Summary - Global distress location and search and rescue operation may utilize Omega VLF navigation signals for position determination. The rescue radio would retransmit the Omega signals to a satellite for relay to a search and rescue center, where the position of the retransmitter would be determined. Since only the phases of the Omega signal are required, preprocessing prior to retransmission can have several advantages, including reduced bandwidth, transmission time, transmitter power, antenna size, and error rate. Both phase measurement and averaging can be accomplished by a simple counting phase detector. The characteristics of an Omega preprocessor using a set of counting detectors are described.

Introduction - Preprocessing of the Omega signals received by a distress location/rescue radio provides an alternative to linear retransmission such as was used in the Omega position location experiment. There is an important difference between full processing and preprocessing. Full processing, as in an Omega receiver, must produce actual phase estimates for position finding. Preprocessing need only produce data which can be used to produce the actual phase estimates. This allows a preprocessor to be considerably less complex than an actual Omega receiver.

A preprocessor must reduce the received Omega signals to a small number of bits of data containing information about the signal phases and carrier-to-noise ratios. The use of digital data will allow considerable flexibility in retransmission. Data may be transmitted slowly in a small bandwidth, or rapidly in bursts in a wide bandwidth, as fits the channel. Transfer from one relay link to another can be accomplished easily and with negligible corruption of the data.

Centicycle accuracy in a phase measurement requires retransmission of at least 7 bits \((2^7 = 128)\). Global position finding will require measurement of the phases of five frequencies from each of three Omega stations. Thus, as few as 105 bits can identify the location. However, an actual preprocessor is likely to transmit more than the minimum 105 bits. To reduce basic measurements to the minimum number of bits would require a complex processor, capable of deciding which stations to monitor and how to correct for such things as drift, clock frequency errors and carrier-to-noise ratio.

Counting Detector Concept - A digital counter can both detect and average the phase of a received Omega carrier. A local reference signal starts a counter, which is toggled at a frequency 100 or more times the carrier frequency (Figure 1). The received Omega signal is clipped into a logic-like signal which stops the counter. The number of counts is then proportional to the phase.

This work was supported under Air Force Contract F33657-72-C-1066.

Dr. Raab is with Cincinnati Electronics Corporation.
difference between the reference and the received signal. Both the local reference and the counting frequency can be derived from the same stable oscillator (clock).

Omega transmissions have duration of approximately one second, which allows a noise bandwidth of approximately 1 Hz. However, the passive filters used in the front end of an Omega receiver have a nominal bandwidth of 100 Hz. The preprocessor must, therefore, take a number of samples and average them to produce the 1 Hz bandwidth. The nominal 100 Hz bandwidth suggests that approximately 100 uncorrelated phase samples can be taken during a one second transmission.

A sampling rate of 100 per second can be accomplished simply by making the local reference signal 100 Hz. A phase sample will be taken whenever a rising edge of the reference signal occurs (Figure 2). All of these samples must be coherent; i.e., in the absence of noise, all phase samples must be the same. To ensure this, the reference or sampling frequency must be an integral submultiple of the received frequency. Both the 10.2 and 13.6 kHz Omega carriers can be sampled at exactly 100 Hz because it is an integral submultiple of each (f/102 and f/136, respectively). The 11.3 kHz and proposed 10.88 kHz carriers must be sampled at slightly different frequencies, for example, 101.19 Hz (f/112) and 100.74 Hz (f/108), respectively.

The counter is set to zero only at the beginning of the one second interval. Successive phase samples simply add to the previous samples. At the end of the one second interval, the counter has a number stored in it proportional to the sum of the counts in each sample. If scaling and quantizing are ignored, this sum can be described by

\[ s(k) = y(1) + y(2) + \ldots + y(k), \quad (1) \]

where \( y(j) \) represents the \( j \)th phase sample. The average phase is then recovered by division by the number of samples:

\[ \bar{y} = \frac{1}{k} s(k) . \quad (2) \]

If 100 samples are taken during a one second transmission and there are 100 counts per cycle, the number of counts cannot exceed 10,000 (214 = 16,384). However, only 7 or 8 of the 14 bits must be transmitted. The others are insignificant because of the noise errors.

In the absence of noise, the average measured by the counter is the true phase difference between the received signal and the referenced signal. If the atmospheric noise is Gaussian, the carrier-to-noise ratio \( R \) is good (= 10 dB or better), and the true phase \( x \) is not near the ends of the measurement range (0 and \( 2\pi \)), the phase noise \( n(k) \) is essentially additive Gaussian noise:

\[ y(k) = x + n(k) \quad (3) \]

\[ \bar{y} = x + \frac{1}{k} \left[ n(1) + n(2) + \ldots + n(k) \right] . \quad (4) \]

The variance of the noise in a particular sample is

\[ \sigma_n^2 = \frac{1}{2R} , \quad (5) \]

where \( R \) is measured in the 100 Hz bandwidth input to the processor. The averaging process reduces the noise variance according to

\[ \bar{\sigma}_n^2 = \frac{1}{k} \sigma_n^2 . \quad (6) \]
One can see that 100 samples taken during a 1 second interval with a 100 Hz bandwidth is equivalent to a 1 Hz noise bandwidth.

In general use, preprocessing of Omega signals with carrier-to-noise ratios as low as 0 or -5 dB (in the 100 Hz bandwidth) may be required. At these carrier-to-noise ratios, gaussian atmospheric noise produces non-gaussian phase noise. In addition, actual VLF noise is impulsive, rather than gaussian. An analytical description of this impulse noise was not available. In addition, the averages of a large number of samples tend to be gaussian. A gaussian noise model was therefore used for its analytical convenience in the preliminary evaluation described here. Since in final processing, a computer converts preprocessor data to phase data using a noise statistics table, it will be possible to change the noise model quite easily.

The generalized phase probability density for gaussian noise with carrier-to-noise power ratio $R$ is given by Beckman \(^1\) as

$$p(\varphi) = \frac{1}{2\pi} e^{-\frac{R}{2}} \left[ 1 + G \frac{e^{G^2}}{\sqrt{\pi}} (1 + \operatorname{erf} G) \right], \quad (7)$$

where

$$G = \sqrt{R} \cos \varphi \quad (8)$$

and the true phase $x$ is zero. For other values of $x$, the probability density for a given measured phase $y$ can be found by using the mapping

$$y = \left\langle y - x \right\rangle \quad (9)$$

where the operator $\langle \rangle$ adds or subtracts $2\pi$ as necessary to insure that

$$0 \leq y \leq 2\pi \quad (10)$$

Note that when the true phase is nearly aligned with 0 or $2\pi$, some measurements will be near 0 and some near $2\pi$, producing an erroneous average of $\pi$, even for good carrier-to-noise ratios. These effects are included in this density.

The characteristics of this phase detector with gaussian atmospheric noise can now be evaluated. The expected value of a phase sample or average can be determined by evaluating

$$\mu = \mathbb{E}[y] = \frac{1}{2\pi} \int_0^{2\pi} y p_y(y) dy. \quad (11)$$

The integration is most easily done by numerical techniques. Simpson's rule integration was used, and full details are described in (2). The resulting curves for $\mu$ as a function of $x$ and $R$ are shown in Figure 3. Note that at high values of $R$, the curves follow closely the $\mu = x$ line. For all values of $R$, as the phase value nears 0 or $2\pi$, the average is pulled toward $\pi$.

The variance of the phase increases not only as $R$ decreases, but as $x$ nears 0 or $2\pi$. The variance was also evaluated numerically, and the standard deviation is plotted in Figure 4. Note that as carrier-to-noise ratio improves, $\sigma$ stays relatively constant except when $x$ is close to 0 or $2\pi$. For very low values of $R$, $\sigma$ approaches that of a uniform phase density, showing little dependence on signal phase. Inaccuracies in this preprocessing technique (other than errors due to atmospheric noise) arise from quantization and the non-linear relationship between true phase and average phase. The maximum quantization error that can occur is

$$\left| \frac{n}{Q} \right|_{\text{max}} = \frac{2\pi}{M}, \quad (12)$$
where $M$ is the number of counts per cycle. A noisy signal will cause some quantization errors to be positive and some to be negative, resulting in an average quantization error much smaller than the maximum.

Evaluation of errors due to the non-linear phase-to-phase average transfer curve requires determination of the slope $\frac{\partial \mu}{\partial x}$ for specified values of $R$. Note that a noise error $\Delta y$ will convert to a phase error $\Delta x$ according to

$$\Delta x = \frac{\Delta y}{\partial \mu / \partial x},$$

so the phase estimate error variance will be

$$\sigma_x^2 = \frac{\sigma_y^2}{(\partial \mu / \partial x)^2}. \quad (13)$$

A decrease in the slope then causes an increase in the estimation error variance.

The slope can be evaluated by differentiation of (11), since all functions are continuous.

$$\frac{\partial \mu}{\partial x} = \frac{1}{2\pi} \int_0^{2\pi} y \frac{\partial p_y(y)}{\partial x} \, dy. \quad (15)$$

This is a straightforward (although somewhat painful) numerical integration, and is described in (2). The resulting values of $\frac{\partial \mu}{\partial x}$ were combined with $\sigma_y$ for various $x$ and $R$ values and the resulting estimation error standard deviation is shown in Figure 5. It is apparent that the most accurate measurements are obtained when the phase is close to $\pi$, and the least accurate measurements occur when the phase is at a turning point on the transfer curves.

It is also apparent that a single counting detector is not sufficient to guarantee accurate phase estimates. The use of two or more counters operating at different reference phases will allow accurate measurements to be made by at least one of two counters. The amount of data which must be averaged to produce a specified accuracy with a given number of counters can now be determined. From Figure 5, the worst case equivalent $\sigma_{X1}$ for a single phase sample with $R = -5$ dB is 5.36 for two counters and 2.52 for three counters. The standard deviation of the average can be found by dividing these numbers by the square root of the number of samples. To guarantee a $\sigma_{X1}$ of one centi-cycle ($\frac{2\pi}{100}$) requires 7285 samples (about 12.1 minutes) with two counters and 1610 samples (about 2.7 minutes) with three counters. The use of three counters is thus preferred for the nominal three minute data collection time in this application.

**Estimating the carrier-to-noise ratio** – There are two related questions which must be answered before the three phase averages can be converted to a phase estimate. First, the most accurate (mid-range) average must be identified. Secondly, if the carrier-to-noise ratio is less than about +5 dB, it must be estimated to allow an accurate phase estimate to be made. Three techniques for resolving these problems which have been considered are average absolute difference, interval variance and curve fitting.

The average absolute difference technique adds some simple circuitry to the preprocessors. This circuitry determines the absolute difference between two successive samples and accumulates a sum of the absolute differences. The expected value of the absolute difference increases as the carrier-to-noise
ratio decreases or as the phase approaches $0$ or $2\pi$, and can therefore be used to identify the most accurate counter and to estimate the carrier-to-noise ratio. While any measure (such as squared difference) of the difference between phase samples can be used to estimate the carrier-to-noise ratio, absolute difference can be implemented more easily.

Evaluation of the expected value of the absolute difference is necessary if $R$ is to be estimated. The average absolute difference is generated by the following operations:

\[
\begin{align*}
a(2) &= |y(2) - y(1)| \\
a(3) &= |y(3) - y(2)| \\
\ldots \\
\bar{a} &= \frac{1}{k-1} q(k).
\end{align*}
\]

(16)

(17)

First, it is necessary to evaluate the probability density $p_a(a)$, from which the statistics of $a$ can be determined. This requires a convolution interval. Noting that a value $a$ can arise two ways:

\[
a = \begin{cases} 
  y_1 - y_2, & y_1 \geq y_2 \\
  y_2 - y_1, & y_1 < y_2
\end{cases}
\]

(18)

the convolution integral becomes

\[
p_a(a) = \int_0^{2\pi} p_y(y) \left[ p_y(u-a) + p_y(u+a) \right] du.
\]

(19)

This is a straightforward numerical integration, but it must be done for many different values of $x$ and $R$.

The next step is evaluation of the expected value

\[
E \left[ a \right] = a = \frac{1}{2\pi} \int_0^{2\pi} a \cdot p_a(a) \, da,
\]

(20)

which is graphed in Figure 6.

An algorithm using the average absolute difference for carrier-to-noise ratio estimation requires six inputs: three counter averages and three average absolute differences. The counter having the lowest average absolute difference is selected as best, and the other two sets of data are discarded. The phase and carrier-to-noise ratio are interrelated, so an iterative procedure must be used. The algorithm first assumes that $\pi = \bar{x}$ (center of the selected counter), and interpolates in a table of $x$ values to estimate $R$ from $\bar{a}$. It then assumes this value of $R$ and interpolates in a table of $\mu$ values to estimate $x$ from $\bar{y}$. This value of $\hat{x}$ replaces $\pi$ and the process is repeated until both $\hat{x}$ and $\hat{R}$ have converged. Figure 7 shows the results of several simulated runs of this detector and algorithm.

While the use of average absolute difference is straightforward, it requires additional circuitry in the preprocessor. If the carrier-to-noise ratio can be extracted without this additional hardware, it will simplify the preprocessor. In this application, the preprocessor will be balloon-borne and drifting. Since drift can cause phase shifts which reduce the accuracy of the average, it will be desirable to transmit data after each Omega transmission so that balloon drift can be estimated. The variance of this set of phase averages can be used to
estimate carrier-to-noise ratio. In this technique, the input data are three sets of (nominally) eighteen phase averages, denoted \( z_1(1), z_2(1), z_3(1), \ldots, z_1(J), z_2(J), \text{ and } z_3(J) \). From these,

\[
\bar{z}_i = \frac{1}{J} \sum_{j=1}^{J} z_i(j) \tag{21}
\]

\[
u_i = \frac{1}{J} \sum_{j=1}^{J} \left( z_i(j) - \bar{z}_i \right)^2 \tag{22}
\]

(Balloon drift is ignored here.) Now \( u_1, u_2, \text{ and } u_3 \) increase as \( x \) approaches 0 or 2, so the smallest value of \( u_i \) indicates the best counter. This value \( u_i \) is then an estimate of the variance of \( z_i \), and is therefore related to \( \sigma_y \) according to

\[
E \left[ u_i \right] = \frac{1}{K} \sigma_y^2 \tag{23}
\]

where \( K \) is the number of samples taken during a one second Omega transmission.

An algorithm using this technique would operate in much the same manner as the one for average absolute difference, iterating until \( \hat{X} \) and \( \hat{R} \) converge. The accuracy possible can be found by comparison with standard tables, since \( u_i \) is a chi-squared random variable.

It appears that a 95 percent confidence interval allows -2.5 dB to +2.1 dB errors in \( R \) for 18 intervals (3 minutes) and -2.1 dB to +1.6 dB for 30 intervals (5 minutes). If \( R \) is +5 dB or better, these errors are of little importance. However, at 0 or -5 dB they will affect the accuracy of the phase estimate.

The third technique uses the phase averages of all three counters, rather than just that of the best one. Since phase average depends on carrier-to-noise ratio, as well as phase, it can be used to estimate carrier-to-noise ratio. Figure 8 shows how a phase produces three averages \( \bar{y}_1, \bar{y}_2, \text{ and } \bar{y}_3 \). Also plotted are phase average curves for two other carrier-to-noise ratios. Note that neither could produce the observed set of averages. By evaluating the squared error between the measured averages and the expected averages for several values of \( x \) and \( R \), the \( x \) and \( R \) which best fit the data can be selected.

The previous algorithm can be used to set limits for this algorithm. At the time of writing, this algorithm had not yet been evaluated.

Other Aspects - The preprocessor must establish synchronization with the Omega transmitter commutation pattern so that it can start and stop phase averaging at the proper times. This can be done by two blocks of circuitry. One of these is a sequencer, which derives commutation times from a 10 Hz signal derived from the clock oscillator. The 10 Hz signal triggers a counter and the numbers in the counter indicate whether to start or stop sampling. The other circuit is a synchronizer which starts the sequencer in the right place. To do this, it is necessary to measure the lengths of the 10.2 and 13.6 kHz transmissions from the strongest station. Logic circuits can then load an appropriate position number into the sequencer.
Such a preprocessor might ultimately be fabricated in the form of three special purpose integrated circuits. The three circuits would probably be the commutator, the detector, and the formatter. This will allow flexibility in the use of the IC's in varied applications. For example, changing the formatter allows the same basic preprocessor to be adapted to a fixed receiver.

As currently envisioned, this preprocessor will make measurements on each Omega frequency during each transmission interval. Data will be transmitted during the following interval. With five Omega frequencies and three counters (7 bits each), this amounts to about 105 bits per second. If modulation is added to the fourth Omega frequency and it can be detected by using four detectors for that carrier, each operating at different parts of the modulation cycle.

Conclusions - The use of counting detectors as an Omega preprocessor appears feasible. The use of three detectors with phase references separated by $2\pi /3$ guarantees at least one accurate phase average. If the detector is to be used with carrier-to-noise ratios of less than +5 dB (in a nominal 100 Hz bandwidth), carrier-to-noise ratio must be estimated to estimate phase accurately. There are several ways this can be done, but a combination of interval-to-interval variance and three counter curve fitting requires the least hardware in the preprocessor. Work is continuing at the time of writing.

The author wishes to thank Jerry Waechter for his valuable assistance in much of the computer programming.

References


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Figure 1. Phase measurement.

Figure 2. Phase averaging.
Figure 3. Phase average.

Figure 4. Standard deviation of phase average.

Figure 5. Phase accuracy.

Figure 6. Average absolute difference.

Figure 7. Simulated phase estimations. $R_a=50\text{B}, T=500$.

Figure 8. Curve fitting.