ACOUSTIC SOURCE LOCALIZATION IN AN ANISOTROPIC PLATE WITHOUT KNOWING ITS MATERIAL PROPERTIES

by

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DEDICATION

For my lovely family; Eun Hae, Lena and Hannah
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Abstract

Acoustic source localization (ASL) is pinpointing an acoustic source. ASL can reveal the point of impact of a foreign object or the point of crack initiation in a structure. ASL is necessary for continuous health monitoring of a structure. ASL in an anisotropic plate is a challenging task. This dissertation aims to investigate techniques that are currently being used to precisely determine an acoustic source location in an anisotropic plate without knowing its material properties. A new technique is developed and presented here to overcome the existing shortcomings of the acoustic source localization in anisotropic plates. It is done by changing the analysis perspective from the angular dependent group velocity of the wave and its straight line propagation to the wave front shapes and their geometric properties when a non-circular wave front is generated.

Especially, ‘rhombic wave front’ and ‘elliptical wave front’ are dealt with because they are readily observed in highly anisotropic composite plates. Once each proposed technique meets the requirements of measurement, four sensor clusters in three different quadrants (recorded by 12 sensors) for the rhombus and at least three sensor clusters (recorded by 9 sensors) for the ellipse, accurate Acoustic Source Localization is obtained. It has been successfully demonstrated in the numerical simulations. In addition, a series of experimental tests demonstrate reliable and robust prediction performance of the developed new acoustic source localization technique.
1 Introduction

1.1 Acoustic source localization

The importance of structural health monitoring (SHM) of aircrafts, aerospace/aeronautical structures and civil structures (See Figure 1) is continuously increasing. SHM is needed to mitigate severe damages or system failures caused by a sudden impact of foreign objects on any surface or internal crack formation. SHM is capable of in-situ monitoring in the manner of being non-destructive, passive and continuous. Continuous monitoring can provide early detection of the degradations/damages and thus prevent catastrophic failure of the entire structure. Structural health monitoring has three components.

- Localizing: Identifying damage initiation region using acoustic emission technique
- Diagnosis: Sensing/characterizing damage by non-destructive testing to get an idea of its kind, size, severity and so on
- Prognosis: Predicting remaining life based on knowledge of fracture mechanics, failure mechanism and constitutive modeling

Figure 1 Civil structures and aerospace/aeronautical structures demand structural health monitoring to prevent irreparable failure by internal damages or an external strike.
Acoustic source localization (ASL) identifies the source point of an acoustic emission in the reverse order of wave propagation phenomenon, see Figure 2. A sudden release of the strain energy caused by internal/external stress generates elastic vibrations of the material in ultrasonic or sonic wave band, then the energy propagates outward from the initiation point in the form of an elastic wave in a solid material. Then one can capture the elastic wave using an acoustic receiver, for instance, piezo-electric sensor. ASL starts from the captured signal. After analyzing the signal, the behaviors of the propagated waves are characterized. Theoretical or geometrical interpretations of the propagated wave can conclude where the acoustic wave source was generated.

![Figure 2 Wave propagation vs. Acoustic source localization](image)

Acoustic wave propagation in isotropic plates have been explored extensively and is known as *Lamb wave* [1], strictly speaking, *Leaky Lamb Wave* because the plate is immersed in air and a small amount of energy is leaked out into the air [2]. Based on extensive theoretical solutions and experimental data for the isotropic plates [3, 4, 5], one can conclude that waves travel in straight lines in all angular directions regardless of their frequency. Angular dependent group velocity of the wave is identical in all directions so that it forms a circular wave front in an isotropic plate. Such straight line propagation
behavior makes acoustic source localization in isotropic plate relatively simple. In the next section, simple acoustic source localization techniques are reviewed.

1.2 Acoustic source localization in isotropic plates

The simplest way to demonstrate the acoustic source localization is the use of one-dimensional problem as shown in Figure 3. Two piezoelectric sensors $S_1$ and $S_2$ are placed on two points to record incoming signals continuously. When the velocity ($c$) of the wave propagation is known, one can formulate Equation (1) where the distance between $S_1$ and $S_2$ ($L$) is measured and the time difference of arrival (TDOA) is computed from two captured signals. By solving the equations for $x_1$ and $x_2$, one can localize the acoustic source in the one dimensional domain.

\[
\begin{align*}
\{ x_1 - x_2 &= c(t_1 - t_2) \\
\quad x_1 + x_2 &= L
\}
\end{align*}
\]

The acoustic source localization technique can be extended to two-dimensional isotropic material without much difficulty, and it requires at least three sensor points depicted in Figure 4. Assuming constant group velocity of the wave generated at an
unknown location, exact traveling times \((t_1, t_2, \text{ and } t_3)\) between the source and each sensor can be used to obtain the radius of three circles. Draw three circles with radius, \(ct_i\) \((i = 1, 2 \text{ and } 3)\), then the source is located at the intersection point of these three circles.

However, not knowing the exact time of the strain energy release event defies acquisition of \(t_1, t_2, \text{ and } t_3\). Time difference of arrival, \(Δt_{21}\), and \(Δt_{31}\), can be utilized to form two circles with their centers located at the sensor positions \(S_2\) and \(S_3\), shown as dotted circle in Figure 5. The next step is to add surrounding concentric circles on top of the dotted circle and around sensor \(S_1\) until there is a common intersection point of the three circles. While these techniques are straightforward calculation, assumptions such as knowing material properties of the plate and constant group velocity limit their versatility of usage in practical applications of acoustic source localization.

![Diagram of source localization](image)

Figure 4 Principle of two-dimensional source localization on an isotropic plate. The source location is marked by a dot which is the intersection point of the three circular wave fronts.
Figure 5 Time difference of arrivals are utilized to form two base circles (dotted), then a common intersection point can be obtained by adding surrounding concentric circles (solid) on top of the base circles.

1.3 Motivation

- Acoustic source localization is inevitable for configuring structural health monitoring (SHM) system. Large anisotropic composite plates that are in high demanded for aircrafts and aeronautical structures badly need this technology. These plates are prone to damage and degradation due to fiber breakage, layer delamination and aging. Early detection of the degradation can prevent catastrophic failure of the entire structure.

- Currently available techniques are limited by a key assumption of straight line propagating path. Acoustic source localization technique using curved wave propagation path has not been reported yet in the literature, therefore this new
One major motivation of this research is to deal with the wave propagation in an anisotropic plate structure where it does not propagate along a straight line and the wave velocity in different directions is not constant anymore. Is it possible to consider curved wave propagation path properly even when the material properties are known?

By observing particular wave front shape formed by non-straight propagating waves, would it be possible to localize the acoustic source accurately? In addition, could the source localization technique perform in the absence of material properties or pre-acquired information such as dispersion curves, power spectral densities or time-difference map? Acoustic source localization technique without knowing the plate material properties has the promise of being versatile and robust for any applications.

In order to achieve a reliable and cost effective structural health monitoring system for in-situ applications, would it be possible to estimate accurate source location with efficient usage of computing power and minimum installation cost, namely fast computation and less number of piezoelectric sensors?
1.4 **Research objectives**

Given the motivations above, the objectives of this dissertation are:

1) To observe the wave front shape of propagation in an anisotropic plate in the numerical simulation.

2) To develop new acoustic source localization technique without knowing the material properties of the anisotropic plate by avoiding conventional analysis methodologies.

3) To numerically validate estimation performance of the developed technique

4) To experimentally validate the reliability and the robustness of the technique

1.5 **Novelty of the work**

This research has developed a new technique of source localization in highly anisotropic thin plates. The proposed technique can accurately predict the impact point without knowing the plate’s material properties. The main contribution from this research to the acoustic source localization research community is that the proposed technique is the first method where the analysis perspective has been changed from the angular dependent group velocity of the propagating wave to geometric properties of the wave front shape itself. As a result, it avoids the necessity of knowing material properties such as density, orthotropic elastic moduli and Poisson’s ratio as long as basic requirements are met such as minimum number of sensors and location configurations.
1.6 Organization of the dissertation

To accomplish the objectives of this work, the research employs technical review of currently available techniques for an anisotropic plate with and without consideration for material properties. Chapter 2 reviews the mathematical formulation of currently available techniques of acoustic source localization for anisotropic plates. After their limitations and difficulties are presented in the end of Chapter 2, the proposed new acoustic source localization technique is introduced in Chapter 3. Two different techniques according to specific wave front shape such as rhombus and ellipse are discussed in separate sections, 3.3 and 3.4. In Chapter 4, simulated wave propagation in a numerically modeled anisotropic plate is performed using a commercially available computer software, and the results are analyzed to validate the proposed technique. Chapter 5 details experimental test results to validate the technique’s reliability and the robustness. Finally, Chapter 6 addresses concluding remarks and remaining future works for subsequent research are proposed.
2 Background

Mathematical formulations for acoustic source localization (ASL) in anisotropic plates are briefly presented in this section and then their limitations are discussed to justify development of a new technique. The source localization formulations with known velocity profile and material properties are different from those techniques developed for anisotropic structures with unknown material properties. These techniques are discussed separately under two sub-sections, 2.1 and 2.2.

2.1 Available ASL techniques with known material properties

Various techniques for acoustic source localization have been reviewed by Kundu [5]. The acoustic source localization technique in an anisotropic plate with known velocity profile are available in the literature [9-12]. The formulation presented below is based on these publications.

The distance \( d_i \) between the acoustic source location \((x_A, y_A)\) and the sensor location \((x_i, y_i)\) is related to \( c(\theta) \) [the group velocity in the direction of wave propagation from the source to the sensor] and \( t_i \) [the time of travel of the wave from the acoustic source to the sensor] in the following manner.

\[
d_i = \sqrt{(x_i - x_A)^2 + (y_i - y_A)^2} = c(\theta_i)t_i
\]

(2)

If there are three receiving sensors placed in directions \( \theta_1, \theta_2 \) and \( \theta_3 \) (see Figure 6) then following the steps given in Reference [5] one can show that minimization of any one of the following three expressions of the objective function \( E(x_A, y_A) \) (also known as the
merit function or the error function) can give the acoustic source coordinates \((x_A, y_A)\) [6, 7, 9].

\[
E(x_A, y_A) = \left( \frac{c(\theta_1)\sqrt{(x_1 - x_A)^2 + (y_1 - y_A)^2} - c(\theta_2)\sqrt{(x_2 - x_A)^2 + (y_2 - y_A)^2}}{t_{12}} \right)^2 + \left( \frac{c(\theta_1)\sqrt{(x_1 - x_A)^2 + (y_1 - y_A)^2} - c(\theta_2)\sqrt{(x_3 - x_A)^2 + (y_3 - y_A)^2}}{t_{23}} \right)^2 + \left( \frac{c(\theta_1)\sqrt{(x_1 - x_A)^2 + (y_1 - y_A)^2} - c(\theta_2)\sqrt{(x_4 - x_A)^2 + (y_4 - y_A)^2}}{t_{31}} \right)^2
\]

\[
E(x_A, y_A) = \left( \frac{t_{31}c(\theta_1)\sqrt{(x_2 - x_A)^2 + (y_2 - y_A)^2} - c(\theta_2)\sqrt{(x_3 - x_A)^2 + (y_3 - y_A)^2}}{t_{31}} \right)^2 + \left( \frac{t_{31}\sqrt{(x_3 - x_A)^2 + (y_3 - y_A)^2} - c(\theta_2)\sqrt{(x_4 - x_A)^2 + (y_4 - y_A)^2}}{t_{31}} \right)^2 + \left( \frac{t_{31}\sqrt{(x_1 - x_A)^2 + (y_1 - y_A)^2} - c(\theta_2)\sqrt{(x_2 - x_A)^2 + (y_2 - y_A)^2}}{t_{31}} \right)^2
\]

\[
E(x_A, y_A) = (c(\theta_1)c(\theta_2)t_{12} - \sqrt{(x_1 - x_A)^2 + (y_1 - y_A)^2}c(\theta_2))^2 + \sqrt{(x_2 - x_A)^2 + (y_2 - y_A)^2}c(\theta_1))^2 + \sqrt{(x_3 - x_A)^2 + (y_3 - y_A)^2}c(\theta_1))^2 + \sqrt{(x_4 - x_A)^2 + (y_4 - y_A)^2}c(\theta_1))^2
\]

where \(t_{ij} (= t_i - t_j)\) is the time difference of arrival (TDOA) between sensors \(i\) and \(j\).
Figure 6 An acoustic source generates acoustic waves in a plate-like structure and the wave can be captured with acoustic sensors at different location. The wave velocity can be varied in the different direction so one can express the velocity as $c(\theta_i)$.

Note that the angle $\theta_i$ of the wave propagation direction from the source $(x_A, y_A)$ to the $i$-th sensor $(x_i, y_i)$ is measured from a reference axis (typically the horizontal axis) and can be obtained from the following equation

$$\theta_i = \tan^{-1}\left(\frac{y_i - y_A}{x_i - x_A}\right)$$  \hspace{1cm} (6)

The above equation is valid for all possible combinations of $(x_A, y_A)$ and $(x_i, y_i)$ for which the computed $\theta_i$ values should vary between $-\pi/2$ and $+\pi/2$. Since the wave velocity in $\theta_i$ and $(\theta_i + \pi)$ directions should be the same, it is sufficient to consider $-\pi/2 < \theta < +\pi/2$ for computing the wave velocity in all possible directions between $-\pi/2$ and $+3\pi/2$.

One shortcoming of Equation (3) is that for certain values of the unknown coordinates $(x_A, y_A)$ the denominators can be zero. Therefore, for those values of $(x_A, y_A)$
the objective function becomes infinity and special care must be taken during the computation of the objective function to avoid these singular points. This problem is avoided in Equations (4) and (5) and therefore they are preferred over Equation (3). If the number of receiving sensors is increased from 3 to \( n \) then Equations (4) and (5) take the following general forms:

\[
E(x_A, y_A) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=1}^{n} \sum_{l=k+1}^{n} \left[ g_i c(\theta_i) c(\theta_j) (d_i c(\theta_i) - d_j c(\theta_j)) - t_{ij} c(\theta_i) c(\theta_j) (d_i c(\theta_i) - d_j c(\theta_j)) \right]^2
\]  

(7)

\[
E(x_A, y_A) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left[ c(\theta_i) c(\theta_j) t_{ij} - d_i c(\theta_j) + d_j c(\theta_i) \right]^2
\]  

(8)

In Equations (7) and (8) \( d_i \) denotes the distance between the source and the \( i \)-th sensor, as given in Equation (2). Note that for \( n \) number of sensors there are \( \frac{n(n-1)}{2} \) unique pairs of sensor and as a result \( \frac{n(n-1)}{2} \) terms appear in Equation (8). However, in Equation (7) the number of terms in the summation series is \( \sum_{m=1}^{\left[ \frac{n(n-1)}{2} - 1 \right]} m \). Therefore, for 3, 4 and 5 receiving sensors the number of terms in Equation (8) should be 3, 6 and 10, respectively whereas in Equation (7) these numbers are 3, 15 and 45, respectively. Clearly, the computational efficiency increases significantly for greater than three receiving sensors when the objective function given in Equation (8) is used instead of that in Equation (7). Therefore, Equation (8) is preferred over Equation (7) for acoustic source localization. The impact point \( (x_A, y_A) \) can be obtained by minimizing the above error function by some optimization scheme (such as simplex algorithm [13] or genetic algorithm [14, 15]).
2.2 Available ASL techniques when material properties are unknown

The technique described above requires the knowledge of the direction dependent velocity profile in the anisotropic plate for source localization. Kundu [16] proposed a technique by which an acoustic source could be approximately localized in an anisotropic plate with the help of six receiving sensors. This technique neither requires the knowledge of the plate properties (such as the direction dependent velocity profile in the plate) nor needs to solve a system of nonlinear equations. Kundu et al. [17] experimentally verified this technique for plates made of both isotropic (requiring 4 sensors) and anisotropic materials (requiring 6 sensors). Ciampa et al. [18, 19] also proposed a technique for source localization in anisotropic plates without knowing the plate properties. Their technique required the solution of a system of nonlinear equations.

Baxter et al. [20] and Xiao et al. [21] also proposed techniques for source localization when the structural properties are unknowns. The technique proposed by Baxter et al. [20] requires initial training of the structure by a system of known acoustic sources at different locations and then use that knowledge for localizing the future acoustic events by comparing the recorded arrival time difference (that they called Delta-T) between different sensor pairs with those from the training dataset. For large structures the training part can be very time consuming.

Xiao et al. [21] used two arrays of sensor in mutually perpendicular directions (x and y) and localized acoustic source from these two arrays by beam forming method. Since for beam forming method the localization accuracy is not very sensitive to the velocity in the array direction the authors showed that by considering two perpendicular arrays the
acoustic source can be localized with reasonable accuracy in a plate like structure even when there is uncertainty about the propagating wave velocity.

In the technique proposed by Kundu et al. [16, 17] three receiving sensors $S_1$, $S_2$ and $S_3$ are mounted on the plate as shown in Figure 7. If the coordinates of three receiving sensors $S_1$, $S_2$ and $S_3$ are $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$, respectively then it is clear that $x_2 = x_1 + d$, $x_3 = x_1$, $y_2 = y_1$ and $y_3 = y_1 + d$. The coordinate values of the acoustic source (A) are given by $(x_A, y_A)$.

![Figure 7](image)

Figure 7 Three sensors at positions $S_1$, $S_2$ and $S_3$ are needed to get the direction of the source by the method described in Section 2.2

The distance $d$ between the sensors is much smaller than the smallest distance $D$ between the acoustic source A and the $i$-th sensor $S_i$ ($i = 1, 2$ or $3$). Therefore, the inclination angle $\theta$ of lines $AS_1$, $AS_2$ and $AS_3$ (see Figure 7) can be assumed to be approximately the
same. Because of this assumption the received signals at these three sensors should be almost identical but slightly time shifted and the wave velocity from the source point A to sensors S₁, S₂ and S₃ should be almost same even for an anisotropic plate, but the wave arrival times can still be different because of the small differences in the travel path lengths. Angle $\theta$ can be expressed as,

$$
\theta = \tan^{-1} \left( \frac{y_1 - y_A}{x_1 - x_A} \right) \approx \tan^{-1} \left( \frac{y_2 - y_A}{x_2 - x_A} \right) \approx \tan^{-1} \left( \frac{y_3 - y_A}{x_3 - x_A} \right)
$$

(9)

If the wave front is assumed to be perpendicular to the wave propagation direction (which is true at some points for an anisotropic plate while for a weakly anisotropic plate it can be approximately assumed to be true at all points) then after arriving at sensor S₁ the time taken by the wave front to reach sensors S₂ and S₃ can be denoted as $t_{21} = t_2 - t_1$ and $t_{31} = t_3 - t_1$, respectively. These two time delays are given by,

$$
t_{21} = \frac{d \cos \theta}{c(\theta)}
$$

(10)

$$
t_{31} = \frac{d \sin \theta}{c(\theta)}
$$

(11)

From Equations (10) and (11) one can easily obtain,

$$
\theta = \tan^{-1} \left( \frac{t_{31}}{t_{21}} \right)
$$

(12)

From Equation (10)

$$
c(\theta) = \frac{d \times \cos \theta}{t_{21}} = \frac{d \times t_{21}}{t_{21} \sqrt{t_{21}^2 + t_{31}^2}} = \frac{d}{\sqrt{t_{21}^2 + t_{31}^2}}
$$

(13)
In the above equation $\cos \theta = \frac{t_{21}}{\sqrt{t_{21}^2 + t_{31}^2}}$ is obtained from the following consideration. From Figure 7 it is clear that $(AS_2 - AS_1) = c(\theta)t_{21}$ and $(AS_3 - AS_1) = c(\theta)t_{31}$. Three lines $AS_1$, $AS_2$ and $AS_3$ are assumed parallel, which should be the case when the source is far away from the sensors. Note that the two triangles $S_1S_2P$ and $S_1S_3Q$ are similar triangles when $AS_1$, $AS_2$ and $AS_3$ are parallel. The lines $AS_2$ and $AS_3$ intersect the wave front going through sensor $S_1$ at points $P$ and $Q$, respectively. Therefore,

\[
\cos \theta = \frac{PS_2}{S_1S_2} = \frac{PS_2}{\sqrt{PS_2^2 + PS_1^2}} = \frac{PS_2}{\sqrt{PS_2^2 + QS_3^2}} = \frac{c(\theta)t_{21}}{\sqrt{c(\theta)^2t_{21}^2 + c(\theta)^2t_{31}^2}} = \frac{t_{21}}{\sqrt{t_{21}^2 + t_{31}^2}}
\]

From Equations (12) and (13) the wave propagation direction and the wave velocity in that direction are obtained in terms of experimentally measured values $t_{21}$ and $t_{31}$.

If three more sensors $S_4$, $S_5$ and $S_6$ are mounted near another corner of the plate as shown in Figure 8 then the wave propagation direction $\theta_4$ from the acoustic source to sensor $S_4$ and the wave speed in that direction $c(\theta_4)$ can be obtained in the same manner from $t_{54}$ and $t_{64}$ using the following equations:

\[
\theta_4 = \tan^{-1} \left( \frac{t_{64}}{t_{54}} \right)
\]

\[
c(\theta_4) = \frac{d}{\sqrt{t_{54}^2 + t_{64}^2}}
\]

From Equations (9) and (12) for the $S_1$, $S_2$, $S_3$ sensor cluster, and from similar two equations for the $S_4$, $S_5$, $S_6$ sensor cluster one can write
Figure 8 Acoustic source can be localized from the intersection point of two lines generated by two clusters of sensors (S1-S2-S3 and S4-S5-S6). A third cluster (S7-S8-S9) can be used to investigate if the third line also goes through the intersection point of the other two lines to reconfirm the prediction.

Equations (17) and (18) give a system of two linear equations with two unknowns $x_A$ and $y_A$ that can be uniquely solved. In other words, two straight lines with inclination angles $\theta_1$ and $\theta_4$ going through sensors $S_1$ and $S_4$ intersect at a point that should be the
acoustic source point. A third cluster can be added as shown in Figure 8 to make sure that the predicted directions from all three clusters meet at one point which is the acoustic source location. Recently Kundu et al. [22] proposed a hybrid technique combining the above two techniques described in Sections 2.1 and 2.2 to reduce the prediction error. First, from Equations (12), (13), (15) and (16) the approximate values of the acoustic source coordinates \((x_A, y_A)\) and the two wave speeds \([c(\theta_1) and c(\theta_4)]\) from the source to the two sensor clusters (see Figure 8) are estimated. Then starting with these initial estimates the exact source coordinates and the two wave speeds are predicted by minimizing the following objective function,

\[
E(x_A, y_A) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left[ c(\theta_i^*) c(\theta_j^*) t_{ij} - d_i c(\theta_j^*) + d_j c(\theta_i^*) \right]^2
\]

where \(\theta_i^*\) (or \(\theta_j^*\)) is equal to \(\theta_1\) [for \(i\) (or \(j\)) = 1, 2 or 3] or \(\theta_4\) [for \(i\) (or \(j\)) = 4, 5 or 6]. It should be noted that this expression is almost identical to Equation (8) with only one difference; \(\theta_i\) and \(\theta_j\) of Equation (8) are substituted by \(\theta_i^*\) and \(\theta_j^*\), respectively in Equation (19). With this substitution, the number of independent variables or parameters in the above expression is reduced from eight (six wave speeds and two coordinate values) to only four, \(x_A, y_A, c(\theta_1)\) and \(c(\theta_4)\). These four parameters can be independently varied or adjusted to minimize \(E(x_A, y_A)\). The above expression can be minimized by any optimization scheme. In order to converge to the global minimum and avoid converging to the local minima, the initial estimates of the unknown parameters [in this case \(x_A, y_A, c(\theta_1)\) and \(c(\theta_4)\)] should be as close to the final values as possible.
Compared to Newton’s method, the simplex algorithm [13] is much easier to implement for solving a system of nonlinear equations and was followed in Reference [19]. With good initial estimates the simplex algorithm should converge quickly to the global minimum. The advantage of this hybrid technique is that it should converge faster because it starts with some good initial estimates as opposed to the method proposed by Ciampa et al. [18, 19] that did not have any way of generating good first estimates of the unknown parameters.

2.3 Limitations

Techniques described in Sections 2.1 and 2.2 have their limitations. Formulation presented in Section 2.1 requires the knowledge of the direction dependent velocity profile $c(\theta)$ and the technique described in Section 2.2 assumes propagating wave front to be perpendicular to the straight line connecting the acoustic source to the sensor. It is true for an isotropic plate but for an anisotropic plate it is true only at some but not all points. Therefore, for an anisotropic plate the source can be localized exactly or approximately depending on the acoustic source location relative to the sensor clusters. The technique proposed by Ciampa et al. [18, 19] also assumes the wave energy to be propagating in the direction of a straight line connecting the acoustic source and the sensor with some average group velocity value.

All cluster based techniques require accurate determination of time difference of arrival (TDOA) or $t_{ij}$ between $i$-th and $j$-th sensors that are placed in close proximity. Since all calculations are based on $t_{ij}$ shown in Equations (12) and (13), it is important to
measure it accurately. The small time difference $t_{ij}$ can be measured accurately by the cross correlation technique, as described in References [5, 23, 24]. Although the hybrid technique [22] improved the acoustic source localization technique by reducing the prediction error as shown in Figure 9 and Figure 10 (pictures are adopted from [22] for explanation), it could not eliminate the error completely for anisotropic plates. These figures show the predicted acoustic source points in a composite plate from two techniques for 5 different experiments. Locations of the two sensor clusters in the plate are shown by solid circles. Predicted points from the intersection of two straight lines from the two clusters (as described in Reference [17], also see Figure 8.) are shown by cross markers. A second set of predictions (open square markers) are obtained by the hybrid technique [22]. Note that in both Figure 9 and Figure 10 the predicted points by the hybrid technique are closer to the true acoustic source point (solid square marker). The prediction error almost disappeared in Figure 9 but not in Figure 10.
Figure 9 Predicted acoustic source locations in a composite plate by the technique described in References [17] are shown by cross markers. These are the intersection points of the straight lines drawn from the two clusters of sensor (shown by solid circles). Predicted source locations by the hybrid technique (Reference [22]) are shown by the hollow square markers and the true position of the acoustic source is shown by the solid square marker. Clearly hybrid technique gives better results.
Figure 10 Same as Figure 9 but for a different acoustic source position. Note that both techniques have prediction errors. However, hybrid technique has relatively less error.
Two-step hybrid technique [22] and other cluster based techniques [17-19] can never be able to eliminate the prediction error problem completely because their basic assumption that the wave can be assumed to be radiating from the acoustic source to different directions with the direction dependent average group velocities is not true. This shortcoming is illustrated with the help of Figure 11. In a unidirectional fiber reinforced composite plate (with fibers running in the x-direction), if the acoustic source is located at the origin O, then waves propagate along curved lines forming approximately an elliptical wave front as shown in Figure 11. Wave propagation lines are shown only in first and third quadrants to keep the diagram simple. Using the mirror symmetry about x and y axes the wave propagation lines in 2nd and 4th quadrants can be also generated.

Figure 11 Nonlinear wave propagation paths in an anisotropic plate form an elliptical wave front in an anisotropic plate. The wave front FAH at sensor A is not perpendicular to line OA. Perpendicular line to this wave front AQ does not go through the acoustic source.
One cluster of three sensors A, B and C are attached to the plate as shown. The actual wave front at point A is defined by line FAH. After reaching point A, the wave front strikes sensor C and then B since sensor C is closer to the wave front. In all cluster based techniques (17-19, 22) it is assumed that there is an average group velocity of the wave in direction OA and it can be safely assumed that the wave propagates in direction OA with this average group velocity. Although one can calculate the average group velocity simply by dividing the path length OA by the time of travel, it should not be assumed that the wave energy propagates in direction OA when it travels through the cluster A-B-C. Clearly, if the wave energy propagates along line OA, then after sensor A it should strike sensor B and then C since EB is smaller than GC. However, for the propagating waves shown in Figure 11, sensor C is struck first and then B. From the TDOA values the wave propagation direction and the group velocity in that direction can be obtained from Equations (12) and (13), respectively. However, the wave propagation direction QA does not go through the acoustic source and therefore, the acoustic source cannot be localized from the intersection point of two straight lines from two clusters as was suggested earlier. A new technique is proposed in this paper to overcome this shortcoming and discussed in the following chapter.
3 Proposed new technique

The present chapter contains the theory, results, and conclusions of the recently submitted journal paper [26]. The following is a reproduction of the paper as a part of this dissertation.

3.1 Direction vectors measured by sensor clusters

Kundu et al. [17] introduced a simple right-angle configuration of three sensors placed orthogonally with a distance \((d)\) from the middle sensor \((O)\) as shown in Figure 12. The three-sensor configuration is called a ‘Sensor cluster’. The cluster is mounted on the surface of the plate to record incoming wave signals synchronously. The distance \(d\) between the sensors in a cluster should be small enough to neglect dispersion over distance \(d\). In addition, the distance \(d\) is much smaller than the distance from the source location to the cluster so that the angle of propagation from the source to the three sensors can be assumed to be the same. Large distance from the source to the acoustic cluster also helps to separate different guided wave modes over the travel distance. This justifies using a single value of wave front velocity \((v)\) shown in Figure 12 and corresponds well with Figure 13 where the first part of each signal displays good agreement in shape. As a result, the wave front passing through the cluster can be assumed to be a plane wave front as depicted in Figure 12.
Figure 12 Plane wave front at the sensor cluster location, sensors $P_1$ and $P_2$ are located at a distance $d$ from Sensor $O$. Direction vector ($\vec{w}$) and parallel vector ($\vec{w} \parallel$) of the plane wave front are obtained by measuring TDOAs between two pairs of sensors ($P_1-O$ and $P_2-O$) and then computing their ratio, 

$$\tan \theta = \frac{\Delta t_{20}}{\Delta t_{10}}.$$ 

The direction vector ($\vec{w} \perp$) and the parallel vector ($\vec{w}$) of the plane wave front toward the point $O$ from the first quadrant of the local x-y coordinates can be described as

$$\begin{bmatrix} \vec{w} \perp \\ \vec{w} \end{bmatrix} = \begin{bmatrix} -\cos \theta \\ -\sin \theta \\ -\cos \theta \\ \cos \theta \end{bmatrix}$$

(20)

where $\theta$ follows the sign convention: CCW is positive, and it is obtained from four-quadrant inverse tangent with two TDOAs

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\nu \Delta t_{20}/d}{\nu \Delta t_{10}/d} = \frac{\Delta t_{20}}{\Delta t_{10}} = \frac{t_2 - t_0}{t_1 - t_0}$$

(21)

$$\theta = \tan^{-1} \left( \frac{t_2 - t_0}{t_1 - t_0} \right)$$

(22)
Figure 13 Signal $F(t)$ is measured at sensor $O$, and signal $G(t)$ is measured at sensor $P_1$. Note that the signal patterns are very similar in the time range 1100 μs to 1500 μs.

With the plane wave front approximation, one can assume that the three sensors receive identical signal patterns near the first arrival and the signals are dispersed afterwards. TDOA is a time shift between two received signals which is computed by the cross-correlation technique, by plotting the product of two signals when for one signal the time shift is continuously changed. Two signals recorded by two sensors are displayed together in Figure 13. Similar signals with a small time shift are found in the range between 1100 μs and 1500 μs. The time shift between them can be easily found by the cross-correlation technique. This method examines the similarity between the two given signals by computing:

$$[F(t) \ast G(t)](\tau) = \int_{lower \ bound}^{upper \ bound} F(t)G(t + \tau)dt$$

(23)

The maximum value of the cross-correlation plot corresponds to the time shift or TDOA as shown in Figure 14.
Figure 14 Cross-correlation between the two signals in the time range (1100 to 1500 μs) is computed and displayed. The maximum correlation exists at -1.75 μs which corresponds to the time shift between the two signals.

3.2 Observation of wave front shape in an anisotropic plate

A 500mm × 500mm × 2mm thin anisotropic plate is modeled by cuLISA3D software [25] for numerical simulation with orthotropic plate material properties as given below:

- Mass density: $1.5 \times 10^{-9}$ tonnes/mm$^3 = 1.5$ gm/cc
- Elastic moduli: $E_1 = 66400$ MPa, $E_2 = 6000$ MPa, and $E_3 = 6000$ MPa
- Poisson’s ratios: $\nu_{12} = 0.2$, $\nu_{23} = 0.25$, and $\nu_{31} = 0.25$
- Shear moduli: $G_{12} = 1400$ MPa, $G_{23} = 2100$ MPa, and $G_{31} = 2100$ MPa
In the model, eight elements through the plate thickness – i.e. $\Delta z = 0.25$ mm, with in-plane element size $\Delta x = \Delta y = 0.5$ mm, were used. The total number of elements in the model was 8 M. The time step, $\Delta t$, was taken 0.025 us to ensure stability of the explicit time integration scheme. Acoustic source is located at the center of the plate (250, 250) and excited by a two-period sine signal modulated by Gauss window. Free boundary conditions are used in the model. Figure 15 shows resultant wave fronts of the numerical simulation at $t = 100 \mu s$. Different sensor-clusters can be placed at various positions as shown in Figure 16. Sensors in each cluster are 15 mm apart in both horizontal and vertical directions.

Figure 15 In the numerical simulation an acoustic source generates various wave fronts from the acoustic source point. This plot is obtained at $t = 100 \mu s$. 
Figure 16 Six clusters at different locations record the wave signal. Sensors in each cluster are 15 mm apart along both horizontal and vertical directions. For example, three sensors of cluster 1 are located at (35,50), (50,50), and (50,35).

All 18 sensors record incoming signals and store them until the simulation ends. One received signal (dark black solid line) is plotted in Figure 17. When we take absolute value of the signal and plot it in logarithmic scale then three different magnitude levels are revealed. The lowest level indicates numerical errors in the context of spurious modes existing in dynamic transient simulations. This aspect can be found in Figure 18 as well.
First wave group, a symmetric guided wave shown in Figure 19 (a), hits the sensor carrying relatively small energy but propagating faster while second wave group, a antisymmetric guided wave shown in Figure 19(b), arrives later with higher level of energy (see gray solid line in Figure 17).

Figure 17 A signal recorded by one sensor is plotted in dark black solid line. When absolute value of the signal is plotted in logarithmic scale as shown in gray solid line, three significantly different levels of signal energy are noticed that represent white noise or numerical error, first wave group arrival and second wave group arrival.
Figure 18 Two different wave groups are recognized in the snapshot at $t = 100 \mu s$ of the numerical simulation.
Figure 19 (a) The first wave group represents a symmetric guided wave containing lower energy contents but fast propagation. (b) The second wave group represents an antisymmetric mode containing higher energy contents but relatively slow propagation. This picture is adopted from Reference [2].

All $\vec{w}^{\perp}$ described in Equation (20) are computed by measuring TDOAs at every cluster for first and second wave groups. Then lines along each $\vec{w}^{\perp}$ are drawn in Figure 20 and Figure 21. The lines should go through the acoustic source if the wave front is circular [5]. However, for the anisotropic plate considered here the lines do not go through a common point. In Figure 20, almost two sets of parallel lines are observed on two sides of the acoustic source location. In Figure 21, multiple crossing points at various locations can be observed. This is an interesting finding and underlines the need for a new source localization technique without knowing material properties. The new technique is discussed in the next section.
Figure 20 First wave group: rhombus wave front: Lines parallel to each $\vec{w}^{\perp}$ are displayed where $\vec{w}^{\perp}$ are obtained from measured TDOAs at every cluster using Equation (20).
Figure 21 Second wave group: non-circular (almost elliptic) wave front. Lines parallel to each $\overrightarrow{w}$ are displayed where $\overrightarrow{w}$ are obtained from measured TDOAs at every cluster using Equation (20).
3.3 Rhombus wave front

In anisotropic plates propagating waves form non-circular wave fronts – typically rhombus or ellipse. When rhombus wave front is formed then measured wave propagation direction vectors ($\vec{w}$) for two adjacent clusters become parallel regardless of the cluster’s location as shown in Figure 20. In this example the rhombus wave front propagates faster than the other wave front so the sensors detect it first without any interference with the reflected waves from the boundary. For acoustic source localization the concentric rhombus wave fronts depicted in Figure 22 are analyzed instead of trying to find the intersection point of a set of straight lines from sensor clusters. In other words, geometric properties of the concentric rhombus are utilized to determine the acoustic source location. All concentric rhombuses share a vertical diagonal and a horizontal diagonal, and the intersection of the two diagonals is the Acoustic Source location as shown in Figure 22.

There are two basic requirements for this method. First, shape velocity or the velocity of the rhombus wave front is constant, and two sensor clusters, for example $S_r$ and $S_I$, are placed such that the shape velocity can be obtained from these two clusters. The distance between the two sensor clusters $S_r$ and $S_I$ along the wave propagation direction is denoted by $d_{r1}$. TDOA between these two sensor clusters is $(t_r - t_I)$. Then the shape velocity $\mu = \frac{d_{r1}}{(t_r - t_I)}$. In order to compute the shape velocity, the two clusters $S_r$ and $S_I$ must be located in the same quadrant; in other words, one plane wave front must pass through these two sensor clusters.

The second requirement is that the minimum number of required sensor clusters is four. One diagonal of the rhombus can be obtained by simple vector analysis with direction
vectors and TDOA \((t_2 - t_1)\) between clusters \(S_I\) and \(S_{II}\). Similarly, the second diagonal of the rhombus can be obtained from direction vectors and TDOA \((t_3 - t_1)\) between clusters \(S_I\) and \(S_{III}\). These two diagonals will be denoted as vertical and horizontal diagonals. Detail derivation is given below. After two diagonals are obtained from the concentric rhombuses, the acoustic source location is identified from the intersection point of the two diagonals.

As a result, only geometric properties of the rhombus shape and direction vector measurements at four cluster positions are needed to predict the exact source location without knowing any material properties or direction dependent velocity profile of the anisotropic plate.

Figure 22 Rhombus wave front is generated by an acoustic source and expands with a shape velocity (or wave front velocity) \(\mu\). At \(t = t_1\), sensor cluster \(S_I\) captures the wave front first. Then clusters \(S_{II}, S_{III},\) and \(S_r\) receive signals sequentially at \(t_2, t_3,\) and \(t_r\), respectively. Simple vector analyses with direction vectors at all clusters and TDOAs allow to form the diagonals of the concentric rhombuses. The intersection point of the two diagonals gives the final acoustic source location.
Direction vectors and parallel vectors at the sensor clusters were discussed in section 3.1. Cross-correlation method for getting TDOAs between any two sensors was also described earlier. Now the vector analysis for localizing the acoustic source will be introduced.

Let us denote the direction vector at $S_r$ and $S_I$ as $\overrightarrow{u}$ and the parallel vector at $S_I$ as $\overrightarrow{u}$. The distance between wave front $L_1$ and $L_r$ is $d_{1r}$, then the shape velocity or the wave front velocity can be obtained in the following manner:

$$\mu = \frac{d_{1r}}{t_r - t_1} = \frac{\|proj_{\overrightarrow{u}}(\overrightarrow{S_R} - \overrightarrow{S_I})\|}{t_r - t_1} = \frac{\|\overrightarrow{S_R} - \overrightarrow{S_I} \cdot \overrightarrow{u} \overrightarrow{u}}{\overrightarrow{u} \cdot \overrightarrow{u}}}{(t_r - t_1)} \quad (24)$$

Figure 23 $V_{tr}$ is obtained from direction vectors and TDOAs at four clusters.
Another direction vector and parallel vector at $S_{II}$ are denoted as $\vec{v}^\perp$ and $\vec{v}$, respectively.

Next we introduce parametric representation for line $L_1$ on 2D plane, it is given as

$$L_1 = \{ \vec{S}_I + \kappa_1 \vec{u} \mid \kappa_1 \in \mathbb{R} \}$$

(25)

where $\mathbb{R}$ is real number space and all vectors have two coordinate components, $(x, y)^T$. If TDOA between $S_I$ and $S_{II}$ is zero, the bisector $V_{bi}$ in Figure 23 becomes the vertical diagonal of the rhombus directly. Otherwise, the line $L_1$ should be shifted by $d_{12}$ to align the rhombus at $t = t_2$ in order to find the true vertical bisector, $V_{tr}$. Using the given shape velocity, the shift is simply

$$d_{12} = \mu(t_2 - t_1)$$

(26)

The shifted line ($L'_1$) may be derived from the following parametric representations involving the shift:

$$L'_1 = \left\{ \vec{S}_I + d_{12} \frac{\vec{u}^\perp}{\|\vec{u}^\perp\|} + \kappa'_1 \vec{u} \mid \kappa'_1 \in \mathbb{R} \right\}$$

(27)

And the line ($L_2$) passing through $S_{II}$ is given in the same manner,

$$L_2 = \{ \vec{S}_{II} + \kappa_2 \vec{v} \mid \kappa_2 \in \mathbb{R} \}$$

(28)

Now we solve $L'_1 = L_2$ to find the intersection point ($P_\nu$) of $L'_1$ and $L_2$, denoted as

$$\vec{S}_I + d_{12} \frac{\vec{u}^\perp}{\|\vec{u}^\perp\|} + \kappa'_1 \vec{u} = \vec{S}_{II} + \kappa_2 \vec{v}$$

(29)

By rearranging Equation (29) as follows, we may solve the simultaneous equations of two unknowns, $\kappa'_1$ and $\kappa_2$:
\[
\begin{bmatrix}
\kappa_1' \\
\kappa_2
\end{bmatrix} = [u \ -v]^{-1} \left( \frac{S_H - S_I - d_{12}}{\|u\|} \right)
\]  

(30)

\(P_V\) is calculated by either substituting \(\kappa_1'\) into Equation (27) or substituting \(\kappa_2\) into Equation (28) in the parametric formulation of the true bisector \(V_{tr}\) (solid line in Figure 23),

\[
V_{tr} = \{ \overrightarrow{P_V} + \kappa_{V} \overrightarrow{b_V} \mid \kappa_{V} \in \mathbb{R} \}
\]

(31)

where \(\overrightarrow{b_V}\) is parallel to the bisector and is obtained by adding two unit vectors:

\[
\overrightarrow{b_V} = \frac{\overrightarrow{u}^\perp}{\|\overrightarrow{u}^\perp\|} + \frac{\overrightarrow{v}^\perp}{\|\overrightarrow{v}^\perp\|}
\]

(32)

For obtaining the true horizontal diagonal \((H_{tr})\) of the rhombus wave front, one can follow exactly the same procedure as described in Equations (26) to (32).

Figure 24 \(H_{tr}\), are obtained from direction vectors and TDOAs at four clusters.
The new shift \( d_{13} \) and the new shifted line \( L_1'' \) are computed based on the TDOAs between \( S_I \) and \( S_{III} \) as shown in Figure 24)

\[
d_{13} = \mu(t_3 - t_1)
\]  

and

\[
L_1'' = \left\{ \overrightarrow{S_I} + d_{13} \frac{\overrightarrow{u}}{||\overrightarrow{u}||} + \kappa_1'' \overrightarrow{\mu} \left| \kappa_1'' \in \mathbb{R} \right. \right\}
\]  

The other line passing through \( S_{III} \) is expressed using the direction vector \( \overrightarrow{w} \) as

\[
L_3 = \left\{ \overrightarrow{S_{III}} + \kappa_3 \overrightarrow{w} \left| \kappa_3 \in \mathbb{R} \right. \right\}
\]  

For obtaining the intersection point \( P_H \), we solve

\[
L_1'' = L_3
\]  

and denote Equation (36) as

\[
\overrightarrow{S_I} + d_{13} \frac{\overrightarrow{u}}{||\overrightarrow{u}||} + \kappa_1'' \overrightarrow{\mu} = \overrightarrow{S_{III}} + \kappa_3 \overrightarrow{w}
\]  

By rearranging Equation (37) one gets the following system of simultaneous equations from which two unknowns, \( \kappa_1'' \) and \( \kappa_3 \) can be solved:

\[
\begin{bmatrix}
\kappa_1'' \\
\kappa_3
\end{bmatrix} = \begin{bmatrix}
\mu & -w
\end{bmatrix}^{-1} \left( \overrightarrow{S_{III}} - \overrightarrow{S_I} - d_{13} \frac{\overrightarrow{u}}{||\overrightarrow{u}||} \right)
\]
\( P_H \) is calculated by either substituting \( \kappa''_1 \) into Equation (34) or substituting \( \kappa_3 \) into Equation (35) and is used in the parametric formulation of the bisector \( H_{tr} \) (solid line in Figure 24),

\[
H_{tr} = \{ \overrightarrow{P_H} + \kappa_H \overrightarrow{b_H} \mid \kappa_H \in \mathbb{R} \} \tag{39}
\]

where \( \overrightarrow{b_H} \) is parallel to the bisector:

\[
\overrightarrow{b_H} = \frac{\overrightarrow{u_\perp}}{\|\overrightarrow{u_\perp}\|} + \frac{\overrightarrow{w_\perp}}{\|\overrightarrow{w_\perp}\|} \tag{40}
\]

Since the two true bisectors are diagonals of the rhombus wave front, we can conclude that the final intersection point of \( V_{tr} \) and \( H_{tr} \) must be the acoustic source location. The final step to examine the source position (\( P_S \)) is to solve the following relation

\[
V_{tr} = H_{tr} \tag{41}
\]

After substituting Equation (31) and Equation (39) into Equation (41) we get

\[
\overrightarrow{P_V} + \kappa_V \overrightarrow{b_V} = \overrightarrow{P_H} + \kappa_H \overrightarrow{b_H} \tag{42}
\]

After some algebraic manipulation one gets,

\[
\begin{bmatrix} \kappa_V \\ \kappa_H \end{bmatrix} = \begin{bmatrix} \overrightarrow{b_V} & -\overrightarrow{b_H} \end{bmatrix}^{-1} (\overrightarrow{P_H} - \overrightarrow{P_V}) \tag{43}
\]

Estimated acoustic source location (\( P_S^* \)) shown in Figure 25 is finally determined by either substituting \( \kappa_V \) from Equation (43) into Equation (31) or substituting \( \kappa_H \) into Equation (39).
Figure 25 Two computed bisectors ($V_r$ and $H_{tr}$) are drawn together. The intersection of two lines (star symbol) indicates the final acoustic source position obtained after a series of vector analysis involving direction vectors and TDOAs.

3.4 **Elliptical wave front**

Another common non-circular wave front for anisotropic plates is an ellipse as shown in Figure 18. Figure 21 shows how the source localization technique with circular wave front assumption fails for anisotropic plate for both rhombus and elliptic wave fronts. In our example the elliptic wave front propagates behind the rhombus wave front (see Figure 18) but the propagating energy is much higher for the elliptic wave front and therefore gives higher signal to noise ratio in the recorded signal.

For acoustic source localization the concentric ellipse wave fronts depicted in Figure 26 are analyzed by establishing analytic approach instead of trying to find the
intersection point of a set of straight lines from sensor clusters. Again it is assumed that the separation between the sensors in a cluster is much smaller than the distance from the source location to the cluster. Therefore, the wave front passing through the cluster can be considered as a plane wave front. The second assumption is that the two principal axes of the anisotropic thin plate are aligned with 2D XY Cartesian coordinates. For most anisotropic materials this assumption can be satisfied without any difficulty.

Figure 26 Elliptical wave front is generated by an acoustic source and propagates outward forming concentric ellipses. At $t = t_1$, sensor cluster $S_I$ receives the wave front first. Then cluster $S_{II}$ and $S_{III}$ receive signals at times $t_2$ and $t_3$, respectively.
The equation of an ellipse with unknown center and eccentricity can be given by,

\[
\left(\frac{x - C_x}{a}\right)^2 + \left(\frac{y - C_y}{b}\right)^2 = 1 \tag{44}
\]

where the ellipse center is \((C_x, C_y)\), ‘a’ is semi-major axis, and ‘b’ is semi-minor axis. At \(t = t_1\), sensor cluster \(S_I\) receives the wave front first. Then clusters \(S_{II}\) and \(S_{III}\) receive signals at time \(t_2\) and \(t_3\), respectively. Following Equations (20) and (22), one can define parallel vectors \([\vec{u} \quad \vec{v} \quad \vec{w}]\) which are tangents to the three wave fronts, shown in Figure 26 as:

\[
[\vec{u} \quad \vec{v} \quad \vec{w}] = \begin{bmatrix} -\sin \theta_I & -\sin \theta_{II} & -\sin \theta_{III} \\ \cos \theta_I & \cos \theta_{II} & \cos \theta_{III} \end{bmatrix} \tag{45}
\]

Slope of parallel vector on \(i^{th}\) cluster \(= \frac{\cos \theta_i}{-\sin \theta_i} = -\cot \theta_i \tag{46}\)

As a result, the derivative of the ellipse at every sensor cluster can be denoted in terms of \(\theta_i\):

\[
\begin{bmatrix} \frac{dx}{dy} \end{bmatrix}_i = -\frac{a^2}{b^2} \frac{(y_i - C_y)}{(x_i - C_x)} = -\frac{1}{\gamma} \frac{(y_i - C_y)}{(x_i - C_x)} = -\cot \theta_i \tag{47}
\]

Note that the concentric ellipses have identical major axis/minor axis ratio \((a/b)\), so one can replace the two unknowns by one, \(\gamma\). Consequently, three equations are established to solve three unknowns \((\gamma, C_x, C_y)\),

\[
\begin{cases}
\gamma C_x - \gamma x_I - \tan \theta_I \cdot C_y = -\tan \theta_I \cdot y_I \\
\gamma C_x - \gamma x_{II} - \tan \theta_{II} \cdot C_y = -\tan \theta_{II} \cdot y_{II} \\
\gamma C_x - \gamma x_{III} - \tan \theta_{III} \cdot C_y = -\tan \theta_{III} \cdot y_{III} \tag{48}
\end{cases}
\]

where \(\tan \theta_i\) is the ratio of two TDOAs as given in Equation (21).
Since Equation (48) is nonlinear, the damped least squares optimization method, so called Levenberg-Marquardt Algorithm (LMA) is used to solve this system of equations. Objective function for the optimization is

\[ \Phi = \sum_i (\gamma C_x - \gamma x_i - \tan \theta_i \cdot C_y + \tan \theta_i \cdot y_i)^2 \quad i = I, II, and III \]  

(49)

and LMA finds \((\gamma, C_x, C_y)\) by forcing \(\Phi\) converging to zero.
4 Numerical validation of the proposed technique

4.1 Overview

Two approaches for acoustic source localization have been proposed in the previous chapter. In the present chapter, wave fronts generated by the numerical simulation are utilized to validate the proposed methods. Thanks to high-performance computing by graphical processing units, elastic wave propagation in complex media for large models can be simulated efficiently where the local interaction simulation approach [27] and a parallel algorithm architecture [28, 29] are incorporated.

An anisotropic plate, 500mm × 500mm × 2mm, is modeled in the simulation framework [30, 31] to observe behavior of propagating waves generated by the HSU source, so called pencil-lead-break acoustic source. The orthotropic plate material properties listed in section 3.2 ensure that the wave experiences anisotropic characteristics during its propagation. In the model, eight elements through the plate thickness – i.e. Δz = 0.25 mm, with in-plane element size Δx = Δy = 0.5 mm, were used. The total number of elements in the model was 8 M. The time step, Δt, was taken 0.025 us to ensure stability of the explicit time integration scheme.

All z-displacements at 12 sensing points in 4 sensor clusters for the rhombic technique and 18 sensing points in 6 sensor clusters depicted in Figure 27(a) and (b), respectively, were stored to validate the proposed acoustic source localization technique. For source localization requirements of the rhombus, two of sensor clusters are closely positioned in the third quadrant and other two sensor clusters are placed in the second and fourth quadrant, respectively. As the ellipse technique requires three sensor clusters, 20
possible combinations out of 6 sensor clusters allow to evaluate more accurate source location. Sensors in each cluster are 15 mm apart along both horizontal and vertical directions. For example, three sensors of the cluster $S_{III}$ in Figure 27(a) are located at (35,450), (50,450), and (50,465). XY coordinates of all sensor nodes in Figure 25 are tabulated in Table 1.

Figure 27 Z-displacement of each selected node in the LISA numerical simulation are recorded, then two proposed acoustic source localization techniques are employed. (a) Total 12 sensor nodes in 4 sensor clusters are selected for the rhombic wave front where two cluster must reside in the same quadrant (the third in the picture) and the rest of them should be located in each different quadrant (the second and the fourth). (b) Among 6 sensor clusters one can make 20 possible combinations with 3 sensor clusters because the elliptic wave front requires only three. They will allow to evaluate more accurate source location.
Table 1 XY coordinates of all sensor nodes in Figure 27 are presented.

<table>
<thead>
<tr>
<th></th>
<th>Figure 27(a)</th>
<th>Figure 27(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_I$</td>
<td>$S_{II}$</td>
<td>$S_{III}$</td>
</tr>
<tr>
<td>x-coord.</td>
<td>120</td>
<td>200</td>
</tr>
<tr>
<td>y-coord.</td>
<td>200</td>
<td>130</td>
</tr>
</tbody>
</table>

As the recorded signal at all sensors have two distinct arrivals as shown in Figure 17, the method appropriate for the rhombus wave front is employed first, and then the technique for the elliptic wave front is employed later. The following results were obtained by analyzing different parts of the time signals, namely the first arrival for the rhombus wave front and then the second arrival for the elliptic wave front. Each analysis result is presented in the section 4.2 and 4.3, respectively.

4.2 Rhombus wave front

Three sensor clusters ($S_I, S_{II}, S_{III}$) are placed in three different quadrants and the reference sensor cluster $S_r$ is placed in the third quadrant with $S_I$. The analysis starts with the calculation of direction vectors, $\overrightarrow{u^\perp}$, $\overrightarrow{v^\perp}$, and $\overrightarrow{w^\perp}$ at the three-sensor cluster positions. The rhombus shape velocity ($\mu$) is obtained from $\overrightarrow{u^\perp}$, TDOA value $t_{r1}$ between $S_r$ and $S_I$. TDOA value $t_{21}$ and $\overrightarrow{u^\perp}$ allow us to obtain the first shifted line ($L'_1$), then $P_V$ is obtained, which is the intersection point of lines $L'_1$ and $L_2$. Then the vertical diagonal (the vertical
solid line $V_{tr}$ in Figure 28) of the rhombus wave front can be obtained by drawing a straight line which is parallel to the bisecting vector $\vec{b}_V$ and going through point $P_v$. In a similar manner with two sensors, $S_I$ and $S_{III}$, the horizontal diagonal (the solid line $H_{tr}$ in Figure 28) can be drawn. Finally, the intersecting point of the two diagonals is the estimated acoustic source location. All computed vectors, points from the numerical simulation and vector analysis are listed in Table 2. Note that the prediction error [or the distance of the predicted point ($P^*_S$) from the actual acoustic source location ($P_S$)] is only 2.68 mm, while the distances between the source and four clusters vary from 130 to 283 mm.

The estimated source location $P^*_S$ by the rhombus technique is denoted in Figure 28, by a solid circle and relevant straight lines obtained from the vector analysis given in Equations (24) to (43) are plotted below.

Figure 28: Computed lines, vectors and points obtained from the vector analysis for the rhombic wave front from numerical simulation results are shown. The estimated location ($P^*_S$) of the acoustic source (250.714, 252.581) is only 2.68 mm away from the true source location ($P_S$).
Table 2 Computed vectors and points from the numerical simulation and the vector analysis. Note that the presented vectors are not normalized.

<table>
<thead>
<tr>
<th>Coordinates[mm]</th>
<th>Computed vectors</th>
<th>Computed vectors or points</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_l ) (200, 130)</td>
<td>( \vec{u} = (-1 -4.1429)^T )</td>
<td>( \vec{b}_V ) (0 (-1.9442))^T</td>
</tr>
<tr>
<td>( S_H ) (350, 90)</td>
<td>( \vec{v} = (1 -4.1429)^T )</td>
<td>( \vec{b}_H ) (-0.5006 (-0.0081))^T</td>
</tr>
<tr>
<td>( S_{III} ) (50, 450)</td>
<td>( \vec{w} = (-1 3.6250)^T )</td>
<td>( P_V ) (250.714, 66.034)</td>
</tr>
<tr>
<td>( S_r ) (120, 200)</td>
<td>( \vec{u} = (-1 -4.1429)^T )</td>
<td>( P_H ) (-722.667, 236.851)</td>
</tr>
</tbody>
</table>

Rhombus shape velocity \( \mu \) 2.0112 km/s

Final estimation of acoustic source \( P_\Sigma \) (250.71, 252.58)

The prediction performance of the rhombus technique is sensitive to how accurately the direction vector at each sensor cluster is defined. As the direction vector described in Equation (20) is the ratio of two TDOAs, the quality of the cross-correlation expressed in Equation (23) influences the predication performance. In order to see sensitivities of the prediction performance of the rhombus technique, a perturbed error is intentionally added into TODAs. The error are randomly generated in the range of \( \pm 0.025 \) μs. Figure 29 shows the sensitivity analysis result of the rhombus wave front technique. The open black circles are the error induced estimations and the original source location is plotted by a solid diamond marker. Perturbed source locations are dispersed within 235 to 260 mm in x coordinate and 246 to 260 mm in y coordinate. The averaged source location (250.04 mm, 252.50 mm) of the perturbations is plotted in a yellow square where the XY error bar represents standard deviation, \( \sigma_x = 6.30 \) mm and \( \sigma_y = 3.68 \) mm, respectively. When the simulation time step is smaller than the current time step size, 0.025 μs, the uncertainty is decreased. The averaged source location and the standard deviation show relatively small error when compared with the distances between the source and the four clusters that vary from 130 to 283 mm.
Figure 29 Sensitivity analysis result of the rhombus wave front technique. The estimated source location under random perturbation as TDOA error within ±0.025 μs is plotted as black open circles and the original source location is plotted by a solid diamond marker. Perturbed source locations are dispersed within 235 mm to 260 mm in x coordinate and 246 mm to 260 mm in y coordinate. The averaged source location (250.04, 252.50) of the perturbations is plotted in a yellow square where the XY error bar represents standard deviation, $\sigma_x = 6.30$ mm and $\sigma_y = 3.68$ mm, respectively.

4.3 Elliptical wave front

Figure 30 shows source localizations obtained from the elliptic wave front analysis. As mentioned before minimum three sensor clusters are required for this analysis. A total of twenty different combinations of three clusters of sensor from the six clusters shown in Figure 27(b) are possible. The system of non-linear equations given in Equation (48) are solved with the computed angle of the elliptical wave front listed in Table 3 for all these
20 combinations. Thirteen of these twenty predicted source locations are plotted as black cross markers in Figure 30. The LMA optimization scheme did not converge for the other seven cases. The point $P^*_S$ shows the centroid of the predicted source locations. The prediction error [or the distance of the predicted point ($P^*_S$) from the actual acoustic source location ($P_S$)] is 18.8 mm, while the distances between the source and various sensor clusters vary from 130 to 283mm.

Figure 30 Computed source locations obtained from the elliptic wave front analysis, using the LMA optimization scheme are shown by black cross markers. The centroid ($P^*_S$) of the predicted source locations is (250.0, 268.8) which is 18.8 mm away from the true location ($P_S$).
Table 3 The computed wave front angle with Equations (21) and (22) are listed.

<table>
<thead>
<tr>
<th>Cluster #</th>
<th>Position</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(50,50)</td>
<td>33.69 °</td>
</tr>
<tr>
<td>2</td>
<td>(200,130)</td>
<td>64.70 °</td>
</tr>
<tr>
<td>3</td>
<td>(120,200)</td>
<td>14.04 °</td>
</tr>
<tr>
<td>4</td>
<td>(350,230)</td>
<td>160.87 °</td>
</tr>
<tr>
<td>5</td>
<td>(350,90)</td>
<td>121.11 °</td>
</tr>
<tr>
<td>6</td>
<td>(450,50)</td>
<td>146.31 °</td>
</tr>
</tbody>
</table>

The localization uncertainty for the elliptic wave front is due to two reasons - 1) the simulated wave front is not perfectly ellipse, and 2) some sensor clusters that are located close to the plate boundary records the interference of two wave fronts – the elliptic wave front coming from the acoustic source and the boundary reflected fast propagating rhombus wave front. Interference between these two wave fronts introduces some error in the TDOA measurements. If an elliptic wave front is generated in a weakly anisotropic large plate, for example in a cold rolled metallic plate, then this technique is expected to give better predictions.
5 Experimental validation

5.1 Overview

One crucial step for non-destructive testing (NDT) technique and structural health monitoring (SHM) is acoustic source localization (ASL) which identifies the initiation point of elastic energy propagation. As the wave can be initiated by the sudden impact of a foreign object or internal cracking of the structure, accurate ASL is inevitable for configuring structural health of the system. Large anisotropic composite plates used in aircrafts and aeronautical structures are prone to damage and degradation due to fiber breakages, layer delamination and aging. Early detection of the degradations can prevent further catastrophic failure of the entire structure.

Various researchers have focused on ASL for anisotropic plates [1-12] where either material properties, dispersion curves, angular-dependent group velocities or a Delta-t contour map should be obtained empirically/explicitly. Optimization and computation based approaches such as simplex algorithm [13], genetic algorithm [14], and neural network [32,33] have been introduced to enhance reliability of ASL. Mathematical remedies such as wavelet transform [34] at a certain frequency, band pass filtering [35], and probabilistic error reduction [36] have been developed as well. While the currently available ASL techniques work well for weakly anisotropic plates since the wave path does not deviate significantly from the straight line propagation directions they fail miserably for highly anisotropic plates because waves in anisotropic solids propagate along curved lines and form non-circular wave fronts.

In Chapter 3, the new ASL technique has been introduced, originated in the previous approach by Kundu et al. [20, 21] where three piezoelectric sensors placed orthogonally
measure time-difference-of-arrivals (TDOA) to acquire the incoming propagation angle of the wave front. The new method focuses on the wave front shape in an anisotropic plate such as concentric rhombus and its geometric properties. Note that all rhombic wave fronts share same two diagonals. First three normal directions for three rhombic wave fronts are obtained by Kundu’s approach using sensor clusters of three sensors. Then one can incorporate these three directions into a series of explicit vector analysis to define not only two diagonals of the rhombus but also their intersection point which is the acoustic source location. By taking advantage of LISA numerical simulation [27-31] specific wave front shapes in an anisotropic plate are observed - rhombus and ellipse. Similar rhombic and elliptical wave fronts were found in other numerical simulation results of Lamb waves propagating in a fabric panel [33]. Both simulation results indicate that the rhombic wave front is generated by the symmetric Lamb mode, containing low energy but propagates faster than the anti-symmetric wave mode. One can detect the arrival time by noticing sudden rise of the signal from the background noise.

The present chapter is focused on demonstrating feasibility of the new ASL technique by testing commercially available off-the-shelf composite plates. Far from the numerical simulation, it is difficult to set accurate time of arrival of the wave front manually using captured acoustic signals. They are easily contaminated by unexpected errors and uncertainties. In order to improve determining onset time of the wave front, the Akaike information criteria (AIC) is adopted. It is widely used in the seismological research to pick seismic primary wave (P-wave) accurately [37, 38]. Three highly anisotropic composite plate samples were selected based on preliminary comparison of AIC curves,
then the new ASL technique was employed on each sample. Limited by the performance of the test system, the experimental validation of only the rhombus wave front is presented.

5.2 Test with anisotropic plates using Akaike Information Criteria

First set of experiments with anisotropic plates were designed to select highly anisotropic composite plates among commercially available off-the-shelf samples. Table 4 lists currently available composite plates for the experimental investigation. Sample #1 and #2 are made from a PREPREG woven and are nominally called quasi-isotropic plate. Although it would not be expected as an anisotropic plate but it can be used as a good reference during the preliminary anisotropy tests. Sample #3 is expected to exhibit some degree of anisotropy in 45-degree direction, although it is relatively small. Sample #4 is a good candidate to be tested as an anisotropic plate. Dimension of sample #5 is large enough and different responses in the 0-degree, 45-degree, and 90-degree directions are expected. The last sample #6 is identical to the sample #5 but it has been aged. The sample #6 was subjected to cyclic thermal loading ranging from the room temperature to 175 °C for 250 hours in order to cause degradation of the material properties of the sample. Comparison of acoustic source localization results for sample #5 and #6 should be interesting for SHM of composite structures in terms of robustness of the technique.

For acquisition of ultrasonic signal in anisotropic plates, a passive piezoelectric sensor shown in Figure 31 was utilized. Its frequency range is from 100 kHz to 450 kHz plotted in Figure 32 and its frequency response is characterized by a peak at 150 kHz where it exhibits a resonance. The vendor mentioned that it is suitable for almost all AE
applications and especially suited for integrity inspection of metallic plates as well as composite plates.

Table 4  Total 6 different composite plates are available to perform acoustic source localization test.

<table>
<thead>
<tr>
<th>Samples</th>
<th>Details</th>
<th>Dimension (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>one-dimensional carbon Hexply 552/34%/134/AS4(12K) layer thickness 0.11 + 0.14 mm</td>
<td>500 x 500 x 4</td>
</tr>
<tr>
<td>#2</td>
<td>Fabric carbon, layer thickness (0.20 + 0.24 – with 10 layers in 0/90/0/90/0 arrangement.</td>
<td>500 x 500 x 2</td>
</tr>
<tr>
<td>#3</td>
<td>carbon fiber reinforcement [0/90]</td>
<td>250 x 250 x 1.5</td>
</tr>
<tr>
<td>#4</td>
<td>one-dimensional composite reinforced with carbon fiber</td>
<td>300 x 500 x 1</td>
</tr>
<tr>
<td>#5</td>
<td>carbon fiber reinforcement, symmetric [0/45/90]</td>
<td>260 x 800 x 2</td>
</tr>
<tr>
<td>#6</td>
<td>carbon fiber reinforcement, symmetric [0/45/90], aged by cyclic thermal load</td>
<td>260 x 470 x 2</td>
</tr>
</tbody>
</table>

Figure 31 Passive piezoelectric sensor from Vallen Systeme, VS150-M (Image is adopted from the vendor’s website [39]). Its contact diameter is 20.3 mm. It is also lightweight, 24g, to minimize gravity induced error.
Figure 32 Frequency response of the VS150-M (Image is adopted from the vendor’s website [39].)

Every sample listed in Table 4 is tested under identical geometrical conditions of the sensor configuration presented in Figure 33. Distances between each sensor and HSU excitation point are kept constant at 150 mm whenever possible for every test. Photograph of every test and three recorded signals by the piezoelectric sensors are presented in Figure 34 to Figure 39.

Figure 33 (a) Schematic diagram of the acoustic source and sensors’ locations (b) Three passive piezoelectric sensors are placed on a composite plate
Figure 34 Anisotropy test results for sample #1
Figure 35 Anisotropy test results for sample #2
Figure 36 Anisotropy test results for sample #3
Figure 37 Anisotropy test results for sample #4
Figure 38 Anisotropy test results for sample #5
Figure 39 Anisotropy test results for sample #6
Although all three recorded acoustic signals for every sample are plotted in the same graph (on top of each other), there are some difficulties in identifying the degree of anisotropy with raw acoustic signals. Incorporating Akaike Information Criterion (AIC) based on information theory [40] is a way to evaluate quality of a given data so that the given signal can be transformed to AIC curve to compare shape of each AIC qualitatively. This technique is applied in the next section for accurately picking the arrival time of the acoustic waves. Maeda’s AIC implementation [28] is widely used,

\[
AIC(t) = t \cdot \log[\text{var}(S(1:t)) + (T - t - 1) \cdot \log[\text{var}(S(t + 1:T))]]
\]  

where \(S\) is the given signal, \(t\) is time, \(T\) is the total time of the given signal, and \(\text{var}\) is the variance defined as

\[
\text{var}(S(a:b)) = \frac{1}{b - a} \sum_{i=a}^{b} \left(S(i) - \frac{1}{b - a + 1} \sum_{i=a}^{b} S(i)\right)^2
\]

Figure 40 shows the original acoustic signal on the top and the corresponding AIC curve on the bottom. One can clearly recognize deeps and hills in the AIC curve at the moment of changing contents of the incoming signal, so it can be treated as a characteristic function of the given acoustic signal. All AIC curves have been calculated and plotted in Figure 41 to Figure 44. Detailed explanation for identifying degree of anisotropy can be found in the figure captions.
Figure 40 AIC (bottom) is computed from the given data (top). Time in the horizontal axis is not calibrated. Deeps at around $t=200$, $550$, and $1050$ represent changes in the contents of the acoustic signal. For example, a symmetric guided wave which is fast and has low energy arrive at $t=200$ and an anti-symmetric guided wave which has higher energy but slow arrive at $t=550$. Then various boundary reflected waves arrive at the piezoelectric sensor after $t=1050$. These patterns such as the number of deeps and hills can be utilized to identify the degree of anisotropy of the sample by comparing the positions of AIC dips and peaks for signals propagating in different direction of the sample.
Figure 41 Sample #1: AIC (bottom) is computed from the given data (top). Time in the horizontal axis is not calibrated. AIC patterns of the signals are very similar. This indicates that this composite sample is quasi-isotropic. The symmetric guided wave arrives at around $t=250$, but sensor 2 records slightly delayed arrival. The symmetric wave experiences anisotropy in the $45^\circ$-direction. The anti-symmetric guided wave arrives at $t = \text{about 600}$. Arrival times are very close therefore the anti-symmetric wave propagates almost along a straight line and forms almost a circular wave front.
Figure 42 Sample #2: AIC (bottom) is computed from the given data (top). Time in the horizontal axis is not calibrated. AIC patterns of the signals are very similar. It indicates this composite sample is also quasi-isotropic. The symmetric guided wave arrives at around t=230, but the sensors 1 shows slightly delayed arrival time. Therefore, the symmetric wave shows some anisotropy. The anti-symmetric guided wave arrives at t=600. Arrival times are very close for the anti-symmetric wave therefore this wave front should be close to a circular wave front.
Figure 43 Sample #3: AIC (bottom) is computed from the given data (top). Time in the horizontal axis is not calibrated. AIC patterns of the signals are similar until t=750. This indicates that this composite sample is quasi-isotropic. However, after t=750, sensors 1 and 3 maintain relatively higher AIC than sensor 2 due to the reflected waves from the free edge. (See Figure 36). The symmetric guided wave arrives at around t=250, but sensors 1 and 3 show slightly delayed arrival. The symmetric wave experiences anisotropy. The anti-symmetric guided wave arrives at t=600. Arrival times are very close for the anti-symmetric so this wave front should be close to a circular wave front.
Figure 44 Sample #4: AIC (bottom) is computed from the given data (top). Time in the horizontal axis is not calibrated. AIC patterns of the signals are quite different from one another over the entire time period. This indicates that this composite sample is highly anisotropic. The symmetric and anti-symmetric guided waves arrive at different times. Therefore, this sample is highly anisotropic and should have non-circular wave fronts.
Figure 45 Sample #5: AIC (bottom) is computed from the given data (top). Time in the horizontal axis is not calibrated. AIC patterns of the signals are different in three directions. This indicates this composite sample is anisotropic. The symmetric guided wave arrives at around t=250, but the sensors 1 and 2 show slightly delayed arrival. The anti-symmetric guided wave arrives between t=600 and t=750. Arrival times are quite different. Therefore, this sample is highly anisotropic and should generate non-circular wave front.
Figure 46 Sample #6: AIC (bottom) is computed from the given data (top). Time in the horizontal axis is not calibrated. Comparing AIC patterns of these signals to those shown in Figure 45, it can be stated that the cyclic thermal load significantly transformed the internal structure of sample #6 because AIC pattern is significantly different in this figure. AIC pattern in sample #6 are similar except in the time period between t=500 and t=700 where a deep in sensor 1 disappears unlike the other two sensors. In the original wave form, it shows similar energy level between the symmetric guided wave and the anti-symmetric guided wave in the 90-degree direction. This is the result of thermal load induced material degradation. The symmetric guided wave arrives at around t=250, but they are slightly separated. The symmetric wave experiences weak anisotropy. The anti-symmetric guided wave arrives between t=600 and t=750. Arrival times are different in different directions indicating strong anisotropy for the anti-symmetric wave mode and should produce non-circular wave front.
From the anisotropy tests, samples #4 and #5 are chosen for validating the proposed acoustic source localization technique since those are highly anisotropic plates. In addition, sample #6 will demonstrate that the proposed acoustic source localization technique is robust and reliable and should work equally well even when the material properties change due to cyclic thermal loads.

5.3 Improved time difference of arrival determination

An accurate determination of the arrival time of the incoming acoustic wave is critical for the acoustic source localization. This issue also arises in seismological applications for recording the arrival time of the primary wave (P-wave) and the secondary wave (S-wave). Although obtaining the arrival time (also known as “onset time picking”) can be carried out manually by human operators, it is prone to error since it relies on the human decision. To overcome this error-prone limitation, seismologists have developed automatic onset time picking techniques from raw data of seismic waves. They have proved that the AIC-picker is superior than manual picking or Hinkley-picker [29]. Maeda’s AIC implementation [28] described in Equation (50) is widely used for many onset time picking applications.

Taking advantage of the AIC method, a modified cross-correlation approach presented in Equation (52) has been adopted for enhancing the quality of arrival time picking.

\[
[F(t) \ast G(t)](\tau) = \int_{\text{lower bound}}^{\text{upper bound}} [F(t) \cdot W_F][G(t + \tau) \cdot W_G] \, dt \quad (52)
\]

where \(W_F\) and \(W_G\) are window filter functions,
\[ W_F(t) = H \left( t - t_{F}^{AIC} + \frac{W}{2} \right) - H \left( t - t_{F}^{AIC} - \frac{W}{2} \right) \]  

(53)

\[ W_G(t) = H \left( t - t_{G}^{AIC} + \frac{W}{2} \right) - H \left( t - t_{G}^{AIC} - \frac{W}{2} \right) \]  

(54)

\[ H(t) = \int_{-\infty}^{t} \delta(s) \, ds \]  

(55)

\( H(t) \) and \( \delta(t) \) are Heaviside step function and Kronecker delta function, respectively; \( w \) is the width of the filter. \( t_{F}^{AIC} \) and \( t_{G}^{AIC} \) are onset times of arrival, determined when AIC forms stiff valley, see Figure 47. In Figure 48(a), three acoustic signals from the sensors have been filtered by AIC based window filter defined in Equation (53). By adopting the new approach described in Equation (52), the improved time difference of arrival between sensors A and B is computed. \( \Delta t_{BA} \) at the peak is the time difference of arrival. Similarly \( \Delta t_{CA} \) for sensors A and C can be set. These cross-correlation calculation are plotted in Figure 48(b) and (c).

This section can be summarized with the following sentences. Highly anisotropic composite samples are selected from the anisotropy tests and an improved method to obtain an accurate time difference of arrival has been chosen. The next section addresses how experiments are designed.
Figure 47. \( t^\text{AIC}_F \) and \( t^\text{AIC}_G \) are determined from the local minima of AIC plot at the stiff valley. The first minimum value represents the arrival time of the symmetric guided wave and the second minimum value represents the arrival time of the anti-symmetric guided wave.
Figure 48 (a) Three acoustic signals from the sensors have been filtered by AIC based window filter defined in Equation (53). (b) Using the new method described in Equation (52), the improved time difference of arrival between sensors A and B is computed. $\Delta t_{BA}$ at the peak is the time difference of arrival. (c) Same as part (b) but for sensor A and C.
5.4 **Experiment design**

In order to demonstrate the proposed technique developed for rhombus and elliptical wave fronts, three samples are chosen from the anisotropy test results described in the previous section. These are sample #4 (the most anisotropic sample), sample #5 (highly anisotropic), and sample #6 (weakly anisotropic but internal composite structure has been changed by the thermal aging process). Total 12 piezoelectric sensors (20.3 mm in diameter) in 4 clusters are mounted on the sample surface as shown in Figure 49, Figure 50, and Figure 51, respectively, where the distance between two sensors in a cluster is set at 30 mm. Location details described in Cartesian coordinates are listed in Table 5. Dispersion over such distance is expected to be negligible. Three clusters (required number of clusters for the elliptical wave front) are mounted at each corner of the sample to meet large distance requirement from the acoustic source for justifying a single value of wave front velocity or plane wave approximation. At least 25 mm margin from the edge is required to minimize signal contamination due to reflections from the free edges or boundaries. In samples #5 and #6, all sensor positions are kept same to avoid position dependent errors. One additional sensor cluster (making total 4 clusters), is located in the first quadrant as a reference for the rhombus wave front analysis. A pencil-lead-break acoustic source is produced in the center of each sample that defines the origin of Cartesian coordinates of the experiments.
Figure 49 Schematic layout (not to the scale) of all sensor positions and the acoustic source for sample #4. The four solid circles show four sensor positions that are employed for signal synchronization.

Figure 50 Schematic layout (not to the scale) of all sensor positions and the acoustic source for sample #5. The four solid circles show four sensor positions that are employed for signal synchronization.
Figure 51 Schematic layout (not to the scale) of all sensor positions and the acoustic source for sample #6. The four solid circles show four sensor positions that are employed for signal synchronization.

Table 5 All sensor positions in the Cartesian coordinate whose origin is the center of each sample

<table>
<thead>
<tr>
<th>Units[mm]</th>
<th>Sample #4</th>
<th>Sample #5, #6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X Coord.</td>
<td>y Coord.</td>
</tr>
<tr>
<td>Source</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$S_{R}^{A}$</td>
<td>50</td>
<td>125</td>
</tr>
<tr>
<td>$S_{R}^{B}$</td>
<td>20</td>
<td>125</td>
</tr>
<tr>
<td>$S_{R}^{C}$</td>
<td>50</td>
<td>95</td>
</tr>
<tr>
<td>$S_{I}^{A}$</td>
<td>220</td>
<td>125</td>
</tr>
<tr>
<td>$S_{I}^{B}$</td>
<td>190</td>
<td>125</td>
</tr>
<tr>
<td>$S_{I}^{C}$</td>
<td>220</td>
<td>95</td>
</tr>
<tr>
<td>$S_{II}^{A}$</td>
<td>-200</td>
<td>-95</td>
</tr>
<tr>
<td>$S_{II}^{B}$</td>
<td>-230</td>
<td>-95</td>
</tr>
<tr>
<td>$S_{II}^{C}$</td>
<td>-200</td>
<td>-125</td>
</tr>
<tr>
<td>$S_{III}^{A}$</td>
<td>220</td>
<td>-95</td>
</tr>
<tr>
<td>$S_{III}^{B}$</td>
<td>190</td>
<td>-95</td>
</tr>
<tr>
<td>$S_{III}^{C}$</td>
<td>220</td>
<td>-125</td>
</tr>
</tbody>
</table>
Three piezoelectric sensors in a cluster must be synchronized to get precise time difference of arrival between different sensor clusters. The technique for elliptical wave front is free of the synchronization requirement for the three clusters because it only uses the incoming wave front’s angle then the optimization method comes in. However, the technique for the rhombus wave front assumes a constant shape velocity so that the time difference of arrival between clusters is critical when the velocity is computed. If at least one sensor from each of the four clusters, for instance \((S^A_I, S^A_{II}, S^A_{III}, S^A_R)\), is synchronized precisely, then it is possible to acquire time difference of arrivals between any two clusters. In Figure 49, Figure 50, and Figure 51, solid filled circles show the sensors that are employed for measurement synchronization during the system calibration process. Figure 52 depicts synchronized signals from the four sensors, \((S^A_I, S^A_{II}, S^A_{III}, S^A_R)\).
5.5 Experiment results

Using the experimental setup described in the previous section, all signals recorded by the twelve piezoelectric sensors have been stored and used in the new acoustic source localization algorithm. The sampling rate is 10 MHz. The algorithm configures the AIC plot from every raw time history data set to pick onset arrival time automatically as shown in Figure 53. In order to determine the time difference of arrival accurately, the cross-correlation of window-filtered signals is employed. Then the angle of the wave front is obtained from the ratio of two time differences as discussed earlier. Next, these angles are used in either the geometrical vector analysis for the case of rhombus wave front or the optimization process for the elliptic wave front. Finally, a single point for the acoustic source location is obtained.
Figure 53 All AIC plots are presented. The automatic onset time picking algorithm facilitates the proper determination of the arrival times with minimal human input. The black dots are onset times. Such a steep rise right after the sharp drop indicates some changes of the characteristics of the incoming acoustic waves, two distinct valleys are expected. The first valley indicates the arrival of the symmetric guided wave mode and the second valley shows the arrival of the anti-symmetric guided wave mode. However, only the reference sensor clusters ($S^A_R$, $S^B_R$, and $S^C_R$) captured both symmetric and anti-symmetric waves because the reference sensor cluster is the closest to the acoustic source. The other three clusters missed the second arrival since they are located further away from the acoustic source and the recorded time history is probably not long enough to capture the arrival of the second wave.
In Figure 54, Figure 55 and Figure 56, the estimated source location and relevant straight lines obtained from the vector analysis given in Equations (24) to (43) are plotted for the samples #4, #5 and #6, respectively. Three sensor clusters \((S_I, S_{II}, S_{III})\) are placed in three different quadrants and the reference sensor cluster \(S_r\) is placed in the third quadrant with \(S_I\). The analysis starts with the calculation of direction vectors, \(\vec{u}^\perp\), \(\vec{v}^\perp\) and \(\vec{w}^\perp\) at the three-sensor cluster positions. The rhombus wave front velocity or shape velocity \((\mu)\) is obtained from \(\vec{u}^\perp\) and TDOA value \(t_{r1}\) between \(S_r\) and \(S_I\). TDOA value \(t_{21}\) and \(\vec{u}^\perp\) allow us to obtain the first shifted line \((L'_1)\), then \(P_V\) is obtained, which is the intersection point of lines \(L'_1\) and \(L_2\). Then the vertical diagonal (the vertical solid line \(V_{tr}\) in Figure 28) of the rhombus wave front can be obtained by drawing a straight line which is parallel to the bisecting vector \(\vec{b}_V\) and going through point \(P_v\). In a similar manner with two sensors, \(S_I\) and \(S_{III}\), the horizontal diagonal (the solid line \(H_{tr}\) in Figure 28) can be drawn. Finally, the intersection point of the two diagonals is the estimated acoustic source location.

The measured direction vectors, \(\vec{u}^\perp\), \(\vec{v}^\perp\) and \(\vec{w}^\perp\) at the three-sensor cluster positions are varied during the tests due to the following uncertainties; 1) the sensor has a relatively large diameter \((20.3\ mm)\) so that arrival time uncertainty takes place in the experiment. 2) There are position uncertainties of all sensor due to the sensor size. So upper limits and lower limits of the incoming wave front’s angles for each direction vector are set during the calculation. Then thirty samples are randomly selected within the limits to confirm how much the measurement errors are. Figure 57, Figure 58, and Figure 59 show measurement errors in each sample. The black circles are the error induced in the source location predictions. The averaged source location is displayed as a yellow square with the
XY error bar of the standard deviations. Note that the prediction error [or the distance of the predicted point \( P_s^* \) from the actual acoustic source location (0, 0)] are only 5.98 mm, 2.98 mm and 13.42 mm, respectively.

The sample #6 is identical to sample #5, but it has been aged by a cyclic thermal load for a long time which created internal damages in the plate and changed its material properties. While conventional acoustic source localization techniques need to get additional information about the aged sample requiring additional time, resources and efforts, the technique proposed here is capable of detecting the acoustic source accurately despite the sample being damaged. From Figure 55 and Figure 56, one can conclude that the new technique is reliable and robust, and can handle changes in material properties and internal defects.
Figure 54 ASL prediction for sample #4 - the computed lines, vectors and points obtained from the vector analysis for the rhombic wave front are shown. The estimated location ($P_s^*$) of the acoustic source (4.7752 2.6728) is only 5.47 mm away from the true source location ($P_S$).
Figure 55 ASL prediction for sample #5 - the computed lines, vectors and points obtained from the vector analysis for the rhombic wave front are shown. The estimated location ($P_s^*$) of the acoustic source (-0.3486 - 0.1569) is only 0.38 mm away from the true source location ($P_s$).
Figure 56 ASL prediction for sample #6 - the computed lines, vectors and points obtained from the vector analysis for the rhombic wave front are shown. The estimated location ($P_s^*$) of the acoustic source (-0.8066 -12.9022) is only 12.93 mm away from the true source location ($P_s$).
Figure 57 Various predictions for sample #4 generated due to measurement errors are plotted as open circles while the original source location is shown by the diamond marker. The obtained source locations are dispersed within -40 and 60 mm in x coordinate while the y coordinate varies between -15 mm and 4 mm. The average source location (2.78, -5.30) of the scattered source positions is shown by a square marker where the XY error bars represent standard deviations, $\sigma_x = 29.56$ mm and $\sigma_y = 4.64$ mm. The averaged source location is only 5.98 mm away from the true source location (0,0).
Figure 58 Various predictions for sample #5 generated due to measurement errors are plotted as open circles while the original source location is shown by the diamond marker. The obtained source locations are dispersed within -1.2 and 0 mm in x coordinate while the y coordinate varies between -5.8 mm and 0 mm. The average source location (-0.44, -2.95) of the scattered source positions is shown by a square marker where the XY error bars represent standard deviations, $\sigma_x = 0.25$ mm and $\sigma_y = 2.44$ mm, respectively. The averaged source location is only **2.98 mm** away from the true source location (0, 0).
Figure 59 Various predictions for sample #6 generated due to measurement errors are plotted as open circles while the original source location is shown by the diamond marker. The obtained source locations are dispersed within -1.8 and 0.3 mm in x coordinate while the y coordinate varies between -16.5 and -10 mm. The average source location (-0.61, -13.41) of the scattered source positions is shown by a square marker where the XY error bars represent standard deviations, $\sigma_x = 0.40$ mm and $\sigma_y = 1.84$ mm, respectively. The averaged source location is only **13.42 mm** away from the true source location (0, 0).
6 Concluding remarks

6.1 Summary and research contribution

The research work presented in this dissertation shows how a new technique of acoustic source localization in highly anisotropic thin plates is developed. The proposed technique can accurately predict the impact point without knowing the plate’s material properties. The main contribution of this research to the acoustic source localization research community is that the proposed technique is the very first method where the analysis perspective has been changed from the consideration of the angular dependent group velocity of the propagating waves and their paths to geometric shapes of the wave fronts and their geometric properties. Thus it avoids the necessity of knowing material properties such as density, orthotropic elastic moduli and Poisson’s ratio as long as the basic requirements are met such as minimum number of sensors and location configurations. Since waves do not propagate along a straight line in anisotropic media all existing methods that assume straight line propagation of waves from the source to the sensor fail for highly anisotropic plates.

Numerical simulation results of the proposed techniques demonstrated that the existing difficulties on source localizing method for highly anisotropic plates are overcome by utilizing the wave front shape for source localization instead of considering the propagation path from the source to the sensor. In addition, the experimental investigation validated that the new technique is robust and reliable – it can handle changes in material properties due to aging and material degradation.
6.2 Recommendations for future works

- From Figure 53, one can see that the AIC algorithm was not able to pinpoint the arrival of the second wave front which is the anti-symmetric guided wave mode in sensors $S^A_1$, $S^A_II$, and $S^A_III$ probably because the recording time is not long enough to receive the second wave front. Once the full lengths of the recorded signals are delivered, the proposed technique for analyzing the elliptical wave front can be verified experimentally.

- As the new technique identifies acoustic source location without any need to have prerequisite information such as dispersion curves, time difference map or comprehensive simulation model, it is suitable to implement for in-situ applications. Once the angles of the incoming wave front are obtained, computation time is less than a few milliseconds. The developed algorithm can be integrated in the experimental test setup; then acoustic source localization will provide estimated source location in real time.

- Wave fronts in periodic structured plates may form some polygon which is different from rhombus or ellipse. Acoustic source localization technique for such periodic structures or for any anisotropic plate that generates wave fronts different from rhombus or ellipse equations presented in this dissertation will not work. However, in those cases new equations can be developed following similar approach presented here but specializing it to those specific wave fronts.
• The sensor cluster is comprised of three sensors that work for 2 dimensional structures such as plates. Taking one additional sensor in each cluster and forming four sensors clusters allows us to localize the acoustic source in 3-dimensional structures, following similar steps.
7 References


[40] https://en.wikipedia.org/wiki/Information_theory

[41] DV Hinkley, “Inference about the change-point for cumulative sum tests,” Biometrika 58, 509–523, 1971