VITERBI AND SERIAL DEMODULATORS FOR PRE-CODED
BINARY GMSK

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ABSTRACT

Three different demodulators applicable to the coherent demodulation of binary Gaussian Minimum Shift Keying (GMSK) signal are described and their performance compared. These include a near-optimal trellis demodulator, which utilizes two matched filters and Viterbi algorithm to carry out maximum likelihood sequence estimation, and a single-filter threshold demodulator with and without pulse equalization. The performance of these demodulators in noise and adjacent channel interference (ACI) are compared for several signal BT products. The equalized threshold demodulator is shown to perform nearly as well as the near-optimal trellis demodulator in additive white Gaussian noise (AWGN), and substantially outperform the trellis demodulator under severe ACI condition.

KEY WORDS

GMSK, Trellis Demodulator, Serial Demodulator, LMS Equalizer, ACI, BER Performance.

INTRODUCTION

The optimal demodulator for a GMSK signal in AWGN is one that implements the maximum likelihood sequence search algorithm. This is known as the trellis demodulator when the sequence search is carried out by way of the Viterbi algorithm. The implementation of the optimal trellis demodulator calls for a bank of \( Q=2^{L-1} \) Laurent[4] filter functions and a \( 2^L \)-state signal trellis, where \( L=\lceil 1/BT \rceil \) is the time span of the pulse response of the Gaussian shaping filter with bandwidth \( B \), \( T \) is the bit period, and \( \lceil x \rceil \) denotes the smallest integer greater than or equal to \( x \). A family of sub-optimal trellis demodulators utilizing only \( F=2^i \) \((0\leq i<Q)\) filter functions has previously been proposed by Kaleh[1], and, in particular, a 2-filter trellis demodulator was shown to be near-optimal for binary GMSK signal with \( BT=1/4 \). The performance of this near-optimal 2-filter trellis demodulator was also shown to be achievable by a serial MSK-type demodulator in conjunction with a transversal equalizer. Since the serial demodulator uses only one Laurent filter function, it is linear and relatively simpler to implement than
the 2-filter trellis demodulator. The use of the serial demodulator also facilitates the
generation of soft decision metrics which are required for soft error decoding in
communication systems that employ forward error correcting code. In this paper we
extend the investigation of the trellis and serial demodulators for the binary GMSK signal
to other BT values of interest: $1/2$, $1/3$, $1/4$, $1/5$, and $1/6$. The tap weights of a near-
optimal equalizer for the serial demodulator for different BT values are reported.
Performance of these demodulators in AWGN, and in AWGN plus adjacent channel
interference (ACI), are compared.

DEMODULATOR DESCRIPTIONS

Trellis Demodulator
The optimal demodulator for signals received in AWGN is known to be a matched filter
demodulator applied to the sequence of transmitted signal and such a demodulator can be
efficiently mechanized via the Viterbi Algorithm. Figure 1 shows an illustration of the
trellis demodulator (TD) for the GMSK signal. The TD consists of a bank of $2^{L-1}$
matched filters followed by a Viterbi algorithm. These filters are matched to the
respective Laurent pulse functions of the GMSK signal. We refer the reader to a detailed
description of these filters in a paper[5] presented earlier in this session. As shown in the
figure, the received base-band complex envelope $z_b(t)$ of the GMSK signal is first filtered
by this filter bank to reject channel noise. The filter outputs are then sampled
synchronously at bit times to form $2^{L-1}$ complex sample streams which in turn are applied
to a Viterbi algorithm (VA). The VA searches for the data sequence with maximum
correlation to the input sampled signal sequences and, at bit decision time instant $nT$,
declares the $\hat{d}_{n-L-1}$ of this sequence as the demodulated data bit. For non-precoded
transmitted data, differential decoding on the demodulated data bit is also necessary. A
discussion of data pre-coding can be found in two companion papers[5, 6] presented in
this session.

Since the signal memory span $L$ is $\left\lceil 1/BT \right\rceil$, and since the number of states of the VA is $2^L$,
the implementation of optimal trellis demodulator for partial response GMSK appears to
require a large number of filters as well as many memory elements and decision circuitry.
Fortunately, near-optimal demodulation performance can be achieved for a TD with only
two of the filters, \( h_0(-t) \) and \( h_1(-t) \), and a 4-state VA, even for BT values as small as 1/6. In the range of bit SNR from 0 to 10 dB, we have found the performance loss of this 2-filter TD to be at most 0.5 dB from that of the all-filter implementation for the BT values considered. Figure 2 shows a trellis diagram [1] of the 2-filter TD along with the formula for computing the branch metrics. Note that the computation of the branch metric involves only addition or subtraction of the matched filter outputs so the VA is actually quite simple to implement. However, since the TD outputs only hard decisions, generating soft decision metric for decoding an error correction coded GMSK signal is not straightforward, and an approach to this problem requires further investigation.

**Serial Demodulator**

Despite the simplicity of the 2-filter 4-state TD discussed above, further simplification of the demodulator is possible by omitting the \( h_1(-t) \) filter from the 2-filter TD. The resulting structure – known as serial, or threshold, demodulator – is shown in Figure 3. With the use of data pre-coding at the transmitter, data decisions of the serial demodulator are simply made by taking the sign of the in-phase and quadrature component of the matched-filter output alternately. The advantages of the serial demodulator over the TD is its implementation simplicity as well as its ease of generating metric for soft decision decoding. The decoding metric can be obtained by directly quantizing into the required number of bits the respective in-phase or quadrature component of the filter output at sampling instants. The disadvantage of the serial demodulator is that its performance may be severely limited by inter-symbol interference (ISI) – particularly for small BT products. However, the deleterious effect of ISI can be mitigated with the use of an equalizer, which we now describe.
**Serial Demodulator with Equalization**

ISI refers to the spreading of a data pulse over several bit intervals. It is an inherent characteristic of a smooth and spectrally compact modulation waveform (such as GMSK) and a direct consequence of the reciprocal spreading theorem of Fourier transform (which states that a pulse that is long in time is necessarily narrow in frequency, and vice versa).

For a GMSK signal, each data pulse is spread among the set of $Q=2^{L-1}$ Laurent pulses with pulse duration ranging from $(L+1)T$ for the dominant Laurent component $h_0(t)$ to $T$ for the weakest Laurent component. Upon reaching the detection filter of the serial demodulator, these Laurent pulses are further spread by the detection filter $h_0(-t)$, resulting in a total data pulse spread as large as $2(L+1)$ bit periods. Since the autocorrelation function of $h_0(t)$ vanishes at $\pm (L+1)T$, a signal sample taken at bit time $nT$ is corrupted by $L$ preceeding data bits as well as $L$ succeeding data bits, resulting in ISI. The presence of ISI in the signal sample causes the signal to randomly deviate from its expected value and could greatly impair the decision device’s ability to make correct bit decisions – even in the absence of receiver noise. The effect of ISI on bit detection is generally data pattern dependent and can be effectively mitigated using a Least Mean Square (LMS) equalizer[3], which seeks to minimize the square of the error of the signal sample from its expected value. An LMS equalizer, also known as a transversal equalizer, can be implemented using a tap delay line, a set of multipliers and a multiple-input summer. For $BT$ values of $1/3$ to $1/6$, we have determined that no more than 3 taps are needed in each case to provide the near-optimal performance of the LMS equalizers. These tap weights are shown in Table 1. We have found that, for $BT$ values of $1/2$ [2] and greater and over the bit SNR range of 0 to 10 dB, the performance of the serial demodulator without an equalizer is nearly the same as coherent BPSK performance and equalization is thus deemed unnecessary for these larger $BT$ values. Figure 4 depicts a serial demodulator equipped with a 3-tap LMS equalizer. It should be noted that the delay spacing between tap weights is $2T$ due to the alternating in-phase/quadrature nature of the serial demodulator.

<table>
<thead>
<tr>
<th>$BT$</th>
<th>$C_2$</th>
<th>$C_0$</th>
<th>$C_2$</th>
</tr>
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<tr>
<td>1/3</td>
<td>-0.032</td>
<td>0.866</td>
<td>-0.032</td>
</tr>
<tr>
<td>1/4</td>
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<td>0.873</td>
<td>-0.064</td>
</tr>
<tr>
<td>1/5</td>
<td>-0.101</td>
<td>0.887</td>
<td>-0.101</td>
</tr>
<tr>
<td>1/6</td>
<td>-0.139</td>
<td>0.907</td>
<td>-0.139</td>
</tr>
</tbody>
</table>

Table 1. Weights of LMS Equalizers for Serial GMSK Demodulation.
PERFORMANCE RESULTS

AWGN Performance
The simulated BER performance in Gaussian noise of the 2-filter trellis demodulator, the non-equalized serial demodulator, and the serial demodulator with LMS equalization are compared in Figures 5a to 5d for signal BT product of 1/3 to 1/6, respectively. The tap weights of the LMS equalizer used for each BT value are as given in Table 1. In each figure the bottom dashed curve is the BER of coherently demodulated BPSK and is included for comparison. The thin solid curve above the BPSK curve is the 2-filter trellis demodulator performance curve, the thick solid curve is the serial demodulator performance with equalization, and the top curve is the serial demodulator performance without equalization. For BT=1/3, we see that there is practically no difference among the performance of the 2-filter trellis demodulator, the equalized serial demodulator, and ideal BPSK signaling over the bit SNR range of 0 to 10 dB. There is also negligible difference between the performance of the non-equalized serial demodulator and BPSK performance for BER greater than or equal to $10^{-3}$. However, the performance of the non-equalized serial demodulator starts to degrade from the BPSK performance curve as BER becomes smaller. For BT=1/4, Figure 5b shows that both the 2-filter trellis demodulator and the equalized serial demodulator perform nearly as well, and their performance remain quite close to BPSK performance, whereas the performance of the non-equalized serial demodulator departs more and more substantially from BPSK performance with increasing bit SNR. Finally, for BT values of 1/5 and 1/6, the BER curves of the 2-filter trellis demodulator and equalized serial demodulator begin to degrade increasingly from the BPSK curve with increasing SNR but still remain relatively close to each other. However, the departure of the non-equalized serial demodulator BER curve from the equalized serial demodulator BER curve becomes markedly larger. From these figures one can see that, for BT values of 1/4 or larger and channel BER requirement of 0.05 or higher, there is really no need for a trellis demodulator nor an equalized serial demodulator – a simple non-equalized serial demodulator suffices. On the other hand, for small BT values and low BER a serial demodulator equipped with an LMS equalizer can perform significantly better than one without an equalizer. For an AWGN channel, the equalized serial demodulator performs only a fraction of a dB worse than the 2-filter trellis demodulator but requires lesser complexity to implement. As it turns out, in an FDMA signaling system where significant power difference could exist among the
neighboring channels, the use of an equalized serial demodulator can have substantial performance advantage over the trellis demodulator. We now turn to a discussion of ACI performance of these demodulators.

ACI Performance

In a frequency division multiple access (FDMA) communication system, the system bandwidth is shared among a group of carrier signals. Typically these carrier signals have the same data rate and adjacent channels are separated by a fixed frequency spacing. Suppose the carrier signals were originated from different terminals distributed over a wide geographical area and these signals were transmitted to and demodulated at a satellite receiver. Then at the satellite receiver the adjacent channels will interfere with each other. The extent and severity of the mutual interference will depend on the difference in terminal transmit power and propagation loss of these signals as well as the channel frequency spacing. Since the interference from the nearest adjacent channels is expected to be dominant among the channels in the system it is sufficient to consider only the two nearest channels in the analysis of an interference situation. Figure 6 illustrates this adjacent channel interference (ACI) scenario where the signal of interest is interfered by neighboring signals each having a power advantage of $\Delta p$ dB over the signal of interest. The level $\Delta p$ will be referred to as the ACI level.
Figures 7a and 7b show the bit SNR required by the demodulators to provide a channel BER of 1% as a function of channel frequency separation, for ACI levels of 0 dB and 20 dB, respectively. The BT value of the GMSK signal is 1/5. Figure 7a shows that, irrespective of the demodulator used, the frequency spacing of adjacent FDMA channels is limited to no less than 0.7Rb. It also shows that, even with 0 dB ACI level, the 2-filter trellis demodulator still performs somewhat better than the equalized serial demodulator just as in AWGN-only channel. However, as seen in Figure 7b, the equalized serial demodulator performs better than the 2-filter trellis demodulator under 20 dB ACI level. The reason that the 2-filter trellis demodulator performs poorer than the equalized serial demodulator for large ACI level is attributed to the wider bandwidth of the second matched filter used in the trellis demodulator, which admits more interference power to the data detector than the serial demodulator.

Figure 6  Adjacent Channel Interference Scenario

Figure 7a. ACI Performance (Δp=0 dB).
Lastly, Figure 8 compares the BER performance of the 2-filter trellis demodulator and the equalized serial demodulator for BT=1/6 with a 30 dB ACI level. Also included for comparison are the performance of these demodulators with AWGN only. The channel frequency separation used in the performance simulation is 0.9Rb. At channel BER of 3%, it is seen that, with AWGN only, the equalized serial demodulator performs only a small fraction of a dB worse than the trellis demodulator whereas, with both AWGN and ACI, the serial demodulator outperforms the trellis demodulator by about 3.5 dB.
CONCLUSIONS

In this paper we have described a near-optimal Viterbi algorithm-based 2-filter trellis demodulator, a simple non-equalized serial demodulator, and a LMS-equalized serial demodulator, all applicable to the coherent demodulation of binary pre-coded GMSK signals. The near-optimal weights of the LMS equalizer for various BT values of interest were reported. The performance of the equalized serial demodulator and a 2-filter trellis demodulator were compared for a channel with noise only, as well as for a channel with both noise and adjacent channel interference. Simulation results show that the trellis demodulator performs better than the equalized serial demodulator when the ACI level is low, whereas the latter may be the preferred demodulator to use when the ACI level is high.

REFERENCES


