ANALYSIS ON THE OPTIMUM GROUP SYNCHRONIZATION CODE OF TIROS SATELLITE

Xie Qiu-Cheng       Cao Jie
(Nanjing Aeronautical Institute)
Nanjing, China

ABSTRACT

In this paper, the group synchronization code (length n = 60 bit) of the TIROS Satellite was analysed. It seems to us the code isn’t optimization.

A series of optimum group sync codes (n = 60) have been searched out with error tolerance E = 1, 2, 3, 4, 5, 6 and 10, 12. Their error sync probabilities are less than the error sync probability of the TIROS code (from two times to two order of magnitudes about). These optimum or qansi-optimum codes will be presented for application in the second generation of the Meteorological Satellites of China.

KEY WORDS, Optimum Code, Group Synchronization Code, Error Synchronization Probability.

INTRODUCTION

The group sync code (length n = 60 bit) of the information transmission system of the TIROS Meteorological Satellite (USA) is as following,

1010,0001,0110,1111,1101,0111,0001,1001,1101,1000,0011,1100,1001,0101.

By hexi-decimal signs, this pattern can be abbreviated to “A116,FD71,9D83,C95”. The mane of this code is signed to S in this paper.

Is this a optimum group sync code?

That is a valuable or interest question.

When n = 60 bit, the number of binary codes (N = 2^n -1 = 2^{60} -1 ≈ 1.152921504E+18) is very large. Under existing calculation speed of digital computer, it is very hard to use the
classical exhaustion technique for searching out the optimum group sync code in such
great set of binary codes. Which like to fish for a little pin in the Pacific Ocean.

Fortunately, the code length (n = 60) is very near to the word length of m sequences
(n=63), so that we can try to use a confined exhaustion method for searching out quasi-
 optimum or suboptimum code within the bounds of several smaller sets of the truncated (or
cut-short) codes from the m sequences. Thereby, the work toad for searching the quasi-
 optimum or suboptimum group sync codes will be decreased in greatly.

A SHORT CUT

Just as an old Chinese saw was said. “Would rather coming home to weave a fishing
nets than standing along the sea coast to envy the fishes”. We can find a way after all.

The m sequences have three primitive polynomials as following,

\[ f_1(x) = x^6 + x^5 + x^2 + x + 1 \]  
\[ f_2(x) = x^6 + x^5 + 1 \]  
\[ f_3(x) = x^6 + x^4 + x^3 + x + 1 \]

Based on these polynomial and by a repeating technique, one by one, step by step, to
alternately cut out 3 bits from the m sequences (n = 63 bit), then we can get a set of the
truncated codes of length n = 60 bit. The total sum of these truncated codes is 189 (63x3).

For distinction, among the mentioned above 189 truncated codes, the first set of 63
codes do be signed as S which are generated by \( f_1(x) \), and the second set of 63 codes do
be signed as S which are generated by \( f_2(x) \), and the third set, S, by \( f_3(x) \). In S, S
and S, i (the order number of truncated codes) = 1, 2, 3, 4,...,63.

For comparison, under the bit error probability \( P_o = 0.1 \) and the error toelrance \( E = 1,2,3,4,5,6 \) and 10,12 separately, the error sync probabilities of mentioned cut-short
codes have been calculated by computer progrom according to the formula (4).

\[
P_{fs} = 2 \sum_{k=0}^{n} \sum_{l=0}^{\min(k,E)} \left( \sum_{j=0}^{\min(k,\frac{E}{i})} \binom{k}{i} \binom{k-j}{i-j} (1-P_o)^{k-j} P_o^{k-j+2i-j} \right) 
\]

Where

E = number of error tolerance
\( P_o \) = the probability that the element of group sync code will be changed by noise
\( \psi(K) = \) agreement vector
\( n = \) the length of group sync code.

**RESULTS AND COMPARISON**

Among the above mentioned 189 cut-short codes, according to their error sync probabilities \( P_e \) from small to large sequencely, under \( E = 1, 2, 3, 4, 5, 6 \) up to \( E = 10 \) and 12, the first good code (the best code) are \( S_{16/II} \), and the \( S_{21/I} \) (i.e. TIROS group sync code) is No.156 (at \( E = 1 \)) No.149 (at \( E = 2 \)), No.149 (at \( E = 3 \)), No.143 (at \( E = 4 \)), No.149 (at \( E = 5 \)), No.137 (at \( E = 6 \)), No.129 (at \( E = 10 \)), No.123 (at \( E = 12 \)).

The pattern of \( S_{16/II} \) (the best code) is as following,

\[
1111,0101,0110,1110,1101,0010,0111,0001,0111,1001,0100,0110,0001,0000.
\]

By hexadecimal sign, this pattern can be abbreviated to “F566, ED27, 1794, 810”.

The autocorrelation function of this best code (\( S_{16/II} \)) is,

\[
-1, -2, -3, -4, -1, -2, -1, -2, -5, 0, -3, 2, -1, -2, -1, -4, -3, 0, -5, 2, -5, 0, -1, 2, 1, -2, -3, 2,
-7, 2, 7, -2, -5, 2, -1, 2, -3, 0, 1, 6, 1, -4, 3, 0, -3, 2, 1, 2, -1, 4, 1, 2, 1, 2, 5, 0, -1, -2,-34, 60.
\]

The autocorrelation function of \( S_{21/I} \) (the group sync code of TIROS) is,

\[
1, -2, 3, -4, 3, -4, 1, 2, -5, 4, -1, 4, -1, 4, -1, 0, -5, -2, 5, -2, 3, 2, -3, 2, 1, 0, -1, 4, -9, 0, 5,
0, -7, 0, -1, -2, 3, -6, 5, 6, -1, -2, -7, 0, -7, 0, -3, 4, -3, -4, 5, -6, -1, -4, 1, 0, -3, 2, -1, 60.
\]

The limit of subpeak of the autocorrelation function of the best code (\( S_{16/II} \)) is (-7, 7) and \( S_{21/I} \) is (-9, 6).

The comparison of the error sync probabilities for both codes (\( S_{16/II} \) and \( S_{21/I} \)) are listed in Table.
Table 1. The comparison of $P_{f_2}$

<table>
<thead>
<tr>
<th>$E$</th>
<th>$S_{21/I}$ (TIROS)</th>
<th>$S_{16/II}$ (Best)</th>
<th>Rate $P_{f_{2a}} / P_{f_{2/I}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5808E-15 (No.156)</td>
<td>0.2082E-16 (NO.1)</td>
<td>27.90</td>
</tr>
<tr>
<td>2</td>
<td>0.1786E-13 (No.149)</td>
<td>0.7672E-15 (No.1)</td>
<td>23.28</td>
</tr>
<tr>
<td>3</td>
<td>0.3618E-12 (No.149)</td>
<td>0.1848E-13 (No.1)</td>
<td>19.58</td>
</tr>
<tr>
<td>4</td>
<td>0.5425E-11 (No.143)</td>
<td>0.3280E-12 (No.1)</td>
<td>16.54</td>
</tr>
<tr>
<td>5</td>
<td>0.6421E-10 (No.149)</td>
<td>0.4574E-11 (No.1)</td>
<td>14.04</td>
</tr>
<tr>
<td>6</td>
<td>0.6245E-9 (No.137)</td>
<td>0.5226E-10 (No.1)</td>
<td>11.95</td>
</tr>
<tr>
<td>10</td>
<td>0.1234E-5 (No.129)</td>
<td>0.1945E-6 (No.1)</td>
<td>6.35</td>
</tr>
<tr>
<td>12</td>
<td>0.2678E-4 (No.123)</td>
<td>0.5778E-5 (No.1)</td>
<td>4.64</td>
</tr>
</tbody>
</table>

CONCLUSION

(1). When $E$ changes from 1 to 12, the TIROS group sync code $S_T$ (i.e. $S_{21/I}$) is all not first good code in the sets of the above mentioned truncated codes. The error sync probability $P_{f_{2/I}}$ of $S_T$ is all greater than $P_{f_{2/I}}$ of the best code $S_{16/II}$ by 28 times to 7 times approximately. So, the TIROS group sync code $S_T$ (i.e. $S_{21/I}$) is really not a optimum group sync code in the set of binary codes of length $n = 60$ bit.

(2). At different $E$, the first good code is generated by the primitive polynomial $f_2(x)$, and $S_T$ (i.e. $S_{21/I}$) is generated by $f_1(x)$, $f_1(x)$ can not get better codes. Perhaps, we would surmise that the designer of $S_T$ is not careful consideration when to choose the group sync code.

(3). Sence the searching work is only carried on the partial set of the binary codes for $n = 60$ bit, it goes without saying that the first good code $S_{16/II}$ is probably not a optimum group sync code but quasi-optimum or suboptimum group sync code. However, this code ($S_{16/II}$) is better than TIROS group sync code $S_T$, and so, this code ($S_{16/II}$) will be recommended for application in the information transmission systems of the second generation of Meteorological Satellites China.

REFERENCE