APPLICATIONS OF A TELEMETRY SYSTEM USING 
DSB-AM SUBCARRIERS

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Summary The advantages of the DSB-AM subcarrier for wideband telemetry requirements have been discussed in previous papers. The purpose of this paper is to consider the specific performance of an FM telemetry r-f link when modulated by a frequency multiplex of DSB subcarriers.

The performance is evaluated by constructing an appropriate model of the subcarrier multiplex based on a predicted noise power spectrum at the r-f demodulator output. The model is used to specify the individual subcarrier amplitude values that constitute the baseband signal, which will modulate the FM transmitter.

The carrier power required to produce a useful signal-to-noise ratio at the outputs of the individual subcarrier demodulators is considered in general. The relationship between the degree of transmitter deviation, receiver bandwidth and the carrier power is derived.

The carrier power required for operation over typical test range distances is determined in terms of the appropriate variables.

The performance of two specific examples is calculated to illustrate the use of the several formulae that are derived. The examples also serve to relate and compare the performance of the DSB system configurations to the more traditional applications. Finally, the advantage of using increased r-f bandwidth is discussed.

Operation of a DSB Frequency Multiplex System with FM Carrier

The Baseband Signal Proper operation of a multichannel subcarrier telemetry system with FM carrier requires that the baseband signal be composed in such a manner to provide uniform noise performance at the outputs of the individual subcarrier demodulators. In addition to the consideration of thermal noise, allowance must be made for intermodulation products that result due to the nonlinearity of elements in the carrier, channel. Specifically, the nonlinearity is produced by a non-uniform phase characteristic in the receiver r-f section and by the nonlinear transformation characteristics of the r-f
modulator and demodulator. It is not intended to critically specify these elements for the system being considered. The analysis will therefore, be predicated on the use of telemetry r-f components that are commercially available and adequately specified for the customary applications.

Consider the composition of a baseband signal consisting of $K$ independent subcarriers. The value of each subcarrier will be specified to conform to the anticipated noise power density at the input of each subcarrier demodulator. In order to consider this, a profile of the noise power density at the output of the r-f demodulator is assumed. A generalized curve of the function in given if Figure 1. The curve in the region specified by $-\frac{\omega_m}{2} < \omega < \omega_m$ is derived by using the well known result, valid at high signal-to-noise ratio, for FM demodulation in thermal noise,

$$N_o = \frac{n_o \omega^2}{A}, \tag{1}$$

where

- $N_o =$ baseband noise power density in per second units
- $n_o =$ carrier channel noise power density in watt-seconds (uniform)
- $A =$ carrier power in watts.

The curve in the region specified by $0 < \omega \leq \frac{\omega_m}{2}$ is obtained by considering a reduction in the thermal noise spectral density with decreasing frequency to be predominated by intermodulation noise that produces a uniform spectral density. This approximation has been justified by experiment with several multichannel systems using FM carrier and typical r-f components. The use of highly linear r-f elements would produce a nominal reduction in the frequency of predominant intermodulation (cross talk) noise. Appendix A shows that, for a uniform threshold condition at the input of each subcarrier demodulator, the subcarrier levels should be programmed according to the following formulae:

$$a_{i}^2 = \frac{\Delta \omega_i}{S^2} \bigg[ \frac{\sum_{i=1}^{k} \Delta \omega_i}{\omega_m^2} + \frac{4}{\omega_m^2} \sum_{i=k+1}^{K} \omega_i^2 \Delta \omega_i \bigg], \quad 0 \leq \omega \leq \frac{\omega_m}{2} \tag{2}$$

and

$$a_{i}^2 = \frac{\omega_i^2 \Delta \omega_i}{S^2} \bigg[ \frac{\sum_{i=1}^{k} \Delta \omega_i}{\omega_m^2} + \frac{4}{\omega_m^2} \sum_{i=k+1}^{K} \omega_i^2 \Delta \omega_i \bigg], \quad \frac{\omega_m}{2} < \omega_i \leq \omega_m \tag{3}$$
where

\[ a_i = \text{the rms value of the subcarrier, when the modulation is unity, in radians per second.} \]

\[ S = \left[ \sum_{i=1}^{K} a_i^2 \right]^{1/2}, \text{the fully modulated rms value of the composite baseband signal in radians per second.} \]

\[ \Delta \omega_i = \text{the bandwidth occupied by the subcarrier in radians per second.} \]

\[ \omega_m = \text{the maximum baseband frequency in radians per second.} \]

In applying (2) and (3) to a particular system, the ratio of the individual subcarrier signal to the total baseband signal is determined for each subcarrier. As the formulae are in terms of subcarrier predetection bandwidth, the value of each subcarrier will vary accordingly. When different types of subcarrier modulation are used in the same baseband, an appropriate factor should be applied to each ai to provide uniform noise performance for all channels.

**Noise Performance Versus Transmitter Deviation Ratio** The general composition of a baseband signal suitable for modulating an FM carrier was considered in the preceding section. In this section, the degree of transmitter modulation by this signal will be considered. The appropriate degree of transmitter modulation depends principally on the following:

a) The allowed carrier channel bandwidth.

b) The highest subcarrier frequency to be transmitted.

c) The desired signal-to-noise ratio in the demodulated subcarrier output.

d) The received carrier power to noise power density ratio.

The carrier power to noise power density ratio for the i\textsuperscript{th} subcarrier is

\[ \frac{A}{n_0} = \frac{\omega_i^2}{N_{0i}}, \quad \frac{\omega_m}{2} < \omega_i < \omega_m. \]  

(4)

As the subcarrier signals have already been properly adjusted to provide uniform noise performance upon demodulation, the performance of one subcarrier will suffice to measure the performance of all others. For convenience, the highest subcarrier is chosen. For the highest subcarrier,
where the subscript K indicates the highest subcarrier frequency.

The deviation ratio of the transmitter is defined as

\[ D = \frac{\delta_p}{\omega_K} \]  

(6)

where \( \delta_p \) is the statistical peak, or \( (3\sigma) \) transmitter deviation given by

\[ \delta_p = 3\left( \sum_{i=1}^{K} a_i^2 \sigma_i^2 \right)^{1/2} \]  

(7)

where

\[ \sigma_i = \text{the rms value of the modulation.} \]

the signals are assumed Gaussian and independent. The formula for detection of the AM subcarrier is

\[ \left( \frac{S}{N} \right)_{AM} = \frac{2\pi a_i^2 \sigma_i^2}{2N_0 \Omega_i} \]  

(8)

where \( \left( \frac{S}{N} \right)_{AM} \) = The signal-to-noise power ratio in the bandwidth \( \Omega_i \) the post detection bandwidth of the \( i^{th} \) subcarrier. Combining (5), (6), (7) and (8) gives the carrier channel power to noise power density ratio required for the \( K^{th} \) subcarrier as

\[ \frac{A}{n_o} = \left( \frac{S}{N} \right)_{AM} \frac{9\Omega_K S^2}{\pi D^2 a_K^2} \]  

(9)

This result gives the received carrier power requirements for permissible operating conditions of the r-f link. The deviation ratio \( D \) can be increased, with a resultant saving of carrier power, until the threshold point of the r-f carrier demodulator is reached. Threshold points for conventional demodulation and feedback demodulation are given below. The conventional discriminator threshold point is given approximately by

\[ \frac{A_t}{n_o} \geq 8(D+1) \omega_m \]  

(10)
The feedback demodulation threshold point (FMFB and PLL) is given approximately by

\[ \frac{A_t}{\eta_0} \geq 10 (D)^{1/2} \omega_m \]  

\[ \text{(11)} \]

**Bandwidth Requirement** The required carrier channel bandwidth for an FM system is a nonlinear function of the deviation ratio. It is customary to define the bandwidth requirement in terms of the percentage of the total signal power included within the nominal passband. The degree to which signal power is excluded is a principle cause of the distortion discussed in the section on bandwidth signal. The usual constraint applied is to avoid exclusion of greater than 1% of the signal power. An analytical determination of the signal power distribution is quite difficult for all but the simplest of modulating signals, and will not be attempted. However, it has been demonstrated experimentally that, for deviation ratios \( D \) a bandwidth allocation of

\[ BW = 2(D+1)\omega_m \quad , \quad D > 1 \]

\[ \text{(12)} \]

is sufficient to limit distortion to the degree provided for in the section on bandwidth signal. For \( D < 1 \), distortion increases rapidly if the bandwidth is constrained as in equation (12), and a provision for additional r-f bandwidth over equation (12) is required to produce a satisfactory system. Since it is not generally good practice to operate an FM system with \( D < 1 \) a thorough experimental determination, with the actual r-f elements to be used, should be undertaken for these systems.

In considering an application, the choice of bandwidth is usually restricted by the availability of receivers and/or channel allocation considerations. Telemetry receivers with selectable bandwidths of 100, 300, 500, 750, 1000, 1500, 2400 and 3300 kilocycles are available at some missile test activities. Once the choice of bandwidth for a particular baseband multiplex is made, the consequent deviation ratio can be determined from equation (12). The received power to noise power density ratio requirement is then given by equation (9). From the standpoint of transmitter power conservation, it is seen to be advantageous to use as wide a bandwidth as practicable. Power requirements for several systems are discussed in the next section.

**Transmitted Power Requirements for Systems Using DSB Subcarriers**

**The Received Signal Power** The received signal power is given as,

\[ A = \frac{P_t G_r G_t L C^2}{(4\pi)^2 R^2 f^2} \]

\[ \text{(13)} \]
where
\[
P_t = \text{transmitted power in watts}
\]
\[
G_t = \text{transmitting antenna gain}
\]
\[
G_r = \text{receiving antenna gain}
\]
\[
L = \text{loss factor to account for polarization, cable losses, and equipment degradation.}
\]
\[
C = 1.619 \times 10^5 \text{ n.m./sec}
\]
\[
R = \text{range in nautical miles}
\]
\[
f = \text{carrier frequency in cps}
\]

**Range Versus Transmitted Power**

To calculate \( \frac{A}{n_o} \), the noise power density is taken as
\[
n_o = [KT_r]F
\]

(14)

where
\[
T_r = 290 K \text{ (standard receiver temperature)}
\]
\[
K = 1.38 \times 10^{-23} \text{ watt-sec/K (Boltzmans constant)}
\]
\[
F = 2.5 \text{ (receiver noise factor of 4 db)}
\]

A table of range versus 10 log \( \frac{A}{n_o} \) for a 1-watt transmitter is given in Table 1. The table is constructed using

\[
G_r = 21 \text{ and 30 db}
\]
\[
G_t = 0 \text{ db}
\]
\[
f = 200 \times 10^6 \text{ cps}
\]

With the range specified and \( \frac{A}{n_o} \) determined from equation (9), the required transmitter power relative to 1-watt can be determined from Table 1.

**Application Examples (Figure 2)**

**Performance of 8 DSB Subcarriers with 3 kc Data Bandwidth**

The performance of 8 DSB 3 kc data bandwidth channels will be evaluated for an r-f link occupying 500 kc bandwidth. The 500 kc bandwidth is customary for VHF telemetry links and is recommended for use, whenever possible, by the current IRIG standards.

Table 2 lists the appropriate subcarrier frequencies and their normalized full modulation values in accordance with equations (2) and (3) of the section on bandwidth signals. The values for calculation of
\[
\frac{a_i^2}{S^2}
\]
are
The required is determined using equation (9) with,

\[
\omega_m = 2\pi(104) \times 10^3
\]

\[
k = 4
\]

The required \( \frac{A}{n_0} \) is determined using equation (9) with,

\[
[S/N]_{AM} = 30 \text{ DB}
\]

\[
\Omega_K = 2\pi(3000)
\]

\[
D = 1.3
\]

From Table 2

\[
\frac{a_K^2}{S^2} = 0.262
\]

The value required is

\[
10 \log \frac{A}{n_0} = 86.5 \text{ db}
\]

The peak FM carrier deviation is 135 kc which is compatible with most standard VHF telemetry transmitters. In setting the transmitter deviation a true rms voltmeter reading corresponding to 45 kc should be obtained with all subcarriers modulated to full scale.

Using the results of Table 1 for \( G_r = 30 \text{ db} \), a 1-watt transmitter will produce the specified operation at ranges of up to 400 n.m.

A 16-channel system with DSB subcarriers of 1.2 kc data bandwidth will yield approximately the same performance.

**Performance of a Hybrid PCM-DSB System**  Another configuration, compatible with an r-f bandwidth of 500 kc, provides the advantages of a moderately high bit rate PCM format in conjunction with 8 DSB subcarriers. This system, with frequency allocation shown in Table 3, operates with an NRZ-PCM rate of 40 K bits per second. Eight standard DSB subcarriers, with 4 providing 1.2 kc data bandwidth and 4 providing 3 kc data bandwidth, are used. The principle advantage of the configuration is that the usual narrowband data requirements that are well suited to PCM techniques can be combined with the wideband data requirements on the same r-f link. This results in a considerable reduction in complexity and equipment cost.
The baseband composition is given in Table 3. For the PCM, only the first order spectral components are transmitted. Using a bit error rate of \( P_e = 10^{-5} \), the required energy per bit to one-sided noise power density ratio is determined from reference sources. \(^9\) For the case where the PCM signal spectrum is limited to the bit rate,

\[
\frac{S}{N}_\text{PCM} = \frac{E}{N_o}
\]

(15)

where

\[
\frac{S}{N}_\text{PCM}
\]

\( E = \text{the energy per bit} \)

\( N_o = \text{the baseband noise power density} \)

Obtaining \( \frac{E}{N_o} = 13 \text{ db} \) from the reference curves,

\[
\frac{S}{N}_\text{PCM} = 20
\]

In constructing Table 3, the procedure is the same as the preceding case. The PCM is treated as an analog channel with \( \Delta \omega_1 \) adjusted to provide for the lower signal-to-noise ratio requirement of the PCM signal. An additional 3 db is allowed over the value calculated for the PCM, because of the critical dependence of the PCM error rate on channel signal-to-noise ratio. The adjusted PCM bandwidth is taken as

\( \Delta \omega_1 = 1600 \)

With the example constants remaining approximately the same as the previous case, the power performance remains the same. In this example, a system employing 12 DSB subcarriers with 1.2 kc data bandwidth would yield nearly equal performance.

**Cases Using Wider Bandwidth** Referring again to equation (9), note that a substantial improvement in power performance is obtained by using an increased deviation ratio. If \( D \) is increased to 2.6 in the two examples given, the power requirement is reduced by 6 db, or, alternately, the range is doubled. The r-f bandwidth requirement to obtain this improvement is 750 kc.

The maximum deviation ratio that can be used is given by equations (10) and (11). The improvement attainable by approaching the conventional discriminator threshold point can be estimated easily by noting that the right-hand side of equation (10) is 4 times the
bandwidth allowance of equation (12). Thus a deviation ratio of 4 times 1.3, or 5.2, would still be above threshold. This would yield a 12 db reduction in required transmitter power over the example, or alternately, 4 times the range. An r-f bandwidth of 1.3 megacycles would be required.

**Conclusions** The principle conclusion is drawn from the two examples. In the first example, it is determined that under conservative conditions, 8 channels of 3 kc wideband data or 15 channels of 1.2 kc wideband data can be efficiently transmitted using a standard IRIG, 500 kc bandwidth, r-f link. This is approximately 4.5 times the data bandwidth encompassed by an extended (channels 1-19) IRIG FM/FM multiplex with a comparable baseband bandwidth.

In the second example, it is shown that 8 wideband DSB channels can be combined with a moderate bit rate PCM format and still be contained within a 500 kc r-f bandwidth. The advantage of using a hybrid system of this type is that it provides a means of effectively handling both narrowband and wideband data requirements on the same r-f link. A requirement situation of this type is frequent in rocket telemetry.

It can be generally concluded that less transmitter power is required when the DSB/FM system is operated at a higher deviation ratio than that used in the examples. The performance using a 750, kc bandwidth is indicated in the section that deals with cases using wider bandwidths. A practical limit for conventional discriminator r-f demodulation is also given; resulting in a carrier power reduction of 12 db for the example cases.

**REFERENCES**

2. A. O. Roche, Addendum to (1), available from the author.
7. RCA, op. cit.
### TABLE 1

**RANGE VERSUS 10 LOG A/n_o (1WATT TRANSMITTER)**

<table>
<thead>
<tr>
<th>RANGE (n.m.)</th>
<th>10 log ( \frac{A}{n_o} )</th>
<th>( G_r = 21 \text{db} )</th>
<th>( G_r = 30 \text{db} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>92</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>86</td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>80</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>74</td>
<td>82</td>
<td></td>
</tr>
<tr>
<td>1600</td>
<td>68</td>
<td>76</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 2

<table>
<thead>
<tr>
<th>CHANNEL NUMBER (i)</th>
<th>1W</th>
<th>2W</th>
<th>3W</th>
<th>4W</th>
<th>5W</th>
<th>6W</th>
<th>7W</th>
<th>8W</th>
</tr>
</thead>
<tbody>
<tr>
<td>FREQUENCY (kc)</td>
<td>17</td>
<td>29</td>
<td>41</td>
<td>53</td>
<td>65</td>
<td>77</td>
<td>89</td>
<td>101</td>
</tr>
<tr>
<td>BANDWIDTH 2( w ) (kc)</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>( \frac{a^2_i}{s^2} )</td>
<td>.068</td>
<td>.068</td>
<td>.068</td>
<td>.068</td>
<td>.108</td>
<td>.152</td>
<td>.204</td>
<td>.262</td>
</tr>
</tbody>
</table>
### TABLE 3

| CHANNEL NUMBER (i) (Std.) | FREQUENCY (kc) | BANDWIDTH 2
\( \omega \) (kc) | \( \frac{a_i^2}{S^2} \) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 PCM 40KB/Sec.</td>
<td>1.6*</td>
<td>.022</td>
<td></td>
</tr>
<tr>
<td>2 SN 38</td>
<td>2.4</td>
<td>.033</td>
<td></td>
</tr>
<tr>
<td>3 MN 44</td>
<td>2.4</td>
<td>.033</td>
<td></td>
</tr>
<tr>
<td>4 7N 50</td>
<td>2.4</td>
<td>.033</td>
<td></td>
</tr>
<tr>
<td>5 8N 56</td>
<td>2.4</td>
<td>.038</td>
<td></td>
</tr>
<tr>
<td>6 5W 65</td>
<td>6.0</td>
<td>.125</td>
<td></td>
</tr>
<tr>
<td>7 6W 77</td>
<td>6.0</td>
<td>.175</td>
<td></td>
</tr>
<tr>
<td>8 7W 89</td>
<td>6.0</td>
<td>.235</td>
<td></td>
</tr>
<tr>
<td>9 8W 101</td>
<td>6.0</td>
<td>.300</td>
<td></td>
</tr>
</tbody>
</table>

*Adjusted (Refer to paragraph 4.2)

### APPENDIX A

**Subcarrier Amplitude Values**  Consider the rms value of the sum of \( K \) independent fully modulated subcarriers

\[
S^2 = \sum_{i=1}^{K} a_i^2
\]

where

- \( S \) = the rms value of the sum
- \( a_i \) = the rms value of the ith subcarrier

It is desired to adjust the value of each subcarrier to produce a uniform output signal-to-noise ratio with the noise power density that each will encounter in reception. The noise power density will be considered in 2 regions of the baseband as follows:

\[
N_{oi} = C_i \frac{\omega_i^2}{4}, \quad 0 \leq \omega \leq \frac{\omega_m}{2}
\]

\[
N_{oi} = C_i \omega_i^2, \quad \frac{\omega_m}{2} < \omega \leq \omega_m
\]
This corresponds to a uniform noise power density in the region 0 to $\frac{\omega_m}{2}$ and a square law relationship in the region $\frac{\omega_m}{2}$ to $\omega_m$. The total signal $S^2$ will be considered as two partial sums,

$$S^2 = S_1^2 + S_2^2$$

and

$$S_1^2 = N_o \sum_{i=1}^{k} \Delta \omega_i$$

$$= C_1 \frac{\omega_m^2}{4} \sum_{i=1}^{k} \Delta \omega_i$$

$$0 \leq \omega \leq \frac{\omega_m}{2}$$

$$i \leq k$$

$$S_2^2 = N_o \sum_{i=k+1}^{K} \Delta \omega_i$$

$$= C_1 \sum_{i=k+1}^{K} \omega_i^2 \Delta \omega_i$$

$$\frac{\omega_m}{2} < \omega \leq \omega_m$$

$$i > k$$

where,

$$\Delta \omega_i = \text{the predetection bandwidth of the } i^{th} \text{ subcarrier.}$$

As each amplitude depends on $N_o$ and $\Delta \omega_i$,

$$a_i^2 = N_o \Delta \omega_i$$

therefore,

$$a_i^2 = C_1 \frac{\omega_m^2}{4} \Delta \omega_i$$

$$0 \leq \omega \leq \frac{\omega_m}{2}$$

$$i \leq k$$
and,

\[ a_i^2 = C_1 \omega_i^2 \Delta \omega_i, \quad \frac{\omega_m}{2} < \omega < \omega_m. \]

Forming the ratio of \( a_i^2 \) to \( S^2 \) produces the normalized result:

\[ \frac{a_i^2}{S^2} = \frac{\omega_m^2 \Delta \omega_i}{4 \sum_{k=1}^{K} \omega_i^2 \Delta \omega_i + \sum_{i=k}^{K} \omega_i^2 \Delta \omega_i}, \quad 0 \leq \omega \leq \frac{\omega_m}{2} \]

\[ i \leq k \]

\[ \frac{a_i^2}{S^2} = \frac{\omega_m^2 \omega_i \Delta \omega_i}{\omega_m^2 \sum_{k=1}^{K} \omega_i \Delta \omega_i + \sum_{i=k}^{K} \omega_i^2 \Delta \omega_i}, \quad \frac{\omega_m}{2} < \omega < \omega_m. \]

\[ i > k \]