SIMPLIFIED 2-state Detectors for SOQPSK-TG and SOQPSK-MIL

Balachandra Kumaraswamy
Department of Electrical Engineering & Computer Science
University of Kansas
Lawrence, KS 66045
balachk@ku.edu

Faculty Advisor:
Erik Perrins

ABSTRACT

We study simple trellis-based detectors for SOQPSK that have a minimal level of complexity. In particular, we show that the state complexity can be cut in half relative to previous approaches—from 4 states down to 2—with asymptotically optimum performance. We give two possible means of achieving this: the pulse amplitude modulation (PAM) technique and the pulse truncation (PT) technique; both of these techniques make use of recent advances in SOQPSK technology based on a continuous phase modulation (CPM) interpretation of SOQPSK. The proposed simplifications are significant since trellis-based SOQPSK detectors are 1–2 dB superior to widely-deployed symbol-by-symbol detectors. These performance gains come at the expense of complexity, and the proposed 2-state detectors minimize this expense. Thus, these simple detection schemes are applicable in settings where high-performance and low complexity are needed to meet restrictions on power consumption and cost.

Introduction

Shaped-offset QPSK is a type of continuous phase modulation (CPM) [1] which is highly bandwidth efficient. While CPMs have a number of advantages, one advantage in particular is responsible for its widespread deployment: it has a constant signal envelope. This makes it compatible with nonlinear power amplifiers, which are highly efficient in converting limited (i.e. battery) power into radiated power. This in turn allows for a smaller physical size and lower cost for the transmitter. To date, SOQPSK has been incorporated into military and aeronautical telemetry standards, although wider use is merited since it is applicable in any setting where bandwidth-efficient constant-envelope modulations are needed.

In addition to its name, SOQPSK shares a number of similarities with conventional offset QPSK (OQPSK). These similarities are exploited at the receiver, where OQPSK-type detectors are the most commonly deployed means of detecting SOQPSK. The advantage of the OQPSK interpretation of SOQPSK is its simplicity. Thus, the low-cost advantage of SOQPSK can be shared by the transmitter and the receiver. However, the disadvantage of OQPSK-type detectors is that they are suboptimal by 1–2 dB,
depending on the details of their implementation [2]. This is a significant loss since it erodes some of the power advantages enjoyed by SOQPSK in the first place.

Since SOQPSK is a modulation with memory, its optimal detector must be trellis-based. Such detectors were first studied in [3], where a cross-correlated trellis-coded quadrature modulation (XTCQM) [4, 5, 6] viewpoint was taken for military-standard “SOQPSK-MIL” [7] which resulted in an optimal detector. Recently in [8], a CPM interpretation of SOQPSK was applied at the receiver. This also resulted in an optimal detector for SOQPSK-MIL and opened the door for two reduced-complexity methods for detecting the more complicated version of SOQPSK adopted by the telemetry group, “SOQPSK-TG” [9]. These two techniques, pulse amplitude modulation (PAM) [10] and the pulse truncation (PT) [11, 12] result in 4-state detectors for SOQPSK-TG that are within 0.2 dB of the impractical 512-state optimum detector.

In this paper, we show the size of the trellis can be reduced to its minimum—2 states—for both SOQPSK-MIL and SOQPSK-TG. This is accomplished by a novel concatenation of the differential encoder and SOQPSK precoder which leads to a simplified representation of the transmitter’s state memory. This simplified transmitter model combined with decision-feedback at the receiver yields the overall state reduction. We show how this state reduction can be implemented using the PAM and PT techniques. In both cases, the 2-state detectors have no asymptotic losses relative to their 4-state counterparts; however, for moderate signal-to-noise ratios, the PT technique results in a negligible loss on the order of 0.1 dB.

This state reduction is significant since the major drawback of trellis-based detectors is their complexity compared to their symbol-by-symbol cousins. Since the proposed detectors reduce the state complexity to its minimum of 2 states, these detectors represent the most attractive means of realizing the 1–2 dB advantage trellis-based detectors have over symbol-by-symbol detectors.

In the next section we describe the signal model for SOQPSK. In Section B, we discuss trellis models for SOQPSK-MIL and SOQPSK-TG. We develop the 2-state detectors in Section B.. In Section D, we study the performance of the detectors and give simulation results in Section G..

**Description of SOQPSK**

A. CPM Signal Model

The complex-baseband representation of SOQPSK as a form of CPM [1] is

\[
    s(t; \alpha) \triangleq \exp \{ j\phi(t; \alpha) \}
\]

where the phase is a pulse train of the form

\[
    \phi(t; \alpha) \triangleq 2\pi h \sum_i \alpha_i q(t - iT)
\]

and \(\alpha_i \in \{-1, 0, +1\}\) is a transmitted symbol, \(T\) is the duration of each \(\alpha_i\), and \(h = 1/2\) is the modulation index. The phase pulse \(q(t)\) is usually thought of as the time-integral of a frequency pulse \(f(t)\) with area 1/2 and duration LT. When \(L = 1\) the signal is *full-response* and when \(L > 1\) it is *partial-response*. Due
to the constraints on \( f(t) \) and \( q(t) \), and assuming a rational modulation index \( h = K/p \), the phase may be expressed as

\[
\phi(t; \alpha) = 2\pi h \sum_{i=-n-L+1}^{n} \alpha_i q(t - iT) + \pi h \sum_{i=0}^{n-L} \alpha_i
\]

where \( nT \leq t < (n + 1)T \). The phase state \( \theta_{n-L} \in \{0, \pi/2, \pi, 3\pi/2\} \) can assume only four distinct values when taken modulo-2\(\pi\), which gives \( e^{j\theta_{n-L}} \in \{\pm1, \pm j\} \).

In this paper we will discuss two versions of SOQPSK. The first, SOQPSK-MIL, is full-response with a rectangular shaped frequency pulse \[7\]. The second, SOQPSK-TG, is partial-response with \( L = 8 \) and a frequency pulse shape defined in \[9\]

### B. SOQPSK Precoder

With SOQPSK, the channel symbols \( \alpha \) are not the underlying information sequence, but are related to the original data sequence \( \alpha \) by the series of operations shown if Figure 1(a). The first of these operations is a double differential encoder \[13\] given by the equation

\[
u_i = a_i \oplus u_{i-2}, \quad a_i, u_i \in \{0, 1\}
\]

The second operation in Figure 1(a) is the precoder, which converts the double differentially encoded \( \{u_i\} \) into ternary data \( \alpha_i \in \{-1, 0, +1\} \) according to the rule \[14\]

\[
\alpha_i(u) = (-1)^{i+1} (2u_{i-1} - 1)(u_i - u_{i-2})
\]

Figure 1(b) shows an alternate precoder representation where the double differential encoder and the precoder are combined to form a *differential precoder*. It was shown in \[15\] that the differential precoder

![Diagram of precoders](image-url)
has the form

\[ \alpha_n = (-1)^S_n a_n. \]  \hspace{1cm} (6)

where the sign state is

\[ S_{n+1} = (S_n + \alpha_n + 1) \mod 2. \]  \hspace{1cm} (7)

We point out that the binary-valued sign state \( S_n \) is the only state variable required by the differential precoder.

In either precoder representation, Figure 1(a) or Figure 1(b), the output of the precoder is connected to an ordinary CPM modulator with \( h = 1/2 \) and the desired pulse shape \( f_{\text{ML}}(t) \) or \( f_{\text{TG}}(t) \). For the special case of full-response CPM \( (L = 1) \), the only memory within the CPM modulator is the phase state \( \theta_{n-1} \). The interaction between the memory of the precoder(s) and the memory of the CPM modulator is discussed next.

**Trellis Representation of SOQPSK**

Figure 2 shows the 4-state time-varying trellis that describes the SOQPSK precoder in Figure 1(a) [8]. The labels along each branch of the trellis show the input bit/output symbol pair, \( a_n/\alpha_n \), for the given branch.

The advantage of the 4-state trellis is that its state variables \( u_{n-1} \) and \( u_{n-2} \) have a *one-to-one correspondence* with the phase state \( \theta_{n-1} \) of the full-response CPM modulator that follows the precoder. The
mapping from precoder trellis states to CPM phase states is [8]

\[
\begin{align*}
00 & \leftrightarrow \frac{3\pi}{2}, & 01 & \leftrightarrow \pi, \\
10 & \leftrightarrow 0, & 11 & \leftrightarrow \frac{\pi}{2}.
\end{align*}
\] (8)

Figure 3 shows the 2-state time-invariant trellis that describes the differential SOQPSK precoder in Figure 1(b) and equations (6) and (7). The state variable is simply the sign state \( S_n \), and the labels along each branch specify the input bit/output symbol pair \( a_n/\alpha_n \) for the given branch.

The obvious advantage of the 2-state trellis is its simplicity with respect to the 4-state time-varying trellis in Figure 2. Unfortunately, this simplification does not also manifest itself with the CPM phase state \( \theta_{n-1} \). Thus, a 4-state trellis is still required to fully (i.e. optimally) describe the entire system in Figure 1(b). In the next section we will show how a simple decision feedback scheme can be employed at the detector; this technique allows the simple 2-state trellis to be successfully applied to SOQPSK and yields near-optimal performance.

SOQPSK Detectors

C. Received Signal Model

The received signal model is

\[ r(t) = s(t; \alpha) + n(t) \] (9)

where \( n(t) \) is complex-valued additive white Gaussian noise (AWGN) with single-sided power spectral density \( N_0 \). Since the transmitted signal \( s(t; \alpha) \) has memory, the optimal detector must perform maximum likelihood sequence detection (MLSD). This is efficiently implemented via the Viterbi algorithm (VA). In the following discussion, we refer to estimated and hypothesized values of a quantity \( w \) as \( \hat{w} \) and \( \tilde{w} \) respectively. Also, \( \hat{\hat{w}} \) and \( \tilde{\tilde{w}} \) can assume the same values as \( w \) itself.

A cumulative metric \( \lambda_n(\tilde{S}_n) \) is maintained for each state \( \tilde{S}_n \) in the trellis. These metrics are extended along the branches from starting states \( \tilde{S}_n \) to ending states \( \tilde{E}_n \) via the update

\[
\lambda_{n+1}(\tilde{E}_n) = \lambda_n(\tilde{S}_n) + z(n, [\tilde{a}_n, \tilde{S}_n])
\] (10)
where \( z(n, [\tilde{a}_n, \tilde{S}_n]) \) is the branch metric increment and is a function of the starting state \( \tilde{S}_n \) and the branch symbol \( \tilde{a}_n \); we refer to \( [\tilde{a}_n, \tilde{S}_n] \) as the branch vector. In the case of SOQPSK, there are two branches that merge into each ending state \( \tilde{E}_n \). The branch with the maximum metric is declared as the survivor and its metric is stored for later use in the next round of updates.

One technique for reducing the complexity of SOQPSK-TG at the receiver is known as pulse truncation (PT) [11, 12]. The branch metric increment for pulse truncation (PT) is explained in [11, 12] and for the PAM representation is explained in [16].

**D. 2-State Detectors for SOQPSK**

As mentioned earlier, the difficulty with the two state trellis in Fig. 3 is that a one-to-one correspondence between the sign state \( S_n \) and the CPM phase state \( \theta_{n-1} \) does not exist. This problem is overcome by using decision feedback.

As mentioned above, at the end of each time step, a surviving branch is declared at each ending state \( \tilde{E}_n \) in the trellis. We use \( \hat{\alpha}_n(\tilde{E}_n) \) to denote the symbol associated with the surviving branch at each ending state \( \tilde{E}_n \). In the modified VA, a cumulative phase \( \hat{\theta}_n(\tilde{S}_n) \) is maintained for each state \( \tilde{S}_n \) in the trellis, in addition to the above-mentioned cumulative metric \( \lambda_n(\tilde{S}_n) \). Once the survivors have been declared, the cumulative phase for each ending state is updated via the recursion

\[
\hat{\theta}_n(\tilde{E}_n) = \left[ \hat{\theta}_{n-1}(\tilde{S}_n) + \pi h \hat{\alpha}_n(\tilde{E}_n) \right] \mod 2\pi. \tag{11}
\]

As it turns out, in the case of the 4-state detector the cumulative phase \( \hat{\theta}_{n-1}(\tilde{S}_n) \) is identical to the phase state \( \tilde{\theta}_{n-1} \) provided the four cumulative phases are initialized according to (8) at the start of the algorithm. This is equivalent to saying that, given the proper initialization, the two branches merging at each ending state in the 4-state trellis will result in the same value for the cumulative phase. This is true by definition of the phase state in (3) and the cumulative phase in (11). Thus, the decision feedback does not introduce any sub-optimality to the 4-state detectors.

In the case of the 2-state detector, using \( \hat{\theta}_{n-1}(\tilde{S}_n) \) instead of \( \tilde{\theta}_{n-1} \) does make the detector suboptimal, but it is a necessary step in order to implement the detector in the first place. The impact of decision feedback on the performance of the 2-state detectors is now studied.

**Performance Analysis**

The bit-error probability of SOQPSK in AWGN is described using error events and minimum distance concepts. The normalized squared Euclidean distance of CPM is [1]

\[
d^2 = \frac{\log_2 M_{\text{info}}}{2T} \int |s(t; \alpha_{Tx}) - s(t; \alpha_{Rx})|^2 dt \tag{12}
\]

where \( \log_2 M_{\text{info}} \) is the number of bits per symbol (for SOQPSK we have \( M_{\text{info}} = 2 \)).
E. Minimum Distance Error Event

The minimum distance error event for the 4-state SOQPSK detectors is where the transmitted and received bit sequences satisfy $a_{Tx} = ..., a_{e-1}, a_{e}, a_{e+1}, a_{e+2}, a_{e+3}, ...$ and $a_{Rx} = ..., a_{e-1}, a_{e}, a_{e+1}, a_{e+2}, a_{e+3}, ....$ In words, this is a double bit error event where the first error occurs at some arbitrary bit location $a_e$ and the second error occurs with bit $a_{e+2}$. In [17] it was shown that when $a_{e+1} = 1$, the precoded symbol sequences satisfy $\pm \gamma_0$, where $\gamma_0 = \alpha_{Tx} - \alpha_{Rx} = ..., 0, -1, 0, +1, 0, ...$ and a squared distance of $d_0^2$ results. It was also shown in [17] that when $a_{e+1} = 0$, the precoded symbol sequences satisfy $\pm \gamma_1$, where $\gamma_1 = \alpha_{Tx} - \alpha_{Rx} = ..., 0, -1, -2, +1, 0, ...$ and a squared distance of $d_1^2$ results. These cases are easily verified by examining the 4-state trellis in Fig. 2.

F. Additional Error Event for 2-State Detectors

For the 2-state detectors, an additional error event is introduced where the transmitted and received bit sequences satisfy $a_{Tx} = ..., a_{e-1}, a_{e}, a_{e+1}, a_{e+2}, ..., a_{e+3}, ...$ and $a_{Rx} = ..., a_{e-1}, a_{e}, a_{e+1}, a_{e+2}, ..., a_{e+3}, ...$. In words, this is a double bit error event where the first error occurs at some arbitrary bit location $a_e$ and the second error occurs with the following bit $a_{e+1}$. In this case, it is easily verified from Fig. 2 that the precoded symbol sequences satisfy $\pm \gamma_2$, where $\gamma_2 = \alpha_{Tx} - \alpha_{Rx} = ..., 0, +1, +1, 0, ...$ resulting in squared distance $d_2^2$. 

Figure 4: Performance of reduced-complexity detectors for SOQPSK-MIL, AND, Performance of reduced-complexity detectors for SOQPSK-TG.
Table 1: Range of distance values in the set \( \{d_{2,l}^2\} \) for the 2-state SOQPSK detectors.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( \min {d_{2,l}^2} )</th>
<th>( \max {d_{2,l}^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIL-MF ( (d_0^2 = 1.73) )</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>MIL-PAM ( (d_0^2 = 1.73) )</td>
<td>2.83</td>
<td>3.03</td>
</tr>
<tr>
<td>TG-PT ( (d_0^2 = 1.60) )</td>
<td>1.71</td>
<td>2.23</td>
</tr>
<tr>
<td>TG-PAM ( (d_0^2 = 1.60) )</td>
<td>2.57</td>
<td>3.35</td>
</tr>
</tbody>
</table>

G. Probability of Bit Error

The PT and PAM approximations discussed earlier result in mismatched detectors \([11, 12]\), i.e. the detector is no longer matched to the transmitted signal. When the 4-state PT detector is used, the distance is slightly influenced by the values of the bits surrounding the error event on each side, \( \{a_{e-k}\}_{k=0}^{5} \) and \( \{a_{e+k}\}_{k=-3}^{5} \). This results in a set of distance values \( \{d_{0,l}^2\}_{l=0}^{63} \) that are clustered around the value \( d_0^2 = 1.60 \) and range from 1.38 to 1.77. The methods for calculating these distances are discussed in \([11, 12, 18]\).

Taking this behavior into account, the final expression for the union bound on the bit-error probability of the 4-state detectors is

\[
P_{b,4} \leq \frac{1}{|d_{0,l}\{d_{0,l}^2\}|} \sum_{d_{0,l}} Q\left(\sqrt{d_{0,l}^2 \frac{E_b}{N_0}}\right) + \frac{1}{|d_{1,l}\{d_{1,l}^2\}|} \sum_{d_{1,l}} Q\left(\sqrt{d_{1,l}^2 \frac{E_b}{N_0}}\right)
\]  \hspace{1cm} (13)

where \( E_b/N_0 \) is the bit energy to noise ratio, \(| \cdot |\) denotes the cardinality (number of elements) of a given set, and

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-u^2/2} du.
\]  \hspace{1cm} (14)

For example, with the MF detector for SOQPSK-MIL, we have singleton sets of \( d_0^2 = 1.73 \) and \( d_1^2 = 2.36 \), so (13) simplifies to a summation of only two terms. In the case of SOQPSK-TG, (13) contains the 128 terms in \( \{d_{0,l}^2\}_{l=0}^{63} \) and \( \{d_{1,l}^2\}_{l=0}^{63} \) that are clustered around the values \( d_0^2 = 1.60 \) and \( d_1^2 = 2.59 \).

For the 2-state detectors, the bit-error probability is the same as that of the 4-state detectors but with an additional summation, i.e.

\[
P_{b,2} \leq P_{b,4} + \frac{1}{|d_{2,l}\{d_{2,l}^2\}|} \sum_{d_{2,l}} Q\left(\sqrt{d_{2,l}^2 \frac{E_b}{N_0}}\right)
\]  \hspace{1cm} (15)

From Table 1 we observe that, with all four of the 2-state configurations, the distances in \( \{d_{2,l}^2\} \) exceed the value of \( d_0^2 \), i.e. the minimum distance is not worsened by the 2-state detectors. This means that the 2-state detectors each have a performance that is asymptotically equivalent (large \( E_b/N_0 \)) to their 4-state counterpart. The second observation from Table 1 is that the PAM-based detectors have values in \( \{d_{2,l}^2\} \) that are far greater than \( d_0^2 \), while the MF and PT detectors have values that are relatively close to \( d_0^2 \); thus, even for moderate ranges of \( E_b/N_0 \) we would expect the PAM-based detectors to have performance identical to the 4-state detectors, while the MF and PT detectors should have minor losses for moderate values of \( E_b/N_0 \). These expectations are borne out in the simulation results we present next.
Simulation Results

There are two modulation types (SOQPSK-MIL and SOQPSK-TG), two trellis sizes (2-state and 4-state), and two branch metric types (MF or PT, and PAM) that have been discussed above. This yields a total of eight detector configurations. Fig. 4 shows performance curves for the four SOQPSK-MIL and SOQPSK-TG configurations. In the low $E_b/N_0$ region of the figure, the 2-state union bounds given by (15) are not necessarily tight with respect to the simulation points (shown as points only, with no connections between points); however, the union bounds and the simulation points show close agreement rapidly as $E_b/N_0$ increases. Furthermore, the results anticipated in the previous section are confirmed. The 2-state PAM-based detector shows no observable degradation with the 4-state detector (across the entire simulation range of $E_b/N_0$), while the 2-state MF-based detector shows a slight performance degradation that narrows and is near zero at the large end of the simulated $E_b/N_0$.

Conclusion

We have successfully developed 2-state detectors for SOQPSK-MIL and SOQPSK-TG using pulse amplitude modulation (PAM) and pulse truncation (PT) techniques. Using performance analysis, we have shown that these 2-state detectors each have performance that is asymptotically equivalent to their 4-state counterparts. This is a satisfying result due to the minimal 2-state level of complexity achieved by these detectors. These simple detection schemes are applicable in settings where high-performance and low complexity are needed to meet restrictions on power consumption and cost.

REFERENCES


