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THE STOCHASTIC EVOLUTION OF ASTEROIDAL REGOLITHS AND THE ORIGIN OF BRECCIATED AND GAS-RICH METEORITES

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THE STOCHASTIC EVOLUTION OF ASTEROIDAL REGOLITHS AND
THE ORIGIN OF BRECCIATED AND GAS-RICH METEORITES

by

Kevin Richard Housen

A Dissertation Submitted to the Faculty of the
DEPARTMENT OF PLANETARY SCIENCES
In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

1981
As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Kevin Richard Housen entitled \textit{THE STOCHASTIC EVOLUTION OF ASTEROIDAL REGOLITHS AND THE ORIGIN OF BRECCIATED AND GAS-RICH METEORITES} and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

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SIGNED:  

Kevin Richard Housen
This dissertation is dedicated to my brother Doug, an unusually creative individual.
ACKNOWLEDGMENTS

During my stay in graduate school I have never once visited a proctologist. Yet these two experiences are similar in the sense that they both have the potential for being "une grande douleur dans mon derriere". I am very grateful, therefore, to the few individuals who have made the past few years very worthwhile intellectually and emotionally.

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ABSTRACT

A model is constructed which views regolith evolution on asteroids as a stochastic process. Average values are shown to be poor descriptors of regolith depth. Large deviations from the average are expected to occur due both to variations in the depth over the surface of a body and to stochastic fluctuations in the variables which determine regolith depth, e.g. the number of craters produced on an asteroid. The utility of the average depth is not significantly increased by avoiding large craters or thick ejecta deposits; a procedure adopted in previous regolith studies. The statistical uncertainty associated with regolith depth severely limits the power of regolith models in predicting parent-body size for brecciated meteorites. Virtually any rocky asteroid larger than 100-200 km in diameter could have produced the abundance of brecciated material observed in the achondritic meteorites. Bodies which are composed of weaker materials and which have diameters greater than 20 km could have produced the abundance of breccias observed in the chondrites.

A Monte Carlo algorithm is used to simulate the random walks and corresponding charged-particle irradiation histories of grains in regoliths. On rocky asteroids, only about 20% of the grains are exposed to solar cosmic ray ions. These grains typically spend a few thousand years in the upper 100 microns of the regolith and acquire
particle track densities of $10^7$-$10^9$/cm$^2$ at their surfaces. Only about 5% of the grains acquire track densities greater than $10^8$/cm$^2$. Grains which reach the surface are exposed to galactic cosmic rays for roughly $10^6$y. Weak asteroids with diameters less than a few tens of kilometers have very immature regoliths because of short collisional lifetimes and the ejection of heavily irradiated grains to space. Only a few percent of the grains are exposed at the surface and these acquire track densities of $10^7$-$10^8$/cm$^2$. Exposure times and the fraction of grains irradiated should increase for larger weak bodies due to longer collisional lifetimes. These results, which are based on present-day conditions in the asteroid belt, agree well with irradiation features observed in gas-rich meteorites; an origin during epochs of early solar system evolution is not required.
CHAPTER 1

INTRODUCTION

Modern theories of solar system evolution, although varied and still speculative, typically begin with a nebular cloud of gas and dust which ultimately collapses into a disk (Safronov, 1972; Goldreich and Ward, 1973; Cameron, 1979). The mechanism which initiated collapse, for instance a nearby supernova, probably gave rise to turbulence. Dissipation of the gas occurred with time because of mass inflow into the growing Sun and also, at some point, hydrodynamic expansion of the upper layers of the disk. As the gas dissipated, turbulence died out allowing the dust to settle into a thin disk at the central plane of the nebula. The disk became gravitationally unstable, and broke into localized dust concentrations which formed planetesimals of roughly kilometer dimensions. There is some dispute as to whether these planetesimals directly accreted to form the planets or whether they merely added mass to protoplanetary cores created during earlier epochs as the result of ring instabilities in the nebula (Safronov, 1979; Cameron, 1979). But there seems to be general agreement that the planetesimals were the building blocks for asteroids.
In subsequent evolution large-scale geological processes arose on the terrestrial planets and effectively erased chemical and physical records of the early solar system. On the other hand, the asteroids should be more representative of primordial material because they are too small to have supported geological or geomorphological processes. However, they have experienced an extensive impact history. Before we can read the records stored in asteroids (and the meteorites derived from them) we must understand how they have been modified by impact cratering. This dissertation is a study of the impact-driven evolution of asteroidal surfaces.

During accretion material impacting protoasteroid surfaces was fractured and comminuted, although not extensively because the impact velocities had to be low enough to result in net accumulation. Asteroidal embryos grew by building up layers of broken, weakly bonded rocky debris, i.e. layers of regolith. Part of the regolith may have since been "destroyed" by being converted into cohesive material. For example, some asteroids are believed to have experienced at least one heating event (maybe two; Wilkening, 1979), possibly driven by the decay of radioactive nuclides (Urey, 1955; Fish et al., 1960) or by electromagnetic induction (Sonett et al., 1968). The heating effects ranged from slight thermal metamorphism to the differentiation of silicates and metals. Moreover, self gravity may have effectively compacted and bonded together materials in the interiors of large bodies. These mechanisms undoubtedly increased the internal strength of some asteroids. Still, many bodies, perhaps the small ones or
those which escaped thermal metamorphism, should have survived as being composed mostly of regolith. Such massive collections are sometimes referred to as asteroidal "megaregoliths".

The evolution of asteroids changed from relatively gentle accretion to destructive fragmentation when their relative velocities were increased to the presently observed value (5 km/s). The velocities may have been pumped up by gravitational perturbations from Earth-size planetesimals which Jupiter scattered through the asteroid belt. The population of asteroids was probably depleted by catastrophic mutual collisions (Davis et al., 1979) or by collisions with Jupiter-scattered planetesimals (Safronov, 1979).

The onset of high relative velocities also changed the evolution of regoliths. High velocity debris striking an asteroid's surface produce craters which comminute and eject surface material. The impact velocities are high enough to launch part of the crater ejecta to escape velocity. In fact, most asteroids are presently experiencing net erosion rather than accretion. Even so, if an asteroid is large enough to retain a non-negligible fraction of its ejected debris, then the continual bombardment of its surface results in the formation of a regolith layer. Of course, for those bodies which accreted and retained a primordial regolith-like surface by escaping thermal metamorphism or differentiation, the bombardment merely serves to further comminute the extant regolith. On bodies with consolidated surfaces, regolith is created when large craters penetrate the existing debris layer and excavate "pristine" material.
One can therefore picture a modern-day asteroidal regolith as a surficial layer containing a size spectrum of debris ranging from small grains to possibly large coherent blocks. These debris are further comminuted, transported through the regolith and possibly ejected from the asteroid by subsequent impacts.

The evolution of a regolith ends if a collision occurs which fragments and disperses the asteroid. This may occur rather quickly for a small body. On the other hand, a large asteroid may repeatedly experience collisional events which are sufficiently energetic to cause major internal fracturing but not dispersal of the fragments against their mutual gravitational field. During such events, surficial layers of regolith may be mixed into the asteroid's interior. Thus, prior to dispersal, large bodies should evolve into gravitationally-bound balls of regolith, which Housen et al. (1979a) have referred to as asteroidal "megaregoliths". As mentioned earlier, this terminology has also been applied to the accumulations of regolith formed during accretionary epochs. In order to avoid confusion we will refer to the ancient accumulations as "accretionary megaregoliths".

The story ends when fragments of asteroids find their way to Earth as meteorites. At some point in an asteroid's evolution, a large impact occurs which either scoops out and ejects a sample of surface material or fragments the asteroid and disperses the newly formed meteoroids. Theories have been constructed regarding the mechanisms that transport material into Earth-crossing orbits and thus
possibly transform meteoroid into meteorite. These transfer mechanisms generally rely on gravitational effects (e.g. close planetary encounters or resonances) because the shock effects resulting from collisional transfer alone would exceed those observed in meteorites and might even cause destruction of the meteoroid. These mechanisms now appear to be able to account for, at least to an order of magnitude, observed galactic cosmic ray exposure ages (which partly measure a meteorite's transit time in space, see below) and the total mass influx observed on the Earth (e.g. Zimmerman and Wetherill, 1973; Williams, 1973; Wetherill, 1974).

Samples of asteroidal regoliths might, therefore, be expected to reach the Earth. Indeed, we have "brecciated" meteorites in our museums. These meteorites are unique in that they are composed of angular rock fragments, or clasts, which are embedded in a finer ground mass. The mixture of fragments and "matrix" material was welded into a coherent assemblage, presumably by impact-generated shock waves. Brecciated meteorites are represented in nearly every class of meteorites. The abundances, which vary from class to class, are discussed in more detail in Chapter 6. Wahl (1952) divided the brecciated meteorites into two types: (1) monomict breccias, where the matrix and clasts are similar in the sense that they could have been derived from the same material, and (2) polymict breccias, where the matrix and clasts do not appear to be genetically related. Wasson (1974) proposed a third type, genomict breccias, for intermediate cases where the clasts and matrix are chemically related but differ in
the details of petrography and mineralogy. The structure of most brecciated meteorites suggests a regolith origin, where clasts and matrix might be derived from the crushing and fragmentation of pre-existing rocks. However, we cannot eliminate the possibility that some monomict breccias were derived from a substrate which underlies a regolith layer. The brecciation textures could have been produced by impact-generated shock waves. On the other hand, many polymict and genomict breccias exhibit other regolithic traits, which we now discuss.

A subset of the polymict and genomict meteorites, the so-called gas-rich meteorites, exhibit high concentrations of noble gases. The fact that the gases have the same isotopic composition as the solar wind and that they are confined to regions very near the surfaces of mineral grains in the meteorites indicates that the gases were implanted by low energy ions in the solar wind. The solar origin was confirmed by the discovery of charged-particle tracks in the same grains which contain noble gases. Particle tracks are latent, roughly tubular, areas of solid state damage which can be chemically enlarged so as to become visible in optical microscopes. Tracks are caused by the ionizing effects of energetic particles with atomic number greater than about 20 (Fleisher et al., 1975). Actually most tracks are due to iron-group nuclei because heavier species are rather rare. The number density of tracks was observed to decrease rapidly in the first 100 microns beneath a grain's surface. The steep gradient reflects the steep energy distribution of ions emitted by the Sun, which are
referred to as solar cosmic rays. The coexistence of these particle tracks and the implanted noble gases strongly suggests a solar origin. Tracks are also produced by galactic cosmic rays, which originate outside the solar system. (Chapter 2 contains a more-detailed description of the track producing particles.) Because these ions are much more energetic than the solar cosmic rays the gradient in their track density is quite small, the density varying little over distances of centimeters. These tracks essentially form a low background which is superposed on the more abundant solar cosmic ray tracks.

The presence of the irradiation features in gas-rich meteorites is consistent with a regolith origin. Because solar gases and tracks are due to low energy ions, they can only be acquired when a mineral grain is exposed to space with little or no shielding. This will happen periodically during a grain's travels through a regolith. We would expect some grains to be exposed more than others, and some not at all. This expectation concurs qualitatively with the observation that only a fraction (typically 10%-30%, see Chapter 6) of the grains in gas-rich meteorites have been irradiated. Galactic cosmic ray tracks are acquired when a grain resides within a meter or so of the surface. These tracks should also be accumulated during a meteorite's transit from its parent body to the Earth. Of course, whether or not a regolith origin can quantitatively account for the observed irradiation features can only be determined from detailed calculations.
An alternative to the regolit origin has been suggested. Subsequent to the observation of article tracks some meteoritic grains were reported to be evenly irradiated around their borders. This prompted the suggestion that the grains were irradiated while they floated freely in space, prior to accretion onto asteroidal embryos (Lal and Rajan, 1969; Pelas et al., 1969). However, comparative studies with samples of the lunar regolith (Wilkening, 1971) and improved spatial resolution in track counting, which showed that grains were not always isotropically irradiated (Macdougall et al., 1974), supported the hypothesis of a regolith origin. Additional observations showed the existence of microcraters, glasses, agglutinates and foreign fragments in gas-rich meteorites (Brownlee and Rajan, 1973; Wilkening, 1973; Raj et al., 1974). These features are all found in samples of the lunar regolith, so a regolith origin seems likely for most gas-rich meteorites and certain for at least some (see e.g. Anders, 1978; Wasson and Wetherill, 1979).

The conclusion that meteorites are fragments of asteroids was first arrived at by a process of elimination (Anders, 1964). The Apollo-program lunar samples demonstrated that meteorites are not from the Moon. In general, planetary surfaces are not good candidates because the ejection velocity needed to escape is so large that the material would experience extreme shock, fragmentation and even vaporization. (However, the possibility remains open that a few meteorites may have been derived from the surface of Mars; Wasson and Wetherill, 1979.) Research and experimentation in the past decade have
provided several reasons for favoring an origin in the asteroid belt: (a) The continual collisions in the asteroid belt generate a large quantity of debris which would be a good source for meteorites. Furthermore, as mentioned above, some major problems of transporting material from the asteroid belt to the Earth have been solved. (b) Matches between the reflection spectra of most asteroids and meteorite types can be made (Chapman, 1976), although there appears to be a dearth of asteroids with spectra corresponding to the most abundant type of meteorites; the ordinary chondrites. (c) The measured surface exposure ages and the content of implanted noble gases in gas-rich meteorites and lunar soils imply a formation location, for the meteorites, between 1 and 8 AU in a region where the cratering rate is two to three orders of magnitude higher than at 1 AU (Anders, 1975). These requirements point to the asteroid belt. (d) The rates at which parent bodies cooled subsequent to heating episodes have been deduced from the metal alloy compositions in meteorites. These cooling rates imply parent bodies of asteroidal size (Wood, 1964; 1979). (e) In the past, the preconception of some was that because asteroids are small bodies experiencing net erosive collisions their regoliths must be mere coatings of dust (see e.g. Hapke, 1971) and therefore incapable of producing any significant quantity of meteorites. Recent modeling (Housen et al., 1979a,b) and the work presented here show this idea to be incorrect, i.e. asteroidal regoliths may be common and quite deep.
Although the view that asteroids are parent bodies for most meteorites seems well founded, some fundamental questions still need to be answered. For example, which asteroids produced the brecciated and gas-rich meteorites? In the following chapters we construct a model for the depth of an asteroidal regolith in order to calculate the relative amounts of regolith and freshly excavated pristine material that a given size of body liberates to space. Results are compared to observed abundances of brecciated meteorites with the hope of eliminating certain sizes of objects from the list of possible parent bodies. Similar calculations and comparisons are performed for the exposure of regolith grains to charged-particle irradiation. The fraction of grains that are irradiated and the track densities in these grains are computed for various sizes of asteroids. Another fundamental question which we will address is, did the brecciated and gas-rich meteorites evolve in modern-day asteroidal regoliths or, as some have suggested (e.g. Wasson, 1972; Poupeau et al., 1974; Chapman, 1976), were the meteorites necessarily formed during early, accretional epochs? We find that the permissible size range of parent bodies for the meteorites is rather broad. Furthermore, we conclude that the observed properties of the meteorites could have arisen from an evolution in modern-day regoliths.

In the next chapter we consider the various "input parameters" required by regolith models. Chapter 3 is a review of previous regolith studies. A new approach, which is a statistical one as opposed to existing determinate models, is given in Chapter 4. We
find that stochastic fluctuations, which are inherent in regolith evolution, limit the power of of regolith models in predicting meteorite parent-body size. Numerical results from the statistical model are described in Chapter 5. The results are discussed and compared with observations in Chapter 6.
CHAPTER 2

THE CRATERING, COLLISIONAL AND CHARGED PARTICLE ENVIRONMENT OF ASTEROIDS

Before the mechanics of regolith evolution can be understood in any detail, the physical quantities which form the "input" to regolith models must be described. These input parameters are the subject of this chapter.

Both the macroscopic properties (e.g. the depth) and the microscopic properties (e.g. the duration of exposure to space irradiation) of a regolith are controlled by the cratering process. The depth of regolith is largely determined by the frequency with which various sizes of craters form on an asteroid. The irradiation histories of regolith grains are additionally dependent on both the geometry of crater bowls and ejecta deposits, which either expose or shield the grains from irradiation and on the flux of charged particles which are responsible for the irradiation. Moreover, all properties of a regolith are dependent on how long an asteroid can survive without experiencing a catastrophic collisional event. This chapter is divided into five sections which cover the size-frequency distribution of craters, the shape of crater bowls, the shape of ejecta blankets, the flux of charged particles in space and the fragmentation and dispersal of asteroids.
All of the above-mentioned quantities depend on the compositional strength assumed for an asteroid. Two hypothetical kinds of asteroids are considered here: (a) relatively "strong" cohesive objects whose strength is akin to rocky materials and (b) "weak" objects whose strength is equivalent to that of loosely bound regolith. Although actual internal strengths of asteroids are poorly known, the properties of both types of hypothetical bodies are derived from experiments with materials whose parameters probably bracket the real asteroids. In general, values for strong and weak bodies are obtained from impacts into basalt and quartz sand or regolith, respectively.

**Crater Size-Frequency Distribution**

Unlike the lunar case, there are no direct observations or measurements of the size distribution of craters on asteroids. However, the distribution can be derived indirectly from knowledge of the impact velocity of debris which strike an asteroid's surface, the mass distribution of debris and the size of crater produced by an impact of given energy. The quantities involved in the construction of the crater flux are now discussed.

**Relative Velocities**

The debris complex in the asteroid belt is characterized by two components: (1) material formed during cratering and fragmentation of asteroids themselves, and (2) a so-called cometary component. The distinction relevant to this study between these components lies in
the relative velocities. Because cometary orbits tend to be rather eccentric the velocity of cometary debris tends to be higher, on the average, than that of asteroidal material. In this study we adopt 5 km/s as the mean impact velocity for collisions among asteroids (Dohnanyi, 1969) and 14 km/s for impacts of cometary debris (Dohnanyi, 1976).

Mass-Frequency Distribution

Observations of near-Earth debris span the entire range of masses from micrometeoroids to kilometer-size objects (Dohnanyi, 1972; Hughes, 1978), but the flux further out in interplanetary space is much more uncertain. At present, information regarding the flux in the asteroid belt is derived from satellite cell-penetration data, telescopic observations of large asteroids and the zodiacal light, and theoretical modeling (Figure 1). The Pioneer 10 and 11 spacecraft observed a nearly constant spatial density of dust between 1 and 5 AU (Humes et al., 1974). This indicates negligible asteroidal contribution at small masses in agreement with pioneer observations of the zodiacal light (Hanner et al., 1976). Therefore, the small-particle flux in the asteroid belt can be approximated by the known flux of presumably cometary particles at 1 AU. A similar approximation at large masses is not used here because for large objects the flux of cometary and asteroidal components vary with heliocentric distance and the variation is not known to better than an order of magnitude (Kessler, 1970). Moreover, the dependence of mass distribution shape on radial distance is unknown. Soberman et al.
Figure 1. Estimates of the number density of projectiles the asteroid belt. All estimates represent the cumulative number of projectiles with mass greater than m. The solid curve (Dohnanyi, 1976) is used here when modeling the exposure of regolith grains to space irradiation. When computing regolith depth, a straight line is fit to the large-mass (i.e. mass > 1 kg) portion of Dohnanyi's curve.
(1974) tried to measure the spatial density of particles smaller than 10 centimeters with the asteroid/meteoroid detector on board Pioneer 10 but calibration difficulties have plagued these observations (Auer, 1974) so they are not used here. For the upper end of the mass spectrum, telescopic observations of asteroids were summarized by Chapman (1976) and are shown in Figure 1. Also shown is an upper limit for the flux which Kessler (1968) established using the observed intensity of the gegenschein.

Observations in the range $1 \text{ g}$ to $10^{15} \text{ g}$ are rare. In this region we rely on modeling of the collisional evolution of particle populations (Dohnanyi, 1976). Briefly, Dohnanyi's model implies that the number (per cm$^3$) of objects with mass between $m$ and $m+dm$ should be

$$dN(m) = K_m a_m m^{a_m-1} dm$$  \hspace{1cm} (2.1)$$

where $K_m$ and $a_m$ are constant over a given mass range. For large masses, a value of $a_m$ is derived for a theoretical steady-state collisional evolution of asteroids. $K_m$ is determined from observations of the number of large asteroids. At $m=1$ kg, the flux given by Dohnanyi bends over because collisions with high velocity cometary debris begin to deplete asteroidal material. For even smaller masses, the cometary component dominates by orders of magnitude. For $m<4\times10^{-2}$ g, the flux plotted is the observed (cometary) flux at 1 AU, which should reasonably approximate the small-particle environment in the asteroid belt, as discussed above. Dohnanyi's model is summarized in Table 1.
Table 1
Parameters Involved in the Projectile Mass Flux (cgs units)

<table>
<thead>
<tr>
<th>mass</th>
<th>$K_m$</th>
<th>$a_m$</th>
<th>$v_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m &lt; 10^{-7}$</td>
<td>$8.6 \times 10^{-20}$</td>
<td>$-\frac{1}{2}$</td>
<td>$1.4 \times 10^6$</td>
</tr>
<tr>
<td>$10^{-7} \leq m &lt; 4 \times 10^{-2}$</td>
<td>$8.0 \times 10^{-25}$</td>
<td>$-\frac{7}{6}$</td>
<td>$1.4 \times 10^6$</td>
</tr>
<tr>
<td>$4 \times 10^{-2} \leq m &lt; 10^3$</td>
<td>$1.6 \times 10^{-23}$</td>
<td>$-\frac{1}{2}$</td>
<td>$5.0 \times 10^5$</td>
</tr>
<tr>
<td>$10^3 \leq m$</td>
<td>$9.4 \times 10^{-23}$</td>
<td>$-\frac{5}{6}$</td>
<td>$5.0 \times 10^5$</td>
</tr>
</tbody>
</table>
When computing regolith depth it will be convenient to approximate the entire mass distribution by a straight line fit to Dohnanyi's curve for masses greater than a kilogram. This is done because, as discussed at the end of this section, small projectiles are relatively ineffective in generating new regolith, compared to large objects. The "segmented" flux model is used in exposure-time calculations, where small impacts are important.

Crater Size

Hypervelocity impact cratering is now a rather well studied phenomenon, both experimentally and theoretically. As a result, quantitative predictions can be made of the crater sizes produced by impacts under a variety of physical conditions. The cratering physics and sources of information relevant to this study are now briefly reviewed.

Upon impact, a roughly hemispherical shock wave is generated which transfers energy, from projectile to target, both in the form of kinetic energy and heat. The pressures generated in hypervelocity impact are sufficiently large to exceed the strengths of most materials, so the early phases of cratering are characterized by fluid flow. As the shock wave travels out across the target face, rarefaction waves are generated because a free surface cannot maintain a state of stress. The rarefaction waves propagate back into the target and deflect material motions from a more or less radial direction up toward the target surface (see e.g. Gault et al., 1968). This represents the onset of the ejection phase. The upward-deflected
material flows out tangent to the upper wall of the expanding crater bowl and is ejected as an inverted conical-shaped curtain. The angle between the curtain and the target surface is observed to remain constant during much of the excavation stage. The energy density of the shock wave decreases as the wave propagates into the target. Moreover, some energy is transferred into heat. Thus, the kinetic energy transferred to the target decreases and results in a corresponding decrease in ejection velocity. That is, ejection velocities diminish with increasing distance from the impact point.

The fracture and comminution of target material, and hence the formation of the crater bowl, ends when shock-generated stresses decrease to values comparable to target material strength. At this point, flow velocities are so low that surface material, rather than being ballistically launched, is merely pushed upward to form a structural uplift. If local gravitational forces are small compared to material strength, then the crater bowl assumes its final dimensions at the point on the surface where comminution and ejection of target material cease, because all ejected material is ballistically launched to locations beyond this point. This case is usually referred to as "strength scaling" for craters. On the other hand, when gravitational forces are large, some of the low velocity material is ejected to points inside the edge of the equivalent strength-scaled crater and thus serve to decrease the apparent crater radius (see e.g. Ivanov, 1976). This is referred to as "gravity scaling" for craters.
The two most important variables in determining crater size, therefore, appear to be material strength and gravitational acceleration. This was recognized and quantified through a dimensional analysis by Gault and Wedekind (1977). They demonstrated that the transition between strength scaling and gravity scaling occurs when $s/\rho g D = 1$ where $s$ and $\rho$ are the target's material strength and density, $g$ is the local gravitational acceleration and $D$ is crater diameter. They also derived functional relationships between crater diameter, impact energy, material strength and gravity for the two limiting cases of $s/\rho g D$ being either very large or small. A general expression for crater diameter, which will be referred to as a "cratering law" is

$$D = K_D \left( \frac{1}{2} m v_i^2 \right)^{a_D} h(g)$$

(2.2)

where $K_D$ is a constant for a given target material (i.e. strong or weak), $v_i$ is the impact velocity and $h(g)$ is a function of gravity. In the strength-scaling regime, with the ratio $s/\rho g D$ being much larger than unity (e.g. for strong targets, low gravity or small craters) $a_D = 1/3$ and $h(g) = 1$. For gravity scaling, with $s/\rho g D$ approaching zero, $a_D = 1/4$ and $h(g) = g^{-1/4}$. Because the limiting case of zero strength is never attained in practice (except possibly for a liquid target), more appropriate values for gravity scaling are $a_D = 0.29$ and $h(g) = g^{-1/6}$ as suggested by many cratering experiments (cf. Nordyke, 1962; Gault, 1974; Gault and Wedekind, 1977).
$K_D$ is determined from cratering experiments. The sources of information used to evaluate $K_D$ for the four combinations of target strengths (weak or strong) and cratering laws (gravity or strength) are now discussed. The estimates for $K_D$ are given in Table 2 at the end of this section.

**Weak Target and Gravity Scaling.** Cratering data, for weak target materials, spanning twenty orders of magnitude in energy and six in diameter are shown in Figure 2. Vedder (1972) impacted micron-size particles, with velocities from 2.5 km/s to 12 km/s, into weakly cohesive mineral dust targets. The results are plotted as a rectangular region because Vedder presented only average values for a number of experiments involving a range of crater diameters and impact velocities. Gault and Wedekind (1977) and Stoffler et al. (1975) performed impact experiments into unbonded quartz sand and obtained craters of centimeter dimensions. Crater diameters produced by "inert" missile impacts into dry, weakly cohesive, sand are given by Moore (1976). These oblique impacts were adjusted to normal incidence by using the expressions found in Gault (1974). Observations of spacecraft impacts into the lunar regolith (Whitaker, 1972) are scaled down to terrestrial conditions by using $g^{-1/6}$ scaling. The correction for non-normal incidence is also applied. At larger diameters, results of two explosion craters formed in alluvium are plotted. These events are used because their depth to diameter ratios (0.2 and 0.23) are close to the value used here (0.2, discussed below) for impact craters. In order to obtain a value of $K_D$ for a weak target
Figure 2. Crater diameter vs. impact energy for weak targets. As discussed in the text, the relationship between diameter and energy (for a given target material) is expected to depend on gravity and crater size. At large diameters a gravity scaling law applies whereas strength scaling applies to small craters. This figure pertains to terrestrial gravity conditions. As gravity decreases, the point of transition between strength scaling and gravity scaling moves upward and to the right along the gravity scaling line.
with gravity scaling, a line of slope 0.29 was regressed to the data shown in Figure 2. The data from Vedder (1972) were excluded from the regression because, as we now discuss, they lie in the strength scaling regime.

**Weak Target and Strength Scaling.** For terrestrial gravity, a transition is made from gravity scaling to strength scaling at a crater diameter of roughly 10 cm (for a reasonable weak-material strength of \(5 \times 10^4\) d/cm\(^2\)), so the data of Vedder (1972) lie in the strength scaling regime. The uncertainties in Vedder's data preclude determination of an exact value for \(K_D\) in this case. The value is chosen to make the two cratering laws agree at \(D=10\) cm, i.e. \(K_D\) is found from \(10=\frac{1}{3}E_{10}^{1/3}\), where \(E_{10}\) is the energy corresponding to a diameter of 10 cm (computed from the gravity scaling law).

**Strong Target and Gravity Scaling.** This case is rather unconstrained due to a lack of data (Figure 3). There are no man-made craters (in cohesive media) which are large enough to be gravity scaled and for which formation energies are known. As a first approximation we use the gravity scaling law derived for the weak-target case because, by definition, gravity scaling laws are independent of target strength. This is only an approximation because, as mentioned earlier, pure gravity scaling is probably never attained. If this is true then the weak-target law should give upper limits for crater diameters because crater size decreases as target strength increases.
Figure 3. Crater diameter vs. impact energy for strong targets.
Strong Target and Strength Scaling. Bloch et al. (1971) simulated microcraters found on lunar rocks by impacting small particles into glass targets (Figure 3). Results of impact cratering in basalt and granite targets over an energy range of $10 - 10^{12}$ ergs are given by Gault (1972). Maurer and Rinehart (1960) fired small steel spheres into sandstone and granite resulting in centimeter-size craters. However, these data may not be directly comparable to say, Gault's results, because impact velocities were not high enough to cause hydrodynamic flow or disruption of the projectile. That is, these events may be near the velocity threshold that divides cratering from simple rebound. Missile impacts in sandstone targets are shown in the figure although these results may also be near the cratering threshold (Moore, 1976). The four explosion events shown were formed in dry basalt and should serve as reasonable analogs for impact craters (all have depth to diameter ratios near 0.2 except Danny Boy which has a ratio of 0.29).

Of all the data plotted in Figure 3, except Gault's (1972), the explosion craters are probably the most reliable, hence $k_D$ is determined for this case by fitting a line, of slope $1/3$, to the explosion data. Gault's data are not used because they strictly apply only to small craters. The value $a_D=1/3$ is suggested both theoretically (Gault and Wedekind, 1977) and experimentally (Dence et al., 1977) for large craters, which dominate regolith growth.
The adopted parameter values for the cratering laws are summarized in Table 2.

The Size-Frequency Distribution

The number of craters (per area per time) in a diameter interval \( dD \) is found from equations (2.1) and (2.2) as

\[
dN(D) = K a D^{a-1} dD
\]

where

\[
K = \frac{h K_v}{m^\left(\frac{v_i^2}{2}\right)^m \left(K_D h(g)\right)^a}
\]

and

\[
a = a_m a_D.
\]

The factor of \( 1/4 \) is included in \( K \) because impacts on an asteroid's surface are spread over an area which is four times the collisional cross-sectional area.

Using the parameter values specified in Tables 1 and 2 for large impacts one can show that the total volume excavated by craters in a diameter interval, centered on an arbitrary diameter, is smaller than the total volume for craters in an equal interval centered on a larger diameter. Large craters are therefore more important than small ones when modeling regolith growth, a fact which is important to remember in subsequent discussions.
**Table 2**

*Values of $K_D$*

<table>
<thead>
<tr>
<th></th>
<th>strength scaling</th>
<th>gravity scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>weak target</td>
<td>$8.0 \times 10^{-3}$</td>
<td>$6.36 \times 10^{-2}$</td>
</tr>
<tr>
<td>strong target</td>
<td>$2.75 \times 10^{-3}$</td>
<td>$6.36 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
Before leaving the subject of crater distributions, an alternate method for finding the distribution should be mentioned. In an attempt to characterize the asteroidal cratering environment without having to rely on estimates of the projectile mass distribution at 3 AU, Langevin and Maurette (1980) used crater scaling laws to "invert" the lunar crater distribution to find the mass distribution of projectiles at 1 AU. They assumed this mass distribution to have the same shape as that at 3 AU (although the rate of impacts was taken to be 30 times higher at 3 AU) and again used scaling laws to find the crater distribution in the asteroid belt, taking into account differences in impact velocity and target strength between the Moon and the asteroids. While this method does not rely on a specific estimate of the projectile distribution at 3 AU, there may be significant uncertainties introduced by the assumption of similar mass distributions for impactors of the Moon and asteroids. The distribution of asteroidal debris is the result of a mutual collisional evolution over a period of time. Conversely, much of the debris delivered to the Moon may have originated in single catastrophic events (Wetherill, 1976). The mass distributions resulting from discrete fragmentation events and from multiple events (i.e. fragmentation of fragments) may not be the same.

Crater Shape

The geometry of crater bowls has been studied over a wide range of sizes. Lunar craters with diameter < 1 mm are typically bowl-shaped pits with depth to diameter ratio, \( \mu \), ranging from 0.2 to
but concentrated near 0.6 (Brownlee et al., 1975). Centimeter to meter-size craters in basalt are roughly conical in shape with $\mu$ near 0.24 (Gault, 1972), while craters in unconsolidated sand under a variety of gravitational accelerations exhibit a ratio of 0.2 (Stoffler et al., 1975; Gault and Wedekind, 1977). Fresh craters in the diameter range 0.2 km to 10 km on Phobos, Deimos, and the Moon are bowl shaped with $\mu$ about 0.2 (Pike, 1977; Thomas, 1978).

For our calculations, craters are assumed to have the shape of spherical-caps with $\mu=0.2$. The volume of such a crater is

$$\text{Volume} = \pi \mu (3 + 4\mu^2) \frac{r^3}{3} = c r^3$$

(2.4)

where $r$ is the crater radius.

**Ejecta Blanket Shape**

During a cratering event, material is launched along roughly ballistic trajectories. The launch speed decreases with increasing distance from the impact point while the angle between the initial velocity vector and the surface remains nearly constant. We now determine the thickness of ejecta at a point outside a crater by specifying the initial conditions for ejected material. The ejecta velocity distribution is considered first.

Langevin and Maurette (1980) assumed that the fraction $f_e(v)$ of ejecta with initial launch speed $> v$ is equal to $(v_r/v)^2$ where $v_r$ is the velocity of material at the crater rim. We adopt a more general form of this expression,
\[ f_e(v) = \left( \frac{v}{v_r} \right)^{a_e} \]  \hspace{1cm} (2.5)

where \( a_e \) is a constant. The Langevin and Maurette expressions for \( v_r \) are used here. For strength scaled craters,

\[ v_r = 0.18 \, s_t^{0.55} \]  \hspace{1cm} (2.6)

where \( s_t \) is the target strength (d/cm\(^2\)). For gravity scaled craters,

\[ v_r = \sqrt{gr/6} \]  \hspace{1cm} (2.7)

The exponent \( a_e \) is considered as a variable here because the experimentally determined ejecta velocity distributions, upon which Langevin and Maurette based their value of \( a_e=2 \), are somewhat uncertain. Measurements of \( f_e(v) \) are very difficult to make, especially at low ejecta velocities. The only available experimental results are summarized by Greenberg et al. (1978) and are shown in Figure 4 as solid curves. The 'strong target' curve was constructed from a single impact into basalt (Gault et al., 1963). The 'weak target' curve was derived from an impact into unbonded sand (Stoffler et al., 1975). For the strong target case, where strength scaling applies, a reasonable fit to the high-velocity tail of the distribution is obtained using equations (2.5) and (2.6) with \( a_e=2 \) and \( s_t=1.4\times10^8 \) d/cm\(^2\) (see the dashed line labeled \( a_e=2 \)). These parameter values appear to be inadequate to describe the slowest 70% of the ejecta. However, examination of the original data shown in Gault et
Figure 4. The velocity distribution of crater ejecta. The vertical axis shows the fraction, $f_e(v)$, of material with ejection speed greater than $v$. The solid lines are derived from cratering experiments (Gault et al., 1963; Stoffler et al., 1975). The dashed lines are fits of a theoretical relationship, discussed in the text, with two different parameter values.
al. (1963) shows that the portion of the curve pertaining to slow ejecta is rather uncertain. If the turnover in the curve at low velocities is real then the slow ejecta could be described better by the values $a_e=1$ and $s_e=5\times10^7$ (shown as the dashed line labeled $a_e=1$). Similar results are found for the weak-target case using the gravity scaling model, i.e. equations (2.5) and (2.7). (Note that the weak-target data are on the borderline between strength and gravity scaling and so could also be fit with the strength scaling equations.) It is interesting to note that other studies (Ivanov, 1976; Maxwell, 1977; Austin et al., 1980; Croft, 1980) suggest values of $a_e$ which vary between 1 and 2. A nominal value of $a_e=2$ is adopted here but, due to the uncertainties involved, the effects of assuming $a_e=1$ are also investigated. We now consider how $f_e(v)$ can be used to find the ejection velocity as a function of distance from the impact point.

Theoretical studies (e.g. Maxwell, 1977; Austin et al., 1980) imply that material motions in a cratering event are characterized by fluid flow along streamlines, one of which is depicted in Figure 5. At a distance $x_0$ from the impact point, the launch velocity is $v(x_0)$. The ejecta with $v>v(x_0)$ is just the material inside the shaded region in the figure because velocity decreases as $x_0$ increases. If we make the reasonable assumption that the streamlines are geometrically similar to one another in shape then the volume of the shaded region is proportional to the total volume of ejected material. If the total volume is equated to the crater volume (this is not strictly correct because some material is "compacted", i.e. driven down into the crater
Figure 5. An example of the trajectories of ejected material. At launch point $x_0$, the launch speed is $v(x_0)$. All debris inside the shaded region have ejection speeds greater than $v(x_0)$. 
bowl rather than being ejected), then the fraction of ejecta with \( v > v(x_0) \) is just \( (x_0/r)^3 \). Using equation (2.5), the launch velocity can be written as

\[
v(x_0) = v_r \left( \frac{x_0}{r} \right)^{-3/a_e}.
\] (2.8)

Letting the (constant) ejection angle be 45 degrees (Stoffler et al., 1975), then equation (2.8) can be used to find the ballistic range, \( x \), as a function of \( x_0 \):

\[
x = x_0 + \left( \frac{x_0}{r} \right)^{-6/a_e} \frac{v_r^2}{g}.
\] (2.9)

Equation (2.9) implies that the range assumes a minimum value, \( x_m \),

\[
x_m = (a_e + 6)(6 \frac{v_r^2}{a_e g r})^{a_e/(a_e + 6)} r/6.
\] (2.10)

For gravity scaled craters, the minimum range is just beyond the rim \( (x_m = 1.1r \text{ to } 1.2r \text{ for } a_e = 1 \text{ to } 2) \). Actually this expression only applies to gravity scaled craters. One can show that for crater diameters appropriate to the strength scaling regime the launch point corresponding to the value of \( x_m \) in equation (2.10) lies outside the crater rim. Thus, for these craters the minimum range is instead given by

\[
x_m = r + \frac{v_r^2}{g}.
\] (2.11)
Let us now consider the shape of an ejecta blanket at points far from the rim, i.e. \( x \gg r \) or equivalently \( x_0 \ll r \). In this case, the first term on the r.h.s. of equation (2.9) is small. If we let \( V \) be the total volume of material ejected beyond range \( x \), then \( V \) is proportional to \( x^3 \). Equation (2.9) now implies that \( V \) is proportional to \( x^{-a_e/2} \). The thickness of an ejecta blanket, \( B(x) \), at range \( x \) can be obtained by noting that

\[
x B(x) \propto \frac{dV(x)}{dx}
\]

so

\[
B(x) \propto x^{-a_e/2 - 2}
\]

That is, for large \( x \), the thickness of ejecta decreases as the \(-2.5\) power for \( a_e = 1 \) and as the \(-3\) power for \( a_e = 2 \).

The behavior of \( B(x) \) near the crater rim is harder to determine because the first term in equation (2.9) is no longer negligible. This means equation (2.9) cannot be inverted analytically to find \( V(x) \) as before. Of course the inversion could be done numerically but this would greatly complicate subsequent calculations. Instead we adopt a simplified model for ejecta blankets. For gravity scaled craters, \( B(x) \) is assumed proportional to the \(-a_e/2-2\) power of \( x \) for all \( x > r \). Actually equation (2.10) implies that \( B(x) \) should be zero for \( x_m > r \) but ejecta are observed to extend all the way up to the rims of gravity scaled craters. This means the model is not quite correct at low velocities for this case. The same shape is used for strength scaling except now \( B(x) = 0 \) for \( x_m > r \). The region of zero
thickness is expected to exist for this case because \( v_r \) for strength scaled craters is usually much larger than for gravity scaled craters.

The proportionality constants can be evaluated by equating the volume of an ejecta blanket to the volume of nonescaping ejecta. The resulting expression for \( B(x) \) is

\[
B(x) = \frac{(a_e c/4\pi)}{(1-\gamma)} r^3 \left( x_m - x \right) \left( a_e^2 - a_e^2/2 \right) x > x_m
\]

\[
= 0 \quad x < x_m
\]  

(2.12)

where \( x_m \) is given by equation (2.11) for strength scaling and \( x_m = r \) for gravity scaling. The fraction, \( \gamma \), of ejecta which escapes an asteroid of radius \( R_A \) is given by

\[
\gamma = \left( \frac{v_r}{\sqrt{2gR_A}} \right)^a
\]

where \( v_r \) is found from equation (2.6) or (2.7) for strength scaling or for gravity scaling, respectively.

The Flux of Charged Particles at 3 AU

Interplanetary space is inhabited by a wide spectrum of energetic ions. The ones which are particularly relevant to studies of regolith evolution and the origin of meteorites can be broadly classified into three groups (Table 3). (1) Solar wind (SW) ions are continuously emitted by the Sun. Their energies are usually only a few keV/nucleon so they are just deposited on the surfaces of material
### Table 3

**Radiation Affecting Meteorites**

<table>
<thead>
<tr>
<th>Name</th>
<th>Energy (MeV/nucleon)</th>
<th>Range (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Galactic cosmic rays</td>
<td>Peaks near 100</td>
<td>$1-2 \times 10^2$</td>
</tr>
<tr>
<td></td>
<td>extends to $&gt;10^{15}$</td>
<td></td>
</tr>
<tr>
<td>Solar cosmic rays (heavy nuclei)</td>
<td>1-10 sharply decreasing for higher energy</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>Solar wind</td>
<td>$10^{-3}$</td>
<td>$10^{-6}$</td>
</tr>
</tbody>
</table>
exposed directly to space. Some of these ions are responsible for the high abundance of noble gases observed in gas-rich meteorites. (2) Solar cosmic rays (SCR) are more energetic than SW ions, having energies on the order 1-10 MeV/nucleon. Particles with atomic number greater than about 20 penetrate silicate materials to depths of less than roughly 100 microns and leave paths of solid state damage. These damaged areas, or particle tracks, can be chemically etched so as to become visible in optical microscopes. (3) Galactic cosmic rays (GCR) are energetic ions which enter from outside the solar system. Their energies peak near 100 MeV but can be much larger than 1 GeV. These ions produce radioactive nuclides in spallation reactions and also form particle tracks. The energies involved allow penetration to depths of a few meters.

The production rate of particle tracks (both SCR and GCR) as a function of depth in a target has been measured at 1 AU by counting the number of tracks at a given depth in lunar rocks having known exposure ages (Bhandari et al., 1973; Hutcheon et al., 1974; Blanford et al., 1975). The estimated track production rates (Figure 6) are quite discordant, differing by two orders of magnitude at depths below 1 mm. The differences are in part due to nonstandardized track counting techniques and to differences in the degree of surface erosion in the samples used (Lal, 1977). Various supporting evidence and arguments can be cited for each production rate (Morrison and Zinner, 1977; Lal, 1977; Zinner, 1980) but there seems to be no compelling reason for choosing one over the others. We will use the
Figure 6. The production rate of charged-particle tracks. — All estimates pertain to 1 AU. In this dissertation the Blanford et al. (1975) estimate is used and is approximated by the function shown in Table 4.
Blanford et al. (1975) estimate in our calculations, if for no other reason than because it represents an intermediate case. We must remember, however, that the track production rate may be uncertain by as much as an order of magnitude at small depths. For computational convenience the Blanford et al. profile is approximated by straight line segments over three depth intervals. The adopted function in given in Table 4. We must now consider how this production rate varies between 1 and 3 AU.

Because the GCR particles (which become the dominant source of tracks below about 1 cm depth) are of such high energy their spatial density does not change drastically between 1 and 3 AU. The cosmic ray telescopes on board Pioneer 10 and 11 showed that the GCR flux increases by only a few percent per AU between the Earth and Jupiter. Thus we will assume that the 1 AU flux is also pertinent for the asteroid belt. The variation of SCR flux with heliocentric distance is more complex. This flux can be broken into two components. Solar flares produce SCR ions in bursts which can vary greatly in intensity from flare to flare. The fluence of flare-produced protons is observed to decrease by roughly a factor of 10-20 between 1 and 3 AU (Zwickl and Webber, 1977) in reasonable agreement with analytical treatments based on standard diffusion-convection-adiabatic deceleration theory (Lee, 1976; Hamilton, 1977). The second component of the SCR flux arises from long lived streams of low energy (<5 MeV) particles which corotate with the interplanetary magnetic field. These streams are apparently uncorrelated with optical flares or radio
Table 4

Particle-Track Production Rate (# cm\(^{-2}\text{-Myr}^{-1}\))
Adapted from Blanford et al. (1975)

<table>
<thead>
<tr>
<th>depth, d, in target (cm)</th>
<th>type of radiation</th>
<th>production rate at 1 AU</th>
<th>production rate at 3 AU</th>
</tr>
</thead>
<tbody>
<tr>
<td>d &lt; 0.22</td>
<td>SCR</td>
<td>(2.7 \times 10^5 d^{-1.8})</td>
<td>(2.7 \times 10^4 d^{-1.8})</td>
</tr>
<tr>
<td>0.22 \leq d &lt; 10</td>
<td>primarily GCR</td>
<td>(9.1 \times 10^5 d^{-1.3})</td>
<td>(9.1 \times 10^5 d^{-1.3})</td>
</tr>
<tr>
<td>10 \leq d</td>
<td>GCR</td>
<td>(6.0 \times 10^9 d^{-5.1})</td>
<td>(6.0 \times 10^9 d^{-5.1})</td>
</tr>
</tbody>
</table>
emissions and are emitted more or less continuously. Contrary to expectations the intensity of these streams was observed to increase with heliocentric distance out to 4 AU (McDonald et al., 1975). The reason for this behavior is not well understood although interplanetary acceleration appears to be a viable explanation. In any event we are primarily interested in the magnitude of the increase between 1 and 3 AU, which is roughly a factor of ten for 1-5 MeV protons (Van Hollebeke et al., 1978).

Before we can estimate the SCR track production rate at 3 AU we must determine the relative magnitudes of the contributions from flares and from corotating streams. The time averaged flux of flare-produced protons at 1 AU can be obtained from Lanzerotti and Maclennan (1973) while the flux due to corotating streams is given in McDonald et al. (1975). Comparison of these data reveals that the flare produced particles dominate by two orders of magnitude. A similar conclusion can be reached by noting that the average particle flux (for energies > 10 MeV) due to flares is about 100 particles/cm²-s (Lanzerotti and Maclennan, 1973). This is comparable to the SCR particle flux, averaged over time periods ranging from a few years to 10⁶ years, obtained from studies of the glass in the Surveyor 3 spacecraft camera (Crozaz and Walker, 1971), from balloon and satellite measurements and radiochemical studies of lunar rocks (Damon, 1977; Reedy, 1980). Note that we have used the proton flux here whereas particle tracks are formed by heavier nuclei, e.g. Fe. This is a reasonable procedure because all of the ions have roughly
the same charge to mass ratio, so the radial dependence of the flux should be insensitive to the ionic species. Thus at 1 AU solar flares are the major source of SCR tracks. Consequently we will assume that the SCR track production rate is a factor of ten smaller at 3 AU than at 1 AU. Actually the rate may be somewhat larger than this because, while the fluence of flare associated ions decreases by a factor of ten, the component from corotating streams increases by a factor of ten at 3 AU. Hence the two components there should be roughly equal. But in light of the uncertainties involved in the measured rate at 1 AU and in the radial dependence of the flux, a factor of 2 seems worthy of little consternation. The adopted flux model is shown in Table 4.

Catastrophic Impact Events

An asteroid is fractured into a multitude of smaller objects, which may or may not remain mutually gravitationally bound, when an impact occurs with energy exceeding a critical value. We will distinguish between two types of catastrophic events: fragmentation (or rupture), and dispersal. In the former case an asteroid is highly fractured but remains at least gravitationally bound. In the latter case a body is impacted with sufficient energy to cause fragmentation and dispersal of the fragments against their mutual gravitational field. For the moment we discuss only the energy required to fragment a body. The subject of dispersal is taken up in Chapter 4.
The critical projectile energy per unit target volume, $s_i$, required to cause fragmentation has been determined experimentally for various target materials. Moore and Gault (1965) fired small (0.5 cm diameter) spherical basalt projectiles into basalt spheres of diameter 5 cm at impact velocities of 1.4, 2 and 2.5 km/s. The lower velocity impacts resulted only in cratering of the targets with some spall fragments ejected in the later stages. The more energetic events (2.5 km/s) fragmented the target, the largest fragment being somewhat bigger than half the diameter of the original sphere. The ratio of projectile energy to target volume for the 1.4 km/s events was roughly $2 \times 10^7 \text{erg/cm}^3$ while the catastrophic events corresponded to a ratio of $6 \times 10^7 \text{erg/cm}^3$; hence the critical ratio for fragmentation must lie between these two values.

Gault and Wedekind (1969) simulated the destruction of tektites by firing projectiles into glass spheres of diameter 4.5 to 10 cm at velocities up to 7.5 km/s. Events with energy per unit volume less than $3 \times 10^6$ produced craters while higher energies caused spallation at the antipodal point of impact. Complete fragmentation occurred when the ratio reached $3 \times 10^7 \text{erg/cm}^3$.

Fujiwara et al. (1977) performed similar experiments using cubic blocks of basalt. Cratering occurred for ratios less than $10^7 \text{erg/cm}^3$. Slightly more energetic impacts chipped off surfaces or
broke the targets into a few pieces. For ratios in the range $3 \times 10^7$ - $3 \times 10^8$, spallation of the target surface was observed while an intact core remained. The targets were completely shattered when the energy per volume exceeded $10^8$.

Hartmann (1978) performed experiments with various materials in a low-velocity regime (0.5 to 50 m/s). Basalt balls and igneous rocks were found to catastrophically fragment (defined to occur when the mass of the largest fragment is half of the original mass) when the energy per volume exceeded roughly $10^7$ - $3 \times 10^7$. Water ice was disrupted for ratios of $3 \times 10^5$ - $10^6$ while "dirt clods", used to represent weakly cohesive carbonaceous-chondrite-like material, fragmented with ratios of $10^4$ - $10^5$ erg/cm$^3$.

The two types of materials used here, i.e strong and weak, are meant to simulate very strong and very weak asteroids, thus bracketing the real range of asteroidal material strengths. As such, we adopt a value of $s^1 = 3 \times 10^7$ for strong targets and $s^1 = 10^4$ for weak targets. Note that $s^1$ is used here to determine only collisional lifetimes, so any uncertainties in impact strength will appear as uncertainties only in asteroid fragmentation lifetimes.

Having described the sources of input for regolith models we now review previous studies of asteroidal regolith evolution.
To date, models of asteroidal regoliths have been concerned largely with two aspects of regolith evolution: the thickness of a debris layer as a function of time and the accumulation of charged-particle irradiation effects. The main thrust has been to understand the origin of gas-rich meteorites, although some effort has been devoted to the interpretation of optical and radar observations of asteroids. The methodologies, computational results and conclusions reached in each of the modeling efforts are now reviewed. First we consider a collection of theories which provided early, preliminary estimates of regolith properties and then we examine two more recent, refined models.

Early Models

The earliest model of asteroidal regoliths was developed by Chapman (1971) in order to understand the texture of the uppermost microns of asteroid surfaces as inferred from astronomical observations and was later used (Chapman, 1976) in speculations on the origin of brecciated meteorites. Chapman emphasized the erosive effects of numerous small impacts on the wide-spread ejecta deposits of large craters. He concluded that asteroidal regoliths are very
thin, except for those on the largest bodies and that even large bodies must have regoliths only "skin deep" (< 100 m). This conclusion, in conjunction with (1) the fact that a significant fraction (tens of percent) of all meteorites are brecciated and (2) the assumption that most meteorites are derived from large cratering events which sample parent bodies to kilometer depths, led Chapman to believe that the formation and irradiation of meteorites must have occurred during very early epochs. Only the accretionary megaregoliths were believed to be thick enough to produce the observed ratio of brecciated to nonbrecciated meteorites. In the following chapters we will find that regoliths are probably thicker than indicated by Chapman's model. Hence an early irradiation is not a necessity.

In order to deduce the formation location of gas-rich meteorites, Anders (1975, 1978) constructed a simple model using the observation that these meteorites have shorter cosmic ray exposure ages and lower gas contents than do lunar soils. He argued that the amount of implanted solar gases should be inversely proportional to the square of the distance from the Sun and directly proportional to the mean residence time of a grain at the surface. Anders ascribed the differences between the gas contents and exposure ages of meteorites and lunar soils to differences in cratering rates. In fact, he argued that mean residence times are inversely proportional to mean cratering rates. Thus the data implied to Anders that the meteorites have come from an environment in which the cratering rate
is 1 to 3 orders of magnitude greater than in the vicinity of the Moon and at a location a few times farther from the Sun, i.e. the meteorites have come from the asteroid belt. Anders isolated the critical parameters which differentiate between the lunar and asteroidal environment but his model did not show how the irradiation process may vary between asteroids. A Monte Carlo algorithm which simulates the exposure histories of regolith grains is described in the next chapter.

Borg et al. (1975) modified a Monte Carlo program, designed to simulate the charged-particle irradiation of the lunar regolith, and applied it to an asteroidal case, the first statistical consideration of the irradiation of regoliths on asteroids. Simulations of the irradiation of grains contained in the Kapoeta meteorite suggested an origin on a small body (diameter=50 km) with a regolith stirring rate roughly 15 times greater than that on the Moon. The details of this model have not been published so it is difficult to comment on it, but it is clear that more work is needed in this area. Borg et al. considered only a single size of asteroid and did not comment on the sensitivity of their results to uncertainties in input parameters.

In a preliminary model, Housen (1976) determined regolith depth at a given time by computing the average distance below an asteroid's surface such that only one excavation had occurred. Rocky bodies 200 km in diameter were found to accumulate debris layers roughly 100 m thick over a period of 1 Gyr. Although these calculations gave regolith thicknesses a factor of ten below those
required by Anders (1975, 1978) to explain the observed prevalence of
gas-rich meteorites, they did show that regoliths on bodies much
smaller than the Moon are considerably more than the thin coatings of
dust Chapman (1971, 1976) had assumed. Refined modeling techniques
and improved input parameters have since revised estimates of regolith
thicknesses upward, as discussed later.

Matson et al. (1977) took a unique approach to the
consideration of asteroidal regoliths. They addressed the question of
whether lunar-regolith-like processes occur on asteroids by examining
asteroid spectral data for the tell-tale signs of optical
"maturation", which produces an anticorrelation between redness and
albedo of surface materials. Finding no such evidence, Matson et al.
considered several possible explanations for the differences between
asteroids and the Moon. They concluded that asteroidal regoliths are
thinner and coarser than the lunar regolith and that they are created
by impacts at velocities too low to produce much glass. However,
neither the evidence cited by Matson et al. nor the other
remote-sensing data about asteroid surfaces is really capable of
addressing the all-important question of how deep asteroidal regoliths
may be.

Cintala et al. (1978, 1979) considered the effects of impact
cratering on small bodies. They pointed out that the small radii of
curvature and weak gravity fields will result in widely spread ejecta
deposits. Also, because the coarsest debris ejected from a crater are
those with the lowest velocity, regoliths on small asteroids, where
escape velocities are low, should be quite blocky whereas larger bodies will retain the high velocity fine grained material. Cintala et al. (1979) suggest that impact generated seismic waves may loft surface debris over a large part of an asteroid. On small bodies this mechanism may contribute to regolith erosion. Seismic shaking may enhance regolith gardening on larger bodies. Higher energy events are expected to cause spallation of asteroids.

Horz and Schaal (work in preparation) considered the formation of agglutinates in asteroidal regoliths. They noted that 5 km/s impacts into porous, fine grained, targets result in the formation of agglutinates, which are ubiquitous in lunar soils but are rare in brecciated meteorites. They then addressed mechanisms which might prevent asteroidal regoliths from becoming fine grained. Based on cratering theory and experiments Horz and Schaal proposed that energetic impacts on asteroids should generate seismic waves of sufficient intensity to cause spallation at free surfaces. The spallation products were expected to be very blocky and slow moving, with speeds of the order of meters per second. Moreover, it was suggested that the volume of spalled ejecta may be an order of magnitude larger than the crater volume. This mechanism was expected to make asteroidal regoliths more coarse-grained than the lunar regolith, thus reducing the agglutinate content of meteoritic breccias. This may also increase the burial rate of surface debris beyond that computed in existing regolith models.
Although Cintala et al. (1978, 1979) and Horz and Schaal (work in preparation) pointed out some processes which might play a role in regolith evolution, their work was extremely qualitative. Just how important seismic waves are can only be assessed through quantitative calculations. Seismic effects are not considered in this dissertation but may be an important area for future research.

The Housen et al. Model

Although regolith depths vary over the surface of an asteroid, all models to date have attempted to characterize this distribution by the mean, or average, regolith depth. Early studies (e.g. Housen, 1976) computed the average by considering all sizes of impact craters. Housen et al. (1979a,b) realized that the impacting debris population in the present-day asteroid belt is dominated in number and cross-section by small objects and dominated in mass by larger bodies. These large bodies produce a few big craters whose effects are not typical of the more uniform terrain resulting from saturation of a surface by numerous small craters. An average value based on all sizes of craters probably would represent neither the uniform terrain nor the "atypical" regions (created by large impacts) very well. Therefore, in order to compute a meaningful average, only the portions of the surface saturated with small craters were considered in the averaging by Housen et al. The relatively uniform terrain was called the "typical region".
The model of Housen et al. (1979a) applies only to asteroids sufficiently small such that crater ejecta are very widely spread over the surface. Although the large sparsely scattered craters themselves are not part of the typical region, their highly dispersed ejecta contribute to regolith buildup and so should be included when computing the average regolith depth. This contrasts to the case of large asteroids (or small weak bodies with correspondingly lower ejecta velocities) where ejecta are retained in the local vicinity of a crater. On these bodies, crater ejecta deposits may produce nonuniform (atypical) terrain, hence they are not included as part of the typical region until a time when the ejecta blankets saturate the surface. A model for large asteroids (Housen et al., 1979b) is summarized later.

Model for Small Asteroids

In order to determine which craters were to be included in the typical region Housen et al. devised a criterion for crater saturation and defined $D_g(t)$ to be the diameter of the largest craters which had saturated the surface by time $t$. The crater population was thus divided into two parts.

1. Craters smaller than $D_s(t)$ comprise the typical region because they saturate the surface. The fraction of their ejecta which escapes the asteroid depletes the regolith.

2. The craters larger than $D_s(t)$, up to a diameter $D_r$ of a crater that catastrophically ruptures the asteroid, are widely dispersed and are excluded from the typical region. However they do
generate a substantial amount of ejecta. The portion of ejecta which is not lost to space is spread widely over the surface and onto the typical region. The effects of all craters are summarized in Figure 7a.

The expression for $D_s(t)$ was used to compute the surface elevation of the typical region as a function of time. Early in an asteroid's evolution, only the very smallest craters are sufficiently numerous to saturate the surface. These small craters remove very little regolith. Most sizes of craters, being larger than $D_s$, deposit a lot of material onto the typical region, causing the elevation to initially increase. With passing time, $D_s$ increases (i.e. moves to the right in Figure 7a) so that larger craters are incorporated into the typical region, resulting in more erosion. Simultaneously, the rate of deposition decreases because the diameter range, $D_s$ to $D_r$, shrinks. These two effects cause the elevation to level off and to eventually decrease. Finally, at some point in time a crater of diameter $D_r$ or larger forms and ruptures the asteroid. At this point, regolith evolution suffers a large discontinuity, so the analytical treatment is stopped.

After computing the surface elevation the actual regolith depth was found in the following manner. For asteroids where the surface elevation continues to increase until catastrophic fragmentation (Figure 8 shows an example), the regolith depth was approximated by the surface elevation. This was deemed reasonable because in such cases, impacts which penetrate the debris layer and
Figure 7. The effects of various sizes of craters on regolith evolution in a "typical region" on an asteroid's surface. — N is the number (per area and time) of craters of diameter D or larger. At time t only craters smaller than some diameter $D_s(t)$ saturate the surface and so compose a "typical region". These craters garden and deplete (i.e. eject to space) regolith in the typical region. In Figure 7a, which applies to small asteroids where ejecta are globally distributed, the regolith is thickened by all craters outside the typical region (i.e. $D < D_s$, where $D_s$ represents the smallest crater that can catastrophically rupture the asteroid). Figure 7b applies to large bodies, where ejecta are not widespread. Only those craters whose ejecta blankets saturate the surface [$D > D_e(t)$] add regolith to the typical region. This figure is from the work of Housen et al. (1979b).
Figure 8. The surface elevation in a typical region on a 300 km diameter rocky asteroid. — The dashed lines are contours of constant excavation rate (1/yr). A regolith depth of roughly 3.5 km is developed before the asteroid is catastrophically fragmented by a large impact. After fragmentation, the asteroid may continue on as gravitationally bound debris. This figure is from the work of Housen et al. (1979b).
commence underlying bedrock are very rare. For smaller bodies, which lose more ejecta, the surface elevation may reach a maximum and then decrease below the initial (t=0) value. For these bodies the regolith thickness was determined by computing the depth below the surface where one excavation is expected to have occurred.

The calculations of Housen et al. (1979a) pertain to the two asteroid types, i.e. strong and weak, described in Chapter 2. Rocky (i.e. strong) bodies with diameter < 10 km maintain only thin dusty coatings because of their very small gravity fields. Regolith depth increases with asteroid size because more impact ejecta are retained. Thus, a 300 km body accumulates 3.5 km of regolith (Figure 8). Larger bodies, where ejecta are not globally distributed, were treated using the methods discussed in the next section. Ejecta are not widespread on weak bodies larger than about 10 km. Weak asteroids of diameter 1 km to 10 km develop centimeters to meters of regolith.

Housen et al. also considered the charged-particle irradiation of regolith material. Because typical penetration depths (in silicate materials) of SCR and GCR particles are roughly 100 microns and 1 m respectively (Table 3), the radiation effects acquired are dependent on how long regolith grains are exposed at or near the surface of an asteroid. Thus, one needs to know how extensively the regolith is churned or "gardened" by craters. This was characterized by comparing the formation rate of ejecta blankets of a specific depth to the formation rate of craters which excavate to the same depth. For all the asteroids modeled, regolith gardening was found to be minimal.
because ejecta blankets occur faster than excavations. Grains in the regolith are excavated very few times, if at all. This is in sharp contrast to the lunar regolith where reworking of material is the dominant process. The relative immaturity (e.g. lower agglutinate and glass content, lower exposure ages, etc.) of gas-rich meteorites compared with lunar breccias was explained by the high rate of blanketing compared with excavation. Housen et al. (1979a) concluded that moderate-size asteroids 100-300 km in diameter would provide a suitable environment in which gas-rich meteorites could be formed. In this environment, blanketing by widespread ejecta occurred at a rate sufficient to provide 1 m of shielding of buried surface materials from GCR irradiation within 2 Myr and also permitted adequate exposure durations of $10^3 - 10^4$ yr for lower energy SCR particles and solar wind gases. In order to account for the relatively large fraction of meteorites that are gas-rich, Housen et al. required that several generations of surficial regolith be incorporated into the interior of a parent body prior to catastrophic dispersal and delivery of meteorite fragments to Earth. These conclusions differ markedly from those of earlier investigators (e.g. Borg et al., 1975 or Matson et al., 1977) who assumed that the immaturity of asteroidal regoliths, compared to the lunar regolith, is due to a higher stirring rate for asteroids and the escape of shocked and glassy materials from small-body regoliths.
Model for Large Asteroids

On a large asteroid ejected debris are restricted more to the proximity of a crater so it is no longer reasonable to consider a uniform layer of ejecta surrounding the body. The formation and superposition of these ejecta blankets results in a spectrum of surface elevations just as the superposition of crater bowls produced a distribution of elevations in the small-body model. In order for the average regolith depth to be meaningful, Housen et al. retained the concept of a typical region over which most of the ejecta was considered to be approximately of uniform depth. A roughly uniform surface elevation was "assured" by requiring that ejecta blankets had overlapped one another extensively, i.e. saturated the surface, before they were included in the typical region. This was analogous to the earlier treatment of small asteroids where craters were not included until saturation had occurred. This method allowed the characterization of regolith depth on large bodies without actually specifying the shape of ejecta blankets. The only requirement was that, when ejecta blankets had saturated, they combined to form a relatively uniform layer of debris.

In order to define ejecta saturation, Housen et al. constructed annuli around craters. Ejecta blankets were said to saturate the surface only if their associated annuli were sufficiently numerous to meet a mathematical criterion. Once this criterion was met, the surface elevation was computed under the assumption that all ejecta were spread in a uniform layer. It is in this sense that the Housen et al. model gave the average surface elevation.
As discussed by Housen et al., the size of an ejecta annulus is difficult to decide upon. The size chosen determines how well the average surface elevation characterizes the true distribution of elevations. Two models for the radial width (i.e. outer radius minus inner radius) of an annulus were adopted: (1) the width is a constant $A_1$ independent of crater diameter, as implied by some experimental and theoretical studies of ejecta velocity distributions, and (2) the width is a constant, $A_2$, times the crater diameter, as suggested by some studies of ejecta blanket topographic profiles (see discussion, below). These two models were referred to as Type 1 and Type 2 ejecta blankets respectively. Values for $A_1$ and $A_2$ were determined from experimental ejecta velocity distributions.

The largest crater diameter whose ejecta annuli had saturated by time $t$ was defined to be $D_{s,i}^*(t)$, where $i$ ($i=1,2$) indicates the ejecta blanket model used. Notice that together, $D_s(t)$ and $D_{s,i}^*(t)$ split the crater population into three parts (Figure 7b).

1. Craters smaller than $D_s(t)$ remove regolith from the typical region.

2. The craters larger than $D_s(t)$ but smaller than $D_{s,i}^*(t)$ do not saturate the surface and so do not reside in the typical region. However, the ejecta blankets of these craters are saturated and so contribute to regolith buildup.

3. Craters larger than $D_{s,i}^*(t)$ do not affect regolith evolution in the typical region because neither the craters themselves nor their ejecta annuli saturate the surface.
The expressions for $D_s$ and $D'_s$ were used to find the surface elevation as a function of time. For a 1000-km diameter strong asteroid the surface elevation at fragmentation was found to be 1.2 km. A 25% variation in the size of ejecta annuli caused the surface elevation to change by 1.5%. Housen et al. found that regolith grains may be excavated a few times over the lifetime of the asteroid. Similar results were found for a 500 km diameter asteroid because, even though more ejecta escape, the ejecta were assumed to be more widespread than on the 1000 km body.

Note that the computed regolith depths for large asteroids are smaller than those for moderate size bodies. This is due primarily to the less widespread ejecta blankets on large asteroids. Thus, much of the ejecta that is widespread (and so builds up the regolith in the typical region) on moderate size bodies is excluded from the typical region on large asteroids. Furthermore, as asteroid size increases, more craters tend to be gravity scaled as opposed to the predominant strength scaled craters on smaller rocky bodies. Compared to strength scaling, a gravity scaled population of craters has relatively less volume concentrated in large diameters and more in small ones. Thus, gravity scaled craters are less efficient in generating new regolith and more efficient in reworking existing debris. This transition in scaling laws tends to make the regolith in the typical region on large bodies thinner and perhaps more extensively gardened as compared to smaller asteroids.
Comparison of Regolith Evolution on Asteroids and the Moon

In order to check their model and to help delineate the difference between regolith evolution on asteroids and the Moon, Housen et al. used their large-body model to compute the regolith depth expected to develop on lunar maria. Depths of 7-10 m were found to accumulate over 3.5 Gyr depending on the assumed size of ejecta annuli. Although the calculated depths are slightly higher than the observed value (about 5 m), the agreement was thought to be satisfactory given the precision of the input parameters; therefore, Housen et al. concluded that their large-asteroid model worked reasonably well.

The depth of the lunar regolith is of the order of a few meters. Conversely, asteroids, which are smaller than the Moon and so retain less crater ejecta, can have regoliths as deep as a few kilometers. This somewhat paradoxial situation arises for two reasons.

Differences in Crater Flux. Consider for example, a Vesta-size object at 3 AU. Roughly 1.5 km of regolith is developed in 3.5 Gyr. If this body were moved to 1 AU it would accumulate only 30 m of regolith in 3.5 Gyr due solely to the reduced flux at 1 AU. Obviously, for equal evolution times, if the cratering flux is reduced then the amount of regolith is also reduced. The small mass flux and higher impact velocity at 1 AU result in a cratering rate which is a factor of 100 below that at 3 AU. The reason that the regolith depth only decreases by a factor of 50, rather than by a factor of 100, is because regolith growth is not linear in time.
Size of the Body. Comparing the Vesta-size asteroid at 1 AU with the Moon, we see that the Moon develops the lesser amount of regolith. This occurs for the same reasons that the largest asteroids have deeper regoliths than slightly smaller bodies.

Some Critical Remarks

While the model of Housen et al. represents the first major contribution to our understanding of asteroidal regoliths and their connection to gas-rich meteorites, it is lacking in five respects.

1. Regolith evolution was modeled over a portion (roughly 2/3) of an asteroid's surface; the so-called typical region. If meteorites are derived primarily from the catastrophic dispersal of asteroids then the whole regolith, not just the part corresponding to the typical region, is sampled. Also, if some meteorites are derived from large but noncatastrophic events, some or all of the ejecta may be sampled from the part of the surface not described by the model. Information about the entire surface would be helpful.

2. For large bodies Housen et al. defined saturation of craters or ejecta annuli to occur when the craters or annuli had occupied 2/3 of an asteroid's surface. However, the area occupied by craters is not necessarily the same area occupied by ejecta, i.e. atypical ejecta deposits can exist on typical craters and vice-versa. Therefore, on large asteroids the size of the typical region will be less than 2/3. In order to assess the magnitude of this effect, a Monte Carlo computer program was written which generated atypical craters and
ejecta on a square grid (representing the asteroid's surface) according to the saturation criteria of Housen et al. The actual size of the typical region turned out to be $< 1/2$ rather than $2/3$. The "typical" region therefore occupies less than 50% of the surface.

3. Housen et al. actually computed the average surface elevation, not the average regolith depth. As noted earlier in this chapter, equating elevation and depth seems reasonable in cases where the elevation continually increases until fragmentation (e.g. Figure 8) because large impacts which excavate below the debris layer are rare. However, a subtle difference between regolith depth and surface elevation is that regolith depth depends on the order in which cratering and blanketing events occur. This point is discussed further in Chapter 6.

4. The regolith depth in the typical region is characterized by the average value. But the present-day asteroid belt is such that regolith growth is significantly affected by a few large impacts. The size, number, and time of formation of these large events and the evolution time (lifetime) of the asteroid are subject to large stochastic fluctuations. Therefore there may be a large variance associated with the computed regolith depth. The magnitude of this variance should be determined in order to assess the validity of conclusions based solely on average depths.

5. The exposure times, for regolith grains, to solar and galactic cosmic rays were also evaluated as average values. But these exposure times will differ among grains because each grain performs its own
random walk through the regolith. A statistical treatment (as opposed to the determinate model of Housen et al.) of the irradiation processes can be used to determine the distribution of exposure times one might expect to find in the grains comprising a meteorite derived from a particular parent body. This distribution will yield not only mean exposure time but also will give an important quantity which Housen et al. did not obtain: the fraction of regolith grains which are expected to be irradiated. Moreover, the regolith mixing calculations of Housen et al. may contain considerable errors due to incorrect modeling of small-scale impact events. They approximated the entire crater diameter distribution by a single power-law function. The function they used applied to the largest craters, which dominate regolith growth. However, the exposure histories of regolith grains are strongly dependent on the small scale events. Calculations which incorporate variations in the mass distribution of projectiles and changes in the cratering laws at small scales should be performed in order to correctly determine exposure times.

The Orsay Models

The other major regolith model, which is actually a collection of separate efforts using the same modeling techniques, has been constructed recently by a group of authors in Orsay, France, and differs fundamentally from the work of Housen et al. Unfortunately the details of the model have not been published yet, so in-depth descriptions and critiques are not possible at this time.
The Duraud et al. Model

Duraud et al. (1979) briefly described a model for the equilibrium depth of regolith on small bodies. The erosive and depositional effects of cratering on an existing regolith layer were considered. For a 20 km diameter asteroid, an equilibrium depth of 2 m was computed. For a 200 km body the depth was found to be 200 m.

In order to assess the characteristic time required to reach equilibrium, Duraud et al. assumed the flux of debris in the asteroid belt to be 10 times higher than at 1 AU. Using this flux they found that the 20 km body reached equilibrium in less than 100 Myr and that the regolith subsequently receded in toward the center of the asteroid at a rate of 10 m/Gyr. The rate for a 200 km asteroid is 2 m/Gyr.

Emplacement of crater ejecta is envisioned to occur as a "steady rain" on small bodies, rather than in discrete layers as observed on the Moon. Hence the small-body regoliths should be devoid of layers.

I have two comments on this model: (1) The equilibrium regolith thickness is claimed to be independent of any assumptions concerning the mass distribution of impacting debris. While the equilibrium value should not depend on the impact rate, it is not clear why the shape of the projectile distribution should not affect the equilibrium thickness. Furthermore, in an environment where a few large impacts can cause major changes in regolith thickness, it is not obvious how a state of equilibrium can be reached before the body is catastrophically fragmented. This point is discussed further in
Chapter 6. (2) The factor of ten increase in mass flux between 1 AU and 3AU is very low. A more reasonable assumption for the increase in flux of large bodies is $10^2$ or perhaps $10^3$ (Kessler, 1970; Dohnanyi, 1972; Chapman, 1976). Note that the rate of small-asteroid surface erosion calculated ($10^m$/Gyr) is roughly 50 times smaller than that computed by Housen et al. (1979a) for a 10 km diameter asteroid. A higher impact rate in the Duraud et al. model would produce agreement between the two models.

The Dran et al. Model

Dran et al. (1979) applied the calculations of Duraud et al. to the irradiation of regolith material. The rate at which the regolith recedes into an asteroid's interior dictates the time of exposure of regolith grains to charged particles. Dran et al. predicted, for the 20 km body, that GCR exposure effects for regolith material should be depleted by a factor of $>10$ relative to lunar soils. The densities of SCR particle tracks should be roughly 300 times less than those observed for the Moon. They suggested that their predictions fit observations of irradiated grains in the Kapoeta meteorite, i.e. a recent origin on a small body was proposed. Early origins were discarded. Irradiation while the grains floated freely in space prior to parent body accretion was rejected because the exposure times to SCR and GCR particles would be similar, contrary to observations. Irradiation during accretion apparently results in exposure times smaller than those observed, due to the presumably high flux of impacting debris during accretion.
The erosion rate calculated by Duraud et al. for the 20 km asteroid results in an exposure time of $10^8$ yr for regolith material to galactic cosmic rays. This is two orders of magnitude greater than the observed exposure ages in meteorites (Anders, 1975). Unless another mechanism is called on to limit the exposure times (e.g. catastrophic rupture), the claim that model results are in accord with observations is clearly unfounded. Note however that, as discussed above, if a more reasonable value for the impact flux in the asteroid belt is assumed, the erosion rate increases (thus the exposure times decrease) by a factor of 10 to 100 relative to those given by Dran et al. Clearly this would improve the calculated exposure times.

The Model of Langevin and Maurette

Langevin and Maurette (1980) (denoted by L-M) developed a regolith model in order to isolate the input parameters which are the most critical in determining regolith thickness. They constructed an estimate of the asteroidal crater size-frequency distribution based on the observed lunar crater distribution, as discussed in Chapter 2. The other important input parameter addressed was the velocity distribution of ejecta. This was also discussed in Chapter 2.

The evolution of regolith depth at a point on the surface was modeled by evaluating the amount of depletion due to local cratering and the buildup due to deposition of crater ejecta, although ejecta from the largest craters were excluded. The radial distance, $R_c$, such that a given percentage (they did not clearly state whether a value of 50% or 90% was used) of ejecta lie within $R_c$, was computed. The
ejecta from craters of a given size were not included until a time when at least one crater of the given size or larger formed at a distance less than \( R_c \) from the surface-point in question.

Although both the projectile mass distribution and the crater ejecta velocity distribution are poorly known, L-M concluded that the velocity distribution is the most critical parameter in regolith evolution. This distribution governs the extent of localization of ejecta (i.e. determines \( R_c \)), which dictates when ejecta are included in the regolith depth computations. For example, large ejecta velocities imply large values of \( R_c \). In this case, ejecta are quickly included, so relatively thick regoliths result.

Regolith depths, at the time of asteroid fragmentation, were computed for various sizes and strengths of bodies. Small (17.5 km diameter) weak bodies accumulated 70 m of regolith during their 700 Myr collisional lifetime. Larger weak bodies built up thinner debris layers due to the higher gravity fields which resulted in more-localized ejecta deposits. Small and moderate-size strong objects (diameter<150 km) developed little regolith because of the escape of most ejecta. Larger bodies, of diameter 300 km, developed roughly 800 m of regolith over a collisional lifetime greater than 4 Gyr. Regolith thickness on the Moon was estimated to be a few to several meters depending on the material strength assumed.
As discussed in the next section, the model of L-M describes the average regolith depth in a "typical region" and is therefore subject to some of the same criticisms as the Housen et al. model. Briefly, a description of the entire surface area is desirable including the statistical uncertainties associated with the average depth computed. The details of any irradiation calculations have not been published and so cannot be critiqued at this time.

Comparison of the Two Recent Models

The results of the two recent models (Housen et al. 1979a,b; Langevin and Maurette, 1980) are somewhat disparate. We now compare these two studies, concentrating on the differences between estimates of regolith depth. As discussed earlier, discrepancies in exposure-time calculations are probably due to discrepancies in the assumed impact fluxes.

In general, Housen et al. predict thicker regoliths than those of L-M. For example, Housen et al. (1979a,b) computed regolith depths, at the time of fragmentation, of 3.5 km, 1.2 km and 1.2 km for 300 km (diameter), 500 km and 1000 km asteroids. L-M found depths of roughly 800 m, 500 m and 250 m.

For the 300 km body, the difference arises because Housen et al. assumed ejecta to be globally distributed while L-M used more localized ejecta deposits. Housen et al. justified global dispersion by computing the radial distance (410 km) which encompassed 90% of the ejecta. Thus 90% of the ejecta was spread over 95% of the surface area. However, the ballistic range for the 50th percentile of ejecta
is 45 km which corresponds to only 2% of the surface area. Although the use of either the 50th or the 90th percentile cannot be rigorously justified, the fact that 50% of the ejecta covers only 2% of the surface implies that the Housen et al. assumption is not correct for this body. If the ejecta is considered to be localized, according to the L-M ejecta velocity model, then the depth decreases to 1.5 km because debris from large craters are now excluded from the typical region. The assumption of globally distributed ejecta should be reasonable for asteroids smaller than 300 km.

For the 500 km and 1000 km bodies both models considered ejecta to be localized. We now address three possible reasons for discrepancy in the large-body calculations.

Definitions of Saturation

Housen et al. computed the average regolith depth in a typical region on an asteroid's surface by excluding the effects of craters or ejecta blankets until they had saturated the surface. L-M also excluded the largest craters and so, in a sense, also modeled the evolution in a typical region. Essentially, both models required knowledge of the diameter $D_s(t)$ of the largest craters which saturated the surface by time $t$ and the diameter $D'_s(t)$ of the largest craters whose ejecta had saturated. Because these two diameters dictate the amount of erosion and buildup of regolith (see Figure 7b), we now consider the methods used to find $D_s(t)$ and $D'_s(t)$. 
Housen et al. computed \( D'_s \) by requiring that the fraction of surface area exterior to the ejecta annuli of craters with unsaturated ejecta (i.e. craters larger than \( D'_s \)) was \( f \). That is, \( f \) represented the size of the typical region. A nominal value of \( f=0.65 \) was used. The size of the typical region was related to the actual area taken up by the atypical ejecta annuli (including the statistical effects of overlap) through the expression

\[
\int_{D'_s}^{D_r} A(D) \, dN(D) = -\ln(f) = 0.43 \quad (3.1)
\]

where \( D_r \) is the diameter of the smallest crater which can fragment the asteroid, \( A(D) \) is the area of an ejecta blanket (i.e. annulus) for a crater of diameter \( D \), \( t \) is the time and \( dN(D) \) is the number of craters produced per unit area per unit time in the diameter interval \( dD \). Equation (3.1) was used to find \( D'_s(t) \). \( D'_s(t) \) was found in a similar manner by replacing, in equation (3.1), \( D'_s \) by \( D \) and by replacing \( A(D) \) by the area of a crater, \( \pi D'^2/4 \).

L-M used a different method to compute \( D'_s(t) \). They required

\[
t \nu R_c^2(D'_s) \, N(D'_s) = 1 \quad (3.2)
\]

where \( N(D) \) is the cumulative number of craters per area per time larger than \( D \) and \( R_c(D) \) is the radial distance, from the center of a crater of diameter \( D \), which encompasses a specified percentage of
crater ejecta. This is analogous to the expression used by Housen et al.

The expressions used to compute \( D'_s \) differ in two respects. First, note that the l.h.s. of both equations (3.1) and (3.2) represent a total number of impacts in a given area. The fact that Housen et al. require the number to be 0.43 rather than 1 tends to produce a larger value of \( D'_s \) than that calculated by L-M. For example, in the L-M model, suppose that at some time only one impact had occurred for a crater of diameter \( D'_s \). Then one would have to consider a crater diameter larger than \( D'_s \) in order that only 0.43 impacts had occurred. The larger values of \( D'_s \) used by Housen et al. resulted in the inclusion of ejecta from larger craters, which tended to produce thicker regoliths than L-M. Second, note that the l.h.s. of equations (3.1) and (3.2) differ. L-M multiplied the area \( \pi R_c^2 \) (with \( R_c \) evaluated for craters of diameter \( D'_s \)) by the cumulative number of impacts, per area per time, larger than \( D'_s \). This is incorrect because the area over which impacts are collected is obviously a function of crater diameter (i.e. \( R_c \) is a function of diameter) so an integral of the type used by Housen et al. should be employed. Moreover, the area used should be that of the ejecta annulus, not the area \( \pi R_c^2 \), because blanketing occurs only when crater centers form in the annular area. The fact that Housen et al. used the integrated area means that they computed larger values of \( D'_s \) while their use of the annular area (rather than \( \pi R_c^2 \)) results in smaller values of \( D'_s \), compared to L-M.
Similar discrepancies seem to exist in the methods used to find the diameter $D'$. L-M did not explicitly describe the method used although they did state that "regolith loss through local cratering is computed using the same concepts..." (as those used to find $D^g$). Thus $D^g(t)$ was undoubtedly computed from

$$t \pi (D'^2/4) N(D^g) = 1 \quad (3.3)$$

while Housen et al. found $D_s$ from

$$t \int_{D_s}^{D'} (\pi D'^2/4) \ dN(D) = 0.43. \quad (3.4)$$

Therefore, by reasoning analogous to that given above for $D'$, the use of 0.43 on the right side of equation (3.4) and an integral on the left side resulted in larger values of $D_s$ for Housen et al.

In short, the methodology used by Housen et al. tends to produce larger values of both $D'$ and $D_s$, relative to L-M. How does this affect regolith growth? Large values of $D'$ obviously favor thick debris layers. Large values of $D_s$ result in thin regoliths, due mainly to shrinkage of the range of crater diameters (i.e. $D_s < D < D'$) which contribute to regolith buildup (see Figure 7b). In order to determine the magnitude of these effects the regolith depth for a 500 km strong asteroid was computed, using the same input parameters as Housen et al., but with the L-M expressions for $D'$ and $D_s$. At the
time of fragmentation \( (t = 3.4 \text{ Gyr}) \) the regolith depth from the Housen et al. model was 1.2 km while \( D'_s \) and \( D_s \) were 105 km and 18 km respectively. Using the L-M expressions, \( D'_s \) and \( D_s \) were found to be 46 km and 2 km while the regolith depth was 2.1 km, due to the larger range of crater diameters contributing to regolith growth. Their methodology for computing \( D'_s \) and \( D_s \) therefore produces deeper regoliths than that of Housen et al. Obviously this does not explain why L-M predict a regolith depth depth of 500 m rather than the 1.2 km reported by Housen et al. The difference must lie in the input parameters used.

Crater Flux

The crater-size distributions, described earlier in this chapter, are apparently similar in shape except for a relative tenfold depletion in the L-M flux near a crater diameter of 1 km. The exact expression they used is not stated. Their lower flux should produce less regolith than Housen et al. but the magnitude of the effect cannot be determined until a complete description of the flux model is available.
Velocity of Crater Ejecta

The ejecta blanket model of L-M is similar to the Housen et al. annuli. Using their ejecta velocity model (described in Chapter 2) one can show that, for gravity scaled craters, 90% of ejecta lie within a distance $R_c = D$. This corresponds to the Type 2 model of Housen et al. with $A_2 = 0.5$. For strength scaled craters $R_c$ is a weak function of $D$, but for values of $D$ appropriate to strength scaling, $R_c$ and also the width of a Housen et al. annulus, are roughly constant. Thus, strength scaling corresponds to a Type 1 annulus.

Although the strength and gravity scaling cases of L-M are equivalent to Type 1 and 2 annuli, the assumed sizes for ejecta deposits differ. For example, for the 500 km body, where gravity scaling applies, the L-M model implies $A_2 = 0.5$ but Housen et al. used $A_2 = 1.4$. The reason for this discrepancy is that Housen et al. determined annuli widths from ejecta velocity data that pertained to strength scaling (where velocities are high) rather than gravity scaling. Their regolith depths are larger because the more widespread ejecta is incorporated into the typical region faster than in the L-M model.

The lower ejecta velocities of L-M also mean that more ejecta are retained on asteroids. Moreover, the fraction of escaping ejecta for gravity scaled craters is dependent on crater size (Housen et al. assumed the fraction was constant).
By repeating the Housen et al. calculations, with $A_2=0.5$ and a diameter-dependent fraction of escaping ejecta for gravity scaled craters, a regolith depth of 900 m was calculated for both a 500 km and a 1000 km asteroid.

Summary

The main difference between the two regolith models is in the assumed velocity distribution of ejecta. Housen et al. somewhat overestimated regolith depths by overestimating ejecta velocities. The Housen et al. saturation criterion results in thinner regoliths than L-M. Neither model is more "correct" than the other because the "degree of saturation" required to produce a nearly uniform debris layer in the typical region (so that the average value meaningfully characterizes the actual distribution of regolith depths) is itself arbitrary. However, the Housen et al. approach is preferable because the exact portion of the surface modeled is clearly defined. The Housen et al. model (with the same ejecta velocities as L-M) still gives deeper regoliths than L-M. This may be due to factors not considered above, e.g. the fragmentation lifetimes of asteroids differ between the two models. The method used by L-M in computing lifetimes has not yet been published, so a comparison cannot yet be made.

The models described above give estimates of the average regolith depth over a portion of an asteroid's surface. Also, Housen et al. attempted to compute exposure times of regolith grains to charged-particle irradiation by assessing the degree to which a regolith is mixed or gardened. In the next chapter we address some of
the criticisms made of these models. A statistical model will be constructed for regolith depth (as opposed to surface elevation) which describes the whole surface of an asteroid. In order to determine how well the average value characterizes regolith depth we compute the variance of the depth. Furthermore, a Monte Carlo computer program is described which simulates exposure of regolith grains to charged-particle irradiation.
CHAPTER 4
THE STOCHASTIC EVOLUTION OF ASTEROIDAL REGOLITHS

In Chapter 3 the properties of regoliths were described from a determinate point of view. In that approach, two asteroids of the same size and strength were considered to be indistinguishable in terms of depth of regolith and irradiation of surface debris. Furthermore, the regolith depth, which varies from point to point on the surface, was characterized by the average value taken over a restricted portion of the surface.

In reality, regolith evolution is dictated by the laws of chance. Consider for example a fictitious population of a large number of bodies of common size and strength. When averaged over the entire population, the number of craters of a given size produced per asteroid will be the same as that given by the average crater flux in the determinate models. However, any one asteroid may experience a flux somewhat different than the mean value. In particular, the number of relatively large (thus rare) craters produced on a body is subject to major statistical fluctuations. Because regolith evolution is dominated by the large impacts, we might expect regoliths to differ considerably among bodies in the population. There are other, probably less important, factors which should be included. For
example, the relative positions of craters and their order of occurrence are random variables which should affect regolith depth.

The actual population of bodies of the given size and strength that exists in the asteroid belt today is envisioned as being a random sample from the larger fictitious population. This is merely a convenient way to conceptualize the fact that each asteroid has been subjected to a unique impact history.

In this chapter a statistical approach is used to model regolith evolution. This allows us to evaluate both the degree of variability among similar-size asteroids and also the amount of variation in regolith depth over an asteroid's surface. If the statistical variability associated with regolith depth is large, then the utility of the determinate models becomes questionable, and the conclusions drawn from such models should be re-evaluated. In any event, a statistical analysis of the problem will indicate how well simple average values characterize regolith evolution.

The stochastic model is broken into three parts. The evolution of regolith depth is considered first. The depth of regolith is then used to determine how much brecciated material a given size of asteroid liberates to space. Finally, a Monte Carlo program is described which simulates the exposure of regolith grains to space irradiation.
The Depth of an Asteroidal Regolith

The evolution of regolith depth, for an asteroid of some given size and strength, is modeled by selecting a body at random from the large (fictitious) population. In order to be precise in our definition, we consider the evolution at a single point selected at random on the surface of the body. As is discussed later in this section, knowledge of the evolution at a random point allows us to make statements regarding the "mean regolith depth on an asteroid". This permits useful comparisons between determinate and stochastic models.

Both the surface elevation and the regolith depth at the randomly selected point change with time: they increase when debris from a nearby impact blankets the point, and they decrease when a crater forms over the point, removing regolith. These blanketing and excavation events are referred to as "jumps". The size of a jump depends on the crater size and its distance from the point. The jump size is, therefore, a random variable because in any given impact event the crater size and position are both random variables. Note also that the total number of events which have occurred up to any particular time is random because the time interval between jump events is random.

For some time \( t \), the surface elevation \( Z(t) \) and regolith depth \( X(t) \), being the result of a random number of random jumps, are both random variables themselves, i.e. both \( Z(t) \) and \( X(t) \) random walk in time. One possible realization, and the relationship between \( Z(t) \) and
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X(t), is shown in Figure 9. The difference between surface elevation and regolith depth is apparent. Although a positive jump increases Z(t) and X(t) by the same amount, a large negative jump may cause only a small decrease in X(t). This occurs when a large crater forms over the point removing all the regolith.

All of the information regarding X(t) is contained in the cumulative distribution function (c.d.f.) F(x,t),

\[ F(x,t) = \text{Prob} \{ X(t) < x \} \]

An exact expression for F(x,t) is not obtained here. However, an approximation based on a diffusion process is found to perform well. Before deriving the approximation a brief comment is given on a classical method that sometimes yields exact results.

A Note on the Forward Equation

A standard textbook-method for finding the probability distribution of a random variable defining a stochastic process is to solve the so-called forward equation for the process. However when one strays from the "simple" textbook cases the solutions often become extremely difficult. For the present case the forward equation is an integro-partial-differential equation which seems to elude the methods typically used to obtain analytical solutions. A numerical solution is possible by converting the forward equation into a system of ordinary differential equations; however, the number of equations
Figure 9. The relationship between elevation and regolith depth. --
The surface elevation, $Z(t)$, at a point on an asteroid's surface,
decreases when a crater forms over the point and increases when a
nearby crater blankets the point. At any given time the regolith
depth, $X(t)$, is the thickness of deposited debris which overlies the
undisturbed substrate. Note that a large excavation event may remove
all the regolith from the surface point (e.g. at the times indicated).
required to obtain an accurate solution is so large that computer run-time costs are prohibitive. Because an exact solution does not appear possible, we turn our attention to approximate methods.

A Diffusion Approximation

In this section an approximation is derived for the c.d.f. of regolith depth at time $t$. This expression is used to estimate the time required for a regolith to reach a state of statistical equilibrium. The distribution of depth at the time of catastrophic fragmentation of the asteroid is then derived and is followed by an assessment of the accuracy of the approximation.

The Regolith Depth at a Fixed Time, $t$. One of the more famous theorems in probability and statistics, the central limit theorem, roughly states that the sum of a large number of random variables is approximately normally distributed. Note that for large $t$ the surface elevation $Z(t)$ is the sum of many random jumps. Thus we can expect the distribution of $Z(t)$ to approach the normal distribution if regolith evolution proceeds for a "long" time. The fact that $Z(t)$ tends toward normality allows approximation of the actual random walk shown in Figure 9 by another type of random walk; a diffusion called the Wiener process. At any given time the random variable defining the Wiener process has a normal distribution. Consequently, for large $t$, $Z(t)$ can be approximated by the Wiener process.

Let us consider how knowledge of the distribution of $Z(t)$ determines the distribution of regolith depth $X(t)$. By referring to Figure 9 it is obvious that $X(t)$ can be written as
\begin{equation}
X(t) = Z(t) - \min[ Z(\tau): 0 < \tau < t ].
\end{equation} \tag{4.1}

That is, the regolith depth at time \( t \) is just the current surface elevation minus the lowest value of the surface elevation recorded in the time interval \((0,t)\). We now make use of a rather surprising result from queueing theory (see Feller, 1971 for a short proof)

\begin{equation}
F(x,t) = \text{Prob} \left\{ \text{max}[ Z(\tau): 0 < \tau < t ] < x \right\}.
\end{equation} \tag{4.2}

The basis for a stochastic model of regolith depth is now established, i.e. for large \( t \) the probability distribution of regolith depth, \( X(t) \), has approximately the same distribution as the maximum value in the Wiener process.

Note that equations (4.1) and (4.2) apply to general random walks, i.e. in principle we could find the distribution of regolith depth resulting from the actual random walk (Figure 9). The reason for approximating the real process by a diffusion is that the distribution of the maximum (or even the distribution of the random walk itself) is very difficult to obtain except in very few cases, the Wiener process being one of them. The paper of Marcus (1970) presents a good example. Marcus attempted to derive the distribution of \( Z(t) \) for planetary surfaces (in particular, the Moon). After a lengthy analysis, he found analytic expressions only under rather specific conditions, e.g. the slope of the crater size-frequency distribution was restricted to a set of three specific values and the largest
crater was taken to be infinitely large. Note that the distribution of the maximum, for the case studied by Marcus, would be even harder to find. The central limit theorem is indeed a useful tool.

The distribution of the maximum in the Wiener process (i.e. \( F(x,t) \), the distribution of regolith depth, as shown above) with mean \( \mu \) and variance \( \sigma^2 \) is given by Cox and Miller (1972) as

\[
F(x,t) = \Phi\left( \frac{(x-\mu t)}{\sigma \sqrt{t}} \right) - e^{2Bt/\sigma^2} \Phi\left( -\frac{(x+\mu t)}{\sigma \sqrt{t}} \right) \tag{4.3}
\]

where \( \Phi(\cdot) \) is the standard normal integral. Baxter and Donsker (1957) derived a similar expression for the probability density function (p.d.f.) of \( X(t) \) but their result appears to contain a typographical error.

In order to use equation (4.3) we must determine the mean and variance of our random walk. At any time \( t \), \( Z(t) \) can be written as

\[
Z(t) = Y_1 + Y_2 + \cdots + Y_{N(t)} \tag{4.4}
\]

where \( Y_i \) is the size of the \( i \)th jump experienced at the surface point in question and \( N(t) \) is the random number of events which occur in time \( t \). The occurrence of jump events is assumed to form a Poisson process with parameter \( \lambda \), i.e. \( N(t) \) is Poisson distributed with mean \( \lambda t \). The parameter \( \lambda \) is just the mean rate of occurrence of excavation or burial events. An expression for \( \lambda \) is given below.
Now, the $Y_i$'s are independent and identically distributed random variables and are independent of $N(t)$. If $m_1$ denotes the mean size of a jump, then

$$E[Z(t)] = m_1 \lambda t$$

where $E$ denotes the expectation operator. The variance can be easily shown to be (e.g. Feller, 1971)

$$\text{Var}[Z(t)] = E[N(t)] \text{Var}[Y_i] + E^2[Y_i] \text{Var}[N(t)].$$

If we let $m_2$ be the second moment (about the origin) of the jump size and note that the variance of a Poisson random variable is equal to its mean, then

$$\text{Var}[Z(t)] = m_2 \lambda t.$$

We now substitute in equation (4.3), $\beta = m_1 \lambda$ and $\sigma^2 = m_2 \lambda$, and recast it in a slightly different form as

$$F(x,t) = \frac{1}{2} \left\{ \text{erf}\left[\frac{(x-m_1 \lambda t)}{\sqrt{2m_2 \lambda t}}\right] + 1 \right\} + e^{2m_1 x/m_2} \text{erfc}\left[\frac{(x+m_1 \lambda t)}{\sqrt{2m_2 \lambda t}}\right]$$

(4.4)

where, in order to simplify subsequent manipulations, the standard normal integral $\Phi(\cdot)$ has been replaced with the error function $\text{erf}(\cdot)$, using the relationships
We now have, in equation (4.4), an approximation for the c.d.f. of regolith depth at time \( t \) for a point chosen at random on the surface of an asteroid chosen at random from a population of bodies of a given size and strength. The size and strength of the asteroid in question, the assumed geometry of craters and ejecta blankets and the assumed size-frequency distribution for crater formation determine the values of \( m_1 \) and \( m_2 \). So as not to disrupt the continuity of this section, the computations of \( m_1 \) and \( m_2 \) are deferred to Appendix 1.

The mean and variance of \( X(t) \), which will be of use in later discussions, can be found from the moment generating function \( \Omega(s,t) \),

\[
\Omega(s,t) = \int_0^\infty e^{sx} \frac{\partial F(x,t)}{\partial x} \, dx \tag{4.5}
\]

\[
\Omega(s,t) = \{ (m_1 + m_2 s) \exp\left[ (2m_1 + m_2 s) m_2 \psi^2/m_1 \right] \text{erfc}\left[ -(m_1 + m_2 s) \psi/m_1 \right] \\
+ m_1 \text{erfc}\left[ \psi \right] \} / (2m_1 + m_2 s)
\]

where

\[
\psi = m_1 \sqrt{\lambda t/2m_2}
\]

See Appendix 2 for details of the integration. The mean value of \( X(t) \) can be found from
\[ E[X(t)] = \frac{\partial \Omega(s,t)}{\partial s} \bigg|_{s=0} \]
\[ = \left( \frac{m_2}{m_1} \right) \left\{ \psi^2 \text{erfc}[-\psi] + \frac{1}{2} \text{erf}[\psi] + \psi e^{-\psi^2/\sqrt{\pi}} \right\}. \tag{4.6} \]

The second moment of \( X(t) \) is

\[ E[X^2(t)] = \frac{\partial^2 \Omega(s,t)}{\partial s^2} \bigg|_{s=0} \]
\[ = \left( \frac{m_2}{m_1} \right)^2 \left\{ 2(\psi^2+1)\psi^2 \text{erfc}[-\psi] - \frac{1}{2} \text{erf}[\psi] \right. \]
\[ \left. + (2\psi^2+1)\psi e^{-\psi^2/\sqrt{\pi}} \right\}. \tag{4.7} \]

The variance of \( X(t) \) is obtained from equations (4.6) and (4.7).

The Equilibrium State of a Regolith. The approximation given in equation (4.4) allows us to investigate the long term behavior of a regolith. Notice that when the average size of a jump in surface elevation is negative (i.e. \( m_1 < 0 \)), as is the case for most asteroids due to the net loss of material to space, the c.d.f. of regolith depth has the equilibrium distribution,

\[ F(x,\infty) = 1 - e^{-\frac{2m_1 x}{m_2}}. \]

A characteristic time to reach equilibrium can be estimated by requiring that the argument of \( \text{erf}(\cdot) \) in equation (4.4) be \( > 1 \) and the argument of \( \text{erfc}(\cdot) \) be \( < -1 \). If we construct the dimensionless space and time variables
\[
\begin{align*}
\xi &= -2m_1 x/m_2 \\
\tau &= 2m_1^2 \lambda t/m_2
\end{align*}
\]

then the above condition on the error function is roughly satisfied when \( \tau = 1 \). Thus a characteristic equilibrium time \( t_e \) for the process is

\[
t_e = m_2 / 2m_1 \lambda.
\]

(4.8)

The Regolith Depth at Fragmentation. The expression for \( F(x,t) \) gives the distribution of regolith depth at time \( t \). This is useful in predicting regolith depths for bodies that are large enough to have escaped catastrophic fragmentation. However, for the vast majority of the asteroids there comes a time when an energetic impact causes major fracturing. This introduces a great discontinuity and forces us to end our analytic treatment of the evolution. Furthermore, it is convenient to speak of the depth at fragmentation because this quantity is independent of the (relatively poorly known) rate of impacts. Therefore, for small and moderate-size asteroids, it is more useful to speak of the regolith depth at fragmentation. Notice that the depth at fragmentation cannot be obtained by setting \( t \) equal to the fragmentation time in the expressions derived above because the time of fragmentation is a random variable. The c.d.f. and the moments of regolith depth at fragmentation are now derived.
Because the occurrence of craters is assumed to form a Poisson process, the waiting time until the first impact capable of rupturing the asteroid is exponentially distributed with parameter, say, $\lambda_r$. If $w(t)$ is the probability density function (p.d.f.) of the fragmentation time then

$$w(t) = \lambda_r e^{-\lambda_r t}.$$  \hspace{1cm} (4.9)

The flux of craters of diameter $D$ or larger is assumed to be of the form $N = K D^a$, where $K$ and $a$ are constants (see Chapter 2), thus

$$\lambda_r = 4\pi R_A^2 K D^a$$  \hspace{1cm} (4.10)

where $R_A$ is the radius of the asteroid and $D_r$ is the smallest crater which can rupture the body.

The c.d.f. of the regolith depth, $X_r$, at rupture can now be written as

$$F_r(x) = \int_0^\infty F(x, t) w(t) \, dt$$

\hspace{1cm} (4.11)

$$= 1 - \exp[(m_1 x/m_2)(1 - \text{sign}(m_1)\sqrt{1 + 2m_2 \lambda_r/m_1^2 \lambda})].$$

See Appendix 2 for details of the integration.

The regolith depth at fragmentation is, therefore, exponentially distributed so the mean depth is just the negative reciprocal of the expression inside the square brackets in equation (4.11),
E[X_r] = (m_1 λ/2λ_r)[1 + \text{sign}(m_1)\sqrt{1 + 2m_2λ_r/m_1^2λ}]. \quad (4.12)

Note, because $X_r$ has an exponential distribution, the standard deviation of $X_r$ is equal to the mean depth.

**How good is the diffusion approximation?** The utility of these results is of course dependent on how well the diffusion approximates the true random walk. We now investigate the accuracy of the approximation by comparing the expressions for the regolith depth at fragmentation with Monte Carlo simulations of the actual process.

A Monte Carlo (computer) algorithm was constructed which followed the evolution of regolith depth at a point on the surface of a 300 km diameter strong asteroid until the asteroid was catastrophically fragmented by a large impact. Craters were positioned randomly on the surface and the change in regolith depth, at the modeled point, caused by each event was recorded. Figure 10 shows the Monte Carlo results when strength scaling was used for craters and their ejecta blankets (see the ejecta model described in Chapter 2).

Following the evolution at a single point until fragmentation represented one Monte Carlo "trial". The c.d.f. of regolith depth, shown in Figure 10, was constructed by performing 1000 trials. The stippled region in the figure represents a 95% confidence band derived from the Kolmogorov-Smirnov test statistic. The curve labeled "Approximation # 1" is the c.d.f. given by the diffusion approximation (equation 4.4) using the same cratering relationships and diameter
Figure 10. A test of the diffusion approximation. $F_r(x)$ is the probability that the regolith depth, at a randomly selected point on an asteroid's surface, is less than $x$ at the time of catastrophic rupture. The series of connected points is the result of a Monte Carlo simulation of regolith growth. A 95% confidence band is shown as a stippled region. Results from two analytical approximations to $F_r(x)$ are also plotted. The approximations are discussed in the text.
range used in the Monte Carlo calculations. Clearly the approximation is not very good for predicting regolith depth. If the mean depths implied by the respective probability distributions are computed, one finds that the diffusion gives values which are roughly a factor of 2-3 larger than the true (i.e. Monte Carlo) mean. Similar discrepancies are obtained when gravity scaling is used instead of strength scaling.

The reader may wonder why a Monte Carlo approach was not used to find the c.d.f. of regolith depth, rather than the method used to find an analytic expression. The problem is that when gravity scaling must be applied to craters, computing costs become large because a very large number of craters must be generated in each run in order to account for most of the mass in the impacting debris population. The analytic approach is, therefore, a matter of practicality.

An Improved Approximation

The diffusion approximation described above is merely a method by which the surface elevation is approximated by a normally distributed random variable. The elevation represents a sum of a large number of random jumps. But the normal approximation is expected to do best when many random variables are summed. The poor performance of the approximation is due to the fact that blanketing events are much more likely to occur than are excavation events because ejecta blankets cover a much bigger area than do crater bowls. Therefore, because the mean jump size is less than zero, the typical size of a positive jump is rather small compared to that of a negative
jump. That is, the evolution of surface elevation is characterized by many, relatively small, positive jumps and a few, rather large, negative jumps. One might guess that the excavation events are responsible for the bad performance of the normal approximation. A simple procedure is now described in which the largest excavation events can be "removed" from the approximation and treated separately.

The main effect of the occasional large excavation is to reset the regolith depth, at the modeled point, to zero. The depth at any time is just the amount of regolith accumulated since the last occurrence of a large excavation. In order to incorporate this fact into our model we can remove the large excavations from the diffusion and now let the regolith evolution proceed for a duration equal to the elapsed time since the last "large" event. The evolution time is discussed below, but first we define more precisely what is meant by a "large" excavation event.

We will define a crater diameter, $D^*$, and assume that if a crater of diameter larger than $D^*$ excavates the surface point then the regolith depth is reset to zero. In order to be precise we should let $D^*$ be a function of $X(t)$. However, this would make $m_1$ and $m_2$ functions of time and so would greatly complicate the analysis. Thus it seems worthwhile to try, as a first step, a value of $D^*$ that is constant in time. A reasonable procedure is to let

$$D^* = \frac{E[X]}{\mu} \quad (4.13)$$

where $\mu$ is the depth to diameter ratio assumed for craters and $E[X]$ is
the average regolith depth for either the fixed-time case (equation 4.6) or the case of fragmentation (equation 4.12). For the fixed-time case, $E[X]$ is evaluated at the time, $t$, of interest. This probably makes an upper bound for $D^*$ because the average regolith depth increases with time, so, early in the evolution craters smaller than $E[X]/\mu$ could reset the depth. There is of course nothing wrong with using an upper limit for $D^*$. In doing so we merely fail to exclude a few large excavations which are capable of zeroing the regolith depth. This is fine, as long as the diffusion approximation can handle these events. The accuracy of the improved approximation is considered below.

There is an obvious problem with equation (4.13): how can $E[X]$ be determined until $D^*$ is known? An iterative procedure could be used, however a much simpler method is to use the estimate of $E[X]$ given by the "first-order" approximation (equation 4.6 or 4.12). Before proceeding to compute the evolution time we briefly consider the rate at which events occur.

Because some of the large excavation events are to be removed from the random walk, the rate of occurrence of events must be adjusted. If the rate of excavation events from craters larger than $D^*$ (and of course smaller than $D_r$) is $\lambda^*$, then

$$\lambda^* = \int_{D^*}^{D_r} \left(\pi D^2/4\right) \kappa a D^{a-1} \, dD$$
because $K d^{a-1} dD$ is the number of craters produced (per area per time) in the diameter interval $dD$ (see Chapter 2). The event rate is therefore just the total production rate of craters minus $\lambda^*$,

$$\lambda = 4\pi R_A^2 K \left( D_1^a - D_r^a \right) - \lambda^* \quad (4.14)$$

where $D_1$ is the smallest crater which forms on the asteroid.

Note also that the effects of large excavations must be excluded when calculating $m_1$ and $m_2$. The procedure for doing this is described in Appendix 1. We now separately consider the case of regolith depth at a fixed time $t$ and the case of depth at the time of fragmentation.

The Regolith Depth at a Fixed Time, $t$. The large excavation events are excluded from the random walk itself but now, so as not to ignore the effects of these events, a new evolution time for the random walk must be computed. The random variable $T_k$ is used to denote the random time of occurrence of the last event which resets the regolith depth (at the surface-point) to zero. The subscript $k$ indicates that $k$ reset-events occurred in the time interval $(0,t)$. The new evolution time $T^*$ is, therefore, just $t - T_k$ (see Figure 9). The p.d.f. of $T^*$ is now derived. The probability that $t - T_k < s$ is just the probability that $T_k < t < T_{k+1}$, $T_k = y$, and $t - T_k < s$ for some combination of $k$ and the dummy variable $y$. That is
\[
\text{Prob} \left\{ T_\ast < s \right\} = \text{Prob} \left\{ t - T_k < s \right\}
\]
\[
= \sum_{k=0}^{\infty} \int_{\max(0, t-s)}^{t} \text{Prob} \left\{ T_k \geq dy, T_{k+1} - T_k > t-y \right\}.
\]

(4.15)

The upper limit of integration in equation (4.15) appears because we require \( y < t \) otherwise \( t \) would not lie between \( T_k \) and \( T_{k+1} \). We also require \( t-y < s \) because \( y \) represents a value for the random variable \( T_k \) and we want the probability that \( t-T_k < s \). But \( y \) must be positive because \( T_k \) is always positive, so two cases need to be considered for the lower limit of integration. When \( t > s \) then \( y = t - s \) is the lower limit. When \( t < s \) then \( y = 0 \) is the lower limit. Note also that the \( k=0 \) term must be treated as a special case. Physically, \( k=0 \) means that fragmentation occurs before the regolith is ever reset to zero, i.e. \( k=0 \) is equivalent to \( T_k=0 \). So again two cases must be considered: when \( t > s \) the \( k=0 \) term is zero because \( t - T_k > s \). When \( t < s \) the term is just \( \exp(-\lambda^* t) \), the probability that the first reset-event occurs after time \( t \).

The random variables \( T_k \) and \( T_{k+1} - T_k \) are independent, hence their joint distribution given on the r.h.s. of equation (4.15) can be written as the product of their respective marginal distributions. The occurrence of events which reset the depth forms a Poisson process with parameter \( \lambda^* \), so \( T_k \) is gamma distributed and \( T_{k+1} - T_k \) is exponentially distributed. Therefore
\[
\text{Prob} \{ T_k \leq dy \} = \frac{\lambda^*}{(k-1)!} (\lambda^*)^{k-1} e^{-\lambda^* y} dy
\]

\[
\text{Prob} \{ T_{k+1} - T_k > t-y \} = e^{-\lambda^* (t-y)}.
\]

Substitution of equations (4.16) into (4.15) yields

\[
\text{Prob} \{ T^* < s \} = \begin{cases} 
1 - e^{-\lambda^* s} & s \leq t \\
1 & s > t
\end{cases}
\]

(4.17)

The p.d.f., \( f^*(s,t) \), of \( T^* \) is seen to contain an atom of weight at \( s=t \),

\[
f^*(s,t) = \begin{cases} 
\lambda^* e^{-\lambda^* s} & s < t \\
e^{-\lambda^* s} & s = t \\
0 & s > t
\end{cases}
\]

(4.18)

Note that \( f^*(s,t) ds \) is just the probability that the evolution time, i.e. the elapsed time between the last reset event and time \( t \), is between \( s \) and \( s + ds \).

The improved approximation for the c.d.f. of regolith depth at time \( t \) can now be obtained from
\[ F(x,t) = \int_{0}^{t} F_0(x,s) f^*(s,t) \, ds + F_0(x,t)e^{-\lambda^* t} \] (4.19)

where, for notational convenience, we have used \( F_0(x,t) \) to denote the "first approximation" to \( F(x,t) \) given in equation (4.4). Unfortunately, the integral in equation (4.19) must be done numerically.

The mean regolith depth can be found from

\[ E[X(t)] = \int_{0}^{t} E_0[X(s)] f^*(s,t) \, ds + E_0[X(t)]e^{-\lambda^* t} \] (4.20)

where, again, \( E[X(t)] \) represents the first approximation to the mean depth given in equation (4.6). The variance of \( X(t) \) is again found from the second moment, which is found from equation (4.7) and an expression analogous to equation (4.20).

The Regolith Depth at Fragmentation. For this case the p.d.f. of \( T^* \) differs from that given above because the fragmentation time \( T_r \) is a random variable. The c.d.f. can be written as

\[ \text{Prob} \{ T^* < s \} = \int_{0}^{s} \text{Prob} \{ T^* < s | T_r = t \} w(t) \, dt. \] (4.21)

Substituting equations (4.17) and (4.9) into (4.21) yields
\[ \text{Prob} \{ T^* < s \} = 1 - e^{-(\lambda^* + \lambda_r)s}. \]

Thus, we now have the remarkable result that the new evolution time, i.e. the time period between fragmentation and the last event to reset the regolith depth, is exponentially distributed with parameter \( \lambda^* + \lambda_r \). This is fortunate because the improved approximation for the regolith depth c.d.f.,

\[ F_r(x) = \int_0^x F_0(x,s) f^*(s) \, ds \]

can be obtained from equation (4.11) with no additional work. The functional form of \( f^*(s) \) is the same as that of \( w(s) \), so we can immediately write down the improved approximation for \( F_r(x) \) by replacing \( \lambda_r \) in equation (4.11) by \( \lambda^* + \lambda_r \),

\[ F_r(x) = 1 - \exp\left[ (m_1 x/m_2)(1 - \text{sign}(m_1) \sqrt{1 + 2m_2(\lambda^* + \lambda_r)/m_1^2 \lambda}) \right]. (4.22) \]

Similarly the expected value is

\[ E[X_r] = \{m_1 \lambda/2(\lambda_r + \lambda^*)\}(1 + \text{sign}(m_1) \sqrt{1 + 2m_2(\lambda^* + \lambda_r)/m_1^2 \lambda}). \] (4.23)

Again, the standard deviation is equal to the mean and so the variance can be obtained by squaring the r.h.s. of equation (4.23).
The steps followed in implementing the new approximation are:

1. Compute from equation (4.14) and compute \( m_1 \) and \( m_2 \) (Appendix 1) using a value of \( D^* = D_r \) so that \( \lambda^* \), the rate of formation of reset events, is zero; 
2. Compute the first approximation for the mean regolith depth using either equation (4.6) or (4.12) depending on whether the depth at time \( t \) or the depth at fragmentation is desired; 
3. Compute \( D^* \) using the mean depth and equation (4.13); 
4. Recompute \( \lambda, m_1 \) and \( m_2 \) using the new value of \( D^* \); 
5. Find the improved approximations for \( F(x,t) \) or \( F_r(x) \) by using equation (4.19) or (4.22).

Using this procedure the accuracy of the new approximation was tested. The results can be compared to the Monte Carlo simulations (discussed in the previous section) in Figure 10. Clearly the approximation has been significantly improved. The c.d.f. computed from equation (4.19) lies well within the 95% confidence band. The values of \( E[X_r] \) were found to differ by only +1%. Similar results were found when gravity scaling was examined.

Physical Interpretation of the Regolith Depth Equations

In the preceding section, approximations were developed for the c.d.f., the mean and the variance of regolith depth at a single point chosen at random on the surface of an asteroid randomly chosen from a large fictitious population of bodies, all with common size and strength. As discussed earlier, the depth varies over the surface of a given asteroid. Furthermore, each asteroid's regolith is unique, e.g., the mean depth on an asteroid changes depending on which body is chosen. At first glance, modeling the regolith depth at a single
point seems to have little practical application. However, because the asteroid and the surface point were selected at random, information about other points on the surface and about the surfaces of other bodies in the population should be implicit in the regolith depth equations. We now examine the stochastic model for clues concerning the global properties of an asteroid's regolith and how much "interasteroid variation" can be expected.

The C.D.F. of Regolith Depth. Suppose, for the sake of argument, that the surface of an asteroid could be sampled enough times to construct the distribution (i.e. the c.d.f.) of regolith depth on the asteroid. If this is done for a large number of asteroids of common size then the distributions for each body could be combined into an "overall" c.d.f. for the population. Taking a random sample of size one from this overall distribution is precisely equivalent to measuring the regolith depth at a randomly selected point on a randomly selected asteroid. The c.d.f. for regolith depth, derived in the previous section, is therefore the overall distribution of depths for the fictitious population.

The Expected Regolith Depth. Let us now arbitrarily assign an identifying number to each asteroid in the population such that once a number is specified, then the distribution of regolith depth on that asteroid is known. If a body is selected at random from the population and its identifying number is denoted by I, a uniformly distributed random variable, then, for example, the mean regolith depth at fragmentation is given by the conditional expectation
The mean depth on a randomly selected asteroid is, therefore, a random variable whose average value is $E[E(X_r | l)] = E[X_r]$, where the expectation outside the square brackets is taken with respect to the distribution of $I$. The expected regolith depth can now be interpreted as the mean depth on an asteroid, averaged over many bodies of the same size and strength.

The Variance of Regolith Depth. The variance of the depth (for example, at fragmentation) can be written as the sum of two components,

$$\text{Var}[X_r] = E[\text{Var}(X_r | l)] + \text{Var}[E(X_r | l)] \quad (4.24)$$

where $\text{Var}(X_r | l)$ is a random variable representing the variance of depth on a particular (but unknown) asteroid. The first component is the variation in depth over an arbitrary asteroid's surface, averaged over the bodies in the fictitious population. The second component represents the variation in the mean depth among bodies.

The determinate models, discussed in Chapter 3, attempted to reduce the total variance (in order to make the average a meaningful descriptor of regolith depth) by minimizing the first component only. This was done by excluding areas on an asteroid's surface that were occupied by large craters. The amount by which the determinate models reduce the first component, and the relative magnitude of the two components, is discussed in the next two chapters. In any event, the
total variance of regolith depth can be used as an indicator of how well the average value can characterize the variation of depth over one asteroid's surface and the variation in depth among asteroids.

The Amount of Brecciated Material Liberated from Asteroids

One observable property of the brecciated meteorites, which could prove useful as an indicator of parent body size, is the volumetric fraction of brecciated material in a given group of meteorites. Assuming that brecciated material was derived from asteroidal regoliths (as opposed to an origin in the zones of fracturing beneath crater bowls), the regolith-depth equations described in the previous section can specify the amount of brecciated material liberated from an asteroid. The computed quantity can be compared to observed values in order to eliminate certain sizes of asteroids from the list of possible parent bodies.

Asteroids release debris to space in essentially two ways. Low-energy impacts produce craters which excavate surface material, part of which is able to escape the asteroid. In this case the fraction of ejecta which is brecciated is determined by the size of the crater and the regolith depth at the impact point. High-energy impacts can fragment and disperse an entire asteroid. Here the fraction of brecciated debris is determined by the volume of an asteroid's regolith. These two cases, i.e. noncatastrophic and catastrophic events, are treated separately.
Debris Liberated in Cratering Events

Consider the case where a group of meteorites is derived from some number, \( n \), of cratering events on the surface of a given size asteroid. (The possibility of deriving meteorites from more than one impact on more than one asteroid is discussed in a later chapter.) The fraction \( Q(n) \) of brecciated ejecta resulting from \( n \) impacts can be written as

\[
Q(n) = \frac{\sum_{j=1}^{n} v_j}{\sum_{j=1}^{n} V_j}
\]  

(4.25)

where \( v_j \) is the volume of brecciated material which escapes the asteroid during the \( j \)th impact event and \( V_j \) is the total volume of ejecta escaping the asteroid during the \( j \)th impact.

A complete statistical analysis of the problem, e.g. finding the p.d.f. of \( Q(n) \), is exceedingly difficult. In order to make calculations tractable we must to make some simplifying assumptions. Even so, expressions can be derived for the mean and variance of \( Q(n) \) which are sufficiently accurate for present purposes.

We assume that \( v_j \) can be expressed as

\[
v_j = \pi D_j^2 X_j q_j / 4
\]

(4.26)

where \( D_j \) is the crater diameter for the \( j \)th event, \( X_j \) is the regolith depth at the point of impact, and \( q_j \) is the fraction of ejecta which escapes. Note \( q_j \) may depend on crater diameter. Equation (4.26) is not strictly correct, for three reasons. (1) The equation implies
that craters have the shape of flat-bottomed cylinders. In reality craters are roughly bowl-shaped. However, for a bowl-shaped geometry, \( v_j \) contains quadratic and cubic terms in \( X_j \). The presence of these terms precludes the derivation of an expression for the variance of \( Q(n) \), hence the simple expression is adopted. (2) When the crater depth is less than the regolith depth at the impact point, equation (4.26) no longer applies. The total volume of regolith ejected from a crater is equal to the crater volume, independent of \( X_j \). In order to avoid this problem, craters too small to puncture through the regolith layer, i.e. craters for which equation (4.26) is incorrect, are ignored. This is not a bad approximation because, as discussed in Chapter 2, the largest craters contain more mass than the smaller ones. This tends to make the computed values of \( Q(n) \) slightly low. (3) Regolith- and nonregolith-derived ejecta are assumed to have the same chance of escaping. Actually, the regolith is more likely to escape because ejection velocities tend to be highest near the surface. Again, this means \( Q(n) \) will be slightly underestimated.

These three circumstances make the expressions for \( Q(n) \) approximate. However a more detailed model probably would not drastically change the conclusions reached. We now proceed with the derivation of the mean and variance of \( Q(n) \).

The volume of escaping debris, both regolith and nonregolith, can be expressed as
where \( c \) is given in equation (2.4). From equations (4.25), (4.26) and (4.27) \( Q(n) \) can be written as

\[
Q(n) = b \sum_{j=1}^{n} D_j^2 q_j / \sum_{j=1}^{n} D_j q_j
\]

where \( b = 2\pi/c \).

The Expected Value of \( Q(n) \). Consider an asteroid of radius \( R_A \) selected at random from a large (fictitious) population of such bodies, each labeled by an identifying number. If \( I \) is a random variable representing the identifying number of an asteroid under consideration, then the mean value of \( Q(n) \) is

\[
E[Q(n)] = \int \cdots \int \sum_{I} E[Q(n) | D_1 = u_1, \ldots, D_n = u_n, I = i] \cdot \text{Prob} \{ I = i \} \prod_{j=1}^{n} f(u_j) \, du_j
\]

where \( f(u_j) \) is the p.d.f. for the random variable \( D_j \). Equation (4.28) allows us to write
Note that \( E[X_j | I=i] \) is assumed to be the same for all \( j \), i.e. we have assumed that the formation of one crater doesn't substantially affect the regolith over the rest of the asteroid. Also, because the regolith depth is time dependent, we assume that the \( n \) craters are not widely separated in time. If equation (4.30) is substituted back into equation (4.29) and the summation over \( I \) is performed, we obtain

\[
E[Q(n)] = b \ E[X] \int \cdots \int \left( \phi_2 / \phi_3 \right) \prod f(u_j) \ du_j. \tag{4.31}
\]

The method used to evaluate the \( n \)-fold integral in equation (4.31) is described later.

The Variance of \( Q(n) \). The second moment of \( Q(n) \) is

\[
E[Q^2(n)] \equiv \int \cdots \int \sum_I E[Q^2(n) | D_1 = u_1, \ldots, D_n = u_n, I=i] \tag{4.32}
\]

\[
\times \text{Prob } \{ I=i \} \prod f(u_j) \ du_j.
\]

If we square the expression given for \( Q(n) \) in equation (4.28) then
\[ E[Q^2(n)|D_1=u_1, \ldots, D_n=u_n, I=i] = \left(\frac{b}{\phi_3}\right)^2 \left\{ E[X^2|I=i] \phi_4 + E^2[X|I=i] \phi_2^2 \right\} \]

Substituting this expression into equation (4.32) yields

\[ E[Q^2(n)] = b^2 E[\text{Var}(X|I)] \int \cdots \int \left\{ \phi_4/\phi_3^2 + \left(\frac{\phi_2}{\phi_3}\right)^2 \sum_I E^2[X|I=i] \text{Prob}\{I=i\} \right\} \prod f(u_j) \, du_j. \]

If we add and subtract the quantity

\[ b^2 \left( \sum_I E[X|I=i] \text{Prob}\{I=i\} \right)^2 \int \cdots \int \left(\frac{\phi_2}{\phi_3}\right)^2 \prod f(u_j) \, du_j \]

on the r.h.s. of equation (4.34) and note that the term which is summed over I in this quantity is just EX, then

\[ E[Q^2(n)] = b^2 \int \cdots \int \left\{ \phi_4 E[\text{Var}(X|I)] + \phi_2^2 \text{Var}[E(X|I)] \right\} \prod f(u_j) \, du_j. \]
In order to simplify the expressions let us write

\[ \text{Var}[X] = k_1 E^2[X] \]

and

\[ \text{Var}[E(X|I)] = k_2 E[\text{Var}(X|I)] \]

where \( k_1 \) and \( k_2 \) are constants for a given size of asteroid. We can now substitute equations (4.36) into equations (4.35), subtract \( E^2[Q(n)] \) from equation (4.35), and simplify to obtain

\[
\text{Var}[Q(n)] = \left\{ \left( bE[X] \right)^2 / (1+k_2) \right\} \int \cdots \int \left\{ [1 + k_2 (1+k_1)] \phi_2^2 + k_1 \phi_4 \phi_3^{-2} \prod f(u_j) \, du_j \right\} - E^2[Q(n)].
\]

The values of the constants \( k_1 \) and \( k_2 \) are estimated from Monte Carlo experiments described in Chapter 5.

In order to use equations (4.31) and (4.37) the \( n \)-fold integrals must be evaluated. The functional form of \( f(u_j) \) precludes an analytic evaluation. Numerical methods can be used but become extremely expensive for \( n>3 \). However there is a very simple way to perform the integrals. Note that the integrals always appear as a function of the \( n \) crater diameters multiplied by the joint p.d.f. of the diameters. That is, the integrals are really just expected values of the functions. For example, the integral in equation (4.31) is the expected value of the function \( \phi_2/\phi_3 \). These expectations can easily be evaluated by Monte Carlo methods. The integral in equation (4.31)
can be evaluated if we generate $n$ crater diameters at random (following the joint p.d.f.), evaluate the function shown above and then repeat this process many times to get an average value. A similar method can be applied to the integral in equation (4.37).

The Monte Carlo calculations were performed as follows. The formation rate of craters of diameter $D$ or larger was assumed to be proportional to $D^a$ where $a$ is a constant (see Chapter 2 for details). The probability of getting a crater in a given size range was taken as proportional to the formation rate weighted by the volume of material the craters liberate to space. This weighting was used because the chance that a meteorite was liberated from its parent body by a crater of some specified diameter depends on the volume of debris contributed by all of the craters of the given size. The smallest crater generated was one whose depth was equal to the mean regolith depth because, as mentioned earlier, the expressions for $Q(n)$ are valid only for those craters which puncture through the regolith layer.

The fraction of ejecta which escapes an asteroid was taken to be a constant (for a given size of asteroid) when strength scaling was applicable to craters. When gravity scaling was appropriate $q_j$ was taken to be proportional to $D_j$ in accord with the ejecta velocity distributions discussed in Chapter 2.

The results of calculations are given in Chapter 5. We now consider the volumetric fraction of brecciated debris liberated when an asteroid is catastrophically fragmented.
Debris Liberated in Catastrophic Events

When an asteroid is finally ruptured and dispersed by an energetic impact, the fraction of debris which is brecciated is the total volume of regolith divided by volume of the asteroid. The volume of regolith on an asteroid of radius \( R_A \) chosen at random from a large (fictitious) population of such bodies is

\[
\text{vol. of regolith} = \int_0^\infty (\text{surface area with depth } \leq dx) \cdot x \\
= \int_0^\infty 4\pi R_A^2 \text{Prob} \{ X_r \leq dx \mid I \} \cdot x \\
= 4\pi R_A^2 E[X_r \mid I] 
\]

where \( X_r \) is the regolith depth at fragmentation and \( I \) is the (random) identifying number of the asteroid chosen. The fraction \( Q_r \) of brecciated material is

\[
Q_r = 4\pi R_A^2 \frac{E[X_r \mid I]}{(4\pi R_A^3/3)} = \frac{3}{R_A} E[X_r \mid I]. 
\]

\( Q_r \) is, therefore, a random variable whose expectation is

\[
E[Q_r] = \frac{3}{R_A} E[X_r] 
\]

and whose variance is

\[
\text{Var}[Q_r] = \frac{9}{R_A^2} \text{Var}(E[X_r \mid I]). 
\]

If we use equations (4.36) and note that, as shown earlier in this
chapter, the mean regolith depth at fragmentation is equal to the standard deviation (i.e. \( k_1 = 1 \) in equation 4.36), then the standard deviation of \( Q_r \) is

\[
\sigma_{Q_r} = \sqrt{k_2/(1+k_2)} \mathbb{E}[Q_r] \tag{4.42}
\]

Equations (4.40) and (4.42) give the mean and standard deviation of \( Q_r \) at the time when an asteroid is catastrophically fragmented. But, as mentioned in the Introduction and in Chapter 2, fragmentation does not necessarily imply that the fragments are dispersed. Therefore, an asteroid may undergo a number of fragmentation events before complete dispersal occurs. This effect could increase \( Q_r \) significantly.

The manner in which a large impact affects regolith evolution is poorly known. Impacts which are energetic enough to cause fragmentation (but not dispersal) may still eject a large quantity of material from an asteroid. Perhaps the uncohesive regolith is preferentially lost in such impacts; that regolith which is retained may be mixed into the asteroid's interior.

Keeping in mind the uncertainties involved, let us make a crude estimate of the mean and standard deviation of \( Q \) upon dispersal of an asteroid. We will estimate the number of times an asteroid experiences fragmentation before dispersal and assume that regolith evolution proceeds as "usual", i.e. as described in the first section of this chapter, in the time intervals between fragmentation events.
Fragmentation occurs when a mass larger than \( m_r \) strikes an asteroid, thus forming a crater of diameter \( > D_r \). Dispersal is defined to occur when a mass larger than say, \( m_d \), occurs. The value of \( m_r \) is determined by setting the kinetic energy of a projectile of mass \( m_r \) equal to the product of an asteroid's volume and the impact strength, \( s_i \), for the asteroid. The parameter \( s_i \) was described in Chapter 2. We will estimate \( m_d \) by equating the projectile kinetic energy to the sum of the energy required for fragmentation and the energy required to disperse the fragments, which is taken to be the gravitational binding energy of the asteroid. Note that some of the impact energy should be partitioned into heating of the target and the angular momentum of the fragments. Thus our estimate of both \( m_d \) and the number of fragmentation events before dispersal will be lower limits.

We now have

\[
m_r = \frac{(2/v_1^2)}{4\pi R_A^3 s_i/3}
\]

and

\[
m_d = \frac{(2/v_1^2)}{[4\pi R_A^3 s_i/3 + 16G\pi^2 \rho R_A^5/15]}.
\]

The second term in the expression for \( m_d \) is the gravitational binding energy for an asteroid of radius \( R_A \) and density \( \rho \). \( G \) is the gravitational constant.
Recall that $\lambda_r$ is the rate at which an asteroid is impacted by masses larger than $m_r$. We denote by $\lambda_d$ the rate of impacts of masses larger than $m_d$. In any catastrophic event, i.e. in any impact of mass larger than $m_r$, the probability that the impact is a dispersal event is $\lambda_d/\lambda_r$. The number, $N_r$, of fragmentation events that occur before dispersal can be viewed as a geometrically distributed random variable with parameter $\lambda_d/\lambda_r$. Thus the mean and variance of $N_r$ are

$$E[N_r] = \frac{\lambda_r}{\lambda_d} - 1$$

and

$$\text{Var}[N_r] = \left(\frac{\lambda_r}{\lambda_d}\right) \frac{(\lambda_r - \lambda_d)/\lambda_d}{\lambda_d}$$

(4.44)

Now if $S_j$ is the volume of regolith developed between the $j$-th and the $j$-th fragmentation events then the total volume of regolith, $S$, at dispersal is

$$S = S_1 + \cdots + S_{N_r+1}$$

(4.45)

To make the problem tractable, the random variables $S_j$ are assumed independent of one another and of $N_r$. This assumption will break down when $N_r$ becomes large, i.e. when an asteroid is fragmented so many times that it is composed mostly of regolith. Taken to the extreme, $S$ will become larger than the volume of the asteroid.
The value of $Q$ at dispersal, denoted by $Q_d$, is obtained by dividing both sides of equation (4.45) by the volume of the asteroid. In doing so we find the mean and variance of $Q_d$ to be

$$E[Q_d] = E[N_r + 1] E[Q_r]$$

and

$$\text{Var}[Q_d] = E[N_r + 1] \text{Var}[Q_r] + E^2[Q_r] \text{Var}[N_r + 1].$$

Using the expressions for the mean and variance of $Q_r$ (equations 4.40 and 4.41) and the constant $k_1$ introduced earlier, the mean and standard deviation of $Q_d$ are found to be

$$E[Q_d] = \left( \frac{\lambda_r}{\lambda_d} \right) E[Q_r]$$

and

$$\sigma_Q = E[Q_d] \sqrt{\frac{k_2}{1+k_2} + \frac{\lambda_r - \lambda_d}{\lambda_r} \frac{\lambda_d}{\lambda_r}}.$$ 

Finally we note that, because the flux of projectiles of mass larger than $m$ is proportional to $m^a$, the ratio $\lambda_r/\lambda_d$ is just $(m_r/m_d)^a$.
A Monte Carlo Algorithm for the Irradiation of Regolith Grains

As discussed in the previous chapter, the charged particle irradiation of regolith grains has not yet been simulated in detail. Borg et al. (1975) gave a brief account of a Monte Carlo simulation of regolith grains but their simulation was far from comprehensive. Housen et al. (1979a) considered the extent to which a regolith grain is gardened by impacts and, by using mean excavation and burial rates, estimated the amount of time a regolith grain might spend near the surface. But they used the projectile flux and cratering laws applicable to large craters. While large craters dominate regolith growth, the small impacts play a very important role in determining the exposure history of regolith grains. In this section a Monte Carlo computer program is described which simulates the random walks and irradiation histories of regolith grains.

In the program a regolith grain starts out at a specified initial (time=0) depth below the surface. At exponentially weighted time intervals the grain experiences random jump events. The size of a jump is determined by randomly generating a crater diameter and the distance between the grain and the crater center. In order to account for the "kinks" in the projectile mass distribution, the differing impact velocities for cometary and asteroidal debris, and changes in scaling laws with crater size, the crater diameter distribution is divided into a number of bins. Within each bin the diameter distribution is a power law which is determined by the projectile mass
distribution, the impact velocity and the scaling law appropriate to the particular bin (equation 2.3).

Depending on the crater diameter and location, the grain can either (a) move further beneath the surface if it is buried by ejecta, (b) move toward the surface if the crater removes some material above the grain, or (c) excavate the grain from the regolith. When an excavation occurs the grain is either randomly emplaced in an ejecta blanket or ejected from the asteroid.

The type (a) events deserve a little more explanation. For gravity scaled craters or large strength scaled craters the magnitude of a burial event is determined from the ejecta blanket model described in Chapter 2. For small strength scaled craters a slightly different procedure is used because these craters are not expected to have continuous ejecta blankets. For example, according to the ejecta model described in Chapter 2, a 1 cm crater on a 300 km diameter asteroid should have a continuous ejecta deposit with a maximum thickness of 0.3 microns. In order to model small scale ejecta deposits in a more realistic manner the debris from small strength scaled craters are assumed to occur as discrete fragments. The number of fragments with mass in a small interval dm is taken to be proportional to \( m^{a_f - 1} \). The proportionality constant and the mass, \( m_L \), of the largest fragment generated are determined by setting the total mass of fragments equal to the nonescaping mass of ejecta and by setting the number of fragments of mass \( m > m_L \) equal to 1. The value of \( a_f \) and the smallest size of fragment are free parameters. A nominal
value of $\alpha = 0.67$ is adopted as suggested by various impact experiments (e.g. Hartmann, 1969). The smallest size of fragment considered is 1 micron. Variations from the nominal values are also investigated.

Note that, in reality, a grain will experience a great many jumps due mainly to the large number of small cratering events. Lest the Monte Carlo program run slower than the actual process, only a part of the jumps are handled on an individual basis. Impact events which change the grain depth by less than a specified percentage, $p$, are assumed to form a continuous process. In the time intervals between large discrete jumps the effects of the small events are added in. The choice of $p$ is somewhat arbitrary. A nominal value of 1% is used and the effects of variation from this value are investigated.

Using this procedure a grain undergoes random walk either until it is ejected from the asteroid or until a specified time limit is exceeded. The program keeps track of the total time spent at various depths beneath the surface so that an exposure age can be computed for the grain. By using the charged-particle flux described in Chapter 2 and by using many trial grains, a distribution of particle-track densities can be determined.

The results from this Monte Carlo program and from the expressions derived earlier are presented in the next chapter.
CHAPTER 5

RESULTS

The expressions derived in Chapter 4 have been evaluated for various sizes of both strong and weak asteroids. The results pertaining to regolith depth are presented first. The calculation of $Q$ and the results from the Monte Carlo irradiation program are given in the second and third sections.

Regolith Depth

The Depth as a Function of Time

Equation (4.20) was used to find the mean regolith depth as a function of time for several sizes of asteroids. The cratering laws and ejecta model described in Chapter 2 were adopted as nominal parameters and the effects of variations from these values were tested. Figure 11 shows the mean depth for four sizes of strong asteroids. The dashed lines in the figure represent plus and minus one standard deviation from the mean value. The collisional fragmentation lifetime of each asteroid is indicated by a vertical line.
Figure 11. Depth of regolith vs. time for rocky asteroids. — The solid line in each plot represents the mean regolith depth. The dashed lines indicate plus and minus one standard deviation from the mean. The time of catastrophic fragmentation is shown as a vertical line (dash-dot).
A 1000 km diameter asteroid acquires a mean depth of roughly 7 km over its 5 Gyr collisional lifetime. Note that the regolith depth is highly variable, as indicated by the large standard deviation. The source of the large variance is discussed below. Smaller asteroids develop less regolith because less crater ejecta are retained and because of shorter lifetimes. Bodies smaller than 100 km diameter are not shown in the figure because they lose nearly all crater ejecta and therefore develop negligible regoliths.

Gravity scaling was used for cratering on the 500 km and 1000 km asteroids because the largest craters, i.e. the ones most important for regolith growth, are gravity scaled (see discussion in Chapter 2). Both gravity and strength scaling were applied to the 300 km asteroid, however the results were nearly identical. Strength scaling was used for the 100 km body.

The computed depths were examined for sensitivity to changes in the cratering laws and the ejecta velocity distribution. For example if \( a_e \), the exponent in the ejecta velocity distribution, is varied from the nominal value of 2 to a value of 1, then the depth decreases by a factor of 1.5, primarily because \( a_e = 1 \) means more crater ejecta is able to escape (see equation 2.5). If the proportionality constants in the cratering laws are varied by 25% then the depth changes by roughly a factor of 2 in the same direction as the change in the cratering constants. Thus the depth is fairly sensitive to uncertainties in the cratering laws.
Calculations for two weak asteroids are shown in Figure 12. Meter-scale and centimeter-scale regoliths are developed on asteroids of diameter 10 km and 2 km. Smaller bodies lose nearly all crater ejecta. Strength scaling was used for both sizes of bodies. Gravity scaling is marginally applicable for the 10 km body; it produces regoliths roughly twice as thick as strength scaling. For these weak bodies, the sensitivity of the results to variations in $a_e$ and the cratering constants are the same as those for strong asteroids.

Results for larger weak asteroids are not shown because the adopted impact strength for weak targets produces unrealistically low fragmentation lifetimes and regoliths depths (see Table 5, discussed below). For large weak bodies, high internal lithostatic pressures will probably produce impact strengths much larger than those adopted. If, as a limiting case, the strength is assumed to be approximately equal to that adopted for strong targets, then the regolith depths computed for large weak asteroids would be comparable to those for large strong ones because, when in the gravity scaling regime, cratering laws are relatively independent of target strength.

The Time Required to Reach Equilibrium

The characteristic time required for a regolith to reach an equilibrium state (equation 4.8) is shown in Table 5. The equilibrium times and depths, which were computed from the first approximation derived in Chapter 4, are meant only to be rough estimates. Judging by the accuracy of the first approximation, these depths (and times?) may be a factor of 2-3 too high. In order to test the accuracy of
Figure 12. Depth of regolith vs. time for weak asteroids.
Table 5
Regolith Depth at Fragmentation and at Equilibrium

**strong asteroids**

<table>
<thead>
<tr>
<th>asteroid diam.</th>
<th>crater scaling law</th>
<th>equil. time</th>
<th>equil. depth</th>
<th>mean frag time</th>
<th>mean depth at frag.</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 km</td>
<td>strength</td>
<td>0.88Gyr</td>
<td>2km</td>
<td>1.5Gyr</td>
<td>0.23km</td>
</tr>
<tr>
<td>300</td>
<td>strength</td>
<td>22</td>
<td>23</td>
<td>2.6</td>
<td>3.6</td>
</tr>
<tr>
<td>300</td>
<td>gravity</td>
<td>100</td>
<td>39</td>
<td>2.6</td>
<td>3.7</td>
</tr>
<tr>
<td>500</td>
<td>gravity</td>
<td>240</td>
<td>65</td>
<td>3.4</td>
<td>4.6</td>
</tr>
<tr>
<td>1000</td>
<td>gravity</td>
<td>780</td>
<td>130</td>
<td>4.8</td>
<td>6.1</td>
</tr>
</tbody>
</table>

**weak asteroids**

<table>
<thead>
<tr>
<th>asteroid diam.</th>
<th>crater scaling law</th>
<th>equil. time</th>
<th>equil. depth</th>
<th>mean frag time</th>
<th>mean depth at frag.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 km</td>
<td>strength</td>
<td>3.4Myr</td>
<td>7.4m</td>
<td>0.27Myr</td>
<td>15cm</td>
</tr>
<tr>
<td>10</td>
<td>strength</td>
<td>1.1Gyr</td>
<td>0.47km</td>
<td>0.61</td>
<td>3.5m</td>
</tr>
<tr>
<td>10</td>
<td>gravity</td>
<td>4.5</td>
<td>1.3</td>
<td>0.61</td>
<td>7.6</td>
</tr>
<tr>
<td>100</td>
<td>gravity</td>
<td>220</td>
<td>13.2</td>
<td>1.9</td>
<td>16</td>
</tr>
<tr>
<td>300</td>
<td>gravity</td>
<td>1500</td>
<td>40</td>
<td>3.3</td>
<td>24</td>
</tr>
<tr>
<td>500</td>
<td>gravity</td>
<td>3500</td>
<td>67</td>
<td>4.3</td>
<td>28</td>
</tr>
<tr>
<td>1000</td>
<td>gravity</td>
<td>12000</td>
<td>136</td>
<td>6.2</td>
<td>36</td>
</tr>
</tbody>
</table>
these estimates, a Monte Carlo simulation of a 300 km strong asteroid was performed. The results are shown in Figure 13. The asteroid was allowed to evolve, without the occurrence of catastrophic fragmentation events, for a period of 50 Gyr. The regolith depth at a surface point was measured every 1 Gyr. This experiment was repeated 100 times so that an average depth as a function of time could be constructed. The predicted equilibrium time for this asteroid is 22 Gyr (Table 5) which appears to be in reasonable agreement with Figure 13. As suggested above, the actual equilibrium depth is roughly a factor of 2-3 lower than that given in Table 5.

Small bodies have shorter equilibrium times than do large asteroids, for two reasons. First, small bodies do not retain much ejecta so large craters cannot cause large fluctuations in regolith depth which might slow down the approach to equilibrium. Secondly, on large asteroids, the equilibrium time is highly dependent on the time interval between the occurrence of the biggest allowable craters, which increases with increasing asteroid size.

Weak asteroids take longer to reach equilibrium than do strong bodies of equal size. Because ejecta velocities are quite low in uncohesive targets, weak asteroids retain more crater ejecta than strong asteroids. Thus the value of $m_1$, i.e. the mean size of a jump in surface elevation, approaches zero, so $t_e$ gets large (see equation 4.8). Physically this means the surface elevation (and regolith depth) may take very large positive excursions. Thus a long time is required for the random walk to settle into equilibrium.
Figure 13. The approach of the mean depth to an equilibrium value. The mean depth was computed from Monte Carlo simulations of regolith growth on a 300 km rocky asteroid. The time required to reach equilibrium is several times larger than both the asteroid's fragmentation lifetime and the age of the solar system. As discussed in the text, this conclusion applies to most sizes of asteroids.
For nearly all the asteroids shown in the table, equilibrium times are at least as large as fragmentation lifetimes and, in general, exceed the age of the solar system. In fact for all bodies, except the 100 km strong one, fragmentation occurs at a factor of ten more quickly than does equilibrium.

The Depth at Fragmentation

The mean regolith depth at the time when an asteroid is catastrophically fragmented was computed from equation (4.23). Results are given in Table 5. Note, from the expressions derived in Chapter 4, the standard deviation of regolith depth at fragmentation is equal to the mean value.

The regolith depth on large strong asteroids is generally in the kilometer range. The depths computed for weak bodies are much smaller due to the short collisional lifetimes. For the larger weak bodies, the actual lifetimes are undoubtedly larger that those shown and therefore serve as extreme lower limits. If the impact strength for these bodies is similar to that of the strong asteroids, then the computed depths would be approximately equal to those expected for the strong asteroids, as discussed above.

Not surprisingly, the mean depth at fragmentation is not equal to the mean depth evaluated at the mean fragmentation time (cf. Table 5 and Figures 11 and 12). Differences as large as 30% occur. This fact is generally not considered in other current models of regolith depth at fragmentation.
The Variance of Regolith Depth

Both the regolith depth at fixed time and at fragmentation exhibit a large variance. As shown in Chapter 4, the variance can be broken into two components: (1) the average variation in depth over any one asteroid's surface and (2) the variation in the average depth among bodies of equal size. We now consider the relative magnitude of these two components in order to see if one or the other is primarily responsible for the large total variance.

A Monte Carlo computer program was written in order to simulate regolith evolution. In the program the surface of an asteroid was represented by a square grid. Craters were generated on the grid until a large impact resulted in catastrophic fragmentation. The regolith depth at each grid point was followed throughout the evolution. For each asteroid, the mean and standard deviation of regolith depth was computed. This experiment was repeated many times so that the average variation in depth over a surface and the interasteroid variation in the average depth could be found.

Simulations were performed for two generic types of bodies: "small" asteroids, on which low gravity fields result in widespread ejecta deposits, and "large" asteroids, on which ejecta are constrained to lie near craters. The actual diameter division between small and large asteroids, i.e. the division between global and nonglobal ejecta deposits, depends on the material strength of an asteroid because ejecta velocities depend on material strength (see discussion in Chapter 2). Thus, as noted by Housen et al. (1979a),
The output for a "typical" small asteroid is shown in Figure 14a. In the figure, the regolith depth, i.e. the thickness of the debris layer, is plotted as a function of position on the surface grid. The Monte Carlo experiments showed that, for small asteroids, the average variation in depth over a surface was roughly equal to the variation in mean depth between bodies. That is, the two components of variance in equation (4.24) were about equal. A similar plot for a large asteroid is shown in Fig. 2b. (Note, in order to graphically illustrate the effects of differing ejecta morphologies on large and small bodies, the same sequence of crater sizes and positions, used in part a of the figure, were used in part b.) The experiments for large bodies showed that the average surficial variation in depth (i.e. the first component in equation 4.24) was 1.5 to 2 times larger than the
Figure 14. Regolith depth as a function of position on an asteroid's surface. — Part (a) applies to an asteroid which is sufficiently small such that crater ejecta are globally spread over the surface. Part (b) represents a larger body, where ejecta are constrained to lie near craters. The increased variability in depth is due to the localization of ejected debris.
variance of the mean depths (the second component in equation 4.24). This is due to the increased roughness caused by the localization of ejecta. Thus, it would appear that neither component vastly dominates the other.

The Amount of Brecciated Material Released to Space

Recall that, for n impacts on an asteroid, Q(n) is the volume fraction of escaping ejecta that is brecciated, i.e. derived from the regolith. \( Q_r \) is the volume fraction at the time when an asteroid is catastrophically ruptured and \( Q_d \) is the fraction resulting from dispersal of a body. Results for strong asteroids and weak asteroids are shown in Figures 15, 16 and 17. The open circles in the figures represent the mean value of Q(n), \( Q_r \) or \( Q_d \) when gravity scaling is applied to craters. The solid dots indicate strength scaling. Plus and minus one standard deviation from the mean are indicated by the vertical "error bars". In Figures 16 and 17 these appear as dashed lines when the mean time for fragmentation or dispersal exceeds 4.5 Gyr.

Results for the case of a single impact (n=1) on strong asteroids are shown at the top of Figure 15. The n-fold integrals required when calculating Q(n) (see equation 4.31) were evaluated using the Monte Carlo technique described in Chapter 4. The required mean and standard deviation of regolith depth were found from the expressions for depth as a function of time, evaluated at a time of 3.5 Gyr. The mean and standard deviation were used to evaluate the constant, \( k_1 \), introduced in Chapter 4. The constant, \( k_2 \), which
Figure 15. The amount of brecciated material with escapes rocky asteroids during cratering events. $Q(n)$ is the fraction of regolith-derived debris amongst the total volume of material which escapes a rocky asteroid during $n$ cratering events. $Q(n)$ is a random variable because both the regolith depth at the impact sites and the diameters of the $n$ craters are random. The open circles represent the mean value of $Q(n)$ when gravity scaling applies to craters. The filled circles represent strength scaling. The vertical "error bars" indicate plus and minus one standard deviation from the mean. In the text the computed values of $Q(n)$ are compared to the observed abundances of brecciated meteorites.
describes the relative magnitudes of the two components of variance of regolith depth, was estimated from the Monte Carlo experiments described in the previous section. A value of $k_2 = 1$ was used for strength scaling, which typically applies to smaller asteroids on which ejecta are widespread. A value of $k_2 = 0.5$ was adopted for gravity scaling, which applies to large asteroids on which ejecta are fairly localized. The results were found to be only weakly dependent on $k_2$. For the smaller bodies in Figure 15 the average value of $Q$ increases with asteroid size because regolith depth increases. For larger bodies (diameter $> 200-300$ km) the mean value decreases. Note that, in the gravity scaling regime, large craters launch more ejecta beyond escape velocity than do small craters because ejection velocities increase with crater size. For the small, strength scaled, bodies ejecta velocity is independent of crater size. Thus on large asteroids, where gravity scaling applies, the mean value of $Q$ is more heavily weighted by large craters, which tend to lower $Q$ because they tend to excavate mainly bedrock.

For larger values of $n$ both the mean and standard deviation of $Q$ decrease. Note that if we generate a sequence of $n$ craters then the size of the largest craters in the sequence should grow as $n$ increases. The largest craters, which are important because they represent most of the volume of the impacting population, tend to make $Q$ small because they excavate a lot of bedrock. Thus, the mean value of $Q$ decreases with increasing $n$. The standard deviation decreases because the more times a particular asteroid is sampled, the
better-known its surficial distribution of regolith becomes. That is, as \( n \) gets large, the first component of variance in equation (4.24) becomes less important. Only the interasteroidal variation in regolith depth contributes to the standard deviation.

The values of \( Q_r \) for strong asteroids are shown in Figure 16a. The mean values here are smaller than those in Figure 15 because now the brecciated material is diluted by a much larger volume of unbrecciated "substrate". Note that, as discussed in the previous chapter, an asteroid may experience many fragmentation events before it is finally impacted with enough energy to disperse the fragments against their mutual gravitational field. A rough estimate of \( Q \) at dispersal, i.e. \( Q_d \), is given in Figure 16b, following the procedure outlined in Chapter 4. The mean number of fragmentation events before dispersal and the lifetimes against dispersal are shown in Table 6. As is obvious from the figure, the mean value of \( Q \) at dispersal is significantly higher than that at fragmentation, especially for big asteroids whose large gravitational binding energies allow many fragmentation events prior to dispersal. Note that as \( Q_d \) approaches unity the assumptions used to derive equations (4.31) and (4.37) break down. Thus, unrealistically high values of \( Q_d \) are obtained for large bodies. Even so, the calculations indicate that large bodies should be composed largely of regolith before they are dispersed by an energetic impact. Of course the large bodies have dispersal lifetimes which are significantly greater than the age of the solar system. Hence the values of \( Q_d \) for these large asteroids are irrelevant,
Figure 16. The amount of brecciated material released upon fragmentation or dispersal of rocky asteroids. $Q$ is the ratio of the volume of regolith to the volume of the asteroid. The mean values are represented by circles, as in Figure 15. The vertical lines, which represent plus and minus one standard deviation, are dashed when the lifetimes against fragmentation or dispersal exceed the age of the solar system. Bodies larger than a few hundred kilometers in diameter are expected to be composed mostly of regolith when they are finally dispersed.
### Table 6

The Number of Fragmentation Events and the Time Required to Disperse an Asteroid

#### Strong asteroids

<table>
<thead>
<tr>
<th>asteroid diameter</th>
<th>mean number of frag. events before dispersal</th>
<th>mean time until dispersal</th>
</tr>
</thead>
<tbody>
<tr>
<td>100km</td>
<td>2</td>
<td>3Gyr</td>
</tr>
<tr>
<td>300</td>
<td>8</td>
<td>21</td>
</tr>
<tr>
<td>500</td>
<td>18</td>
<td>61</td>
</tr>
<tr>
<td>1000</td>
<td>57</td>
<td>270</td>
</tr>
</tbody>
</table>

#### Weak asteroids

<table>
<thead>
<tr>
<th>asteroid diameter</th>
<th>mean number of frag. events before dispersal</th>
<th>mean time until dispersal</th>
</tr>
</thead>
<tbody>
<tr>
<td>2km</td>
<td>2</td>
<td>0.54Myr</td>
</tr>
<tr>
<td>10</td>
<td>21</td>
<td>13</td>
</tr>
<tr>
<td>100</td>
<td>960</td>
<td>1.8Gyr</td>
</tr>
<tr>
<td>300</td>
<td>6000</td>
<td>20</td>
</tr>
<tr>
<td>500</td>
<td>14000</td>
<td>60</td>
</tr>
<tr>
<td>1000</td>
<td>44000</td>
<td>270</td>
</tr>
</tbody>
</table>
assuming that the flux of projectiles was not significantly higher in the past. A higher impact rate would decrease the fragmentation and dispersal lifetimes (but would not change the value of \( Q_d \)).

The values of \( Q_d \) for weak asteroids are shown in Figure 17. Results for \( Q(n) \) are not presented because weak asteroids are expected to be fragmented rather frequently. The fragmentation events tend to mix surficial layers of regolith into the interiors of these bodies. Thus the concept of the regolith existing as a surficial layer, i.e. the concept upon which the expressions for \( Q(n) \) were founded, is not valid in this case. Results for \( Q_r \) are not presented because they are not needed in subsequent discussions. Inspection of Figure 17 shows that weak asteroids as small as 10 km can produce a significant quantity of brecciated material when they are dispersed. As can be seen from Table 6, weak asteroids are expected to undergo many fragmentation events before dispersal. Note that the impact strength of the large asteroids shown in the table may be higher than that used in the calculations because of high lithostatic pressures. Thus the number of fragmentation events experienced by these bodies is probably lower than those listed in the table. In any event, bodies larger than a few tens of kilometers should be composed mostly of regolith when they are finally dispersed.

The Irradiation Histories of Regolith Grains

Results from the Monte Carlo irradiation program are presented in Figures 18-24. The main result of the program is the prediction of particle-track densities due to irradiation by solar cosmic ray (SCR)
Figure 17. The amount of brecciated material released upon dispersal of weak asteroids.
and galactic cosmic ray (GCR) ions. In the Monte Carlo simulations, we essentially random walk a "point" through a regolith. The track density is computed from the total time spent at various depths below the surface and from the track production rate described in Chapter 2. This track density is assumed to be equivalent to the track density at the surface of a finite-size grain. This would not be a good procedure if grains were constantly impacted, eroded or reoriented as grains are in the lunar regolith. However, grains in asteroidal regoliths tend to be buried, i.e. protected from impacts, rather quickly so erosion and reorientation should not be a problem here. We will also find it convenient to discuss the track density at the center of a grain of given size. Note that the penetration depth of SCR ions is so low that the track production rate can change drastically with depth below a grain's surface. The additional shielding at the center of a grain is accounted for when central track densities are computed. Much of the data presented in the literature on meteorites pertain to 100 micron diameter grains. Therefore, when central track densities are presented here they will apply to a grain size of 100 microns.

Figure 18 shows the particle track densities accumulated at the surfaces of grains which reside in the regolith of a 300 km strong asteroid. Results are given for four different starting (i.e. time=0) depths in order to illustrate how a grain's initial depth below the surface affects its exposure history. For each starting depth 100-200 random walks were performed so that a distribution of track densities
Figure 18. Particle-track distributions using various starting depths for regolith grains. -- The track densities are those produced at the surfaces of grains in the regolith of a 300 km diameter rocky asteroid. In each plot the histogram (read on the left ordinate) gives the distribution for grains which acquired track densities above $10^7$/cm$^2$. Grains with lower densities are considered to be unirradiated. The smooth curve (read on the right ordinate) shows the fraction of all grains which acquired a given track density or less. The solid square on the right ordinate indicates the fraction of grains which were exposed to solar cosmic ray irradiation. The peak in a histogram at low track densities is generally due to galactic cosmic ray ions. The higher density peak is the result of solar cosmic rays.
could be constructed. In general, the starting depth was varied from 10 microns to 100m or 1 km in logarithmic steps. The grains were allowed to random walk for a period equal to the mean fragmentation lifetime of the asteroid. Each distribution shown in Figure 18 contains three pieces of information. (1) The histogram gives the fraction of grains (read on the left vertical axis) which accumulated a given density of tracks. Actually, only the grains with densities greater than $10^3$/cm$^2$ are shown in the histogram. Grains with smaller densities spent a negligible part of their time in the upper meter of the regolith and, therefore, were for all practical purposes unirradiated. These unirradiated grains were excluded from the histogram in order to facilitate comparisons with observed track-density data given in the literature. (2) The "smooth" curve in each figure represents the fraction of grains (read on the vertical axis on the right) with a given track density or less. This curve applies to all grains, even those with densities less than $10^3$. (3) The fraction of grains which approached close enough to the surface to be irradiated by SCR particles is indicated by the solid square on the right vertical axis.

As an example, consider Figure 18a which applies to grains starting out 10 microns below the surface. The track density histogram exhibits two peaks. The peak centered near $3\times10^7$ is primarily due to the high flux of SCR particles. Even though all of these grains begin their evolution quite close to the surface, some of them are quickly buried to depths below a millimeter or so, thus
shielding them from SCR particles. However, they may spend a considerable amount of time in the upper meter of the regolith and thus acquire a significant number of tracks from GCR ions. The GCR irradiation is largely responsible for the second peak in the histogram, near $5 \times 10^5$. Because all of these grains started out near the surface they all received some SCR irradiation, as indicated by the solid square at the top of the vertical axis on the right side of the figure.

Figure 18b shows the distribution of tracks for grains which started out 1mm beneath the surface. The SCR peak is less pronounced here, compared to Figure 18a, because the larger starting depths prohibit some grains from reaching the uppermost surface regions.

Figures 18c and 18d apply to starting depths of 10cm and 10m and further illustrate the effect of increasing the initial depth of a grain. Roughly 10-15% of the grains starting at 10cm are essentially unirradiated (track density $< 10^3$) by either SCR or GCR ions and only 40% are SCR irradiated. Note also that the average density of GCR tracks decreases with increasing starting depth. This is to be expected because, the deeper a grain starts, the less time it will spend in the upper meters of the regolith. Conversely, the average density of SCR tracks is seen to be rather insensitive to starting depth. This is a result of the low penetration depths of SCR ions compared to GCR ions. In order to accrue SCR tracks a grain must be exposed essentially at the surface; "close approaches" to the surface do not contribute much. On the other hand, GCR ions penetrate to
meter depths. Thus grains originating from great depths can still acquire GCR effects by making close approaches to the surface, but the density of tracks in such grains will be low.

One of the problems involved in modeling the irradiation histories of grains in meteorites is that we have no idea where the grains originated within their parent body. While the SCR track density is insensitive to initial depth, the fraction of grains which are irradiated depends quite heavily on the depth. One possible remedy is to combine all of the results, for the various initial depths, into a single track distribution, thereby "averaging out" the dependence on initial grain depth. In order to implement such a procedure we must find a method for combining the distributions. Note that most meteorites are probably derived in large cratering events, which scoop out massive sections of regolith and bedrock, or perhaps in catastrophic impact events, in which all parts of the regolith might be sampled with equal probability. In both cases the entire vertical extent of the regolith is sampled. Hence it seems reasonable to assume that all starting depths, from zero down to the thickness of the regolith layer, contribute equally to the distribution of track densities for grains. Actually, on bodies which lose most of their ejecta, grains in the regolith may originate from depths much larger than the thickness of the regolith because as the asteroid is eroded its surface (and regolith) recede in toward the center of the body. But this should not be too important because most of the bodies which we will consider retain most of their crater ejecta. Besides, the
Monte Carlo simulations show that the exposure histories are not very sensitive to the initial depth for depths below 100 m or so. Therefore, we will construct a single track distribution by averaging together the distributions, for the various initial depths, and weighting them by the volume of debris they are responsible for. Note that the distributions for small starting depths will contribute very little in the averaging because very little debris originates from such shallow depths.

Figure 19 shows the depth-averaged distributions for a 300 km strong asteroid. As seen in part (a) of the figure, which applies to the surfaces of grains, some 70% of the grains are never irradiated and only about 15% are SCR-irradiated. Part (b) shows the distribution of track densities at the center of 100 micron grains. Note the GCR peak does not change between the two figures, as we would expect. The SCR track densities are all lowered due to the extra distance the ions must penetrate while a grain is exposed at the surface. Table 7 lists some additional information regarding the depth-averaged distributions. The mean (± one standard deviation) exposure times to SCR and GCR irradiation are roughly indicated by the total time the "point" in the Monte Carlo program spent at depths less than 100 microns and at depths less than 1 meter. The table also gives the number of times that the point entered these two depth regimes and the number of times it was excavated. The numbers in parentheses apply only to those points that received SCR irradiation.
Figure 19. Track densities for grains on a 300 km diameter rocky body. The distributions were obtained by averaging together the distributions for various starting depths. The method used in the averaging is discussed in the text. Results are shown for the density of tracks both at the surfaces of grains and at the centers of 100 micron diameter grains.
Table 7

Some Results from the Simulated Exposure Histories of Regolith Grains*

<table>
<thead>
<tr>
<th>case</th>
<th>total time (yr) spent in region</th>
<th>no. times entered region</th>
<th>no. times excavated</th>
<th>total time (Myr) spent in region</th>
<th>no. times entered region</th>
<th>total no. of times excavated</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 km</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>strong</td>
<td>350±1200 (2700±2400)</td>
<td>0.67±2.20</td>
<td>0.39±1.50</td>
<td>0.41±0.78 (1.10±0.94)</td>
<td>1.40±2.50</td>
<td>3.20±5.40</td>
</tr>
<tr>
<td>100 km</td>
<td>260±1100 (3100±2500)</td>
<td>0.54±2.30</td>
<td>0.40±1.70</td>
<td>0.23±0.65 (1.30±1.00)</td>
<td>1.30±1.00</td>
<td>1.50±5.00</td>
</tr>
<tr>
<td>strong</td>
<td>230±880 (2200±1900)</td>
<td>0.39±1.40</td>
<td>0.12±0.64</td>
<td>0.44±0.84 (1.20±1.00)</td>
<td>0.97±1.50</td>
<td>2.80±3.80</td>
</tr>
<tr>
<td>500 km</td>
<td>540±1700 (2600±2900)</td>
<td>0.95±2.70</td>
<td>0.61±2.20</td>
<td>0.89±1.60 (1.90±1.90)</td>
<td>1.60±2.70</td>
<td>5.10±7.10</td>
</tr>
<tr>
<td>strong</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000 km</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>strong</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 km</td>
<td>11±72 (300±280)</td>
<td>0.06±0.40</td>
<td>0</td>
<td>0.30±0.05 (0.30±0.05)</td>
<td>0</td>
<td>0.05±0.26</td>
</tr>
<tr>
<td>weak</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 km</td>
<td>33±172 (54±502)</td>
<td>0.14±0.65</td>
<td>0.04±0.28</td>
<td>0.03±0.08 (0.18±0.14)</td>
<td>0.16±0.39</td>
<td>0.21±1.00</td>
</tr>
<tr>
<td>weak</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*entries not in parentheses are average values ± 1 standard deviation for all grains

entries in parentheses pertain only to SCR irradiated grains
Figure 20 shows the results for a 100 km diameter strong asteroid. The track distributions for this case are strikingly similar to those in Figure 19. The only difference is that the SCR track densities are slightly higher, on the average, for this asteroid. This is probably due to the fact that grains at the surface of a 100 km asteroid are not buried quite as heavily as they are on larger bodies, where more ejecta are retained.

The track distributions for a 500 km and a 1000 km body are shown in Figures 21 and 22 (note the change of scale on the vertical axes in some plots). Again the results are very similar to the ones discussed earlier, except for the fact that a slightly higher percentage of grains were exposed to SCR irradiation on the 1000 km body compared to the smaller ones.

Figures 23 and 24 are plots of the density distributions for weak asteroids of diameter 2km and 10km. Because the fragmentation lifetimes of these bodies are typically only a million years (see Table 5), grains starting at centimeter to meter depths essentially are never irradiated. This is consistent with the fact that these bodies buildup very little regolith (on the order of 10cm and 1m for the 2km and 10km bodies, respectively) over their collisional lifetime. Grains starting below 10cm-1m are rarely excavated and brought to the surface before catastrophic fragmentation occurs. Thus, the track distributions shown in Figures 23 and 24 were obtained by averaging over depths ranging from zero to 10cm, for the 2 km body, and to 1m for the 10 km asteroid. Figure 23 implies that roughly 2/3
Figure 20. Track densities for grains on a 100 km rocky asteroid.
Figure 21. Track densities for grains on a 500 km rocky asteroid.
Figure 22. Track densities for grains on a 1000 km rocky asteroid.
Figure 23. Track densities for grains on a 2 km weak asteroid.
Figure 24. Track densities for grains on a 10 km weak asteroid.
of the grains on kilometer-size weak bodies are irradiated by GCR ions. The reason that such a large percentage of grains are irradiated is that they all started at depths of 10 cm or less. Only a few percent acquire SCR tracks. Furthermore, the grains that are irradiated have very low track densities. High densities of SCR tracks are absent because grains that make it to the surface are easily ejected from the asteroid's weak gravity field. The story is quite different for a 10 km weak asteroid (Figure 24). Whereas the majority of GCR irradiated grains on the 2 km body have track densities near $10^3$, most grains on the 10 km asteroid have densities less than $10^3$ (because they started out further beneath the surface), so these grains are not shown in the histogram in Figure 24. Thus the distribution is made up mostly of SCR irradiated grains. Note that the comparatively high gravity field of this asteroid allows it to retain the grains which were heavily irradiated by SCR ions.

Additional Monte Carlo experiments were performed in order to examine the sensitivity of the computed track distributions to variations in uncertain parameters. For the most part, only the parameters which pertain to small-scale impacts were varied because the small impacts are primarily responsible for determining exposure histories. A series of six experiments, each involving a 300 km strong asteroid, was performed.

1. The flux of low-mass (cometary) debris was lowered by a factor of three below that shown in Table 1. A factor of three was chosen because this is the level of uncertainty involved in
the Pioneer-spacecraft observations of interplanetary dust (Humes et al., 1974).

2. The track distributions shown in Figures 18-24 were based on the assumption that small craters (diameters less than about 1 cm) formed in regolith. However, blocks of rocks should be present, so small craters may often form in a "strong" target rather than in a "weak one". As an extreme case, a distribution of track densities was generated under the assumption that the small craters formed in rock.

3. As discussed in Chapter 2, the size of craters produced in weak targets, when strength scaling is appropriate, is not well known. For the asteroids this regime includes craters with diameters less than 1-10m. A test case was run using a value of $K_D$ which was twice as large as the value shown in Table 2 for strength scaling in a weak target.

4. The slope of the mass distribution of ejecta fragments produced in an impact event (see the discussion of the Monte Carlo irradiation algorithm in Chapter 4) was varied from the nominal value of $a_f=-0.67$ to -0.9. Note that the nominal value really only applies to the ejecta from a single impact event. Craters which excavate previously fragmented debris might produce a relatively "steeper" mass distribution of fragments, i.e. a distribution with a higher proportion of small fragments compared to the nominal distribution. Thus, a value of $a_f=-0.9$ was also tried.
5. The size of the smallest fragment produced was varied from the nominal value of 1 micron to 10 microns.

6. The parameter, $p$, that determined which impact events were to be handled individually and which were to be considered as a continuous process, was lowered, by a factor of ten, from the nominal value of 0.01.

All of these experiments produced track distributions which were essentially identical to those shown in Figure 19. However, there were some small differences. By lowering the flux of small projectiles (experiment number 1), the fraction of SCR-irradiated grains decreased to roughly a few percent because grains near the surface were not excavated as often as those in Figure 19. On the other hand, increasing the sizes of craters produced in small impacts (experiment number 3) allowed roughly 25% of the grains to be irradiated by SCR ions. Also, the maximum track density observed increased by a factor of 5. Overall, the results were found to be relatively insensitive to the parameters which were varied.

This concludes the discussion of the numerical results from our regolith model. We now consider some of the implications of these results.
CHAPTER 6

IMPLIEDS FOR RECENT MODELS AND THE ORIGIN OF METEORITES

Having described the results of the stochastic model for regolith evolution, we now discuss the implications of these results in connection with previous regolith models and the origins of brecciated and gas-rich meteorites.

Regolith Depth

Relationship with Recent Studies

We begin with a brief comment on how the calculations of the time required for a regolith to reach equilibrium bear on the model of Duraud et al. (1979). The equilibrium times shown in Table 5 imply that most asteroids are not able to reach an equilibrium state before they are catastrophically fragmented. This seems quite obvious in light of the fact that large craters tend to dominate regolith growth. Equilibrium cannot be expected to be reached until formation of "several" large impacts, by which time fragmentation is likely to have occurred. The equilibrium model of Duraud et al. therefore appears exceedingly suspect. In general, asteroidal regoliths are not able to attain equilibrium before being fragmented.
The mean depth of regolith at the time of fragmentation was computed and described in Chapters 4 and 5. Housen et al. (1979a,b) also computed the mean depth, but by a different method. We now take a moment to compare these two results. Using the same set of input parameters and ejecta blanket model as adopted by Housen et al., the mean depths were again computed. For 100 and 300 km strong asteroids the depths found from equation (4.23) were roughly a factor of 3 and 1.15 larger than those computed by Housen et al. This seems rather perplexing because while the present calculations are based on all sizes of craters on the surface, the Housen et al. calculations excluded the areas occupied by large crater bowls, where the depth should be small. This should make their depths the larger of the two. Note, however, that Housen et al. actually computed the surface elevation and argued that it should closely approximate regolith depth. There is a subtle and interesting facet of regolith evolution which makes regolith depth distinct from surface elevation: elevation is independent of the order in which cratering events occur whereas regolith depth is not. As a simple illustration, suppose we are sitting at a point on a surface (where the initial elevation is zero) and four impact events occur. Further suppose that the four events all cause equal changes in the surface elevation, although two increase it and the other two decrease it. The elevation at the point does not depend on the order of the events; it is always zero. But the regolith depth at the point will be larger if the negative changes occur preferentially early rather than if they occur late. The fact
that regolith depth should exceed elevation is supported by equation (4.1) because the minimum surface elevation is always negative.

For asteroids of diameter 500 and 1000 km the Housen et al. depths are smaller than those computed from the present model by a factor of 2. The difference between elevation and depth again applies but there is an additional effect. In modeling a "typical region" on large bodies Housen et al. excluded the ejecta of large craters whereas equation (4.23) includes the effects of all craters. The exclusion of large-crater ejecta tends to make the Housen et al. depths lower.

Which method is the correct one for computing regolith depth? Clearly the present model has the advantage of actually computing the depth, not the surface elevation. But the Housen et al. model has the apparent advantage of modeling a "typical region". Recall that the typical region avoids the parts of the surface occupied by large craters or their ejecta. This procedure was adopted in order to make the average value a meaningful descriptor of regolith depth. Housen et al. conjectured that the average would misrepresent most of the surface if the large crater bowls, where the regolith was thought to be "atypically" thin, and the large-crater ejecta, where the regolith was atypically deep, were included in the averaging.

Housen et al. were right in their contention that the regolith is highly variable in depth when all parts of the surface are considered. This was demonstrated in Chapter 4 where the standard deviation of depth was shown to be equal to the mean. However, we
also showed in that chapter that the variance of depth consists of two components, one due to the variation in depth over an asteroid's surface and a second component which arises from statistical fluctuations in the quantities which determine regolith depth (i.e. the number of craters on an asteroid, their order of occurrence, etc.). In Chapter 5 these two components were found to be of comparable magnitude. The Housen et al. procedure should reduce the first component of variance. It may also reduce the second component because part of the interasteroidal variability arises from the statistical uncertainty in the number of large craters on a body. Thus avoiding the areas occupied by large craters may reduce the interasteroidal component.

Let us now consider the effectiveness of modeling a typical region by seeing how much this reduces the variance of regolith depth. Using the same regoliths that were generated by the Monte Carlo program described in the previous chapter (see section on the variance of regolith depth) the mean and variance of the depth were recomputed. However, this time the mean and variance were computed over the same parts of the surface considered by Housen et al. For small asteroids the bowls of large craters were avoided, where "large" was determined from the saturation criterion of Housen et al. (1979a) described in Chapter 3. This caused the mean depth, and standard deviation, to increase. The coefficient of variation (standard deviation divided by the mean) decreased slightly but was still found to be 1. Thus, avoiding the parts of the surface occupied by large crater bowls does
not significantly decrease the uncertainty associated with regolith depth. For large asteroids, the ejecta deposits of big craters were also avoided according to the Housen et al. (1979b) definition of ejecta saturation. This caused the mean depth to decrease, however, the coefficient of variation was still roughly unity. In both cases the two components of variation were again observed to be of equal magnitude.

It would appear that modeling a typical region does not significantly improve the utility of the average value in describing regolith depth on asteroids. That is, the Housen et al. model describes the regolith depth in the typical region no better than does a model which computes the mean depth by averaging over the entire surface of an asteroid. Note that these comments apply equally well to the model of Langevin and Maurette (1980).

While on the subject of the variability of regolith depths, we might reasonably ask if there are any processes which might tend to reduce the variability below that calculated. For example, if an asteroid does not conform to a shape of hydrostatic equilibrium then components of the gravity field which are tangent to the local surface may redistribute the regolith, thus smoothing the debris layer to conform to the geoid (see discussion in Cintala et al., 1979). This mechanism may have played a role in forming the smooth appearance of Deimos' surface (Thomas, 1979).
It is, however, worthwhile to note that a smooth surface does not guarantee a regolith of uniform depth; variability also arises from the topography at the bottom of a regolith. To illustrate this we can consider the extreme case where the regolith is sufficiently mobile to redistribute itself into gravitationally "low" spots. In order to estimate how smoothing affects the variance of depth, the computer-generated regoliths, referred to earlier, were allowed to "relax" so as to produce a smooth, planar, surface.

After the surfaces were smoothed, the standard deviation of the surficial depth distribution (i.e. the square root of the first component of variance in equation 4.24) was observed to increase by roughly a factor of 1.7. Figure 25 shows an example. Parts a and b of the figure are plots of the surface elevation before and after smoothing. Parts c and d show the regolith depth before and after. The reason for the increased variance is clear; by removing the regolith from the high surface elevations and accumulating it in the low spots we tend to add more weight to the tail of the regolith-depth distribution.

The above simulation was meant to be a simple example of the effects of smoothing an asteroid's surface. The actual process is difficult to simulate because in reality the extent to which a regolith is redistributed depends on how far the asteroid deviates from an equilibrium figure, on the cohesive properties of its regolith and on how frequently impacts "shake" the regolith (Cintala et al., 1979; Langevin and Maurette, 1981). However, the simulations do
Figure 25. An illustration of how gravitational settling of debris might affect the variability of regolith depth. — In order to illustrate the effects of debris settling, the regoliths generated by a Monte Carlo program were allowed to "relax" so as to produce a smooth top surface. Part (a) shows the elevation of the top surface of a regolith before smoothing. The surface elevation on the same asteroid, after the regolith has been redistributed, is plotted in part (b). Part (c) shows the regolith depth, i.e. the thickness of the debris layer, corresponding to the surface in part (a). The regolith depth, after the surface was smoothed, is shown in part (d). Smoothing an asteroid's surface can actually increase the variance of the regolith depth distribution because, by removing regolith from the high surface elevations and accumulating it in the gravitationally low spots (e.g. the bottoms of large craters), more weight is added to the tail of the regolith depth distribution.
indicate that, while an asteroid (or planetary satellite) may have a smooth surface, its regolith may be quite variable in depth.

Implications for the Brecciated Meteorites

As discussed in the Introduction, many of the meteorites now in our museums are composed of angular fragments of rock embedded in a finer ground mass. The structure of these brecciated meteorites is indicative of a regolith origin. The observed abundance of brecciated meteorites is determined largely by the prevalence of regolith-derived material in the total volume of debris that is ejected from an asteroid. The volume fraction, Q, of escaping debris that is derived from a regolith was shown in the previous chapters to be a function of asteroid size. We now compare the calculated values of Q with the values observed in meteorite collections in order to determine the extent to which regolith models can constrain parent body size for the brecciated meteorites. The basaltic achondrites are considered first and are followed by a discussion of the chondritic meteorites.

Of the basaltic achondrites we will concentrate primarily on the eucrites, howardites, diogenites and aubrites. The other subclasses, e.g. the chassignites, ureilites, nakhlites, shergottites and angrites are not considered because they are rare. Nearly all of the basaltic achondrites are brecciated. For example, the eucrites, howardites, diogenites and aubrites are, by volume, roughly >98%, 100%, 80% and 100% brecciated (Mason, 1962; Heymann et al., 1968; Wasson, 1974; Hutchison et al., 1977). Based on a variety of cosmochemical and petrological arguments, the eucrites, howardites and
diogenites appear to be genetically related and are often suggested as having originated on one, or perhaps a few, parent bodies (e.g. Duke and Silver, 1967; Mason, 1967; Ganapathy and Anders, 1969; Jerome and Goles, 1971; McCarthy et al., 1972; Stolper, 1975; Dreibus et al., 1977; Takeda, 1979). Consolmagno and Drake (1977) modeled the evolution of rare earth element abundances in eucrites and concluded that these meteorites were likely the result of 5%-15% partial melting of an essentially chondritic parent body. Based on the observed meteoritic abundances and asteroid reflection spectra they further concluded that the eucrite parent body is still intact and that the most likely source body is asteroid 4 Vesta. (The Vesta scenario has been embellished by Hostetler and Drake, 1978 and has been disputed by Wasson and Wetherill, 1979.) Furthermore, Ganapathy and Anders (1969) suggest that the eucrites and howardites were liberated from their parent body in a few impacts.

With the above considerations in mind let us now compare the observed values of Q for the eucrites, howardites, and diogenites with the calculated values of Q given in Figure 15. If the meteorites were liberated in a single impact then, because of the large standard deviation associated with the calculated values of Q, virtually any size of asteroid larger than 100 km could have produced the fraction of brecciated material observed in the basaltic achondrites. If a few impacts (n<5?) were responsible then the most probable size of parent body is 200-300 km. Note however that the values shown in Figure 15 may be somewhat low because of the assumptions made in deriving Q (see
Chapter 4). Thus, for the case of a few impacts, even a Vesta-size asteroid is probably capable of producing the values of Q observed for the basaltic achondrites. In short, it would appear that the eucrites, howardites and diogenites were derived from a body whose diameter is $> 100-200$ km.

The Consolmagno and Drake (1977) conclusion that the eucrite parent body is still intact was based on the observation that no meteorites have been found with the expected composition of the parent body mantle. The calculated 5%–15% melting would imply a 4–13 km-thick crust of eucritic basalts (for a uniform layer on a 500 km body). We can check to see if our regolith depth calculations are consistent with the Consolmagno and Drake model by determining the extent to which the mantle material might be mixed into the regolith. This can be roughly estimated by computing the volume of material excavated from beneath a crust of specified thickness. If we divide this volume by the total volume of regolith, obtained from the calculations described in Chapter 5, then the fraction of regolith material which is derived from the mantle can be calculated. For crustal thicknesses of 5, 10, 15 and 20 km the calculated abundances of mantle material are roughly 60%, 30%, 15% and 7%. Taken at face value these numbers would suggest that the eucrite parent body crust should have been at least 15 or 20 km thick. A thinner crust would likely result in meteorites composed largely of mantle material, contrary to observation. We can obtain another estimate of the thickness of the crust by computing the amount of material which is
excavated from the mantle and ejected from the asteroid (along with debris excavated from the crust) in meteorite-producing impact events. This is easily found from the expressions derived for $Q(n)$ in Chapter 4. If the crustal thickness is substituted for the mean regolith depth in equation (4.31) then the fraction of meteoritic material derived from the parent body mantle (this is analogous to the fraction of material derived from the beneath the regolith) can be found. For a 500 km asteroid we find that, unless the crust is thicker than about 15 km, a significant fraction (more than a few tens of percent) of the debris liberated from the asteroid will be derived from the mantle. Hence, the crust on the eucrite parent body must have been thicker than roughly 15 km. This is a bit larger than the thickness computed by Consolmagno and Drake. In this regard it is interesting to note that Fukouka et al. (1977), using a slightly different rare earth element abundance for the eucrite parent body than that used by Consolmagno and Drake, estimated that the eucrites may have been formed by 8-24% partial melting. This would imply a crust of thickness 7-22 km.

In the above discussion of parent body size as inferred from the observed values of $Q$ we assumed that the eucrites, howardites and diogenites were derived from a common parent body. This assumption is often a convenient one but cannot be rigorously justified. In fact, geochemical arguments can be made for the case of several parent bodies (Delaney et al., 1981). However, even if several parent bodies were involved, our conclusions regarding parent body size are
unchanged. The virtual ubiquity of brecciated material among the basaltic achondrites argues for parent bodies larger than 100-200 km in diameter. This conclusion also applies to the aubrites, for which $Q$ is observed to be very nearly unity.

So far our discussion has been based on the assumption that the basaltic achondrites were liberated in a few, nondestructive, cratering events. Although this is supported by geochemical arguments (Consolmagno and Drake, 1977), there is another point to consider. That is, volumetrically speaking, should we expect cratering events to generate more debris than catastrophic dispersal events? For the moment let's assume that only bodies smaller than 200 km in diameter have dispersal lifetimes less than 4.5 Gyr (Table 6). If we compute the total volume of material liberated during cratering events on all sizes of asteroids and the total volume liberated during dispersal of bodies smaller than 200 km, then we find that the dispersal events dominate by about an order of magnitude. But figure 16b implies that the bodies which are disrupted (diameters less than 200 km) are expected to produce very little brecciated material. This would seem hard to reconcile with the observed prevalence of brecciated material among the achondrites. Of course there are caveats. First, the dispersal lifetimes shown in Table 6 are lower limits because, as mentioned in Chapter 4, some of the impact energy which was assumed to be partitioned into the dispersal of the fragments is actually taken up in heating of the target, etc. Davis et al. (1979) have computed dispersal lifetimes in some detail and find that only asteroids
smaller than about 10 km are dispersed over the age of the solar system. This will considerably reduce the contribution of debris from dispersal events. Secondly, Horz and Schaal (1981, work in preparation) have suggested that impact-generated shock waves may spall off the regolith-laden outer layers of rocky asteroids. The volume of debris produced by spallation may exceed the volume produced by craters. If, in the future, this mechanism is shown to be quantitatively important then the quantity of material derived from dispersal events will be judged as even less significant. If we are willing to accept these somewhat circumstantial arguments then the assumption that the achondrites were derived in nondestructive cratering events is reasonable. We now turn our attention to the undifferentiated meteorites, i.e. the chondrites.

As a group, the chondritic meteorites are much more abundant than the achondrites. The literature on these meteorites is quite voluminous but tends for the most part to avoid the question of whether or not a particular chondrite is brecciated. However, there are a few sources of information pertinent to the values of Q. Wahl (1952) described the brecciated structure of many stony meteorites but his survey, now quite old, is rather incomplete. In a more recent work, Binns (1967) conducted a petrological study of many noncarbonaceous chondritic meteorites and found that roughly 15%, 33%, 20% and 60% of the enstatite chondrites, H chondrites, L chondrites and LL chondrites are brecciated. While these represent percentages by number of meteorites, rather than the percentages by volume, they
do indicate that brecciated material is less common among the chondrites than among the basaltic achondrites. Actually, as we shall see, these qualitative estimates of Q are sufficiently accurate for the purposes at hand.

We will compare these observations with the values of Q estimated for the dispersal of weak asteroids. As mentioned in the previous chapter, it is difficult to compute Q(n) for these asteroids. Besides, catastrophic events may actually contribute more debris than cratering events. Because of low ejection velocities less debris escapes a weak asteroid than from a strong body. Furthermore, the spallation mechanism which was called upon to augment the quantity of debris liberated in noncatastrophic events for strong asteroids, should be less effective in weak materials. Thus, dispersal may be the dominant source of meteorites from weak asteroids. Inspection of Figure 17 shows that virtually any asteroid larger than 20 km in diameter could have produced enough brecciated material to be a parent body for the enstatite, H, L, or LL chondrites. This is a good example of why it is important to consider the stochastic nature of regolith evolution. If only the average values of Q are considered one might conclude that a 20 km body is unable to develop enough regolith to produce the LL chondrites or that bodies larger than 100 km produce more breccias than observed for the enstatite chondrites. The large variance of Q renders such conclusions invalid.
Descriptions of the textures of carbonaceous chondrites are even more scarce than for the other chondrites. All of the CI chondrites are observed to be gas-rich (Anders, 1978) and, therefore, are also brecciated. Several C3V chondrites are brecciated (McSween, 1977), the most notable member being Allende. The large mass of Allende (2000 kg) results in a value of Q near unity for these meteorites. The C30 chondrites are described by Methot et al. (1975) as being typically brecciated. Thus it would appear that Q for the carbonaceous chondrites, both as a group and within each subgroup, approaches one. Figure 17 would suggest a parent body whose diameter is greater than a few tens of kilometers. Again, the statistical uncertainties inherent in the calculated values of Q preclude a more precise estimate of parent body size.

To summarize, the ubiquity of brecciated material among the basaltic achondrites suggests a parent body (or bodies) with diameter greater than 200-300 km. If Consolmagno and Drake (1977) are correct in their identification of Vesta as the eucrite parent body then basaltic flows on Vesta should be thicker than about 15 km. The observed proportion of brecciated material among the enstatite and ordinary chondrites is consistent with an origin on bodies of diameter greater than 10-20 km. The high value of Q observed for the carbonaceous chondrites suggests a parent body of diameter greater than a few tens of kilometers.
The Exposure Histories of Gas-Rich Meteorites

We saw in Chapter 5 that the predicted charged-particle track distributions for regolith grains on rocky asteroids are quite different from those expected on weak bodies. However, we also found that the distributions do not vary much from one size of rocky asteroid to another. Thus, it would appear that we cannot use these track data to precisely pinpoint the size of meteorite parent bodies. Taking a somewhat more optimistic view, perhaps there is no unique size of parent body that can be associated with a given class of meteorites, i.e. meteorites may indeed originate from a wide size range of asteroids. We now compare the Monte Carlo irradiation results described in Chapter 5 with the observed irradiation features in gas-rich meteorites to see what we can infer about the origins of these meteorites.

Measurements of the particle-track density distributions in meteorites are not overly abundant in the literature. One group of stony meteorites, the aubrites, has been studied in some detail. Roughly 1/3 of the aubrites are gas-rich (Anders, 1978). Typically, in the gas-rich members, 20% of the grains have surface densities of SCR tracks of $>10^8$/cm$^2$, but this fraction can vary from 3% to nearly 50% (Poupeau and Berdot, 1972; Poupeau et al., 1974). Thus, of all the grains in the aubrites, about 6% (varying from 1% to 15%) have SCR track densities greater than $10^8$. This is consistent with the track density distributions shown in Chapter 5. For rocky asteroids between 100 and 1000 km diameter the fraction of grains with track densities...
greater than $10^8$ is about 3-5% (cf. Figures 18-22). There is a slight discrepancy in that Poupeau and Berdot (1972) observed maximum track densities of $10^9$ whereas the maximum densities resulting from the Monte Carlo simulations was about $5 \times 10^8$. However, this maximum density would probably increase if a larger number of grains were tested in the simulations. Poupeau and Berdot also observed a peak in the GCR track density at about $5 \times 10^5$, which they attributed to irradiation while the meteorites were in transit to the Earth. This peak is rather large and undoubtedly envelopes the GCR peak near $10^4$ produced in the Monte Carlo model. The number of tracks with densities between $10^6$ and $10^8$ was observed to be comparable to the number above $10^8$, whereas the predicted distributions show a somewhat smaller fraction of grains above $10^8$. This could easily be explained by the uncertainties in the assumed track production rate discussed in Chapter 2. Poupeau et al. (1974) estimated that the SCR irradiated crystals spent no more than $10^3-10^4$ years in the upper 10-100 microns of the regolith. From the density of GCR tracks, the time spent in the upper 10cm-1m was estimated to be less than $10^6$ yr. These estimates agree very well with the data presented in Table 7. Poupeau et al. suggested that the exposure times of the aubritic grains, which are very short compared to lunar soils, could only result from evolution in a regolith where the stirring rate was high compared to that in the lunar regolith. The assumption that such a high mixing rate existed only long ago in the asteroid belt led Poupeau et al. to the conclusion that the aubrites must have formed 3.8-4 Gyr ago. The
Monte Carlo simulations, which employ present-day estimates of input parameters, clearly demonstrate that the aubrites could be the result of a more recent regolith origin.

Numerous studies of another group of stony meteorites, the howardites, have been made (Lal and Rajan, 1969; Pellis et al., 1969; Wilkening et al., 1971; Wilkening, 1971; Barber et al., 1971; Macdougall et al., 1974; Rajan, 1974; Price et al., 1975; Anders, 1978). Some of the basic observations, which are pertinent to our discussions, are (a) SCR tracks are most often observed to be $10^8$ at grain surfaces and $10^7$ in the central regions; (b) 50-90% of the track-rich grains (defined as those grains with central track densities $>10^8$) exhibit nonuniform densities of tracks around their borders; (c) roughly 1/3 of the howardites are gas-rich; and (d) typically 5-20% of the grains are track-rich. The observation that many grains have surface densities near $10^8$ agrees roughly with the computed track distributions. The observation that most irradiated grains have higher track densities on one side than on another is also consistent with our calculations. Table 7 indicates that regolith grains should be reoriented only a few times during their exposure histories. Observations (c) and (d) imply that about 1-6% of the regolith grains should have central densities greater than $10^8$. On the other hand, Figures 18b-22b exhibit central densities which rarely exceed $5 \times 10^6$ to $10^7$. This discrepancy might be explained by uncertainties in the assumed production rate of tracks. For example, the production rate of Hutcheon et al. (1975) is about a factor of ten
higher than the adopted rate (see Chapter 2). We could also explain the discrepancy in track densities if the howardites were irradiated during a period of solar system evolution when the ion flux was higher than at present. This, however, is purely speculative because there is no conclusive evidence for a higher SCR flux in the past (Crozaz, 1980). Alternatively, the heavily irradiated grains may have acquired their tracks prior to being chipped off of larger "parent rocks", as suggested by Borg et al. (1975).

A few studies of the irradiation histories of ordinary chondrites have been made (Shultz et al., 1972; Lorin and Pellas, 1979) but details of the track density distributions are not available. We will just note that the observed abundances of gas-rich members among the ordinary chondrites (about 10%; Anders, 1978) is in rough agreement with our results.

Because of their peculiar nature, the carbonaceous chondrites have been fairly well scrutinized. They stand out among the meteorites in that they exhibit very low track densities. For example, most grains have surface track densities of roughly $10^7$ and only rarely exceed $10^8$ (Goswami et al., 1976; Macdougall and Phinney, 1977; Korotkova et al., 1979). Around 60-80% of the carbonaceous chondrites are gas-rich (Anders, 1978) but only a few percent (ranging from 1-20%) of the grains have central track densities greater than $10^5$ (Goswami et al., 1976), and only about 30% of these irradiated grains have densities exceeding $10^8$ (Goswami and Lal, 1979). Additionally, only a few percent of the heavily irradiated grains show
evidence of irradiation on more than one side (Macdougall, 1976; Goswami and Lal, 1979). By comparing these observations to our results for small weak asteroids (Figures 23-24) we see that, first of all, these small bodies should produce rather immature meteorites. The primary reason is, of course, that catastrophic fragmentation occurs rather quickly. Such impacts cause major internal fracturing and undoubtedly will mix much of the surficial material into the interior, thus shielding it from further irradiation. Indeed only a few percent of the regolith grains acquire central track densities higher than $10^5$ (Figures 23b and 24b), in agreement with the observed values. Furthermore, Table 7 implies that grains emplaced on the surface hardly ever change their orientation and thus should develop very anisotropic track distributions. There is, however, a marked disparity between the Monte Carlo results and the observations: roughly 0.5-3% of the grains should have central densities higher than $10^8$ but none are observed in the computed track distributions. This might be written off as a result of the statistics of small numbers (only 100 test grains were used) but the lack of any grains with densities greater than $10^6$ makes this an unlikely possibility. Goswami and Lal (1979) noted this problem and suggested that the grains in carbonaceous chondrites were irradiated while they resided in centimeter-size clumps prior to accretion onto their parent bodies. The Monte Carlo results seem to indicate that a regolith scenario is
incapable of producing the carbonaceous chondrites. However, we should again note that a higher charged-particle flux could remove the discrepancy.

Overall, the Monte Carlo simulations of the exposure histories of regolith grains agree quite well with observations of irradiated grains in gas-rich meteorites. The calculations presented remove the necessity to invoke an early irradiation of meteorites as some have suggested in the past (Lal and Rajan, 1969; Poupeau et al., 1974). The near constancy of the model distributions for different sizes of asteroids suggests that any given group of meteorites may originate from a rather wide size range of parent bodies. This contrasts with the conclusions of Housen et al. (1979a,b) who suggested that large asteroids should have better-mixed and rather mature regoliths compared to smaller bodies. The predicted central track densities are somewhat lower than those observed, but this discrepancy can be accounted for by uncertainties in the assumed track production rate. Determining whether regolith models can account for the details of the observed irradiation features cannot be done until better estimates of the various input parameters, especially the production rate of tracks, become available.
Models of asteroidal regoliths are important tools in the study of the origins of brecciated and gas-rich meteorites. Previous investigations in this area were reviewed in Chapter 3. The two most detailed models are those of Housen et al. (1979a,b) and Langevin and Maurette (1980). Both of these models concentrated primarily on the depth of an asteroid's regolith and attempted to characterize the depth by an average taken over a restricted part of the surface. In order that the average might reasonably describe at least part of the surface, only the areas over which the regolith was thought to be relatively uniform in depth, i.e. the areas "saturated" by numerous small impacts, were modeled. Housen et al. (1979 a,b) collectively referred to these areas as the "typical region". Note that modeling a typical region implies that some parts of an asteroid's surface were ignored: For example, in the Housen et al. model, roughly 1/3 of the surface area of small bodies and at least 1/2 of the area on large bodies were ignored. Of course, there is no reason to expect meteorites to be derived only from the typical regions on asteroids. Furthermore, Housen et al. and Langevin and Maurette could not estimate the effectiveness of modeling a typical region, i.e. they
could not determine the degree to which the average value characterized the distribution of regolith depths over the modeled parts of the surface.

Another drawback of previous studies is that regolith evolution was considered from a determinate point of view, i.e. asteroids of identical size were assumed to have identical regoliths. Actually, every similar-size asteroid should develop a unique regolith because of the stochastic nature of the cratering process. For instance, even though we might precisely specify the average formation rate of craters, we cannot predict with certainty the number of craters that will form on any given body. These stochastic effects are important when regolith models are used in speculations on the origin of meteorites.

A third drawback of earlier studies is that none have considered, in detail, the exposure histories of regolith grains to space irradiation. A few calculations of exposure times have been performed but the investigations were far from exhaustive.

The stochastic evolution of asteroidal regoliths and the connections with brecciated and gas-rich meteorites were considered in Chapters 4, 5 and 6. A model of regolith growth was constructed which has an advantage over previous models in that it describes the entire surface of an asteroid. Furthermore, the distribution of regolith depths, not just the mean value, was obtained. A Monte Carlo algorithm was constructed and used to simulate the charged-particle irradiation histories of regolith grains. The main results from the statistical modeling are now summarized.
1. The computed distribution of regolith depths was found to have a rather large standard deviation (approximately equal to the mean depth) because of variations in depth over the surface of any given asteroid and because of stochastic fluctuations in the average regolith depth among similar-size bodies. Avoiding the parts of a surface that were occupied by large craters or thick ejecta deposits did not change the magnitude of the standard deviation relative to the mean. Thus, modeling a typical region, as opposed to modeling the entire surface, does not significantly improve the utility of the average value in describing asteroidal regolith depths.

2. The large statistical uncertainty associated with regolith depth limits the power of regolith models in predicting the sizes of meteorite parent bodies. For example, virtually and rocky asteroid of diameter larger than 100-200 km could have produced the quantity of brecciated material observed in the achondritic meteorites. Smaller bodies can be ruled out because they do not retain enough ejecta to develop regoliths of the required depth. Weak asteroids larger than about 20 km could have produced the observed abundance of brecciated material in the chondrites.

3. Most asteroids are collisionally fragmented before their regoliths can attain a state of statistical equilibrium. Moreover, the equilibrium times are generally greater than the age of the solar system. Thus, equilibrium models of regolith evolution (e.g. Duraud et al., 1979) are not valid.
4. Monte Carlo simulations showed that the irradiation history of a regolith is quite independent of asteroid size, for those bodies large enough to generate regoliths. Thus, Housen et al. (1979a,b) were wrong in their conclusion that large asteroids should have highly-gardened and rather mature regoliths compared to smaller bodies. The irradiation of grains is much more extensive on rocky asteroids than on weaker bodies, primarily because of the short fragmentation lifetimes of weak asteroids. This is in qualitative agreement with the observed differences between the irradiation features in the achondrites and the chondrites. The computed fraction of grains which were exposed to space irradiation agrees well with the fraction observed in gas-rich meteorites. The range of densities of charged-particle tracks in regolith grains is comparable to that observed in meteorites, although the computed fraction of heavily irradiated grains is significantly smaller than that observed. This discrepancy could be explained by uncertainties in the assumed production rate of tracks. Alternatively, the heavily irradiated grains might be due to irradiation while the grains were part of larger "parent rocks", as suggested by Borg et al. (1975). In general, the irradiation features observed in gas-rich meteorites do not require an origin during early epochs of solar system evolution, as previously suggested.
Modeling asteroidal regolith evolution is currently hampered by inadequate knowledge of some of the necessary input parameters and certain aspects of the cratering process. The most critical gaps in our knowledge are the mass-frequency distribution of matter and the flux of track-producing ions in the asteroid belt, now and in the past, and the physics of the impact process in low gravity environments. These deficiencies are amenable to some improvement by possible, albeit difficult, observations or experiments.

In short, regolith models have reached a level of detail which is roughly similar to the precision of the required input parameters. Further advances in our understanding of regoliths on asteroids must come from further experimentation.
APPENDIX 1

THE MOMENTS OF A JUMP IN SURFACE ELEVATION

We now derive expressions for \( m_1 \) and \( m_2 \), the first two moments of the size of a jump in surface elevation. The methodology involved in the "improved approximation" described in Chapter 4 is used. That is, excavations by craters larger than a diameter, \( D^* \), are excluded from the calculations. The moments for the "first approximation" can be found by setting \( D^* = D_1 \), where \( D_1 \) is the diameter of the smallest crater assumed to form on the asteroid. In doing so the excavation effects of all sizes of craters are included.

If \( Y \) is the (random) size of a jump in the surface elevation at a randomly selected point on an asteroid's surface then the \( n \)th absolute moment of \( Y \) can be written as

\[
E[Y^n] = \int_{D_1}^{D_R} G_n^-(u) C^-(u) H(u) \, du + \int_{D_1}^{D_R} G_n^+(u) C^+(u) H(u) \, du
\]

\[
\equiv E[Y^n]_{\text{excavation}} + E[Y^n]_{\text{burial}}. \tag{Al.1}
\]

Note that \( m_1 \) and \( m_2 \) are obtained from equation (Al.1) by setting \( n=1 \) or \( n=2 \) respectively. \( G_n^- \) is defined as the conditional expectation of \( Y^n \) given that a crater of diameter \( u \) has formed over the surface.
point. \( C(u) \) is the probability that the crater forms over the point given that the crater diameter is equal to \( u \). \( H(u)du \) is the probability that the crater diameter is in the interval \( du \). The symbols with + superscripts are defined similarly for burial events. The excavation and burial terms in equation (A1.1) are now considered separately.

**Excavation**

If a crater of diameter \( D \) forms on the surface and the distance between the crater center and a surface point is \( R \) then the change, \( Y \), in the surface elevation at the point is

\[
Y = -c_1 D - \sqrt{c_2 D^2 - R^2}
\]

(A1.2)

for \( R < D/2 \). The constants \( c_1 \) and \( c_2 \) are given by

\[
\begin{align*}
    c_1 &= \frac{(4\mu^2 - 1)}{8\mu} \\
    c_2 &= \left[\frac{(4\mu^2 + 1)}{8\mu}\right]^2
\end{align*}
\]

where \( \mu \) is the depth to diameter ratio for craters.

In computing \( G_n(u) \) we are given that \( D \) has the value \( u \). Thus, \( Y \) is a function of the single random variable \( R \). \( G_n(u) \) can be evaluated by noting that the probability that \( R \) is in the interval \( dx \) is

\[
\text{prob} \{ R \in dx \} = 2\pi x \ dx/(\pi u^2/4) = 8x/u^2 \quad 0 \leq x \leq u/2.
\]

Thus, we can write
This integral can be easily evaluated by substituting a new variable for the expression in brackets on the r.h.s. of equation (A1.3). After some algebra we obtain

\[ G^{-\frac{u}{2}}(u) = \int_{0}^{u} \frac{(8x/u^2)}{(-c_1u - \sqrt{c_2u^2 - x^2})^n} \, dx. \]  

Recall that \( C^{-}(u) \) is the probability that an excavation occurs given the formation of a crater of diameter \( u \). Thus

\[ C^{-}(u) = \begin{cases} \frac{(\pi u^2/4)}{4\pi R^2} & \text{for } u < D^* \\ 0 & \text{for } u > D^* \end{cases} \]

\[ C^{-}(u) \] is zero when \( u > D^* \) because craters larger than \( D^* \) are not allowed to excavate (see Chapter 4).

The crater size-frequency distribution which was adopted in Chapter 2 allows us to to express the p.d.f. of crater diameter as

\[ H(u) = a u^{a-1} / (D_r^a - D_1^a) \quad D_1 < u < D_r. \]

Substituting equations (A1.4), (A1.5) and (A1.6) into the expression for \( E[Y_n] \) given in equation (A1.1) we find

\[ E[Y_n]_{\text{excavation}} = \frac{(-\mu)^n \left[ n(4u^2+1) + 2 \right]}{(n+1)(n+2)} \frac{a}{16R_A^2 (D_r^a - D_1^a)} \frac{D_r^{n+2+a}}{(D_r^a - D_1^a)^{n+2+a}}. \]
Burial

The burial term in equation (Al.1) is now evaluated. Because ejecta blanket morphology for gravity scaling differs from that of strength scaling (see Chapter 2), these two cases are considered separately.

Gravity Scaling

Let us now compute \( G_n^+(u) \), the nth moment of the size of a jump given that a crater of diameter \( u \) has buried a surface point. \( G_n^+(u) \) can be found by multiplying the nth power of the expression for the thickness, \( B(x) \), of ejecta at a distance \( x \) from a crater (equation 2.12) times the probability that the crater forms at a distance \( x \) from the surface point and then integrating over all \( x \). The p.d.f. of the distance is

\[
\frac{2\pi x}{(4\pi R_A^2 - \pi a^2/4)} = \frac{x}{(2R_A^2 - a^2/8)}.
\]

Thus, we can write

\[
G_n^+(u) = \int_{u/2}^{2R_A} \left\{ \left( a e^{c/4\pi} \right)^n \left[ 1 - \left( u/24R_A \right)^{a e/2} \right] \left( u/2x \right)^{a e/2 + 3} x \right\} \frac{x}{2R_A^2 - u^2/8} \, dx
\]

\[
= \left\{ \left( a e^{c/4\pi} \right)^n \left[ 1 - \left( u/24R_A \right)^{a e/2} \right] \left( u/2 \right)^{a e/2 + 3} \right\} \frac{(2R_A)^{e_1} - (u/2)^{e_1}}{(2R_A^2 - u^2/8)^{e_1}}
\]

(A1.8)

where \( e_1 = -n(a e/2+2)+2 \). The probability, \( C^+(u) \), of getting a burial event, given the formation of a crater of diameter \( u \), is
Now, $E[Y^n]_{\text{burial}}$ is found from

$$E[Y^n]_{\text{burial}} = \int_{D_1}^{D_r} G_n^+(u) C^+(u) H(u) \, du. \quad (A1.10)$$

Equations (A1.8), (A1.9) and (A1.6) are substituted into equation (A1.10). By expanding the expression in equation (A1.8), in a binomial series, substituting in the expression for $c$ given in equation (2.4), and performing some straightforward algebra we find

$$E[Y^n]_{\text{burial}} = \frac{[n(a_e(3+4\mu^2)/24)]^n}{[e_1(D_r^a - D_r^a) 8R_A^2]} \sum_{i=0}^{n} \binom{n}{i} (-1)^i \frac{1}{(24R_A^2)} \left\{ \frac{-1a_e/2}{(4R_A^2)^{e_1}(D_r^e - D_r^e)/e_2} \right\} \right)$$

where

$$e_2 = 1a_e/2 + n(a_e/2 + 3) + a$$

and

$$e_3 = 1a_e/2 + n + 2 + a.$$
Strength Scaling

We can evaluate $G_n^+(u)$ in the same way that we found equation (A1.8). But note that, for strength scaling, the p.d.f. for the distance between the surface point and the crater center is

$$2\pi x/(4\pi R_A^2 - \pi x_m^2).$$

Using the expression for $B(x)$ in equation (2.13) we find

$$G_n^+(u) = \int_{x_m}^{2R_A} [B(x)]^n [2x/(4R_A^2 - x_m^2)] dx$$

$$= \left\{(a e c/4\pi)(1-\gamma)(u/2)^3 a e/2 \right\}^n \frac{(2R_A e_1 - x_m e_1)}{(2R_A^2 - x_m^2/2) e_1}.$$  \hspace{1cm} (A1.12)

For strength scaling the fraction, $\gamma$, of escaping ejecta is independent of crater diameter. An expression for $\gamma$ is given in Chapter 2. The minimum ballistic range, $x_m$, is a function of $u$.

For strength scaling a burial event only occurs when a crater forms at a distance greater than $x_m$ from the surface point. Thus, we can write $C^+(u)$ as

$$C^+(u) = (4\pi R_A^2 - x_m^2)/4\pi R_A^2.$$  \hspace{1cm} (A1.13)

Substituting equations (A1.12), (A1.13) and (A1.6) into equation (A1.10) yields
\[ E[Y^n]_{\text{burial}} = \left[ a e^{u(3+4u^2)} (1-\gamma)/12 \right]^n a/[(D_r^a - D_1^a)2R_A e_1] \]

\[ \int_{D_1}^{D_r} \left[(u/2)^n x_m e/2 \left[(2R_A - x_m^e_1) u^{-1} \right] u \right] du. \]

The integral in equation (A1.14) is evaluated numerically.

We now have all the expressions required to find the moments of the jump size. For gravity scaling, \( E[Y^n] \) is found by summing equations (A1.7) and (A1.11). For strength scaling we sum equations (A1.7) and (A1.14). The moments \( m_1 \) or \( m_2 \) can be found by setting \( n=1 \) or \( n=2 \) in the expressions for \( E[Y^n] \).
APPENDIX 2

DETAILS OF SOME DERIVATIONS

We now outline the derivations of the expressions for the moment generating function, $\Omega(s,t)$, of regolith depth at time $t$ (equation 4.5) and the c.d.f., $F_r(x)$, of regolith depth at catastrophic rupture (equation 4.11).

The Moment Generating Function of Regolith Depth at Time $t$

Equation (4.5) involves the p.d.f. of regolith depth at time $t$. The p.d.f. is obtained by differentiating equation (4.4) with respect to $x$. By substituting the resulting expression into the integral in equation (4.5) we can write

$$
\Omega(s,t) = \sqrt{2/\pi m_2 \lambda t} \int_0^\infty \exp[-(x-m_1 \lambda t)^2/2m_2 \lambda t + sx] \, dx
$$

$$
- (m_1/m_2) \int_0^\infty \exp[2m_1 x/m_2 + sx] \text{erfc}[(x+m_1 \lambda t)/\sqrt{2m_2 \lambda t}] \, dx
$$

(A2.1)

The first integral in equation (A2.1) can be found in the table of integrals by Gradshteyn and Ryzhik (1980). We will find that an integral of this kind is also useful in evaluating the second term in equation (A2.1). The integral can be written in general form as
\[ \int_0^\infty \exp[-a(x+b)^2 + cx] \, dx = \sqrt{\pi/4a} \exp[(c/4a - b)c] \exp\left[(2ab-c)/2\sqrt{a}\right] \]  
(A2.2)

where \( a, b \) and \( c \) are constants and \( a > 0 \). Thus, the first term in equation (A2.1) can be written as

\[ \exp[(2m_1 + m_2 s) s\lambda t/2] \, \text{erfc}[-(m_1 + m_2 s)\sqrt{\lambda t/2m_2}] \]  
(A2.3)

The second integral in equation (A2.1) can be integrated by parts to yield

\[ -(m_1/m_2) \int_0^\infty \exp[2m_1 x/m_2 + sx] \, \text{erfc}[(x + m_1 \lambda t)/\sqrt{2m_2 \lambda t}] \, dx \]

\[ = \left[ \frac{m_1}{(2m_1 + m_2 s)} \right] \left\{ \frac{\text{erfc}[m_1 \lambda t/\sqrt{2m_2 \lambda t}]}{\sqrt{2m_2 \lambda t}} \right\} \]  
(A2.4)

\[ - \left( \frac{2}{\sqrt{2m_2 \lambda t}} \right) \int_0^\infty \exp[-(x + m_1 \lambda t)^2/2m_2 \lambda t + (2m_1/m_2 + s)x] \, dx \}

The integral on the r.h.s. of equation (A2.4) can be evaluated by using equation (A2.2). Combining the resulting expression with expression (A2.3) we find

\[ \Omega(s, t) = \{ (m_1 + m_2 s) \exp[(2m_1 + m_2 s)s\lambda t/2] \, \text{erfc}[-(m_1 + m_2 s)\sqrt{\lambda t/2m_2}] \\
+ m_1 \, \text{erfc}[m_1 \lambda t/2m_2] \} / (2m_1 + m_2 s) \]

which is equivalent to equation (4.5) presented in Chapter 4.
The C.D.F. of Regolith Depth at Fragmentation

According to equations (4.9) and (4.11) the c.d.f., \( F_r(x) \), can be written as

\[
F_r(x) = \int_0^\infty F(x, t) \lambda_r e^{-\lambda_r t} dt \tag{A2.5}
\]

If we substitute equation (4.4) into equation (A2.5) and express the function \( \text{erf}(\cdot) \) in terms of \( \text{erfc}(\cdot) \) then we find

\[
F_r(x) = 1 + \left( \frac{\lambda_r}{2} \right) \int_0^\infty \text{erfc}\left[ \frac{x-m_1 \lambda t}{\sqrt{2} m_2 \lambda t} \right] e^{-\lambda_r t} dt \tag{A2.6}
\]

\[
- e^{\frac{2m_1 x}{m_2}} \left( \frac{\lambda_r}{2} \right) \int_0^\infty \text{erfc}\left[ \frac{x+m_1 \lambda t}{\sqrt{2} m_2 \lambda t} \right] e^{-\lambda_r t} dt
\]

If we now make the simple transformation \( s^2 = t \) then the two integrals in equation (A2.6) can be expressed in a form which can be found in Gradshteyn and Ryzhik (1980), i.e.

\[
\int_0^\infty \text{erfc}[as + b/s] e^{(a^2-c)s^2} \, s \, ds = \exp[-2b(a+\sqrt{c})]/2\sqrt{c}(\sqrt{c} + a) \tag{A2.7}
\]

where \( a, b, \) and \( c \) are constants and \( b, c > 0 \). The two integrals in equation (A2.6) can be evaluated, using equation (A2.7), combined and simplified to yield

\[
F_r(x) = 1 - \exp\left[ (m_1 x/m_2) \left( 1 - \text{sign}(m_1) \sqrt{1 + 2m_2\lambda_r/m_1^2 \lambda} \right) \right]
\]

which is the desired result.
LIST OF REFERENCES


Moore, H.J. (1976). Missile impact craters (White Sands Missile Range, New Mexico) and applications to lunar research. USGS Professional Paper 812-B.


