in Region IA.

The corresponding dispersion relations are shown in Figure 7.

High Frequency Waves in Region II

In this region the dielectric can be approximated as

$$\varepsilon_1(k,\omega) = \frac{\alpha c^2}{\tilde{\omega}_e v_a^2} \left[\frac{1}{1 - \tilde{\omega}_e} + \frac{\tilde{k}_e^2}{(1 - \tilde{\omega}_e)^2}\right] \tag{2.12}$$

$$\varepsilon_2(k,\omega) = \frac{\alpha c^2}{v_a^2} \sqrt{\frac{\pi}{2}} \frac{1}{\tilde{\omega}_e |\tilde{k}_e|} \exp - \frac{1}{2} \left(\frac{1 - \tilde{\omega}_e}{\tilde{k}_e}\right)^2 \tag{2.13}$$

To obtain this result it has been assumed that $\omega \gg (m_i T_i / m_e T_e)^{1/2} \Omega_i$ and $v_a^2 / \alpha c^2 \ll 1$, where $\alpha = m_e/m_i$. The dimensionless variables are $\tilde{\omega}_e = \omega/\Omega_e$ and $\tilde{k}_e = k v_{te}/\Omega_e$.

Using these expressions for the dielectric, the dispersion relation can be solved to obtain

$$\tilde{k}_e^2 = \frac{1}{2} \beta \frac{T_e}{T_i} \left[\frac{(1 - \tilde{\omega}_e)^2}{(1 - \tilde{\omega}_e)^3 - \frac{1}{2} \beta \frac{T_e}{T_i} \tilde{\omega}_e}\right] \tag{2.14}$$

$$\frac{\Gamma}{\omega} = \frac{-\sqrt{\frac{\pi}{2}} \exp - \frac{1}{2} \left(\frac{1 - \tilde{\omega}_e}{\tilde{k}_e}\right)^2}{\tilde{\omega}_e |\tilde{k}_e| \left[\frac{\tilde{k}_e^2}{\tilde{\omega}_e^2} \frac{2}{\beta} \frac{T_i}{T_e} + \frac{1}{(1 - \tilde{\omega}_e)^2} + \frac{3\tilde{k}_e^2}{(1 - \tilde{\omega}_e)^4}\right]} \tag{2.15}$$

Graphs of these functions are presented in Figures 9 and 10.

The characteristics of the dispersion relation can again be seen by considering the denominator of equation 2.14. Define $\omega_d = (m_i T_i / m_e T_e)^{1/2} \Omega_i$, then finite β effects are negligible whenever the inequalities $\omega \gg \omega_d$ and $\beta \tilde{\omega}_e \ll 2(T_i/T_e)(1 - \tilde{\omega}_e)^3$ can be simultaneously satisfied.

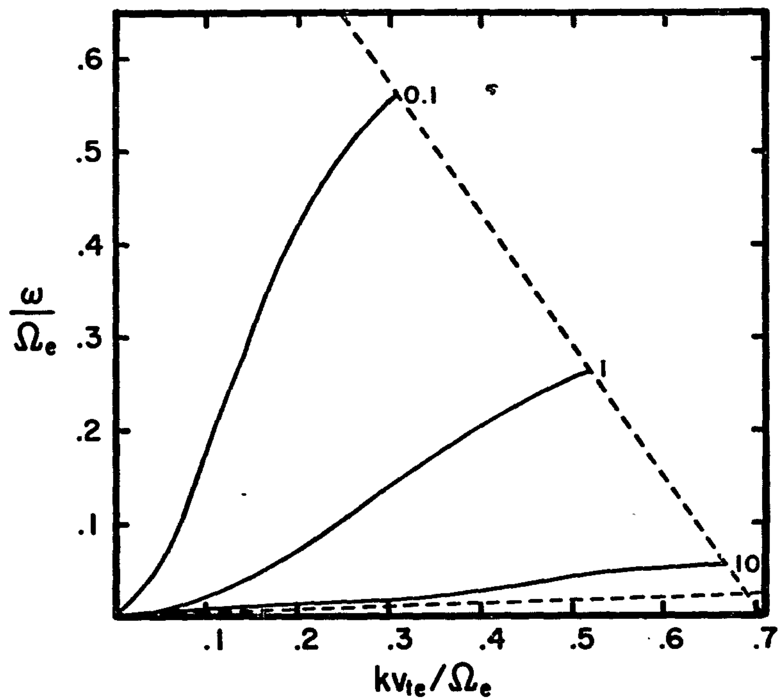


Figure 9. Dispersion Relation for RH Wave in Region IB.

Damping rates are shown in Figure 10.

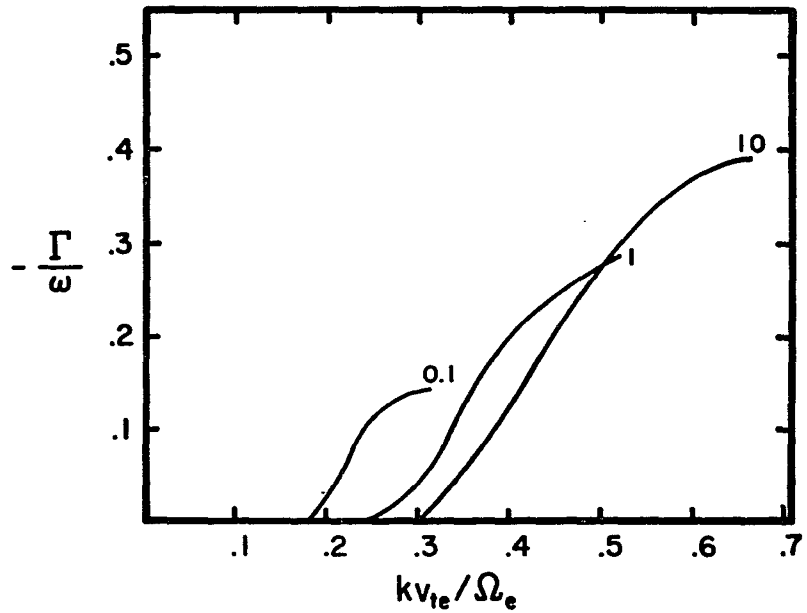


Figure 10. Damping Rate for RH Wave in Region IB.

Damping rates are shown in Figure 9..

For these frequencies the dispersion relation can be approximated by the usual cold plasma result.

As larger and larger frequencies are considered the denominator becomes smaller until at a frequency defined by $\beta\,\tilde{\omega}_e \sim 2(T_i/T_e)\;(1-\tilde{\omega}_e)^3$ the wave frequency becomes independent of k. This is analogous to the electron cyclotron turnover in a cold plasma. For sufficiently high β this turnover occurs for $\tilde{\omega}_e << 1$, and then the turnover frequency is given by

$$\omega_t = (2/\beta)\;(T_i/T_e)\;\Omega_e \tag{2.16}$$

For $\beta >> 1$ this can occur at a frequency considerably below the electron cyclotron frequency. There are no solutions to the dispersion relation for ω greater than the turnover frequency.

Finally, whenever the turnover frequency, ω_t, becomes of order ω_d there are no solutions to the dispersion relation in the frequency range $\omega >> \omega_d$. By equating ω_d and ω_t, it can be shown that this occurs whenever

$$\beta \gtrsim 2(m_i T_i/m_e T_e)^{1/2} \tag{2.17}$$

This is consistent with the results obtained in the previous section for low frequency waves in this region of the ω,k plane. There it was found that for $\beta > 10$, the frequency of the RH wave remained below $(m_i T_i/m_e T_e)^{1/2}\Omega_i$ until k became large enough so that the assumption $\omega+\Omega >> \sqrt{2}\;k\;v_{ti}$ was violated. This leads to the preliminary conclusion that in a high β there are no solutions to the dispersion relation with $\Omega_i \lesssim \omega \lesssim \Omega_e$. This result will be confirmed in subsequent sections when waves with larger values of k are considered.

B. Solution of the Dispersion Relation in Region II

The inequalities defining region II, $\omega \pm \Omega_i \ll \sqrt{2}\, kv_{ti}$ and $\omega \pm \Omega_e \gg \sqrt{2}\, kv_{te}$, can be rigorously satisfied only for a narrow range of k. Nevertheless, it is necessary to investigate waves in this region to determine whether they are consistent with the results of the previous sections. In addition, these "first approximation" solutions exhibit properties which are important for discussing the propagation of high energy particles through a thermal background plasma. A more accurate exposition of the wave properties in this region is desirable, but it will probably not be available unless numerical solutions are done.

The dielectric can be written

$$\varepsilon(k,\omega) = \frac{c^2}{v_a^2}\left\{\pm\frac{1}{\tilde{\omega}_i} + i\sqrt{\frac{\pi}{2}}\left[\left(\frac{T_i m_i}{T_e m_e}\right)^{1/2}\frac{1}{\tilde{\omega}_i|\tilde{k}_i|}\right.\right. \tag{2.18}$$

$$\left.\left.\exp -\frac{1}{2}\left(\frac{m_i T_i}{m_e T_e}\right)\frac{1}{\tilde{k}_i^{\,2}} + \frac{1}{\tilde{\omega}_i|\tilde{k}_i|}\right]\right\} \tag{2.19}$$

where it was again assumed $v_a^2/c^2 \ll 1$.

When this result is substituted into the dispersion relation, one can solve for the frequency and the damping rate:

$$\tilde{\omega}_i = \pm\frac{2}{\beta}\;\frac{\tilde{k}_i^{\,2}}{1 + (\Gamma/\omega)^2} \tag{2.20}$$

$$\frac{\Gamma}{\omega} = -\sqrt{\frac{\pi}{2}}\left[\left(\frac{T_i m_i}{T_e m_e}\right)^{1/2}\frac{1}{|\tilde{k}_i|}\exp -\left(\frac{T_i m_i}{T_e m_e}\right)\frac{1}{2\tilde{k}_i^{\,2}} + \frac{1}{\tilde{k}_i}\right] \tag{2.21}$$

These results are valid for any value of Γ, and are unlike many previous treatments, not subject to the restriction $\Gamma << \omega$. (See however Morrison et al. 1981).

First, notice that there is no solution to the dispersion relation for LH waves in this region having $\omega > 0$. This is a manifestation of the thermal nature of the plasma and it will be discussed in more detail in the following section.

Graphs of the RH wave dispersion relation and damping rate are shown in Figures 11 and 12. Note that the results obtained here are consistent with those of previous sections in that RH waves do not exist in this region of the ω,k plane unless $\beta \gtrsim 10$. In addition, the damping rate Γ/ω is large over most of this region.

C. The Maximum k Limitation; Application to LH Waves In Region III and RH Waves in Region IV

In general, both the RH and LH wave propagating through a plasma exhibit cyclotron-like turnover, where by definition the wave frequency becomes independent of k. In a cold plasma this turnover in the dispersion relation occurs at $\omega = \Omega_e$ and $\omega = \Omega_i$ for the RH and LH circularly polarized wave. As we have found in the previous sections, the turnover frequencies for the RH and LH waves are considerably lower in a high β plasma. The RH wave has a turnover at $\omega \simeq (2/\beta)\ \Omega_e$ while the LH wave has a turnover at $\omega \simeq (2/\beta)\ \Omega_i$. For any plasma of finite temperature, i.e. $T \neq 0$, these resonances do not extend out to infinite values of k.

It is found that, in a high β plasma, solutions to the dispersion relation do not exist for wavelengths smaller than the gyro-radius of a thermal electron (proton) for RH(LH) polarized waves. However, it is important to note that this minimum wavelength limitation is an

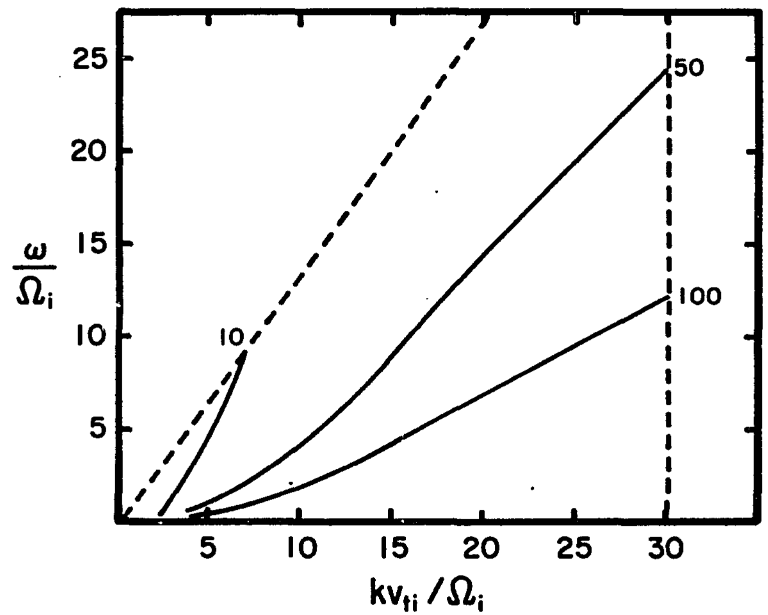


Figure 11. Dispersion Relation for RH Wave in Region II.

Damping rates are shown in Figure 12.

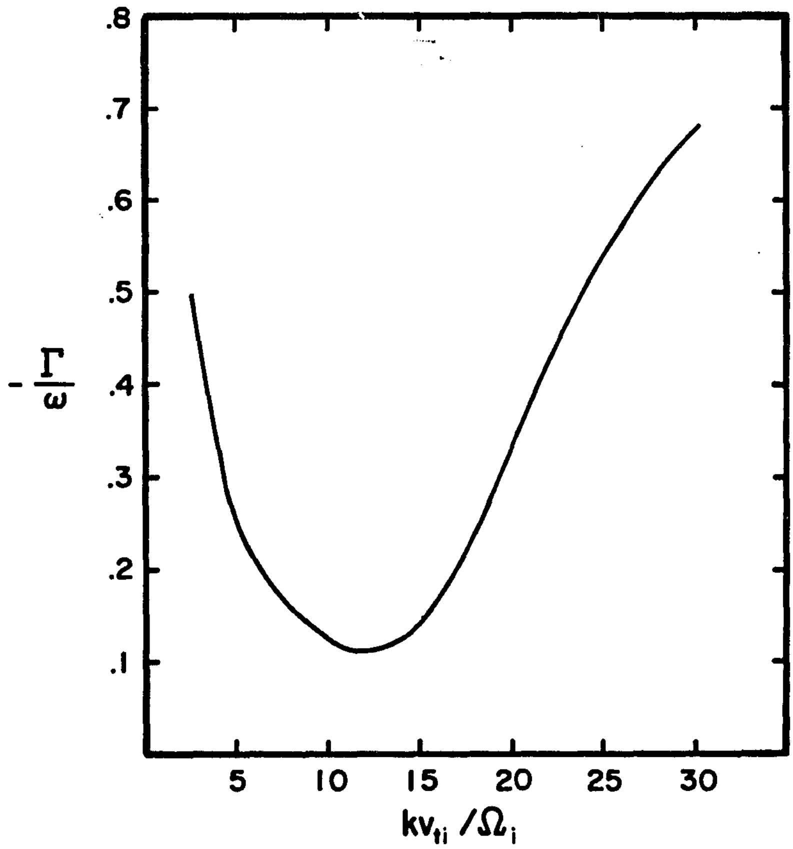


Figure 12. Damping Rate for RH Wave in Region II.

Dispersion relations are shown in Figure 11.

important characteristic of plasmas of finite temperature regardless of the value of β.

To see how this effect comes about, we will follow arguments similar to those given by Stix (1962 p. 195). In general, the dispersion relation for a Maxwellian plasma of a finite temperature is given by equation 2.2 . In the limit $T \to 0$, this can be written as

$$\frac{k^2c^2}{\omega^2} = 1 - \frac{\omega_e^{\,2}}{\omega(\omega \pm \Omega_e)} - \frac{\omega_i^{\,2}}{\omega\ \omega \pm \Omega_i} \tag{2.22}$$

which is the usual cold plasma dispersion relation. Now, as $\omega \to \Omega_e$ or $\omega \to \Omega_i$ we see that $k \to \infty$ and there is no maximum limit on k.

In the region defined by $Z_m^j >> \sqrt{2}$ the dispersion relation for a plasma of finite temperature is very similar to the cold plasma dispersion relation. However, the assumption $Z_m^j >> \sqrt{2}$ cannot be satisfied for all values of $0 \le k \le \infty$ for plasma with $T > 0$. In fact, as long as $T \neq 0$ there is a region where the expansion for f_o in the limit $Z_m^j << \sqrt{2}$ must be used. From the expression for f_o in this limit given in equation 2.4 we see that as $k \to \infty$, $Z_m^j \to 0$ and f_o approaches a maximum value given by $\left|(f_o) \max\right| = (\pi/2)^{1/2}$. This fact, which is entirely a finite temperature effect, gives rise to the maximum k limitation.

Consider the LH wave in the region $\omega << \Omega_e$. By substituting the maximum value of f_o into the dispersion relation one obtains

$$k^3_{max} \simeq \sqrt{\frac{\pi}{2}}\ \frac{\omega_i^{\,2}\ \omega_t}{c^2 v_{ti}} \tag{2.23}$$

where ω_t = turnover frequency. For a low β plasma $\omega_t \simeq \Omega_i$, and for a high β plasma $\omega_t \simeq (2/\beta)\Omega_i$. Using these in equation 2.23

$$k^3_{max} \approx \sqrt{\frac{\pi}{2}}\ \frac{\omega_i\,\Omega_i}{c^2 v_{ti}} \qquad \text{for } \beta \ll 1 \tag{2.24}$$

$$k^3_{max} \approx \sqrt{\frac{\pi}{2}}\ \frac{2\omega_i^{\,2}\Omega_i}{\beta c^2 v_{ti}} \qquad \text{for } \beta \gg 1 \tag{2.25}$$

In terms of the variables used in the previous section we can rearrange the high β result to obtain

$$\left(\frac{kv_{ti}}{\Omega_i}\right)^3_{max} \approx \sqrt{\frac{\pi}{2}} \tag{2.26}$$

Based on this result one would not expect to obtain solutions to the dispersion relation for the LH polarized wave in region II. This is consistent with the results obtained in the previous section.

Similar considerations for RH waves in region III give;

$$k^3_{max} \approx \sqrt{\frac{\pi}{2}}\ \frac{\omega_e^{\,2}\Omega_e}{c^2 v_{te}} \qquad \text{for } \beta \ll 1 \tag{2.27}$$

$$k^3_{max} \approx \sqrt{\frac{\pi}{2}}\ \frac{2\omega_e^{\,2}\Omega_e}{\beta c^2 v_{te}} \qquad \text{for } \beta \gg 1 \tag{2.28}$$

Again, the high β result can be reformulated in the variables used in previous sections to give:

$$\left(\frac{kv_{te}}{\Omega_e}\right)^3_{max} \approx \sqrt{\frac{\pi}{2}} \tag{2.29}$$

Therefore, we would not expect the RH wave to exist in region III. By analogy with the LH wave result, one would not expect to obtain solutions to the dispersion relation for RH waves in region IV.

CHAPTER 3

RESONANCE OF ENERGETIC PARTICLES WITH PLASMA WAVE MODES

In general, the kinetic theories developed to quantitatively describe the particle pitch angle scattering process have invoked various levels of approximation. These can be broadly classed as linear, quasi-linear or non-linear. Fundamental to all these approximations, however, is the notion of wave/particle resonance. Therefore, to understand the physical processes which affect streaming of high energy particles, one must develop a thorough understanding of wave/particle resonance. So, in this chapter, the wave/particle resonance condition is examined in considerable detail.

Proper application of the wave/particle resonance condition is particularly crucial in the region near $\mu = 0$. Holman, Ionson and Scott (1979) pointed out that, in a high β plasma, the short wavelength waves responsible for particle scattering in this region are strongly damped by the background plasma. They conclude that this damping leaves a gap in μ-space with a width $\Delta\mu \simeq v_{ti}/c$, where the scattering rate is severely depressed. This gap effectively prevents particles having $\mu > 0$ from scattering into the $\mu < 0$ hemisphere, and hence the relaxation of a streaming particle distribution to isotropy may occur on time scales much longer than previously thought.

In this chapter, the application of the resonance condition is examined in considerable detail, particularly in the region near $\mu = 0$.

First, the approximation of magnetostatic turbulence which has been widely used in the past is considered. It is found that this approximation is inappropriate when considering small values of μ. A method is then developed which allows simple consideration of the full electromagnetic resonance condition. This method is applied to the problem of wave particle resonance in a high β plasma, and more exact expressions for the size of the resonance gap of Holman, Ionson and Scott (1979) are obtained. When this method is used to examine wave/particle resonance in a low β plasma, one can easily show that there are resonance solutions which are not predicted by magnetostatic theory. Specific examples are the resonance of relativistic protons with LH. waves, relativistic electrons with RH waves and protons with whistlers.

In addition it is found that a resonance gap exists ($\Delta\mu \sim v_a/v$) which is similar in many respects to the one mentioned above for a $\beta >> 1$ plasma.

All calculations assume parallel propagating waves, i.e. the usual picture of turbulence in the solar wind. The implications of these conclusions for flare propagation are discussed in later chapters.

A. Some Preliminary Resonance Considerations

The resonance condition which must be satisfied for high energy particles of type α to interact with circularly polarized waves is

$$\omega - k_{||} v_{||} = \nu\Omega_\alpha/\gamma_\alpha \tag{3.1}$$

where $\Omega_\alpha = |q_\alpha| B_\alpha c$ is the non-relativistic gyrofrequency and γ_α is the usual Lorentz factor. The parameter ν is chosen in accordance with the prescription

$$\nu = \frac{q_\alpha}{|q_\alpha|} \times \begin{cases} -1 & \text{For resonance with RH waves} \\ +1 & \text{For resonance with LH waves} \end{cases} \tag{3.2}$$

The wave frequency is determined by the dispersion relation $\omega = \omega(\vec{k})$.

Solution of the resonance condition is at this point a conceptually simple process. The dispersion relation is used to eliminate ω, and k_{res} can be solved for directly. However, in practice, solution of the full electromagnetic resonance condition can be algebraically complicated due to the complex dependence of ω on the wave number, k. This has led many authors to adopt the simplifying assumption of magnetostatic turbulence.

It is demonstrated below that this assumption leads to incorrect conclusions regarding particle propagation in many astrophysical settings. Further, it is pointed out that this assumption is not necessary for the solution of the problem, if one approaches the resonance question with the graphical method developed below.

B. Limits of the Magnetostatic Approximation

The magnetostatic approximation is obtained by assuming that $\omega \ll k_{||}v_{||}$, so that temporal variations of the wave can be neglected. In this approximation the resonant wave number, k, is inversely proportional to μ, i.e. $k_{||} = \nu\Omega_\alpha/\gamma_\alpha \mu v$, where we have used $v_{||} = \mu v$. In this picture, one sees that as μ decreases the resonant wave number, k, becomes larger, and presumably in the limit $\mu \to 0$ one must require the resonant value of $k \to \infty$. If, on the other hand, one considers the full electromagnetic resonance condition, equation 3.1, in the limit $\mu \to 0$ (i.e. $v_{||} \to 0$) one obtains a result which is entirely independent of k, namely $\omega = \nu\Omega_\alpha/\gamma_\alpha$.

This apparent paradox can be resolved by noting that whenever $v_{||} \simeq \omega/k_{||} \simeq v_{ph}$, the inequality $\omega << k_{||}v_{||}$, necessary for validity of the magnetostatic approximation, is violated. It can be shown from the results of Chapter 2 that in a high β plasma v_{ph} can be of order v_{ti}, while in a low β plasma $v_{ph} \sim v_a$. Therefore, it is obvious that for pitch angles near 90° ($|\mu| \lesssim v_{ph}/v$) the magnetostatic approximation to the resonance condition breaks down, and that an alternate method of determining resonant wave parameters is required. Proper evaluation of particle scattering in this region is crucial to a qualitative understanding of particle propagation in plasma turbulence.

C. Electromagnetic Resonance

Simultaneous solution of the electromagnetic resonance condition and the wave dispersion relation can be accomplished analytically using the procedure previously outlined. However, to obtain expressions for k_{res}, one must usually solve polynomial equations which are of cubic or higher order. But the algebraic complexity of the solutions obtained in this manner makes their interpretation and application difficult. As an alternative to this procedure, a simpler semi-graphical method of solution is proposed. The primary virtue of this method is that the complex relationship between the dispersion relation, particle pitch angle and particle energy can be conveniently and concisely visualized without resorting to undue approximations. Then analytic results can be obtained easily which define limiting or critical values of the resonance parameters.

To solve the electromagnetic resonance condition, first rearrange equation 3,1 to obtain

$$\frac{\omega}{\Omega_i} = \frac{k_{||}u}{\Omega_i}\left(\frac{v_{||}}{u}\right) + \frac{\nu}{\gamma_\alpha}\left(\frac{\Omega_\alpha}{\Omega_i}\right) \tag{3.3}$$

where u is some fundamental speed of signal propagation in the plasma and α denotes particle type.

For example, in a low β plasma the disperson relation is practically independent of β when k is measured in units of Ω_i/v_a. Therefore, the most logical choice is $u = v_a$, so that the resonance condition is also independent of β. In a plasma with $\beta > 10$, the dispersion relation is independent of β if k is measured in units of Ω_i/v_{ti}. Therefore, when high β plasmas are considered $u = v_{ti}$ will be chosen.

Notice that the resonance equation is a linear relation of the form $\tilde{\omega} = a\tilde{k} + \nu b$ with independent variable $\tilde{k} = ku/\Omega_i$; and dependent variable $\tilde{\omega} = \omega/\Omega_i$. The slope of this expression is related to μ through $\mu = a(u/v)$ where v is the particle speed. Therefore, one can consider a and μ as physically equivalent parameters which differ <u>only</u> by the scale factor u/v. This scale factor is nearly constant for relativistic particles having $v \simeq c$. The intercept, νb, is inversely related to the particle energy. For protons $b \leq 1$ and for electrons $b \leq (m_i/m_e)$ with the equalities corresponding to non-relativistic i.e. $\gamma \simeq 1$, particles. For extremely relativistic particles $\gamma \to \infty$, and $b \to 0$.

To obtain the resonant wave parameters ω_{res} and k_{res} (or equivalently $\tilde{\omega}_{res}$ and $\tilde{k}_{res}$), one makes a plot of the dispersion relation, rewritten in the form $\tilde{\omega} = \tilde{\omega}(\tilde{k})$. Then on the same graph plot the resonance condition $\tilde{\omega} = a\tilde{k} + \nu b$ for a given particle energy (γ) and pitch angle (μ). The intersections of these two functions define the resonant wave parameters. Particle resonance with backward traveling waves, i.e. $\vec{k} \cdot \vec{v}_{||} < 0$ can be examined by simply noting that $\tilde{\omega}(\tilde{k}) = \tilde{\omega}(-\tilde{k})$.

To illustrate the utility of this procedure two situations of astronomical interest will be considered, (1) particle streaming in a $\beta >> 1$ plasma such as the hot interstellar medium and (2) particle streaming in a plasma representative of the solar wind, $\beta \sim 1$. The results obtained here will be later applied to questions concerning the cosmic ray particle escape from supernova remnants and the propagation of flare particles in the interplanetary plasma.

D. Wave/Particle Resonance in a $\beta >> 1$ Plasma

Consider a distribution of energetic particles all having $\mu \geq 0$ streaming parallel to the magnetic field in a background plasma having $\beta >> 10$. We will assume that the density in the beam is sufficiently low so that the normal modes of the plasma/beam system are well approximated by the normal modes of the background plasma alone, (c.f. Morrison et al. 1981) and that only scattering by self-generated turbulence is significant. Since only waves with $\vec{k} \cdot \langle\vec{v}\rangle > 0$ are amplified, where $\langle\vec{v}\rangle$ is the average velocity of the beam particles, resonances with backward traveling waves can be neglected.

At some value of $\tilde{k} = \tilde{k}_d$, where $\tilde{k} = kv_{ti}/\Omega_i$, the wave damping due to the background particles is exactly balanced by wave growth due to the beam particles. Then, only waves with $\tilde{k} \leq \tilde{k}_d$ are amplified above the thermal background level and, since the scattering rate is proportional to the power of the resonant wave, one can ignore scattering by waves having $\tilde{k} > \tilde{k}_d$.

First consider the resonances having $\nu = -1$. This corresponds to resonance of protons (electrons) having energy $E_\alpha = \gamma_\alpha m_\alpha c^2$ with RH (LH) polarized waves. Solution of the resonance condition and dispersion relation is illustrated schematically in Figure 13 . Since small values of the slope, a, correspond to small values of μ, particles near pitch angles of 90° are resonant with waves of highest k. This is easily seen by considering Figure 13. As the slope of the resonance line becomes smaller the intersection between it and the dispersion relation occur at larger values of $\tilde{k}$. Eventually, as this process is continued, one arrives at the point where $\tilde{k}_{res} = \tilde{k}_d$. This condition defines the minimum value of a, and hence the minimum value of μ. This minimum value of μ where resonances can occur can be easily calculated as

$$\mu_2^\sigma = (v_{ti}/v_\alpha) \frac{\tilde{\omega}_d^\sigma + b_\alpha}{\tilde{k}_d{}^\sigma}$$

where the superscript σ denotes the applicable wave mode, and $\tilde{\omega} = \omega/\Omega_i$. The maximum value of the slope is determined by the condition $0 \leq \mu \leq 1$. This gives $a_{max} = v_\alpha/v_{ti}$. Resonances for intermediate values of μ fall between these two limiting values.

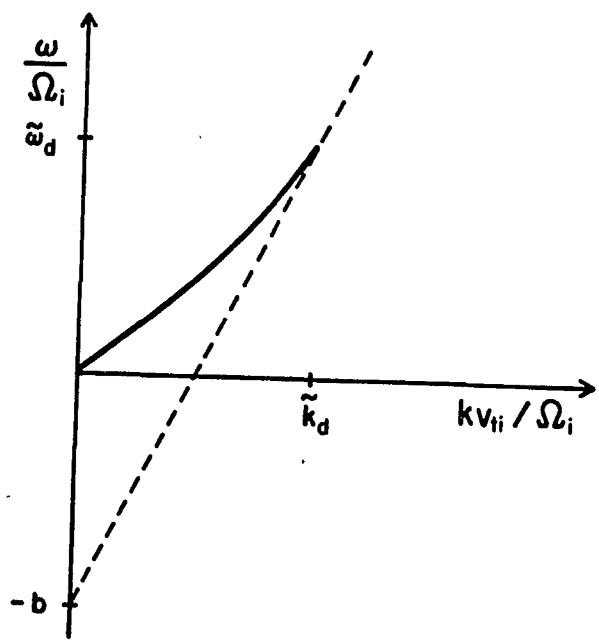


Figure 13. Graphical Solution of the Resonance Condition in a $\beta >> 1$ Plasma for Resonances having $\nu = -1$.

This condition corresponds to resonance of protons (electrons) with RH (LH) waves. The particular value of the slope shown is chosen to illustrate the minimum value of μ where scattering by these waves is possible.

Figure 14 shows a schematic of the solution of the resonance condition for $\nu = +1$, i.e. protons (electrons) resonant with LH (RH) waves. In this case, the resonance picture is slightly more complicated. First, since by assumption there are no particles at $\mu < 0$, only resonances having slopes $a \geq 0$ are allowed. Therefore resonance can only occur for waves having $\omega^{\sigma}/\Omega_i \geq b$. This is especially significant for electrons since in the high β case all waves have $\omega < \Omega_i$ which requires $\gamma_e \geq m_i/m_e$.

Secondly if we assume that the particles are energetic enough for resonance to occur, i.e. $\omega^{\sigma}/\Omega_i \geq b$, then resonances are possible only for very small values of μ. This is also best seen by considering Figure 14. The inequality $\omega^{\sigma}/\Omega_i \geq b$ is easily obtained by simply considering the resonance condition with a slope of zero, i.e. horizontal in the graph. Then it is easily seen that there are no intersections between the resonance condition and the dispersion relation unless the above inequality is satisfied.

The fact that these resonances occur only for small values of μ can be seen by starting with an intercept small enough (i.e. assume a particle which is energetic enough) so that the inequality above is well satisfied. Then as larger and larger slopes are considered, one finds that the largest occurs when $\tilde{k}_{res} = \tilde{k}_d$. In fact, from the figure one can show that resonances exist only for values of μ satisfying $0 \leq \mu \leq \mu_1$, where

$$\mu_1 = (v_{ti}/v_{\alpha}) \; \frac{\tilde{\omega}_d^{\sigma} - b_{\alpha}}{\tilde{k}_d^{\sigma}} \tag{3.5}$$

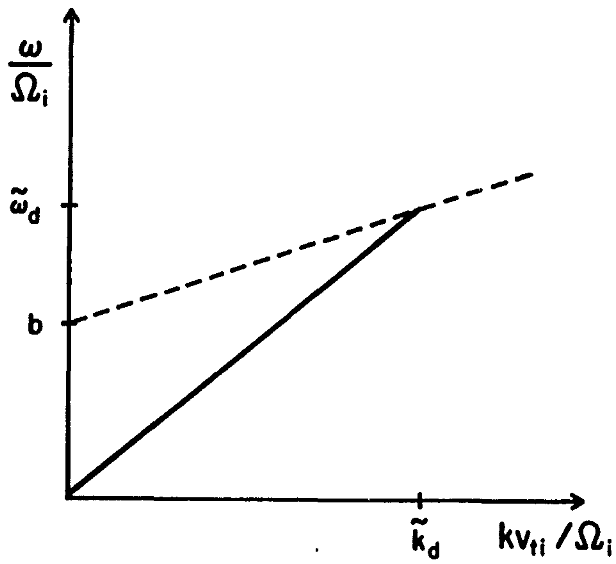


Figure 14. Graphical Solution of the Resonance Condition in a $\beta >> 1$ Plasma for Resonances having $\nu = +1$.

This condition corresponds to the resonance of electrons (protons) with RH (LH) waves. The slope shown illustrates the maximum value of μ where scattering by these waves is possible.

These results can be summarized as follows. Particles streaming along magnetic field lines with $\mu \simeq 1$ can be resonantly scattered down (i.e. decreasing slope) to some minimum value $\mu_2 > 0$ by resonances having $\nu = -1$. The exact value of μ_2 depends on the type of particle, the particle energy and the wave spectrum, i.e. $\tilde{k}_d^\sigma$ and $\tilde{\omega}_d^\sigma$. For low energy particles, $\gamma_\alpha < \tilde{\omega}_d^\sigma$, there are no additional resonances in the range $\mu < \mu_2$. In other words, there is a gap in μ-space, near $\mu = 0$, where resonant scattering cannot operate, because the resonance condition cannot be satisfied for these particle energies. This is in direct contrast with magnetostatic theories which predict resonances for all $|\mu| > 0$.

In addition, for very energetic particles ($\gamma_\alpha > \tilde{\omega}_d^\sigma$), there are solutions of the resonance condition for particles having $0 \leq \mu \leq \mu_1$. These resonance solutions are not obtained in magnetostatic theories. The precise effect of these resonant interactions on the evolution of the particle distribution function is not clear however, since resonant pitch angle scattering at constant particle energy is no longer the dominant process. In this region, wave/particle interactions can also cause systematic changes of the resonant particle energy. The details of the precise interplay between particle energy change and pitch angle scattering in this range of μ have not been fully explored. Therefore, firm conclusions relative to the evolution of the streaming particle distribution function cannot be drawn.

Nevertheless, it is possible that a scattering mechanism will be found which can operate within the resonance gap. Holman, Ionson and Scott

(1979) examined several possibilities and concluded that all of them were ineffective. It is reasonable to assume that whatever the dominant mechanism for scattering within the gap is found to be, it will operate on a time scale much longer than the time scale associated with resonant scattering. If this presumption is correct, then the time scale for relaxation to isotropy would depend strongly on the details of the scattering mechanism in the resonance gap.

E. Wave/Particle Resonance in a $\beta \lesssim 1$ Plasma

As a second illustration, the resonance of energetic particles with low β plasma modes will be considered. This problem has been considered before, but only under the assumption of magnetostatic turbulence. In this section the semi-graphical methods developed earlier in this chapter will be used to examine wave/particle resonance while incorporating the full electromagnetic character of the waves.

Assume β is small enough so that the wave modes in the plasma are reasonably well approximated by the cold plasma modes. Since the primary application of these results will be to particle scattering in the solar wind, it can also be assumed that all waves propagate with $k > 0$. The resonance condition is now written as $\tilde{\omega} = a\tilde{k} + \nu b$, where $\tilde{\omega} = \omega/\Omega_i$, $\tilde{k} = kv_a/\Omega_i$ and $b = (\Omega_\alpha/\gamma_\alpha\Omega_i)$. The slope is now related to μ through $a = \mu v/v_a$. It is convenient, in this case, to consider proton and electron resonances separately, so let us begin with consideration of energetic protons.

For protons the intercept of the resonance condition is negative (positive) for RH(LH) circularly polarized waves. This leads to the graphical solutions shown schematically in Figure 15. Notice that on this figure, three special values of the slope have been indicated for the RH wave and two for the LH wave.

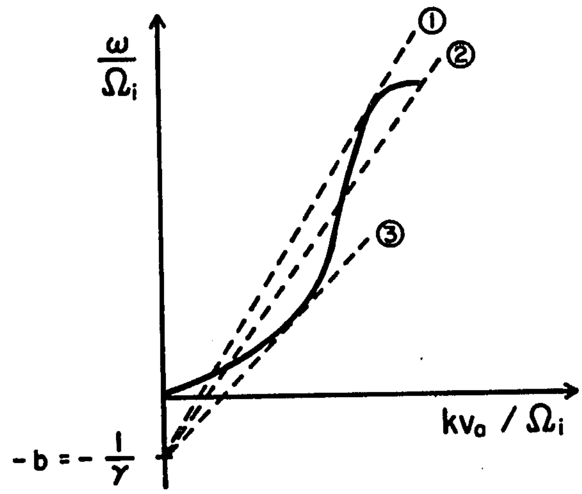
Consider first the RH wave. For $a > a_1$, only resonance with low frequency waves is possible. For $a = a_1$, there are two values of $\tilde{\omega}$ and $\tilde{k}$ which are resonant with the proton. For $a_1 > a \geq a_2$ there are three resonance solutions for a given value of μ. In the range $a_2 > a > a_3$ there are two solutions to the resonance condition and for $a = a_3$ there is only one solution. Now consider particles with $a_1 > a$, there are no resonances in this region.

To show that the proper order has been chosen for these slopes, i.e. $a_1 > a_2 > a_3$, one must solve for the slopes explicitly. The solution for a_1 and a_3 is facilitated by noting that at these points one can impose an additional condition. That is, one must not only require that the dispersion relation and the resonance condition be satisfied simultaneously, one must also require that their derivatives are equal. By imposing this additional condition the value of $\tilde{k}_1$, $\tilde{\omega}_1$, a_1 and $\tilde{k}_3$, $\tilde{\omega}_3$, a_3 can be uniquely determined. The dispersion relation for waves in a cold plasma is

$$\tilde{\omega} = \tilde{k}\ (\sqrt{1 + \tilde{k}^2/4} \pm \tilde{k}/2) \qquad \omega << \Omega_e \qquad (3.6)$$

$$\tilde{\omega} = \tilde{k}^2\ /\ (1 + \alpha\tilde{k}^2) \qquad \omega >> \Omega_i \qquad (3.7)$$

where $\alpha = m_e/m_i$. The results of the solution are shown in Table 2. The values of $\tilde{\omega}_3$ and a_3 are given only in the relativistic ($\gamma >> 1$) and non-relativistic ($\gamma \sim 1$) limits.



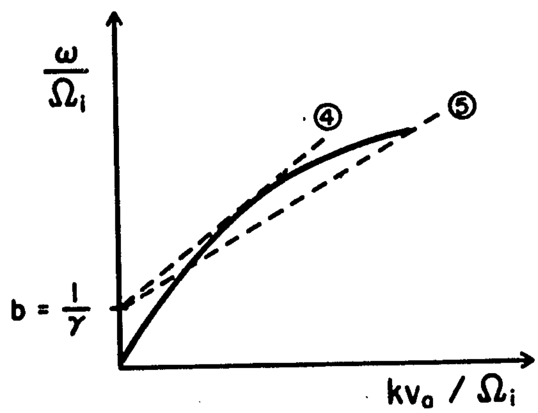


Figure 15. Graphical Solution of Proton Resonance Condition in a $\beta << 1$ Plasma.

The upper (lower) portion of the figure shows resonance of protons with RH (LH) waves. The numbers on the figures refer to specific values of μ discussed in the text. Expressions for these values of μ are shown in Table 2.

A lower limit to the value of a_2 can be obtained in the following way. In Chapter 2, it was demonstrated that there are no solutions to the dispersion relation for RH waves for $k \gtrsim (\pi/2)^{1/2}\ \omega_e^2\Omega_e/c^2 v_{te}$, i.e. there are no RH waves with k larger than this value. By inserting this value into the resonance condition, and assuming $\omega \simeq \Omega_e$, one can obtain an estimate for the slope a_2. This result is also shown in Table 2. This is, of course, only a lower limit for a_2 since wave damping effects can become significant at smaller values of k than the one used in this estimate. The main point is, however, that a_3 is smaller than even this lower limit on a_2 and hence the value of a_3 determines the smallest value of μ resonant with the waves.

In other words, it is easy to show that $a_1 > a_2 > a_3$ as long as $(\pi T_i/T_e)\ (m_e/m_i)^3 < \beta < \pi T_i/64T_e$. This is an incredibly large range spanning about eight orders of magnitude. Whenever $\beta \simeq 1$ one would expect finite β effects to become significant in the high frequency region due to the depression of the electron cyclotron resonance frequency. This effect was discussed in detail in Chapter 2. Unfortunately, it appears that β is large enough in the solar wind so that the cold plasma results derived here for resonance with waves having $\omega >> \Omega_i$ are not rigorously applicable. The results derived for resonance in the low frequency region are applicable since RH waves are not very sensitive to changes in β as long as $\beta \lesssim 2$.

Solutions for the resonance of protons with LH polarized waves can be obtained in a similar manner. The results for points 4 and 5 of Figure 15 are shown in Table 2.

Table 2. Summary of Special Values of μ for Proton Resonance.
The point numbers are defined in the text and in Figure 15.

Point	$\tilde{k}_{res}$	$\tilde{\omega}_{res}$	μ_{res}
1	$(m_i/m_e)^{1/2}$	$1/2\ m_i/m_e$	$2\ (v_a/v)\ (m_i/m_e)^{1/2}$
2	$\sim (m_i/m_e)^{1/2}\ (\pi T_i/\beta T_e)^{1/6}$	$\sim\ m_i/m_e$	$\sim (v_a/v)\ (m_i/m_e)^{1/2}\ (\beta\ T_e/\pi\ T_i)^{1/6}$
3	$\left\{\frac{2}{\gamma}\frac{(1-1/4\gamma)}{(1-1/\gamma)}\left[1-\sqrt{1-\frac{1-1/\gamma}{(1-1/4\gamma)^2}}\right]\right\}^{1/2}$	$\frac{2}{\sqrt{\frac{2}{\gamma}}}$	$3\ (\sqrt{3}\ /2)\ (v_a/v)\quad (\gamma \sim 1)$ $(1+\sqrt{1/2\gamma}\ )\ (v_a/v)\quad (\gamma >> 1)$
4	$\left\{\frac{2}{\gamma}\frac{(1-1/4\gamma)}{(1-1/\gamma)}\left[1+\sqrt{1-\frac{1-1/\gamma}{(1-1/4\gamma)^2}}\right]\right\}^{1/2}$	$\frac{1}{\sqrt{\frac{2}{\gamma}}}$	$0\quad (\gamma \sim 1)$ $(1-\sqrt{1/2\gamma}\ )\ (v_a/v)\quad (\gamma >> 1)$
5	$\sim\ (\pi/\beta)^{1/6}$	1	$(\beta/\pi)^{1/6}\ (1-1/\gamma)\ (v_a/v)$

There are several important points to note in these results. First, we see that contrary to previous magnetostatic analyses (e.g. Melrose 1980) protons can reasonantly interact with high frequency RH polarized waves.

Secondly, protons of a given energy and pitch angle can sometimes resonantly interact with waves having several values of k. Since the rate of interaction is proportional to the wave power $P(k_{res})$, and since $P(k)$ is typically a strongly decreasing function of k, resonance with the wave of smallest k would dominate the interaction.

For non-relativistic protons, there are essentially no resonances between the particles and LH polarized waves. The resonance at $\mu = 0$ can usually be ignored since these waves are highly damped for any finite plasma temperature.

When the particles are sufficiently relativistic however, proton resonance with lightly damped LH waves is possible. This result is again in direct conflict with magnetostatic theories which predict that protons can never reasonantly interact with forward travelling LH waves.

The most important feature of these results, however, is that in all cases there is a gap in μ-space between the smallest value of μ resonant with RH waves and the largest value of μ resonant with LH waves. Therefore, unless additional particle scattering mechanisms are found which operate in this range of μ, particles <u>cannot resonantly scatter through $\mu = 0$</u> as has been assumed in the past. This gap can be very small $\Delta\mu \sim \sqrt{2/\gamma_i}\ (v_a/c)$ for relativistic protons, or relatively large, $\Delta\mu \simeq 3\sqrt{3/2}\ (v_a/v)$ for non-relativistic protons.

Now consider the resonance of electrons with waves in a low β plasma. The resonance condition is of the same form, $\tilde{\omega} = a\tilde{k} \pm b$, where the slope $a = \mu\ (v/v_a)$. The intercept, however, is $b = (m_i/m_e)\ (1/\gamma_e)$, different from the proton value by a factor of m_i/m_e. A schematic representation of the graphical solution is presented in Figure 16. Again several special points are labeled which separate μ-space into well defined zones. The values of a_1, a_2 and a_3 can be determined from considerations analogous to those used for protons. The results of these calculations are shown in Table 3.

Note that unless the electrons are at least mildly relativistic ($\gamma_e > 1$) there are no resonances with the RH wave. Even for mildly relativistic electrons there are no resonances for $a > a_1$. In the region $a_1 > a \geq a_2$ there are two waves resonant with the particle and for $a_2 > a \gtrsim 0$, there is only one resonance solution. Resonance of electrons with LH waves is considerably simpler, there are resonance solutions only for $a \gtrsim a_3$.

Again these results show that a resonance gap exists near $\mu = 0$. To see this, consider electrons having $\gamma << m_i/m_e$. Then, particles having $\mu > 0$ can resonantly scatter only if they have pitch angles such that $\mu > (\beta/\pi)^{1/6}\ (m_i/m_e)\ (1/\gamma_e)\ (v_a/v)$. This gap can be large for non-relativistic electrons, i.e. $\gamma \lesssim (m_i/m_e)\ v_a$.

As one considers electrons of higher and higher energy the size of the resonance gap decreases. This is due to two factors. First the value of the slope a_3 decreases, i.e. μ decreases indicating that particles can resonantly scatter to lower values of μ, and secondly scattering by the RH wave near $\mu = 0$ becomes important. This also

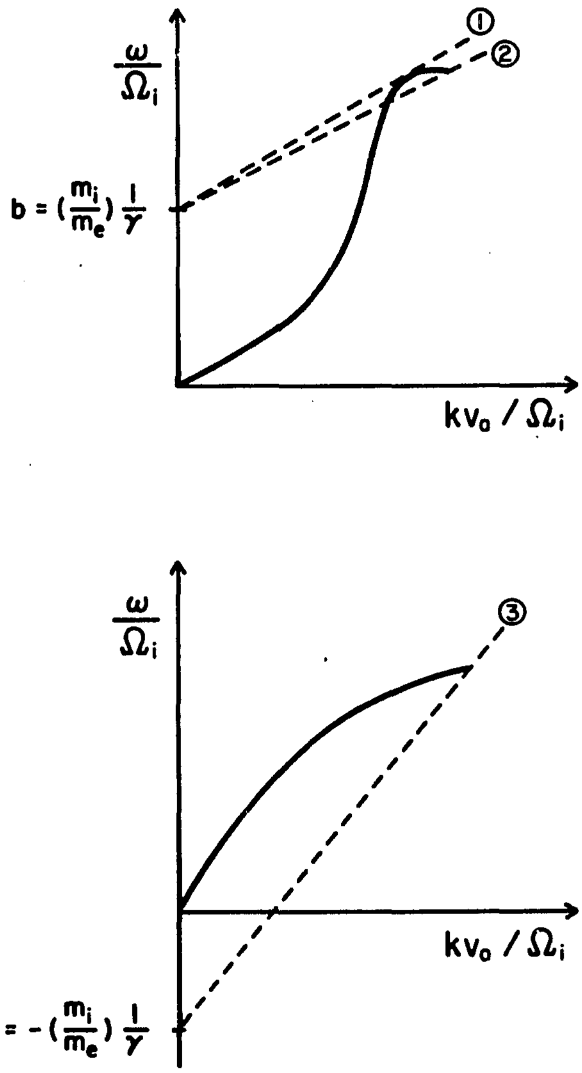


Figure 16. Graphical Solution of Electron Resonance Condition in a $\beta << 1$ Plasma.

The upper (lower) portion of the figure shows resonance of electrons with RH (LH) waves. The numbers refer to specific values of μ given in Table 3 and discussed in the text.

Table 3. Summary of Special Values of μ for Electron Resonance.

The point numbers are defined in the text and in Figure 16.

Point	$\tilde{k}_{res}$	$\tilde{\omega}_{res}$	μ_{res}
1	$\left\{\frac{1}{2}\left(\frac{m_i}{m_e}\right)\frac{1+2/\gamma}{1-1/\gamma}\left[+1\sqrt{1+\frac{4(1-1/\gamma)}{\gamma(1+2/\gamma)}}\right]\right\}^{1/2}$	m_i/m_e	$\frac{v_a}{v}\left(\frac{m_i}{3m_e}\right)^{1/2}(1-1/\gamma)^{3/2}$
		$\frac{1}{2}\, m_i/m_e$	$\frac{1}{2}\; v_a/v\;(m_i/m_e)^{1/2}$
2	$\sim\left(\frac{m_i}{m_e}\right)^{1/2}\left(\pi T_i/\beta T_c\right)^{1/6}$	$\sim m_i/m_e$	$\sim\left(\frac{v_a}{v}\right)(\beta T_c/\pi T_i)^{1/6}(m_i/m_e)^{1/2}(1-1/\gamma)$
3	$(\pi/\beta)^{1/6}$	~ 1	$\sim\frac{v_a}{v}\;(\beta/\pi)^{1/6}\left[m_i/v_e m_e+1\right]$

helps to close the gap. Eventually as one considers particles of higher and higher energy the gap can be completely closed allowing electrons to resonantly scatter from $\mu > 0$ to $\mu < 0$. This occurs for $\gamma_e \gtrsim 2 (m_i/m_e)^{1/2}$.

CHAPTER 4

ESCAPE OF COSMIC RAY PARTICLES FROM SUPERNOVA REMNANTS

In the past, several authors have suggested that high energy particles cannot freely stream through a magnetized background plasma. This suggestion is based on the fact that high energy particles amplify pre-existing waves in the background plasma, which in turn, resonantly interact with the streaming particles reducing their anisotropy through pitch angle scattering. In this picture, pitch angle scattering continues until the high energy particle distribution has evolved to isotropy in a frame moving with the waves.

This suggested evolution to near isotropy is surprisingly rapid. In fact, the scattering mean free path, as calculated from standard pitch angle scattering theory, is so small compared to the size of typical astronomical objects (e.g. supernova remnants, galaxies, etc.) that it has been widely assumed that the bulk propagation speed of the high energy particles streaming out of these objects was limited to a speed on the order of the local Alfven speed, $v_a = (B^2/4\pi nm)^{1/2}$. These results seemed to imply that supernovae could not be the source of cosmic ray particles observed in the solar system.

This conclusion was based on the following argument. Supernova remnants are observed to contain relativistic particles. However, if these particles were to stream away from the remnant, the streaming particles would cause rapid growth of the ambient plasma turbulence, and this turbulence would quickly isotropize the particles. Once isotropized, the relativistic particles would rapidly lose their energy as the supernova shell expanded. It was concluded, therefore, that the vast majority of the observed cosmic ray particles must be generated elsewhere.

Holman, Ionson and Scott (1979) questioned several aspects of this picture, however. They pointed out that recent observations show that most of the interstellar medium consists of a very hot ($T \sim 10^6$ °K) high β ($\beta \gtrsim 100$) plasma. Since this hot medium fills most of the volume of interstellar space, they concluded that most supernova remnants should be immersed in the $\beta \gtrsim 100$ medium. However previous analyses of particle escape from supernova remnants have assumed a cold ($\beta << 1$) plasma.

Further, they pointed out that proper consideration of wave damping in a $\beta >> 1$ background plasma, such as the hot interstellar medium, could significantly alter conventional particle scattering theories. They noted that for the streaming speed to be significantly reduced on short time scales, particles must be able to scatter efficiently through pitch angles near 90°. However, the waves of short wavelength responsible for scattering in this region are strongly damped by the background plasma and hence must be present only at thermal levels. They concluded that the scattering rate, which is directly proportional to the wave power, is so severely reduced in the region around $\mu = 0$ (μ = cosine of particle pitch angle) that, to a good approximation, particles cannot resonantly scatter from the forward hemisphere ($\mu > 0$) into the backward hemisphere ($\mu < 0$), and therefore that a bulk propagation speed much larger than the Alfven speed was possible. They obtained these results, however, by using the damping rate and growth rate obtained for a $\beta << 1$ plasma.

Foote and Kulsrud (1979) considered the propagation of waves having frequencies, ω, far below the ion cyclotron frequency, Ω_i, in a $\beta >> 1$ plasma. They also applied their results to the problem of high energy particle streaming, and concluded that pitch angle scattering by self-generated turbulence can effectively contain the high energy particles and force them to propagate at a bulk speed near v_a. However, in the application of the theory they neglected wave damping in the background plasma and thereby missed the major relevant effect of the high β plasma.

Achterberg (1981) combined these two approaches by considering low frequency ($\omega << \Omega_i$) waves in a $\beta >> 1$ plasma along with the proper damping rates. In addition he used the non-linear theory of Volk (1973) to examine pitch angle scattering near $\mu = 0$ and concluded that particles with energies below a few GeV cannot be contained by self-generated turbulence, in general agreement with Holman, Ionson and Scott (1979).

In this chapter, the specific problem of particle escape from a supernova remnant (SNR) is considered. The results obtained are slightly different from those obtained earlier by Achterberg (1981) and Holman, Ionson and Scott (1979). It is found that only electrons having $\gamma \gtrsim 10^5$ can be contained within the SNR by self-generated turbulence. Protons, on the other hand, cannot be contained within the SNR by resonant pitch angle scattering regardless of the particle energy. The primary reason for the difference between these and previous results is the proper consideration of the electromagnetic nature of the turbulence. Based on these results, it is not possible to rule out supernovae as sources of cosmic ray particles.

A. Calculation of the Wave Growth Rate

When particles are scattered by self-generated turbulence alone, only waves having $k \gtrsim k_d$ are effective. The value of k_d is determined by equating the growth rate of the waves, which is due to the relativistic particle beam, to the wave damping rate. For $k > k_d$ damping dominates, so there is little wave power at these wavelengths, and hence particle interactions with waves having $k > k_d$ are negligible. The damping rates for waves in a $\beta >> 1$ plasma were calculated in Chapter 2. In this section the growth rates necessary to estimate k_d are calculated.

The relativistic particles contribute an imaginary term to the dielectric (Montgomery and Tidman 1964) given by

$$\epsilon_b = - \frac{\pi}{2} \sum_\alpha \frac{n_{b_\alpha}}{n_\alpha} \frac{\omega_\alpha^2}{\omega} m_\alpha c^2 \int_0^\infty dE \frac{E^2}{c^3} \int_{-1}^{+1} (1-\mu^2)\ \delta(\omega kc\mu - \nu\Omega_\alpha/\gamma_\alpha)$$

$$\left\{\frac{\partial f}{\partial E} + \frac{1}{E}\left(\frac{kc}{\omega} - \mu\right)\frac{\partial f}{\partial \mu}\right\} d\mu \tag{4.1}$$

where $\frac{n_{b_\alpha}}{n_\alpha}$ is the ratio of beam particle density to the density of the background plasma, $E = \gamma mc^2$ and μ is the cosine of the particle pitch angle. In addition it has been assumed that the growth rate of the waves due to the beam, $\Gamma_b \ll \omega$, so that the Plemelj formula can be used to separate out the imaginary part of the dielectric, i.e. the particle density is small enough so that the effect demonstrated by Morrison et al. (1981) is negligible. With these assumptions the growth rate can be calculated from the dispersion relation using the standard Taylor expansion technique discussed in Chapter 2.

A distribution of the form $f(\mu, E) = F(E)\, G(\mu)$ is assumed with

$$F(E) = \begin{cases} 0 & E < E_* \\ \frac{c^3}{E^2} \, \frac{s-1}{E_*} \, \frac{E}{E_*}^{-s} & E > E_* \end{cases} \tag{4.2}$$

$$G(\mu) = \begin{cases} 0 & -1 \leq \mu < 0 \\ (n+1)\mu^n & 0 \leq \mu \leq 1 \end{cases} \tag{4.3}$$

It is assumed that this distribution function is representative of at least the initial distribution of cosmic ray particles streaming out of a supernova remnant, i.e. before backward scattering becomes significant.

By substituting this distribution function into equation 4.1 and performing the integral over μ one obtains

$$\varepsilon_b = -\frac{\pi}{2}\sum_{\alpha}\frac{n_{b_\alpha}}{n_\alpha}\frac{\omega_\alpha^2}{\omega|k|c}\frac{s-1}{\gamma_*}\int_{\gamma_o/\gamma_*}^{\infty} d\left(\frac{\gamma}{\gamma_*}\right)\left(\frac{\gamma}{\gamma_*}\right)^{-(s+1)}(1-\mu_{res}^2)$$

$$\left[\frac{nkc}{\omega\mu_{res}} - (n+s+1)\right] G\,(\mu_{res}) \tag{4.4}$$

where

$$\mu_{res} = \frac{\omega}{kc} - \frac{\nu\Omega_\alpha}{\gamma kc} \tag{4.5}$$

is the solution of the delta function. The lower limit on the energy integral is chosen by requiring that $0 \leq \mu_{res} \leq 1$. For resonances having $\nu = 1$, this inequality requires that γ_o is chosen as the greater of γ_* or Ω_α/kc, and for resonances having $\nu = +1$, γ_o is chosen as the greater of γ_* or Ω_α/ω. Only particles with $\gamma \geq \gamma_o$ can resonantly interact with a wave of given ω and k.

Finally, define

$$I_j\left(\frac{\gamma_o}{\gamma_*}\right) = \int_{\gamma_o/\gamma_*}^{\infty} d\left(\frac{\gamma}{\gamma_*}\right)\left(\frac{\gamma}{\gamma_*}\right)^{-(s+1)} \mu_{res}^{\,j}\left(\frac{\gamma}{\gamma_*}\right) \tag{4.6}$$

then the relativistic particle contribution to the dielectric can be written as

$$\varepsilon_b = -\frac{\pi}{2}\sum_{\alpha}\frac{n_{b_\alpha}}{n_\alpha}\frac{\omega_\alpha^2}{\omega|k|c}\frac{(s-1)\;(n+1)}{\gamma_*}\left\{\frac{nkc}{\omega}\left(I_{n-1} - I_{n+1}\right) - \right.$$

$$\left.(n+s+1)\;(I_n - I_{n+2})\right\} \tag{4.7}$$

In general, an analytic expression for I_j can be obtained for any integer value of j by partial integration. For this example, it will be assumed n = 1. This corresponds to a situation where classical pitch angle scattering has already begun to redistribute the particles in pitch angle space, but scattering through $\mu = 0$ is negligible. In addition, $kc \gg \omega$ and only the first term in the growth rate need be considered. With these assumptions

$$\varepsilon_b = -\frac{\pi}{2}\sum_\alpha \frac{n_{b\alpha}}{n_\alpha}\frac{\omega_\alpha^2}{\omega^2}\frac{k}{|k|}\frac{2(s-1)}{\gamma_*}\left(\frac{\gamma_o}{\gamma_*}\right)^{-s}\left\{\frac{1}{2}-\frac{1}{S+2}\left(\frac{\nu\Omega_\alpha}{\gamma_o kc}\right)^2\right\} \qquad (4.8)$$

In this expression all terms involving ω/kc have been neglected since they are always small compared to 1/s for the case considered here.

B. Calculation of the Resonance Gap Width

In this section, the wave growth rate derived above will be used to estimate $\tilde{k}_d$. From this, the size of the resonance gap for electrons and protons can be estimated. Before doing this however, it is necessary to consider some of the relevant physical parameters needed for this analysis.

The electron density within a typical supernova remnant can be estimated from radio observations. Using the data presented by Woltjier (1972), one can estimate the density of relativistic electrons as 10^{-4}-10^{-7} cm^{-3}. The spectral index for these objects indicates an electron distribution with $n(E) \propto E^{-s}$ where $s \sim 2.0$-2.7. The rest of the calculations presented in this chapter will assume

$s = 2.7$. The results obtained are, however, very insensitive to the exact value of this parameter.

Also from radio observations, one can demonstrate that the electrons responsible for the observed emission have $\gamma \gtrsim 10^3$. However, it is probable that less energetic electrons are also present. It will be assumed that the electron power law distribution extends down to $\gamma_* = 10$. This assumption is a reasonable one, and is not in conflict with the observation above since emission by the low energy electrons is strongly attentuated by the Earth's atmosphere.

The density of relativistic protons within these objects is essentially unknown. However, local observations within the solar system show that $n_i/n_e \simeq 100$ at 1 GeV. This means that within the supernova remnant one can estimate

$$\frac{n_i}{n_e} \simeq 100 \left(\frac{\gamma_{*e}\, m_e}{\gamma_{*i}\, m_i}\right)^{s-1} \simeq 10^{-3} \tag{4.9}$$

For the numerical estimate, the values $s = 2.7$ and $\gamma_{*i} = 2$ were assumed. These are again based on local cosmic ray observations.

Using this ratio, the density of relativistic protons can be estimated as 10^{-7}-10^{-10} cm^{-3} within the remnant.

In a plasma having $\beta >> 1$, off-axis waves with propagation angles $\theta > (2\beta)^{-1/4}$ are strongly transit time damped by the background plasma. In this case, it is a good first approximation to assume that all waves present in the background medium have $\theta \approx 0$, i.e., on axis waves, and therefore that the results of the previous chapters can be applied directly.

With these assumptions, the growth rate for RH waves is dominated by the contribution from the proton portion of the energetic particle beam and can be written as

$$\frac{\Gamma_b^R}{\omega} = \frac{1}{D}\left[\frac{n_{bi}}{n_i \gamma_{*i}} \frac{\pi(s-1)}{s} \left(\frac{\Omega_i}{\omega}\right)^2\right] \tag{4.10}$$

where n_{bi} is the density of protons in the relativistic beam, and n_i is the density of ions in the background plasma. For the calculations presented below $n_i = 0.1\ \text{cm}^{-3}$, a typical value for the hot interstellar medium. In this expression the inequality $\gamma_{*i} \gg \Omega_i/kc$ was also used. This approximation is valid for all but the smallest values of k. The coefficient D is defined as

$$D = \frac{c^2}{v_a^2} \frac{1}{\omega^2} \frac{\partial}{\partial\omega} \omega^2 \varepsilon_1 \tag{4.11}$$

The growth rate for LH waves is dominated by the electron beam component and is given by

$$\frac{\Gamma_b^L}{\omega} = \frac{1}{D}\left[\frac{n_{be}}{n_e \gamma_{*e}} \frac{\pi(s-1)}{S} \frac{m_i}{m_e} \left(\frac{\Omega_i}{\omega}\right)^2\right] \tag{4.12}$$

In this expression n_{be} is the density of relativistic electrons and $n_e = 0.1\ cm^{-3}$ is the density of electrons in the background plasma.

The values of $\tilde{k}_d^\sigma$ and $\tilde{\omega}_d^\sigma$ are determined by requiring that the magnitude of the damping rate and the growth rate be equal. This procedure is illustrated graphically in Figures 17 and 18 for various values of the relativistic particle density.

Only supernova remnants with $n_{be} \lesssim 10^{-6}$ can be accurately discussed using this analysis. This is because whenever $n_{be} > 10^{-5}$, $\tilde{k}_d^L$ is large enough so that the assumption of weak damping is no longer rigorously justified. ($\Gamma/\omega = 0.1$ at $\tilde{k} = 0.36$ for LH waves). Therefore, the results presented below pertain only to these low density remnants, but it is anticipated that calculations performed for arbitrary damping and growth rates would result in conclusions similar to those presented below.

An electron density of $n_{be} \sim 10^{-5}$-10^{-7} implies a proton density within the remnant of $n_{bi} \sim 10^{-8}$-10^{-10}. Using these particle densities and Figures 17 and 18, one can estimate $\tilde{k}_d^\sigma$ as

$$\tilde{k}_d^R = 0.125 \qquad \tilde{k}_d^L = 0.24 \tag{4.13}$$

Assuming $\beta = 100$, typical of the hot interstellar medium, these values of $\tilde{k}_d$ correspond to

$$\tilde{\omega}_d^R = 0.027 \qquad \tilde{\omega}_d^L = 0.015 \tag{4.14}$$

These values can be inserted into the expressions for μ_1 and μ_2 derived in Chapter 3. The width of the resonance gap, $\Delta\mu = \mu_2 - \mu_1$,

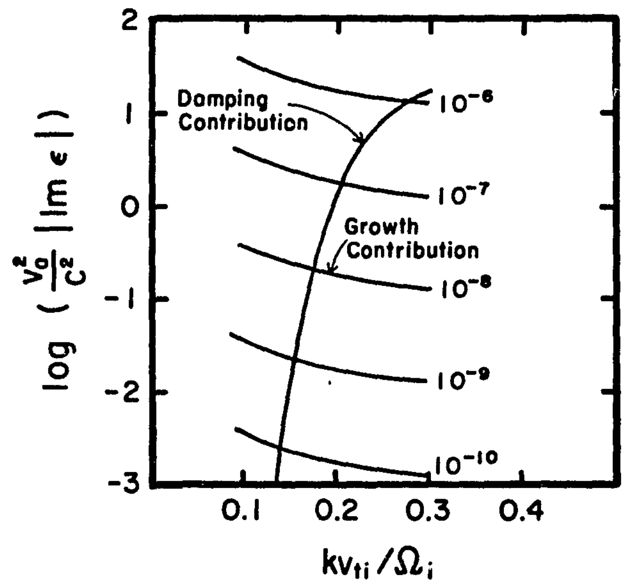


Figure 17. Comparison of the Dielectric Contribution from Damping and Growth Rates for RH Waves.

Curves are shown for several values of the relativistic electron density n_{be}.

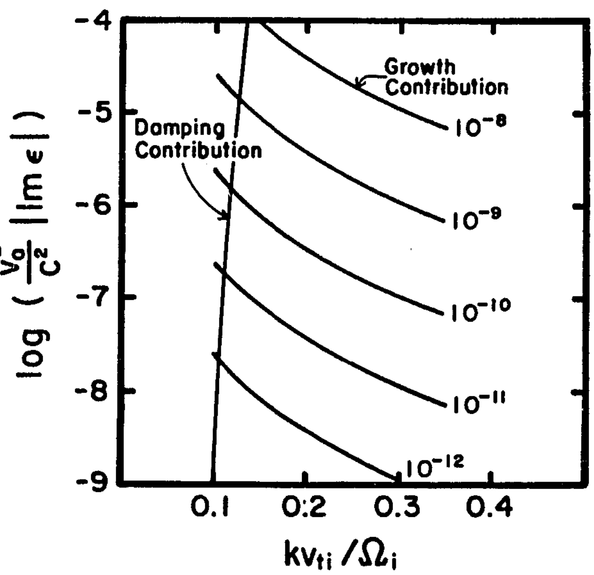


Figure 18. Comparison of the Dielectric Contribution from Damping and Growth Rates for RH Waves.

Curves are shown for several values of n_{bi}, the relativistic ion density.

can then be calculated. The results of these calculations are discussed below. First, the resonance gap for protons will be discussed.

Protons

Using the procedure described above the width of the proton resonance gap can be estimated as

$$\Delta\mu = \frac{v_{ti}}{c} \ (o.16 + 12/\gamma) \tag{4.15}$$

For $\gamma >> 75$, this width $\Delta\mu \sim 0.16\ v_{ti}/c$ is approximately independent of the particle energy. This width is a factor of five smaller than the one given by Holman, Ionson and Scott (1979). For $\gamma \lesssim 75$, however, the width of the gap does depend on the particle energy. For example, the resonance gap for 1 GeV protons is $\Delta\mu \sim v_{ti}/c$.

Regardless of the particle energy, however, resonant pitch angle scattering cannot cause protons to scatter through $\mu = 0$. One must conclude, therefore, that energetic protons cannot be contained within a supernova remnant by self-generated turbulence.

This conclusion must, of course, be modified if another scattering mechanism is found which can operate on particles within the resonance gap. Two such mechanisms have been considered in the past. Holman, Ionson and Scott (1979) considered particle mirroring on compressive waves excited by the energetic particle beam. Achterberg (1981) considered resonance broadening (non-linear) effects. Each concluded that these mechanisms could not provide significant particle scattering across the resonance gap.

Electrons

The size of the gap for electrons can be determined in a similar manner. Electrons with $\gamma < 7\times10^4$ (and $\mu \geq 0$) cannot resonantly interact with RH waves. For these particles then, the width of the gap can be estimated as

$$\Delta\mu = \frac{v_{ti}}{c}\,(0.0625 + 7.69\times10^3/\gamma) \qquad (4.16)$$

From this expression it is obvious that the width of the resonance gap for electrons also depends on the particle energy. At low energies, $\gamma \sim 10$, the resonance gap can be very large $\Delta\mu \sim 10^3\ v_{ti}/c$. But as one considers particles of higher and higher energy the resonance gap becomes smaller. Until finally, when electrons having $\gamma \gtrsim 10^5$ are considered, it is found that the resonance gap has completely closed.

This closing of the gap is caused by two effects. First the resonant interaction of the energetic electrons with LH waves can occur at smaller values of μ. This effect is described by equation 4.16. Secondly, resonance of electrons with RH waves becomes possible whenever $\gamma > (\tilde{\omega}_d^R)^{-1}\ (m_i/m_e) \sim 10^5$. These additional resonances also help to close the gap.

The net result is that electrons with $\gamma \gtrsim 10^5$ <u>can</u> resonantly scatter through $\mu = 0$. It is therefore, probable that these particles are trapped by self-generated turbulence within the supernova remnant. Lower energy electrons, however, <u>cannot</u> resonantly scatter through

$\mu = 0$. Therefore, unless a scattering mechanism which operates in the gap can be found, it seems reasonable to conclude that electrons with $\gamma \lesssim 10^5$ cannot be trapped by self-generated turbulence within a supernova remnant.

CHAPTER 5

ENERGETIC PARTICLE PROPAGATION IN THE INTERPLANETARY MEDIUM

In the previous chapter, it was demonstrated that resonance gaps can have significant impact on conventional theories of particle propagation in turbulent plasmas. There it was demonstrated that, in a $\beta \gg 1$ plasma, a resonance gap inhibits the scattering of energetic particles through $\mu=0$. Therefore, the time for a streaming particle distribution to relax to isotropy can be significantly longer than predicted by conventional scattering theories.

Those basic ideas, developed to describe particle scattering near a supernova remnant, can be applied in many astrophysical environments. Examples include propagation of cosmic rays in the Galaxy and the transport of energetic particles in radio sources.

In addition, the results of Chapter 4 have significant implications for models of particle acceleration near interstellar shocks (Jokipii 1966, Fisk 1971, Bell 1978) and for the calculation of transport coefficients (e.g. heat flux instability) in high temperature plasmas. Before considering these additional problems, however, one would like to have some experimental confirmation of theoretical predictions regarding particle propagation in a medium where a resonance gap is predicted.

Supernova remnants are, of course, not accessible for direct observation of particle propagation. However, the efects of scattering in the interplanetary medium can be observed directly. Solar flares provide a unique opportunity to compare the observed properties of particle propagation in a turbulent plasma with the properties predicted by "resonance gap" theory.

In this chapter, the problem of particle scattering in the interplanetary medium is considered. It is found that "resonance gap" theory can account for the observed magnitude and dependence of the particle scattering mean free path, and it provides a natural explanation for scatter-free electron events. This discussion is begun in the following paragraphs with a brief review of the apparent conflict between conventional theory and observation.

The propagation of solar flare particles in the interplanetary medium is usually regarded as a diffusive process. In this picture energetic particles released near the surface of the sun resonantly interact with the interplanetary turbulence as they propagate outward. This interaction causes the particle pitch angle to change in a random way, resulting in phase space diffusion. The pitch angle diffusion coefficient $D_{\mu\mu}$ is related directly to the spectrum of the turbulence by conventional quasi-linear theory.

The spatial diffusion coefficient for propagation parallel to the zero order field, $K_{||}$, is then obtained by properly averaging $D_{\mu\mu}$ over μ. Several such averaging schemes have been developed (Jokipii 1971), but they all seem to give similar results. The following expression is that given by Earl (1974)

$$\kappa_{||} = \frac{v^2}{4} \int_0^1 \frac{(1 - \mu^2)^2}{D_{\mu\mu}} \, d\mu \tag{5.1}$$

It demonstrates the basic property of all averaging schemes, i.e. as $D_{\mu\mu}$ increases the spatial diffusion coefficient $\kappa_{||}$ decreases.

In principle, once the turbulent spectrum is measured, one can calculate $D_{\mu\mu}$ and $\kappa_{||}$. Then the calculated value of $\kappa_{||}$ can be compared to the experimental value of $\kappa_{||}$ inferred from observations of flare particle propagation in the interplanetary medium.

The results of these comparisons have not been encouraging (Ma Sung and Earl 1978, Zwickl and Webber 1977). Scattering theory predicts that the mean free path of energetic particles in the solar wind should increase with increasing rigidity. Experimentally it is found that the scattering mean free path is either constant or decreases slightly with increasing particle rigidity. In addition, $\kappa_{||}$ calculated from scattering theory is approximately one to two orders of magnitude smaller than $\kappa_{||}$ estimated from observations. (This, of course, implies a similar discrepancy for the mean free path).

Furthermore, the discrepancy between theory and observation cannot be resolved by considering higher order, i.e. non-linear, scattering terms in the usual way. This is because non-linear scattering terms cause the delta function resonance of quasi-linear theory to be broadened. This broadening is unimportant except in the regions near $\mu = 0$ and $\mu = 1$ where the quasi-linear scattering coefficient vanishes.

The net result is that consideration of non-linear terms increases $D_{\mu\mu}$ and hence decreases $\kappa_{||}$ (equation 5.1). Since $\kappa_{||}$ calculated from quasi-linear theory is already an order of magnitude smaller than the observed value, incorporation of non-linear scattering terms only increases the disagreement between theory and observations.

Goldstein (1980) suggested that many of these problems could be reconciled if two assumptions were made. First, he assumed that resonant scattering by transverse waves could not scatter particles through $\mu = 0$. This meant that the mean free path could be significantly longer than predicted by resonant scattering theory. Secondly, he assumed that all scattering through $\mu = 0$ was due to particle mirroring on compressional fluctuations of the interplanetary magnetic field.

In this chapter, it is demonstrated that these assumptions follow naturally from the proper consideration of resonant scattering in electromagnetic turbulence. It is found that both electrons and protons have a resonance gap, similar to the one obtained for scattering in a high β plasma. Within this gap (near $\mu = 0$) resonant scattering processes cannot operate and scattering through $\mu = 0$ is significantly inhibited. The width of the gap is not the same for electrons and protons however. It is found that for electrons with $T \leq 300$ keV the gap is wide enough so that the particles are not scattered at all. This provides a natural explanation for the scatter free electron events observed in the interplanetary medium. Goldstein's (1980) analysis does not predict differences between electron and proton propagation.

One can begin to understand these differences when it is recognized that two of the basic assumptions underlying the theoretical results discussed above cannot be justified for particle scattering in the solar wind. These are: (1) the assumption of magnetostatic turbulence, and (2) the related assumption that ω and k of the turbulence are related by $\omega = kv_a$. Both of these assumptions break down when scattering at small values of μ is considered.

Failure of the magnetostatic assumption was discussed in Chapter 3. Basically whenever ω becomes of order $kv_{||}$, i.e. $v_{||} \sim v_{ph}$, the temporal variation of the wave cannot be ignored in the solution of the resonance condition. For most waves $v_{ph} \sim v_a$.

In addition, particles having $v_{||} \sim v_a$ do not resonantly interact with Alfven waves, i.e. waves having $\omega = kv_a$. For example, consider non-relativistic protons having $v_{||} \sim v_a$. These particles resonantly interact with waves having $k \sim \Omega_i/v_a$ but Alfven waves exist only for $k << \Omega_i/v_a$. It is clear, therefore, that protons which have small values of μ do not resonantly interact with Alfven waves, and the assumption $\omega = kv_a$ is simply not appropriate. Waves having $k \sim \Omega_i/v_a$ are in fact in the transition region between Alfven waves and whistlers.

Further, it is well known that whenever $k \sim \Omega_i/v_{ti}$ cyclotron wave damping becomes important. In the solar wind one can estimate $v_{ti} \sim v_a$ for nominal plasma parameters. So one can expect damping to be significant for waves having $k \sim \Omega_i/v_a$, but again these are precisely the waves which interact with particles having $v_{||} \sim v_a$.

From these preliminary considerations it is obvious that current notions of particle scattering in magnetostatic turbulence with $\omega = kv_a$ and $0 \le k \le \infty$ are inadequate. This is particularly true for particle scattering near $\mu = 0$. In this chapter, particle scattering in a plasma where these assumptions are relaxed is considered.

It is worthwhile to emphasize that rejection of the hypothesis of magnetostatic turbulence does not imply a rejection of quasi-linear scattering theory. Quasi-linear theory has been developed for electromagnetic turbulence by Hall and Sturrock (1967) and Melrose (1980). But it does imply that many of the applications of quasi-linear scattering theory which have assumed magnetostatic turbulence should be reconsidered. In this chapter only one such example will be considered; particle scattering in the interplanetary medium.

First, some of the relevant features of proton and electron scattering in electromagnetic turbulence will be discussed. It is found that both protons and electrons exhibit a gap near $\mu = 0$ where resonant scattering processes do not operate. The implications of this gap for particle scattering theory are then discussed, and it is shown that reconcillation of theory and observation is possible if the non-resonant scattering rate in the gap can be calculated.

Finally, in the last section of this chapter, the non-resonant scattering rate due to particle mirroring is estimated. It is shown that the combination of resonant scattering outside the gap and mirroring within the gap can account for the observed particle mean free path in the interplanetary medium.

In addition, it is demonstrated that cyclotron wave damping is important for the analysis of electron propagation. Incorporation of wave damping leads naturally to an explanation of scatter free electron events.

A. Particle Scattering in the Interplanetary Medium

Except for the items mentioned above, a conventional view of scattering in the interplanetary medium will be adopted. Only scattering by waves having $\vec{k} \parallel \vec{B}_o$ will be considered. In addition, it will be assumed that all turbulence is generated near the sun so that only outward propagating waves are present. In the solar wind plasma $\beta \lesssim 1$, so that finite β effects can essentially be ignored in the dispersion relation, except for cyclotron damping which is important when electron propagation is considered. Only particles having $\gamma \sim 1$ will be considered. The generalization of this discussion to more relativistic particle energies is straightforward, but unnecessary for the vast majority of solar flare events. Finally, only scattering in a frame moving with the solar wind will be discussed. First consider proton scattering.

Proton Scattering

In Chapter 3, it was demonstrated that non-relativistic protons with $\mu \geq 0$ resonantly interact only with RH polarized waves, and that these interactions do not occur for all values of $\mu \geq 0$. In fact, there is a minimum value $\mu_{min} = (3\sqrt{3}/2)\ v_a/v$ such that in the range $\mu_{min} \geq \mu \geq 0$ there are no solutions to the resonance condition. This means that there is a resonance gap of width $\Delta\mu \sim \mu_{min}$ in which resonant particle scattering processes do not operate.

This result is not obtained in magnetostatic theories, and can be interpreted in the following way. The order of magnitude of $\mu_{min} \sim v_a/v$ is determined by the characteristic propagation speed of the waves while the coefficient, $3\sqrt{3}/2$, is determined by the details of the dependence of ω on k. There are two important differences between this result and the predictions of magnetostatic theory.

First, magnetostatic theories predict that $D_{\mu\mu} = 0$ only in an infinitesimally small region around $\mu = 0$. This is because particles having $\mu \rightarrow 0$ are required to resonantly interact with waves having $k \rightarrow \infty$ in these theories. Since the power spectrum of the turbulence, P_k, is assumed to vanish for $k \rightarrow \infty$, one must then conclude that $D_{\mu\mu} \rightarrow 0$ as $\mu \rightarrow 0$.

When the electromagnetic nature of the turbulence is properly accounted for it is found that $D_{\mu\mu} = 0$ in a finite region around $\mu = 0$, i.e. for $\mu_{min} \geq \mu \geq 0$. This result is independent of the wave power spectrum and comes about simply because the resonance condition cannot be satisfied in this range of μ. The finite size of the resonance gap is an important point which will be considered again when discussing the proper calculation of $\kappa_{||}$ in electromagnetic turbulence.

The second major difference between magnetostatic and electromagnetic scattering theories also involves scattering near $\mu = 0$. In magnetostatic theories, when $\mu \to 0$ the coherence time, t_c, between the waves and the particle can become very large. This, of course, violates one of the fundamental assumptions of quasi-linear theory which is that $t_c \ll t_s$, where t_s is the scattering time. This point has been widely discussed in both derivations and criticisms of quasi-linear theory (Jokipii 1971 and Volk 1973). It comes about because of the approximation $v_{ph} = 0$, where v_{ph} is the phase velocity of the wave.

When the full electromagnetic resonance condition is considered, it is found that

$$\left| v_{ph} - v_{||} \right| = \Omega_i/k \tag{5.2}$$

This means that whenever the resonance condition is satisfied, one is assured that $v_{ph} \neq v_{||}$, and therefore t_c is finite. This does not guarantee $t_c \ll t_s$, but it does show that this condition is not automatically violated for small values of μ.

Electron Scattering

Electrons with $\mu \gtrsim 0$ and $\gamma \sim 1$ are scattered almost exclusively by LH polarized waves. However these waves are subject to strong ion cyclotron damping in the interplanetary thermal plasma. This damping can significantly modulate the power spectrum of the waves and hence is one of the major factors to be considered when discussing electron propagation in the heliosphere. Therefore, this discussion is begun with a consideration of cyclotron damping of LH polarized waves.

In Chapter 2, it was shown that waves having $\tilde{\omega} = \omega/\Omega_i \sim 0.4$ and $\tilde{k} = kv_a/\Omega_i \sim 0.25$ are damped out in approximately ten wave periods in a plasma having $0.1 \leq \beta \leq 1$. This range of β is similar to that typically observed in the solar wind. Waves which are damped this rapidly will not propagate far into the interplanetary plasma before being absorbed. Since, by assumption, the turbulence responsible for particle scattering in the solar wind is generated somewhere near the solar surface, one must conclude that LH waves having $\tilde{k} \geq 0.25$ will not be available for particle scattering throughout most of the interplanetary medium.

The absorption of LH waves having large values of k has significant impact on the scattering of electrons. This can be seen in the following way. First, waves having large values of k are resonant with particles having small values of μ. However wave damping causes the wave power to be severely depressed for large k. Since the scattering rate is proportional to the wave power, the scattering rate is severely depressed for small values of μ. This leads to a resonance gap which is somewhat analogous to the gap discussed in Chapter 4 for a high β plasma.

It should be emphasized that the discussion above applies only to LH waves. For the small values of β observed in the solar wind the RH wave does not experience significant wave damping until $\tilde{k} \sim v_{te}/\Omega_e$. Therefore the damping of LH waves with $\tilde{k} \gtrsim 0.25$ is consistent with observations of interplanetary turbulence which show significant wave power at these values of k.

Returning now to the discussion of electron scattering, once the value of $\tilde{k}_{max}$ is determined, the value of μ_{min} can be estimated from the resonance condition

$$\mu_{min} \sim \frac{v_a}{v} \frac{m_i}{m_e} \frac{1}{\gamma \tilde{k}_{max}} \sim \frac{1.23}{\gamma(v/c)} \tag{5.3}$$

where it was assumed that $\gamma << \frac{m_i}{m_e}$. For the numerical estimate it was assumed that $v_a = 5\text{x}10^6$ cm/sec, a typical value for the Alfven speed at 1 AU.

The significance of this result is that the resonance gap can be very large for low energy electrons. In fact for $v/c \leq 0.78$ one can estimate $\mu_{min} \sim 1$ and the particles are not scattered at all. This corresponds to electrons having kinetic energies $T \leq 300$ keV, and provides a natural explanation for the scatter free electron events observed in the solar wind. (See McDonald, Fitchel and Fisk (1974) for a discussion of these events.) Such events are not explained by theories which ignore cyclotron wave damping.

Observational evidence seems to indicate that scatter free events are possible only for low energy electrons. Typically, scatter-free propagation is observed for $T \lesssim 500$ keV. This is in good agreement with the energy range derived above.

B. Non-Linear Scattering in the Resonance Gap

The problem of scattering particles through $\mu = 0$ is not a new one. Previous authors (Volk 1973, Goldstein 1976, and Jones, Birmingham and Kaiser 1978) have approached the problem by considering non-linear scattering processes. They find that non-linear scattering leads to a broadening of the wave/particle resonance which is unimportant (compared to quasi-linear terms) except in the region of small μ where $D_{\mu\mu}$ becomes very small. However, this is also where the assumption of magnetostatic turbulence, universally invoked in these theories, breaks down.

In the previous section, it was shown that when the electromagnetic nature of the turbulence is properly incorporated into quasi-linear scattering theory, there is a gap of finite size, near $\mu = 0$, where resonant scattering processes are ineffective. Based on this, it is almost certain that existing formulations of non-linear scattering theory are not directly applicable in electromagnetic turbulence. Nevertheless, it is worthwhile to consider whether the resonance broadening mechanism discussed by these authors could result in significant scattering for particles having values of μ within the resonance gap if the assumption of magnetostatic turbulence were relaxed. It is concluded that significant scattering by this mechanism is not likely. To see this one must consider the physical mechanism of resonance broadening.

In non-linear theories, it is assumed that the actual particle trajectory is essentially just the product of two components. The first component is due simply to the helical motion of the particle in the zero order field. This component of the motion is completely determined by the initial conditions, i.e. $\mu(t) = \mu(t=0)$.

The second component, due to the interaction of the waves on the particle, can be regarded as essentially a probability distribution. This factor accounts for the small random changes in μ due to the resonant interaction of the particle with the waves. The magnitude of this effect is proportional to the rate of pitch angle scattering , $D_{\mu\mu}$. When $D_{\mu\mu}$ is large, the particle is more likely to have been scattered away from its initial value of μ than when $D_{\mu\mu}$ is small.

In this picture, therefore, a particle with a given value of $\mu = \mu_o$ at $t = 0$ has a finite probability to be in a range of μ (near μ_o) at some later time $t > 0$. Since the resonant wave number, k, is a function of μ, this finite range of possible values for μ means that the particle which had μ_o at $t = 0$ can resonantly interact with waves in a finite range of k.

In other words, resonance broadening is an inherently statistical process which assumes that the primary non-linear effect of the fluctuating fields on particle motion is to cause a slow "difussion" of the particle away from its unperturbed helical trajectory. This "diffusion" introduces uncertainty in the value of μ for $t > 0$, and this uncertainty is translated, via the resonance condition, to uncertainty in the value of k. This is the physical mechanism responsible for resonance broadening.

Now consider particles within the resonance gap. These particles cannot resonantly interact with any component of the turbulence present in the plasma. Therefore, the unperturbed trajectory used in quasi-linear theory is an excellent approximation to the actual particle trajectory, i.e. $\mu(t) = \mu_o$ is an excellent approximation to the true orbit of the particle. Based on this, one would not expect resonance broadening to be significant for particles within the resonance gap.

Current formulations of resonance broadening theory, therefore, almost certainly overestimate the scattering rate for small values of μ. The primary reason for this is that existing theories assume that there are resonant interactions at all values of $\mu > 0$, i.e. they assume magnetostatic turbulence. The calculations presented below assume that resonance broadening is negligible for particles in the resonance gap.

C. Qualitative Picture of Particle Scattering in Electromagnetic Turbulence

It has been demonstrated above that resonant wave/particle interactions cannot cause particles to scatter through $\mu = 0$. This is because both electrons and protons have a gap in μ-space, near

$\mu = 0$, where the resonant scattering process does not operate. For electrons, this gap is due to collisionless wave damping, and for protons it is due to the fact that there are no solutions to the resonance condition for $\mu_{min} > \mu > 0$.

Based on these results, one can develop the following qualitative picture of particle propagation in the interplanetary medium. Flare particles, released near the sun, scatter rapidly in velocity space throughout most of the $\mu > 0$ hemisphere. This scattering occurs on the very short time scale characteristic of conventional pitch angle scattering theory, i.e. within approximately 0.01 AU of the sun. However, the gap prevents particles from resonantly scattering through $\mu = 0$. Observational evidence on the other hand indicates that particles do scatter through $\mu = 0$, but that this scattering occurs on time scales much longer than those predicted by resonant pitch angle scattering theories.

The purpose of the following section is to show how these qualitative ideas can be incorporated into a quantitative picture of particle propagation in the interplanetary medium.

D. Calculation of Scattering Mean Free Path in Electromagnetic Turbulence

To compare the theoretical results to the observational data, one must calculate $\kappa_{||}$ or equivalently the mean free path λ. This can be accomplished by using equation 5.1. It will be assumed that $D_{\mu\mu}$ can be divided into two parts.

$$D_{\mu\mu} = D^R_{\mu\mu} + D^{NR}_{\mu\mu} \tag{5.4}$$

The first portion, $D^R_{\mu\mu}$, represents the pitch angle scattering due to resonant processes, while the second, $D^{NR}_{\mu\mu}$, represents the non-resonant scattering, i.e. all scattering due to other processes. In regions of μ-space where both processes operate, it will be assumed

$$D^R_{\mu\mu} >> D^{NR}_{\mu\mu} \tag{5.5}$$

In the conventional picture of scattering in magnetostatic turbulence, resonant scattering operates at all values of μ except $\mu = 0$. Non-resonant scattering has also been assumed to occur, but for any value of μ it has been assumed that the scattering rate is dominated by the resonant scattering processes. As a result, the mean free path calculated from this theory is practically independent of the non-resonant scattering rate and is given by

$$\lambda \lesssim \frac{v}{4} \int_0^1 \frac{(1-\mu^2)^2}{D^R_{\mu\mu}} \, d\mu \tag{5.6}$$

The equality applies when $D^{NR}_{\mu\mu} = 0$. This upper limit to the mean free path is at least an order of magnitude too small to explain the observed values of λ for low rigidity particles.

When particle scattering in electromagnetic turbulence is considered, one finds that there is a gap of finite size near $\mu = 0$ where $D^R_{\mu\mu} = 0$. In this region only non-resonant interactions contribute to the scattering rate. Therefore, contrary to previous results, the proper incorporation of non-resonant scattering is essential to the calculation of λ in electromagnetic turbulence.

For example, if one neglects non-resonant scattering, $D_{\mu\mu} = 0$ for $\mu_{min} > \mu \geq 0$. It is then obvious that the integral which defines λ diverges. This is equivalent to the more intuitive statement that in the absence of particle scattering through $\mu = 0$ one must find that the mean free path, $\lambda \to \infty$. This behavior is clearly not observed for scattering in the interplanetary medium, and therefore, non-resonant scattering processes must be able to scatter particles that have values of μ within the resonance gap.

From these arguments it is clear that the expression for λ must be rewritten as a sum of two integrals.

$$\lambda = \frac{v}{4} \int_{\mu_{min}}^{1} d\mu \; \frac{(1-\mu^2)^2}{D_{\mu\mu}^{R}} + \frac{v}{4} \int_{0}^{\mu_{min}} d\mu \; \frac{(1-\mu^2)^2}{D_{\mu\mu}^{NR}} \tag{5.7}$$

The inequality, 5.5, has been used to obtain this result. For most particles $\mu_{min} << 1$, and λ can be estimated as

$$\lambda \sim \lambda_R + \lambda_{NR} \tag{5.8}$$

where

$$\lambda_R \simeq \frac{v}{4} \int_{\mu_{min}}^{1} \frac{(1-\mu^2)^2}{D_{\mu\mu}^{R}} \; d\mu \tag{5.9}$$

and

$$\lambda_{NR} \simeq \frac{v}{4} \; \frac{\mu_{min}}{D_{\mu\mu}^{NR}} \tag{5.10}$$

The magnitude of the first term λ_R, is approximately equal to the mean free path calculated from conventional magnetostatic theory, e.g. Jokipii (1971). This term is entirely due to resonant wave/particle interactions. The second term, λ_{NR}, is due entirely to the interactions within the resonance gap. This term is not obtained in conventional theories.

There are several features of this result which should be emphasized. First, if the scattering rate $D_{\mu\mu}^{NR}$ is small enough, the second term can dominate the first and the magnitude of λ is determined almost entirely by the non-resonant scattering term. Therefore, λ can be much larger than previous estimates indicate.

Secondly, since μ_{min} is proportional to v^{-1}, the mean free path is approximately independent of particle energy provided $D_{\mu\mu}^{NR}$ is also independent of particle energy. Particle mirroring provides such a scattering mechanism. The mean free path for mirroring is estimated later in this chapter.

And finally, the magnitude of the non-resonant term remains approximately constant with increasing particle energy. But the magnitude of the resonant contribution increases with particle energy. So in the limit of relativistic particle energies, one would expect the mean free path to approach the value calculated from magnetostatic theory. All of these properties are observed for particle propagation in the interplanetary medium.

The last step in this analysis is to estimate the contribution of non-resonant scattering to the mean free path, and to compare these results with observations. This is done in the following section.

E. Estimate of Mean Free Path for Mirroring and Comparison with Observations

There are several sources of non-resonant scattering which could contribute to the particle scattering rate inside the resonance gap. These include particle mirroring, backward propagating waves, off-axis waves and magnetic field discontinuities near shocks or the equatorial neutral sheet. In this section only the scattering rate due to particle mirroring is estimated.

This approach was chosen primarily to provide a concrete example of the application of the theory developed above. However, the remarkable agreement between the observed value of λ and the one estimated for particle mirroring suggests that mirroring may be the dominant process by which particles are scattered through the resonance gap. First mirroring of protons will be considered.

Protons

The magnetic field fluctuations in the interplanetary medium are primarily non-compressive. Measurements indicate that the power in transverse (non-compressive) waves is approximately an order of magnitude larger than the power in fluctuations in the magnitude of $\vec{B}$. Nevertheless, even at this relatively low power level, the compressive fluctuations are strong enough to mirror protons across the resonance gap. The condition required for mirroring to occur is

$$\frac{\delta |B|}{|B|} > \left(\frac{v_{||}}{v_{||0}}\right)^2 \sim \left(\frac{v_a}{v}\right)^2 \tag{5.11}$$

In this expression the initial velocities near the edge of the resonance gap have been used. For typical non-relativistic protons one can estimate that mirroring can occur when

$$\frac{\delta |B|}{|B|} \gtrsim 10^{-6} \tag{5.12}$$

Measurements of the power spectrum (Burlaga 1972) of $\delta|B|$ show that this condition is easily satisfied in the interplanetary medium.

The other condition which must be satisfied for mirroring to occur is that the scale size of the compressional fluctuation should be large compared to the particle gyro-radius. This means that only waves whose

$$k \ll \frac{2\pi\Omega_i}{v_\perp} \sim 10^{-9}\ \text{cm}^{-1} \tag{5.13}$$

can effectively mirror protons in the resonance gap. For this numerical estimate typical values of $\Omega_i = 0.5$ rad/sec and $v_\perp = 0.1c$ were used. Again, observations show that waves with values of k which satisfy the inequality above are present in the interplanetary medium.

These considerations show that magnetic compressions of sufficient strength and proper scale are available in the interplanetary medium for mirroring protons in the resonance gap. The mean free path for scattering on these fluctuations can be estimated in the following way.

First, note that protons near the edge of the gap have parallel velocities on the order of $2v_a$. The compressional waves have phase velocities typically of order $v_{ph} \sim v_a$. So that the relative velocity between the waves and the protons is of order v_a. For a proton to be mirrored it must travel a distance, measured in the wave frame, on the order of the wavelength of the fluctuation. This takes a time $t \sim 2\pi/kv_a$.

During this time the motion of the particle through the interplanetary medium is primarily due to convection in the solar wind, i.e. $v_{11} \lesssim v_a << v_w$, where v_w is the wind speed. Therefore, one can estimate the mean free path as

$$\lambda_{NR} \simeq tv_w \simeq \frac{2\pi v_w}{kv_a} = 0.3 \text{ A.U.} \tag{5.14}$$

For the numberical estimate $v_a = 5x10^6$ cm/sec and $v_w = 4x10^7$ cm/sec were used. In addition, it was assumed that $k \sim 10^{-11}$ cm^{-1}. Larger values of k are allowed by the inequality, however waves with k on this order represent the dominant scale of the magnetic turbulence, and hence contain most of the wave power.

Although the estimate described above is admittedly a crude one, a mean free path for particle mirroring of this magnitude is not without precedent. Goldstein's (1980) more sophisticated numerical study resulted in a similar value. The difference between the analysis presented here and previous results is that at low proton energies $\lambda_{NR} >> \lambda_R$, i.e. the non-resonant contribution to the mean free path dominates the resonant scattering contribution (see equation 5.8). Therefore, one can estimate the total mean free path for low energy protons as

$$\lambda \sim \lambda_{NR} \sim 0.3 \text{ A.U.} \tag{5.15}$$

The mean free path above is independent of the proton rigidity. This is because non-relativistic protons can pitch angle scatter only until $v_{||} = (3\sqrt{3}/2)\ v_a$. This minimum value of $v_{||}$ is independent of the proton rigidity. This fact makes the scattering time and hence the mean free path independent of rigidity.

Furthermore, the estimated magnitude of λ (see Figure 19) is only a factor of two larger that the observational results of Ma Sung and Earl (1978) and Zwickl and Webber (1977). Consideration of additional sources of scattering would lower the value predicted for λ bringing it into even closer agreement with the observations. The estimated value is also consistent with the lower limits obtained by Zwickl and Webber (1978). Previous theoretical estimates of λ based on magnetostatic resonance theory are more than an order of magnitude too small at rigidities ~ 300 MV, and the discrepancy becomes larger at lower rigidities.

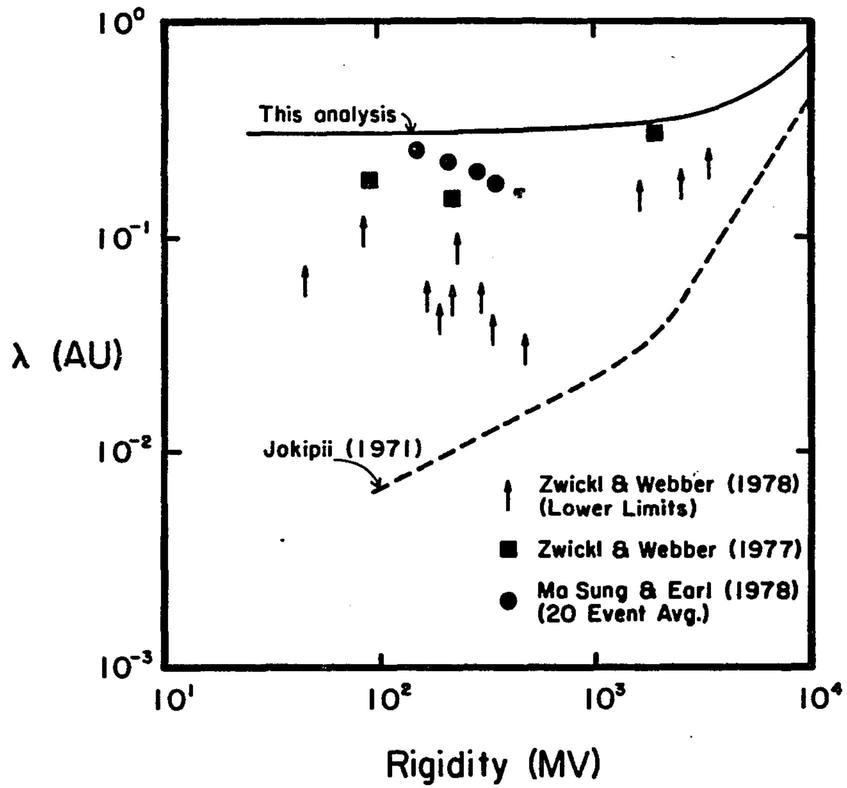


Figure 19. Comparison of the Estimated Proton Mean Free Path to the Observational Values.

Electrons

As discussed above, non-relativistic electrons ($T \lesssim 300$ keV) experience scatter free propagation in the interplanetary medium. However, as one considers electrons of higher and higher kinetic energies, it is obvious that the resonance gap becomes narrower. Eventually, an energy will be reached where particle mirroring will cause electrons to scatter across the resonance gap. This energy can be estimated in the following way.

The condition for mirroring to occur can be written, assuming $\delta|B|/|B| << 1$, as

$$\mu_{min} \lesssim \left(\frac{\delta|B|}{|B|}\right)^{1/2} \tag{5.16}$$

where μ_{min} represents the smallest value of μ where resonant pitch angle scattering is effective, i.e. μ at the end of the resonance gap.

Using the expression for μ_{min} given in equation 5.3, one can show that mirroring across the gap can occur only for electrons with

$$\gamma \gtrsim 1.23 \left(\frac{\delta|B|}{|B|}\right)^{-1/2} \tag{5.17}$$

The value of $\frac{\delta|B|}{|B|}$ varies somewhat with solar activity, but is typically on the order of 0.06. With this value one can estimate that mirroring can cause electrons to cross the resonance gap only when

$$\gamma \gtrsim 5 \tag{5.18}$$

This corresponds to kinetic energies $T \gtrsim 2$ MeV.

For electrons mirroring can be effective only for waves having

$$k << 10^{-7}\ \gamma^{-1}\ cm^{-1} \tag{5.19}$$

This again is easily satisfied by interplanetary turbulence.

The mean free path can be estimated in a mannter similar to that used for protons. The significant difference however is that convection is completely negligible for most electrons of interest since $v_{||} >> v_w$. The result is then

$$\lambda \sim \lambda_{NR} \sim \frac{2\pi}{k} \sim 0.04 \quad AU \tag{5.20}$$

This result is roughly an order of magnitude smaller than the mean free path estimated for protons. However, again there is reasonable agreement between the estimated and observed value for electrons. (see Figure 20) The mean free path estimated for particle mirroring is approximately a factor of two lower than the observational result.

From their analysis of flare particle propagation, Ma Sung and Earl (1978) obtained an electron mean free path $\lambda \sim 0.08$ AU. Zwickl and Webber (1978) derive a lower limit of $\lambda \sim 0.04$ AU, although they consider $\lambda \sim 0.1$ AU a more likely average value. Both of these results are in reasonable agreement with the estimate given above.

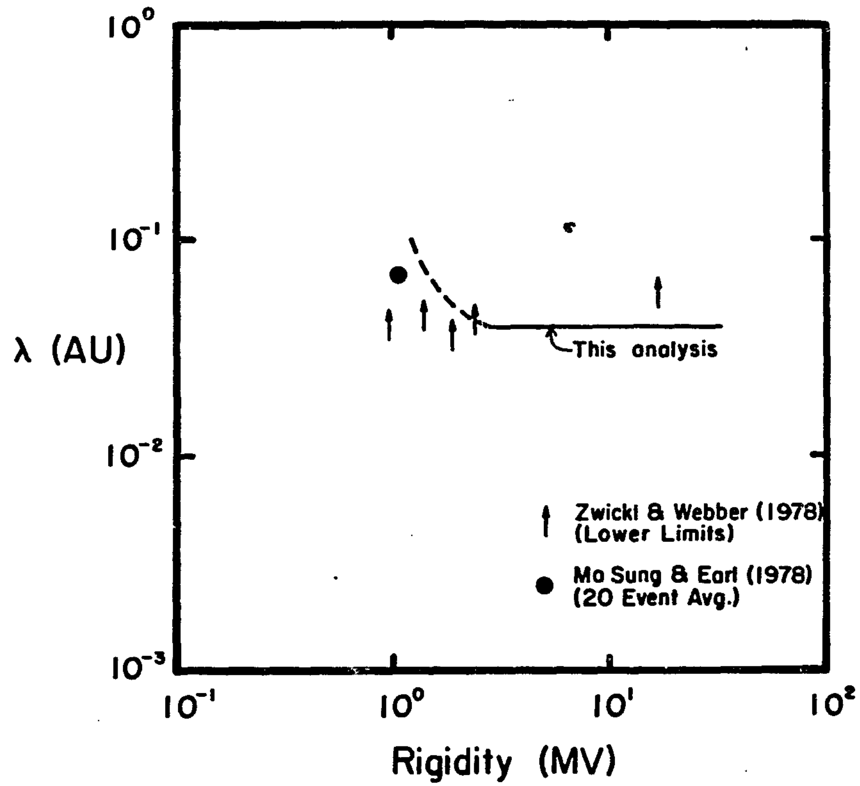


Figure 20. Comparison of the Estimated Electron Mean Free Path to the Observational Values.

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