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EXPLICIT ELEMENT VALUE EQUATIONS FOR THE TWO ZERO-SIX POLE DOUBLY AND SINGLY TERMINATED INVERSE CHEBYSHEV FILTER

The University of Arizona
M.S. 1985

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# EXPLICIT ELENENT VALUE EQUATIONS FOR THE TWO ZERO-SIX POLE DOUBLY AND SINGLY TERMINATED INVERSE CHEBYSHEV FILTER 

by<br>Alfredo Mendoza Garcia

A Thesis Submitted to the Faculty of the DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

In Partial Fulfillment of the Requirements For the Degree of MASTER OF SCIENCE WITH A MAJOR IN ELECTRICAL ENGINEERING

In the Graduate College THE UNIVERSITY OF ARIZONA

1985

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SIGNED:


## APPROVAL BY THESIS DIRECTOR

This thesis has been approved on the date shown below:


WILLIAM J. KERWIN
Professor of Electrical and Computer Engineering

## DEDICATION

To: The memory of my mother
My father
Rosalinda
Alfredo
Cecilia Irene
Mario Alberto

## ACKNOWLEDGENENTS

I would like to express my most sincere thanks to professor William J. Kerwin for his invaluable assistance and guidance throughout the development of this work. I would also like to acknowledge the great support of Mr . Jorge Garcia Revilla, the Sistema de Institutos Tecnologicos and in particular the one in Nogales in my pursuit of this degree. Special thanks are due for my wife Rosalinda for all the help and understanding she has given me these years.
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#### Abstract

General aspects of the two zero-n pole inverse Chebyshev filter are introduced in chapter one. Two equations were developed to compute the coefficients of the transfer function for any position of the zero. For the two zero-six pole (2z-óp) doubly terminated filter a set of explicit element value equations were developed that were within $0.7 \%$ in the -3 dB cutoff frequency in the worst case within the usable range of zero positions. A correction factor was determined that reduced the error to a negligible level. A second solution (approximation) for the two zero-four pole, ( $2 z-4 p$ ) doubly terminated filter was obtained applying the same procedure as in the $2 z-6 p$ case. No correction factor was needed in this instance.

A set of exact design equations were developed for the two zero-six pole singly terminated configuration.


## CHAPTER 1

## INTRODUCTION

This thesis is concerned primarily with the behavior and design of the two zero-six pole (2z-6p) (sixth order) inverse Chebyshev passive filter. However, since the method proposed here to determine the element values for the sixth order doubly terminated case applies as well to the fourth order ( $2 \mathrm{z}-4 \mathrm{p}$ ) filter, explicit element value equations for this circuit will also be derived to generate a second solution which will be compared with the one already existing.

In a certain way, this work may be also considered as a complement to the one presented by Mr. David B. Henry in his M.S. thesis where, besides giving a complete description of the general aspects of all two zero inverse Chebyshev filters, he focused his attention to the odd-ordered configurations, specifically, the third, fifth and seventh order filters.

For the sake of clearness, some of the most significant characteristics of this family of filters (Kerwin, 1981) are briefly mentioned in this introductory chapter. The general low pass two zero inverse Chebyshev circuit has a maximally flat magnitude (MFM) response in
the pass band and a peak return in the stop band, with a zero located somewhere in between the pass band and the peak return. The -3 dB cutoff frequency is normalized to one rps. The magnitude $\left(\left|T\left(j \omega_{\max }\right)\right|\right)$ and the frequency $\left(\omega_{\text {max }}\right)$ of the peak return are functions of the order of the filter ( $n$ ) and the frequency of the zero $\left(\omega_{z}\right)$. For a given order, the greater $\omega_{z}$, the smaller the magnitude of the peak return and the same effect is observed when the order of the filter is increased for a given position of $\omega_{z}$. The next two equations (Henry, 1983) summarize the above explanation:

$$
\begin{gather*}
\omega_{\max }=\omega_{z} \sqrt{\frac{n}{n-2}}  \tag{1.1}\\
T\left(j \omega_{\max }\right)=-10 \log \left[\frac{\frac{1}{4} n^{n} \omega_{z}^{2 n-4}\left(\omega_{z}^{2}-1\right)^{2}}{(n-2)^{n-2}}\right] d B \tag{1.2}
\end{gather*}
$$

The general low pass doubly terminated $2 z-n p$ transfer function is:

$$
\begin{equation*}
T_{T}(s)=\frac{V_{0}}{V_{i}}=\left(\frac{1}{2}\right) \frac{a s^{2}+1}{b s^{n}+c s^{n-1}+d s^{n-2}+\ldots .+1} \tag{1.3}
\end{equation*}
$$

The only difference between the doubly terminated and the singly terminated transfer function is the value of the D.C. component, which instead of being one half is one.

## CHAPTER 2

THE TWO ZERO-SIX POLE INVERSE CHEBYSHEV FILTER

The main objective of this work is to develop a set of explicit element value equations for the doubly and singly terminated two zero-six pole inverse Chebyshev filter in terms of the coefficients of the sixth order transfer function (2.1). The doubly and singly terminated passive circuit configurations selected to produce the desired responses are the sixth order ladder networks shown infigures 2-1 and 5-1 respectively. It can be observed that the transmission zeros in the transfer function (1.3) are produced by the parallel tuned circuit formed by the elements $L_{2}$ and $C_{3}$, which unfortunately causes an extra reactive component in the ladder (seven instead of six).

The resistors $R_{i}$ and $R_{o}$ are equal in value in the doubly terminated circuit considered herein. In the singly terminated case $R_{i}$ will take any required value while $R_{0}=\infty$.

## Sixth order coefficient development procedure

As previously stated, one of the characteristics of all inverse Chebyshev filters is an MFN response in the pass band. In order to achieve this requirement, it is necessary that the coefficients of the squared magnitude of


Fig. 2-1. Two zero-six pole doubly terminated circuit.
the transfer function meet certain conditions. Specifically, every coefficient in the denominator with the only exception of the coefficient of the highest power term, must be equal to the one of the same power in the numerator (weinberg, 1962).

The starting point to obtain an MFM response is the two zero-six pole doubly terminated transfer function shown next:

$$
\begin{equation*}
T_{T}(s)=\frac{1}{2} \frac{a s^{2}+1}{b s^{6}+c s^{5}+d s^{4}+e s^{3}+f s^{2}+g s^{+1}} \tag{2.1}
\end{equation*}
$$

From (2.1) the squared magnitude response can be obtained by letting $s=j \omega$, squaring the real and imaginary parts, and grouping common terms. Next, the final form of this equation is shown:

$$
\begin{align*}
& \left|T_{T}(j \omega)\right|^{2}=\frac{1}{4} \frac{a^{2} \omega^{4}-2 a \omega^{2}+1}{b^{2} \omega^{12}+\left(c^{2}-2 b d\right) \omega^{10}+\left(d^{2}-2 c e+2 b f\right) \omega^{8}+\left(e^{2}-\right.} \\
&  \tag{2.2}\\
& \frac{2 d f+2 c g-2 b) \omega^{6}+\left(f^{2}-2 e g+2 d\right) \omega^{4}+\left(g^{2}-2 f\right) \omega^{2}+1}{}
\end{align*}
$$

Applying the previously stated MFM conditions to equation (2.2), it can be seen that the coefficients of $\omega^{10}, \omega^{8}$, and $\omega^{6}$ must be zero while the coefficients of $\omega^{4}$ and $\omega^{2}$ must be equal to $\mathrm{a}^{2}$ and -2 a respectively. Equations (2.3)
through (2.7) (Henry, 1983) have to be satisfied in order for the magnitude response to be maximally flat.

$$
\begin{align*}
& c^{2}-2 b d=0  \tag{2.3}\\
& d^{2}-2 c e+2 b f=0  \tag{2.4}\\
& e^{2}-2 d f+2 c g-2 b=0  \tag{2.5}\\
& f^{2}-2 e g+2 d=a^{2}  \tag{2.6}\\
& g^{2}-2 f=-2 a \tag{2.7}
\end{align*}
$$

Another important relation is obtained by normalizing the -3 dB cutoff frequency to one rps. Substituting the right hand side of equations (2.3) through (2.7) into (2.2) and letting $\omega=1 \mathrm{rps}$ the squared magnitude response should be equal to $1 / 8$ and (2.2) becomes:

$$
\begin{equation*}
T_{T}(j 1)^{2}=\frac{1}{4} \frac{1-2 a+a^{2}}{b^{2}+a^{2}-2 a+1}=\frac{1}{8} \tag{2.8}
\end{equation*}
$$

It can easily be seen that for equation (2.8) to hold true it is necessary that:

$$
\begin{equation*}
b^{2}=(1-a)^{2} \tag{2.9}
\end{equation*}
$$

Looking at the numerator of equation (2.2), it can be observed that the values of $\omega$ that make the magnitude response equal to zero are $\pm \sqrt{1 / a}$, which correspond to the
position of the zeros (Kerwin, 1981) in all two zero inverse Chebyshev filters. Thus

$$
\begin{equation*}
\omega_{\mathrm{z}}= \pm \sqrt{1 / \mathrm{a}} \tag{2.10}
\end{equation*}
$$

Solving for a from (2.10)

$$
\begin{equation*}
a=\frac{1}{\omega_{z}^{2}} \tag{2.11}
\end{equation*}
$$

This important result sets the first step in the procedure to determine the coefficients of the transfer function that is selecting the position of the zero ( $\omega_{z}$ ). Once this is done, from equations (2.11) and (2.9) the coefficients $a$ and $b$ become known. Therefore, the only ones left to be solved for are: $c, d, e, f$, and $g$.

The following algebraic manipulation is used to determine the above mentioned unknowns. For the purpose of clearness only the most significant steps of this algebraic procedure are shown.

Solving for from (2.7)

$$
\begin{equation*}
f=\frac{g^{2}}{2}+a \tag{2.12}
\end{equation*}
$$

Now substituting (2.12) into (2.6)

$$
\begin{equation*}
\frac{g^{4}}{4}+a g^{2}-2 e g+2 d=0 \tag{2.13}
\end{equation*}
$$

Solving for d from (2.3)

$$
\begin{equation*}
d=\frac{c^{2}}{2 b} \tag{2.14}
\end{equation*}
$$

Substituting (2.12) and (2.14) into (2.4)

$$
\begin{equation*}
\frac{c^{4}}{4 b^{2}}-2 c e+b g^{2}+2 a b=0 \tag{2.15}
\end{equation*}
$$

Now, substituting (2.14) into (2.13)

$$
\begin{equation*}
\frac{g^{4}}{4}+a g^{2}-2 e g+\frac{c^{2}}{b}=0 \tag{2.16}
\end{equation*}
$$

Substituting equations (2.12) and (2.14) into (2.5) and solving for e the following relation is obtained:

$$
\begin{equation*}
e=\sqrt{\frac{c^{2}}{2 b}\left(g^{2}+2 a\right)+2(b-c g)} \tag{2.17}
\end{equation*}
$$

Now, substituting (2.17) into (2.15), (2.18) is generated

$$
\begin{equation*}
\frac{c^{4}}{4 b^{2}}-2 c \sqrt{\frac{c^{2}}{2 b}\left(g^{2}+2 a\right)+2(b-c g)}+b\left(g^{2}+2 a\right)=0 \tag{2.18}
\end{equation*}
$$

Again, substituting (2.17) but this time into (2.16) generates the next equation:

$$
\begin{equation*}
\frac{g^{4}}{4}+a g^{2}-2 g \sqrt{\frac{c^{2}}{2 b}\left(g^{2}+2 a\right)+2(b-c g)}+\frac{c^{2}}{b}=0 \tag{2.19}
\end{equation*}
$$

The original system of equations has been reduced to two (2.18 and 2.19) in two unknowns ( $c$ and g). Unfortunately the resultant equations are not linear, which makes it difficult to explicitly solve for the two coefficients. Without much choice, an iterative solution will be forced. Once these two relations are iterated and a solution with the desired accuracy is met, it is very easy to determine the values of the rest of the unknowns by using (2.12), (2.14) and (2.17).

It should also be mentioned that the former algebraic procedure is not unique and a different pair of final equations could be reached depending upon which two coefficients one would like to solve for.

The iterative method used to solve the system of equations (2.18) and (2.19) will be covered in chapter 3.

## Sixth order doubly terminated transfer function

Once the numerical values of the coefficients of the two zero-six pole transfer function can be determined for any position of the zero, the next objective is to obtain the same coefficients in terms of the elements of the circuit by solving for the voltage transfer function ( $\mathrm{V}_{0} / \mathrm{V}_{\mathrm{i}}$ ) of the sixth order doubly terminated circuit shown in figure 2-1. A total of seven equations and seven unknowns should be obtained from the transfer function corresponding to every coefficient and reactive component respectively.

As previously stated, $R_{i}$ and $R_{o}$ will be equal in value. To truly simplify the algebraic work both resistors will be made equal to one ohm; later, by the proper impedance scaling the resistors could take any required value. Since the circuit is a ladder network it would probably be much easier to use linearity to solve it than any other method.

Again, for the sake of clearness only the most significant steps of this procedure will be presented. Refer to figure 2-2 for current directions and reference voltages. Let $V_{0}=1$ volt

$$
\begin{align*}
& I_{5}=s C_{5}+1  \tag{2.20}\\
& V_{2}=s I_{3} I_{5}+1=s^{2} L_{3} C_{5}+s L_{3}+1  \tag{2.21}\\
& I_{4}=s C_{4} V_{2}=s^{3} I_{3} C_{4} C_{5}+s^{2} L_{3} C_{4}+s C_{4}  \tag{2.22}\\
& I_{3}=I_{4}+I_{5}=s^{3} I_{3} C_{4} C_{5}+s^{2} I_{3} C_{4}+s\left(C_{4}+C_{5}\right)+1  \tag{2.23}\\
& V_{1}=V_{2}+I_{3}\left(\frac{s L_{2}}{s^{2} L_{2} C_{3}+I}\right) \tag{2.24}
\end{align*}
$$



Fig. 2-2. Current directions and reference voltages in the sixth order doubly terminated circuit.

$$
\begin{align*}
V_{1}= & {\left[s^{4}\left(I_{2} I_{3} C_{3} C_{5}+I_{2} I_{3} C_{4} C_{5}\right)+s^{3}\left(I_{2} I_{3} C_{3}+I_{2} I_{3} C_{4}\right)+s^{2}\left(I_{2} C_{3}+\right.\right.} \\
& \left.\left.I_{2} C_{4}+I_{2} C_{5}+I_{3} C_{5}\right)+s\left(I_{2}+I_{3}\right)+I\right] / s^{2} L_{2} C_{3}+1 \tag{2.25}
\end{align*}
$$

$$
\begin{align*}
I_{2}=s C_{2} V_{1}= & {\left[s^{5}\left(I_{2} I_{3} C_{2} C_{3} C_{5}+I_{2} I_{3} C_{2} C_{4} C_{5}\right)+s^{4}\left(I_{2} I_{3} C_{2} C_{3}+L_{2} I_{3} C_{2} C_{4}\right)+\right.} \\
& s^{3}\left(I_{2} C_{2} C_{3}+I_{2} C_{2} C_{4}+I_{2} C_{2} C_{5}+I_{3} C_{2} C_{5}\right)+ \\
& \left.s^{2}\left(I_{2} C_{2}+I_{3} C_{2}\right)+s C_{2}\right] / s^{2} L_{2} C_{3}+1  \tag{2.26}\\
I_{1}=I_{2}+I_{3}= & {\left[s^{5}\left(I_{2} I_{3} C_{2} C_{3} C_{5}+I_{2} I_{3} C_{2} C_{4} C_{5}+I_{2} I_{3} C_{3} C_{4} C_{5}\right)+\right.} \\
& s^{4}\left(I_{2} I_{3} C_{2} C_{3}+I_{2} I_{3} C_{2} C_{4}+I_{2} I_{3} C_{3} C_{4}\right)+ \\
& s^{3}\left(I_{2} C_{2} C_{3}+I_{2} C_{2} C_{4}+L_{2} C_{2} C_{5}+I_{2} C_{3} C_{4}+I_{2} C_{3} C_{5}+L_{3} C_{2} C_{5}+\right. \\
& \left.I_{3} C_{4} C_{5}\right)+s^{2}\left(I_{2} C_{2}+I_{2} C_{3}+I_{3} C_{2}+I_{3} C_{4}\right)+ \\
& \left.s\left(C_{2}+C_{4}+C_{5}\right)+1\right] / s^{2} L_{2} C_{3}+1 \tag{2.27}
\end{align*}
$$

$$
\begin{equation*}
V_{i}=\left(s L_{1}+1\right) I_{1}+V_{1} \tag{2.28}
\end{equation*}
$$

$$
\begin{align*}
& V_{i}=\left[s^{6}\left(I_{1} L_{2} I_{3} C_{2} C_{3} C_{5}+I_{1} I_{2} I_{3} C_{2} C_{4} C_{5}+I_{1} I_{2} I_{3} C_{3} C_{4} C_{5}\right)+\right. \\
& s^{5}\left(\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{3}+\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{4}+\mathrm{L}_{1} \mathrm{~L}_{2} \mathrm{I}_{3} \mathrm{C}_{3} \mathrm{C}_{4}+\right. \\
& \left.\mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{C}_{5}+\mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{4} \mathrm{C}_{5}+\mathrm{L}_{2} \mathrm{I}_{3} \mathrm{C}_{3} \mathrm{C}_{4} \mathrm{C}_{5}\right)+ \\
& { }_{5}{ }^{4}\left(\mathrm{~L}_{1} \mathrm{~L}_{2} \mathrm{C}_{2} \mathrm{C}_{3}+\mathrm{L}_{1} \mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{4}+\mathrm{L}_{1} \mathrm{~L}_{2} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{L}_{1} \mathrm{~L}_{2} \mathrm{C}_{3} \mathrm{C}_{4}+\mathrm{L}_{1} \mathrm{~L}_{2} \mathrm{C}_{3} \mathrm{C}_{5}+\right. \\
& \mathrm{L}_{1} \mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{L}_{1} \mathrm{I}_{3} \mathrm{C}_{4} \mathrm{C}_{5}+\mathrm{L}_{2} \mathrm{~L}_{3} \mathrm{C}_{2} \mathrm{C}_{3}+\mathrm{L}_{2} \mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{4}+\mathrm{L}_{2} \mathrm{I}_{3} \mathrm{C}_{3} \mathrm{C}_{4}+ \\
& \left.\mathrm{L}_{2} \mathrm{~L}_{3} \mathrm{C}_{3} \mathrm{C}_{5}+\mathrm{L}_{2} \mathrm{I}_{3} \mathrm{C}_{4} \mathrm{C}_{5}\right)+ \\
& \mathrm{s}^{3}\left(\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{2}+\mathrm{L}_{1} \mathrm{I}_{2} \mathrm{C}_{3}+\mathrm{L}_{1} \mathrm{I}_{3} \mathrm{C}_{2}+\mathrm{I}_{1} \mathrm{I}_{3} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{3}+\mathrm{L}_{2} \mathrm{I}_{3} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{3}+\right. \\
& \left.\mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{L}_{2} \mathrm{C}_{3} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{C}_{3} \mathrm{C}_{5}+\mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{L}_{3} \mathrm{C}_{4} \mathrm{C}_{5}\right)+ \\
& \mathrm{s}^{2}\left(\mathrm{I}_{1} \mathrm{C}_{2}+\mathrm{I}_{1} \mathrm{C}_{4}+\mathrm{L}_{1} \mathrm{C}_{5}+\mathrm{I}_{2} \mathrm{C}_{2}+\mathrm{I}_{2} \mathrm{C}_{4}+\mathrm{L}_{2} \mathrm{C}_{5}+\mathrm{I}_{3} \mathrm{C}_{2}+\mathrm{I}_{3} \mathrm{C}_{4}+\mathrm{I}_{3} \mathrm{C}_{5}+\right. \\
& \left.\left.2 \mathrm{~L}_{2} \mathrm{C}_{3}\right)+\mathrm{s}\left(\mathrm{~L}_{1}+\mathrm{L}_{2}+\mathrm{L}_{3}+\mathrm{C}_{2}+\mathrm{C}_{4}+\mathrm{C}_{5}\right)+2\right] / \mathrm{s}^{2} \mathrm{~L}_{2} \mathrm{C}_{3}+\mathrm{I} \tag{2.29}
\end{align*}
$$

It can be observed that equation (2.29) is the desired transfer function to the minus one. In other words, one would only need to exchange the numerator and denominator to obtain $\mathrm{V}_{\mathrm{o}} / \mathrm{V}_{\mathrm{i}}$. However, there is no need to do this because the seven coefficient equations in terms of the components of the circuit can be obtained directly from (2.29) and are shown next:

$$
\begin{equation*}
2 b=\left(I_{1} I_{2} I_{3} C_{2} C_{3} C_{5}+I_{1} I_{2} I_{3} C_{2} C_{4} C_{5}+I_{1} I_{2} I_{3} C_{3} C_{4} C_{5}\right) \tag{2.30}
\end{equation*}
$$

$$
\begin{align*}
& 2 c=\left(I_{1} I_{2} I_{3} C_{2} C_{3}+I_{1} I_{2} I_{3} C_{2} C_{4}+I_{1} I_{2} L_{3} C_{3} C_{4}+\right. \\
&\left.L_{2} I_{3} C_{2} C_{3} C_{5}+I_{2} I_{3} C_{2} C_{4} C_{5}+I_{2} I_{3} C_{3} C_{4} C_{5}\right) \tag{2.31}
\end{align*}
$$

$$
2 \mathrm{~d}=\left(\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{3}+\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{4}+\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{3} \mathrm{C}_{4}+\right.
$$

$$
\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{3} \mathrm{C}_{5}+\mathrm{L}_{1} \mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{L}_{1} \mathrm{~L}_{3} \mathrm{C}_{4} \mathrm{C}_{5}+\mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{3}+
$$

$$
\begin{equation*}
\left.\mathrm{L}_{2} \mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{3} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{3} \mathrm{C}_{5}+\mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{4} \mathrm{C}_{5}\right) \tag{2.32}
\end{equation*}
$$

$2 \mathrm{e}=\left(\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{2}+\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{3}+\mathrm{I}_{1} \mathrm{I}_{3} \mathrm{C}_{2}+\mathrm{I}_{1} \mathrm{I}_{3} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{3}+\right.$

$$
\begin{align*}
& \mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{3}+\mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{4}+\mathrm{L}_{2} \mathrm{C}_{2} \mathrm{C}_{5}+ \\
& \left.\mathrm{L}_{2} \mathrm{C}_{3} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{C}_{3} \mathrm{C}_{5}+\mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{I}_{3} \mathrm{C}_{4} \mathrm{C}_{5}\right) \tag{2.33}
\end{align*}
$$

(2.30) through (2.36) are the coefficients equations of the sixth order doubly terminated inverse Chebyshev filter.

## CHAPTER 3

## EXPIICIT ELEMENT VALUE EQUATIONS FOR THE SIXTH ORDER DOUBLY TERMINATED FILTER

The main objective in this chapter is to obtain a set of explicit element value equations in terms of the coefficients of the transfer function for the two zero-six pole doubly terminated inverse Chebyshev filter by making use of equations (2.30) through (2.36) developed in chapter two, and the relations (2.3) through (2.7) generated as a result of applying the MFM conditions to the sixth order magnitude squared transfer function.

## Manipulation of coefficient equations

The first approach to find a component solutionwill be to algebraically manipulate the above indicated set of equations in a brute force manner. Even though it does not look very promising it may help and save time later. Once again, only the most significant algebraic steps of this procedure will be shown.
Dividing (2.31) by (2.30) the next expression is
obtained:

$$
\begin{equation*}
\frac{c}{b}=\frac{1}{C_{5}}+\frac{1}{L_{1}} \tag{3.1}
\end{equation*}
$$

Rearranging (2.34)

$$
\begin{equation*}
2 f=\left(I_{1}+I_{2}+L_{3}\right)\left(C_{2}+C_{4}+C_{5}\right)+2 L_{2} C_{3} \tag{3.2}
\end{equation*}
$$

Now, rearranging (2.35)

$$
\begin{equation*}
\mathrm{C}_{2}+\mathrm{C}_{4}+\mathrm{C}_{5}=2 \mathrm{~g}-\left(\mathrm{I}_{1}+\mathrm{I}_{2}+\mathrm{I}_{3}\right) \tag{3.3}
\end{equation*}
$$

Substituting (3.3) and (2.36) into (3.2) and rearranging terms, the next expression is found:

$$
\begin{equation*}
\left(L_{1}+L_{2}+L_{3}\right)^{2}-2 g\left(L_{1}+L_{2}+L_{3}\right)+2 f-2 a=0 \tag{3.4}
\end{equation*}
$$

It can be seen that (3.5) is a quadratic equation in $\left(I_{1}+I_{2}+I_{3}\right)$ whose solution is given by:

$$
\begin{equation*}
\left(L_{1}+L_{2}+I_{3}\right)=g \pm \sqrt{\left(g^{2}-2 f+2 a\right)} \tag{3.5}
\end{equation*}
$$

By observing equation (2.7) it is obvious that the square root term in the above equation is equal to zero, then (3.5) simply becomes:

$$
\begin{equation*}
g=I_{1}+L_{2}+L_{3} \tag{3.6}
\end{equation*}
$$

Next, applying exactly the same procedure as before, but
this time solving for $\left(\mathrm{C}_{2}+\mathrm{C}_{4}+\mathrm{C}_{5}\right)$, the following relation is obtained:

$$
\begin{equation*}
\mathrm{g}=\mathrm{C}_{2}+\mathrm{C}_{4}+\mathrm{C}_{5} \tag{3.7}
\end{equation*}
$$

Now, substituting (3.6), (3.7) and (2.36) into (2.32) and regrouping terms the next expression is found:

$$
\begin{align*}
2 d= & a g\left(I_{1}+L_{3}\right)+I_{1} I_{2} C_{2} C_{4}+I_{1} I_{2} C_{2} C_{5}+I_{1} I_{3} C_{2} C_{5}+ \\
& I_{1} I_{3} C_{4} C_{5}+I_{2} I_{3} C_{2} C_{4}+I_{2} I_{3} C_{4} C_{5} \tag{3.8}
\end{align*}
$$

Again, substituting (3.6), (3.7) and (2.36) but now into (2.33) and regrouping terms, the following equation is generated:

$$
\begin{align*}
& 2 \mathrm{e}=\mathrm{ag}+\mathrm{a}\left(\mathrm{I}_{1}+\mathrm{I}_{3}\right)+\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{2}+\mathrm{I}_{1} \mathrm{I}_{3} \mathrm{C}_{2}+\mathrm{L}_{1} \mathrm{I}_{3} \mathrm{C}_{4}+ \\
& \mathrm{I}_{2} \mathrm{~L}_{3} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{I}_{3} \mathrm{C}_{4} \mathrm{C}_{5} \tag{3.9}
\end{align*}
$$

Rearranging (3.8)

$$
\begin{align*}
& \mathrm{L}_{2} \mathrm{C}_{2} \mathrm{C}_{4}+\mathrm{L}_{2} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{I}_{3} \mathrm{C}_{4} \mathrm{C}_{5}= \\
& \frac{I}{\mathrm{I}_{1}}\left[2 \mathrm{~d}-\mathrm{ag}\left(\mathrm{I}_{1}+\mathrm{I}_{3}\right)-\mathrm{L}_{2} \mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{4}-\mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{4} \mathrm{C}_{5}\right] \tag{3.10}
\end{align*}
$$

Now, reorganizing (3.9)

$$
\begin{align*}
& \mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{I}_{3} \mathrm{C}_{4} \mathrm{C}_{5}= \\
& 2 \mathrm{e}-\mathrm{ag}-\mathrm{a}\left(\mathrm{I}_{1}+\mathrm{I}_{3}\right)-\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{2}-\mathrm{I}_{1} \mathrm{I}_{3} \mathrm{C}_{2}-\mathrm{I}_{1} \mathrm{I}_{3} \mathrm{C}_{4}-\mathrm{I}_{2} \mathrm{C}_{3} \mathrm{C}_{4} \tag{3.11}
\end{align*}
$$

Equating the right hand sides of (3.10) and (3.11), equation (3.12) is formed.

$$
\begin{align*}
& 2 e I_{1}-a g I_{1}-a I_{1}\left(I_{1}+I_{3}\right)-I_{1}\left(I_{1} I_{2} C_{2}+I_{1} I_{3} C_{2}+I_{1} I_{3} C_{4}+I_{2} I_{3} C_{4}\right)= \\
& 2 d-a g I_{1}-a g I_{3}-I_{2} I_{3} C_{2} C_{4}-I_{2} I_{3} C_{4} C_{5} \tag{3.12}
\end{align*}
$$

Simplifying and rearranging terms in (3.12) one gets:

$$
\begin{align*}
& I_{1}\left(2 e-a I_{3}\right)-I_{1}^{2}\left(a+I_{2} C_{2}+I_{3} C_{2}+I_{3} C_{4}\right)+ \\
& I_{2} I_{3} C_{4}\left(C_{5}-I_{1}\right)+L_{2} I_{3} C_{2} C_{4}-2 d=0 \tag{3.13}
\end{align*}
$$

Now, from (2.31) one can see that:

$$
\begin{equation*}
\mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{4}=\frac{2 \mathrm{c}}{\mathrm{I}_{1}+\mathrm{C}_{5}}-a \mathrm{I}_{3}\left(\mathrm{C}_{2}+\mathrm{C}_{4}\right) \tag{3.14}
\end{equation*}
$$

Substituting (3.14) back into (3.13) the next equation is obtained:

$$
\begin{align*}
& 2 e I_{1}-a I_{1}^{2}-a I_{1} I_{3}-2 d-a g I_{3}-I_{1}^{2} I_{3}\left(C_{2}+C_{4}\right)-I_{1}^{2} I_{2} C_{2}+ \\
& I_{2} I_{3} C_{4}\left(C_{5}-I_{1}\right)+\frac{2 c}{I_{1}+C_{5}}-a I_{3}\left(C_{2}+C_{4}\right)=0 \tag{3.15}
\end{align*}
$$

Rearranging terms in this last expression one obtains:

$$
\begin{align*}
& 2 \mathrm{eI}_{1}-\mathrm{aI}_{1}^{2}-a \mathrm{I}_{1} \mathrm{I}_{3}-2 \mathrm{~d}+a g I_{3}-\mathrm{I}_{1}^{2} \mathrm{I}_{2} \mathrm{C}_{2}+\frac{2 \mathrm{c}}{\mathrm{I}_{1}+\mathrm{C}_{5}}+ \\
& \mathrm{L}_{2} \mathrm{I}_{3} C_{4}\left(\mathrm{C}_{5}-\mathrm{I}_{1}\right)-\left(\mathrm{C}_{2}+\mathrm{C}_{4}\right)\left(\mathrm{I}_{1}^{2} \mathrm{I}_{3}+a \mathrm{I}_{3}\right)=0 \tag{3.16}
\end{align*}
$$

Now, from equations (3.6) and (3.7) it can be seen that:

$$
\begin{align*}
& C_{2}+C_{4}=g-C_{5}  \tag{3.17}\\
& C_{4}=g-C_{2}-C_{5}  \tag{3.18}\\
& I_{2}=g-I_{1}-I_{3} \tag{3.19}
\end{align*}
$$

Substituting (3.17) through (3.19) inclusive, into (3.16) one gets:

$$
\begin{align*}
& 2 e L_{1}-a I_{1}^{2}-a I_{1} I_{3}+a g L_{3}-\operatorname{agI}_{3}-I_{1}^{2} C_{2}\left(g-I_{1}-L_{3}\right)-g I_{1} I_{3}+I_{1} L_{3} C_{5}+ \\
& a L_{3} C_{5}+L_{3}\left(g-I_{1}-I_{3}\right)\left(g-C_{2}-C_{5}\right)\left(C_{5}-L_{1}\right)-2 d+\frac{2 c}{L_{1}+C_{5}}=0 \tag{3.20}
\end{align*}
$$

Regrouping and cancelling common terms, (3.20) simplifies to:

$$
\begin{align*}
& \mathrm{I}_{1}^{3} \mathrm{C}_{2}+\mathrm{I}_{1}^{2}\left(\mathrm{~L}_{3} \mathrm{C}_{5}-\mathrm{gC}_{2}+\mathrm{L}_{3} C_{2}-g \mathrm{I}_{3}-a\right)+\mathrm{I}_{1}\left(2 \mathrm{e}-\mathrm{aL}_{3}\right)+a \mathrm{I}_{3} C_{5}+ \\
& \mathrm{I}_{3}\left(g-\mathrm{I}_{1}-\mathrm{I}_{3}\right)\left(g-C_{2}-C_{5}\right)\left(C_{5}-\mathrm{I}_{1}\right)-2 d+\frac{2 \mathrm{c}}{\mathrm{I}_{1}+C_{5}}=0 \tag{3.21}
\end{align*}
$$

It is still possible to solve for $C_{5}$ in terms of $L_{I}$ from (3.1) and substitute it back into (3.21) to eliminate $\mathrm{C}_{5}$. However, at this stage actually performing this step serves no purpose. The best result obtained is one equation in three unknowns ( $L_{1}, L_{3}$ and $C_{2}$ ) which is still very far from the desired result of explicitely solving for any reactive component of the circuit in terms of the coefficients of the transfer function.

To have been able to obtain equation (3.21) many hours of work were spent not only in the procedure developed in this section but also in other possible routes that were explored as well.

It is then clear that at this point one should try other alternatives to satisfactorily solve the problem. One of them is treated in the next section.

## Developing a pattern

As previously stated, the algebraic manipulation of coefficient equations did not produce satisfactory results for the sixth order doubly terminated filter. Therefore,
as an alternative one should look closer into the already existing solutions of this family of filters hoping to find a clue that can help to unravel this yet complicated problem.

It is at this stage convenient to mention that the solutions for the odd-ordered doubly terminated configurations are fairly easily found due to the obvious symmetry of the circuits, because by making the the symmetrical elements in the ladder equal (see figures 3-2 and 3-3) the complexity of the problem is greatly reduced.

It was found that for the fifth order filter (Garcia, 1981) the first series reactive component in the circuit namely $I_{1}$, is related to some of the transfer function coefficients in the following form:

$$
\begin{equation*}
I_{1}=I_{3}=\frac{2 b}{c} \tag{3.22}
\end{equation*}
$$

For the seventh order filter (Henry, 1983) the ladder network is structured a little differently (fig. 3-3). However, the first parallel reactive component which is $C_{1}$, is related to the transfer function coefficients in exactly the same way $I_{1}$ is for the fifth order; in other words

$$
\begin{equation*}
C_{1}=C_{5}=\frac{2 b}{c} \tag{3.23}
\end{equation*}
$$



Fig. 3-1. Two zero-four pole doubly terminated circuit.


Fig. 3-2. Two zero-five pole doubly terminated circuit.


Fig. 3-3. Two zero-seven pole doubly terminated circuit.

For the fourth order filter (Garcia, 1981) it was found that only the first series reactive element (not symmetric network) $L_{I}$ is equal to:

$$
\begin{equation*}
L_{1}=\frac{c}{d} \tag{3.24}
\end{equation*}
$$

Apparently there is no defined pattern in the relation of the first reactive component in the previously mentioned circuits to some of the coefficients. However, from equation (2.3) it can readily be seen that:

$$
\begin{equation*}
\frac{c}{d}=\frac{2 b}{c} \tag{3.25}
\end{equation*}
$$

Suddenly, this has become a very defined pattern where the first reactive parallel or series element in the circuit, despite the order of the filter, is equal to $2 b / c$ and certainly it will be worth trying.

If $I_{1}$ in the sixth order filter is made equal to $2 b / c$ then from expression (3.1) it can easily be seen that:

$$
\begin{equation*}
L_{1}=C_{5}=\frac{2 b}{c} \tag{3.26}
\end{equation*}
$$

This simple result seems to be very powerful. The first step in this new approach is to let $\mathrm{L}_{1}=\mathrm{C}_{5}$ and substitute it
back in equation (3.21) generating the following expression:

$$
\begin{equation*}
L_{1}^{2}\left(I_{1} C_{2}+I_{1} I_{3}+I_{3} C_{2}-g C_{2}-g L_{3}-a^{2}+\frac{2 e}{I_{1}}-\frac{2 d}{I_{1}^{2}}-\frac{c}{I_{1}^{3}}\right)=0 \tag{3.27}
\end{equation*}
$$

Even though (3.21) was greatly reduced in complexity, still there are two unknowns in a single equation $\left(L_{3}\right.$ and $\mathrm{C}_{2}$ ).

With certainty the most significant clue one can obtain from (3.26) is the fact that the first reactive series element ( $I_{1}$ ) is equal to the last reactive parallel component $\left(\mathrm{C}_{5}\right)$. This opens a possibility never considered in this type of two zero filter before. Recall that in the even-ordered Butterworth and Chebyshev filters (Weinberg, 1962) That is exactly the pattern the values of their elements follow. In other words, the first series reactive element in the ladder is equal in value to the last reactive parallel component; The first parallel reactive element is also equal in value to the last series reactive component in the ladder and so on. Applying this pattern to the sixth order filter the following component relations are forced.

$$
\begin{equation*}
\mathrm{I}_{1}=\mathrm{C}_{5} \quad \mathrm{I}_{2}=\mathrm{C}_{4} \quad \mathrm{I}_{3}=\mathrm{C}_{2} \tag{3.28}
\end{equation*}
$$

Now, by letting $\mathrm{C}_{2}=\mathrm{I}_{3}$ and $\mathrm{I}_{1}=2 \mathrm{~b} / \mathrm{c}$ in equation (3.27) the next expression is generated:

$$
\begin{equation*}
I_{3}^{2}+2 I_{3}\left(\frac{2 b}{c}-g\right)+\frac{c e}{b}+\frac{c^{4}}{8 b^{3}}-\frac{c^{2} d}{2 b^{2}}-a=0 \tag{3.29}
\end{equation*}
$$

This is a quadratic equation in $\mathrm{L}_{3}$ whose solution is given by:

$$
\begin{equation*}
L_{3}=g-\frac{2 b}{c}-\sqrt{\left(\frac{2 b}{c}-g\right)^{2}-\left(\frac{c^{4}}{8 b^{3}}+\frac{c e}{b}-\frac{c^{2} d}{2 b^{2}}-a\right)} \tag{3.30}
\end{equation*}
$$

One could be tempted to leave this expression the way it is. However, still further simplification can be achieved. But before any thing is done it should be noted that the radical would have to be equal to $\mathrm{I}_{2}$ to be consistent with equation (3.6). From (2.3) one can see that:

$$
\begin{equation*}
c^{2}=2 b d \tag{3.31}
\end{equation*}
$$

Substituting (3.31) back into the radical one gets:

$$
\begin{equation*}
I_{2}=\sqrt{\left(g-\frac{2 b}{c}\right)^{2}-\left(\frac{d^{2}}{2 b} \frac{c e}{b}-\frac{d^{2}}{b}-a\right)} \tag{3.32}
\end{equation*}
$$

Rearranging and simplifying common terms, the next relation is obtained:

$$
\begin{equation*}
I_{2}=\sqrt{\left(g-\frac{2 b}{c}\right)^{2}-\left(\frac{-d^{2}+2 c e}{2 b}-a\right)} \tag{3.33}
\end{equation*}
$$

From (2.4) one can see that:

$$
\begin{equation*}
2 b f=-d^{2}+2 c e \tag{3.34}
\end{equation*}
$$

Substituting this back into (3.33) the next expression is obtained:

$$
\begin{equation*}
I_{2}=\sqrt{\left(g-\frac{2 b}{c}\right)^{2}-f+a} \tag{3.35}
\end{equation*}
$$

Recall that:

$$
\begin{equation*}
L_{1}=C_{5}=\frac{2 b}{c}=\frac{c}{d} \tag{3.36}
\end{equation*}
$$

(3.35) can be written in terms of $L_{1}$

$$
\begin{equation*}
L_{2}=C_{4}=\sqrt{\left(g-I_{1}\right)^{2}-f+a} \tag{3.37}
\end{equation*}
$$

From (3.6) one can see that:

$$
\begin{equation*}
L_{3}=C_{2}=g-L_{1}-I_{2} \tag{3.38}
\end{equation*}
$$

And finally:

$$
\begin{equation*}
c_{3}=\frac{a}{L_{2}} \tag{3.39}
\end{equation*}
$$

The design equations (3.36) through (3.39) constitute an explicit solution to the two zero-six pole doubly terminated inverse Chebyshev filter.

## Verification of design equations

One way to verify the set of design equations just obtained in the previous section, would be to compute the coefficients of the transfer function for several different positions of $\omega_{z}$. Then determine the values of the components of the circuit using the design relations (3.36) through (3.39) and substitute these results in the original coefficient equations ( 2.30 through 2.36) and observe if the numerical values of the coefficients one started with are obtained.

The proposed procedure was tried for several different positions of $\omega_{\mathrm{z}}$ between 1.7 and 3.0 rps , which is the most usable range for the sixth order filter (Henry, 1983). It was found that the coefficient equations were not satisfied exactly and as a matter-of-fact the error increased as $\omega_{\mathrm{z}}$ was set closer to the -3 dB cutoff frequency. The remaining three coefficient relations were exact.

The results in table 3-1, using the extreme values of the usable range for $\omega_{z}$ illustrate this unexpected problem.

The obvious question now is, how these deviations in the coefficient equations affect the performance of the filter? Basically, the -3 dB cutoff frequency moves from one rps towards a lower value as $\omega_{z}$ is placed closer to the cutoff frequency. The effect on the frequency and magnitude of the peak return is negligible, at least in the usable range of $\omega_{z}$.

Figure $3-4$ clearly illustrates the $-3 d B$ point deviation as a function of $\omega_{z}$. The data corresponding to this plot is shown in table 3-2.

## Determining a correction factor

The exact cause that produces this deviation in the coefficient equations has not yet been determined. However, in the search for such an answer the attempt was made to see if a correction factor could be applied to one of the design equations to reduce the encountered deviation.

By observing the developing procedure of design equations (3.36) through (3.39), one can readily see that $L_{I}$ and $C_{5}$ are determined in a straishtforward manner and both are explicitly independent of the coefficient a. Recall that the error is a function of the position of the zero or the coefficient a. $L_{3}$ is found by a simple subtraction of

Table 3-1. Deviation in coefficient equations for $\omega_{z}$ equal
1.7 and 3.0 rps.

| $\omega_{\mathrm{z}}$ | Eq. | Exact result | Actual result | Error |
| :---: | :---: | :---: | :---: | :---: |
|  | $(2.30)$ | $2 \mathrm{~b}=1.307958478$. | $2 \mathrm{~b}=1.295672502$ | $0.94 \%$ |
| 1.7 | $(2.31)$ | $2 \mathrm{c}=5.289616488$ | $2 \mathrm{c}=5.239929817$ | $0.94 \%$ |
|  | $(2.32)$ | $2 \mathrm{~d}=10.69607448$ | $2 \mathrm{~d}=10.63714327$ | $0.55 \%$ |
|  | $(2.33)$ | $2 \mathrm{c}=13.76284227$ | $2 \mathrm{e}=13.74525881$ | $0.13 \%$ |
|  |  |  |  |  |
|  | $(2.30)$ | $2 \mathrm{~b}=1.777717777$ | $2 \mathrm{~b}=1.776391598$ | $0.078 \%$ |
|  | $(2.32)$ | $2 \mathrm{~d}=13.60564051$ | $2 \mathrm{~d}=13.59948978$ | $0.045 \%$ |
|  |  | $2 \mathrm{c}=16.89158031$ | $2 \mathrm{e}=16.88992459$ | $0.010 \%$ |

Table 3-2. Deviation in the $-3 d B$ cutoff frequency as a function of $\omega_{z}$.

| $\omega_{\mathrm{z}}$ | $-3 \mathrm{~dB}(\mathrm{rps})$ | Error |
| :---: | :--- | :--- |
| 1.7 | 0.9930 | $0.70 \%$ |
| 1.9 | 0.9959 | $0.41 \%$ |
| 2.1 | 0.9974 | $0.26 \%$ |
| 2.3 | 0.9983 | $0.17 \%$ |
| 2.5 | 0.9988 | $0.12 \%$ |
| 2.7 | 0.9992 | $0.08 \%$ |
| 2.9 | 0.9994 | $0.06 \%$ |
| 3.1 | 0.9995 | $0.05 \%$ |



Fig. 3-4. -3dB cutoff frequency deviation as a function of $\omega_{z}$.
$L_{1}$ plus $L_{2}$ from the coefficient g. Again, $L_{3}$ is not a good candidate to receive a correction factor. $C_{3}$ is selected at at the very end of the design procedure to set the zero; so it can not be altered. However, $\mathrm{I}_{2}$ is also directly involved in the setting of $\omega_{z}(3.39)$ and not only that but explicitly its equation (3.37) contains the coefficient a. Since the error increases with $\underline{a}$, a correction factor could be applied to (3.37) to reduce in an adequate amount the effect contributed by the presence of coefficient a in this relation. When a factor greater than one is introduced dividing $a, L_{2}$ decreases which is the right direction to go because $I_{3}$ will increase in value making its contribution overcome this drawback and still make the coefficient equations approach the correct result.

From table 3-1 the equation with greater error is (2.30). Therefore, a will be adjusted to satisfy this equation. The other coefficient relations are not expected to reduce its error as much as (2.30) will, but hopefully, enough so as to be close to an exact solution.

Naming the correction factor $K$, design equation
(3.37) becomes:

$$
\begin{equation*}
L_{2}=\sqrt{\left(g-I_{1}\right)^{2}-f+\frac{a}{K}} \tag{3.40}
\end{equation*}
$$

Remember, the other design equations remain unaltered in form.

The limits of $K$ in the usable range of the filter were found to be 1.0699 and 1.2575 for $\omega_{z}$ equal to 1.7 and 3.0 rps respectively.

It can be observed from table 3-3 that the error was reduced in all equations. Equation (2.31) was deleted from table 3-3 because has the same error than (2.30). Twelve more correction factors were computed to reduce the error very muy like those in table 3-3. This results are plotted in figure 3-5 and data tabulated in table 3-4.

To have an idea how much the -3 dB cutoff frequency is corrected, $\omega_{z}$ of l.7 rps will be used. With the elements without any correction the magnitude of the response at 1 rps is -9.184 dB when it should be -9.0309. Including the correction factor in the calculations of the components the magnitude of the response at 1 rps turned out to be -9.0253 dB . So the error is greatly reduced.

The complete frequency response for several different positions of $\omega_{z}$ are included in chapter 6 .

Table 3-3. Deviation in coefficient equations with

| $\omega_{z}$ | Eq. | Exact result | Actual result | Error |
| :---: | :---: | :---: | :---: | :---: |
|  | $(2.30)$ | $2 \mathrm{~b}=1.307958478$ | $2 \mathrm{~b}=1.307956092$ | $0.00018 \%$ |
| 1.7 | $(2.32)$ | $2 \mathrm{~d}=10.69607448$ | $2 \mathrm{~d}=10.67486538$ | $0.20000 \%$ |
|  | $(2.33)$ | $2 \mathrm{e}=13.76284227$ | $2 \mathrm{e}=13.75501603$ | $0.05700 \%$ |
|  |  |  |  |  |

```
Table 3-4. K as a function
of \(\omega_{z}\).
```

| $\omega_{z}$ (rps) | Correction <br> Factor (K) |
| :--- | :--- |

$1.7 \quad 1.2575$
$1.8 \quad 1.2228$
1.91 .1950
$2.0 \quad 1.1725$
2.11 .1538
2.21 .1382
$2.3 \quad 1.1250$
2.41 .1136
2.51 .1038
2.6 1.0952
$2.7 \quad 1.0877$
2.81 .0810
$2.9 \quad 1.0752$
$3.0 \quad 1.0699$
$3.1 \quad 1.0652$


Fig. 3-5. Correction Factor $K$ as a function of $\boldsymbol{\omega}_{\mathrm{z}}$.

## Determining the transfer functions coefficients

As stated in chapter 2, in order to determine the coefficients of the sixth order transfer function for different values of $\omega_{z}$ within the specified usable range, equations (2.18) and (2.19) had to be iterated because of their nonlinear characteristics. The computer program used to solve for the coefficients $c$ and $g$ is GOSPEL (General Optimization Software Package for ELectrical networks) optimization strategy OPT4 Newton-Raphson (Huelsman, 1968).

The values of $c$ and $g$ obtained from the above mentioned iterative procedure in all cases made equations (2.18) and (2.19) equal to a number no greater than $\pm 10^{-6}$, when both should have been equal to zero. The rest of the coefficients were easily found by using (2.12), (2.14) and (2.17). Substituting these results in the $\mathbb{M F N}$ equations the deviation from zero never was greater than $\pm 10^{-6}$. As an example, for $\omega_{z}$ equal 2 rps $a=0.25, b=0.75$, the GOSPEL program gave the following values for $c$ and $g$ :

$$
\begin{aligned}
& c=2.988299065 \\
& g=3.309576357
\end{aligned}
$$

then $d, e$ and $f$ were calculated to be:

$$
\begin{aligned}
& d=5.953287535 \\
& e=7.538481956 \\
& f=6.408563105
\end{aligned}
$$

Substituting these values in the NFN equations the next results are achieved:

$$
\begin{aligned}
& c^{2}-2 b d=-10^{-9} \\
& d^{2}-2 c e+2 b f=-3.2 \times 10^{-8} \\
& e^{2}-2 d f+2 c g-2 b=0.000000000 \\
& f^{2}-2 e g+2 d-a^{2}=6 \times 10^{-8} \\
& g^{2}-2 f+2 a=0.000000000
\end{aligned}
$$

Each coefficient (except $a$ and b) is plotted as a function of $\omega_{z}$ (figures 3-6 through 3-10). The component values are easily calculated from the design equations once the coefficients are known. Also the elements of the circuit with the correction factor included are plotted as a function of $\omega_{z}$ and shown in figures 3-11 through 3-14.

Fig. 3-6. Coefficient c as a function of $\boldsymbol{\omega}_{\mathrm{z}}$


Fig. 3-7. Coefficient $d$ as a function of $\omega_{z}$


Fig. 3-8. Coefficient $e$ as a function of $\omega_{z}$


Fig. 3-9. Coefficient $f$ as a function of $\omega_{z}$

Fig. 3-10. Coefficient $g$ as a function of $\omega_{z}$


Fig. 3-11. Component $L_{1}=C_{5}$ as a function of $\omega_{2}$


Fig. 3-12. Component $L_{2}=C_{4}$ as a function of $\omega_{z}$


Fig. 3-13. Component $I_{3}=C_{2}$ as a function of $\omega_{2}$


Fig. 3-14. Component $C_{3}$ as a function of $\omega_{z}$

## CHAPTER 4

## A SECOND SOLUTION FOR THE TWO ZERO-FOUR POLE DOUBLY TERMINATED FIITER

As previously stated, there is already a set of explicit element value equations (Garcia, 1981) for the two zero-four pole doubly terminated inverse Chebyshev filter. The method used by Mr. Garcia to solve for the components of the circuit was direct manipulation of the coefficient and MFN equations. From this algebraic manipulation a quartic equation in $\mathrm{I}_{1}$ was obtained, whose complicated solution gave birth to the first set of component relations for the mentioned filter. Recall that this method was applied to the sixth order circuit, but it was virtually impossible even to obtain a sixth order equation to work with much less a solution due to the complicated $2 z-6 p$ coefficient expressions. Therefore, to get satisfactory results other alternatives had to be tried.

Next, Garcia's equations are shown (see figure 3-1 for components position in the ladder).

$$
\begin{equation*}
I_{1}=\frac{c}{d} \tag{4.1}
\end{equation*}
$$

$$
\begin{equation*}
L_{2}=e-L_{1}=e-\frac{c}{d} \tag{4.2}
\end{equation*}
$$

$$
\begin{equation*}
C_{2}=2-C_{4}=\frac{d e-c}{d-a} \tag{4.3}
\end{equation*}
$$

$$
\begin{equation*}
C_{4}=\frac{c-a e}{d-a} \tag{4.4}
\end{equation*}
$$

$$
\begin{equation*}
C_{3}=\frac{a}{I_{2}} \tag{4.5}
\end{equation*}
$$

## A second solution for the fourth order filter

Once an approximate solution for the two zero-six pole doubly terminated circuit was found by applying a method not ever tried before in this particular even-ordered configuration, it would be interesting to see how the fourth order doubly terminated filter behaves when the method of equating inductor and capacitor values is applied to it.

Using linearity in the circuit of figure 3-1 the following set of coefficient equations are obtained:

$$
\begin{align*}
& 2 \mathrm{~b}=\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{4}+\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{3}+\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{3} \mathrm{C}_{4}  \tag{4.6}\\
& 2 \mathrm{c}=\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{2}+\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{C}_{3}+\mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{C}_{2} \mathrm{C}_{3}+\mathrm{I}_{2} \mathrm{C}_{3} \mathrm{C}_{4}  \tag{4.7}\\
& 2 \mathrm{~d}=\mathrm{I}_{1} C_{2}+\mathrm{I}_{1} \mathrm{C}_{4}+\mathrm{I}_{2} \mathrm{C}_{2}+\mathrm{I}_{2} \mathrm{C}_{4}+2 \mathrm{I}_{2} \mathrm{C}_{3}  \tag{4.8}\\
& 2 \mathrm{e}=\mathrm{I}_{1}+\mathrm{I}_{2}+\mathrm{C}_{2}+\mathrm{C}_{4}  \tag{4.9}\\
& \mathrm{a}=\mathrm{I}_{2} C_{3} \tag{4.10}
\end{align*}
$$

Rearranging (4.9)

$$
\begin{equation*}
2 \mathrm{e}-\left(\mathrm{I}_{1}+\mathrm{I}_{2}\right)=\mathrm{C}_{2}+\mathrm{C}_{4} \tag{4.11}
\end{equation*}
$$

Substituting (4.10) into (4.8) and rearranging terms one gets:

$$
\begin{equation*}
2 \mathrm{~d}-2 \mathrm{a}=\left(\mathrm{I}_{1}+\mathrm{I}_{2}\right)\left(\mathrm{C}_{2}+\mathrm{C}_{4}\right) \tag{4.12}
\end{equation*}
$$

Substituting (4.11) into (4.12) one obtains:

$$
\begin{equation*}
\left(L_{1}+L_{2}\right)^{2}-2 e\left(L_{1}+L_{2}\right)+2 \mathrm{~d}-2 \mathrm{a}=0 \tag{4.13}
\end{equation*}
$$

It can be seen that (4.13) is a quadratic equation in $\left(L_{1}+I_{2}\right)$ whose solution is:

$$
\begin{equation*}
\left(L_{1}+L_{2}\right)=e \pm \sqrt{e^{2}-2 d+2 a} \tag{4.14}
\end{equation*}
$$

The following equation (Henry, 1983) comes from the MFM requirements:

$$
\begin{equation*}
e^{2}-2 d+2 a=0 \tag{4.15}
\end{equation*}
$$

Therefore, it can readily be seen that (4.14) reduces to:

$$
\begin{equation*}
e=I_{1}+I_{2} \tag{4.16}
\end{equation*}
$$

Using the same procedure as above but now for $\left(\mathrm{C}_{2}+\mathrm{C}_{4}\right)$ the next expression is obtained:

$$
\begin{equation*}
e=C_{2}+C_{4} \tag{4.17}
\end{equation*}
$$

Applying the pattern proposed in chapter 3 to the circuit in figure 3-1, the following relation between elements is forced:

$$
\mathrm{I}_{1}=\mathrm{C}_{4} \text { and } \mathrm{L}_{2}=\mathrm{C}_{2}
$$

Then, equation (4.6) becomes:

$$
\begin{equation*}
2 b=\mathrm{I}_{1}^{2}\left(\mathrm{I}_{2}^{2+a}\right)+\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{a} \tag{4.18}
\end{equation*}
$$

Rearranging terms

$$
\begin{equation*}
I_{1}\left(I_{2}^{2}+a\right)=\frac{2 b}{I_{1}}-I_{2} a \tag{4.19}
\end{equation*}
$$

Substituting (4.6) into (4.7) letting $\mathrm{I}_{1}=\mathrm{C}_{4}$ and $\mathrm{I}_{2}=\mathrm{C}_{2}$, and rearranging terms, the next expression is found:

$$
\begin{equation*}
I_{1}\left(I_{2}^{2}+a\right)=2 c-\frac{2 b}{I_{1}} \tag{4.20}
\end{equation*}
$$

Equating the right hand sides of $(4,19)$ and (4.20) and reorganizing terms the following equation is obtained:

$$
\begin{equation*}
2 \mathrm{cI}_{1}+\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{a}-4 \mathrm{~b}=0 \tag{4.21}
\end{equation*}
$$

From (4.16) one can see that:

$$
\begin{equation*}
I_{2}=e-I_{1} \tag{4.22}
\end{equation*}
$$

Substituting (4.22) back into (4.21) and rearranging terms one gets:

$$
\begin{equation*}
a L_{1}^{2}-(2 c+a e) L_{1}+4 b=0 \tag{4.23}
\end{equation*}
$$

This is a cuadratic equation in $L_{1}$ whose solution is given by:

$$
\begin{equation*}
L_{1}=\frac{2 c+a e-\sqrt{(2 c+a e)^{2}-16 a b}}{2 a} \tag{4.24}
\end{equation*}
$$

Which can be also put in the following final form:

$$
\begin{equation*}
L_{1}=C_{4}=\frac{c}{a}+\frac{e}{2}-\sqrt{\left(\frac{c}{a}+\frac{e}{2}\right)^{2}-\frac{4 b}{a}} \tag{4.25}
\end{equation*}
$$

From (4.22)

$$
\begin{equation*}
I_{2}=C_{2}=e-I_{1} \tag{4.26}
\end{equation*}
$$

Finally

$$
\begin{equation*}
\mathrm{C}_{3}=\frac{\mathrm{a}}{\mathrm{I}_{2}} \tag{4.27}
\end{equation*}
$$

Equations (4.25), (4.26) and (4.27) constitute a second solution (approximate) to the doubly terminated fourth order filter.

## Verification of design equations

Applying the same method used to verify the design equations for the sixth order filter to the ( $2 z-4 p$ ) configuration one could determine the accuracy of the design equations and at the same time find out if there is a need for a correction factor to be introduced. Again, the most usable range for $\omega_{z}$ (Henry, 1983) for the fouth order filter was determined to be from 2.3 to about 5 rps. For this verification $\omega_{z}$ will be chosen a little below the lower limit and another one somewhere in the middle of the given range, therefore two convenient values for $\omega_{z}$ would be 2 and 3 rps .

Starting with 2 rps the coefficient a can be found from (2.11)

$$
a=\frac{1}{\omega_{z}^{2}}=\frac{1}{4}=0.25
$$

making use of (2.9) b is found to be

$$
b=1-a=0.75
$$

In order to have an MFM response the next set of relations (Henry, 1983) must be satisfied.

$$
\begin{align*}
& e^{2}-2 d=-2 a  \tag{4.28}\\
& d^{2}-2 c e+2 b=a^{2}  \tag{4.29}\\
& c^{2}-2 b d=0 \tag{4.30}
\end{align*}
$$

Expression (4.31) (Kerwin, 1981) is obtained as a result of manipulating these relations.

$$
\begin{equation*}
e^{4}+4 a e^{2}-8 e \sqrt{2(1-a)\left(\frac{e^{2}}{2}+a\right)}+8(1-a)=0 \tag{4.31}
\end{equation*}
$$

Using a hand-held programmable calculator (4.31) was iterated to give a value for e equal to 2.253900266. The rest of the coefficients are very easily determined from (4.28) and (4.30). Results for $\omega_{z}=2$ rps are summarized next:

$$
\begin{aligned}
& a=0.25 \\
& b=0.75 \\
& c=2.045739428 \\
& d=2.790033205 \\
& e=2.253900266
\end{aligned}
$$

Now, making use of (4.25), (4.26) and (4.27) the values of the components are found.

$$
\begin{aligned}
& \mathrm{I}_{1}=\mathrm{C}_{4}=0.668473700 \quad(\mathrm{H}, \mathrm{~F}) \\
& \mathrm{L}_{2}=\mathrm{C}_{2}=1.585426566 \quad(\mathrm{H}, \mathrm{~F}) \\
& \mathrm{C}_{3}=0.157686269 \quad(\mathrm{~F})
\end{aligned}
$$

Substituting these element values in equations (4.6) through (4.10) and dividing by two, the next results are obtained:

$$
\begin{aligned}
& b=0.749939069 \\
& c=2.045557128 \\
& d=2.790033205
\end{aligned}
$$

$$
\begin{aligned}
& e=2.253900266 \\
& a=0.25
\end{aligned}
$$

It can be observed by comparing results that only coefficients $b$ and $c$ show a very small numerical error, so small that probably there is no need for a correction factor to be introduced. In the next chapter the response of this filter will be tested.

For the sake of completeness the values of the components using Garcia's relations are shown below ( $\omega_{z}=2 \mathrm{rps}$ ).

$$
\begin{aligned}
& I_{1}=0.733231212 \mathrm{H} \\
& I_{2}=1.520669054 \mathrm{H} \\
& \mathrm{C}_{2}=1.670339249 \mathrm{~F} \\
& \mathrm{C}_{4}=0.583561018 \mathrm{~F} \\
& \mathrm{C}_{3}=0.164401320 \mathrm{~F}
\end{aligned}
$$

It should also be mentioned that when these results are substituted back in the coefficient equations, practically no numerical error is found in any of the coefficients.

For $\omega_{\mathrm{z}}=3$ rps as expected, the difference between the exact and the actual values was even smaller than the previous one as can be seen below:

| Exact result | Actual result | Error |
| :--- | :--- | :---: |
| $2 b=1.777777777$ | $2 b=1.777773004$ | $0.00027 \%$ |
| $2 c=4.726918605$ | $2 c=4.726905420$ | $0.00027 \%$ |

## CHAPTER 5

## SINGLY TERNINATED TWO ZERO-SIX POLE <br> INVERSE CHEBYSHEV FILTER

Another basic circuit in passive LC filters is the singly terminated case, and although it is much more sensitive than the doubly terminated one, it is still widely used. In the singly terminated configuration $R_{0}=\infty$ (figure 5-1). For the sake of completeness the design equations in terms of the coefficients of the transfer function (coefficients are the same in both, singly and doubly terminated cases) are presented in this chapter.

## A solution for the singly terminated case

Applying linearity to the circuit of figure 5-1, the following coefficient equations are obtained:

$$
\begin{equation*}
\mathrm{b}=\mathrm{I}_{1} \mathrm{I}_{2} \mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{4} \mathrm{C}_{5}+\mathrm{I}_{1} I_{2} I_{3} C_{2} \mathrm{C}_{3} \mathrm{C}_{5}+\mathrm{I}_{1} I_{2} I_{3} \mathrm{C}_{3} \mathrm{C}_{4} \mathrm{C}_{5} \tag{5.1}
\end{equation*}
$$

$c=I_{2} I_{3} C_{2} C_{4} C_{5}+I_{2} I_{3} C_{2} C_{3} C_{5}+I_{2} I_{3} C_{3} C_{4} C_{5}$

$$
\begin{align*}
& d=I_{1} I_{2} C_{2} C_{3}+L_{1} I_{2} C_{2} C_{4}+L_{1} I_{2} C_{2} C_{5}+L_{1} I_{2} C_{3} C_{4}+I_{1} I_{2} C_{3} C_{5}+ \\
& \mathrm{I}_{1} \mathrm{I}_{3} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{I}_{1} \mathrm{~L}_{3} \mathrm{C}_{4} \mathrm{C}_{5}+\mathrm{L}_{2} \mathrm{I}_{3} \mathrm{C}_{3} \mathrm{C}_{5}+\mathrm{L}_{2} \mathrm{I}_{3} \mathrm{C}_{4} \mathrm{C}_{5}  \tag{5.3}\\
& e=I_{2} C_{2} C_{4}+L_{2} C_{2} C_{3}+L_{2} C_{2} C_{5}+L_{2} C_{3} C_{4}+\mathrm{I}_{2} \mathrm{C}_{3} \mathrm{C}_{5}+\mathrm{L}_{3} \mathrm{C}_{2} \mathrm{C}_{5}+\mathrm{I}_{3} \mathrm{C}_{4} \mathrm{C}_{5} \tag{5.4}
\end{align*}
$$

$$
\begin{equation*}
I=I_{1} C_{2}+I_{1} C_{4}+I_{1} C_{5}+I_{2} C_{4}+I_{2} C_{5}+I_{3} C_{5}+I_{2} C_{3} \tag{5.5}
\end{equation*}
$$

$g=C_{2}+C_{4}+C_{5}$
$a=L_{2} C_{3}$

Showing the algebraic steps of this procedure serves no purpose therefore, only the results (design equations) are presented in this section.

$$
\begin{equation*}
L_{1}=\frac{b}{c} \tag{5.8}
\end{equation*}
$$

$$
\begin{equation*}
C_{2}=\frac{g a^{2}-a e+c}{a^{2}+\left(I_{1} g-f\right) a+d-e L_{1}} \tag{5.9}
\end{equation*}
$$



$$
\begin{equation*}
c_{5}=g-c_{2}-c_{4} \tag{5.13}
\end{equation*}
$$

$$
\begin{equation*}
L_{3}=\frac{d-L_{1} e}{C_{5}\left(L_{2} C_{4}+a\right)} \tag{5.14}
\end{equation*}
$$

(5.12)

The design equations (5.8) through (5.14) constitute an explicit solution to the sixth order singly terminated inverse Chebyshev filter.

## Verification of design equations

The same procedure used in previous chapters for verifying the design equations was followed for the singly terminated circuit. $\omega_{z}$ was chosen to be 2 rps . When the element values were substituted back in the coefficient equations the results were exact, proving that the design equations are not in this case, an approximation. The response of this filter is shown in chapter 6.

Fig. 5-1 Two zero-six pole singly terminated circuit.

## CHAPTER 6

## RESULTS AND CONCLUSIONS

Using a hand-held programmable calculator with circuit analysis capability, the frequency response of the doubly terminated sixth order filter was obtained for three different values of $\omega_{z}(1.7 .2 .4$ and 3.0 rps$)$. The design equations used included the correction factor $K$. Figures 6-1, 6-2 and 6-3 show the graphs and tables 6-1, 6-2 and 6-3 the plotted data. Also a frequency response for the ( $2 z-6 p$ ) singly terminated filter is included in figure 6-4 and data in table 6-4.

For the fourth order doubly terminated filter a frequency response (figure 6-5) is obtained using the design equations developed in chapter 4 and compared to the frequency response obtained from the exact design equations (table 6-5).

Table 6-1. Normalized frequency response for $\omega_{1}=1.7$ rps doubly terminated

| $\omega$ (rps) | $\mathrm{V}_{0} / \mathrm{V}_{\mathrm{i}}(\mathrm{dB})$ |
| :---: | :---: |
| 0.2 | - 6.0206 |
| 0.4 | - 6.0224 |
| 0.6 | - 6.0404 |
| 0.8 | - 6.2741 |
| 0.9 | - 6.9773 |
| 1.0 | - 9.0253 |
| 1.2 | - 18.0100 |
| 1.4 | - 29.6622 |
| 1.6 | - 45.6309 |
| 1.7 | -220.0000 |
| 1.8 | - 51.2772 |
| 2.0 | - 46.7555 |
| 2.0821 | - 46.5638 |
| 2.2 | - 46.8358 |
| 2.4 | - 48.0178 |
| 2.6 | - 49.5953 |
| 2.8 | - 51.3210 |
| 3.0 | - 53.0887 |
| 3.2 | - 54.8476 |
| 3.6 | - 58.25116 |
| 4.0 | - 61.4528 |



Fig. 6-1. ( $2 z-6 p$ ) doubly terminated frequency response for $\omega_{z}=1.7 \mathrm{rps}$.

Table 6-2. Normalized frequency response for $\omega_{z}=2.4 \mathrm{rps}$ doubly terminated. ${ }^{2}$

| $\omega_{\mathrm{z}}=2.4 \mathrm{rps}$ |  |
| :--- | :--- |
| $\omega(\mathrm{rps})$ | $\mathrm{V}_{0} / \mathrm{v}_{\mathrm{i}}(\mathrm{dB})$ |
| 0.2 | -6.0206 |
| 0.4 | -6.0210 |
| 0.6 | -6.0313 |
| 0.8 | -6.2808 |
| 0.9 | -7.0360 |
| 1.0 | -9.0277 |
| 1.2 | -16.7305 |
| 1.4 | -25.5470 |
| 1.6 | -42.9627 |
| 1.8 | -50.7840 |
| 2.0 | -61.3869 |
| 2.2 | -627.1000 |
| 2.4 | -66.8713 |
| 2.8 | -66.57671 |
| 2.9393 | -66.6174 |
| 3.0 | -67.1663 |
| 3.2 | -68.0828 |
| 3.4 | -69.18352 |
| 3.6 | -70.3775 |
| 3.8 | -71.6150 |
| 4.0 | -74.1173 |
| 4.4 |  |



Fig. 6-2. ( $2 z-6 p$ ) doubly terminated frequency response for $\omega_{z}=2.4 \mathrm{rps}$.

Table 6-3. Normalized frequency response for $\omega_{z}=3.0 \mathrm{rps}$ doubly terminated.

| $\omega_{\mathrm{z}}=3.0 \mathrm{rps}$ |  |
| :--- | :--- |
| $\omega(\mathrm{rps})$ | $\mathrm{v}_{0} / \mathrm{V}_{\mathrm{i}}(\mathrm{dB})$ |
| 0.2 | -6.0206 |
| 0.4 | -6.0208 |
| 0.6 | -6.0300 |
| 0.8 | -7.28933 |
| 0.9 | -9.05958 |
| 1.0 | -16.42135 |
| 1.2 | -24.71991 |
| 1.4 | -32.40586 |
| 1.6 | -52.79105 |
| 1.8 | -66.87447 |
| 2.2 | -76.45254 |
| 2.6 | -232.40000 |
| 2.8 | -82.83239 |
| 3.0 | -79.69551 |
| 3.2 | -78.88523 |
| 3.4 | -78.83861 |
| 3.6 | -78.94488 |
| 3.6742 | -79.42805 |
| 3.8 | -80.14243 |
| 4.0 | -81.91503 |
| 4.2 | -83.87672 |
| 4.6 | -86.87580 |
| 5.0 |  |
| 5.6 |  |



Fig. 6-3. ( $2 z-6 p$ ) doubly terminated frequency response for $\omega_{z}=3.0 \mathrm{rps}$.

Table 6-4. Normalized frequency response for $\omega_{z}=2.0 \mathrm{rps}$ singly terminated. ${ }^{\text {Z }}$

| $\omega_{z}=2.0 \mathrm{rps}$ |  |
| :---: | :---: |
| $\omega(r p s)$ | $\mathrm{V}_{0} / \mathrm{v}_{\mathrm{i}}(\mathrm{dB})$ |
| $2.0 \times 10^{-1}$ | 0.00 |
| 4.0 " | 0.00 |
| 6.0 " | - 0.0064 |
| 8.0 " | - 0.23163 |
| 10.0 " | - 3.01030 |
| 12.0 " | - 11.2203 |
| 14.0 " | - 20.9205 |
| 16.0 " | 30.8731 |
| 18.0 " | - 42.5591 |
| 20.0 | -220.0000 |
| 22.0 " | - 52.14759 |
| 24.0 | - 50.257562 |
| 24.4 | - 50.212533 |
| 24.5 | - 50.210946 |
| 24.6 | - 50.212825 |
| 26.0 | - 50.52108 |
| 28.0 | - 51.5148 |
| 30.0 " | - 52.8176 |
| 32.0 " | - 54.2567 |
| 34.0 | - 55.7495 |
| 36.0 " | - 57.25257 |
| 38.0 " | - 58.74245 |
| 40.0 " | - 60.2060 |



Fig. 6-4. ( $2 \mathrm{z}-6 \mathrm{p}$ ) singly terminated frequency response for $\omega_{\mathrm{z}}=2.0 \mathrm{rps}$.

Table 6-5. (2z-4p) doubly terminated exact and approximate frequency response.

|  | $\omega_{z}=2.0 \mathrm{rps}$ | $\mathrm{V}_{0} / \mathrm{V}_{\mathrm{i}}(\mathrm{dB})$ |
| :--- | :--- | :--- |
| $\omega($ rps $)$ | Exact response | Approximate |
| 0.0 | -6.0206 | -6.0206 |
| 0.4 | -6.0223 | -6.0224 |
| 0.8 | -6.5658 | -6.5667 |
| 1.0 | -9.0309 | -9.0317 |
| 1.2 | -14.4122 | -14.4119 |
| 1.6 | -28.7486 | -28.7475 |
| 2.0 | -220.0000 | -201.6000 |
| 2.4 | -41.0710 | -41.0699 |
| 2.8 | -39.6509 | -39.6499 |
| 2.8284 | -39.6473 | -39.6463 |
| 3.2 | -40.0730 | -40.0721 |
| 3.6 | -41.0224 | -41.0215 |
| 4.0 | -42.1453 | -42.1444 |



Fig. 6-5. ( $2 \mathrm{z}-4 \mathrm{p}$ ) doubly terminated frequency response for $\omega_{\mathrm{z}}=2.0 \mathrm{rps}$.

## Practical circuit built and tested

A ( $2 \mathrm{z}-6 \mathrm{p}$ ) doubly terminated filter was designed and built using the corrected equations in chapter $3 . f_{z}$ was located at 4000 Hz and the -3 dB cutoff frequency was at 2000 Hz . The filter was impedance scaled to 1000 ohms.

The calculated components (already frequency and impedance scaled) are listed next.

$$
\begin{aligned}
& \mathrm{I}_{1}=39.945 \mathrm{mH} \\
& \mathrm{C}_{5}=39.945 \mathrm{nF} \\
& \mathrm{~L}_{2}=134.352 \mathrm{mH} \\
& \mathrm{C}_{4}=134.352 \mathrm{nF} \\
& \mathrm{I}_{3}=104.986 \mathrm{~m} \\
& \mathrm{C}_{2}=104.986 \mathrm{nF} \\
& \mathrm{C}_{3}=11.784 \mathrm{nF}
\end{aligned}
$$

The components used were measured in the HP 4262A LCR meter $U$ of $A$ ID No. 7148, at a frequency of 1 KHz .
$\mathrm{I}_{1}=39.4 \mathrm{mH}$
$L_{2}=133.5 \mathrm{mH}$
$L_{3}=103 \mathrm{mH}$
$Q_{1}=76.9$
$Q_{2}=100$
$Q_{3}=100$
$\mathrm{C}_{5}=39.9 \mathrm{nF}$
$C_{4}=133.6 \mathrm{nF}$
$\mathrm{C}_{2}=103 \mathrm{nF}$
$\mathrm{C}_{3}=11.8 \mathrm{nF}$
$R i=1005 \Omega$
$R o=1008 \Omega$

Equipment used:
FLUKE 8810A Digital Multimeter U of A ID No. 6790.
FLUKE 7260A Universal Counter/Timer U of A ID No. 6789.
H.P. 339A Distortion Analyser U of A ID No. 7913.
H.P. 1220A Oscilloscope U of A ID No. 6076.

The circuit was fed through a unity gain noninverting opamp (IMT41) so the internal resistance of the oscillator did not affect the performance of the circuit. The input voltage $V_{i}$ (opamp output) was kept at all times at 0.5 V RMS.

The results are tabulated in table 6-6 and the measure response is shown in figure 6-6.

Table 6-6. ( $2 z-6 p$ ) doubly terminated measured response.

| $V_{i}=0.5 \mathrm{~V} . R M S$ | $f_{z}=4000 \mathrm{~Hz}$ |
| :---: | :--- |
| $f(\mathrm{~Hz})$ | $\mathrm{V}_{0} / \mathrm{V}_{\mathrm{i}}(\mathrm{dB})$ |
| 20 | -6.0580 |
| 500 | -6.0918 |
| 1000 | -6.1491 |
| 1600 | -6.5485 |
| 1900 | -8.0991 |
| 1990 | -9.11216 |
| 2000 | -9.2436 |
| 2800 | -26.1427 |
| 3400 | -41.0115 |
| 3800 | -53.5568 |
| 3900 | -57.7215 |
| 4000 | -61.5142 |
| 4100 | -60.3544 |
| 4700 | -54.0665 |
| 4900 | -53.8074 |
| 5200 | -53.9801 |
| 5600 | -54.7042 |
| 6000 | -55.4940 |


Fig. 6-6. ( $2 z-6 p$ ) doubly terminated measured response.

## Conclusions

Since the circuit is of the doubly terminated type it is extremely insensitive to variations in the components (Orchard, 1979). It is this property that made the approximate solution possible.

Even though the solution found for the sixth order doubly terminated filter is not exact, the calculated frequency response and the measured results showed that it is a good approximation.

Another way one can deal with the cutoff frequency deviation problem is to simply frequency scale the circuit to break at a desired frequency using the data of table 3-2 to correct the -3 dB frequency.

The results for the $2 z-4 p$ doubly terminated circuit using the developed desigh equations in chapter 4 were surprisingly good. There was no need for a correction factor to be introduced since the response of the filter with $\omega_{z}=r p s$ was almost free of error, and even that error decreases as $\omega_{z}$ is increased.

The constructed circuit responded very well in the pass band. The -3 dB cutoff frequency was off by about 10 Hz . which is less than $1 \%$ error. However, in the stop band the response did not go beyond -60 dB probably due to some noise at the output of the circuit near the zero frequency.

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