APPLICATION OF LATERAL SHEARING INTERFEROMETRY
TO STOCHASTIC (RANDOM) INPUTS

by

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STATEMENT BY AUTHOR

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ABSTRACT

Lateral shearing interferometry is a simple and accurate method of measuring the shape of an optical wavefront. However, the complex mathematics required in reduction of the interferograms, along with the restriction that evaluation is only possible at positions separated by the shear, has limited the adoption of this interferometric technique.

A data reduction technique using Fourier analysis methods has been developed to make lateral shearing interferogram reduction more flexible. This technique is hypothetically applied to the measurement of wavefronts propagating through a turbulent medium, and demonstrates the simplicity and accuracy of this data reduction method.
CHAPTER 1

INTRODUCTION

Wavefront shearing interferometry (Bates 1947) provides a simple and accurate means for testing the shape of wavefronts. However, the complex mathematics required for data reduction (Martin 1961, pp. 330-334; Bryngdahl 1965, pp. 39-47; Saunders 1967, pp. 16-20; Saunders 1961), and the restriction that evaluation of the wavefront shape is possible at only those points whose separation equals the lateral shear, has limited the adoption of the technique. One simple but accurate technique (Saunders 1970) has been devised, but it is still dependent on evaluation at shear distances.

Therefore, a data reduction technique using Fourier analysis methods has been developed to make lateral shearing interferometric data reduction more flexible. Results can now be obtained by performing a Fourier transformation of the sheared wavefront data, a proper filtration in the frequency domain, and an inverse transformation to obtain the shape of the wavefront. The result is an accurate determination of the wavefront deviations from a best fitting reference sphere. The restriction of evaluation at
only shear distances is not required by this data reduction technique. However, evaluation must be performed at equally spaced points in the interferogram.

This technique is especially well suited for lateral shearing interferometric measurements of stochastic processes, such as random polishing errors of optical surfaces and propagation through a turbulent atmosphere. The Fourier analysis approach easily permits calculation of the power spectrum or autocorrelation function of the stochastic process as part of the data reduction technique and therefore can determine an important statistical parameter of the process. Since the wavefront shape is also reconstructed by this data reduction technique, the probability density function is also derivable from the data.
CHAPTER 2

BACKGROUND

Lateral Shearing Interferometry

The lateral shearing interferometer gives the difference in aberration at pairs of points separated by the shear. When the shear is small, the resulting interferogram is approximately the directional derivative of the aberration.

It has proven to be a most useful instrument, but for some users it is unpopular, as it does not give the wave aberration directly but only differences.

The main disadvantages of this type of interferometer are: no sensitivity to wavefront shapes perpendicular to the shear direction; interferogram covers only part of the aperture; no sensitivity to periodic errors having a period equal to the shear in a direction parallel to it. However, if the shear is small, periodic errors equal to the shear are unlikely to be present, and interferograms can be recorded in which the shear is at right angles to its direction in the first case.

The sheared interferograms are usually interpreted by integration of the wavefront slope. If the shear is in the x direction and the shear is \( \Delta x \), the aberration at \( x \)
for one component meets the aberration at $x+\Delta x$ for the other. Hence, since the aberration of a wavefront is expressible as a function of $x$ and $\gamma$, $W(x,y)$, then by Taylor's theorem,

$$W(x+\Delta x, y) = W(x,y) + \Delta x \frac{\partial W}{\partial x} + \frac{(\Delta x)^2 \partial^2 W}{2! \partial x^2} + ... \quad (2.1)$$

If the shear is small and the rate of change of $\partial W/\partial x$ is not large, the $(\Delta x)^2$ term in Equation 2.1 is negligible in comparison to $\Delta x$, and

$$W(x+\Delta x) - W(x,y) = \Delta x \frac{\partial W}{\partial x} \quad (2.2)$$

As the aberration difference grows, the vertical displacement of the fringe tells us the retardation difference in so many wavelengths, $x_\lambda$, on the scale of the interferogram. Hence, the slope of the wavefront is given by

$$\frac{\partial W}{\partial x} = \frac{W(x+\Delta x, y) - W(x,y)}{\Delta x} = \frac{x_\lambda}{\text{shear}}, \quad (2.3)$$

and

$$W(x,y) - W(x_0, y) = \int_{x_0}^{x} \frac{x_\lambda}{\Delta x} \, dx, \quad (2.4)$$
where $W(x_0, y)$ is the aberration at the reference point $(x_0, y)$ selected for the interferogram.

The aberration of the wavefront is therefore obtained by an integration of the area under a curve showing the number of fringes per unit distance in the direction of shear, and this simplification is only possible when the shear is small and the rate of change of $\partial W/\partial x$ is not large. More complex data reduction techniques are required when these conditions are not satisfied.

Measurement of Random Stochastic Inputs

Experimental investigations of the effects of atmospheric turbulence on the propagation of optical wavefronts have been interferometrically investigated by many authors. Specifically, Michelson (Herrick and Meyer-Arendt 1966; Buck 1967), Mach-Zender (Carnevale, Crosignani, and Di Porto 1968; Bertolotte, Carnevale, Crosignani, and Di Porto 1969; Bertolotte, Muzii, and Sette 1970), reversing front (Bertolotte, Muzii, and Sette 1970; Bertolotte, Carnevale, Muzii, and Sette 1970), grating (Burlamacchi, Consortini, and Ronchi 1967), differential (Ward and Berry 1967), channel spectrum (Erickson 1962), and correlation interferometers (Bertolotte, Carnevale, Muzii, and Sette 1968; Bertolotte, Carnevale, Daino, and Sette 1970) have been used to measure the phase fluctuations of the stochastic process.
The Michelson and Mach-Zender techniques require a reference beam travelling in a homogeneous medium to reconstruct the wavefront shape, which is difficult for vertical or long horizontal paths. With reversing front, differential, channel spectrum, and grating interferometers or correlation techniques, the determination of wavefront shape is impossible, and wavefront statistics as a function of the spatial separation in the beam is usually measured. These measurements, however, cannot determine the probability density function of the random phase fluctuations. Therefore, the stochastic process is usually assumed to be normally distributed, although Bertolotte, Carnivale, Daino, and Sette (1970) have reported on an experimental technique for verifying the assumption of Gaussian distribution of the phase fluctuations.

Since the nature of the problem precludes the use of a plane reference wavefront, a lateral shearing interferometer is ideally suited for measurements of the effects of atmospheric turbulence on optical wavefront propagation. Using this interferometric approach a reference beam is provided by the propagating wavefront. The wavefront shape can be reconstructed from the interferogram using the Fourier data reduction technique, and a complete statistical analysis, including determination of the probability distribution of the random process is experimentally obtainable.
A lateral shearing interferogram of a wavefront of infinite extent propagating through a turbulent medium can be written as

\[ V(x,y; s) = W(x+s/2, y) - W(x-s/2, y) \]

\[ = W(x,y) \ast \frac{2}{s} \int \delta \left( \frac{x - \frac{1}{2} s}{s} \right) \delta \left( \frac{x + \frac{1}{2} s}{s} \right) \]

when the shear, s, is in the x direction. The "\( \ast \)" represents a convolution operation and \( \int \delta \left( \frac{x - \frac{1}{2} s}{s} \right) \delta \left( \frac{x + \frac{1}{2} s}{s} \right) \) is an odd impulse pair function (Bracewell 1965, p. 79).

Equation 3.1 is a convolution integral of the x component of the wavefront shape, \( W(x,y) \), shown in Figure 1.a for a section, \( y = y_c \) = a constant in the interferogram, with an odd impulse pair function, shown in Figure 1.b. This integral represents the difference in phase at pairs of points separated by the shear for an infinite continuous record and is shown in Figure 1.c for the wavefront of Figure 1.a. For small shears, this integral is approximately the directional derivative of \( W(x,y_c) \).
Figure 1. Wavefront Shape, Odd Impulse Pair Function, and Resultant Lateral Shearing Aberration Differences
Since this integral is a convolution integral, its Fourier transform is, by the convolution theorem (Bracewell 1965, pp. 108-110),

\[ \hat{V}(\xi, \eta; s) = \hat{W}(\xi, \eta) \cdot 2i \sin(\pi s \xi), \]  

(3.2)

where \(\xi\) and \(\eta\) are the x and y spatial frequencies respectively, the "\(\hat{}\)" denotes a functions Fourier transform representation, and \(2i \sin(\pi s \xi)\) is the Fourier transform of \(\frac{2}{s} I_0\left(\frac{x}{s}\right)\).

Solving Equation 3.2 for \(\hat{W}(\xi, \eta)\) we obtain

\[ \hat{W}(\xi, \eta) = \frac{\hat{V}(\xi, \eta; s)}{2i \sin(\pi s \xi)}. \]  

(3.3)

Since

\[ W(x, y) = F^{-1}\{\hat{W}(\xi, \eta)\}, \]  

(3.4)

where \(F^{-1}\) is the inverse Fourier transform operation,

\[ W(x, 0) = F^{-1}_x \left\{ \int_{-\infty}^{+\infty} \hat{W}(\xi, \eta) d\eta \right\}, \]  

(3.5)

where \(F^{-1}_x\) is the x inverse Fourier transform operation.

Substituting Equation 3.3 into Equation 3.5,
\[ W(x, o) = F^{-1} \left\{ \frac{\int_{-\infty}^{\infty} V(\xi, \eta; s) \, d\eta}{2i \sin(\pi s \xi)} \right\} \]

\[ = F^{-1} \left\{ \frac{F(V(x, o; s))}{2i \sin(\pi s \xi)} \right\}, \quad (3.6) \]

and we recover the wavefront shape, \( W(x, o) \).

Although the above derivation was performed for the section \( y = o \) in the interferogram, it can be seen that the same arguments hold for any section \( y = y_C \) in the interferogram.

For a lateral shearing interferogram of finite extent measured at intervals \( \Delta x \) along a section \( y = y_C \) in the record, Equation 3.1 is equivalent to a lattice of samples of the function, \( W(x+s/2, y_C) - W(x-s/2, y_C) \). Omitting in the notation the functional dependence on \( y \),

\[ V_s(x; s) = \left\{ \left\{ \text{rect}(\frac{1}{\Delta x} V(x; s)) \right\} - \text{III}(\frac{1}{\Delta x}) \right\} \ast \text{III}(\frac{1}{\Delta x}) \}, \quad (3.7) \]

where \( X = N\Delta x \) is the maximum length of the interferometric data record, \( \text{rect}(x) \) is the rectangle function (Bracewell 1965, p. 52), and \( \text{III}(x) = \sum_{n=-\infty}^{\infty} \delta(x-n) \) is the sampling or replicating function (Bracewell 1965, pp. 77-79). The function \( \frac{1}{\Delta x} \text{III}(\frac{x}{\Delta x}) \) samples, and the rectangle function truncates the infinite continuous record, \( V(x; s) \). The
function, \( \frac{1}{X}III\left(\frac{X}{x}\right) \), produces periodic replicas of the truncated, sampled interferogram, which is necessary for application of the fast Fourier transform algorithm (Cooley, Lewis, and Welch 1969).

Substituting Equation 3.1 into Equation 3.7,

\[
V_s(x; s) = \{{\{\text{rect}\left(\frac{X}{s}\right) W(x) * I_s\left(\frac{X}{s}\right) \}} - III\left(\frac{X}{s}\right) \} * -III\left(\frac{X}{s}\right). \quad (3.8)
\]

Fourier transforming Equation 3.8 we obtain

\[
\hat{V}_s(\xi; s) = \hat{X}\{{\{\text{sinc}(X\xi) \} W(\xi) \cdot 2\xi\sin(\pi s \xi)} \} * III(\Delta x \xi) III(X\xi), \quad (3.9)
\]

where \( \text{sinc}(x) = \sin(\pi x)/\pi x \), and a smoothed sampled spectrum of, \( W(x) * \frac{2}{s} I_s\left(\frac{X}{s}\right) \), is erected about each point, \( \frac{n}{\Delta x} \), on the \( \xi \) axis.

Now, from the sampling theorem (Whittaker 1915, Shannon 1949, Linden 1959), if this spectrum is "band-limited", we can multiply Equation 3.9 by an appropriate rectangle function, specifically, \( \text{rect}(\Delta x \xi) \), and if \( \Delta x \) has been chosen sufficiently small to prevent aliasing in the frequency domain, the zero order of this spectrum can be filtered out to obtain

\[
\hat{V}_s(\xi; s) = \text{rect}(\Delta x \xi) \{{\{X\text{sinc}(X\xi) \} W(\xi) \cdot 2\xi\sin(\pi s \xi)} \} * III(\Delta x \xi) \cdot III(X\xi). \quad (3.10)
\]
Equation 3.10 can be compared with the Fourier transform of the sampled, replicated, and filtered infinite continuous wavefront function forming the lateral shearing interferogram,

\[ W_s(\xi) = \text{rect}(\Delta x \xi) \{ W(\xi) * \text{III}(\Delta x \xi) \} \text{III}(x \xi). \quad (3.11) \]

If the sinc function of Equation 3.10 is sufficiently narrow so that appreciable smoothing does not occur when the convolution is performed, \( W_s(\xi) \) can be obtained from \( V_s(\xi; s) \) by dividing the zero order of each sampled value of \( V(\xi; s) \) at spatial frequency, \( \xi \), by the appropriate value of \( 2\xi \sin(\pi s \xi) \). This restriction requires a long enough record length, represented by the rectangle function of Equation 3.8, so that the sinc function is narrow in its spatial frequency width compared to the fine detail of the wavefronts Fourier transform.

Therefore, with this approximation

\[ W_s(\xi) = \frac{V_s(\xi; s)}{2\xi \sin(\pi s \xi)}, \quad (3.12) \]

which in Dirac delta function notation becomes, by substituting Equation 3.10 into Equation 3.12,
\[ W_s(\xi) = \text{rect}(\Delta x \xi) \{Xsinc(\xi \xi) \ast \sum_{n=\infty}^{\infty} W(\xi - n) \Delta x \}
\]

\[ \cdot 2i\sin\{\pi s(\xi - \text{---})\} \cdot \sum_{m=\infty}^{\infty} \delta(\xi - \text{---}) / 2i\sin(\pi s \xi) \] (3.13)

Inverse transforming Equation 3.13,

\[ W_s(x) = F^{-1}_x \{W_s(\xi)\} \]

\[ = F^{-1}_x \{\text{rect}(\Delta x \xi) \{Xsinc(\xi \xi) \ast \sum_{n=\infty}^{\infty} W(\xi - n) \Delta x \}
\]

\[ \cdot 2i\sin(\pi s(\xi - \text{---})) \cdot \sum_{m=\infty}^{\infty} \delta(\xi - \text{---}) / 2i\sin(\pi s \xi) \} \] (3.14)

and recovery of the wavefront shape can be achieved from an appropriately spaced lattice of the sampled values from the wavefront shearing interferogram. The recovery is accomplished by appropriate filtering in the spatial frequency domain and inverse transforming of the filtered array.
A hypothetical lateral shearing interferogram was mathematically constructed so that the Fourier data reduction technique could be evaluated. The case considered is applicable to the measurement of an optical wavefront shape after propagation over a long path through a turbulent medium.

A random aberration function was constructed as the initial step in the generation of the interferogram. The statistical characteristics of the aberration function were appropriately chosen to approximate the atmospheric stochastic process. An array of uncorrelated normal deviates, \( g(I) \), was convolved with a hyperbolic secant function, \( \text{sech}(I+n) \), for \( n = 1,N \). The resulting array, \( w(n) \), was adjusted for zero mean and a standard deviation of a quarter wavelength. A portion of this aberration function, smoothed between the discrete data points, is shown in Figure 2 and its autocorrelation function is shown in Figure 3.

The lateral shearing interferogram was constructed by calculating the inherent phase difference due to the
ABERRATION (Wavelengths)

Position in Interferogram (cm)

Figure 2. Input Aberration Function
Figure 3. Input Autocorrelation Function

Lag Units $\Delta x = 0.75\text{cm}$
shear, s, at a distance, y, from a point source propagating in a homogeneous medium, and adding in the phase difference due to the aberration function in a line x in the record, as shown in Figure 4.

With the refractive index of the homogeneous medium approximated by one, the pathlength to the point \(x_1 = x + s/2\) is by the Pythagorean theorem,

\[
R_1 = y\sqrt{1 + \frac{(x+s/2)^2}{y^2}}. \tag{4.1}
\]

For \((x+s/2)/y\ll1\), a binomial expansion of Equation 4.1 results in

\[
R_1 \approx y(1 + \frac{(x+s/2)^2}{2y^2}). \tag{4.2}
\]

Similarly, the pathlength for the point \(x_2 = x-s/2\) is

\[
R_2 \approx y(1 + \frac{(x-s/2)^2}{2y^2}). \tag{4.3}
\]

Therefore the phase difference, \(\Delta\phi\), due to propagation in the homogeneous medium is

\[
\Delta\phi \approx \frac{2\pi(R_2 - R_1)}{\lambda} \approx \frac{-2\pi sx}{\lambda y} = \frac{-2\pi s}{\lambda} \tan\theta. \tag{4.4}
\]
Figure 4. Interferogram Construction
An array of $\Delta \phi(n\Delta x)$ was then combined with the aberration function array, $w(n)$. A quadratic interpolation routine was used to calculate the value of the aberration function at the positions $x+s/2$ and $x-s/2$ for all values, $n = 1,1024$, of the array. Designating these interpolated values of the aberration function as $\delta_1/\lambda$ and $\delta_2/\lambda$, the resulting lateral shearing interferogram can be written as,

$$V_s(x,y;s) = W(n\Delta x+s/2,y) - W(n\Delta x-s/2,y)$$

$$= \frac{2\pi}{\lambda} (R_1 - R_2 + \delta_1 - \delta_2).$$

and approximates wave propagation in a turbulent media.

A portion of the sheared interferogram with a shear $s = 1\text{cm}$, a sampling interval $\Delta x = 0.75\text{cm}$, a wavelength $\lambda = 5.0 \times 10^{-5}\text{cm}$, at a distance $y = 1.5 \times 10^6\text{cm}$ along a line from $x = 6000\text{cm}$ to $x = 5808\text{cm}$ from the origin is shown in Figure 5. The computer program used to generate the interferogram, written in extended Fortran IV for use on a CDC 6400 is given in Appendix B.

A second computer program, given in Appendix C, was used to recover the wavefront aberrations from the lateral sheared interferogram. The initial step in this Fourier data reduction program is a straight line least square fit
Figure 5. Lateral Shearing Interferogram
to the lateral shearing interferometric data, \( V_s(x,y;s) \), represented by Equation 4.5, and a calculation of residuals, \( V_{SR}(x,y;s) \), of the fit. This first order fit of the data is justified by the approximation given in Equation 4.4 and removes most of the phase difference in the interferogram due to propagation in a homogeneous medium.

The residuals are then scaled by \( \Delta x \) to conform to the similarity theorem (Bracewell 1965, pp. 101-104). The real array, \( \Delta xV_{SR}(x,y;s) \), is now used to form the complex array, \( (\Delta xV_{SR}(x,y;s), 0) \), for input into a fast Fourier transform subroutine (Cooley, Lewis, and Welch 1969), and the array is transformed.

Since the original array is real, the resulting transformed complex array, \( \tilde{V}_{SR}(\xi,y;s) \), is hermitian (Bracewell 1965, pp. 14-16), and therefore its real part is even and its imaginary part is odd. Consequently, the \( n \)th member of this array, \( (R,I) \), with spatial frequency, \( \xi_n \), is related to the \( (N+2-n) \)th member of the array, \( (R,-I) \), with spatial frequency, \( -\xi_n \), if the complex array has \( N \) components. The \( (N/2 + 1) \)th member of the array has a spatial frequency associated with it of \( \xi = \pm 1/2\Delta x \).

The modulus and phase of the transformed array are shown in Figure 6 and 7, respectively, and indicate that \( \Delta x \) has been chosen sufficiently small in order to prevent aliasing in the frequency domain.
Figure 6. Modulus of Transformed Residuals
Spatial Frequency (Cycles/cm)

Figure 7. Phase of Transformed Residuals
Since for this test case, \( N = 1024, \Delta x = 0.75, \)
the sinc function of Equation 3.3 is sinc \((768\xi)\). The first zeroes of this function are at \( \xi = \pm \frac{1}{768} \) cycles/cm, which is the spatial frequency separation between adjacent members of the array. Therefore, appreciable smoothing does not occur and the approximations leading to Equation 3.12 are valid.

The transformed complex array, \( \tilde{V}_{SR}(\xi,y;s) \), is then filtered by multiplication with the complex array, \((0, -1/2\sin(\pi\xi s))\), to obtain the equivalent of Equation 3.13. This imaginary filter function is shown in Figure 8. Since the filter is odd and imaginary, this filter operation results in another hermetian array, where the real-even and imaginary-odd components of \( \tilde{V}_{SR}(\xi,y;s) \) have been interchanged by the filtration. The filter operation at zero frequency is explained in Appendix A.

The result of this filtration is the approximate Fourier transform of the wavefront aberrations. Results have shown that a small amount of second order curvature remains from the first order fit performed as the initial step of the data reduction. It is therefore preferable to inverse transform the filtered array and remove this second order curvature before calculating the power spectrum and autocorrelation function of the wavefront aberration function, since the variance is about 10% to high if
Figure 8. Imaginary Filter Function
these operations are performed before curvature removal.

The hermetian filtered array is therefore Fourier transformed and results in a complex array which has only a real component. However, since the fast Fourier transform subroutine only performs a transform and not an inverse transform, the recovered aberration function is of the form, \( W_s(-x,y) \), and a proper reordering of the array is required. A second order least squared fit then results in the recovered wavefront aberration. The portion from \( x = 6000 \text{cm} \) to \( x = 5808 \text{cm} \), corresponding to the part of the original wavefront aberration function in Figure 9, is shown in Figure 10. The recovery is excellent, except for a slight increase in the values of the aberration function for about 30cm of the x-axis at both ends of the array. This is probably caused by the parabolic nature of the least squared fit subroutine employed to remove the second order curvature.

In order to obtain the power spectrum of the aberration function, the wavefront is scaled, Fourier transformed, and multiplied by its complex conjugate, and is shown in Figure 11.

The autocorrelation function is then obtained either by Fourier transformation of the power spectrum or
Figure 9. Input Aberration Function
Figure 10. Recovered Aberration Function
Figure 11. Power Spectrum of Wavefront Aberrations
by the equation

$$C(\Delta x) = \frac{1}{X_n - |\Delta x|} \sum_{-(X_n - |\Delta x|)/2}^{(X_n - |\Delta x|)/2} W(x - \frac{\Delta x}{2}, y) W(x + \frac{\Delta x}{2}, y) \, dx \tag{4.6}$$

where $X_n$ is the length of the record, and is usually not used for lags, $\Delta x$, longer than 5 to 10 percent of the length of the record (Blackman and Tukey 1959, pp. 11-12). These two autocorrelation functions are shown in Figure 12 in comparison with the autocorrelation function of the original wavefront aberration calculated by using Equation 4.6. The agreement between original and recovered autocorrelations is excellent, and the standard deviations differ by only .004 waves for the two wavefront functions.

An estimate of the smoothed (average over frequency) power spectrum, $p_i(\xi)$, of the true power spectrum is shown in Figure 13, (Blackman and Tukey 1959). This estimate was obtained by Fourier transforming the apparent autocorrelation function

$$C_i(\Delta x) = D_i(\Delta x) \cdot C(\Delta x), \tag{4.7}$$

where $C(\Delta x)$ is the autocorrelation function calculated using Equation 4.6 for approximately 5% of the record length and $D_i(\Delta x)$ is the lag window.
Figure 12. Input and Output Wavefront Autocorrelation Functions
Figure 13. Smoothed Power Spectrum
While the modified apparent autocorrelation functions, which are obtained by multiplying $C(\Delta x)$ by $D_1(\Delta x)$, are far from being respectable estimates of the true autocorrelation function, their transforms are very respectable estimates of smoothed values of the true spectral density.

$$D_0(\Delta x) = \text{rect}(\frac{\Delta x}{2X_n}).$$

(4.8)
CHAPTER 5

CONCLUSIONS

The Fourier data reduction technique presented here provides a unique new method for interpretation and analysis of lateral shearing interferograms. The method is especially useful since the requirement of evaluation at only shear distances is not required by the technique. This additional flexibility in data reduction permits more flexibility in the parameters used during the recording of the interferogram. The shear value can be chosen to give optimum fringe spacing, and recording of the sheared aberration differences can be recorded at any equally spaced intervals in the interferogram.

The only requirement on the interval spacing is that appreciable aliasing does not occur for the Fourier transform of the sheared wavefront. A complete recovery depends on the concept of a "bandlimited" Fourier transform. However, a more realistic requirement is that the magnitude of the modulus and phase at the higher spatial frequencies are negligible compared to its values at the lower spatial frequencies.
This concept can be useful in determining an optimum recording interval for the sheared wavefront interferogram. An interval can be chosen so that appreciable aliasing is just avoided, and this recording interval is then the minimum required for almost complete recovery. This requirement is similar to the concept of efficient recovery in sampling theory.

An improvement in the computer reduction technique presented here is necessary to take complete advantage of these concepts. The fast Fourier transform subroutine given in Appendix C can only be used for arrays of magnitude, $2^M$. For large $M$ this puts a restriction on the number of data points necessary to completely fill the array to be transformed. However, fast Fourier subroutines are available which do not have as stringent array restrictions, and can be used as substitutes in the computer program to provide more efficient data reduction.
APPENDIX A

DETERMINATION OF INDETERMINATE FILTER OPERATIONS

In the derivation of the Fourier data reduction technique, a problem was encountered in dividing Equation 3.10 by zero at $\xi = \pm n/s$, $n = 0, 1, 2, \ldots, N$. The solution to this difficulty is now considered.

The laterally sheared interferometric data, $V(x, y; s)$, is a real function. Therefore, its Fourier transform can be expressed as

\[
V(\xi, y; s) = \int_{-\infty}^{+\infty} V(x, y; s) \cos(2\pi \xi x) dx - i \int_{-\infty}^{+\infty} V(x, y; s) \sin(2\pi \xi x) dx.
\]  

(A.1)

Since any function can be split unambiguously into odd and even parts, then

\[
V(x, y; s) = O(x, y; s) + E(x, y; s),
\]

(A.2)

and Equation A.1 can be written as

\[
V(\xi, y; s) = \int_{-\infty}^{+\infty} E(x, y; s) \cos(2\pi \xi x) dx - i \int_{-\infty}^{+\infty} O(x, y; s) \sin(2\pi \xi x) dx,
\]

(A.3)
where the first integral is a real and even and the second integral is an imaginary and odd function of $\xi$.

Dividing Equation A.3 by the filter function, $2i\sin(\pi s \xi)$,

$$\hat{V}(\xi, y; s) = \frac{-1}{2\sin(\pi s \xi)} \int_{-\infty}^{+\infty} 0(x, y; s) \sin(2\pi \xi x) \, dx$$

$$- \frac{i}{2\sin(\pi s \xi)} \int_{-\infty}^{+\infty} E(x, y; s) \cos(2\pi \xi x) \, dx.$$ \hspace{1cm} (A.4)

Since the imaginary term of Equation A.4 is now an odd function of $\xi$, its value at the origin is zero as far as an inverse transformation is concerned. Therefore, taking the limit as $\xi \to 0$ of Equation A.4,

$$\hat{V}(\xi, y; s)_{\xi=0} = \frac{1}{2} \lim_{\xi \to 0} \frac{1}{\sin(\pi s \xi)} \int_{-\infty}^{+\infty} 0(x, y; s) \sin(2\pi \xi x) \, dx,$$ \hspace{1cm} (A.5)

and since the integral is not dependent on $\xi$, Equation A.5 can be rearranged to obtain

$$\hat{V}(\xi, y; s)_{\xi=0} = \frac{1}{2} \int_{-\infty}^{+\infty} 0(x, y; s) \lim_{\xi \to 0} \frac{\sin(2\pi \xi x)}{\sin(\pi s \xi)} \, dx.$$ \hspace{1cm} (A.6)
Invoking L'Hospital's rule, Equation A.6 becomes

\[ V(\xi,y;s)_{\xi=0} = \frac{1}{s} \int_{-\infty}^{+\infty} x\phi(x,y;s)dx. \] (A.7)

But by the moment theorem (Bracewell 1965, p. 138), if \( f(x) \) has a Fourier transform \( \hat{f}(\xi) \), then the first moment of \( f(x) \) is equal to \(- (2\pi i)^{-1}\) times the slope of \( f(\xi) \) at \( \xi = 0 \); that is

\[ \int_{-\infty}^{+\infty} x f(x)dx = \frac{\hat{f}'(0)}{-2\pi i}. \] (A.8)

Therefore, using the relationship of Equation A.8 in Equation A.7, we obtain

\[ V(\xi,y;s)_{\xi=0} = \frac{\phi'(0)}{2\pi is}, \] (A.9)

where \( \phi'(0) \) is the slope of the Fourier transform of the odd part of the wavefront aberration function, \( \phi(\xi) \), at \( \xi = 0 \). The value of the right hand of Equation A.9 is real since the transform of a real and odd function is imaginary and odd, and the \( i \) in the denominator will cancel. Since the imaginary term of Equation A.4 was shown to be zero at the origin as far as any inverse transformation is concerned, the evaluation of the indeterminate relationship
in Equation A.5 results in a determinate complex number,
\( \tilde{0}^{-1}(o) / 2\pi i s, o \).

Now, \( O(\xi) \), at \( \xi = 0 \), determines the d.c value of the
wavefront aberration function. However, the absolute value
of phase is of no consequence in the evaluation of the
aberration function. Therefore, when filtration with
\( 2i \sin(\pi s \xi) \) is performed at \( \xi = 0 \), a proper value for \( W_s(o, y) \)
in Equation 3.14 is the complex number \( (0, 0) \).

This approach to the filtering operation at zero
spatial frequency was successfully used in the recovery of
the wavefront aberration function for the hypothetical data
case presented here.

The above argument can not be used for \( \xi = \pm n/s \),
\( n = 1, 2, 3, \ldots, N \). Two alternatives are possible however.
A sampling interval, \( \Delta x \), can be chosen so that the cutoff
of the fast Fourier transform subroutine, \( \xi = \pm 1/2\Delta x \), is
such that \( s/2\Delta x < 1 \), and filter values are within the first
half cycle of the sine function. This is what was done
for the hypothetical data case presented here. Or, a
\( \Delta x \) can be chosen, so that for all members of the array,
\( \xi \neq \pm n/s, n = 1, 2, \ldots, N \). Therefore, the indeterminate, \( 0/0 \),
is not encountered, and all divisions in the array by
filtration are possible.
APPENDIX B

INTERFEROGRAM GENERATING COMPUTER PROGRAM

PROGRAM ATA(INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT, IPUNCH)
DIMENSION A(1152), B(25), AC(50)
C READ ARRAY OF UNCORRELATED RANDOM NORMAL DEVIATES
READ(S,10)(A(I), I=1, 1152)
10 FORMAT(16F5.2)
C CALCULATE HYPERBOLIC SECANT FUNCTION UNTIL VALUE OF
C FUNCTION IS LESS THAN .01. N DETERMINES
C AUTOCORRELATION LENGTH DESIRED FOR SMOOTHED ABERRATION
C FUNCTION.
N = 6
DO 15 J = 1, N
R = (6.0*FLOAT(J))/FLOAT(N)
B(N+1-J) = 2.0/(EXP(R)+EXP(-R))
15 CONTINUE
B(N+1) = 1.0
C REDUCE VALUES OF SECANT FUNCTION SO THAT THE SUM
C OF ALL (2N+1) VALUES EQUALS ONE.
SUM2 = 1.0
DO 16 L = 1, N
SUM2 = SUM2 + 2.0*B(L)
16 CONTINUE
M = N + 1
DO 18 I = 1, M
B(I) = B(I)/SUM2
18 CONTINUE
C CONVOLVE NORMALIZED HYPERBOLIC SECANT FUNCTION WITH
C ARRAY OF RANDOM NORMAL DEVIATES.
DO 20 K = 1, 1028
SUM1 = 0.0
DO 25 J = 1, N
SUM1 = SUM1 + B(J)*(A(K + J - 1) + A(K + 2*N + 1 - J))
25 CONTINUE
A(K) = B(N+1)*A(K+N) + SUM1
20 CONTINUE
C CALCULATE MEAN AND STANDARD DEVIATION OF SMOOTHED
C ABERRATION FUNCTION.
CALL STAT(A, 1028, BMEAN, SDV)
C SET STANDARD DEVIATION REQUIRED FOR SMOOTHED
C ABERRATION FUNCTION.
ADSDV=0.25
C ADJUST STANDARD DEVIATION TO REQUIRED VALUE AND
C MEAN TO ZERO.
DO 35 I=1,1028
A(I)=(ADSDV*(A(I)-BMEAN))/SDV
35 CONTINUE
C CHECK STANDARD DEVIATION AND MEAN OF ADJUSTED
C ABERRATION FUNCTION.
CALL STAT(A,1028,AMEAN,SDV1)
WRITE(6,40)AMEAN,SDV1
40 FORMAT(55X,*MEAN*,10X,*STD DEV*,49X,E15.6,E16.6,
1//,50X,*ACORR OF ORIGINAL ABERRATION FUNCTION*,//)
C CALCULATE AUTOCORRELATION FUNCTION OF SMOOTHED AND
C ADJUSTED WAVEFRONT ABERRATION FUNCTION.
CALL ACORR(A,1028,50,AC)
C WRITE AUTOCORRELATION FUNCTION.
WRITE(6,42) (I,AC(I),I=1,50)
42 FORMAT(5(I10,E15.4))
C WRITE SMOOTHED ABERRATION FUNCTION.
WRITE(6,45)
45 FORMAT(1H1,53X,*SMOOTHED ABERRATION FUNCTION*,//)
WRITE(6,47) (I,A(I),I=1,1024)
47 FORMAT(8(I5,F10.3))
C CALCULATE LATERAL SHEARING INTERFEROGRAM
C APPROXIMATING PROPAGATION IN TURBULENT MEDIUM.
CALL DELPHI(A,1.0D0,1.5D6,5.0E-5,0.75D0,6.0D3)
C WRITE AND PUNCH CARD DECK OF INTERFEROMETRIC VALUES.
WRITE(6,50)
50 FORMAT(1H1,52X,*LATERAL SHEARING INTERFEROGRAM*,//)
WRITE(6,55) (I,A(I),I=1,1024)
55 FORMAT(8(I5,F10.3))
PUNCH 60,(A(I),I=1,1024)
60 FORMAT(8F10.5)
STOP
END
SUBROUTINE STAT(X,N,A,D)
C
C GIVEN N VALUES OF A TABULATED FUNCTION X=X(K), COMPUTE MEAN "A" AND STANDARD DEVIATION "D".
C
DIMENSION X(1152)
D=0.
A=0.
DO 1 I=1,N
  A=A+X(I)
  D=D+X(I)*X(I)
1 CONTINUE
A=A/FLOAT(N)
D=SQRT((D/FLOAT(N))-A**2)
RETURN
END

SUBROUTINE ACORR(X,N,N2,AC)
C
C THE ARRAY TO BE AUTOCORRELATED IS X. COMPUTE AUTOCORRELATION FOR N2 TAU VALUES. N2 SHOULD BE N/10 AT MOST.
C
DIMENSION X(1152),AC(50)
DO 2 J=1,N2
  AC(J)=0.
DO 1 K=J,N
  AC(J)=AC(J)+X(K+1-J)*X(K)
1 CONTINUE
AC(J)=AC(J)/FLOAT(N-J+1)
2 CONTINUE
RETURN
END
SUBROUTINE DELPHI(A,S,Y,W,DX,X0)

CALCULATE LATERAL SHEARING INTERFEROGRAM APPROXIMATING PROPAGATION IN TURBULENT MEDIUM. "A" IS THE ARRAY OF WAVEFRONT ABERRATION VALUES, "S" IS THE SHEAR, "Y" IS THE PERPENDICULAR DISTANCE TO INTERFEROGRAM FROM POINT SOURCE, "W" IS WAVELENGTH OF SOURCE, "DX" IS SAMPLING INTERVAL, AND "X0" IS REFERENCE POINT IN INTERFEROGRAM MEASURED FROM ORIGIN.

DIMENSION A(1152)
DOUBLE PRECISION XPOS,S,Y,T,Q,R1,R2,X0,DX

	W0PI=2.0*3.14159265358979
	T=0.500*S
	Q=Y*Y

X1 AND X2 ARE NORMALIZED COORDINATE VALUES FOR USE IN QUADRATIC INTERPOLATION ROUTINE.

X1=T/DX
X2=-X1

DETERMINE INITIAL SET OF COEFFICIENTS FOR QUADRATIC INTERPOLATION ALGORITHM.

D1=A(5)-A(4)
D2=A(5)-2.0*A(4)+A(3)
D3=A(5)-3.0*A(4)+3.0*A(3)-A(2)
D4=A(5)-4.0*A(4)+6.0*A(3)-4.0*A(2)+A(1)

DO 20 I=1,1024

CALCULATE PATHLENGTH DIFFERENCE ΔEL DUE TO PROPAGATION TO TWO POINTS CENTERED AT (X0-N*DX) AND SEPERATED BY SHEAR AT PERPENDICULAR DISTANCE "Y" FROM INTERFEROGRAM.

T1=I-1
XPOS=X0-DBLE(T1)*DX
R1=DSORT((XPOS+T)*2+Q)
R2=DSORT((XPOS-T)*2+Q)
DEL=R1-R2

CALCULATE INTERPOLATED VALUES OF WAVEFRONT ABERRATION FUNCTION Y1 AND Y2, AT NORMALIZED COORDINATE VALUES.

X1 AND X2.

Y1=D1+((D2*(X1-3.0))/2.0)*((D3*(X1-2.0)*(X1-1.0))/6.0)
1 *((D4*(X1-1.0)*(X1+1.0)*(X1-2.0))/24.0)

Y1=A(I+2)*X1*Y1

Y2=D1+((D2*(X2-3.0))/2.0)*((D3*(X2-2.0)*(X2-1.0))/6.0)
1 *((D4*(X2-1.0)*(X2+1.0)*(X2-2.0))/24.0)

Y2=A(I+2)*X2*Y2
CALCULATE TOTAL PHASE DIFFERENCE AT POINT IN LATERAL SHEARING INTERFEROGRAM DUE TO PROPAGATION AND INTERPOLATED ABERRATION FUNCTION DIFFERENCES. 

\[ A(i) = 2 \pi \left( \frac{\Delta \theta}{w} + y_2 - y_1 \right) \]

DETERMINE NEW SET OF COEFFICIENTS FOR QUADRATIC INTERPOLATION FORMULA AND CONTINUE CALCULATION FOR NEW POINT IN INTERFEROGRAM.

\[ \text{TEMP 1} = D_1 \]
\[ \text{TEMP 2} = D_2 \]
\[ \text{TEMP 3} = D_3 \]
\[ D_1 = A(i+5) - A(i+4) \]
\[ D_2 = D_1 - \text{TEMP 1} \]
\[ D_3 = D_2 - \text{TEMP 2} \]
\[ D_4 = D_3 - \text{TEMP 3} \]

20 CONTINUE
RETURN
END
APPENDIX C

INTERFEROGRAM REDUCTION COMPUTER PROGRAM

PROGRAM AT9(INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT)
DIMENSION DP(1024), C(3), XPOS(1024), AC(50), CC(3)
COMPLEX CDP(1024), D1, D2

C SET VALUE OF SAMPLING INTERVAL "DX".
DX=0.75
PI=3.14159265358979
R1=0.

C SET VALUE OF SHEAR "S".
S=1.0

C READ LATERAL SHEARING INTERFEROMETRIC DATA.
READ(5,10)(DP(I), I=1, 1024)
10 FORMAT(8F10.5)

CALCULATE ARRAY OF POSITION VALUES IN INTERFEROGRAM
FOR USE IN FIRST AND SECOND ORDER FIT ROUTINES.
CALL XVAL(DX, XPOS)

PERFORM FIRST ORDER FIT OF INTERFEROMETRIC DATA
AND CALCULATE RESIDUALS.
CALL LFIT(XPOS, DP, 1024, C)
CALL STATP(XPOS, DP, 1024, C, 2, SD1, AM)

FORM COMPLEX ARRAY FROM SCALED RESIDUALS FOR
FFT SUBROUTINE
DO 22 I=1, 1024
R3=DX*DP(I)
CDP(I)=CMPLX(R3, R1)
22 CONTINUE

PERFORM FAST FOURIER TRANSFORM OF COMPLEX ARRAY.
CALL FFT(CDP, 10*PI)

CALCULATE AND WRITE MODULUS AND PHASE OF TRANSFORM
TO CHECK FOR ALIASING.
DO 25 I=1, 1512
DP(I)=SQRT((REAL(CDP(I)))**2+(AIMAG(CDP(I)))**2)
DP(I+512)=ATAN(AIMAG(CDP(I))/REAL(CDP(I)))
25 CONTINUE

WRITE(6,27)
27 FORMAT(45X, "MOD AND PHASE OF FFT OF INTERFEROGRAM")
WRITE(6,29)(I, DP(I), DP(I+512), I=1, 512)
29 FORMAT(4(I5, 2E14.4))
DETERMINE QUANTITY \(\text{(spatial freq per data point)}\)

\(R^2 = (\pi \times S)/(1024 \times \Delta x)\)

PERFORM FILTRATION OF FFT OF INTERFEROGRAM.

DO 30 I=1,511
\(XX = R^2 \times \text{FLOAT}(I)\)
\(R4 = 0.5/\sin(XX)\)
\(R5 = -R4\)
\(D1 = \text{CMPLX}(R1, R4)\)
\(D2 = \text{CMPLX}(R1, R5)\)
\(\text{CDP}(I+1) = \text{CDP}(I+1) \times D2\)
\(\text{CDP}(1025-I) = \text{CDP}(1025-I) \times D1\)

30 CONTINUE

PERFORM FILTRATION AT ZERO FREQUENCY AND CENTER FREQUENCY.
\(\text{CDP}(1) = \text{CMPLX}(R1, R1)\)
\(XX = 513 \times \Delta x\)
\(R4 = -0.25/\left(\sin(XX) \times \sin(XX)\right)\)
\(D2 = \text{CMPLX}(R1, R4)\)
\(\text{CDP}(513) = \text{CDP}(513) \times D2\)

TRANSFORM FILTERED COMPLEX ARRAY.
CALL FFT(CDP, 10, PI)

RECOVER REAL PART OF COMPLEX ARRAY, SCALE, AND REORDER PROPERLY.
\(R1 = 1.0/(2048 \times \Delta x \times \pi)\)
DO 38 I=1,1024
\(\text{DP}(1025-I) = \text{REAL}(\text{CDP}(I)) \times R1\)

38 CONTINUE

PERFORM SECOND ORDER FIT OF RECOVERED WAVEFRONT FUNCTION AND CALCULATE RESIDUALS TO OBTAIN RECOVERED WAVEFRONT ABBERRATIONS.
CALL QFIT(XPOS, DP, 1024, CC)
CALL STATP(XPOS, DP, 1024, CC, 3, SD2, BM)

CALCULATE AUTOCORRELATION OF RECOVERED ABBERRATION FUNCTION.
CALL ACORR(DP, 1024, 50, AC)

WRITE FIT COEFFICIENTS, MEANS, AND STANDARD DEVIATIONS OF FIRST AND SECOND ORDER FITS.
WRITE(6, 40) AM, SD1, C(1), C(2), BM, SD2, CC(1), CC(2), CC(3)

40 FORMAT(1H1, //, 9E14.4, //, 53X, *ACORR OF ABBERRATIONS* 1, //)

WRITE AUTOCORRELATION FUNCTION OF RECOVERED WAVEFRONT ABBERRATIONS.
WRITE(6, 45) (I, AC(I), I=1, 50)

45 FORMAT(5(I10, E15.4))

WRITE RECOVERED WAVEFRONT ABBERRATION FUNCTION.
WRITE(6, 46)

46 FORMAT(1H1, 52X, *RECOVERED ABBERRATION FUNCTION* 1, //)
WRITE(6,48) (I,DP(I),I=1,1024)
48 FORMAT(8(I5,E11.3))
C CALCULATE POWER SPECTRUM OF WAVEFRONT ABERRATION
C FUNCTION.
R1=0.
DO 50 I=1,1024
R2=DP(I)*DX
CDP(I)=CMPLX(R2,R1)
50 CONTINUE
CALL FFT(CDP,10,PI)
DO 55 I=1,512
D1=CDP(I)
D2=CDP(1+512)
CDP(I)=D1*CONJG(D1)
CDP(1+512)=D2*CONJG(D2)
55 CONTINUE
C WRITE POWER SPECTRUM OF WAVEFRONT ABERRATIONS.
WRITE(6,57)
57 FORMAT(1H1,52X,*POWER SPECTRUM OF ABERRATIONS*,//)
WRITE(6,60)(I,CDP(I),I=1,512)
60 FORMAT(4(I7,E13.3,E13.3))
C CALCULATE AUTOCORRELATION FUNCTION BY FOURIER
C TRANSFORMATION OF POWER SPECTRUM.
CALL FFT(CDP,10,PI)
C SCALE AUTOCORRELATION FUNCTION AND WRITE.
R2=1.0/(1024.*DX)
R3=R2*R2
D2=CMPLX(R3,R1)
DO 65 I=1,512
CDP(I)=CDP(I)*D2
65 CONTINUE
WRITE(6,70)
70 FORMAT(1H1,46X,*ACORR FROM FFT OF POWER SPECTRUM*,//)
WRITE(6,60)(I,CDP(I),I=1,512)
DO 80 I=1,49
R2=AC(I+1)*DX
CDP(I+1)=CMPLX(R2,R1)
CDP(1025-I)=CMPLX(R2,R1)
80 CONTINUE
DO 85 I=50,511
CDP(I+1)=CMPLX(R1,R1)
CDP(1025-I)=CMPLX(R1,R1)
85 CONTINUE
R2=AC(1)*DX
CDP(1)=CMPLX(R2,R1)
CDP(513)=CMPLX(R1,R1)
CALL FFT(CDP,10,PI)
WRITE(6,90)
90 FORMAT(1H1,55X,*SMOOTHED POWER SPECTRUM*,//)
WRITE(6,60)(I,CDP(I),I=1,512)
STOP
END
SUBROUTINE XVAL(DX,XPOS)

C CALCULATE ARRAY OF RELATIVE POSITION VALUES IN
C LATERAL SHEARING INTERFEROGRAM FOR USE IN FIRST
C AND SECOND ORDER FIT ROUTINES.

DIMENSION XPOS(1024)
DO 30 I=1,1024
T=I-1
XPOS(I)=DX*T
30 CONTINUE
RETURN
END

SUBROUTINE LFIT(X,Y,N,C)

FIT A STRAIGHT LINE THROUGH THE N POINTS (X(I),Y(I)),
I=1,N). THE EQUATION OF THE LINE IS Y=C(1)+C(2)*X.

DIMENSION X(1024),Y(1024),C(2)
SX0=N
SYX0=0.
SX1=0.
SYX1=0.
SX2=0.
DO 5 I=1,N
XX=X(I)-X(1)
YY=Y(I)-Y(1)
SYX0=SYX0+YY
SX1=SX1+XX
SYX1=SYX1+XX*YY
SX2=SX2+XX*YY
5 CONTINUE
C(1)=(SYX0*SX2-SYX1*SX1)/(SX0*SX2-SX1*SX1)
C(2)=(SYX0-C(1)*SX0)/SX1
RETURN
END
SUBROUTINE QFIT(X,Y,N,C)

GIVEN "N" TABULATED VALUES OF SOME FUNCTION.
KTH FUNCTION VALUE IS Y(K), FOR X=X(K). FIT A
PARABOLA THROUGH FUNCTION. RETURN COEFFICIENTS IN C.
C(1) IS CONSTANT, C(2) IS LINEAR TERM, AND C(3)
IS QUADRATIC TERM.

DIMENSION X(1024), Y(1024), C(3)
DOUBLE PRECISION SX0, SYX0, SX1, SYX1, SX2, SYX2, SX3,
SX4, DUM, SX2S, SX3S, SX3C, XX, YY

SX0 = N
SYX0 = 0.
SX1 = 0.
SYX1 = 0.
SX2 = 0.
SYX2 = 0.
SX3 = 0.
SX4 = 0.
DO 30 I = 1, N
XX = X(I) - X(1)
YY = Y(I) - Y(1)
SYX0 = SYX0 + YY
SX1 = SX1 + XX
SYX1 = SYX1 + YY * XX
DUM = XX * XX
SX2 = SX2 + DUM
SYX2 = SYX2 + YY * DUM
DUM = DUM * XX
SX3 = SX3 + DUM
SX4 = SX4 + DUM * XX
30 CONTINUE
SX2S = SX2 * SX2
SX3S = SX3 * SX3
SX3C = SX3S * SX3
AA = SX2 * SX4 * SYX0 * SX3
BB = SYX0 * SX3C + SX1 * SX2 * SX3S
DD = SX1 * SYX2 * SX3S - SX1 * SX3 * SYX1 * SX4 - SYX2 * SX3 * SX2S
EE = 2 * SX2 * SX1 * SX3S - SX3 * SX2 * SX2S - SX3 * SX4 * SX1 * SX1
FF = SX2 * SX4 * SX0 * SX3 - SX0 * SX3C
C(1) = (AA + BB + DD) / (EE + FF)
GG = SX1 * SX2 - SX0 * SX3
HH = SX1 * SX3 - SX2S
C(2) = (SYX0 * SX3 - SYX1 * SX2 + C(1) * GG) / HH
C(3) = (SYX0 - C(2) * SX1 - C(1) * SX0) / SX2
RETURN
END
SUBROUTINE STATP(X,Y,N,C,M,S,R)

GIVEN N VALUES OF A TABULER FUNCTION(Y(K) IS VALUE AT X(K)), AND A PREVIOUSLY FITTED POLYNOMIAL OF ORDER M-1 (M COEFFICIENTS STORED IN VECTOR C FROM LOW TO HIGH ORDER). COMPUTE MEAN "R", STANDDARD DEVIATION "S", AND RESIDUALS WHICH ARE STORED BACK IN Y(K).

DIMENSION X(1024), Y(1024), C(3)
R = 0.
S = 0.
Y1 = Y(1)
DO 40 I = 1, N
XX = X(I) - X(1)
F = 0.
K = M
DO 45 J = 1, M
F = F + XX * C(K)
K = K - 1
45 CONTINUE
F = Y(I) - Y1 - F
Y(I) = F
S = S + F * F
R = R + F
40 CONTINUE
R = R / FLOAT(N)
S = SRT(S / FLOAT(N) - R * R)
RETURN
END

SUBROUTINE ACORR(X,N,N2,AC)

THE ARRAY TO BE AUTOCORRELATED IS X. COMPUTE AUTOCORRELATION FOR N2 TAU VALUES. N2 SHOULD BE N/10 AT MOST.

DIMENSION X(1024), AC(50)
DO 2 J = 1, N2
AC(J) = 0.
DO 1 K = J, N
AC(J) = AC(J) + X(K+1-J) * X(K)
1 CONTINUE
AC(J) = AC(J) / FLOAT(N-J+1)
2 CONTINUE
RETURN
END
SUBROUTINE FFT(A,M,PI)
C
CALCULATE FOURIER TRANSFORM OF COMPLEX ARRAY A, OF
DIMENSION 2**M. FOURIER TRANSFORM IS STORED IN
ARRAY "A" AT COMPLETION OF SUBROUTINE.
C
COMPLEX A(1024),U,W,T
N=2**M
NV2=N/2
NM1=N-1
J=1
DO 7 I=1,NM1
IF(I.GE.J) GO TO 5
3 T=A(J)
A(J)=A(I)
A(I)=T
5 K=NV2
6 IF(K.GE.J) GO TO 8
J=J-K
K=K/2
GO TO 6
8 J=J+K
7 CONTINUE
DO 30 L=1,M
LE=2**L
LE1=LE/2
U=(1.0,0.0)
W=CMPLX(COS(PI/LE1),SIN(PI/LE1))
DO 20 J=1,LE1
DO 10 I=J,N,LE
IP=I+LE1
T=A(IP)*U
A(IP)=A(I)-T
A(I)=A(I)+T
10 CONTINUE
U=U*W
20 CONTINUE
30 CONTINUE
RETURN
END
REFERENCES


