Evaluation of Spectral vs Energy Efficiency Tradeoff Considering Transmission Reliability in Cellular Networks

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Abstract
Spectral efficiency (SE), energy efficiency (EE), and transmission reliability are basic parameters to measure the performance of a cellular network. In this paper, spectral efficiency and energy efficiency tradeoff is considered keeping in mind the transmission reliability, where all the three are function of signal to noise ratio (SNR). SNR, in turn is a function of constellation size (or the number of bits per symbol) and data rate. Then, we propose a new power model which is as function of this SNR. Based on the power model, SE-EE trade-off function is evaluated taking transmission reliability in to consideration. Results confirmed that increasing constellation size results an increase in SNR and leads to a significant increase in energy efficiency without changing the transmit power. To demonstrate the validity of our analysis, channel gain and constellation size are varied keeping transmit power constant. The results also indicate that securing transmission reliability, the EE-SE trade-off is optimized by increasing the constellation size.

Keywords: Spectral efficiency; Energy efficiency; Transmission reliability; Tradeoff; cellular network.

Introduction
In communication networks, energy efficiency (EE), spectral efficiency (SE) and transmission reliability are the main metrics that researchers are focusing on to optimize the network performance. SE is defined as the number of bits transmitted within a given bandwidth (in bits/sec/Hz), while EE is the number of bits to be transmitted per unit of energy consumption (in bits/Joule/Hz) [1, 2]. Transmission reliability of a communication link can be described in terms of bit error rate (BER) (that is 1 minus bit error rate) [3]. SE alone, as a metric, indicates how efficiently a limited frequency spectrum is used but fails to provide any insight on how efficiently the energy is consumed [3]. The more energy-efficient communication system, the less energy required to achieve the same task. On the other hand, the more bandwidth- (spectral-) efficient communication system, the more bits per second it can transfer through the same channel.
Maximizing the EE, or equivalently minimizing the consumed energy, while maximizing the SE are conflicting objectives. The concept of EE-SE trade-off has first been introduced for power limited system and accurately defined for the low-power (LP)/ low-SE regime in [1]. Shannon’s groundbreaking work on reliable communication over noisy channels showed that there is a fundamental trade-off between SE and EE. This capacity theorem illustrates that there exists a trade-off between bandwidth, $W$, transmit power, $P_t$, and the coding strategy implemented to achieve a transmission rate, $R$. [1]

Although optimization of EE and SE oriented designs can save the energy and bandwidth respectively, the transmission rate cannot be guaranteed. Indeed, it is of great necessity to balance EE and SE in future wireless systems since both of these two utilities are very important and deserve considerations of cellular network operators [4, 5].

Most of the research works for the optimization of SE-EE tradeoff focuses on the optimization of SE, EE, or both [2, 3, 4, 6, 7]. In order to optimize both energy and spectral efficiency, a unified metric is developed in [2] which can optimize both EE and SE simultaneously. It uses both multi-object optimization (MOO) problem and single object optimization (SOO) problem to find a Pareto optimal point where both EE and SE can be improved simultaneously. A detailed overview and direction for future research initiatives targeted to improve the EE of wireless systems are also provided in [4]. For Orthogonal Frequency Division Multiple Access (OFDMA) based networks, a low complexity algorithm that balances the SE-EE tradeoff is proposed in [6] and validated in a single cell setting without considering interference from other cells. In the case of interference limited environments, which is more representative of real-life densely deployed networks, a multi-channel power allocation of non-cooperative game is studied in [9] to maximize the EE while trading off a certain amount of SE. In [8], given the SE requirement and maximum power limit, a constrained optimization problem is formulated to maximize EE for downlink multiuser distributed antenna systems.

However, to the best of our knowledge, there is no work that relates and analyzes energy efficiency, spectral efficiency and transmission reliability combined, considering the signal to noise ratio. Thus, in this paper, a new relationship for EE, SE and transmission reliability is derived and analyzed the effectiveness numerically and using Matlab simulation tool.

The rest of the paper is organized as follows. The system model is presented in Section II and Section III gives results and discussion. Finally, in Section IV, conclusion and recommendation is presented.

**SYSTEM MODEL**

Consider a downlink cellular network with one BS and $n$ number of users distributed in a cell at different distances as shown in the Figure 1 below. The received signal by the user can be described as:

$$y = \sqrt{p_t} * h * G * S + \sigma^2$$  (1)

Where, $p_t$ is the transmit power from the base station, $G$ is the path loss from the BS to the user, $S$ is the transmit information as a packet or symbols, $h$ denotes the channel coefficient where all entries at different transmission time interval $T$ are independent and identically distributed (i.i.d) complex Gaussian distributed with zero mean and unit variance. The $\sigma^2$ denotes the complex additive white Gaussian noise power. $\sigma^2 = WN_0$, where $N_0$ is noise power spectral density.
Fig. 1. Single base station, and multi-user downlink network model

Assuming that the channel is static, the signal to noise ratio (SNR) of a single mobile station from [4] is:

\[ \gamma(W, P_t, h, b) = \frac{bW h P_t}{R \sigma^2} \]  

(2)

Where, \( b \) is the number of information bits per symbol, \( W \) is the system bandwidth, \( R \) is the symbol transmission rate, \( P_t \) is transmit power, \( h \) is the channel gain (where \( h = \frac{k}{d^\alpha} \), \( k \) is proportionality constant, \( d \) is the distance between the mobile and base station, and \( \alpha \) is path loss coefficient) which is independent of the transmit power.

POWER MODEL AND PROBLEM FORMULATION

From [4], the BS power model is determined with three components of power consumption. The first is power control consumption (\( P_{pc} \)): is due to the effect of power amplifier, feeder loss, and extra loss in transmission related cooling. That is:

\[ P_{pc} = \frac{P_t}{\eta} \]  

(3)

Where \( \eta \) is the power conversion efficiency, accounting for the power amplifier efficiency, feeder loss and extra loss in transmission. The second is Static Power Consumption (\( P_{sta} \)) which includes a power consumption of cooling systems, power supply (\( P_{ac} \)), and battery backup. It is assumed to be constant and independent of \( P_t \), system bandwidth, \( W \) and number of transmit antennas. Finally, dynamic Power Consumption (\( P_{Dyn} \)) is due to the circuit power (\( P_c \)) and signal processing power (\( P_{sp} \)). It is assumed to be dependent on number of transmit antennas, \( n \) and bandwidth but independent of the transmit power.

\[ P_{Dyn} = n(P_c + P_{sp}) \]  

(4)

Hence, from (2) and (3), the total power consumption in the base station is:

\[ P_T = P_{pc} + P_{Dyn} + P_{sta} \]  

(5)

Since the number of transmit antennas is one (i.e. \( m=1 \)) for single input single output (SISO) system, the total power consumption can be given as:

\[ P_T = \left( \frac{P_t}{\eta} + P_{sta} + P_{ac} \right) + W(P_t + P_{sp}) \]  

(6)

The mobile user (MS) power consumption is omitted here because it is negligible with respect to the BS power consumption. We also assumed flat fading channel for a signal period but the channel varies for subsequent signal transmission periods.
A. Transmission Reliability

Transmission reliability function, \( f(\gamma) \) for various modulation schemes is different because it is generally expressed as a function of bit error rate, BER [2, 5],

\[
f(\gamma) = \left( (1 - 2 \times BER)^{2L/b} \right)
\] (7)

It represents the frame success rate, where a user transmits \( L \) information bits in a frame at a rate of \( R \) bits per second using \( P \) watts of power.

For M-ary Phase shift keying (M-PSK) the BER is:

\[
Pe = BER = Q\left(\sqrt{2\gamma}\right), \quad \text{where,} \quad Q(x) = \frac{1}{2} \exp\left(-\frac{x^2}{2}\right).
\]

Thus, the reliability function is:

\[
f(\gamma) = \left( 1 - e^{-\gamma} \right)^{2L/b}
\] (9)

For M-ary Quadrature amplitude modulation (M-QAM), the number of bits transmitted by each symbol is, \( b = \log_2 M \). For square \( M-QAM \) modulation that is \( b \in \{2,4,6,...\} \), \( P_e \) and \( f(\gamma) \) can be expressed as:

\[
P_e = BER = 2 \left( 1 - \frac{1}{\sqrt{M}} Q\left(\frac{3}{M-1}\sqrt{\gamma}\right) \right) \quad \text{and}
\]

\[
f(\gamma) = \left[ 1 - \left( 1 - \frac{1}{\sqrt{2^b}} \right) \exp\left(\frac{3}{2(2^b-1)} \gamma\right) \right]^{2L/b} - 2^{-L}
\] (11)

where \( 2^{-L} \approx 0 \) when \( L \) is large.

B. Spectral Efficiency

If continuous rate adaptation is used, the spectral efficiency for a mobile station \( m \) can be [8]:

\[
f_{SE}(w, p_t, h, b) = \log_2 \left( 1 + \beta \gamma_m(W, p_t, h, b) \right)
\]

Where, \( \beta = -\frac{1.5}{\ln 5 P_{BER}} \)

Thus, the spectral efficiency for \( M \) number of mobile stations is:

\[
f_{SE}(w, p_t, h, b) = \sum_{m=1}^{M} \log_2 \left( 1 + \beta \gamma(w, p_t, h, b) \right)
\] (13)

C. Energy Efficiency

The EE can be characterized with respect to the transmit power and system bandwidth for different modulation schemes, packet size and the channel gain. From [8], the energy efficiency function can be written as:

\[
f_{EE}(w, p_t, h, b) = R \frac{f(\gamma(w,p_t,h,b))}{P_T}
\]

(14)

It can also be written as:

\[
f_{EE}(w, p_t, h, b) = R \frac{W f(\gamma(w,p_t,h,b))}{P_T}
\]

(15)
Since spectral efficiency is expressed as the ratio of the data rate to the given system bandwidth, \( f_{SE} = \frac{R}{W} \), (14) becomes:

\[
f_{EE}(w, p_t, h, b) = W f_{SE}(w, p_t, h, b) \frac{f(y(w, p_t, h, b))}{P_T} \tag{16}
\]

Finally, from (6), (13), and after rearranging (16) with respect to the bandwidth, energy efficiency for all mobile stations in a given cell is obtained as:

\[
f_{EE}(w, p_t, h, b) = \frac{\sum_{m=1}^{M} f_{SE}(w, p_t, h, b) f(y_m(w, p_t, h, b))}{\sum_{m=1}^{M} \frac{1}{W} \left[ \frac{P_T}{W} + P_{st} + P_c \right] + (P_{ac} + P_{sp})} \tag{17}
\]

**RESULTS AND DISCUSSION**

This section provides simulation results and analysis. MATLAB is the simulation tool used to analyze the results. The related system parameters used are indicated in Table 1 (obtained from [4]). We assume a single BS with different users randomly distributed.

**TABLE I. SYSTEM PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Numerical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_{\text{max}} )</td>
<td>20MHz</td>
</tr>
<tr>
<td>( P_{t, \text{max}} )</td>
<td>4watt/MHz</td>
</tr>
<tr>
<td>( P_{st} )</td>
<td>36.6watt</td>
</tr>
<tr>
<td>( P_c )</td>
<td>66.4 watt</td>
</tr>
<tr>
<td>( P_n )</td>
<td>3.32watt/MHz</td>
</tr>
<tr>
<td>( P_m )</td>
<td>1.82watt/MHz</td>
</tr>
<tr>
<td>( L )</td>
<td>64 bits</td>
</tr>
<tr>
<td>( R )</td>
<td>( 10^7 ) bits/sec</td>
</tr>
<tr>
<td>Modulation Schemes</td>
<td>M-PSK, M-QAM</td>
</tr>
<tr>
<td>( H )</td>
<td>0.38</td>
</tr>
<tr>
<td>( N_0 )</td>
<td>5X10^{-21} watt/MHz</td>
</tr>
<tr>
<td>( K )</td>
<td>0.097</td>
</tr>
<tr>
<td>( A )</td>
<td>4</td>
</tr>
<tr>
<td>( D )</td>
<td>1000m</td>
</tr>
</tbody>
</table>

**A. Varying constellation size keeping other parameters constant**
Fig. 2 evaluates the spectral efficiency with respect to the variable transmit power for different modulation orders. Spectral efficiency increases as the transmit power increases. At a fixed lower transmit power, whenever the modulation order increases, proportionally the spectral efficiency increases and the SE gap between different modulation orders is relatively wider. But at higher transmit power; the SE gap is getting narrower.

In Fig. 3, initially, up to the optimum transmit power, the energy efficiency increases. But later it decreases. Unlike other works, when the transmit power get increased beyond the optimum power, because of the reliable data transmission, the energy efficiency is nearly the same for different modulation orders. This implies, taking a specific transmit power in a range near to the optimum power in which almost the EE is constant, without extra power expenditure, SE can be increased by engaging with higher modulation orders. Fig.4 illustrates the transmission reliability and spectral efficiency, where both dependent on the SNR. Fig.5 demonstrates that when the transmission reliability increases (bounded by 0 and 1), the energy efficiency also increases. But whenever the transmission reliability is at the saturation point (i.e. when it
Fig. 5. Energy Efficiency and Transmission Reliability (or Efficiency Function)

approaches 1), the energy efficiency drops automatically even though the modulation order increases. It is because of more power requirement for higher modulation orders. Fig. 6 presents the EE-SE trade-off for different modulation orders.

Fig. 6. Energy Efficiency and Spectral Efficiency with a variable bandwidth.

Fig. 7. Comparison of Spectral Efficiency with Transmit Power Varying Channel Gain
For lower transmit power range, smaller modulation orders are more efficient because the amount of energy per bit required is smaller. But as the modulation order increases the amount of energy required increases, hence the spectral efficiency increases too. In Fig.6, 64-QAM has higher spectral efficiency than 16-QAM but it utilizes more energy than 16-QAM. Hence 64-QAM has lower energy efficiency. Initially when the transmit power increases, both EE and SE increases until they reach the optimal values because of the exponential increase of transmission reliability. But as the transmit power getting larger the trade-off between EE and SE starts. If the SE is set constant at some required value, the EE increases as the number of bits transmitted (modulation order) increases.

**B. The Effect of Varying Channel Gain**

Fig.7 shows spectral efficiency versus transmit power plot for different channel gains. It shows that as the transmit power increases, the SE increases for all channel gains. As the channel gain increases, keeping the transmit power and constellation size constant, the SE also increases. But at higher values of power consumption, even though the channel gain is
increasing, the SE increment for different channel gains gets smaller. In Fig.8 the simulation results show that for the same transmit power, the EE can be increased with an increase in the channel gain at the beginning. Later, even though the channel gain increases, since the transmit power increases, the EE starts to decrease. As shown in Fig.9 the EE increases with an increase in transmission reliability. It also shows that for the same transmit power, the EE can be increased with an increase in the channel gain. Fig.10 compares the EE versus SE among the different channel gains. As the channel gain increases keeping the SE constant, the EE increases. Beyond the optimal values, the EE can no longer be increased, regardless of how much additional energy is provided and channel gain is used.

CONCLUSION

In this work, we have derived the energy efficiency and spectral efficiency relationship considering transmission reliability for downlink cellular network. The approach is properly choosing the transmit power to balance the transmission reliability function and the total power consumption, considering better channel gain and different modulation orders. We focused more on improving the EE-SE trade-off taking the transmission reliability into consideration. The transmission reliability is as a function of the SNR in order to have an exponential increase. As a result the EE increases with out a change of transmit power. We compared the EE performance for different modulation orders and proposed that for a given constant optimum bandwidth, QPSK is more efficient than higher order MQAM for lower transmit powers. The result also indicates that securing transmission reliability, the EE-SE trade-off is optimized by increasing the modulation order.

REFERENCES


