FPGA-based LDPC-coded APSK for optical communication systems

DING ZOU,* CHANGYU LIN, AND IVAN B. DJORDJEVIC
Department of Electrical and Computer Engineering, University of Arizona, 1230 E Speedway Blvd., Tucson, Arizona, 85719, USA
*dingzou@email.arizona.edu

Abstract: In this paper, with the aid of mutual information and generalized mutual information (GMI) capacity analyses, it is shown that the geometrically shaped APSK that mimics an optimal Gaussian distribution with equiprobable signaling together with the corresponding gray-mapping rules can approach the Shannon limit closer than conventional quadrature amplitude modulation (QAM) at certain range of FEC overhead for both 16-APSK and 64-APSK. The field programmable gate array (FPGA) based LDPC-coded APSK emulation is conducted on block interleaver-based and bit interleaver-based systems; the results verify a significant improvement in hardware efficient bit interleaver-based systems. In bit interleaver-based emulation, the LDPC-coded 64-APSK outperforms 64-QAM, in terms of symbol signal-to-noise ratio (SNR), by 0.1 dB, 0.2 dB, and 0.3 dB at spectral efficiencies of 4.8, 4.5, and 4.2 b/s/Hz, respectively. It is found by emulation that LDPC-coded 64-APSK for spectral efficiencies of 4.8, 4.5, and 4.2 b/s/Hz is 1.6 dB, 1.7 dB, and 2.2 dB away from the GMI capacity.

© 2017 Optical Society of America

References and links

7. R. Maher, A. Alvarado, D. Lavery, and P. Bayvel, “Modulation order and code rate optimization for digital coherent transceivers using generalized mutual information,” in ECOC (2015), paper Mo.3.3.4
1. Introduction

With current emerging digital coherent technologies and efficient digital signal processing techniques (DSP), the combination of soft decision forward error correction (SD-FEC) codes and multilevel modulation, known as coded modulation (CM), represents as one of the key enabling technologies for the next-generation high-spectral-efficiency optical communications systems [1, 2]. The FEC operating in large Galois field (GF) that matches the size of multilevel modulation formats, called the non-binary FEC coded modulation, which deals with modulation symbol directly was reported in [3]. Due to the extensively computation complexity of the nonbinary CM, the binary FEC together with a soft de-mapper that maps symbol to bit is widely adopted both in simulation and experimental demonstrations, and it has been shown that capacity approaching performance are possible even with binary CM [4, 5]. On the other hand, the non-uniform probabilistic shaping over conventional square quadrature amplitude modulation (QAM), which theoretically can come closer to the Shannon limit, have been extensively studied recently [7,8]. Meanwhile, amplitude phase shift keying (APSK) modulations that is Gaussian distribution geometrically represent an alternatively way to approach Shannon limit [9]. In the literature, the optimized signal constellations such as [10, 11] can also be used. However, the optimum mapping, especially for medium and large constellation sizes, remains unknown. Given that large latency due to iteration of extrinsic information between a posteriori probability (APP) demapper and LDPC decoders is not tolerable in ultra-high-speed optical communications, we prefer the use of GMI-inspired APSK with simple Gray mapping rule instead.

In this paper, we investigate the usage of the generalized mutual information (GMI) for the design and analysis of coded APSK modulations. The FPGA emulation results indicate that the GMI can be used to predict the code rate region in which APSK can brings benefit over conventional square QAM constellation. Together with the proper designed hardware efficient bit interleaver, which outperforms simple block interleaver based system significantly, the FPGA-based emulation results demonstrate that GMI-inspired LPC-coded 64-APSK, with Gray mapping, is 1.6 dB away from the GMI channel capacity at spectral efficiency (SE) of 4.8 bits/channel use.

The contribution of this paper can be summarized as follows: Firstly, we design gray-mapping APSK constellations with the aid of GMI analysis and provide an in-depth discussion on the SNR operation region in which APSK can bring benefits over conventional square QAM constellations. Then we proposed a hardware-efficient bit interleaver design that is suitable for coded modulation in optical communication systems. To be best of authors’ knowledge, this is the first FPGA implementation of adaptive LDPC-coded GMI-inspired APSK with Gray mapping rule.

The paper is organized as follows. In Section 2, the system model, generalized mutual information inspired APSK design method, together with in-depth discussion on MI and GMI analysis are presented. In Section 3, we compare two different interleaver designs and provide an FPGA-based emulation results. Conclusions are drawn in Section 4.
2. Channel capacity achieving APSK scheme

We consider the optical coded modulation system shown Fig. 1, where the \( m \) parallel encoded data streams are first feed to interleaver, and then the M-QAM or M-APSK mapper takes bits from interleaver and maps every \( m \) bits \( b_i \) into symbols \( S \). At receiver side, the received symbol \( Y \) is first used for symbol log-likelihood ratio (LLR) calculation, then the mapping rule is taken into consideration in bit LLR calculation. Each decoder takes de-interleaved data as inputs and performs decoding processing. In the same Figure, the process of calculation both mutual information (on symbol level) and generalized mutual information (on bit level) is shown as well.

![Fig. 1. LDPC-coded Gray mapping APSK scheme.](image)

2.1 Generalized mutual information

The Shannon theory states that reliable transmission can be achieved if the information rate \( R \) is smaller than the channel capacity \( C \). Combined with higher order modulation formats, capacity-approaching FEC schemes have been extensively studied in the literature and LDPC coding schemes with capacity-approaching performance have been demonstrated [9]. The capacity of arbitrary modulation format \( \chi \) can be numerically calculated by maximizing mutual information (MI) \( I(S;Y) \), as follows

\[
C_{MI} = \max I(S;Y) = m - \mathbb{E}_{s,y}[\log_2 \left( \frac{\sum_{z \in \chi} p(y | z) p(y | s)}{p(y | s)} \right)]
\]

where \( S \) and \( Y \) are the constellation symbols and channel output, respectively. \( \mathbb{E}[\cdot] \) and \( p(y | z) \) represent the expectation operation and conditional probability of the channel outputs \( y \) given the input symbol \( z \). Theoretically, the MI of higher-order modulation formats is expected to more closely approach the Shannon limit. Since the MI measures the similarity of transmitted symbol and detected symbol, the theory bound is highly valuable on FEC scheme that operates on a symbol level, namely for non-binary FECs. Alternatively, high-complexity iterative decoding and optimum bit mapping have been proposed to closely approach the MI capacity for binary FECs [12, 13]. However, neither capacity-achieving non-binary FEC nor iterative demodulation represents a hardware efficient solution.

When a binary FEC is employed, the GMI is more relevant to be exploited as it takes the mapping rule into consideration. This concept is first introduced in [14,15], and it is practically deployed in communication system and it is known as bit interleaved coded modulation (BICM) capacity. The GMI capacity can be defined as:

\[
C_{GMI} = \max \sum_{i=0}^{m-1} I(b_i;Y) = m - \mathbb{E}_{b,y}[\log_2 \left( \frac{\sum_{z \in \chi} p(y | z) \sum_{s \in \chi} p(y | s)}{\sum_{z \in \chi} p(y | z)} \right)]
\]
to capacity loss due to the symbol to bit conversion. To compensate for this problem we typically employ iterative decoding/demapping that exchanges the extrinsic soft information between FEC decoder and demapper. Another way is to determine a Gray-mapping constellation, where the capacity loss of GMI is small, and then do not perform iterative damping to reduce latency, which is highly relevant in high-speed optical communications.

2.2 APSK signaling

While it is known that QAM signaling represents one of the most efficient modulations when operating at very high SNR since it have the largest minimum distance. However, regarding the coded-modulation schemes, the MI is more relevant for non-binary FECs, while the GMI is more relevant for binary FECs. Thus it is highly desired to develop a design method in order to determine the signal constellation that maximizes the MI, while at the same time the corresponding mapping rule maximizes the GMI. One way to that maximizes MI is to design a Gaussian like constellation or probability shaping of a given constellation with Gaussian probability [6]. In this paper, we will design M-APSK constellation that can approach Shannon limit, for sufficiently large constellation sizes, and at the same time the corresponding Gray mapping rule reduces the loss from the MI. An M-APSK constellation is composed of \( R \) concentric rings, each with uniformly spaced PSK points. The M-APSK constellation set \( \chi \) is given

\[
\chi = \begin{cases} 
    r_1 \exp\left(j\left(\frac{2\pi}{n_1}i + \theta_1\right)\right), i = 0, ..., n_1 - 1 \\
    r_2 \exp\left(j\left(\frac{2\pi}{n_2}i + \theta_2\right)\right), i = 0, ..., n_2 - 1 \\
    \vdots \\
    r_R \exp\left(j\left(\frac{2\pi}{n_R}i + \theta_R\right)\right), i = 0, ..., n_R - 1 
\end{cases}
\]

(3)

where \( n_i \), \( r_i \) and \( \theta_l \) \((l = 1, ..., R)\) denote the number of points, the radius, and the phase offset of the \( l \)-th ring, respectively.

Inspired by the non-uniform method in [16], it is recommended in that the radius \( r_l \) is determined as \( r_l = \sqrt{-\ln(1-P_l)} \), where \( P_l \) can be interpreted as the probability of the transmitted signals located within the \( l \)-th ring, which is evaluated as
\[ P_i = \left( \sum_{j=0}^{i-1} n_j + n_i / 2 \right) / M . \]

In order to obtain the Gray mapping for APSK signal constellation, we set a fixed number of points and fixed phase offset in each ring. The resulted 16-APSK and 64-APSK constellations associated with its gray mapping are depicted in Figs. 2(a) and 2(b), where the mapping rule of 16-APSK is labeled by binary number while 64-APSK is labeled by decimal number. For comparison purpose, we also provide the two APSK modulations adopted in digital video broadcasting for satellites (DVB-S2) standards, as shown in Figs. 2(c) and 2(d).

2.3 MI/GMI analysis

With the previously designed APSK constellations, Fig. 3 plots the simulated the MI and GMI capacities of 16-APSK and 64-APSK with different design parameters together as well as the capacity of square 16-QAM and 64-QAM. The benefit of MI/GMI analysis is to separate the design of modulation and coding by assuming that there exist codes that can achieve Shannon limit. It is worth noting that the average energy of the constellation is normalized to unit in the calculation of MI/GMI.

![Fig. 3. MI/GMI for APSK and QAM constellations for: (a) 16-point constellations and (b) 64-point constellations.](image)
As shown in Fig. 3(a), the MI of Gray-mapping-based 16-APSK outperforms the MI of pseudo Gray-mapping-based 16-APSK and 16-QAM when the system is operated at spectral efficiency of 3.3 bits/channel use. The spectral efficiency is calculated by assuming the symbol bandwidth is equal to the WDM channel spacing, similar as in Nyquist-WDM systems. However, the penalty of 0.1 dB of GMI from MI is observed in Gray-mapping-based 16-APSK at code rate of 0.7, which leads to ~0.1 dB SNR improvement over Gray-mapping-based 16-QAM. Since the pseudo gray-mapping-based 16-APSK does not perform well in the code rate region from 0.7 to 0.8, we will only use Gray-mapping-based APSK for 64-point constellation. As shown in Fig. 3(b), the GMI of Gray-mapping-based 64-APSK outperforms Gray-mapping-based 64-QAM, in terms of symbol SNR, by ~0.2 dB and ~0.35 dB at SE of 4.8 and 4.2 bits/channel use, respectively.

3. LDPC-coded APSK

In this section, we first discuss the proposed design and compare two types of interleavers, commonly used in open literature, namely, block interleaver and bit interleaver. Then we present the emulation results and provide in-depth discussion on the relationship of information theoretic results and FPGA emulation results.

3.1 Block interleaved LDPC-coded modulation

One of the straightforward methods to implement the interleaver is so called block interleaver. In Fig. 1, related to the block interleaver, the bits denoted by the same color belong to the same codeword. The block interleaver accepts data from the encoder in row-wise fashion and outputs the data in column-wise fashion to the mapper that accepts \( m \) bits at each time. With \( m \) encoders operating in parallel, this scheme requires negligible hardware resources since it is equivalent to a \( m \)-bit buffer with depth of 1 bit. However, as each codeword is assigned to specific bit in M-ary APSK and each bit in M-ary APSK is protected unequally, thus the performance of this block interleaver based system will be limited by the worst bit. To be more specific, as illustrated in Fig. 4, Gray-mapping-based 16-APSK contains 4 bits per symbol, the first bit possess the smallest decomposed mutual information, the second bit and third bit share the identical decomposed mutual information. This is determined by the mapping rule of 16-APSK and can be easily explained by the constellation diagram, in which blue circle denotes bit 1 while yellow circle denotes the bit 0. In Fig. 5, the same procedure is conducted for pseudo-Gray-mapping-based 64-APSK and Gray-mapping-based 64-QAM.

![Decomposed GMI vs. SNR for Gray-mapping-based 16-APSK, pseudo-Gray-mapping-based 16-APSK and Gray-mapping-based 16-QAM.](image-url)
3.2 Bit interleaved LDPC-coded modulation

While the block interleaver is costless, however, the smallest decomposed GMI limits the system performance. A careful bit interleaver is to be designed to achieve the GMI capacity. As shown in Fig. 1, the cells marked with the same color belong to one codeword, which implies that the bits involved in one codeword are distributed evenly among $m$ bits in either M-APSK or M-QAM. By computer simulation, we randomly place the bits from $m$ codewords into the interleaver with size of $mn$ under the constraint of avoiding the assignment of the worst bits to variable nodes that are connected to the same check nodes, where $n$ is the codeword length and the connections of variable nodes and check nodes are determined by the parity check matrix of LDPC code. In hardware implementation, such an interleaver is implemented by using $m$ m-input muxes to route the encoded data to the right paths and the selection signal is pre stored in the read only memory (ROM) with the size of codeword length and the de-interleaver being properly designed to reverse the interleaving operation.

Differently from the block interleaver, the average GMI feed into each decoder enables it to achieve GMI capacity. Based on the inherent unequal error protection properties for high-order modulation formats, alternative coded modulation scheme is reported in the literature, known as the multilevel coding. The implementation of this scheme, although theoretically much stronger than BICM, is severely affected by: (i) the requirement of implementing different component codes in hardware and (ii) the large latency of its multi-stage decoding process makes it unsuitable for high-speed applications such as optical communications [17].

3.3 FPGA-based emulation results and analysis

We study the performance of the rate-adaptive LDPC-coded modulation in a programmable gate array (FPGA) platform. For the implementation details of LDPC-coded modulation, an interested reader can refer to [18], while the design and implementation of block and bit interleaver are described in Section 3.2. Note that the max-star (max*) operation in [18] is replaced with max operation in bit calculation for the simplicity. In emulations, the mother LDPC code is $(34635, 27710, 0.8)$ code that is constructed based on permutation matrices. Two shortened version LDPC codes $(27708, 20783, 0.75)$, $(23090, 16165, 0.7)$ are obtained by eliminating several block columns in the parity-check matrix (from the right) and used in the following emulation.

![Decomposed GMI vs. SNR for Gray-mapping-based 64-APSK and Gray-mapping-based 64-QAM.](image)
Figure 6 presents both block interleaver based and bit interleaver based coded BER performance as a function of symbol SNR. In the case of block interleaver, shown in (a), the pseudo-Gray-mapping-based 16-APSK is almost overlapped with each other in three different code rates. While the Gray-mapping-based 16-APSK is worse than the other two modulations by 1 dB, 0.8 dB, and 0.7 dB for rates of 0.8, 0.75, and 0.7 at the BER of $10^{-12}$, respectively. This finding is in a good agreement with the decomposed GMI performance shown in Fig. 4, where the SNR gap between worst bit of pseudo-Gray-mapping-based 16-APSK and Gray-mapping-based 16-QAM is negligible; and the SNR gap between the worst bit in Gray-mapping-based 16-APSK and Gray-mapping-based 16-QAM decreases as the code rate decreases. In the case of bit interleaver, shown in Fig. 6(b), we first notice that the employment of bit interleaver provides an SNR improvement of 1.5 dB and 0.7 dB when using the code rate of 0.8 compared to the block interleaver, for Gray-mapping 16-APSK and 16-QAM, respectively. We can observe this phenomenon in Fig. 3(a) and Fig. 4, where the gap between worst decomposed GMI and GMI in Gray-mapping-based 16-APSK is around 1.8 dB at code rate of 0.8. The other useful observation is that the Gray-mapping-based 16-APSK starts to outperform Gray-mapping-based 16-QAM as code rate decreases. This is well
explained by using GMI curves shown in Fig. 3(a), where the GMI gap is negligible at code rate of 0.8 and small gap is observed at code rate of 0.7.

Figure 7 shows both block interleaver based and bit interleaver based coded BER performance vs. symbol SNR for 64-point constellations. Compared to block interleaver system, the bit interleaver system provides an SNR gain of even 3.5 dB and 1.6 dB in 64-APSK and 64-QAM modulation when code rate of 0.8 is applied, while the information theoretic SNR gain of 3.5 dB and 1.9 dB is observed from Fig. 3(b) and Fig. 5. In bit interleaver case, Gray-mapping-based 64-APSK outperforms Gray-mapping-based 64-QAM by ~0.1 dB, ~0.2 dB, and ~0.3 dB for code rates of 0.8, 0.75, and 0.7 at BER of $10^{-12}$, respectively. We observe the same phenomenon as in Fig. 3(b), in which the SNR gap between Gray-mapping-based 64-APSK and 64-QAM is ~0.2 dB, ~0.25 dB, ~0.35 dB as spectral efficiency decreases from 4.8 to 4.2 bits/channel use. With the bit interleaving scheme, we are 1.6 dB, 1.7 dB, and 2.2 dB away from the GMI capacity for code rates of 0.8, 0.75, and 0.7, respectively.

Fig. 7. BER vs. SNR performance for LDPC-coded 64-APSK and 64-QAM when: (a) block interleaver and (b) bit interleaver is used.
In this section, we have designed an efficient bit interleaver and demonstrated its advantages over block interleaver via both information theoretic-based simulation and FPGA emulation. Therefore, with the proper bit interleaver design, the GMI can be used to predict the binary LDPC-coded performance of both APSK signaling and QAM signaling. As it indicated in Fig. 3(a), the APSK signal will only be benefit over QAM signal if its GMI is larger than QAM and an implementation penalty of 0.1 dB is observed in our emulation. To be more specific, the simulation gain is negligible for 16-APSK over 16-QAM when operated at code rate 0.7, while the GMI gain of 0.1 is observed in Fig. 3(a).

4. Conclusion

In this paper, we first use mutual information and generalized mutual information capacities’ analyses as a tool to design a geometrically shaped APSK that mimics an optimal Gaussian distribution with equiprobable signaling and the corresponding Gray-mapping rules. The information theoretic simulation results demonstrated that the APSK signaling can achieve Shannon limit closer than conventional QAM signaling at specific range of FEC overheads for both 16-APSK and 64-APSK. Later, by hardware efficient FPGA-based LDPC-coded APSK emulator, a comparison between block interleaver and bit interleaver has been conducted, and it has been found that the FPGA based emulation predictions have been in excellent agreement with information theoretical simulation results. Both information theoretic simulation results and FPGA-based emulation results confirmed that the proposed LDPC-coded bit interleaver scheme significantly outperforms the corresponding block interleaver based scheme. In bit interleaver-based emulation, it has been found that the performance of LDPC-coded 16-APSK has been comparable to the 16-QAM, while the LDPC-coded 64-APSK outperforms 64-QAM by 0.1 dB, 0.2 dB, and 0.3 dB (in symbol SNR) at spectral efficiencies of 4.8, 4.5, and 4.2 b/s/Hz, respectively. The authors believe that a more significant gain can be observed in fiber-optics transmission system due to the larger minimum distance on the outer ring will lead to a better nonlinearity tolerance over conventional QAM.

Funding

National Science Foundation (NSF) CIAN ERC (EEC-0812072); ONR MURI program (N00014-13-1-0627)