DESIGN OF OPTIMAL NYQUIST AND PARTIAL RESPONSE FIR DIGITAL FILTERS USING LINEAR PROGRAMMING TECHNIQUES

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ABSTRACT

The design of a Nyquist filter for generating a band-limited pulse for data transmission with the zero intersymbol interference is formulated as a linear programming (LP) problem and the Steiglitz program [18] is modified and then used to design this type of pulse shaping filters. The advantage of the present approach, as compared to other methods, with regard to design speed and filter optimality, are described, and illustrated by means of examples.

1. Introduction

The design of a Nyquist filter for generating a band-limited pulse for data transmission with the minimum intersymbol interference has always been an important subject [1-4]. Due to the recent developments in advanced large-scale integrated technology, digital Nyquist filters are playing an important role in digital modem systems [5-8]. By digital design techniques, a pulse shaping transversal filter with an exact zero crossing impulse response, which corresponds to zero intersymbol interference, can be obtained. Let \( H(z) \) denote the transfer function of a digital Nyquist filter, and \( X(z) \) is the transform of input data signals (i.e., input pulse train) whose speed is the Nyquist rate \( F_n \) and the sampling rate for \( H(z) \) and \( Y(z) \) (the transform of output data signals) is \( F_r \), where \( F_r = MF_n \), \( M \) an integer and \( z = e^{j2\pi F/F_r} \). The frequency response of the digital Nyquist filter will have normalized band edges [9]

\[
F_p = \frac{1 - \varphi}{2M},
\]  

(1a)
where \( F_s = \frac{1 + \varphi}{2M} \),

(1b)

where indicates the rolloff rate. The desired FIR Nyquist filter needs to have an impulse response \( h(n) \) exactly zero crossing at the Nyquist rate except for one point \( n = K \).

Nakayama and Mizukami [8] proposed a two-step method to design the Nyquist filter described above. The first step is to design the optimal filter by approximating the desired frequency response directly using the Remex algorithm [10] without time constraint on the filter coefficients. The second step is to apply the iterative Chebyshev approximation methods in [11] to design the desired Nyquist filter by using the coefficients of the optimal filter obtained in the first step as the initial values of the iterative nonlinear optimization algorithm for finding the Chebyshev solution, where the coefficients \( h(n) \) are modified to zero for \( ((n-K_0)_M = 0 \) and \( n \neq K \).

In this chapter, the design of linear phase FIR filters with some of the coefficients constrained to be zero is considered.

The filter design problem is formulated as a linear program so that the LP approach can be employed to design the filter. Examples are presented to illustrate the concept and the efficiency of the design techniques.

In Section 2, the LP formulation and solution of the constrained FIR filter design problem is presented, and it is found that LP techniques are particularly suitable for designing the above types of constrained FIR filters due to their high degree of flexibility. In Section 3, the design of optimal Nyquist and Class 1 partial response FIR pulse shaping filters, both with zero intersymbol interference is considered, and the design efficiency is compared with that of other design methods. Concluding remarks are presented in Section 4.

2. Linear Programming Solution of Constrained - Linear Phase FIR Filter Design

Let the transfer function of a linear phase constrained FIR filter with odd-length \( N \) be of the form

\[
H(z) = \sum_{n \in \mathbb{Z}_c} h(n) z^{-n}
\]

(2)

where \( \mathbb{I}_c \) denotes the set of indices of \( h(n) \)'s which are constrained to be zero. The magnitude response \( H^*(F) \) of the constrained filter can be written in the form [12]
where, and henceforth, we use the abbreviated notation \( P(F) \) for \( P(e^{j2\pi F}) \) obtained from a given function \( P(z) \), and \( h(n) \) can be derived from \( \tilde{h}(n) \) as

\[
\tilde{h}(0) = 2h\left(\frac{N-1}{2}\right) \neq 0 \quad \text{and} \quad \tilde{h}(n) = 2h\left(\frac{N-1}{2} - n\right), \quad n = 1, 2, \ldots, \frac{N-1}{2}, \quad n \notin I_c.
\]

The number of filter coefficients becomes

\[
NFILT = N - N_c
\]

Where \( N \) is the number of elements in \( I_c \) and the number of independent \( \tilde{h}(n) \)’s is thus \( M = (NFILT + 1)/2 \).

The design of a constrained FIR filter described by (2) consists of finding \( \tilde{h}(n) \)’s, \( n = 0, 1, 2, ..., \frac{N-1}{2}, n \notin I_c \), such that \( H^*(F) \) in (3) is the best approximation to the desired frequency response, where by the best approximation we mean the one which minimizes the maximum absolute error between \( H^*(F) \) and \( D(F) \) over the frequency bands to which the approximation applies.

By means of (3), the above design objectives may be formulated as a linear programming problem. By taking \( N_q \) grid points from 0 to \( 2\pi \) radians/sample (usually a grid density of 16 is used), the magnitude response of the filter at the grid point \( k \) is

\[
H^*(F_k) = \sum_{n=0}^{(N-1)/2} \tilde{h}(n) \cos(2\pi n F_k), \quad k = 0, 1, \ldots, N_q/2
\]

Let \( D(F_k) \) and \( W(F_k) \) denote respectively the desired frequency response and the desired weighting value for the approximating error at the grid point \( k \), and let \( \delta \) be the maximum allowable approximation error. Accordingly, it has to satisfy the following set of linear inequalities [13]

\[
-\delta \leq W(F_k) [D(F_k) - H^*(F_k)] \leq \delta, \quad k = 0, 1, \ldots, N_q/2.
\]
Taking into account the fact that $H^*(F)$ is a linear combination of $r$ cosine functions, we may use (7) to formulate the following linear program:

\[
\text{Maximize } (-\delta),
\]

Subject to:

\[(8b) \sum_{n \neq F_c} H(n) \cos(2\pi n F_c) - S \leq W(F_c) D(F_c), \quad k = 0, \ldots, N/2
\]

\[(8c) \sum_{n \neq F_c} H(n) \cos(2\pi n F_c) - S \geq W(F_c) D(F_c), \quad k = 0, \ldots, N/2
\]

This is the “primal problem” with variables $\tilde{h}(n)$’s, $n = 0, 1, \ldots, (N-1)/2, n \in I_c$ and $(-\delta)$. By the duality principle, (8) can be shown to be mathematically equivalent to the “dual problem”, which is a linear program in standard form. The standard form is the most natural form for digital filter design, and thus is commonly used for obtaining the desired numerical solution. The primal problem is transformed into the dual problem by replacing each inequality in (8) by an equality and a slack (nonnegative) variable. Since the nonnegativity constraints may be taken into account without increasing the volume of computations, the preceding transformation may be interpreted as replacing an inequality by an equality at the cost of adding one variable. The dual problem of (8) has one equality constraint for each of the unconstrained variables $\tilde{h}(n)$’s = 0, 1, ...., $(N-1)/2, n \in I_c$, $-\delta$ in (8) and one nonnegative variable for each of the inequality constraints in (8).

The solution to the above LP problem with $(r+1)$ variables and $N_q/2+1$ inequality constraints occurs when at least $(r+1)$ of the $N_q/2+1$ equations are solved with equality (instead of inequality); the remaining inequalities being strict with inequalities. For the optimal filter design problem this implies that there are at least $(r+1)$ frequencies at which the ripple achieves a maximum. The number of variables in the linear program of (8) is $(r+1)$, where $r$ independent coefficients and $\delta$ are variables.

It is usual in these problems to solve the dual problem by the revised simplex algorithm [14]. The tableau used in the revised simplex algorithm for solving the dual problem can be obtained from (8). Based on this tableau, the steps involved in the revised simplex algorithm at each pivot are outlined as follows:

Step 1: Determine an initial program;
Step 2: Price each column by calculating its relative cost $\tilde{c}_j$. If no $\tilde{c}_j$ is negative, the present program is the optimal solution; if not, change the basis; the minimum $\tilde{c}_j$ determines the column $k$ to enter the basis so as to maximize the absolute change of the objective function. This is equivalently accomplished by using the maximum entry criterion.

For implementing the design procedure on a digital computer, a suitable small number must be used in this step to test the optimality of the program. If it is too small, invalid pivots will be induced due to the accumulated roundoff errors; if it is too large, valid pivots will be overlooked and suboptimal solution produced;

Step 3: The secondary variable $x_k$ associated with column $k$ becomes a basic variable and basic variable $x_l$ associated with column $l$ becomes a secondary variable. Generate column $k$ by using an $m \times m$ inverse-basis matrix which is carried along from pivot to pivot. The quantities associated with the old pivot is transformed to the corresponding quantities associated with the new pivot;

Step 4: Use the usual ratio test to choose a row $l$ which then determines the column which is to leave the basis by the minimum exit criterion. If a tie occurs, break the tie by choosing one corresponding to the largest pivot from among the rows where the tie occurs. Same as pricing operation, a suitable small number must be used in the ratio test;

Step 5: Update the $m \times m$ inverse-basis matrix by pivoting operation, then go to step 2.

The two-phase method is used in the program, where in phase 1, one determines whether any feasible solution exists and obtains one if it does exist, and in phase 2, one proceeds from a feasible solution determined by phase to the optimal solution. The program in [18] is modified to allow the design of the constrained filter formulated in (8).

It must be observed that the solution obtained through the LP approach is exactly the same as the one obtained using the Remex algorithm. The well-known simplex algorithm used in the solution of the linear programming problems, can be viewed as a single exchange algorithm. Thus, the linear programming formulation leads to an algorithm less efficient than the Remez multiple exchange algorithm. Both the linear programming approach has the advantage of being more flexible. Its flexibility is based essentially on the fact that other constraints can be considered in addition to (8). For designing the constrained filter discussed in this paper, the constraint inequalities in (8) keep the same form as that in the unconstrained filter design. Details on the revised algorithm for solving the dual program can be found in the references on linear programming (see, e.g., see [15].
3. Design of Pulse Shaping FIR Filters

3.1 Design of FIR Nyquist Filters with Zero Intersymbol Interference

Let the transfer function of the FIR Nyquist filter be \( H(z) \) in (2) and the desired frequency response be \( D(F) \). In the design of \( H(z) \), \( D(F) \) takes the following values:

\[
D(F) = 1 \quad F \quad 0 \quad F \quad \frac{1}{2M} \quad ,
\]

\[
= 0 \quad , \quad \frac{1}{2M} \quad F \quad 0.5 \quad ,
\]

(9)

with the constraints:

\[
h(n) = 0 \text{ for } \left( (n - \frac{N-1}{2}) \right)_{M} = 0 , \text{ and } n = \frac{N-1}{2} , \quad n = 0,1, \ldots, N-1 ,
\]

(10)

where \((x)_M\) denotes modular operation with \( \text{mod} \ M \). The design of this digital filter can be represented as the following minimax approximation problem with (10) as constraints on filter coefficients

\[
\overline{W}(F) \left| \frac{H(F)}{1 - 1} \right| \leq S , \quad 0 \leq F \leq F_{r} ,
\]

(11a)

and

\[
\overline{W}(F) \left| \frac{H(F)}{1} \right| \leq S , \quad F_{s} \leq F \leq 0.5 ,
\]

(11b)

where \( W(F) \) is the weighting function for controlling the ripples in the passband and stopband.

The constrained approximation problem of (10) and (11) can be solved by using the LP techniques described in Section 2.

3.2 Optimal Design of Class 1 Partial Response Filters

Time domain Class 1 partial response data transmission systems have received considerable attention recently since they can be used at an increased bit rate under a prescribed available bandwidth for data transmission [16]. It has been shown that the bit insensitivity of the partial response system is such that the sampling rate of the system can be varied by 43 percent for the given filter configuration without closing the three-level eye pattern [9].

The impulse response of a digital Class 1 partial response filter should be large and have the same value at main two adjacent sample points and the response at other sample points
should be zero for zero intersymbol interference if the process of decision directed
cancellation of intersymbol interference is used [17]. Let the transfer function of the FIR
Class 1 partial response filter be denoted by \( H(z) \). Then \( H(z) \) can be designed by
approximating the desired frequency response under the following constraints on the filter
coefficients:
\[
\begin{align*}
h(n) &= 0, (n - \frac{N-1}{2} - \frac{3M}{2}) M \geq n \geq \frac{N-1}{2} + \frac{3M}{2}, \text{ or } n \leq \frac{N-1}{2} - \frac{3M}{2}, \quad (12)
\end{align*}
\]
where \( M \) is defined in (5). The filter design can be formulated as the following minimax
approximation problem:
\[
\begin{align*}
\mathcal{W}(F) |H(F)| - 1 \leq \delta, \quad \forall \omega \in \Omega \\
\mathcal{W}(F) |H(F)| \leq \delta, \quad \forall \omega \leq 0.5 
\end{align*}
\]
where \( \delta = 1/(2M) < \mathcal{W}(F) \) is the weighting function for controlling the ripples in the
passband and stopband, and \( \delta \) is the maximum weighted approximation error. The LP
techniques described in Section 2 can again be employed to design these Class 1 partial
response filters.

3.3 Design Examples

The first two examples are from [8]. A DEC PDP11/55 computer with double precision
was used for all calculations in designing the filters in this paper.

Example 1:

An FIR Nyquist filter with length \( N = 23 \), \( = 0.3 \), \( M = 4 \) was designed.

The band edges \( F_p \) and \( F_s \) of the filter are calculated from (1) as \( F_p = 0.0875 \) and
\( F_s = 0.1625 \) and the constraint in \( I_c \) are obtained from the given \( N \) and \( M \) values. The LP
technique was modified and thus used to design the filter. Filters with different passband
and stopband ripples were designed by controlling the weighting values of passband and
stopband ripples in (8). The impulse response and frequency response of a resulting filter
with stopband ripple \( A_s = 38.0 \) dB and passband ripple 0.34 dB are shown in Fig. 1(a) and
Fig. 1(b) respectively.

The corresponding optimal filter with length 23 and stopband ripple \( A_s = 38.0 \) dB has a
passband ripple of 0.19 dB which is less than that of the above Nyquist filter. This is
because some of the coefficients of the latter filter with small values but not exactly equal to zero are constrained to zero.

Fig. 1(a) Impulse response of the filter designed in Example 1.

Fig. 1(b) Amplitude response of the filter designed in Example 1.
EXAMPLE 2:

A Nyquist filter same as in Example 1 was designed but with the parameters: $N = 39, q = 0.15$ and $M = 4$. Nyquist filters corresponding to different passband and stopband ripples were designed. Figs. 2(a) and (b) show respectively the impulse response and frequency response of a resultant filter with stopband ripple $A = 33.0$ dB and passband ripple $0.45$ dB. The corresponding optimal filter with length 39 has a passband ripple of $0.35$ dB with the same stopband ripple.

Fig. 2(a) Impulse response of the filter designed in Example 2.

Fig. 2(b) Amplitude response of the filter designed in Example 2.
Example 3:

A Class 1 partial response FIR filter with the following parameters was designed: \( N = 23 \) and \( M = 4 \).

The filter with \( F = 0.125 \) was designed to have equal passband and stopband ripples. Fig. 3(a) and (b) show the impulse response and frequency response with a stopband ripple of 33.2 dB and a passband ripple of 0.19 dB.

3.4 Comparison with the Iterative Chebyshev Approximation Method [8, 11].

In our algorithm, when designing a filter with appropriate filter coefficients constrained to zero, it is only required to run the LP program one time for obtaining the zero intersymbol interference pulse shaping filter with the desired ripple ratio.

Fig. 3(a) Impulse response of the filter designed in Example 3.
4. Concluding Remarks

We have presented LP techniques for the design of digital Nyquist and partial response FIR filters with zero intersymbol interference, which are used in data transmission systems. The advantage of this approach over the two-step iteration Chebyshev method is its high design speed. Examples have illustrated that the LP techniques are suitable for the design of FIR pulse shaping filters with zero intersymbol interference.

REFERENCES


