THE CODING-SPREADING TRADEOFF PROBLEM IN
FINITE-SIZED SYNCHRONOUS DS-CDMA SYSTEMS

by
Zuqiang Tang

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ABSTRACT

This dissertation provides a comprehensive analysis of the coding-spreading tradeoff problem in finite-sized synchronous direct-sequence code-division multiple-access (DS-CDMA) systems. In contrast to the large system which has a large number of users, the finite-sized system refers to a system with a small number of users. Much work has been performed in the past on the analysis of the spectral efficiency of synchronous DS-CDMA systems and the associated coding-spreading tradeoff problem. However, most of the analysis is based on the large-system assumptions. In this dissertation, we focused on finite-sized systems with the help of numerical methods and Monte-Carlo simulations.

Binary-input achievable information rates for finite-sized synchronous DS-CDMA systems with different detection/decoding schemes on additive white Gaussian noise (AWGN) channel are numerically calculated for various coding/spreading apportionments. We use these results to determine the existence and value of an optimal code rate for a number of different multiuser receivers, where optimality is in the sense of minimizing the SNR required for reliable multiuser communication. Our results are consistent with the well-known fact that all coding (no spreading) is optimal for the maximum $a$ posteriori (MAP) receiver.
Simulations of the low-density parity-check (LDPC)-coded synchronous DS-CDMA systems with iterative multiuser detection/decoding (MUDD) and minimum mean-square error (MMSE) multiuser detection/single-user decoding are also presented to show that the binary-input capacities can be closely approached with practical schemes. The coding-spreading tradeoff is examined using these LDPC code simulation results, where agreement with the information-theoretic results is demonstrated.

We extend our work to the DS-CDMA systems on two idealized Rayleigh flat-fading channels: the chip-level flat-fading (CLFF) and the (code) symbol-level flat-fading (SLFF). These models represent ideal fast fading and slow fading channels, respectively. Both information-theoretic results and LDPC code simulation results are presented to show the effects of channel fading on system performance and the coding-spreading tradeoff. It is shown that fast fading can be beneficial to system performance under the condition of perfect channel state information (CSI) at receiver, but slow fading is very harmful. Slow fading also increases the importance of coding greatly, compared to the AWGN and fast fading.

Finally, we present some comparisons with large-system results on AWGN and CLFF channels, which show both consistencies and discrepancies. These results show that it is necessary to perform analyses on finite-sized systems as we have done.
CHAPTER 1
INTRODUCTIONS

In this chapter, we present the fundamentals of direct-sequence code-division multiple-access (DS-CDMA), capacity/spectral efficiency, and low-density parity-check (LDPC) codes. A brief review of the coding-spreading tradeoff problem in DS-CDMA systems is also included. Baseband signals are assumed throughout this dissertation.

1.1 Channel Models

The role of channel models in the analysis and design of communication systems can never be underestimated. One of the simplest and most widely adopted channel models is the additive white Gaussian noise (AWGN) channel. A single-user AWGN channel model in the real domain is simply

\[ y = x + w \]  \hspace{1cm} (1.1)

where \( y \) is the received signal, \( x \) is the transmitted signal, and \( w \) is the additive white Gaussian channel noise.

In a practical communication system, especially in a wireless/mobile communication system, the channel is time-varying and complicated. The variation of
the channel makes it necessary to estimate the channel state information (CSI) at the receiver, which may or may not be accessible to the transmitter. Due to the complexity of the estimation of CSI, theoretically and practically, there are various types of CSI that could be available to the receiver and/or the transmitter, including completely unknown CSI, partially known CSI, and full knowledge of CSI. In our work, we will assume that CSI is completely known to the receiver, but no CSI is available to the transmitter.

Another important property of the real channel is its complexity. In wireless communications, the radio transmission could be severely affected by buildings, mountains, and trees, etc. Unlike most wired channels which are usually stationary and predictable, wireless channels are random. Fortunately, based on a tremendous amount of measurement results, it is possible to construct various channel models for different wireless communication scenarios. These models with their random parameters, which could be estimated at the receiver with varying degrees of accuracy, can be used to analyze the performance of wireless communication systems.

Typically, there are large-scale fading and small-scale fading effects for terrestrial wireless communication channels. In either fading case, many fading statistical models can be assumed according to different applications. In our work, besides the AWGN channel, we will only consider the Rayleigh flat-fading channel, which corresponds to no line-of-sight (dominant and stationary) propagation path and no delay spread (constant gain over the whole communication bandwidth).
A single-user Rayleigh flat-fading channel model in the complex domain can be described as

\[ y_c = \alpha_c x_c + w_c \]  

(1.2)

where \( y_c, x_c, \) and \( w_c \) are the complex received signal, complex transmitted signal, and the complex channel AWGN. \( \alpha_c \) is a circularly symmetric complex Gaussian random variable with mean 0 and variance \( \sigma^2 \), i.e. \( \alpha_c \sim \mathcal{CN}(0, \sigma^2) \), which represents the effect of flat-fading. The effect of \( \alpha_c \) is two-fold: amplitude and phase, i.e. \( \alpha_c = r e^{j\theta} \). For Rayleigh fading, \( r \) has the Rayleigh probability density function (p.d.f.) [1]

\[
p(r) = \begin{cases} 
\frac{2r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} & (0 \leq r < +\infty) \\
0 & r < 0
\end{cases}
\]  

(1.3)

and \( \theta \) has uniform distribution over [0, 2\pi).

1.2 Code-Division Multiple-Access Systems

Though it was back in 1896 when Guglielmo Marconi was granted the world’s first patent for a system of wireless telegraphy, and the system was demonstrated successfully in London, on Salisbury Plain, the wireless communication era could not be born without the help of the technologies and concepts developed during the 1960’s and 1970’s. These technologies and concepts, from large-scale circuit integration, digital and RF circuit fabrication, to the cellular concepts, fostered
an ever-growing wireless communication industry. The ultimate goal is to provide communication services from any person to any person in any place at any time without any delay in any form through any medium by using one pocket-sized unit at minimum cost with acceptable quality and security through using a personal telecommunication reference number [2].

Much has been done, yet much more needs to be done. One of the key problems in telecommunications, whether wireless or wired, is the limited spectrum. This issue becomes more critical in wireless/cellular communications because many users, such as the mobile users in a single cell, must share the same spectrum simultaneously without creating unmanageable interference with one another. Thus, efficient use of the transmitting resource using a so-called multiple access technique, becomes an important topic in the design of wireless communication systems. Depending on the different application, there are many multiple access techniques which have been proposed and implemented. These include time-division multiple-access (TDMA), frequency-division multiple-access (FDMA), code-division multiple-access (CDMA), ALOHA, and carrier-sense multiple-access (CSMA). These techniques can usually be used jointly in a wireless system to exploit their advantages. In these techniques, CDMA is an important multiple access technique used in cellular systems, like the 2nd-generation IS-95 system and the recent 3rd-generation wide-band CDMA systems [3], [4].
Unlike TDMA and FDMA, CDMA distinguishes users in the “code domain”. Each user is assigned a unique code, by which certain properties of the user’s information-bearing signal are affected. The same code is then used by the receiver to extract this user’s information from the combination of all users’ signals. Though there are many varieties of CDMA, we will focus only on DS-CDMA system. Hence, throughout this dissertation, CDMA will refer to DS-CDMA.

1.2.1 Synchronous CDMA systems

Ignoring coding and modulation for now, the baseband model for a $K$-user BPSK-modulated synchronous CDMA system on AWGN channel is shown in Fig. 1.1. At the transmitter, each user’s binary input, $x_k$ for user $k$, is multiplied by a binary signature code $s_k$ of length $N$ and the transmitted amplitude $a_k$ before it is sent across the AWGN channel. This binary code $s_k$ is called a spreading sequence because it increases the signal bandwidth by $N$. All of the spreading sequences have the same length $N$.

So, within the time spanned by the spreading sequence, the discrete-time received signal vector is

$$
\mathbf{r} = \mathbf{SAx} + \mathbf{w}
$$

where $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K]$ includes all of the (normalized) spreading sequences of the $K$ users; $\mathbf{A} = \text{diag}\{a_1, a_2, \ldots, a_K\}$ represents the transmitted amplitudes of the $K$ users; $\mathbf{x} = [x_1, x_2, \ldots, x_K]^T$ is the vector of the (binary) inputs of the $K$
Figure 1.1: Synchronous CDMA model on AWGN channel with $K$ users.

users; $\mathbf{w} \sim N(0, \sigma^2 \mathbf{I})$ is the i.i.d. additive channel Gaussian noise vector. This synchronous CDMA system model will be used in our work.

Theoretically, an asynchronous CDMA system can be viewed as a special synchronous CDMA system [5]. So, though we will not discuss asynchronous systems in this dissertation, our work can be generalized to asynchronous CDMA systems.

In our work, we will also consider the synchronous CDMA systems on two special cases of the flat-fading channels, the CLFF and SLFF channels, which can be treated equivalently as the systems on the AWGN channel. We will not consider
CDMA systems on more complicated channels such as frequency-selective fading channels.

1.2.2 Multiuser Detectors

Given the above model of the received CDMA signal, we now turn to the receiver (or detector) for the CDMA signal. Only receivers for the synchronous CDMA system on AWGN channel are discussed here. These receivers can be generalized to other CDMA systems.

Obviously, if all users’ spreading sequences are orthogonal, the optimal receiver for one user will be a conventional single-user matched-filter (SUMF) detector that is just a correlator matched to the user’s spreading sequence. However, in practical CDMA systems, the perfect orthogonality of the spreading sequences cannot be guaranteed and, furthermore, nonorthogonality is often intentionally introduced in the design. This nonorthogonality will result in interference from other users and make the conventionally optimal SUMF detector suffer from the near-far problem [5]. The interference from other users is called multiuser interference (MUI) or multiple-access interference (MAI) which imposes a major limitation on the performance of a CDMA system. Therefore, all or some of the users have to be detected jointly to overcome the MAI and, hence, multiuser detectors (MUDs) have been proposed [5].
What is the optimal detector for the CDMA signal? Feeding the received signal \( r \), as in (1.4), through a matched-filter bank \( S^T \) which consists of \( K \) linear filters matched to each of the \( K \) users’ spreading sequences. Then the output of the matched-filter bank is

\[
y = RAx + n
\]  

(1.5)

where \( R = S^T S \) is the correlation matrix of the spreading sequences of the \( K \) users; \( n = S^T w \) is the filtered output of the channel noise, \( n \sim N(0, \sigma^2 R) \); \( A \) and \( x \) are as in (1.4).

It can be shown that \( y \) is a sufficient statistic for the detection of \( x \) [5]. Therefore, we can design the optimal MUD [5], in the sense of maximum likelihood, as,

\[
\hat{x} = \arg\max_{x \in [-1, +1]^K} \left\{ 2x^T Ay - x^T ARAx \right\}
\]  

(1.6)

For asynchronous CDMA systems, a similar optimal MUD can be realized efficiently with the Viterbi algorithm [5].

The complexity of the optimal MUD is exponential in the number of users, which is too large to be practical. Thus, various suboptimal MUDs have been proposed which have lower complexity, at the expense of a penalty on performance. Among them, the linear MUDs are very attractive. The most well-known linear MUDs are the MMSE MUD and the decorrelating MUD [5]. Analogous to the equalization problem of intersymbol interference (ISI) signals, the MMSE MUD minimizes the
mean-square error of the estimates much like a MMSE equalizer. The decorrelating MUD aims to completely remove the MAI much like a zero-forcing equalizer.

Consider the sufficient statistics (1.5), the output of the MMSE MUD is

$$\hat{x}_{MMSE} = My$$

(1.7)

where $M$ is a $K \times K$ real matrix representing the linear MMSE MUD and satisfying

$$M = \arg\min_{M'} E[\|M'y - Ax\|^2]$$

(1.8)

It can be shown [5] that

$$M = (R + \sigma^2 A^{-2})^{-1}$$

(1.9)

where $\sigma^2$ is the channel noise variance.

The decorrelating MUD simply multiplies $y$ by $R^{-1}$, assuming $R$ is invertible, yielding the MUD output

$$\hat{x}_{dec} = R^{-1}y = Ax + R^{-1}n.$$  

(1.10)

Note that the decorrelating MUD completely removes the MAI and its performance approaches the performance of the MMSE MUD as $\sigma^2 \rightarrow 0$. However, as in the case with the zero-forcing filter for ISI channels, the decorrelating MUD suffers from noise enhancement.

There are also many other MUDs such as the successive interference cancellation (SIC) MUD, the parallel interference cancellation (PIC) MUD, and blind MUDs ([5] and references therein). We will not discuss these alternative MUDs in this work.
For the synchronous CDMA systems on a flat-fading channel, the MUD structures are similar if the receivers have full CSI, that is, full knowledge of all fading coefficients. We will provide details in Chapter 3.

1.3 Channel Capacity and Achievable Information Rates

In previous sections, we gave brief descriptions of the channels and the synchronous CDMA system model that we consider in this dissertation. It is natural to ask how many users a CDMA system can support, given a specific channel. More precisely, we want to know the information-theoretic capacity or spectral efficiency of a CDMA system.

Basically, for a single-input single-output memoryless system or channel with input $X$ and output $Y$, the capacity is defined as [6]

$$ C = \max_{p(X)} \{H(Y) - H(Y|X)\} \tag{1.11} $$

where $H(Y)$ is the entropy of the output $Y$, $H(Y|X)$ is the conditional entropy of the output $Y$ given the input $X$, and the maximization is over all possible p.d.f.'s of the input $X$.

If we constrain the input $X$ to be equally probably and binary, we will have the constrained capacity (or mutual information),

$$ C_{bi} = H(Y) - H(Y|X) \tag{1.12} $$
Generally, for a multi-input multi-output (MIMO) system such as a CDMA system, we may be interested in the capacity for all users (sum capacity) or for each one of the users (single-user capacity). If the sum capacity is our goal, the CDMA system (1.5), can be treated as a MIMO system and the sum capacity is (assuming optimal processing at the receiver)

\[ C_{\text{sum}} = \max_{p(x)} \{ H(y) - H(y|x) \} \]  

(1.13)

If we want to know the single-user capacity, say for user 1, the capacity formula will be

\[ C_{\text{user } 1} = \max_{p(x)} \{ H(y) - H(y|x_1) \} \]  

(1.14)

where \( x_1 \) is the first entry of \( x \).

Similarly, as for the assumption leading to (1.12), we have formulae of the constrained capacities for CDMA systems. With these formulae, we will be able to obtain the capacities of the CDMA systems with different constraints and assumptions. Moreover, these capacities set upper bounds on the achievable information rates.

1.4 Low-Density Parity-Check Codes

The capacities given in the last section are the maximal information rates that can be arbitrarily reliably transmitted through the CDMA systems. To achieve these limits, special capacity-approaching coding schemes are required. In this
dissertation, we turned to low-density parity-check (LDPC) codes which have been shown to approach the capacity of several channels [7], [8]. We review LDPC codes in the following subsection.

1.4.1 Description of LDPC Codes

An LDPC code is a type of linear block code which is usually defined by its parity-check matrix $H$ which is sparse (“low density”). In our work, we will only consider binary LDPC codes so that all of the entries in $H$ are either ‘0’ or ‘1’. For an $(n, k)$ binary LDPC code, its parity-check matrix $H$ is a binary matrix with $n$ columns and $m = n - k$ rows. Any valid codeword $c$ is a vector of size $1 \times n$ which satisfies the parity-check equation (with addition and multiplication in $GF(2)$)

$$cH^T = 0$$

(1.15)

The original class of LDPC codes [9] are called Gallager codes or regular LDPC codes, the latter name referring to the uniform number of nonzero entries in each column or row of their parity-check matrices. It has been found recently that irregular LDPC codes, which have a varying number of nonzero entries in each column or row, have the potential for better performance than their regular counterparts [10].
1.4.2 Encoding of LDPC Codes

As with other linear block codes, the encoder of an \((n,k)\) LDPC code can be realized by a generating matrix \(G\) with \(n\) columns and \(k\) rows. The \(G\) matrix satisfies (in GF(2))

\[
GH^T = 0 \tag{1.16}
\]

and can be derived from \(H\) using Gauss-Jordan elimination. Mathematically, the encoder outputs a codeword \(c\) computed via

\[
c = uG \tag{1.17}
\]

with \(u\) as the input information vector of size \(1 \times k\). The matrix \(G\) is normally “high-density” - with a large portion of nonzero entries (‘1’ for binary codes). This makes the complexity of encoding very high. Some specially designed LDPC codes, such as the finite-geometry codes [11] and the eIRA codes [12], enjoy the property of low-complexity encoding.

1.4.3 Decoding of LDPC Codes

The decoding of LDPC codes is a critical part in the invention of LDPC codes. Without the existence of the efficient iterative decoding algorithm, LDPC codes would not be considered useful and would not receive the great level of attention they are currently receiving in industry and academia.
A standard decoding algorithm of LDPC codes is the iterative message-passing algorithm which can be dated back to Gallager [9]. To understand the message-passing algorithm, a bipartite graph corresponding to the parity-check matrix $H$ is helpful in the sense that a trellis is helpful in understanding the Viterbi algorithm. This bipartite graph is also known as a Tanner graph [13]. For a parity-check matrix $H$ with $n$ columns and $m$ rows, the corresponding Tanner graph has two kinds of nodes: $n$ “variable” nodes and $m$ “check” nodes. The $n$ “variable” nodes correspond to the $n$ code bits (columns of $H$) and the $m$ “check” nodes correspond to the $m$ parity-check equations (rows of $H$) specified by $H$. Edges in the graph connect variable node to a check node. There is an edge connecting a variable node and a check node if and only if the corresponding code bit appears in the parity-check equation corresponding to the check node. Fig. 1.2 gives an example of Tanner graph for the (7,4) Hamming code.

Given the received signal statistics, the message-passing algorithm iteratively estimates the a posteriori distributions of the variable nodes which correspond to the code bits corrupted by the channel. The decoding process is best viewed on the code’s Tanner graph.

For every iteration, each variable node $v_i$ sends updating message $q_{i,j}$ to each check node $f_j$ which is adjacent to $v_i$. The updating message $q_{i,j}$ approximates $v_i$’s “belief”, given messages received from all other check nodes adjacent to $v_i$ and messages from the channel. The “belief” concerns whether or not $v_i$ is a 0 or a
\[ H = \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 1
\end{bmatrix} \]

Figure 1.2: The \( H \) matrix and Tanner graph of the (7,4) Hamming code.
1. Also, each check node \( f_j \) sends a message \( r_{i,j} \) to each adjacent variable node \( v_i \), given messages received from all other variable nodes adjacent to \( f_j \). The updating message \( r_{i,j} \) approximates the probability of check \( f_j \) being satisfied, conditioned on \( v_i \)'s value. The distributions of all variable nodes are assumed to be independent.

After each iteration, a tentative decision is made on every variable node based on the probability distribution information residing at that node. The message-passing decoder stops if the tentative decision \( \hat{c} \) satisfies the parity-check equation, i.e., \( \hat{c}H^T = 0 \), or if the maximum number of iterations is reached. Even though the convergence to the true \( a \ posteriori \) distributions is not guaranteed, this iterative algorithm has been shown to work very well, i.e., near maximum likelihood, for most LDPC codes.

For binary LDPC codes, the message-passing algorithm can be performed more efficiently and numerically stably in the “log domain”, where the probability pairs are equivalently characterized by the log-likelihood ratios (LLR’s). Considering the decoding of a binary LDPC code on the binary-input AWGN channel as an example, the log-domain algorithm consists of the following steps: (a detailed presentation can be found in [14])

1. **Initialization.**

Suppose the binary code bit \( c_i \in \{0, 1\} \) is mapped into the channel symbol \( x_i = (1 - 2c_i) \in \{+1, -1\} \). The sampled channel output \( y_i \) has the conditional
p.d.f.

\[ p_{i}^{ch}(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left( -\frac{(y_i - x_i)^2}{2\sigma^2} \right) . \]  \hspace{1cm} (1.18)

where \( \sigma^2 \) is the variance of the i.i.d. Gaussian noise sample. The \( y_i \)'s are statistically independent since the noise samples are statistically independent.

Assuming the \textit{a priori} probabilities \( P_r(c_i = 0) = P_r(c_i = 1) = \frac{1}{2} \), the message observed from the channel can be expressed as

\[ LLR_{ch}(x_i) = \log \frac{p_{i}^{ch}(+1)}{p_{i}^{ch}(-1)} = \frac{2y_i}{\sigma^2} . \] \hspace{1cm} (1.19)

2. The \( p^{th} \) Iteration.

(a) Updating messages from variable nodes to check nodes

\[ LLR(q_{ij}) = \sum_{j' \in \text{col}[i] \setminus \{j\}} LLR(r_{ij'}) + LLR_{ch}(x_i) , \] \hspace{1cm} (1.20)

for all \( i, j \) for which the \((j, i)^{th}\) element of \( \mathbf{H} \), \( h_{ji} = 1 \). \( \text{col}[i] \) denotes a set of row indices of \( \mathbf{H} \), \( \{l|h_{li} = 1\} \). In the first iteration, the \( LLR(r_{ij'}) \)'s are initialized to be zero for any \( i, j' \).

(b) Updating messages from check nodes to variable nodes

\[ LLR(r_{ij}) = \Phi\left( \sum_{i' \in \text{row}[j]\setminus\{i\}} \Phi(|LLR(q_{i'j})|) \right) \cdot \prod_{i' \in \text{row}[j]\setminus\{i\}} \text{sgn}(LLR(q_{i'j})) , \] \hspace{1cm} (1.21)
for all $i, j$ for which the $(j, i)^{th}$ element of $H$, $h_{ji} = 1$. $row[j]$ denotes a set of column indices of $H$, $\{l|h_{jl} = 1\}$. The function $\Phi$ is defined as

$$\Phi(x) = -\log \left( \tanh \left( \frac{1}{2}x \right) \right) = \log \left( \frac{e^x + 1}{e^x - 1} \right)$$  \hspace{1cm} (1.22)

(c) Update the a posteriori LLR’s of variable nodes

$$LLR_{\text{posterior}}(x_i) = \sum_{j' \in \text{col}[i]} LLR(r_{ij'}) + LLR_{\text{channel}}(x_i).$$  \hspace{1cm} (1.23)

3. Decision.

$$\hat{c}_i = 1 - \text{sgn}(LLR_{\text{posterior}}(x_i)), \text{ for } i = 1, 2, ..., n$$  \hspace{1cm} (1.24)

- Repeat iterative process until the stopping criterion is satisfied, i.e.,

$$\hat{c}H^T = 0.$$

- A failure is declared if the maximum number of iterations is reached while $\hat{c}$ is not a valid codeword.

1.4.4 eIRA Codes

In our work, we will choose the extended irregular repeat-accumulate (eIRA) class of LDPC codes [12] in our simulation-based studies in subsequent chapters. The eIRA code is a specially designed irregular LDPC code with efficient encoding and low error-rate floors [12]. The $H$ matrix of an eIRA code has two submatrices,

$$H = [ H_1 \ H_2 ],$$  \hspace{1cm} (1.25)
where the matrix $H_2$ has the structure

$$H_2 = \begin{pmatrix}
1 & & & \\
1 & 1 & & \\
& 1 & 1 & \\
& & \cdots & \\
& & & 1 \\
& & & 1 & 1
\end{pmatrix}. \quad (1.26)$$

The corresponding $G$ matrix is therefore

$$G = \begin{bmatrix}
I & H_1^T H_2^{-T}
\end{bmatrix} \quad (1.27)$$

and it can be easily shown that

$$H_2^{-T} = \begin{pmatrix}
1 & 1 & \ldots & \ldots & \ldots & 1 \\
1 & 1 & \ldots & \ldots & \ldots & 1 \\
\ddots & \ddots & \ddots & & & \\
& \ddots & \ddots & \ddots & & \\
& & \ddots & \ddots & \ddots & \\
& & & 1 & 1 & \\
& & & & 1
\end{pmatrix}. \quad (1.28)$$

Obviously, multiplication of this upper triangular matrix as in (1.27) can be realized by a differential encoder $\frac{1}{1\oplus D}$. Considering additionally the low density characteristic of $H_1^T$, a low-complexity encoder with generating matrix $G$ can be depicted as Fig. 1.3.
In addition to the efficient encoding property of eIRA codes, it has also been shown that the eIRA codes have comparably lower error-rate floors than competing random or structured LDPC codes in the moderate-length and high-rate region [12].

With the foregoing sections as background, we are now able to present the problems addressed by this dissertation in the next section.

1.5 The Coding-Spreading Tradeoff Problem in CDMA Systems

In a typical digital communication system, at the transmitter there exist basic elements: a source encoder, a channel encoder and a modulator. For a CDMA system, there is additionally a spreading element. If we do not consider the source encoder and the modulator, the effect of the channel encoder and the spreading element expands over the signal bandwidth. Thus, it is natural to raise a question about how much bandwidth should be allocated to spreading and to channel coding, since the total available bandwidth is always limited. This is the coding-spreading...
tradeoff problem to be studied in this dissertation, and we consider the AWGN channel and the Rayleigh fading channel.

It is very common to treat spreading and coding as two similar parts in the overall bandwidth expansion [15]. For the conventional matched filter receiver [5], this approach is appropriate. However, they are really very different kinds of bandwidth expansion schemes [16][17] whose different effects can be more readily exploited by more advanced multiuser receivers [5].

Let us consider the space of real-valued functions that span $T$ seconds and are approximately band-limited to a baseband bandwidth of $W$ Hz. The value

$$D_F = 2WT$$

(1.29)

is defined as the Fourier dimension of the signal space and $W$ is therefore strictly called Fourier bandwidth.

A complementary concept is the Shannon dimension. Let the set $\mathcal{S}$ consist of $M$ real-valued functions $s_1, s_2, \ldots, s_M$, which have a support of $T$ seconds and are approximately band-limited to a baseband (Fourier) bandwidth of $W$ Hz. The dimension of $\text{span}(\mathcal{S})$, denoted $D_S$, is called Shannon dimension and the value

$$B = \frac{D_S}{2T}$$

(1.30)

is then called Shannon bandwidth of the set $\mathcal{S}$. Clearly, $D_S \leq D_F$ and $B \leq W$.

So what is the difference between these two bandwidths? Consider a system using signal set $\mathcal{S}$ to send information over an AWGN channel with bandwidth $W$. 
Let $B$ be the Shannon bandwidth of $S$. Then the maximum achievable information rate of the system is \cite{16}\cite{17}

$$R = B \log \left( 1 + \frac{P}{N_0 B} \right) \leq W \log \left( 1 + \frac{P}{N_0 W} \right) = C$$  \hspace{1cm} (1.31)

where $P$ is the signal power, $N_0/2$ is the double-sided white Gaussian noise spectral density, and $C$ is the channel capacity. We see that the channel capacity $C$, which is determined by the Fourier bandwidth $W$, acts as the upper bound of the maximum achievable information rate $R$ which is determined by the Shannon bandwidth $B$. In other words, the Shannon bandwidth is the amount of bandwidth that the system needs and the Fourier bandwidth is the amount of bandwidth the system uses \cite{16}.

Given the foregoing formulation, we are able to say that spreading is a very special kind of coding that increases the Fourier bandwidth of the signals, but keeps the Shannon bandwidth unchanged. Moreover, spreading can be seen as a unitary linear mapping which does not change the distances between the signals and thus does not provide coding gain \cite{17}.

Thus, spreading is really a trivial coding. Without surprise, for the CDMA system with optimal multiuser processing, “all coding” (no spreading) will maximize the sum capacity of the system. However, it is still interesting to consider the coding-spreading tradeoff problem in CDMA systems for optimal and suboptimal receivers. Firstly, considering the complexity of optimal processing, tradeoffs with different MUDs are necessary. Secondly, all or some users’ spreading sequences are
usually known to the receiver, this will help to increase the performance gain due to spreading. Also considering the complexity and performance of practical coding schemes and other practical issues, there is still a need to perform the tradeoff between coding and spreading.

1.5.1 Large-System Analysis

In order to perform the coding-spreading tradeoff, researchers have generally adopted as the performance criterion the spectral efficiency of the entire CDMA system, involving all users. There are at least two ways to connect the spectral efficiency problem to the tradeoff problem. One is to find the maximum spectral efficiency with respect to different tradeoffs, under the condition of fixed $E_b/N_0$. The other one is to find the minimum required $E_b/N_0$ with respect to different tradeoffs, under the condition of fixed spectral efficiency. In this section, we give some important results on the spectral efficiencies of large CDMA systems, by which we mean a large number of users. This analysis falls into the first category in which $E_b/N_0$ is fixed.

In [18], the spectral efficiencies of large CDMA systems with random spreading are analyzed. The large system means a CDMA system with the number of users $K$ and the spreading sequence length $N$ going to infinity while maintaining a constant ratio $K/N$. Since the spreading sequences of all users are random, binary or spherical (Gaussian), the spectral efficiency is a random variable. However, with
the large-system assumptions, the distribution of the eigenvalues of the correlation
matrix \( R \) converges to a fixed function as \( K \) and \( N \) go to infinity. This property
facilitates the analysis of large random-spread systems.

The main results of [18] for an AWGN channel are as follows:

1. If optimal processing is employed, the spectral efficiency in bits/chip of a
large CDMA system on an AWGN channel with random spreading is

\[
C_{opt}^* = \frac{\beta}{2} \log_2(1 + SNR - \frac{1}{4} \mathcal{F}(SNR, \beta)) + \frac{1}{2} \log_2(1 + SNR \cdot \beta - \frac{1}{4} \mathcal{F}(SNR, \beta)) - \frac{\log_2 e}{8SNR} \mathcal{F}(SNR, \beta)
\]

where \( \beta = K/N \), \( SNR = 2(E_b/N_0)C/\beta \) with \( C = C_{opt}^* \) for (1.32), and

\[
\mathcal{F}(x, z) \triangleq \left( \sqrt{x(1 + \sqrt{z})^2 + 1 - \sqrt{x(1 - \sqrt{z})^2 + 1}} \right)^2
\]

2. If the single-user matched-filter detector [5] is employed, the spectral efficiency
in bits/chip is

\[
C_{MF}^* = \frac{\beta}{2} \log_2 \left( 1 + \frac{SNR}{1 + SNR \cdot \beta} \right)
\]

3. If the decorrelating MUD [5] is employed, the spectral efficiency in bits/chip is

\[
C_{dec}^* = \frac{\beta}{2} \log_2 (1 + SNR(1 - \beta))
\]

4. If the MMSE MUD [5] is employed, the spectral efficiency in bits/chip is

\[
C_{MMSE}^* = \frac{\beta}{2} \log_2(1 + SNR - \frac{1}{4} \mathcal{F}(SNR, \beta))
\]
The results above are valid for both binary and spherical (Gaussian) random spreading. We reproduce Fig. 1 in [18] as Fig. 1.4 to give some concreteness to the formulae above. We will compare these large-system results with our finite-sized system results in chapter 4.

Figure 1.4: Large-$K$ spectral efficiencies for $E_b/N_0 = 10$ dB. No spreading; orthogonal signatures; random signatures: optimal, matched-filter, decorrelator, MMSE.

The same results can be derived by using the replica method in statistical mechanics [19]. Moreover, this method can provide more details on the behavior of
systems and the spectral efficiencies with a binary-input constraint which are not available in [18]. Therefore, this method is worthy of further research.

If we consider time-varying frequency-flat fading (having time dynamics on the order of a code symbol duration), the formulae are more complex [20] and so we will not discuss these here.

1.5.2 Other Work on the Coding-Spreading Tradeoff

Beside the large-system results presented in the last subsection, there exist other interesting results in the literature.

In [17], the coding-spreading tradeoff problem is well posed in terms of the coding/spreading bandwidth expansion factor $Q$. The synchronous CDMA system on the AWGN channel with matched-filter and linear MMSE front-ends are analyzed in detail. Spectral efficiencies are calculated for finite-sized systems and large systems (asymptotic). No specific coding scheme is considered.

In [21] and [22], the coding-spreading tradeoff problem is considered in synchronous CDMA systems on AWGN channel and Rayleigh flat-fading channel which is similar to the (code) symbol-level fading channel in Chapter 3. Specially chosen rate-compatible punctured turbo/convolutional (RCPT/RCPC) codes are used with the matched-filter and linear MMSE front-ends. Large-system results in [18] and [23] are adopted to evaluate the system performance with RCPT/RCPC codes.
Similar analysis using the large-system results and LDPC codes are considered in [24].

In [25] and [26], serial turbo coded synchronous and asynchronous CDMA systems with iterative multiuser detection and decoding are considered. The coding-spreading tradeoff problems in both systems are analyzed via density evolution technique and simulations. Similar analysis with density evolution technique on finite-sized turbo CDMA systems using LDPC codes are considered in [24].

In our work, we will focus on the finite-sized CDMA systems without using large-system assumptions and will use LDPC codes to approach the system capacities. Both information-theoretic analysis and simulation technique are adopted. Binary random spreading is assumed.
CHAPTER 2

THE CODING-SPREADING TRADEOFF PROBLEM IN FINITE-SIZED SYNCHRONOUS CDMA SYSTEMS ON AWGN CHANNEL

In this chapter, we compute achievable information rates for finite-sized synchronous CDMA systems and study the ability of LDPC codes to approach these rate limits. The associated coding-spreading tradeoff problem is then considered using these results. Assuming binary random spreading sequences, the computed achievable rates are averaged over ensembles of spreading sequences. Unlike most prior work which analyzed the spectral efficiency of large CDMA systems under Gaussianity assumptions (channel inputs and/or MAI), we make no such assumptions. In order to display the coding-spreading tradeoff, we plot the minimum required $E_b/N_0$ for reliable transmission as a function of information rate. It is shown that the coding-spreading tradeoff favors all coding (no spreading) when the optimal joint multiuser detector/decoder (MUDD) is employed, whereas for systems with suboptimal MUDs and single-user decoding, there generally exists an optimal balance between coding and spreading. We also provide simulation results on the performance of LDPC-coded synchronous CDMA systems which approach the information-theoretic limits we have computed.
2.1 Introduction

Direct-sequence CDMA systems have been extensively studied in the last two decades. Their ability to support more users than TDMA and FDMA mobile systems has been widely acknowledged. Hence, CDMA is considered an enabling technology in present and future wireless systems.

In recent years, some important work has been reported on the information-theoretic capacities of CDMA systems. References [18], [20], [27], [28], [29] provide analyses on the spectral efficiencies of large (i.e., large number of users) CDMA systems on AWGN and flat-fading channels. Reference [19] gives an insightful new approach to the analysis of the capacity of large CDMA systems via the replica method in statistical mechanics. However, the assumption underlying large CDMA systems, namely, the Gaussianity of MAI, does not generally hold in real systems. Other approaches include the assumption of Gaussian channel inputs in capacity problems and the adoption of spherical spreading sequences [30], [31], which would facilitate the analysis as well.

In another area, recent developments in channel coding have yielded codes capable of near-capacity operation on many channels, including the binary symmetric channel (BSC), the binary erasure channel (BEC), the AWGN channel, and fading channels. These capacity-approaching codes include turbo, turbo-like, and LDPC codes [32], [33], [34]. It has been more difficult to determine whether or not these
codes are capable of near-capacity performance on binary-input interference channels such as MAI channels in part because it is difficult to compute the capacity of these channels (except for some special cases and under different assumptions which may not be realistic).

It is natural to ask how the capacity of binary-input CDMA (BI-CDMA) systems with random spreading may be achieved with channel coding. A number of researchers have presented analytical and simulation results in the literature to demonstrate the efficacy of channel codes on CDMA channels [35], [36], [37], [25] and the possibility of their approaching the information-theoretic capacity of CDMA systems. In view of the constraints on bandwidths in communication systems, it is also natural to consider the tradeoff in bandwidth expansion due to spreading and that due to coding. References [17], [21], [22] have presented results on this tradeoff problem for large CDMA systems. However, because of the large CDMA system assumption, these works also adopt the Gaussian-input and Gaussian-MAI assumptions to certain extents.

In this chapter, we consider finite-sized CDMA systems with binary random spreading where the Gaussian assumptions do not hold and we constrain the inputs to be binary random variables as is true in actual systems. The achievable information rates we numerically compute may be considered to be binary-input capacity or constrained capacity. Similar numerical methods to those in [38] are adopted to compute the information rates. Lastly, we consider the capacity to
be dependent on the receiver and so we consider a number of different receivers (MUDs), from the optimal multi-user receiver to the decorrelating receiver.

Two LDPC-coded synchronous CDMA systems, with iterative multiuser MUDD and MMSE MUD, are considered. These two implementable structures are used to approach the theoretic capacities we obtained numerically. Irregular LDPC codes, specifically, eIRA codes, are adopted in the two systems. Simulation results show that, in both cases, the systems with LDPC codes have performance close to their capacities.

The rest of this chapter is outlined as follows. Section 2.2 presents the synchronous CDMA model adopted in this work. Section 2.3 provides the analysis of the binary-input capacity of CDMA systems with different MUDs. Section 2.4 provides numerical results for the achievable information rates of finite-sized CDMA systems. Section 2.5 describes the structures of two LDPC-coded synchronous CDMA systems with iterative MUDD and MMSE MUD. Section 2.6 gives simulation results of the LDPC-coded synchronous CDMA systems. Section 2.7 contains conclusions.

2.2 Synchronous CDMA System Model

The baseband model for a $K$-user BPSK-modulated synchronous CDMA system as described in Chapter 1 is adopted. The received CDMA signal can be written
as

\[ r = \sum_{k=1}^{K} a_k x_k s_k + w \]  

(2.1)

where \( a_k, x_k, \) and \( s_k \) are, respectively, the amplitude, the binary code bit, and the binary normalized (\( \| s_k \| = 1 \)) spreading sequence of the \( k \)th user; \( w \) is the channel AWGN vector with mean \( 0 \) and covariance \( \sigma^2 I \). All vectors, \( r, s_k, \) and \( w \), are \( N \times 1 \) vectors, where \( N \) is the length of the spreading sequences. We emphasize that \( x_k \) is a binary code bit since we will consider channel coding together with the synchronous CDMA model.

The discrete-time model for the synchronous CDMA system at the output of the matched-filter bank is [5], as in (1.5),

\[ y = RAx + n \]  

(2.2)

where

\[ R = S^T S = [s_1 \ s_2 \ \cdots \ s_K]^T [s_1 \ s_2 \ \cdots \ s_K] ; \]  

(2.3)

\[ A = \text{diag}\{a_1 \ a_2 \ \cdots \ a_K\} ; \ x = [x_1 \ x_2 \ \cdots \ x_K]^T \in \{-1, +1\}^K ; \text{ and } n = S^T w. \]

To simplify our analysis, we assume all users have the same power at the receiver. Therefore, the matrix \( A \) becomes an identity matrix and can be omitted. Since all users are symmetric under the assumption of random spreading, the “capacity” (achievable information rate), averaged over the ensemble of spreading sequences, is the same for all \( K \) users. Thus, we will only consider the information rate of
the first user in our analysis and simulations. The reason we consider the binary-input achievable information rates for each user, instead of the sum capacity of the entire system, is that it easily leads us to the coding-spreading tradeoff problem and optimal code rates.

In the coding-spreading tradeoff problem, a fixed transmission bandwidth is assumed, and the code rate is traded off with the spreading gain to minimize the $E_b/N_0$ required for reliable communication. The actual transmitted information rate depends on the bandwidth expansion factor $Q$ of the CDMA system, which is defined as the ratio of the spreading gain, $N$, to the code rate, $r$,

$$Q = \frac{N}{r}. \quad (2.4)$$

The optimal code rate yields the solution to the coding-spreading tradeoff problem. Heuristically, we imagine that spreading is used to combat MAI and coding is used to combat the channel white Gaussian noise. (However, as described in [6] and below, for the optimal MUDD, no spreading should be employed.)

2.3 Multiuser Detectors and Their Binary-Input Capacities

Multiuser detectors for CDMA systems have been studied for more than 15 years [5]. Many MUDs, from the optimal MUD to the single-user matched-filter detector, have been proposed as we discussed in Chapter 1. In this section, we will study the binary-input capacities of random-spread synchronous CDMA systems for the
following receivers: optimal MUD, BCJR-once MUD, MMSE MUD, decorrelating MUD, and single-user matched-filter (SUMF or MF) detector. Descriptions of these MUDs may be found in the literature [5], [38], [39]. Below we present the binary-input information rate expressions for each of them.

2.3.1 The Optimal MUD

In this chapter, the optimal MUD refers to the optimal joint multiuser detector/decoder, which we also write as optimal MUDD. (In Chapter 1, the optimal MUD does not consider the decoder for channel coding.) By definition, the optimal MUDD achieves the largest binary-input capacity \( C_{bi} \). Given a randomly selected set of spreading sequences, the per-user expression for \( C_{bi} \) of the synchronous CDMA system with optimal MUDD is given [6] by

\[
I(y; x)/K = \left[H(y) - H(y|x)\right]/K
= \left[H(y) - H(n)\right]/K
\]

(2.5)

where, by definition,

\[
H(y) = -\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} p_Y(z) \log(p_Y(z)) \, dz/K
\]

(2.6)

and

\[
H(n) = \frac{1}{2} \log(2\pi e\sigma^2)^K |R|
\]

(2.7)
for positive definite $R$.

There is no simple closed-form formula for the capacity expression (2.5) which involves a $K$-dimensional integration, unless some Gaussian approximations are made when $K$ is large. However, it is possible to obtain numerical results for such $C_{bi}$ in (2.5) when $K$ is small. In the next section, we will use Monte-Carlo integration to obtain values for $C_{bi}$.

To facilitate the Monte-Carlo integration, we give an equivalent expression for $I(y; x)$ by whitening the Gaussian noise vector $n$ in (2.2). Consider the eigen-decomposition of the correlation matrix $R$. Let

\[
R = U \Lambda U^T
\]

\[
= U \sqrt{\Lambda} \sqrt{\Lambda} U^T
\]

(2.8)

where $U$ is an orthonormal matrix, $\Lambda = \sqrt{\Lambda} \sqrt{\Lambda}$ is the diagonal matrix of the (nonnegative) eigenvalues of $R$, and $\sqrt{\Lambda}$ is the diagonal matrix of the square roots of the eigenvalues of $R$. Using (2.8), a whitened version of (2.2) can be written as

\[
\tilde{y} = \sqrt{\Lambda} U^T X + \sqrt{\Lambda}^{-1} U^T n
\]

(2.9)

Notice that if we assume random spreading, $R$ may have zero-valued eigenvalues. In this case, the corresponding diagonal entries of $\sqrt{\Lambda}^{-1}$ are simply set to zero. This will not affect the information rate values.

Given (2.9), it can be easily shown that

\[
I(y; x) = I(\tilde{y}; x)
\]

(2.10)
Therefore, we can calculate $I(\tilde{y}; x)$ instead of $I(y; x)$. What we have accomplished is that, in (2.9), the elements in the vector $\sqrt{\lambda}^{-1} U^T n$ are Gaussian and mutually independent. This simplifies the numerical integration.

The capacity given in (2.5) (or (2.10)) is a function of the randomly chosen spreading sequences and, hence, is a random variable. By averaging (2.5) over the ensemble of random spreading sequences of length $N$, we obtain the binary-input per-user capacity of the synchronous CDMA system with optimal MUDD. That is,

$$C_{\text{opt}}^* = E_{\{s_1, s_2, \ldots, s_K\}}[I(y; x)/K]$$

This capacity can be approached by a soft-input soft-output (SISO) MUD, which employs a maximum a posteriori (MAP) detector, in cooperation with a SISO decoder [35], [36], [37], [25], [26]. We call this receiver iterative MUDD. The BCJR-once MUD, described in the next subsection, represents a compromise iterative MUDD in which the MAP detector joins only in the first iteration.

2.3.2 The BCJR-once MUD

In [39], a low-complexity suboptimal detector/decoder, called BCJR-once detector/decoder, was introduced for LDPC-coded ISI channels. A similar discussion on the separation of MUD and coding can also be found in [38]. Here we analyze a suboptimal MUD which applies this idea. Unlike the iterative MUDD, the
BCJR-once MUD performs SISO MAP detection only once, so that there are no iterations between the MUD and the following (single-user) decoders. The receiver with the BCJR-once MUD is a single-user receiver in the sense that every user has its own independent encoder and decoder. (BCJR algorithm is not necessary for the synchronous CDMA system. But we will keep the name “BCJR-once”.)

So for the system with BCJR-once MUD, the $C_{bi}$ for user 1 can be expressed as

$$I(y; x_1) = H(y) - H(y|x_1) \quad (2.12)$$

where $H(y)$ is as in (2.6) and

$$H(y|x_1) = H(Rx' + n). \quad (2.13)$$

Here we define $x' \triangleq [0, x_2, x_3, \cdots, x_K]^T$. Similar to the optimal MUDD case, we can calculate $I(\tilde{y}; x_1)$ instead of $I(y; x_1)$, where $\tilde{y}$ is as in (2.9).

So the averaged $C_{bi}$ per user of the synchronous CDMA system with BCJR-once MUD is,

$$C_{BCJR-once}^* = E_{\{s_1, s_2, \cdots, s_K\}}[I(y; x_1)] = E_{\{s_1, s_2, \cdots, s_K\}}[I(\tilde{y}; x_1)] \quad (2.14)$$
2.3.3 MMSE MUD

The other three MUDs that will be discussed are linear MUDs: the MMSE MUD, the decorrelating MUD, and the single-user MF detector. As for the BCJR-once MUD, the receivers with these three MUDs are single-user receivers. Thus there are no iterations between the MUD and the decoders.

Since the three MUDs are all linear MUDs and share the same mathematical form, their binary-input capacities can be expressed in the same form. The general form of the linear MUD for one user is

\[
Z = l^T y \\
= l^T R x + l^T n
\] (2.15)

where \( l \) is a \( K \times 1 \) vector which represents the linear MUD. Therefore, the \( C_{bi} \) for user 1 of the synchronous CDMA system with a linear MUD is

\[
I(Z; x_1) = H(Z) - H(Z|x_1)
\] (2.16)

where

\[
H(Z|x_1) = H(l^T (R x' + n))
\] (2.17)

and \( x' = [0, x_2, x_3, \cdots, x_K]^T \). Here \( H(Z) \) and \( H(Z|x_1) \) both involve infinite integrations and can be calculated numerically. The averaged binary-input capacity per user is then

\[
C_{\text{linear}}^* = E_{\{s_1, s_2, \cdots, s_K\}}[I(Z; x_1)]
\] (2.18)
For the MMSE MUD, $l$ for user 1 is given by [5]

$$l_{\text{MMSE}} = ([1 \ 0 \ 0 \ \cdots \ \ 0] \ (R^2 + \sigma^2 I)^{-1})^T$$ (2.19)

So, for a fixed random spreading sequence set, the $C_{bi}$ per user of the synchronous CDMA system with MMSE MUD is

$$I(Z_{\text{MMSE}}; x_1) = H(Z_{\text{MMSE}}) - H(Z_{\text{MMSE}}|x_1)$$ (2.20)

where

$$Z_{\text{MMSE}} = l_{\text{MMSE}}^T Rx + l_{\text{MMSE}}^T n$$ (2.21)

Thus, the averaged $C_{bi}$ per user is

$$C_{\text{MMSE}}^* = E_{\{s_1, s_2, \ldots, s_K\}}[I(Z_{\text{MMSE}}; x_1)]$$ (2.22)

### 2.3.4 Decorrelating MUD

The decorrelating MUD is a linear detector as in (2.15) with $l = l_{\text{dec}}$, where

$$l_{\text{dec}} = ([1 \ 0 \ 0 \ \cdots \ \ 0] \ R^{-1})^T$$ (2.23)

The expression for the binary-input capacity for user 1 of the synchronous CDMA system with decorrelating MUD is

$$I(Z_{\text{dec}}; x_1) = H(Z_{\text{dec}}) - H(Z_{\text{dec}}|x_1)$$ (2.24)

where

$$Z_{\text{dec}} = l_{\text{dec}}^T Rx + l_{\text{dec}}^T n$$ (2.25)
So that the averaged $C_{bi}$ per user is

$$C_{dec}^* = E_{\{s_1,s_2,\ldots,s_K\}}[I(Z_{dec};x_1)]$$  \hspace{1cm} (2.26)

The decorrelating MUD requires the inverse of the correlation matrix $\mathbf{R}$. Since we are studying random-spread CDMA systems, $\mathbf{R}$ is occasionally not invertible and, therefore, the decorrelating MUD is not implementable. In this case, we assume no information can be transmitted through the channel. Thus, for the decorrelating MUD case, the values we compute with (2.26) are upper bounds on the capacities.

### 2.3.5 Single-User MF Detector

The MF detector is a linear detector as in (2.15) with $\mathbf{l} = \mathbf{l}_{MF}$, where

$$\mathbf{l}_{MF} = [1 \ 0 \ 0 \ \cdots \ 0]^T$$  \hspace{1cm} (2.27)

We then have the expression for the binary-input capacity for user 1 of the synchronous CDMA system with MF detector,

$$I(Z_{MF};x_1) = H(Z_{MF}) - H(Z_{MF}|x_1)$$  \hspace{1cm} (2.28)

where

$$Z_{MF} = \mathbf{l}_{MF}^T \mathbf{R}x + \mathbf{l}_{MF}^T \mathbf{n}$$  \hspace{1cm} (2.29)

Averaging over the sets of random spreading sequences, we obtain

$$C_{MF}^* = E_{\{s_1,s_2,\ldots,s_K\}}[I(Z_{MF};x_1)]$$  \hspace{1cm} (2.30)
To conclude this section, we point out the relationships among the capacities:

\[ C^{\ast}_{\text{opt}} \geq C^{\ast}_{\text{BCJR-\emph{once}}} \geq C^{\ast}_{\text{MMSE}} \geq C^{\ast}_{\text{MF}} \]  

(2.31)

The last inequality is not obvious from the formulae of capacities. But it would be true by the construction of the MMSE and MF MUDs and this will be supported by our numerical results.

Since the decorrelating MUD cannot be theoretically implemented for random spreading CDMA systems, we didn’t include its capacity in the inequality above. But if the spreading gain \( N \) is large, we can expect that the correlation matrix \( \mathbf{R} \) would almost always be positive definite. In this case, the value given by (2.26) can be seen as the true system capacity.

2.4 Numerical Results

In this section, we use Monte Carlo integration [40] to numerically compute the integrals presented in Section 2.3. Due to the high complexity of multi-dimensional numerical integration, we only consider small synchronous CDMA systems with \( K = 4 \) users and two bandwidth expansion factors: \( Q = 20 \) and \( Q = 8 \). The corresponding spectral efficiencies, \( \eta \triangleq K r / N = K / Q \), are 0.2 and 0.5, respectively.

First, we numerically compute the averaged \( C_{bi} \)'s per user for different systems. 2000 binary random spreading sequence sets for the 4 users were generated. The algorithm is as follows. (For the decorrelating MUD, since the correlation matrix \( \mathbf{R} \)

needs to be positive definite, the length of spreading sequences $N$ has to be bigger than or equal to the number of users $K$.)

\textit{Algorithm}

1. Select and fix the number of users $K$ and the bandwidth expansion factor $Q$.

2. For $N = 1$ to $Q$,

\textit{(For the decorrelating MUD: For $N = K$ to $Q$.)}

(a) generate $K$ binary random spreading sequences of length $N$.

(b) for each MUD, use the Monte Carlo integration to calculate the $C_{bi}$’s of CDMA systems with the formulae given in the previous section.

(c) if haven’t done this for 2000 spreading sequence sets, go to step a). Otherwise, continue.

(d) average these 2000 $C_{bi}-E_s/N_0$ curves to obtain the averaged $C_{bi}-E_s/N_0$ curve, where $E_s/N_0 = N/Q \times E_b/N_0$.

(e) find the minimum required $(E_b/N_0)^*$ corresponding to the code rate $r = N/Q$. In this case, the code rate $r$ is the maximum information rate the CDMA users can achieve with the signal-to-noise ratio $(E_b/N_0)^*$ and spreading gain $N$.

3. Plot the $(E_b/N_0)^*-r$ curve obtained from Step 2.
This algorithm leads to the results presented in Fig. 2.1 and Fig. 2.2. From both Fig. 2.1 and Fig. 2.2, we can see that, for systems employing the optimal MUDD, the more bandwidth is given to coding, the less $E_b/N_0$ is required. That is, no spreading/all coding achieves the optimal coding-spreading tradeoff. This is reasonable and consistent with the results of [17], [18] for large systems and [6].

However, things are different for suboptimal receivers. In Fig. 2.1 (corresponding to $Q = 20$), we see that systems with suboptimal detectors, except the decorrelating MUD, follow the same trend as the system with optimal MUDD. Yet the system with decorrelating MUD possesses an optimal code rate greater than the minimum value of $K/Q$.

In Fig. 2.2 (corresponding to $Q = 8$), systems with BCJR-once, single-user MF, and MMSE MUDs all have optimal code rates greater than $1/Q$. However, the system with decorrelating MUD favors the most coding it can have (code rate=$K/Q$).

2.5 Performance of LDPC Codes

In this section, we describe the simulation models adopted to study the capacity-approaching capability of a class of LDPC codes in synchronous CDMA systems. The LDPC codes considered are the extended irregular repeat-accumulate (eIRA) codes [12]. Two receivers are considered: the iterative MUDD receiver and the MMSE MUD receiver. These receivers will be discussed in the next two subsections.
2.5.1 Iterative MUDD

Intuitively, optimal processing should include joint spreading/encoding and joint
detection/decoding. However, this joint processing requires complex code design
and high implementation complexity.

A popular and effective way is to use iterative processing, which performs mul-
tiuser detection and decoding iteratively. In this context, SISO MAP detection
and MAP decoding (e.g., message-passing algorithm for LDPC codes) are gener-
ally used. For practical reasons, each CDMA user is encoded and spread separately
at the transmitter as in Fig. 1.1. At the receiver side, a SISO MAP MUD and a
group of independent MAP decoders are adopted. Iterative processing is achieved
by iteratively exchanging soft information between the SISO MAP MUD and the
group of independent MAP decoders.

In this work, we simplify the system further by using a single LDPC encoder and
decoder for all users. The iterative MUDD system is shown in Fig. 2.4. The data
bits from $K$ users are serialized by a parallel-to-serial (P/S) converter. Then the
whole block of bits is encoded by a single LDPC encoder. The codeword then passes
through a serial-to-parallel (S/P) converter. The total number of parallel paths is
equal to the number of users, $K$. Each path is then spread by a binary random
spreading sequence and transmitted through the synchronous CDMA channel of
Fig. 2.3. At the receiver, the outputs of the matched-filter bank are sent to
a SISO MUD based on the MAP rule. The soft information is then iteratively exchanged between the SISO MAP MUD and the LDPC decoder through a P/S (S/P) converter. After several iterations, the LDPC decoder provides the final decisions on the data bits for the $K$ users.

To be more specific about the iterative MUDD, consider the system model as in (2.9). The SISO MAP MUD generates the log *a posteriori* probability (APP) ratio for $x_1$ as (similar for $x_2, ..., x_K$)

$$LAP P(x_1) \triangleq \log \left( \frac{p(x_1 = +1 | \tilde{y})}{p(x_1 = -1 | \tilde{y})} \right) = \log \left( \frac{p(\tilde{y} | x_1 = +1)}{p(\tilde{y} | x_1 = -1)} \right) + \log \left( \frac{p(x_1 = +1)}{p(x_1 = -1)} \right) = LLR(x_1) + L(x_1)$$

(2.32)

where the log *a priori* probability ratio

$$L(x_1) \triangleq \log \left( \frac{p(x_1 = +1)}{p(x_1 = -1)} \right)$$

(2.33)

is the extrinsic information from the LDPC decoder; the log-likelihood ratio (LLR)

$$LLR(x_1) \triangleq \log \left( \frac{p(\tilde{y} | x_1 = +1)}{p(\tilde{y} | x_1 = -1)} \right)$$

(2.34)

can be computed from (2.9) and the extrinsic information $L(x_1), L(x_2), ..., L(x_K)$. Then the LLR's, $LLR(x_1), LLR(x_2), ..., LLR(x_K)$, are sent to the LDPC decoder as *a priori* probabilities, with which the LDPC decoder performs one iteration of the message-passing algorithm and returns the extrinsic information to the SISO MAP MUD.
2.5.2 MMSE MUD

For the system with MMSE MUD, there will be no iteration between the MMSE MUD and the group of LDPC decoders. When iterations are employed as in [36], the receiver is an approximation to the optimal joint multiuser detection/decoding. At the output of the MMSE MUD for one user, the residual MAI plus filtered Gaussian noise are not Gaussian, especially under the assumption of a small system. Thus, the optimal design would employ specially designed codes for the distribution of MAI plus noise. This is beyond the scope of this dissertation. To simplify our work, we will use the LDPC codes designed for AWGN channel. Because the synchronous CDMA channel is memoryless, these LDPC codes are expected to perform well, even with the penalty due to the non-Gaussian MAI plus filtered noise.

Another issue is that, ideally, we should compute the LLR at the output of the MMSE MUD based on the non-Gaussian distribution. But to further simplify our simulations, as in [36], we will approximate the MAI plus filtered noise as Gaussian noise. Then we can just use its mean and variance to provide soft information to the following LDPC decoders.

The resulting system diagram is shown in Fig. 2.5. Each user will have its own LDPC encoder and their data bits are encoded separately. After being transmitted through the synchronous CDMA channel, the outputs of the matched-filter bank are
processed by a MMSE MUD, followed by a bank of independent LDPC decoders, one for each user.

Considering user 1, let the output of the MMSE MUD be (as in (2.21))

\[ Z_{MMSE} = l_{MMSE}^T R x + l_{MMSE}^T n \]

\[ \triangleq \alpha x_1 + \beta \quad (2.35) \]

Then the LLR for user 1, considering the Gaussian approximation, is

\[ \lambda \{ x_1 \} \triangleq \log \left( \frac{p(Z_{MMSE} | x_1 = +1)}{p(Z_{MMSE} | x_1 = -1)} \right) \]

\[ = \frac{(Z_{MMSE} - \alpha)^2}{2\nu^2} - \frac{(Z_{MMSE} + \alpha)^2}{2\nu^2} \]

\[ = \frac{-2\alpha Z_{MMSE}}{\nu^2} \quad (2.36) \]

where \( \alpha \) is the \((1,1)\)th entry of vector \( l_{MMSE}^T R \), \( \beta \) is the residual MAI plus filtered noise, and \( \nu^2 \) is the variance of \( \beta \). Since \( \beta \) is approximated as a Gaussian random variable, its variance is

\[ \nu^2 = l_{MMSE}^T R R^T l_{MMSE} - \alpha^2 + \sigma^2 l_{MMSE}^T l_{MMSE} \quad (2.37) \]

where \( l_{MMSE}^T R R^T l_{MMSE} - \alpha^2 \) is due to MAI and \( \sigma^2 l_{MMSE}^T l_{MMSE} \) comes from the channel Gaussian noise.

The LLR of (2.36) is then sent to the LDPC decoder for user 1 as a priori probability. Then the LDPC decoder will give the final decisions on the data for user 1. We emphasize that there is no iteration between the MMSE MUD and the LDPC decoders.
2.6 Simulation Results

In this section, we compare the performances of the LDPC-coded systems employing iterative MUDD and MMSE MUD with the capacity results.

At first, we performed simulations to assess the performance of the LDPC-coded synchronous CDMA systems with iterative MUDD. We expect LDPC code performance to approach the performance of the optimal MUDD whose binary-input capacity is given in Section 2.3 and 2.4. To compare with our numerical capacity results, we chose a 4-user synchronous CDMA system with bandwidth expansion factor $Q = 20$. The binary spreading sequences for each user are randomly generated. Eight length-20000 eIRA LDPC codes of rates $4/20$, $6/20$, $8/20$, $10/20$, $12/20$, $14/20$, $16/20$, and $18/20$ are used. These LDPC codes are designed for AWGN channel as described in [12]. The simulated bit error rates (BER’s) are shown in Fig. 2.6.

We have also performed simulations on the performance of the synchronous CDMA systems with MMSE MUD. The simulated BER’s of the $Q = 20$ system with MMSE MUD are shown in Fig. 2.7.

From these two figures, we can get the approximate required $E_b/N_0$’s at BER $10^{-4}$. Combined with previous numerical results on capacities and the performance of eIRA codes on AWGN channel, we have Fig. 2.8.
We have repeated the simulations on the systems with $Q = 8$. The corresponding length-20000 eIRA LDPC codes are of rates $2/8$, $3/8$, $4/8$, $5/8$, $6/8$, and $7/8$. The simulated BERs of the system with iterative MUD are shown in Fig. 2.9 and the simulated BERs of the system with MMSE MUD are given in Fig. 2.10. Also there is comparison of the LDPC simulation results with the capacities, which is shown in Fig. 2.11.

We make following observations regarding Fig. 2.8 and Fig. 2.11:

1. In both figures, the LDPC simulation curves for iterative MUD and MMSE MUD show the shape for coding-spreading tradeoff: optimal rates exist greater than $1/Q$. This is different from the information-theoretic results of Section 2.4, where only the system with MMSE MUD for $Q = 8$ has an optimal code rate greater than $1/Q$. Intuitively, if the coding scheme is less powerful, the optimal coding rate could be even higher.

2. The performance of LDPC codes is quite good even though they were designed for the AWGN channel. Since their performances (curves) should be above the computed capacities and their performances on AWGN channel, we can see that the LDPC simulation curves for iterative MUD are not far away from their limits: for $Q = 20$, they are less than about 2 dB away from the capacities and less than about 1 dB away from the LDPC performances on AWGN channel; for $Q = 8$, both differences are less than about 2 dB. For
the systems with MMSE MUD, the LDPC performances are less than about 2 dB away from the capacities for $Q = 20$ and less than about 3 dB away for $Q = 8$. Furthermore, these performances are expected to be improved by using exact LLR’s instead of (2.36). Considering the memorylessness of the synchronous CDMA channel, the results are reasonable. However, specially designed LDPC codes would be expected (see also [41] and [42]).

3. For $Q = 8$, there are larger gaps between the LDPC simulation results and the theoretic results, and between the MMSE MUD results and the iterative MUDD results, than for the $Q = 20$ cases. This is because of the higher MAI penalty due to the lower total bandwidth expansion.

2.7 Conclusions

In this chapter, the binary-input achievable information rates (capacities) for finite-sized synchronous CDMA systems were studied. The coding-spreading trade-off problem in synchronous CDMA systems was also addressed. Numerical results show that there exist optimal code rates for the synchronous CDMA systems with certain suboptimal MUDs. However, all coding (no spreading) is optimal for the optimal MUDD receiver. Simulations of the LDPC-coded synchronous CDMA systems with iterative MUDD and MMSE MUD are also presented to show that the binary-input capacities can be closely approached with practical schemes.
Figure 2.1: The minimum required $E_b/N_0$ vs. code rate for synchronous CDMA systems with 4 users and bandwidth expansion factor 20.
<table>
<thead>
<tr>
<th>Code Rate (r)</th>
<th>Minimum Required $\frac{E_b}{N_0}$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sync. CDMA with $K=4$ and $Q=8$</td>
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<tr>
<td></td>
<td>Matched Filter</td>
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<tr>
<td></td>
<td>MMSE</td>
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<td></td>
<td>Decorrelating</td>
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<tr>
<td></td>
<td>BCJR−once</td>
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<tr>
<td></td>
<td>Optimal MUDD</td>
</tr>
</tbody>
</table>

Figure 2.2: The minimum required $E_b/N_0$ vs. code rate for synchronous CDMA systems with 4 users and bandwidth expansion factor 8.
Figure 2.3: Synchronous CDMA channel with AWGN.
Figure 2.4: Synchronous CDMA system with iterative MUDD.
Figure 2.5: Synchronous CDMA system with MMSE MUD and separate LDPC encoders/decoders.
Figure 2.6: The performance of one user in the LDPC-coded synchronous CDMA systems with iterative MUDD. The systems have 4 users and bandwidth expansion factor 20. The curves correspond to the systems with eIRA codes of rates $4/20$, $6/20$, $8/20$, $10/20$, $12/20$, $14/20$, $16/20$, and $18/20$. 
Figure 2.7: The performance of one user in the LDPC-coded synchronous CDMA systems with MMSE MUD. The systems have 4 users and bandwidth expansion factor 20. The curves correspond to the systems with eIRA codes of rates 4/20, 6/20, 8/20, 10/20, 12/20, 14/20, 16/20, and 18/20.
Figure 2.8: Comparisons of the minimum required $E_b/N_0$ vs. code rate and the LDPC simulation results. The systems have 4 users and bandwidth expansion factor 20. Also included are the LDPC simulation results on single-user AWGN channel, which give the required $E_b/N_0$'s at BER $10^{-4}$. 
Figure 2.9: The performance of one user in the LDPC-coded synchronous CDMA systems with iterative MUD. The systems have 4 users and bandwidth expansion factor 8. The curves correspond to the systems with eIRA codes of rates 2/8, 3/8, 4/8, 5/8, 6/8, and 7/8.
Figure 2.10: The performance of one user in the LDPC-coded synchronous CDMA systems with MMSE MUD. The systems have 4 users and bandwidth expansion factor 8. The curves correspond to the systems with eIRA codes of rates $2/8, 3/8, 4/8, 5/8, 6/8,$ and $7/8$. 
Figure 2.11: Comparisons of the minimum required $E_b/N_0$ vs. code rate and the LDPC simulation results. The systems have 4 users and bandwidth expansion factor 8. Also included are the LDPC simulation results on single-user AWGN channel, which give the required $E_b/N_0$’s at BER $10^{-4}$. 
CHAPTER 3
THE CODING-SPREADING TRADEOFF PROBLEM IN
FINITE-SIZED SYNCHRONOUS CDMA SYSTEMS ON
RAYLEIGH FLAT-FADING CHANNELS

In this chapter, we compute achievable information rates for finite-sized synchronous CDMA systems with Rayleigh flat-fading. We also compare the performance of LDPC codes to these information-theoretic limits. As in Chapter 2, the associated coding-spreading tradeoff problem is also considered using these results. We still assume binary random spreading sequences and the computed achievable rates are averaged over ensembles of random spreading sequences and channel fading coefficients. Two ideal Rayleigh flat-fading channels are considered: the chip-level flat-fading (CLFF) channel and the (code) symbol-level flat-fading (SLFF) channel. The receiver is assumed to know the channel state information (CSI) perfectly, but no CSI is available at the transmitter. As in Chapter 2, we do not make Gaussianity assumptions on channel inputs and MAI. Then the minimum required $E_b/N_0$’s for reliable transmission as a function of information rates are plotted to show the coding-spreading tradeoff. We also provide simulation results on the performances of the LDPC-coded synchronous CDMA systems with
Rayleigh flat-fading, which approach the information-theoretic limits that we have computed.

3.1 Introduction

Having done the coding-spreading tradeoff analysis in finite-sized synchronous CDMA systems on an AWGN channel, it is quite natural to raise the question: How do things change for fading channels? In [20], the authors presented an analysis on the impact of the frequency flat-fading on the spectral efficiency of CDMA systems, based on their analysis on AWGN channel in [18]. A similar analysis on linear receivers can be found in [23]. In [21], [22], RCPC/RCPT codes are adopted to analyze the spectral efficiency and coding-spreading tradeoff problem in synchronous CDMA systems on a flat-fading channel. Also included in [21], [22] is an analysis on the AWGN channel based on the large-system results of the output signal-to-interference plus noise ratio’s (SINR’s) given in [23], which are also contained in [20].

In this chapter, we analyze finite-sized CDMA systems with binary random spreading on Rayleigh flat-fading channels. As in Chapter 2, the binary-input achievable information rates are numerically computed and the coding-spreading tradeoff problem is discussed. To facilitate the numerical calculation of information rates, two ideal Rayleigh flat-fading channel models are considered: the CLFF
model and the SLFF model. Furthermore, as in Chapter 2, two LDPC-coded synchronous CDMA systems with iterative MUDD and MMSE MUD are considered. These two implementable structures with the help of eIRA codes are again used to approach the information-theoretic capacities obtained numerically.

The rest of this chapter is outlined as follows. Section 3.2 presents the flat-fading channel models and the synchronous CDMA models with flat fading which will be adopted in this chapter. Section 3.3 provides the analysis of the binary-input capacities of the synchronous CDMA systems with different MUDs and fading channel models. Section 3.4 provides numerical results for the achievable information rates given in Section 3.3. Section 3.5 describes the structures of two LDPC-coded synchronous CDMA systems with iterative MUDD and MMSE MUD on flat-fading channels. Section 3.6 gives simulation results of the LDPC-coded synchronous CDMA systems on flat-fading channels. Section 3.7 contains conclusions.

3.2 Synchronous CDMA System Models with Rayleigh Flat-Fading

In this chapter, we will use a baseband model for the $K$-user BPSK-modulated synchronous CDMA system on the Rayleigh flat-fading channel. First, we would like to describe the Rayleigh flat-fading channel models we are using together with the synchronous CDMA system.
3.2.1 Channel Models

In Chapter 1, we introduced the Rayleigh flat-fading channel model for the single-user channel, which will be generalized to the synchronous CDMA multiuser channel below. As mentioned above, to facilitate the theoretical calculation of the information rates, two ideal models of flat-fading are considered: CLFF and SLFF. In CLFF, every CDMA chip of each user experiences Rayleigh fading independently. In SLFF, every (spread) code symbol of each user experiences Rayleigh fading independently. The CLFF model represents very fast fading and the fading coefficients vary independently chip to chip. The SLFF model represents very slow fading and the fading coefficients vary independently symbol to symbol. In practice, there exist situations in which the fading coefficients are constant (or almost constant) for several code symbols. In this case, we can use an interleaver for the code symbols, long enough to make the fading coefficients experienced by neighboring symbols independent, so that these cases fit in the SLFF model. We will consider only Rayleigh fading, thus the channel fading coefficients can be represented by normalized circularly symmetric complex Gaussian random variables.
3.2.2 Synchronous CDMA System Models with CLFF and SLFF

The received synchronous CDMA signal on a Rayleigh flat-fading channel can be written as

\[ r = \sum_{k=1}^{K} a_k x_k \tilde{s}_k + w \]  

(3.1)

where \( a_k, x_k, \) and \( \tilde{s}_k \) are, respectively, the transmit amplitude, the binary code symbol, and the equivalent complex spreading sequence of the \( k^{th} \) user (The "equivalent" is a consequence of fading.); \( w \) is the circularly symmetric complex channel AWGN vector with mean \( \mathbf{0} \) and covariance \( \sigma^2 \mathbf{I} \). We will drop all \( a'_k s \) since we only consider equal-power systems. All vectors, \( r, \tilde{s}_k, \) and \( w \), are \( N \times 1 \) complex vectors, where \( N \) is the length of the spreading sequences.

The equivalent spreading sequence \( \tilde{s}_k \) is the result of the effect of the fading channel on user \( k \)'s binary spreading sequence \( s_k \). Since we are considering flat-fading, the binary spreading sequence and the equivalent complex spreading sequence have the same length \( N \).

In the CLFF case, different CDMA chips experience independent Rayleigh fading. Thus, if let \( \tilde{s}_k = (\tilde{s}_{k1}, \tilde{s}_{k2}, \cdots, \tilde{s}_{kN}) \) and \( s_k = (s_{k1}, s_{k2}, \cdots, s_{kN}) \), we have (for \( k = 1, 2, \cdots, K, j = 1, 2, \cdots, N \))

\[ \tilde{s}_{kj} = c_{kj} \cdot s_{kj} \]  

(3.2)

where the \( c_{kj} \) are i.i.d. circularly symmetric complex Gaussian random variables representing the channel effect on the \( j^{th} \) chip of user \( k \).
In the SLFF case, different code symbols experience independent Rayleigh fading. So we have (for \( k = 1, 2, \ldots, K \))

\[
\tilde{s}_k = c_k \cdot s_k
\]  

(3.3)

where the \( c_k \) are i.i.d. circularly symmetric complex Gaussian random variables representing the channel effect on user \( k \), which changes from code symbol to code symbol independently.

At the receiver, we assume to have full channel state information. Thus, the receiver knows all fading coefficients and a matched-filter bank can be adopted to generate the sufficient statistics for detection and decoding. As for the AWGN channel, the matched-filter bank is matched to the equivalent spreading sequences \( \tilde{s}_k \)'s. Therefore, for the synchronous CDMA system with Rayleigh flat-fading, the discrete-time model at the output of the matched-filter bank (refer to Fig. 2.3) is

\[
y = \tilde{R}x + n
\]  

(3.4)

where \( y \) is the \( K \times 1 \) complex output vector of the matched-filter bank; \( \tilde{R} \) is the complex \( K \times K \) correlation matrix of the equivalent spreading sequences of the \( K \) users,

\[
\tilde{R} = [\tilde{s}_1 \, \tilde{s}_2 \, \cdots \, \tilde{s}_K]^H [\tilde{s}_1 \, \tilde{s}_2 \, \cdots \, \tilde{s}_K];
\]  

(3.5)

\( x = [x_1 \, x_2 \, \cdots \, x_K]^T \) is the binary-input vector of the \( K \) users, \( x_i \in \{-1, +1\} \); and \( n = [\tilde{s}_1 \, \tilde{s}_2 \, \cdots \, \tilde{s}_K]^H w \) is a \( K \times 1 \) complex Gaussian noise vector with mean 0 and
covariance \( \hat{\mathbf{R}} \). We notice that, in both the CLFF and SLFF cases, the complex correlation matrix \( \hat{\mathbf{R}} \) is Hermitian and semi-positive definite.

Since all users are symmetric under the assumption of random spreading and independent flat-fading, the “capacity” (achievable information rate), averaged over the ensemble of binary spreading sequences and Rayleigh fading coefficients, is the same for all \( K \) users. Thus, as in Chapter 2, we will only consider the information rate of the first user. For the coding-spreading tradeoff problem, the bandwidth expansion factor \( Q \) is defined as before as

\[
Q = \frac{N}{r}
\]

where \( N \) is the spreading gain and \( r \) is the code rate.

### 3.3 Multiuser Detectors and Their Binary-Input Capacities

As in Chapter 2, the following MUDs will be studied for the random-spread synchronous CDMA systems on Rayleigh flat-fading channels: optimal MUD, BCJR-once MUD, MMSE MUD, decorrelating MUD, and single-user matched-filter (SUMF or MF) detector. Since the Rayleigh flat-fading channel (CLFF or SLFF) will only affect the correlation matrix \( \hat{\mathbf{R}} \) and induce complex AWGN instead of real AWGN, expressions for MUDs and capacities are similar to those for the AWGN channel.
3.3.1 The Optimal MUD

Again, as in Chapter 2, the optimal MUD refers to the optimal (maximum a posteriori) joint multiuser detector/decoder, which we also write as optimal MUDD. Given a randomly selected set of binary spreading sequences and Rayleigh fading coefficients, the per-user expression for the binary-input capacity $C_{bi}$ of the synchronous CDMA system with optimal MUDD is given by [6]

$$I(y; x)/K = [H(y) - H(y|x)]/K = [H(y) - H(n)]/K$$

(3.7)

where, since $y$ and $n$ are complex and by definition,

$$H(y) = -\int_{\mathbb{C}^K} p_y(z) \log(p_y(z)) \, dz$$

(3.8)

and

$$H(n) = \log(\pi\sigma^2)^K |\tilde{R}|$$

(3.9)

for positive definite $\tilde{R}$.

There is also no simple closed-form formula for the expression (3.7), which is an integration over the $K$-dimensional complex space $\mathbb{C}^K$, unless some Gaussian approximations are made when $K$ is large. We will use Monte-Carlo integration to obtain the values of $C_{bi}$ as was done for the AWGN case. Further, to facilitate the Monte-Carlo integration, we give an equivalent expression for $I(y; x)$ by whitening the Gaussian noise vector $n$ in (3.4).
The complex correlation matrix $\tilde{R}$ is Hermitian and thus all its eigenvalues are real and non-negative. Let its eigen-decomposition be

$$\tilde{R} = U\Lambda U^H$$
$$= U\sqrt{\Lambda}\sqrt{\Lambda}U^H$$

(3.10)

where $U$ is a unitary matrix, $\Lambda = \sqrt{\Lambda}\sqrt{\Lambda}$ is the diagonal matrix of the eigenvalues of $\tilde{R}$, and $\sqrt{\Lambda}$ is the diagonal matrix of the square roots of the eigenvalues (nonnegative real numbers) of $\tilde{R}$.

Thus, a whitened version of (3.4) is

$$\tilde{y} = \sqrt{\Lambda}U^Hx + \sqrt{\Lambda}^{-1}U^Hn,$$

(3.11)

from which we have

$$I(y; x) = I(\tilde{y}; x).$$

(3.12)

By averaging (3.7) over the ensemble of binary random spreading sequences and channel fading coefficients, we obtain the binary-input capacities of the synchronous CDMA systems with optimal MUD, for the CLFF and SLFF channels, respectively,

$$C^{*}_{opt,CLFF} = E_{\{s_k, c_{kj}, k=1,2,\ldots,K, j=1,2,\ldots,N\}}[I(y; x)/K]$$
$$= E_{\{\tilde{s}_k, c_{kj}, k=1,2,\ldots,K, j=1,2,\ldots,N\}}[I(\tilde{y}; x)/K]$$

(3.13)
and

\[ C_{\text{opt,SLFF}}^* = E_{\{s_k, c_k, k=1,2,\ldots,K\}}[I(y;x)/K] \]
\[ = E_{\{s_k, c_k, k=1,2,\ldots,K\}}[I(\tilde{y};x)/K] \]  \hspace{1cm} (3.14)

These capacities can be approached by the iterative MUD.

3.3.2 The BCJR-once MUD

The \( C_{bi} \) for user 1 of the synchronous CDMA system with the BCJR-once MUD can be expressed as

\[ I(y;x_1) = H(y) - H(y|x_1) \]  \hspace{1cm} (3.15)

where \( H(y) \) is as in (3.8) and

\[ H(y|x_1) = H(\tilde{\mathbf{R}}\mathbf{x'} + \mathbf{n}) \]  \hspace{1cm} (3.16)

with \( \mathbf{x'} = [0, x_2, x_3, \cdots, x_K]^T \). As for the optimal MUD case, we can calculate \( I(\tilde{y};x_1) \) instead of \( I(y;x_1) \), where \( \tilde{y} \) is defined in (3.11).

The averaged per-user \( C_{bi} \)'s of the synchronous CDMA systems with BCJR-once MUD are, for the CLFF and SLFF channels, respectively,

\[ C_{BCJR-once,CLFF}^* = E_{\{s_k, c_{kj}, k=1,2,\ldots,K, j=1,2,\ldots,N\}}[I(y;x_1)] \]
\[ = E_{\{s_k, c_{kj}, k=1,2,\ldots,K, j=1,2,\ldots,N\}}[I(\tilde{y};x_1)] \]  \hspace{1cm} (3.17)
and

\[ C_{BCJR-once,SLFF}^* = E_{\{s_k,c_k, \ k=1,2,\ldots,K\}}[I(y;x_1)] \]

\[ = E_{\{s_k,c_k, \ k=1,2,\ldots,K\}}[I(\tilde{y};x_1)]. \quad (3.18) \]

### 3.3.3 MMSE MUD

The general form of the linear MUD for one user is

\[ Z = l^H y \]

\[ = l^H \tilde{R}x + l^H n \quad (3.19) \]

where \( l \) is a \( K \times 1 \) complex vector which represents the linear MUD. Therefore, the \( C_{bi} \) for user 1 of the synchronous CDMA system with linear MUD is

\[ I(Z;x_1) = H(Z) - H(Z|x_1) \quad (3.20) \]

where

\[ H(Z|x_1) = H(l^H \tilde{R}x' + l^H n) \quad (3.21) \]

and \( x' = [0, x_2, x_3, \ldots, x_K]^T \). Here \( H(Z) \) and \( H(Z|x_1) \) both involve integrations over the complex space \( \mathbb{C} \) and can be calculated numerically. Then the averaged binary-input capacities per user are, for the CLFF and SLFF channels, respectively,

\[ C_{linear,CLFF}^* = E_{\{s_k,c_{kj}, \ k=1,2,\ldots,K, \ j=1,2,\ldots,N\}}[I(Z;x_1)] \quad (3.22) \]
and

\[ C_{\text{linear,SLFF}}^* = E_{\{s_k,c_k, k=1,2,\ldots,K\}}[I(Z;x_1)]. \] (3.23)

For the MMSE MUD, \( l \) for user 1 is given by

\[ l_{\text{MMSE}} = ([1 0 0 \cdots 0][\hat{R} + \sigma^2 I]^{-1})^H \] (3.24)

Therefore, given a random spreading sequence set and channel fading coefficients, the \( C_{bi} \) for user 1 of the synchronous CDMA system with MMSE MUD is

\[ I(Z_{\text{MMSE}};x_1) = H(Z_{\text{MMSE}}) - H(Z_{\text{MMSE}}|x_1) \] (3.25)

where

\[ Z_{\text{MMSE}} = l_{\text{MMSE}}^H \hat{R}x + l_{\text{MMSE}}^H n \] (3.26)

Thus, the averaged \( C_{bi} \)'s per user are, for the CLFF and SLFF channels, respectively,

\[ C_{\text{MMSE,CLFF}}^* = E_{\{s_k,c_{kj}, k=1,2,\ldots,K, j=1,2,\ldots,N\}}[I(Z_{\text{MMSE}};x_1)] \] (3.27)

and

\[ C_{\text{MMSE,SLFF}}^* = E_{\{s_k,c_k, k=1,2,\ldots,K\}}[I(Z_{\text{MMSE}};x_1)]. \] (3.28)

### 3.3.4 Decorrelating MUD

The decorrelating MUD is a linear detector as in (3.19) with \( l = l_{\text{dec}} \), where

\[ l_{\text{dec}} = ([1 0 0 \cdots 0][\hat{R}^{-1}]^H \] (3.29)
The expression for the binary-input capacity for user 1 of the synchronous CDMA system with the decorrelating MUD is

$$I(Z_{dec}; x_1) = H(Z_{dec}) - H(Z_{dec}|x_1)$$  \hspace{1cm} (3.30)

where

$$Z_{dec} = l^H_{dec} \tilde{R}x + l^H_{dec} n$$  \hspace{1cm} (3.31)

Thus, the averaged $C_{bi}$'s per user are, for the CLFF and SLFF channels, respectively,

$$C^*_{dec, CLFF} = E_{\{s_k, c_k, j = 1,2,\ldots,N\}}[I(Z_{dec}; x_1)]$$  \hspace{1cm} (3.32)

and

$$C^*_{dec, SLFF} = E_{\{s_k, c_k, j = 1,2,\ldots,K\}}[I(Z_{dec}; x_1)].$$  \hspace{1cm} (3.33)

3.3.5 Single-User MF Detector

The MF detector is a linear detector as in (3.19) with $l = l_{MF}$, where

$$l_{MF} = [1 \; 0 \; 0 \; \cdots \; 0]^H$$  \hspace{1cm} (3.34)

We then have the expression for the binary-input capacity for user 1 of the synchronous CDMA system with MF detector,

$$I(Z_{MF}; x_1) = H(Z_{MF}) - H(Z_{MF}|x_1)$$  \hspace{1cm} (3.35)
where

$$Z_{MF} = \mathbf{l}^H_{MF} \tilde{\mathbf{R}} \mathbf{x} + \mathbf{l}^H_{MF} \mathbf{n}$$  \hfill (3.36)$$

Averaging over the ensemble of binary random spreading sequences and channel fading coefficients, we obtain, for the CLFF and SLFF channels, respectively,

$$C^*_{MF,CLFF} = E_{\{s_k, c_k, j_1, j_2, \ldots, k, j_1, j_2, \ldots, N\}} [I(Z_{MF}; x_1)] \hfill (3.37)$$

and

$$C^*_{MF,SLFF} = E_{\{s_k, c_k, j_1, j_2, \ldots, k, j_1, j_2, \ldots, N\}} [I(Z_{MF}; x_1)]. \hfill (3.38)$$

To conclude this section, we point out the relationships among the capacities:

$$C^*_{opt} \geq C^*_{BCJR-once} \geq C^*_{MMSE} \geq C^*_{MF} \hfill (3.39)$$

which is also partly supported by our numerical results.

3.4 Numerical Results

In this section, we use Monte Carlo integration [40] to numerically compute the averaged capacities per user of the synchronous CDMA systems with optimal MUDD and MMSE MUD, for the CLFF and SLFF channels, with the formulae presented in Section 3.3. We will also consider the synchronous CDMA systems with $K = 4$ users and two bandwidth expansion factors: $Q = 20$ and $Q = 8$, which correspond to the spectral efficiencies, $\eta = Kr/N = K/Q$, 0.2 and 0.5, respectively.
In both the CLFF and SLFF cases, we use the equivalent spreading sequences to describe our numerical algorithm instead of the binary spreading sequences. In our computations with the optimal MUDD, 10000 random equivalent spreading sequence sets for the 4 users were generated, and 20000 random equivalent spreading sequence sets were used for the computations with the MMSE MUD. The algorithm is as follows.

\textit{Algorithm}

1. Select and fix the number of users $K$ and the bandwidth expansion factor $Q$.

2. For $N = 1$ to $Q$,

   (a) generate $K$ random equivalent spreading sequences of length $N$.

   (b) for each MUD, use Monte Carlo integration to calculate the $C_{bi}$ of CDMA systems with the formulae given in the previous section.

   (c) if haven’t done this for required number of equivalent spreading sequence sets (10000 for optimal MUDD and 20000 for MMSE MUD), go to step a). Otherwise, continue.

   (d) average all $C_{bi} - E_s/N_0$ curves to obtain the averaged $C_{bi} - E_s/N_0$ curve, where $E_s/N_0 = N/Q \cdot E_b/N_0$.

   (e) find the minimum required $(E_b/N_0)^*$ corresponding to the code rate $r = N/Q$. In this case, the code rate $r$ is the maximum information rate
the CDMA users can achieve with the signal-to-noise ratio \( (E_b/N_0)^* \) and spreading gain \( N \).

3. Plot the \( (E_b/N_0)^* - r \) curve obtained from Step 2.

The results of this algorithm will be given together with the LDPC simulation results and will be discussed in Section 3.6.

3.5 Performance of LDPC Codes

As in Chapter 2, two simulation models are adopted in this section to study the capacity-approaching capability of the eIRA codes in the synchronous CDMA systems with Rayleigh flat-fading (CLFF and SLFF). The iterative MUDD receiver and the MMSE MUD receiver are considered again.

From Section 3.2, we know that the only difference between the synchronous CDMA model on AWGN channel and those on CLFF and SLFF are the (equivalent) spreading sequences. Therefore, the receiver structures are same instead of the use of equivalent spreading sequences here.

3.5.1 Iterative MUDD

Specifically, the iterative MUDD receiver has the same structure and operating algorithm as in Section 2.5 (Fig. 2.4). The only difference is that the channel is
CLFF or SLFF and the spreading sequences are replaced by the equivalent spreading sequences according to the CLFF or SLFF models. So, the SISO MAP MUD generates the log-APP ratio for $x_1$ as (similar for $x_2, ..., x_K$)

$$LAPP(x_1) \triangleq \log \left( \frac{p(x_1 = +1|\tilde{y})}{p(x_1 = -1|\tilde{y})} \right)$$

$$= \log \left( \frac{p(\tilde{y}|x_1 = +1)}{p(\tilde{y}|x_1 = -1)} \right) + \log \left( \frac{p(x_1 = +1)}{p(x_1 = -1)} \right)$$

$$= LLR(x_1) + L(x_1)$$  \hfill (3.40)

where the log *a priori* probability ratio

$$L(x_1) \triangleq \log \left( \frac{p(x_1 = +1)}{p(x_1 = -1)} \right)$$  \hfill (3.41)

is the extrinsic information from the LDPC decoder; the LLR

$$LLR(x_1) \triangleq \log \left( \frac{p(\tilde{y}|x_1 = +1)}{p(\tilde{y}|x_1 = -1)} \right)$$  \hfill (3.42)

can be computed from (3.11) and the extrinsic information $L(x_1), L(x_2), ..., L(x_K)$. Then the LLR’s, $LLR(x_1), LLR(x_2), ..., LLR(x_K)$, are sent to the LDPC decoder as *a priori* probabilities, with which the LDPC decoder performs one iteration of the message-passing algorithm and returns the extrinsic information to the SISO MAP MUD.

### 3.5.2 MMSE MUD

Similar to the iterative MUDD case, the fading channel system with the MMSE MUD is analogous to the one in Fig. 2.5. Considering user 1, let the output of the
Then the LLR for user 1, considering the Gaussian approximation, is

\[
\lambda \{ x_1 \} \triangleq \log \left( \frac{p(Z_{MMSE}|x_1=+1)}{p(Z_{MMSE}|x_1=-1)} \right)
= \frac{\| Z_{MMSE} - \alpha \|^2}{\nu^2} - \frac{\| Z_{MMSE} + \alpha \|^2}{\nu^2}
= \frac{-4Re\{\alpha^H Z_{MMSE}\}}{\nu^2}
\]  

where \( \alpha \) is the \((1,1)\)th entry of vector \( l_{MMSE}^H \tilde{R} \), \( \beta \) is the residual MAI plus filtered noise, and \( \nu^2 \) is the variance of \( \beta \). Since \( \beta \) is approximated as a complex Gaussian random variable, its variance is

\[
\nu^2 = l_{MMSE}^H \tilde{R}^H l_{MMSE} - \| \alpha \|^2 + \sigma^2 l_{MMSE}^H l_{MMSE}
\]  

where \( l_{MMSE}^H \tilde{R}^H l_{MMSE} - \| \alpha \|^2 \) is due to MAI and \( \sigma^2 l_{MMSE}^H l_{MMSE} \) comes from the channel Gaussian noise.

The LLR of (3.44) is then sent to the LDPC decoder for user 1 as an \textit{a priori} probability. Then the LDPC decoder will give the final decisions on the data for user 1. We again emphasize that there is no iteration between the MMSE MUD and the LDPC decoders.
3.6 Simulation Results

In this section, we compare the performances of the LDPC-coded systems employing iterative MUDD and MMSE MUD with the capacity results. Both chip-level and symbol-level Rayleigh flat-fading channels are considered.

At first we performed simulations to assess the performances of the LDPC-coded synchronous CDMA systems on chip-level Rayleigh flat-fading channel. An equivalent spreading sequence set for all users was randomly generated for each code symbol slot. The detection/decoding method is similar to the one in Chapter 2, except here the system models are complex.

We first simulated 4-user synchronous CDMA systems with a bandwidth expansion factor \( Q = 20 \) on the CLFF channel. Eight length-20000 eIRA codes of rates \( 4/20, 6/20, 8/20, 10/20, 12/20, 14/20, 16/20, \) and \( 18/20 \) are used. The simulated BER’s of the systems with iterative MUDD and MMSE MUD are shown in Fig. 3.1 and 3.2. From these two figures, we may obtain the required \( E_b/N_0 \)'s at BER \( 10^{-4} \). We have plotted in Fig. 3.3 these values together with the capacity-based values computed numerically from the Section 3.4 formulae.

We repeated the CLFF simulations on the systems with \( Q = 8 \). Again the length-20000 eIRA codes are of rates \( 2/8, 3/8, 4/8, 5/8, 6/8, \) and \( 7/8 \). The simulation results and the comparisons with capacity limits are given in Fig. 3.4 (\( Q = 8, \) iterative MUDD), 3.5 (\( Q = 8, \) MMSE MUD), and 3.6 (\( Q = 8, \) Comparisons).
For the systems on the SLFF channel, the simulation results are shown in Fig. 3.7 (\(Q = 20\), iterative MUDD), 3.8 (\(Q = 20\), MMSE MUD), 3.10 (\(Q = 8\), iterative MUDD), and 3.11 (\(Q = 8\), MMSE MUD) respectively. The comparisons are shown in Fig. 3.9 (\(Q = 20\), Comparisons) and 3.12 (\(Q = 8\), Comparisons).

From these figures, we can observe that

1. Both chip-level and symbol-level fading increase the importance of coding relative to AWGN. Figs. 3.3, 3.6, 3.9, and 3.12 all show that more bandwidth should be allocated to coding than in the AWGN case, both for the theoretic-limit curves and the LDPC-simulation curves. The same relationship between the AWGN and SLFF cases is also observed in [22]. This shows that spreading is less effective than coding in combatting fading, especially slow fading (SLFF), though spreading enjoys the advantage of simplicity.

2. Chip-level fading and symbol-level fading decrease the performance gap between the systems with iterative MUDD and MMSE MUD, both in their theoretic limits and LDPC-coded performances. This is especially obvious for the systems with high load (\(K = 4, Q = 8\)) and operating at low rates (see Figs. 2.11, 3.6, and 3.12). This could be seen as a benefit of fading for the MMSE MUD receiver.

To observe more about the effects of fading on CDMA systems, we combine the results in Chapter 2 and Chapter 3 yielding Fig. 3.13, 3.14, 3.15, and 3.16. These
figures give comparisons of the performances of each CDMA system on the AWGN, CLFF, and SLFF channels. We have the following observations:

1. The chip-level (fast) fading is possibly beneficial under the condition that the receiver has full CSI. For the low-loaded systems \((K = 4, Q = 20)\), Fig. 3.13 and 3.14 show that: at low rates, the performance on CLFF is better; at high rates, the performance on AWGN is better. For the high-loaded systems \((K = 4, Q = 8)\), Fig. 3.15 and 3.16 show that the chip-level fading is always beneficial and for lower rates, the improvement is larger. Consider for chip-level fading, the complex model of (3.1), with the \(a_k\)’s dropped for an equal-power system, can be rewritten as

\[
\mathbf{r} = Re(\mathbf{r}) + i Im(\mathbf{r})
\]

\[
= \sum_{k=1}^{K} x_k (Re(\tilde{s}_k) + i Im(\tilde{s}_k)) + Re(\mathbf{w}) + i Im(\mathbf{w}).
\]

Thus, we have an equivalent received signal in real domain,

\[
\tilde{\mathbf{r}} = \begin{bmatrix}
Re(\mathbf{r}) \\
Im(\mathbf{r})
\end{bmatrix}
\]

\[
= \sum_{k=1}^{K} \begin{bmatrix}
Re(\tilde{s}_k) \\
Im(\tilde{s}_k)
\end{bmatrix} x_k + \begin{bmatrix}
Re(\mathbf{w}) \\
Im(\mathbf{w})
\end{bmatrix}.
\]

The model (3.47) can be viewed as a synchronous CDMA system on an AWGN channel with normalized i.i.d. real Gaussian distributed spreading sequences of length \(2N\). Hence, for each user \(k\), its complex equivalent
spreading sequence $\tilde{s}_k$ of length $N$ becomes a real equivalent spreading sequence $[Re(\tilde{s}_k)^T, Im(\tilde{s}_k)^T]^T$ of length $2N$. Further, the users maintain the same code rate $N/Q$ without violating the bandwidth constraint. In other words, the CLFF system enjoys twice the spreading gain that the AWGN system does, with the difference being that the equivalent spreading sequences are i.i.d. real Gaussian distributed and quadrature demodulators are needed in the implementation of the receiver. When $N$ is small, the doubled spreading gain decreases MAI and improves the system performance. When $N$ is increased, the benefit of the doubled spreading gain decreases and the Gaussian distribution of the equivalent spreading sequences becomes influential, thus degrading system performance.

2. The SLFF curves in Figs. 3.13, 3.14, 3.15, and 3.16 have much sharper slope than those for CLFF and AWGN. This means that the slow fading is very harmful to system performance even if the receiver possesses perfect CSI. We conclude that more bandwidth should be allocated to coding in the SLFF case than in the CLFF and AWGN cases.

3. The performance of LDPC codes and their limits in the CLFF case are better or lower than those in the SLFF case. This is because the fast fading in the chip level makes possible the diversity provided by the spreading/despreading. Such diversity becomes more valuable as the spreading gain becomes larger.
which explains larger performance gaps at higher rates than the gaps at lower rates between the CLFF and SLFF curves.

4. The performances of LDPC codes, in both CLFF and AWGN cases, have similar gaps from their limits. But, in the SLFF case, the gaps are larger than those in the CLFF and AWGN cases, especially at high rates. This shows again the drawback of slow fading.

3.7 Conclusions

In this chapter, the binary-input achievable information rates for finite-sized synchronous CDMA systems on Rayleigh flat-fading channels were studied. The chip-level and symbol-level flat-fading models were both considered. The coding-spreading tradeoff problem was also addressed based on the numerical results of the achievable information rates and the simulation results of the LDPC-coded systems. Compared with the AWGN channel results in Chapter 2, it is shown that the existence of both types of fading push the optimal code rates to smaller values. However, under the condition that the receiver has perfect CSI, the chip-level (fast) fading is, on the whole, beneficial to the system, while the symbol-level (slow) fading is detrimental.
Figure 3.1: The performance of one user in the LDPC-coded synchronous CDMA systems with iterative MUDD on chip-level Rayleigh flat-fading channel. The systems have 4 users and bandwidth expansion factor 20. The curves correspond to the systems with eIRA codes of rates 4/20, 6/20, 8/20, 10/20, 12/20, 14/20, 16/20, and 18/20.
Figure 3.2: The performance of one user in the LDPC-coded synchronous CDMA systems with MMSE MUD on chip-level Rayleigh flat-fading channel. The systems have 4 users and bandwidth expansion factor 20. The curves correspond to the systems with eIRA codes of rates $\frac{4}{20}$, $\frac{6}{20}$, $\frac{8}{20}$, $\frac{10}{20}$, $\frac{12}{20}$, $\frac{14}{20}$, $\frac{16}{20}$, and $\frac{18}{20}$. 
Figure 3.3: Comparisons of the minimum required $E_b/N_0$ vs. code rate and the LDPC simulation results. The systems have 4 users and bandwidth expansion factor 20. Also included are the LDPC simulation results on single-user AWGN channel, which give the required $E_b/N_0$’s at BER $10^{-4}$. 
Figure 3.4: The performance of one user in the LDPC-coded synchronous CDMA systems with iterative MUD on chip-level Rayleigh flat-fading channel. The systems have 4 users and bandwidth expansion factor 8. The curves correspond to the systems with eIRA codes of rates 2/8, 3/8, 4/8, 5/8, 6/8, and 7/8.
Figure 3.5: The performance of one user in the LDPC-coded synchronous CDMA systems with MMSE MUD on chip-level Rayleigh flat-fading channel. The systems have 4 users and bandwidth expansion factor 8. The curves correspond to the systems with eIRA codes of rates 2/8, 3/8, 4/8, 5/8, 6/8, and 7/8.
Figure 3.6: Comparisons of the minimum required $E_b/N_0$ vs. code rate and the LDPC simulation results. The systems have 4 users and bandwidth expansion factor 8. Also included are the LDPC simulation results on single-user AWGN channel, which give the required $E_b/N_0$’s at BER $10^{-4}$. 
Figure 3.7: The performance of one user in the LDPC-coded synchronous CDMA systems with iterative MUDD on symbol-level Rayleigh flat-fading channel. The systems have 4 users and bandwidth expansion factor 20. The curves correspond to the systems with eIRA codes of rates $4/20$, $6/20$, $8/20$, $10/20$, $12/20$, $14/20$, $16/20$, and $18/20$. 
Figure 3.8: The performance of one user in the LDPC-coded synchronous CDMA systems with MMSE MUD on symbol-level Rayleigh flat-fading channel. The systems have 4 users and bandwidth expansion factor 20. The curves correspond to the systems with eIRA codes of rates $4/20$, $6/20$, $8/20$, $10/20$, $12/20$, $14/20$, $16/20$, and $18/20$. 
Figure 3.9: Comparisons of the minimum required $E_b/N_0$ vs. code rate and the LDPC simulation results. The systems have 4 users and bandwidth expansion factor 20. Also included are the LDPC simulation results on single-user AWGN channel, which give the required $E_b/N_0$'s at BER $10^{-4}$. 
Figure 3.10: The performance of one user in the LDPC-coded synchronous CDMA systems with iterative MUDD on symbol-level Rayleigh flat-fading channel. The systems have 4 users and bandwidth expansion factor 8. The curves correspond to the systems with eIRA codes of rates $2/8$, $3/8$, $4/8$, $5/8$, $6/8$, and $7/8$. 
Figure 3.11: The performance of one user in the LDPC-coded synchronous CDMA systems with MMSE MUD on symbol-level Rayleigh flat-fading channel. The systems have 4 users and bandwidth expansion factor 8. The curves correspond to the systems with eIRA codes of rates $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, and $\frac{7}{8}$. 
Figure 3.12: Comparisons of the minimum required $E_b/N_0$ vs. code rate and the LDPC simulation results. The systems have 4 users and bandwidth expansion factor 8. Also included are the LDPC simulation results on single-user AWGN channel, which give the required $E_b/N_0$’s at BER $10^{-4}$. 
Figure 3.13: Comparisons of the minimum required $E_b/N_0$ vs. code rate and the LDPC simulation results on AWGN, CLFF, and SLFF channels. The systems have 4 users and bandwidth expansion factor 20.
Figure 3.14: Comparisons of the minimum required $E_b/N_0$ vs. code rate and the LDPC simulation results on AWGN, CLFF, and SLFF channels. The systems have 4 users and bandwidth expansion factor 20.
Figure 3.15: Comparisons of the minimum required $E_b/N_0$ vs. code rate and the LDPC simulation results on AWGN, CLFF, and SLFF channels. The systems have 4 users and bandwidth expansion factor 8.
Figure 3.16: Comparisons of the minimum required $E_b/N_0$ vs. code rate and the LDPC simulation results on AWGN, CLFF, and SLFF channels. The systems have 4 users and bandwidth expansion factor 8.
CHAPTER 4
COMPARISONS, CONCLUSIONS, AND FURTHER RESEARCH

4.1 Comparisons With Large-System Results

In this section, we will compare our results on finite-sized synchronous CDMA systems with the large-system results [18] introduced in Chapter 1, specifically, the formulae (1.32), (1.34), (1.35), and (1.36). Only AWGN and CLFF channels are considered. For the SLFF channel, to our knowledge, no appropriate comparison can be made, so we will not consider it.

At first we need to modify these large-system formulae to make them comparable with our results, which means we need to present the large-system results in terms of the minimum required $E_b/N_0$ versus code rate. At present, the formulae are in terms of spectral efficiency versus SNR. Fortunately, we can obtain the required closed-form results from (1.34), (1.35), and (1.36). By definition [18], we know that

$$SNR = 2N E_b / K N_0^2 C^* = 21 E_b / \beta N_0^2 C^*$$  \hspace{1cm} (4.1)$$

Since $C^*$ is the sum capacity per chip for all $K$ users, the code rate is

$$r \text{(bits/symbol/user)} = C^* N / K = C^* / \beta$$  \hspace{1cm} (4.2)$$

So we have
1. for the single-user matched-filter detector,

$$\frac{E_b}{N_0} = \frac{1}{2r} \frac{2^{2r} - 1}{1 + \beta - \beta 2^{2r}}$$  \hspace{1cm} (4.3)

2. for the decorrelating MUD,

$$\frac{E_b}{N_0} = \frac{2^{2r} - 1}{2(1 - \beta)r}$$  \hspace{1cm} (4.4)

3. for the MMSE MUD, [43]

$$\frac{E_b}{N_0} = \frac{2^{2r} - 1}{2r} \frac{1}{1 - \beta + \beta 2^{-2r}}$$  \hspace{1cm} (4.5)

Though there is no closed-form solution for (1.32), we can solve it numerically. Let us write (1.32) as

$$C_{opt}^* = \mathcal{G}(\beta, SNR)$$  \hspace{1cm} (4.6)

where the function $\mathcal{G}$ represents the right-hand side of (1.32). Then with the help of (4.1) and (4.2), (1.32) can be rewritten as

$$r = \frac{C_{opt}^*}{\beta} = \mathcal{G}\left(\beta, 2 \frac{E_b}{N_0} r\right) / \beta$$  \hspace{1cm} (4.7)

We now need to find the $E_b/N_0$ solution for a fixed $r$ in (4.7). If $r$ is fixed, with an initial value of $E_b/N_0$, we can calculate

$$r^* = \mathcal{G}\left(\beta, 2 \frac{E_b}{N_0} r\right) / \beta$$  \hspace{1cm} (4.8)

If $r^*$ is bigger than $r$, we then decrease $E_b/N_0$ and recalculate $r^*$. If $r^*$ is smaller than $r$, we then increase the $E_b/N_0$ and recalculate $r^*$. In this way, we can see that
$r^*$ gets closer to $r$ and the final stable value of $E_b/N_0$ is the solution to (4.7) for the fixed $r$, where convergence is proved by our numerical results.

The above procedure leads to the comparison results shown in Figs. 4.1, 4.2, 4.3, and 4.4 for different settings. For the low-loaded systems ($K = 4, Q = 20$), Fig. 4.1 and 4.2 show that, for the optimal MUDD, MMSE MUD, and matched-filter receivers, the difference between the large-system results and the finite-sized system results is quite small for low code rates. The larger difference at high rates is due to the different input distributions assumed: binary (finite) versus Gaussian (large). For the decorrelating MUD receiver, though the difference between the curves is large, we observe a similar trend.

However, for the high-loaded systems ($K = 4, Q = 8$), things become more complicated. For the optimal MUDD receiver, the two curves are still in fairly close agreement for low rates (Fig. 4.3). For the MMSE MUD receiver, the difference is larger for low rates, but still less than 1 dB (Fig. 4.3). For the matched-filter receiver (Fig. 4.4), the difference is quite large (larger than 2 dB) for all rates. Particularly for the large-system results, there are no valid solutions for $r \geq 0.5$, which means, under the large-system assumptions, the matched-filter receiver cannot support any code rate larger than or equal to 0.5 with arbitrarily small BER. For the decorrelating MUD receiver, the large and finite curves have opposite trends (Fig. 4.4).
We now make comparisons on the CLFF channel. The large-system results can be derived from the same formulae as on AWGN channel [18], except that the equivalent spreading gain is 2 times the actual spreading gain $N$ as we demonstrated in Section 3.6. The comparison results are shown in Fig. 4.5 and 4.6 for the optimal MUDD and MMSE MUD receivers with $Q = 20$ and $Q = 8$, respectively. We see again that the high-loaded systems ($K = 4, Q = 8$) have bigger discrepancy than the low-loaded systems ($K = 4, Q = 20$). However, comparing with the previous comparison results on AWGN channel, chip-level fading brings the large-system results and the finite-sized system results closer together. The reason is that fading changes the distribution of MAI.

In summary, these comparison results show that there are both consistencies and discrepancies between large-system results and finite-sized system results and therefore, it is necessary for us to do analysis on the spectral efficiency and the coding-spreading tradeoff problem in finite-sized systems as we have done in this dissertation.
Figure 4.1: Comparisons of the finite-sized system results with the large-system results on the AWGN channel. The synchronous CDMA systems have 4 users and bandwidth expansion factor 20.
Figure 4.2: Comparisons of the finite-sized system results with the large-system results on the AWGN channel. The synchronous CDMA systems have 4 users and bandwidth expansion factor 20.
Figure 4.3: Comparisons of the finite-sized system results with the large-system results on the AWGN channel. The synchronous CDMA systems have 4 users and bandwidth expansion factor 8.
Figure 4.4: Comparisons of the finite-sized system results with the large-system results on the AWGN channel. The synchronous CDMA systems have 4 users and bandwidth expansion factor 8.
Figure 4.5: Comparisons of the finite-sized system results with the large-system results on the CLFF channel. The synchronous CDMA systems have 4 users and bandwidth expansion factor 20.
Figure 4.6: Comparisons of the finite-sized system results with the large-system results on the CLFF channel. The synchronous CDMA systems have 4 users and bandwidth expansion factor 8.
4.2 Conclusions

Much work has been done on the analysis of the spectral efficiency and the related coding-spreading tradeoff problem in synchronous CDMA systems. However, most of the work depends on large-system assumptions (large number of users). These assumptions are not valid for a practical/small system and the inconsistency can be large as demonstrated in this research.

In this dissertation, we analyzed the coding-spreading tradeoff problem in finite-sized synchronous CDMA systems. Both AWGN channel and two ideal Rayleigh flat-fading channels are considered. Numerical results are given for the minimum required $E_b/N_0$'s at different coding-spreading tradeoffs in the synchronous CDMA systems with different detection/decoding schemes. Irregular eIRA LDPC codes are adopted to approach these limits and show good performance even without special consideration of CDMA channels in their design.

In Chapter 2, the binary-input achievable information rates for finite-sized synchronous CDMA systems on the AWGN channel are studied. The coding-spreading tradeoff problem is also addressed. Numerical results show that there exist nontrivial optimal code rates for the synchronous CDMA systems with certain suboptimal MUDs. However, all coding (no spreading) is optimal for the system with the optimal MUDD. Simulations of the LDPC-coded synchronous CDMA systems with the
iterative MUDD and MMSE MUD are also presented to show that the binary-input capacities can be closely approached with practical schemes.

In Chapter 3, the binary-input achievable information rates for finite-sized synchronous CDMA systems on Rayleigh flat-fading channels are studied. Both chip-level and symbol-level flat-fading models are considered, which represent ideal fast fading and slow fading, respectively. Their effects on the coding-spreading tradeoff problem are discussed based on the numerical results and LDPC code simulation results. Comparisons with the results in Chapter 2 are also given.

Earlier in this concluding chapter, comparisons of our finite-system results with large-system results on the AWGN and CLFF channels are given which show both consistencies and discrepancies. Comparisons for the SLFF channel require much extra work.

Before we give the suggestions for further research, we note that in both Chapter 2 and 3, the rate-0.2 data points may be a little bit high because the eIRA codes suffer at low rates. We are studying other rate-0.2 LDPC codes right now.

4.3 Suggestions for Further Research

In this section, we identify a few areas that warrant further research.

First, the large-system analyses have achieved a great deal of nice results based on the large random matrix theories [44], [45] and statistical mechanics [19], [46], [47]. More work is needed to incorporate factors like asynchronism, multi-path
fading, power control, modulation, networking, or even high-level protocols. Also, new developments or applications of the knowledge and theories from other areas could result in unexpected achievements. The applications of the large random matrix theories and statistical mechanics on the analysis of CDMA systems gave us excellent examples.

Second, the finite-sized system analysis need go further to bridge the large-system analysis and practical implementations, and to provide a guidance to the design of systems, though, with few powerful analysis tools, the finite-sized system analysis could be even tougher than the large-system analysis. A comparison of finite-sized system and large-system results that we did not do in this work is on the SLFF channel. Our results are for real and binary inputs which are not comparable to the large-system results in the literature [20] where the inputs are assumed to be complex Gaussian. To make them comparable, we need consider quadrature inputs which increases the complexity of finite-sized system analysis, though it would be a meaningful work.

Third, we have seen the good performance of the eIRA codes, though there was no consideration of the CDMA channels in their design. It would be a quite interesting topic to design LDPC codes or any other coding/decoding schemes for the CDMA channels or other multiple access channels. In [41] and [42], special LDPC codes have been designed for CDMA and MIMO channels. These codes are
expected to approach the information-theoretic limits further. Similar techniques
could also be used to design codes for other channels.
REFERENCES


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