SOLAR ENERGETIC PARTICLE TRANSPORT IN THE HELIOSPHERE

by

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DEDICATION

To my parents
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The transport of solar energetic particles (SEPs) in the inner heliosphere is a very important issue which can affect our daily life. For example, large SEP events can lead to the failure of power grids, interrupt communications, and may participate in global climate change. The SEPs also can harm humans in space and destroy the instruments on board spacecraft. Studying the transport of SEPs also helps us understand remote regions of space which are not visible to us because there are not enough photons in those places.

The interplanetary magnetic field is the medium in which solar energetic particles travel. The Parker Model of the solar wind and its successor, the Weber and Davis model, have been the dominant models of the solar wind and the interplanetary magnetic field since 1960s. In this thesis, I have reviewed these models and applied an important correction to the Weber and Davis model. Various solar wind models and their limitations are presented. Different models can affect the calculation of magnetic field direction at 1 AU by as much as about 30%.

Analysis of the onset of SEP events could be used to infer the release time of solar energetic particles and to differentiate between models of particle acceleration near the Sun. It is demonstrated that because of the nature of the stochastic heliospheric magnetic field, the path length measured along the line of force can be shorter than that of the nominal Parker spiral. These results help to explain recent observations.

A two dimensional model and a fully three dimensional numerical model for the transport of SEPs has been developed based on Parker’s transport equation for the first time. “Reservoir” phenomenon, which means the inner heliosphere works like a reservoir for SEPs during large SEP events, and multi-spacecraft observation of peak intensities are explained by this numerical model.
CHAPTER 1

Introduction

In this chapter, various important heliophysical concepts, such as the solar wind and the heliosphere, and their unknown issues, are explained. Then, I briefly review the solar events and the governing theories for the transport of solar energetic particles (SEPs). The goal of this dissertation is presented at the end of this chapter.

1.1 Basic concepts

1.1.1 Solar wind

Let us start with the Sun. The Sun blows out the solar wind (mainly electrons and protons) constantly. The average speed of the solar wind is about 400 km per second, about 1 million miles per hour. At this speed, the proton’s kinetic energy is about 1000 electron volts (eV). A sample observations of the solar wind made by Ulysses during the solar minimum and the solar maximum are shown in Fig. (1.1). During the solar minimum, the solar wind has a almost constant “fast” speed (around 800 km/s) above 30 degrees in latitude, which is called the “fast” solar wind. Close to the ecliptic plane, the solar wind speed is smaller (around 300 km/s) and it has large oscillations. This is the so called “slow” solar wind. During the solar maximum, the solar wind speed oscillates at all latitude.

In 1930s, scientists had determined that the temperature of the solar corona must be around a million degrees Celsius based on the observations during total eclipses. In the mid-1950s, a British mathematician, Sydney Chapman, calculated the properties of a gas at such a high temperature and determined it would be a superb conductor of heat. At approximately the same time, a German scientist, Ludwig Biermann, postulated that the Sun emits a steady stream of particles that
Figure 1.1: The solar wind observations by Ulysses (http://solarprobe.gsfc.nasa.gov/solarprobe_science.htm).
makes the cometary tail point away from the Sun regardless of the comet’s direction of travel.

In Chapman’s model, the corona was static. The thermal pressure at infinity given by this model was around $10^{-5}$ dynes cm$^{-2}$ which is not consistent with the known value of interstellar gas, about $10^{-12}$ to $10^{-13}$ dynes cm$^{-2}$. Inspired by Chapman’s model and Biermann’s observation, Eugene Parker proposed the idea of the “solar wind” and demonstrated this mathematically in his manuscript submitted to the Astrophysical Journal in 1958 which was rejected by two reviewers. (Fortunately, this manuscript was saved by the editor, Subrahmanyan Chandrasekhar). Parker’s model of the solar wind was confirmed by observations made by the Soviet satellite Luna I in January 1959. Three years later, Marcia Neugebauer and her collaborators made a solar wind measurement using the Mariner 2 spacecraft.

In Parker’s model, which will be discussed in more detail in chapter 2, the solar wind is powered by the plasma’s thermal energy. In the 1990s, the SOHO spacecraft observed the fast solar wind emanating from the poles of the sun, and found that the wind accelerates to much faster speeds than can be accounted for by thermodynamic expansion alone. This has to be related to magnetic energy which is important for solar activities and is omitted in Parker’s model.

Parker’s model also predicted that the wind should make the transition to supersonic flow at an altitude of about four solar radii from the photosphere; but the transition (or “sonic point”) observed by SOHO was perhaps only one solar radius above the photosphere, suggesting that some additional mechanism accelerates the solar wind away from the sun (DeForest et al., 2001). Weber and Davis (1967) added the magnetic energy and the kinetic energy related to angular momentum into Parker’s model. In Weber and Davis’s model the transition can be much lower than from Parker’s model, so this model is more realistic than Parker’s model. However as I will show later, their original equation has a “typo” or needs a correction, and their numerical solution was incomplete. I present the correct solution in Chapter 2.
1.1.2 Heliosphere

The solar wind slows down eventually by the termination shock at a distance around 100 AU away from the Sun. The bubble formed by the solar wind in the Milky Way is called the “heliosphere”. The artistic versions of the heliosphere are shown in Fig. (1.2) and Fig. (1.3). Spacecraft Voyager 1 and 2, Pioneer 10 and 11 are also shown in Fig. (1.3). Neither Pioneers spacecraft is currently functional. Voyager 1 crossed the termination shock in 2005. It is believed that Voyager 2 now has crossed the termination shock.

A bow shock is thought to be formed where the interstellar gas hits the heliosphere, but this shock may be very weak and its existence is still uncertain. Maybe the Voyagers can confirm this before they lose power. The heliopause is the location where the solar wind pressure is balanced by the interstellar gas. The region between the bow shock and the heliopause is called the heliosheath. Theories and observations about these regions are not fully developed. The heliosphere is the domain of our interest in this work, although we focus on the inner heliosphere.

1.1.3 Cosmic rays

Cosmic rays are atoms, subatomic particles, neutrons, and electrons, which have much higher energy than the background fluid, the solar wind, inside the heliosphere.

Depending on their origin, cosmic rays can be divided into three categories, galactic cosmic rays (GCRs), anomalous cosmic rays (ACRs), and solar energetic particles. GCRs come from outside of the heliosphere (see Fig. (1.3)), are trapped by galactic magnetic field, and are possibly accelerated by supernova shocks. GCRs travel at close to the speed of light. ACRs originally come from the neutral interstellar wind (see Fig. (1.2)) and are accelerated by the termination shock (the acceleration process is shown in Fig. (1.4)). The neutral interstellar wind flows through the heliosphere with an average speed at tens of kilometers per second and encounters almost no resistance because it is neutral. When these neutral atoms get close to the Sun, some of them become charged due to photo-ionization or
Figure 1.2: Heliosphere 3D configuration (an artistic version, http://ourworld.compuserve.com/homepages/jbstoneking/jbspage5.htm).
Figure 1.3: Heliosphere 2D configuration (an artistic version, http://solarsystem.nasa.gov/multimedia/gallery)
charge exchange. These new-born ions (called “pick-up” ions in the literature) are then carried out to the termination shock by the solar wind and are accelerated there. Because of the diffusion process, these particles can be observed at 1 AU. Compared to GCRs, ACRs are low-energy and low speed particles. The SEPs are particles injected to space by solar flares (discussed later) or associated with coronal mass ejections (CMEs) and accelerated close to the solar surface or accelerated well away from the Sun. These SEPs can be accelerated directly by the electric field through magnetic reconnection or by shocks through diffusive shock acceleration. These processes transform electromagnetic energy into kinetic energy. Typical energy spectrum for each species is shown in Figure (1.5) (copied from Gloeckler (1984)).

1.2 Solar events

After the first solar flare was observed in the mid-19th century, SEPs were assumed to come only from flares for more than one hundred years before the discovery of CMEs in 1970s. Solar flares are violent explosions in the Sun’s atmosphere which last for minutes or tens of minutes. A typical X-ray measurement of solar flares is shown in Fig. (1.6). They produce electromagnetic radiation across the electromagnetic spectrum at all wavelengths from long-wave radio to the shortest wavelength gamma rays. Since flares usually occur in active regions around the sun spots, it is reasonable to assume that all flare-associated SEPs are accelerated by a point source in time and space. The observed properties of SEPs were explained in terms of spherically symmetric transport from a point source. This paradigm is called the “flare myth” in the literature (Gosling, 1993).

CMEs are ejections of material from the corona, usually observed with white-light coronagraph. Fig. (1.7) shows a typical image recorded by a coronagraph. Like solar flares, most CMEs also originate from solar active regions and many have solar flares associated with them. It takes about one to five days for CMEs to reach the Earth (the speed of a CME ranges from 20 to 2000 km per second). The visible
Figure 1.4: ACR acceleration diagram (http://helios.gsfc.nasa.gov/acr.html).
Figure 1.5: Typical differential energy spectra of solar energetic particles.
Figure 1.6: Typical X-ray measurement of solar flares by spacecraft GOES (http://en.wikipedia.org).
impact of a CME on the Earth is an aurora which has been reported in ancient literature including the Old Testament, Greek, and Chinese literature. The cause of the CME eruption is still unclear.

According to their duration, solar events are also called impulsive events or gradual events (Reames, 1999). Impulsive solar events usually last tens of minutes or maybe a few hours and are characterized by large enhancements of \( ^3\text{He}^{++}/^4\text{He}^{++} \) relative to coronal abundances. Gradual events last several days and their average abundances reflect coronal abundances. It is believed that interplanetary shocks are responsible for SEPs during gradual events. A detailed “classic” comparison is shown in Table 1.1 based on old observations. In this two class paradigm, the SEPs are supposed to be scatter-free in most cases (Reames, 1999).

However, we note that recent observations by ACE, WIND, and SOHO during last solar cycle show that larger Fe/O and \( ^3\text{He}/^4\text{He} \) ratio for gradual events than previously reported. What’s more, there are no shocks observed for many gradual events (Lario et al., 2003). All these suggest that this simple two-class paradigm is oversimplified and a combination of effects is taking place (Giacalone and Kóta, 2006). So the sources of SEPs remain uncertain for large SEP events.

1.3 Mechanisms affecting SEPs

SEPs are accelerated from selected thermal particles (it is about 1 keV for solar wind proton) originally from the Sun. Then, they are injected into the expanding solar wind. So the governing theories include acceleration (with selection), injection, convection, adiabatic cooling and diffusion.

1.3.1 Acceleration

Since the flare can be viewed as a point source in space and time, all properties of SEP events were explained in terms of transport from a point source, rather than as characteristics of the acceleration and of the source itself. These properties included intensity-time profiles, the longitude and latitude distributions of the particles, and
Figure 1.7: Typical coronagraph image for a CME (http://en.wikipedia.org).
all variations of abundances with time.

Currently, it is generally believed that there are two distinct acceleration mechanisms. One is called CME-driven shock acceleration. The other one is attributed to resonant wave-particle interactions associated with flares.

For particles accelerated in solar flares, the energy is thought to come from the annihilation of magnetic fields because of magnetic reconnection. Although some theories prefer shocks, most theories suggest magnetohydrodynamics (MHD) waves accelerate particles in solar flares. Particles can also be accelerated directly by the electric field at sites of the magnetic field reconnection where $B \approx 0$.

Shock acceleration includes parallel shock (the shock normal is parallel to the mean magnetic field) acceleration and perpendicular shock (the shock normal is perpendicular to the mean magnetic field) acceleration. For a parallel shock, particles traverse the shock front many times due to the magnetic irregularities. Thus particles are accelerated by the converging flows due to the compression at the shock like a ping pong ball bouncing between ping-pong paddles approaching each other. For a perpendicular shock, there is an additional effect to accelerate the particles, called the shock drift, because particles drift in the inhomogeneous mean magnetic field at the shock front parallel to the electric field ($E = -V \times B/c$). For an oblique shock, both of these acceleration mechanisms act on particles. Since new observations challenge the theories of the gradual events, the importance of interplanetary shock acceleration during gradual events is uncertain.

1.3.2 Injection into the solar wind

The injection problem is very difficult issue because we can not observe this process directly or in situ. Although we can infer the injection profile for SEPs from the observations at 1 AU, many uncertainties remain because we have to take into account transport processes, such as, interplanetary scattering, solar wind convection, adiabatic cooling, which the SEPs are involved before they are detected by spacecraft. Therefore, the injection time of SEPs is crucial to understand the processes the SEPs experienced before they are detected by space probes. This “onset time
analysis” is related to this issue and some numerical results are presented in Chapter 3.

The injection profile is usually assumed in two forms in the literature, a delta function or a Reid-Axford profile (Axford, 1965) which is,

\[
I(t) = \frac{N}{t} \times \exp \left( -\frac{\beta}{\tau} - \frac{t}{\tau} \right)
\]  

where \(N\) is a time independent constant, \(\beta\) is the coronal diffusion time, \(\tau\) is the loss time.

For solar flares, most theories use a point-source spatial injection. The difference is the injection time profile. That is to say, the differences between those models are choose of the values of \(\beta\) and \(\tau\). Because of the difficulty of determining the 3-D structure of a CME shock, it is very difficult to take the injection as a function of space although it is easier to investigate how the particles escape from the shock.

1.3.3 Transport

Generally speaking the transport of SEPs is governed by the Boltzmann equation. Because the density of the solar wind in the interplanetary space is very small, about \(1 \sim 10\) particles per \(\text{cm}^3\) at 1 AU, all the particles are collisionless, but SEPs experience pitch angle scattering in the interplanetary magnetic field. Depending on the effectiveness of the pitch angle scattering, two transport theories are commonly used to describe the SEPs, which are developed by Parker (1965) and by Roelof (1969) respectively.

If the scattering process is strong enough to make the energetic particle distribution nearly isotropic in the plasma frame. In this case, the cosmic ray distribution function satisfies Parker’s equation (Parker 1965),

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial x_i} \left( \kappa_{ij} \frac{\partial f}{\partial x_j} \right) - u_i \frac{\partial f}{\partial x_i} + \frac{1}{3} \frac{\partial u_i}{\partial x_i} \frac{p}{p} \frac{\partial f}{\partial p} + Q .
\]  

(1.2)

Here \(f(p, x_i, t)\) is the omni-directional distribution function, \(x_i\) is the spatial variable, \(p\) is the momentum magnitude, \(u_i\) is the solar wind velocity, and \(Q\) is the source term. \(\kappa_{ij}\) is the spatial diffusion tensor which can be decomposed into two parts,
$\kappa_s$, the symmetric part, and $\kappa_A$, the anti-symmetric part. The divergence of the anti-symmetric part is $\nabla \cdot \kappa_A = pcv(3q)^{-1}\nabla \times \mathbf{B}/B^2$ which is the drift velocity. $\kappa_s$ represents the spatial diffusion parallel $\kappa_\parallel$ and perpendicular $\kappa_\perp$ to $\mathbf{B}$. One challenging problem in recent research is that the estimation of the spatial diffusion coefficient perpendicular to $\mathbf{B}$, $\kappa_\perp$.

We can express $\kappa_{ij}$ as

$$
\kappa_{ij} = \kappa_\perp \delta_{ij} + \frac{B_i B_j}{B^2} (\kappa_\parallel - \kappa_\perp) + \epsilon_{ijk} \kappa_A B_k B .
$$

where $B_i$, $B_j$, and $B_k$ are the magnetic components. This equation, which has most of the effects: diffusion, convection, adiabatic deceleration, and drifts, is used to model the transport of the ACRs, GCRs, and SEP events.

Because the two-class paradigm prevailed, the Parker’s equation was ignored for many years. Most of numerical research is based on Roelof’s equation. The Roelof equation is also called the focused transport equation in the literature, because in this case, the focusing effect is more important. If the scattering is negligible for the SEPs, Roelof’s equation is preferred.

$$
\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial z} + \frac{1 - \mu^2}{2L} v \frac{\partial f}{\partial \mu} = \frac{\partial}{\partial \mu} \left( D_{\mu\mu} \frac{\partial f}{\partial \mu} \right) + Q
$$

where $f(z, \mu, t)$ is the gyro-phase averaged distribution function. $v$ is the particle speed, $z$ is the distance along the magnetic filed line, $\mu = \cos \theta$ is the particle pitch-angle cosine and $t$ is the time. $L(z) = -B(z)/(\partial B/\partial z)$ is called the focusing length in the diverging magnetic field. When the radial magnetic field is considered, $L(z)$ is replaced by $r/2$. $D_{\mu\mu}$ is the pitch-angle diffusion coefficient which represents stochastic forces. $Q$ is the source term which represents the injection of the energetic particles near the Sun. The third term is the focusing effect. There is no perpendicular component in this equation.

This equation is often used to describe the evolution of the energetic particle in solar flares (Dröge, 2000; Ng and Reames, 1994; Ruffolo, 1991).
1.3.4 Adiabatic deceleration

Since the particle is moving along the expanding magnetic field, it is cooled adiabatically (Parker, 1965),

\[
\frac{1}{p} \frac{dp}{dt} = -\frac{1}{3} \frac{\partial v_i}{\partial x_i}
\]

\[
\frac{1}{T} \frac{dT}{dt} = -\frac{n(T)}{3} \frac{\partial v_i}{\partial x_i}
\]

where \(n(T) = 2\) for non-relativistic particles and otherwise \(n(T) = 1\) for extreme relativistic particles where \(T\) is the kinetic energy of particles.

Now let us derive Equation (1.6). First we know that volume expansion rate for an infinitesimal material element is \(\Delta = \nabla \cdot \mathbf{v}\). From thermodynamics, we know that this material element will do work to the environment when it expands. Thus for a non-relativistic case, \(P = 2/3T\) (G. K. Batchelor, An introduction to fluid dynamics, p39, 1.7.3) where \(P\) is pressure for non-relativistic particle.

\[
\frac{dT}{dt} = -P \delta V = -\frac{2}{3} \nabla \cdot \mathbf{v} T
\]

where \(V\) is volume, \(n(T) = 1 + \gamma^{-2}, \gamma = 1/\sqrt{1 - v^2/c^2}, T = (\gamma - 1)mv^2\). For relativistic particles, we know that \(P = T/3\), so Parker’s result for \(n(T) = 1\) is correct.

So the adiabatic cooling comes from the diverging flow of the solar wind. It is important for low energy particles but not important for GeV or TeV GCRs.

1.4 Broader impacts of my research

1.4.1 Roles of SEPs in space weather

Space weather refers to the environmental conditions within interplanetary or interstellar space, which is distinct from the concept of weather within a planetary atmosphere. Coronal mass ejections and their associated shock waves are important drivers of space weather as they can compress the magnetosphere and trigger
geomagnetic storms. SEPs can damage electronics on board spacecraft in earth orbit and outside the earth’s magnetosphere, change radiation doses received by the crew flying in high altitude aircraft over the polar regions, and threaten the life of astronauts. The soft X-ray flux of X class flares increases the ionization of the upper atmosphere, which can interfere with short-wave radio communication, and can increase the drag on low orbiting satellites, leading to orbital decay. The radiation risk posed by solar flares and CMEs is one of the major concerns in discussions of manned missions to Mars or to the Moon. Some kind of physical or magnetic shielding would be required to protect the astronauts. For the astronauts on Mars, they may have as little as 15 minutes to get into a shelter. Federal Aviation Regulations requires reliable communication over the entire flight of a commercial aircraft. It is estimated to cost about $100,000 each time such a flight is diverted from a polar route (http://www.aiaa.org/pdf/public/stills_united.pdf).

Here are several examples of space weather events:

The CME on March 9, 1989 caused the collapse of the Hydro Quebec power network on March 13, 1989. The collapse led to a general blackout, which lasted more than 9 hours and affected 6 million people (http://www.agu.org/sci_soc/eiskappenman.html).

CME on January 7, 1997 caused the loss of the AT&T Telstar 401 communication satellite, a $200 million value satellite. (http://www.solarstorms.org/SWChapter2.html).

A large event happened on August 7, 1972, between Apollo 16 and Apollo 17. The dose of particles would have been deadly or at least life-threatening to astronauts if this event had happened during one of these missions (http://science.nasa.gov/headlines/y2006/01sep_sentinels.htm).

1.4.2 Using SEPs to test basic physics

SEPs are probes of heliosphere structure or properties and conditions in solar active regions, which are inaccessible to spacecraft. For example, the charge state of Fe was found to be $14 \pm 1$ on average in some solar events. This indicated that the
source material had an electron temperature of around 2 MK, a typical temperature of corona. In other events, the charge state of Fe was found to be $20.5 \pm 1.2$ on average, an indication of an electron temperature of about 10 MK. We can also tell the selection and acceleration from the ionic charge state.

This research can help to understand the physics underlying the production of the SEPs. Except the $\gamma$-rays from flares, most the heliospheric source of energetic particles can not be seen with photons because most of the acceleration takes place in very low density regions where the measurable intensity of photons can not be produced. Thus our information on the properties of the cosmic rays comes from the particles themselves.

1.4.3 Summary of this work

Currently situation of the particle transport is that there two well-known mathematic models (Equation (1.2) and (1.4)) deduced from the Boltzmann equation. But they both can not be solved analytically. Most of the research has been done for the Equation (1.4) because this equation is 1D in space in last century. For equation 1.2, efforts have been to solve it in 1D in 1960s and 1970s. With the development of computer technology, a fully 3D calculated is desirable and affordable. As far as I know, there is no successful 2D or 3D code has been reported to solve Equation (1.2) for solar energetic particles. In this dissertation, I present my 2D and 3D numerical solution to Parker’s equation and discuss their applications to the observations, energetic particle observations.

Specifically, this dissertation is organized in this way:

Chapter 2: different interplanetary magnetic field models and their effects on the SEPs are discussed. The first half of this chapter is focused on the Weber and Davis model. I point out the errors made by Weber and Davis (1967) and gives the correct solutions for the isothermal and polytropic corona (Pei et al., 2007a). The last half is based on the idea from Joe Giacalone and J. R. Jokipii. All magnetic field models discussed in this chapter are for the mean interplanetary magnetic field.

Chapter 3: the effect of magnetic turbulence on the on-set time analysis is dis-
cussed (Pei et al., 2006). The magnetic field in this chapter consists of a mean interplanetary magnetic field with superimposed magnetic irregularities. I have shown that because of the nature of the stochastic heliospheric magnetic field, the path length measured along the line of force can be shorter than that of the nominal Parker spiral. This leads to a time interval required for solar-energetic particles to cross 1 AU that is shorter than expected based on a nominal spiral field. Our results may help explain recent observations (Hilchenbach et al., 2003).

Chapter 3: the numerical method to solve Parker’s equation is presented. I also include my treatment of boundary conditions and the initial condition in this chapter. The Parker’s equation is discretized according to the alternating direction implicit (ADI) method in spherical coordinates.

Chapter 4: “reservoir” phenomenon is discussed and numerical simulation is provided. The SEP distribution function $f$ satisfies the transport equation 1.2. In this chapter, I provide a method to solve 2D diffusion for SEPs. Thus I can quantitatively explain both the reservoir phenomenon and the high latitude observations by Ulysses (Pei et al., 2007c).

Chapter 5: multi-spacecraft observations is discussed and compared with numerical solutions. In this chapter I show that a three dimensional solution of Parker’s transport equation (1.2 can explain the multi-spacecraft observations of peak intensity of SEPs (Wibberenz and Cane, 2006). The simulations use the numerical method described in Chapter 4 which reduced the dependence on particle momentum (Pei et al., 2007b).

Chapter 6: conclusions and future work is discussed.
Table 1.1: The comparison of impulsive events and gradual events.

<table>
<thead>
<tr>
<th></th>
<th>Impulsive Events</th>
<th>Gradual Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration time</td>
<td>minutes</td>
<td>hours or days</td>
</tr>
<tr>
<td>(3\text{He}^{++}/\text{He}^{++})</td>
<td>(\sim 1)</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td>Fe/O</td>
<td>(\sim 1)</td>
<td>(\sim 0.1)</td>
</tr>
<tr>
<td>Fe charge state</td>
<td>20</td>
<td>14</td>
</tr>
<tr>
<td>longitude coverage</td>
<td>&lt; 20(^\circ)</td>
<td>160(^\circ)</td>
</tr>
<tr>
<td>radio burst type</td>
<td>III</td>
<td>II, IV</td>
</tr>
</tbody>
</table>
CHAPTER 2

The Interplanetary Magnetic Field and Its Effects

The solar wind and the interplanetary magnetic field provide the forces that govern the motion of SEPs. The interplanetary magnetic field originates from the Sun and is carried away almost radially by the solar wind which is predicted by Parker in 1958 and is confirmed later by USSR spacecraft Luna I in 1959 and by Mariner 2. Currently widely used model is that by Weber and Davis (1967). They added the azimuthal component of the solar wind and the magnetic force into Parker’s model. I will show that Weber and Davis’ original solution is only one of the infinite physically reasonable solutions in their model in this chapter. The last half chapter is focused on the effects of different solar wind models which has not been fully realized by the community.

2.1 Parker’s model

Before Parker, it was generally believed that the corona was static which led to a finite pressure at infinity, about seven or eight orders of magnitude higher than the pressure of the interstellar gas. Realizing this discrepancy, Parker argued that the corona must undergo a steady expansion which forms the solar wind.

Parker treats the corona as a neutral plasma with the mass density of

$$\rho = n(m_p + m_e) \approx nm$$  \hspace{1cm} (2.1)

where $m_p$ and $m_e$ are the mass of the proton and the mass of the electron respectively. Here, we use $m$ to represent the mass of the proton for simplicity. The plasma temperature, $T$, is a constant (assume the corona is isothermal) which is the average of the proton temperature, $T_p$, and the electron temperature, $T_e$.

$$T = \frac{T_p + T_e}{2}.$$  \hspace{1cm} (2.2)
So the plasma pressure, \( P \), is given by the equation of state

\[
P = nk(T_e + T_p) \approx 2nkT
\]  
(2.3)

where \( k \) is the Boltzmann constant.

In this model, mass conservation gives

\[
\rho ur^2 = I
\]  
(2.4)

where \( I \) is a constant. The momentum equation is given by

\[
\rho u \frac{du}{dr} = -\frac{dP}{dr} - \rho \frac{GM_\odot}{r^2}
\]  
(2.5)

where \( G \) is the gravitational constant and \( M_\odot \) is the mass of the Sun.

By using the Equation (2.4), the momentum equation is transformed into

\[
\frac{1}{u} \left( u^2 - \frac{2kT}{m} \right) \frac{du}{dr} = \frac{4kT}{mr} - \frac{GM_\odot}{r^2}
\]  
(2.6)

which can be integrated into

\[
\frac{u^2}{2} - \frac{2kT}{m} \ln(u r^2) - \frac{GM_\odot}{r} = C
\]  
(2.7)

where \( C \) is a constant related to the energy flux. One can recognize this equation as Bernoulli’s equation for the solar wind (Parker, 1963). The term which has a logarithm is because for isothermal case, polytropic index \( \gamma = 1 \) (Note that in Equation (2.11), \( \gamma > 1 \)). This equation has infinite solutions which are shown in a contour plot, Figure (2.1), depending on the different choices of the constant \( C \). The only physically acceptable solution is the one that passes through the critical point and continues to increase based on boundary conditions. So the solar wind is a transonic flow.

The critical point, where the speed of the solar wind is equal to the local sound speed, can be determined from Equation (2.6). By setting both sides of Equation (2.6) equal to zero, we have

\[
u_c^2 = \frac{2kT}{m}
\]  
(2.8)

\[
r_c = \frac{GM_\odot m}{4kT}
\]  
(2.9)
Figure 2.1: The contour plot of Parker’s model for the solar wind.
where \( u_c \) is the critical speed and \( r_c \) is position of the critical point.

If the corona is polytropic,

\[
P = P_0 \left( \frac{\rho}{\rho_0} \right)^\gamma
\]

where \( P_0 \) and \( \rho_0 \) is pressure and density at some reference position. \( \gamma \) is the polytropic index. Bernoulli’s equation becomes

\[
\frac{u^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P_0}{\rho_0} \left( \frac{ur^2}{r_0^2u_0} \right)^{1-\gamma} - \frac{GM_\odot}{r} = C_2
\]

From this equation, we can see that for each \( \gamma \), there is one physical solution.

### 2.2 “Frozen-in” magnetic field

The interplanetary magnetic field is “convected” away from the Sun because the solar wind has a extremely high electrical conductivity and the magnetic field is “frozen” in the plasma. Thus the solar magnetic field expands into the interplanetary region with the solar wind. Mathematically this can be proven. Consider the change of magnetic flux in time through a closed contour, \( L \), moving with the solar wind,

\[
\frac{d\Phi}{dt} = \frac{d}{dt} \int B \cdot dA = \int \frac{\partial B}{\partial t} \cdot dA + \int B \cdot \frac{\partial}{\partial t}(dA)
\]

\[
= \int (-c \nabla \times E) \cdot dA + \oint_L B \cdot (-dl \times V)
\]

\[
= -c \int_L dl \cdot \left( E + \frac{V \times B}{c} \right)
\]

\[
= 0.
\]

Here, we already use Faraday’s law to get rid of the time derivative of the magnetic field. By Ohm’s law, the current, \( J \) is

\[
J = \sigma \left( E + \frac{V \times B}{c} \right).
\]

If the current is finite when \( \sigma \) is almost infinite,

\[
E = -\frac{V \times B}{c}.
\]
So we have $d\Phi/dt = 0$. This means the field lines bundled by $L$ at time $t$ will be bundled by it thereafter. So magnetic field lines move with the fluid. This statement is similar to the Kelvin theorem for inviscid flow which states that the circulation is conserved for a material contour and vortex lines move with the fluid.

Under Parker’s assumption, the solar wind velocity in a frame rotating with the Sun in spherical coordinate is

\begin{align*}
V_r &= u(r), \\
V_\phi &= -\omega r \sin \theta, \\
V_\theta &= 0.
\end{align*}

where $\omega = 2.7 \times 10^{-6}$ radian sec$^{-1}$ is the angular velocity of solar rotation. The azimuthal component is entirely due to the solar rotation. With the “frozen-in” field, the magnetic field line is determined by

\begin{equation}
\frac{1}{r} \frac{d}{d\phi} \left( \frac{dr}{d\phi} \right) = \frac{V_r}{V_\phi} = \frac{u}{-\omega r \sin \theta}
\end{equation}

The numerical solution of Equation (2.7) or (2.11) shows that $u$ is almost a constant after a few times of $r_c$ where $r_c$ is the radial position of the critical point (one typical solution is shown in Figure (2.2)). Set $u$ to a constant, say, $u(r) = u_0$. From the fact that divergence of $\mathbf{B}$ is zero, we have

\begin{equation}
r^2 B_r = r_0^2 B_0 = \text{const.}
\end{equation}

Thus the magnetic field is

\begin{align*}
B_r &= B(r_0, \phi_0, \theta) \left( \frac{r_0}{r} \right)^2 \\
B_\phi &= -B(r_0, \phi_0, \theta) \left( \frac{r_0}{r} \right)^2 \frac{r \Omega \sin \theta}{u_0} \\
B_\theta &= 0.
\end{align*}

This magnetic field is shown in Fig. (2.3) which is called Parker’s Spiral in the literature. If we don’t take $u$ as a constant, we must integrate Equation (2.18) numerically.
Figure 2.2: One sample solution of solar wind speed in Parker’s model.
Figure 2.3: Parker’s Spiral.
2.3 Weber and Davis’ model

In Parker’s model, the magnetic force is not considered in the momentum equation. The azimuthal component of the solar wind is not included either. Weber and Davis (1967) took into account both of these effects in a polytropic case on the ecliptic plane. They concluded that their solution for the radial component, \( u \), is very close to Parker’s solution \( u = u(r) \). Here I will show that their solution is incomplete and misleading. For completeness, I add the discussion of the isothermal corona where the discussion is not limited to the ecliptic plane.

The solar wind velocity in this model is defined as

\[
V = u \hat{r} + v_\phi \hat{\phi} \tag{2.23}
\]

where \( \phi \) is the azimuthal angle.

The magnetic field is defined as

\[
B = B_r \hat{r} + B_\phi \hat{\phi} \tag{2.24}
\]

We also assume that the magnetic field is in a steady state, so \( dB/dt = 0 \). From Maxwell’s equation,

\[
c(\nabla \times E)_\phi = \frac{1}{r} \frac{d}{dr} \left[ r (u B_r - v_\phi B_\phi) \right] = 0 \tag{2.25}
\]

where we use the fact that \( E = -V \times B/c \) which was shown in (2.1). In a frame co-rotating with the Sun, we know \( V' \) is parallel to \( B \) since \( E' = 0 \), thus

\[
r (u B_\phi - v_\phi B_r) = -\Omega \sin \theta r^2 B_r = \text{Const.} \tag{2.26}
\]

where \( \theta \) is the polar angle. The \( \phi \) component of the momentum equation is

\[
\rho \frac{u}{r} \frac{d}{dr} (r v_\phi) = \frac{1}{c} J \times B = \frac{1}{4\pi} [(\nabla \times B) \times B]_\phi = \frac{B_r}{4\pi} \frac{d}{dr} (r B_\phi) \tag{2.27}
\]

By using Equation (2.27), (2.19), (2.26), and (2.4), we have

\[v v_\phi - \frac{r B_r B_\phi}{4\pi \rho u} = L \tag{2.28}
\]
where $L$ is a constant which can be determined at the Alfvénic critical point where the radial Alfvénic Mach number is equal to 1. The radial Alfvénic Mach number is defined as $M_a^2 = \frac{4\pi \rho u^2}{B_r^2}$. From Equation (2.28) and (2.26), we find the azimuthal velocity is

$$v_\phi = \Omega \sin \theta r \frac{M_a^2 L}{r^2 \Omega \sin \theta} - \frac{1}{M_a^2 - 1}$$

(2.29)

Let us denote $r_a$ and $u_a$ as the radius and radial velocity at the Alfvénic critical point, respectively. In Equation (2.29), for a finite value of $v_\phi$ at this point, the constant $L$ must have the value

$$L = \Omega r_a^2 \sin \theta$$

(2.30)

By using quantities at the critical point, we can also get $M_a^2 = \frac{\rho_a}{\rho} = \frac{u r^2}{u_a r_a^2}$. Now we can solve the radial momentum equation

$$\rho u \frac{du}{dr} = -\frac{dP}{dr} - \frac{GM_\odot}{r^2} + \frac{1}{c}(\mathbf{J} \times \mathbf{B})_r + \frac{v_\phi^2}{r}$$

(2.31)

After rearranging Equation (2.31), for a polytropic corona, the radial momentum equation becomes

$$\frac{r}{u} \frac{du}{dr} \times \left[ \left( u^2 - \frac{\gamma P_a}{\rho_a} M_a^{2(1-\gamma)} \right) (M_a^2 - 1)^3 - \Omega^2 r^2 M_a^4 \left( \frac{r_a^2}{r^2} - 1 \right)^2 \right]$$

$$= \left( 2\gamma \frac{P_a}{\rho_a} M_a^{2(1-\gamma)} - \frac{GM_\odot}{r} \right) (M_a^2 - 1)^3$$

$$+ \Omega^2 r^2 \left( \frac{u}{u_a} - 1 \right) \left[ (M_a^2 + 1) \frac{u}{u_a} - 3M_a^2 + 1 \right]$$

(2.32)

Note that this equation is a little different from Weber and Davis’ original equation. I believe that they had a typo in their original equation (there are several other typos in their original paper).

The Bernoulli’s equation for the polytropic corona is

$$\frac{u^2}{2} + \frac{\gamma P_a}{\gamma - 1} M_a^{2(1-\gamma)} - \frac{GM_\odot}{r} + \frac{\Omega^2 r_a^4}{2r^2} \left[ 1 + \frac{(2M_a^2 - 1)(r^2 - r_a^2)^2}{r_a^4 (M_a^2 - 1)^2} \right] = C_4$$

(2.33)

where $C_4$ is a constant related to the energy flux.
At the critical point, the coefficient of the derivative in Equation (2.32) or Equation (2.50) vanishes. Thus we can use the following equations to determine the critical point,

\[
\left( u^2 - \frac{\gamma P_a}{\rho_a} M_a^{2(1-\gamma)} \right) (M_a^2 - 1)^3 = \Omega^2 r^2 M_a^4 \left( \frac{r_a^2}{r^2} - 1 \right)^2 \tag{2.34}
\]

\[
\left( \frac{GM_{\odot}}{r} - \frac{2\gamma P_a}{\rho_a} M_a^{2(1-\gamma)} \right) (M_a^2 - 1)^3 = \Omega^2 r^2 \left( \frac{u}{u_a} - 1 \right) \times \left[ (M_a^2 + 1) \frac{u}{u_a} - 3M_a^2 + 1 \right] \tag{2.35}
\]

Due to the introduction of magnetic field into the momentum equation, there are three critical points in the contour plot of Equation (2.33) shown in Figure (2.4) and Figure (2.5) (suffix c will denote the quantities for the first critical point, a for the second and f for the third). The Equation (2.33) is a non-linear equation which may have

1. a unique solution,
2. no solution, or
3. infinite solutions.

To have a physically reasonable solution, the contour must pass through all three critical points. Physically, it means the energy flux must be conserved along the path which is implied by Equation (2.33). From numerical solutions, we find that there are infinite physically reasonable solutions satisfying this requirement for this problem. To prove this, let us begin with the calculation of Equation (2.33) at the Alfvénic critical point. Because this is a singular point, We need an asymptotic expansion in the neighborhood of this point. Suppose in the neighborhood of the Alfvénic critical point,

\[
u = u_a + \epsilon_a (r - r_a) + \epsilon_a^2 (r - r_a)^2 + \cdots , \tag{2.36}\]

thus

\[
\lim_{M_a \to 1} \frac{(r^2 - r_a^2)}{(M_a^2 - 1)} = 1/(1 + 0.5 \epsilon_a r_a/u_a) . \tag{2.37}\]

Equation (2.33) at this critical point becomes

\[
\frac{u_a^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P_a}{\rho_a} - \frac{GM_{\odot}}{r_a} + \frac{\Omega^2 r_a^2}{2} \left[ 1 + \frac{1}{(1 + 0.5 \epsilon_a r_a/u_a)} \right] = C_4(r_a, u_a) . \tag{2.38}\]
Where \( C_4(r_a, u_a) \) stands for the constant at the Alfvénic point. So depending on the value of \( \epsilon_a \), \( C_4(r_a, u_a) \) takes different values. We can verify this from Figure 2.5 that all the contour lines close to the Alfvénic critical point (the second critical point) are connected to it.

Suppose we define \( C'_4(r_a, u_a) \) as

\[
\frac{u_a^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P_a}{\rho_a} - \frac{GM_\odot}{r_a} + \frac{\Omega^2 r_a^2}{2} = C'_4(r_a, u_a),
\]

(2.39)

which is always smaller than \( C_4(r_a, u_a) \), we can always find a contour that connects the first and the second critical point. We will show that \( C_4(r_f, u_f) \) is always greater than \( C'_4(r_a, u_a) \). Thus the third critical point and the second critical point are always connected to each other. But since \( C_4(r_a, u_a) \) is generally different from \( C_4(r_f, u_f) \), the three critical points are not always connected by the same contour. We need to adjust \( r_a, u_a \) to connect the three critical points. For each different \( \gamma \), we can find at least one pair of \( (r_a, u_a) \) to satisfy this. Table 2.1 shows the numerical results.

We know that the third critical point is very close to the Alfvénic critical point (Figure 2.5). So we can do a similar expansion around the third critical point. Suppose \( u_f = u_a + \epsilon u_a, r_f = r_a + \delta r_a \). So \( u_f^2 \approx (1 + 2\epsilon)u_a^2, M_f^2(r_f, u_f) \approx 1 + \epsilon + 2\delta \) where \( f \) denotes the third critical point quantities. \( C_4(r_f, u_f) \) stands for the constant at the third critical point, which is

\[
\frac{u_a^2(1 + 2\epsilon)}{2} + \frac{\gamma}{\gamma - 1} \frac{P_a}{\rho_a} - \frac{\gamma P_a}{\rho_a}(\epsilon + 2\delta) - \frac{GM_\odot}{r_a}(1 - \delta) + \frac{\Omega^2 r_a^2}{2}(1 - 2\delta) \left[ 1 + \frac{(1 + 2\epsilon + 4\delta)4\delta^2}{(\epsilon + 2\delta)^2} \right] = C_4(r_f, u_f).
\]

(2.40)

If we substitute \( u_f = u_a + \epsilon u_a, r_f = r_a + \delta r_a \) into Equation (2.34) and Equation (2.35), we have

\[
\left( u_a^2(1 + 2\epsilon) - \frac{\gamma P_a}{\rho_a}(1 + \epsilon + 2\delta)^{1-\gamma} \right)(\epsilon + 2\delta)^3 = 4\Omega^2 r_a^2 \delta^2 \\
\times (1 + 2\epsilon + 6\delta)
\]

\[
\left( \frac{GM_\odot}{r_a}(1 - \delta) - \frac{2\gamma P_a}{\rho_a}(1 + \epsilon + 2\delta)^{1-\gamma} \right)(\epsilon + 2\delta)^3 = -2\Omega^2 r_a^2 \epsilon \\
\times \delta(2 - \epsilon + 4\delta)
\]

(2.41)
Figure 2.4: The contour plot of Weber and Davis’ model for the solar wind. This plot is different from the original plot of Weber after several corrections to their model.
Figure 2.5: The zoomed figure of Fig. (2.4) shows the second & the third critical point. The second critical point is the one at the left lower corner. The third one is close to $r = 1.71$ AU.
If we multiply Equation (2.41) by $\epsilon$, multiply Equation (2.41) by $\delta$, and add them up, we have

$$\epsilon u_a^2 - (\epsilon + 2\delta) \frac{\gamma P_a}{\rho_a} + \frac{\delta GM_\odot}{r_a} = -\epsilon \left(2\epsilon u_a^2 - (\epsilon + 2\delta)(1 - \gamma) \frac{\gamma P_a}{\rho_a}\right)$$

$$+ \delta \left(\frac{\delta GM_\odot}{r_a} + 2(\epsilon + 2\delta)(1 - \gamma) \frac{\gamma P_a}{\rho_a}\right)$$

$$+ \frac{10\epsilon^2 \delta^2 + 16\epsilon \delta^3}{(\epsilon + 2\delta)^3} \Omega^2 r_a^2$$

$$\approx \frac{10\epsilon^2 \delta^2 + 16\epsilon \delta^3}{(\epsilon + 2\delta)^3} \Omega^2 r_a^2$$

(2.42)

Thus the difference between $C'_4(r_a, u_a)$ and $C_4(r_f, u_f)$ is given by

$$C_4(r_f, u_f) - C'_4(r_a, u_a) = \epsilon u_a^2 - (\epsilon + 2\delta) \frac{\gamma P_a}{\rho_a} + \frac{\delta GM_\odot}{r_a}$$

$$+ \frac{\Omega^2 r_a^2}{2} \left[(1 - 2\delta)(1 + 2\epsilon + 4\delta)4\delta^2 - 2\delta\right]$$

$$= \frac{2\delta \Omega^2 r_a^2}{(\epsilon + 2\delta)^3} (-16\delta^4 - 16\delta^3 \epsilon - 4\delta^2 \epsilon^2 - 4\delta^3 + 2\delta^2 \epsilon + \delta \epsilon + \delta \epsilon^2 + 2\delta^2 - \epsilon^3)$$

$$\approx \frac{2\delta \epsilon^2 \Omega^2 r_a^2}{(\epsilon + 2\delta)^3} \left[(2 - 4\delta) \left(\frac{\delta}{\epsilon}\right)^2 + (1 + 2\delta) \frac{\delta}{\epsilon} + \delta - \epsilon\right]$$

(2.43)

if we neglect the high order term.

We can estimate the ratio of $\epsilon/\delta$. From Equation (2.41) and (2.41), we have

$$\frac{\delta}{\epsilon} \approx \frac{u_a^2 - \gamma P_a}{\frac{2\gamma P_a}{\rho_a} - \frac{GM_\odot}{r_a}} > \frac{(u_a^2)_{\text{min}} - (\gamma P_a)_{\text{max}}}{(2\gamma P_a)_{\text{max}}} \approx 1.5$$

(2.44)

Thus $\delta > \epsilon$ and the right hand side of Equation (2.43) is always greater than zero which means $C_4(r_f, u_f) > C'_4(r_a, u_a)$. So there is always one contour connecting these two critical points. Actually, as we can see from Figure 2.5, there are two contours connect the second and the third critical point, but as $r$ tends to infinity, only the one with $u$ increasing satisfies the boundary condition.

### 2.3.1 Numerical solution of the Weber and Davis model

The boundary conditions at 1 AU are

$$u_E = 400 \text{ km/sec}$$

(2.45)
\[ B_{rE} = 5 \text{ gamma} \quad (2.46) \]
\[ \rho_E = 11.7 \times 10^{-24} \text{ gm cm}^{-3} \text{ i.e., 7 protons cm}^{-3} \quad (2.47) \]
\[ T_E = 2 \times 10^{50} \text{ K} \quad (2.48) \]

which are the same boundary conditions used by Weber and Davis (1967). The subscript \( E \) indicates values measured near the Earth.

To solve Equation (2.33), we need to determine \( \gamma, \frac{P_a}{\rho_a}, \) and \( C_4 \). We also need to determine the position and speed at three critical points, \( r_c, u_c, r_a, u_a, r_f, \) and \( u_f \). \( GM_\odot \) are constants. \( \frac{P_a}{\rho_a} \) can be determined by the boundary conditions at 1 AU by using Equation (2.4) and Equation (2.10),

\[
\frac{P_a}{\rho_a} = \frac{P_E}{\rho_E} \left( \frac{\rho_a}{\rho_E} \right)^{\gamma - 1} = \frac{2kT_E}{m} \left( \frac{u_{E}r_{E}^2}{u_a r_a^2} \right) = \frac{2kT_E}{m} \left( \frac{B_{rE}}{4\pi\rho_E u_E^2} \right)^{1-\gamma} \quad (2.49)
\]

Thus totally we have eight unknowns, \( \gamma, r_{[c,a,f]}^{[c,a,f]}, u_{[c,a,f]} \), and \( C_4 \). At each critical point except the second one, these unknowns satisfy three equations, Equation (2.34), (2.35), and (2.33). At 1 AU, they also satisfy Equation (2.33) with \( r = r_E \) and \( u = u_E \). So we have seven equations to use. One of the eight unknowns must be a free parameter. Thus we can freely set a value of \( \gamma \) to make the number of unknowns and the number of equations equal. Clearly, for this problem, if we don’t set \( \gamma \), we have infinite solutions which contradict the claim by Weber and Davis. They think at the second critical point we can also use Equation (2.33), which is not true since at this critical point, \( C_4 \) has infinite values as I have shown earlier in this chapter. So we can not use this equation to give more useful information, for example, to determine the value of \( \gamma \).

The numerical solution of these seven equations are shown in Table (2.1) for \( \gamma \) between 1.15 to 1.40 (\( \gamma = 1 \) is for isothermal case). For each \( \gamma \) there is one solution which is similar to Parker’s polytropic solution. \( r_c \), the first critical position is decreasing with \( \gamma \). At \( \gamma = 1.15 \), \( r_c \) is almost 4.2 solar radii. At \( \gamma = 1.40 \), \( r_c \) is only about 1.15 solar radii away from the center of the Sun.

At \( \gamma \) close to 1.3, we notice that \( r_c \) is close to 2 solar radii from Table (2.1) which is consistent with SOHO’s observation that the transonic point is about 1
solar radius above the photosphere. In Weber and Davis’ original solution, $r_c$ is around 3 solar radii.
Table 2.1: New Solutions of Weber & Davis' model

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For the isothermal corona, all the arguments still hold. For completeness, I list equations for the isothermal case. The momentum equation is

\[ \frac{r}{u} \frac{du}{dr} \times \left[ \left( u^2 - \frac{2kT}{m} \right) (M_a^2 - 1)^3 - \Omega^2 r^2 M_a^4 \left( \frac{r_a^2}{r^2} - 1 \right)^2 \right] = \frac{4kT}{m} - \frac{GM_\odot}{r} (M_a^2 - 1)^3 + \Omega^2 r^2 \left( \frac{u}{u_a} - 1 \right) \]

\[ \times \left( M_a^2 + 1 \right) \frac{u}{u_a} - 3M_a^2 + 1 \] (2.50)

The Bernoulli’s equation for the isothermal corona is

\[ \frac{u^2}{2} - \frac{2kT}{m} \ln(\text{ur}^2) - \frac{GM_\odot}{r} + \frac{\Omega^2 r_a^4}{2r^2} \left[ 1 + \frac{(2M_a^2 - 1)(r^2 - r_a^2)^2}{r_a^4(M_a^2 - 1)^2} \right] = C_3 \] (2.53)

2.4 Effects of different solar wind models

First let us define several constants

\[ r_\odot = 7 \times 10^{10} \text{cm} \] (2.54)

\[ r_0 = 10r_\odot \] (2.55)

\[ V_w = 4 \times 10^7 \text{ cm/sec} \] (2.56)

\[ \Omega_\odot = 2.7 \times 10^{-6} \text{radian/sec} \] (2.57)

Based on these constants, the relative errors of the calculation of radius, of the field length, and of the footpoint of the magnetic field line on the solar surface, are discussed in this section. This work initially discussed by Joe Giacalone and J. R. Jokipii. I present my calculations analytically and numerically based on their idea.

2.4.1 Calculation of Radius

There are 3 azimuthal velocities in inertial frame in the literature,

\[ v_\phi = 0 \] (2.58)

\[ v_\phi = r_0 \Omega_\odot \sin \theta \] (2.59)

\[ v_\phi = r \Omega_\odot \sin \theta \left( \frac{r_0}{r} \right)^2 \] (2.60)
Parker's model corresponds to Equation (2.58). Weber & Davis’s model is very close to Equation (2.60). Equation (2.59) physically means that the azimuthal velocity of solar wind is a constant regardless of \( r \). This is not correct although it has been used by many researchers.

In co-rotating frame they become

\[
v_\phi = -r \Omega_\odot \sin \theta \tag{2.61}
\]

\[
v_\phi = r_0 \Omega_\odot \sin \theta - r \Omega_\odot \sin \theta \tag{2.62}
\]

\[
v_\phi = r \Omega_\odot \sin \theta \left( \frac{r_0}{r} \right)^2 - r \Omega_\odot \sin \theta \tag{2.63}
\]

the corresponding \( B_\phi \)'s are

\[
B_\phi = -B_0 \left( \frac{r_0}{r} \right)^2 \frac{r \Omega_\odot \sin \theta}{V_w} \tag{2.64}
\]

\[
B_\phi = -B_0 \left( \frac{r_0}{r} \right)^2 \frac{r \Omega_\odot \sin \theta}{V_w} \left( 1 - \frac{r_0}{r} \right) \tag{2.65}
\]

\[
B_\phi = -B_0 \left( \frac{r_0}{r} \right)^2 \frac{r \Omega_\odot \sin \theta}{V_w} \left[ 1 - \left( \frac{r_0}{r} \right)^2 \right] \tag{2.66}
\]

The corresponding magnetic field lines are given by

\[
r = r_0 - \frac{V_w}{\Omega \sin \theta} (\phi - \phi_0) \tag{2.67}
\]

\[
r = r_0 + r_0 \ln \frac{r}{r_0} - \frac{V_w}{\Omega \sin \theta} (\phi - \phi_0) \tag{2.68}
\]

\[
r = r_0 + r_0 \left( 1 - \frac{r_0}{r} \right) - \frac{V_w}{\Omega \sin \theta} (\phi - \phi_0) \tag{2.69}
\]

Three magnetic field lines are shown in Figure (2.6). The first effect of different models is the calculation of \( r \) according Equation (2.67) to Equation (2.69). The relative error for the calculation of \( r \) can be as large as 8% near 1 AU.

### 2.4.2 Field Length Comparison

Let us define \( a = \left( \frac{\Omega_\odot \sin \theta}{V_w} \right)^2 \). The corresponding magnetic field line length is given by

\[
\int \sqrt{1 + ar^2} \, dr = \frac{r}{2} \sqrt{1 + ar^2} + \frac{\sinh^{-1}(\sqrt{ar})}{2\sqrt{a}} \tag{2.70}
\]
\[
\int \sqrt{1 + a(r - r_0)^2} dr = \frac{r - r_0}{2} \sqrt{1 + a(r - r_0)^2} + \frac{\sinh^{-1}(\sqrt{a}(r - r_0))}{2\sqrt{a}} \quad (2.71)
\]

\[
\int \left[1 + a \left[1 - \left(\frac{r_0}{r}\right)^2\right]\right] r^2 dr \quad (2.72)
\]

The last equation can be integrated analytically. But the solution is kind of tedious. Let us define

\[
Y = \sqrt{r^2 + a(r^2 - r_0^2)} \quad (2.73)
\]

\[
Z = 1 - 2ar_0^2 + 2ar^2 + 2\sqrt{a}Y \quad (2.74)
\]

\[
W = 2ar_0^4 + r^2 - 2ar_0^2r^2 + 2r_0^2\sqrt{a}Y \quad (2.75)
\]

\[
\int \sqrt{1 + a \left[1 - \left(\frac{r_0}{r}\right)^2\right]} r^2 dr = \frac{Y}{2} + \frac{(1 - 2ar_0^2)(\ln Z - \ln 2\sqrt{a}) - 2ar_0^2(\ln W - 2\ln r)}{4\sqrt{a}} \quad (2.76)
\]

So \(a = 4.5563 \times 10^{-27} \text{ cm}^{-2}\) providing that \(\sin \theta = 1\) on the ecliptic plane.

If we take \(r_0 = 10R_\odot\),

\[
L1 = 1.1278 \text{AU} \quad (2.77)
\]

\[
L2 = 1.1179 \text{AU} \quad (2.78)
\]

\[
L3 = 1.1026 \text{AU} \quad (2.79)
\]

Note that if \(V_w\) is smaller or \(r_0\) is larger, the difference between \(L2\) and \(L3\) will be larger. Anyway, we can see here the relative error for the field line is very small, around 1 percent.

2.4.3 Footpoint

The footpoint differences can be determined from Eq. (2.67) to Eq. (2.69),

\[
\phi - \phi_0 = -\frac{(r - r_0)\Omega_\odot \sin \theta}{V_w} \quad (2.80)
\]

\[
\phi - \phi_0 = -\frac{(r - r_0)\Omega_\odot \sin \theta}{V_w} + \frac{r_0\Omega_\odot \sin \theta}{V_w} \ln \frac{r}{r_0} \quad (2.81)
\]

\[
\phi - \phi_0 = -\frac{(r - r_0)\Omega_\odot \sin \theta}{V_w} + \frac{r_0\Omega_\odot \sin \theta}{V_w} \left(1 - \frac{r_0}{r}\right) \quad (2.82)
\]
Figure 2.6: Comparison of the calculation of $r$ for different models. For all three $\phi_0 = 0$. The red line is the Parker’s Model. The green line is the one close to Weber & Davis’ Model. The blue line is the one without physical meaning.
At 1 AU the azimuthal angle difference, can be about 18 degrees. Figure (2.7) shows these differences which cause a relative error around 30%. Because $r_0$ is much smaller than 1 AU, it hard to tell the difference on the solar surface. However, if we take Fig. (2.6), we can see the difference in $\phi$. 

Figure 2.7: Comparison of the footpoint of the magnetic field on the solar surface for all three models. For all three $\phi_0 = 0$. The red line is the Parker’s Model. The green line is the one close to Weber & Davis’ Model. The blue line is the one without physical meaning.
CHAPTER 3

Turbulent effects

3.1 Introduction

One of the fundamental goals in understanding solar energetic particles is to relate observations made at 1 AU to the physics of particle acceleration and injection at the Sun. A significant part of this involves an analysis of the onset times of particle events at 1 AU. Many authors have considered this problem (e.g. Dalla et al., 2003b,a; Bieber et al., 2004; Krucker and Lin, 2000). Using the onset time of a particle event, it is possible to estimate the distance traveled by particles prior to their arrival at a spacecraft (path length), and to infer the release time from their origin, perhaps near the Sun (Dalla et al., 2003b,a; Bieber et al., 2004). This analysis could also be used to differentiate between models of particle acceleration near the Sun (Zhang et al., 2003).

The arrival time for a low rigidity particle of energy $E$ moving from the Sun to the detector at 1 AU in the absence of scattering, may be written

$$t_{\text{arr}} = t_{\text{Sun}} + \int_{S_{\text{Sun}}}^{1\text{AU}} \frac{ds}{v_{\parallel}(E, s)},$$

(3.1)

where $t_{\text{Sun}}$ is the time of particle release at the Sun, $t_{\text{arr}}$ is the arrival time at the detecting spacecraft, $s$ is the distance along the magnetic field, and $v_{\parallel}$ is the particle speed parallel to the magnetic field. Since the first-arriving particles have experienced little or no scattering, I may set $t_{\text{arr}} = t_{\text{onset}}$, the onset time for an energetic particle event. This definition of onset time is slightly different from the form used by Krucker and Lin (2000). In some studies, the effect of the pitch angle with respect to the magnetic field line is considered (Hilchenbach et al., 2003), and in others, the path length is replaced by 1.3 AU (Kahler, 1994). The latter is only valid for one specific solar wind speed and for a detector located at 1 AU. In all
previous papers, the magnetic field line is taken to be that obtained using the Parker spiral.

Recent observations have shown evidence of an onset time that is shorter than the value determined for a simple Parker spiral (Hilchenbach et al., 2003). These authors reported on observations made by SOHO / CELIAS / STOF. While most of the energetic particles follow the “canonical” value of Parker spiral, there are events in which the particles follow a shorter path (see their Figure 4). Torsti et al. (1998) discussed the May 12, 1997 proton event that showed a precursor which may be evidence for the particle moving along a shorter interplanetary field line than the bulk of the earliest arriving particles.

Although there are other explanations for this, I discuss here that this may be related to the stochastic nature of the magnetic field (Jokipii, 1966; Jokipii and Parker, 1968). I use numerical simulations to determine the path lengths and propagation times of solar-energetic particles released impulsively at the Sun. I use a numerical model for the heliospheric magnetic field similar to that discussed in Giacalone (1999) and Giacalone and Jokipii (1996, 2001). The field consists of a stochastically varying magnetic field, generated by super-granule fluid motions near the Sun, added to a nominal Parker spiral. Treatments based on this paradigm began in the 1960’s (e.g. Jokipii, 1966; Jokipii and Parker, 1968, 1969). Here I show that the onset time of an SEP event in a stochastic magnetic field can be sooner than that inferred by assuming the particles move along a simple Parker spiral.

3.2 Model

Charged particles that move in the electric and magnetic fields in the inner heliosphere are subjected to four distinct transport effects: convection, energy change, spatial diffusion, and drift. In this study I are interested in those particles from any given solar-energetic particle event which are the earliest to arrive at an observer located at 1 AU. Since I are interested here in the propagation of high-energy particles that have speeds which are much larger than the solar wind speed, the effect of
convection is small. Energy change, due to adiabatic expansion of the solar wind, is most important near the Sun. Spatial diffusion is not an important effect for our problem since the earliest-arriving particles propagate nearly scatter free. The effect of the curvature drift may be important. The computation which I describe below includes these effects.

3.2.1 Fluctuating electromagnetic field

A simple but useful heliospheric magnetic field model is Parker’s spiral magnetic field which has been discussed in Chapter 2. In reality, the magnetic field fluctuates randomly around a mean given by the Parker spiral. Observations near the ecliptic plane have verified the general correctness of this structure (Smith et al., 1986). At high heliographic latitude, Ulysses measurements provided the evidence of the large-scale transverse magnetic-field variance as well (Jokipii et al., 1995). Jokipii and Parker (1968); Jokipii et al. (1995), suggested that the source of large-scale transverse magnetic fluctuations was the super-granulation in the solar photosphere, which is a large scale (∼ 30,000 km) random transverse motion of the solar photosphere. There are also observations of fine-scale variations in the intensity of solar-energetic particles associated with impulsive solar flares (Mazur et al., 2000) that may be related to large-scale variation in the magnetic field. Giacalone et al. (2000) gave an interpretation which fits the particle observations well.

In this study, I illustrate the effects of a fluctuating magnetic field on the particle transport by using a quantitative model of the heliospheric magnetic field discussed by Giacalone (2001). The magnetic field is composed of an average field (the Parker spiral field) plus a random component, which has zero mean and which is treated statistically. The random field is composed of large-scale fluctuations determined by the movements of the plasma on the solar surface.

I assume that the transverse motion of the plasma at the solar surface is a function of time and position. I further assume that the transverse flow on the surface is divergence-free which means I can introduce the stream function, ψ, which
is conveniently expressed in terms of spherical harmonics as

$$
\psi(\theta, \phi, t) = \sum_{n=1}^{N} \sum_{m=-n}^{n} a_n^m \exp(i\omega_n^m t + i\beta_n^m) Y_n^m(\theta, \phi),
$$

(3.2)

where $a_n^m$ is the amplitude of the $(n, m)$ mode, $\omega_n^m$ is its frequency, and $\beta_n^m$ is a random phase. In this case the radial magnetic field along $r_0$ is independent of position and time. The conclusions in this work should not be sensitive to this approximation. To represent a reasonable turbulent spectrum, the amplitudes are given by

$$
a_n^m = 6V_g G_n^m \left( \sum_{n=1}^{N} \sum_{m=-n}^{n} G_n^m \right)^{-1},
$$

(3.3)

$$
G_n^m = \left\{ \left[ 1 + (n/N_c)^{10/3} \right] \left[ 1 + (\omega_n^m T_c)^{5/3} \right] \right\}^{-1/2},
$$

(3.4)

where $T_c$ is the characteristic time scale of the fluctuations, and $V_g$ is the “rms” speed of the fluctuations. $N_c = \Delta/(\pi r_0)$, where $\Delta = V_g T_c$ is the characteristic size of the “super-granulation cells”. The frequencies are given by $\omega_n^m = 1/\tau_n^m$, where

$$
\tau_n^m = \left( \frac{1}{n} + \frac{1}{m} \right) \frac{\pi r_0}{V_g},
$$

(3.5)

is taken to be a characteristic period associated with the mode $(n, m)$. Then the random motions of the plasma are

$$
V_\theta = \frac{1}{\sin \theta} \frac{\partial \psi}{\partial \phi},
$$

(3.6)

$$
V_\phi = -\frac{\partial \psi}{\partial \theta}.
$$

(3.7)

On the solar surface, because it is frozen-in to the plasma, the magnetic field satisfies the following boundary conditions,

$$
B_r(r_0, \theta, \phi, t_0) = B_0,
$$

(3.8)

$$
\frac{B_\theta(r_0, \theta, \phi, t_0)}{B_0} = \frac{V_\theta(\theta, \phi, t_0)}{V_w},
$$

(3.9)

$$
\frac{B_\phi(r_0, \theta, \phi, t_0)}{B_0} = -r_0 \omega \sin \theta + \frac{V_\phi(\theta, \phi, t_0)}{V_w}.
$$

(3.10)
Faraday’s law then gives

\begin{align*}
B_r(r) &= B_0 \left( \frac{r_0}{r} \right)^2, \\
B_\theta(r, \theta, \phi, t) &= B_0 \left( \frac{r_0}{r} \right) \frac{V_\theta(\theta, \phi, t_0)}{V_w}, \\
B_\phi(r, \theta, \phi, t) &= B_0 \left( \frac{r_0}{r} \right) \frac{-r_0 \omega \sin \theta + V_\phi(\theta, \phi, t_0)}{V_w}.
\end{align*}

(3.11) (3.12) (3.13)

where \( t_0 = t - (r - r_0)/V_w \). This magnetic field is different from those discussed in Chapter 2 which have no \( \theta \) component.

Figure (3.1) and Figure (3.2) illustrate the resulting fluctuating magnetic field near the heliospheric equator (in Figure (3.1), \( V_g = 2 \text{ km s}^{-1} \); in Figure (3.2), \( V_g = 4 \text{ km s}^{-1} \). In both cases, \( N = 50 \)). \( V_g \) is occasionally observed to be as high as several \( 10 \text{ km s}^{-1} \) (Wang et al., 2003; Xu et al., 2004), thus, the values of \( V_g \) I use here are reasonable.

3.2.2 Electric Field

At each particle position, I calculate the electric field which is given by Equation (2.14) since the concept of “frozen-in” magnetic field applies.

\[
\mathbf{E} = -\frac{\mathbf{u} \times \mathbf{B}}{c} = V_w B_\phi \mathbf{i}_\theta/c - V_w B_\theta \mathbf{i}_\phi/c
\]

(3.14) (3.15)

where \( \mathbf{i}_\theta \) is a unit vector in the \( \theta \) direction, \( \mathbf{i}_\phi \) is a unit vector in the \( \phi \) direction, and \( c \) is the speed of light in vacuum. \( B_\phi \) and \( B_\theta \) are given in the previous section.

For the special case of the simple Parker spiral, the total energy of a test particle in the stationary frame of reference is given by

\[
\epsilon = \frac{1}{2} m V^2 - B_0 r_0^2 \epsilon \omega \cos \theta/c
\]

(3.16)

where \( m, e \) and \( V \) is the mass, charge and speed of the test particle. This equation is useful because I can use it to test the numerical accuracy of our numerical integration method (see section (3.3)). Unfortunately, the fluctuating magnetic field is time dependent, and a more general treatment is necessary for this case.
Figure 3.1: Ecliptic projections of the fluctuating magnetic field. In this case, $V_g = 2.0 \text{ km s}^{-1}$. $M = N = 50$. 
Figure 3.2: Ecliptic projections of the fluctuating magnetic field. In this case, $V_g = 4.0 \text{ km s}^{-1}$, $M = N = 50$. 
3.2.3 Particle scattering

The test particles are released at 0.1 AU isotropically in the hemisphere outward from the Sun and at a polar angle of 90°. I follow them until they cross an outer boundary at 5 AU where they are allowed to escape the system. In our model, the particle mean-free path, $\lambda$, is a constant. Since interplanetary mean-free paths at 1 AU for $E = 10–100$ MeV protons are often observed to be of the order of 0.1 AU (Vainio et al., 2000; Dröge, 2000), I take $\lambda = 0.3$ AU in our calculation. The mean scattering time, $\tau_s$, is determined from $\tau_s = \lambda/V_p$, where $V_p$ the particle speed. Applying $V_p = r_g \Omega_i$, where $\Omega_i$ is the cyclotron-frequency and $r_g$ is the gyro-radius (Boyd and Sanderson, 1969), I have

$$\tau_s = \frac{\lambda}{r_g \Omega_i}.$$  (3.17)

Then at each point along the particle trajectory a test is performed to see whether the particle scatters. For example, I can compare a random number with the ratio of time step to the mean scattering time. This method approximates an exponential probability of scattering time. If the test is positive, the particle is scattered isotropically in a frame moving with the solar wind (so called the “plasma frame”) so that the kinetic energy in this frame is conserved. The motivation for using ad hoc scattering is because these particles normally will not be scattered by the magnetic field in this model since the high frequency turbulence which can resonance with the particle is not included.

3.2.4 Integration scheme

I integrate the trajectories of charged test particles moving in the model electromagnetic fields by numerically solving the Lorentz equation. Provided that velocities are low enough that I can omit the relativistic effects, the Lorentz equation (Boyd and Sanderson, 1969) is

$$m\ddot{\mathbf{r}} = e\left[\mathbf{E}(\mathbf{r}, t) + \frac{\mathbf{r} \times \mathbf{B}(\mathbf{r}, t)}{c}\right],$$  (3.18)
where \( m, e \) and \( \mathbf{r} \) are the mass, charge, and position of the particle, \( t \) is the time, and \( c \) is the velocity of light. The numerical integration scheme is known as the Bulirsch-Stoer method, which is fast for a smoothly varying field. Energy is conserved to better than 0.1%.

For simplicity, I consider here protons with energy of 10 MeV. No current sheet is included in the calculation this time. Since particles can not penetrate the current sheet in this simulation, I can assume that the particles are far away from the current sheet. I believe this will not affect our conclusions.

3.3 Results

In our simulation for the special case of a nominal Parker spiral, I find that the total energy, which is defined in Equation (3.16), is conserved very well. And the particles lose energy rapidly as they move in the expanding solar wind in the plasma frame because of adiabatic energy losses (Parker, 1965; Giacalone, 1999). All these are consistent with theory and can be as a partial validation of our codes which use the Bulirsh-Stoer method for integration.

I have carried out twelve simulations of SEPs for both the two cases of \( V_g = 2.0 \) \( \text{km s}^{-1} \) and \( V_g = 4.0 \) \( \text{km s}^{-1} \). Every simulation corresponds to one realization of the magnetic field. In each simulation, thousands of particles are released. By recording the time they first cross 1 AU, I obtain the onset time of each simulation (event). The results are shown in Figure (3.3). For the purpose of comparison, I also show the onset time without any fluctuation \( (V_g = 0) \) in Figure (3.3).

Each point in Figure (3.3) corresponds to one realization of the magnetic field in which a different set of random numbers in Equation (3.2) are chosen. The line without symbols is for the case of a nominal spiral \( (V_g = 0) \) in which \( \Delta t = t_{\text{onset}} - t_{\text{sun}} \) is a constant. I refer this as the standard value. The solid line with circles is for the case \( V_g = 2 \) \( \text{km s}^{-1} \), \( T_c = 1 \) day, and \( N = 50 \). In this figure, there are 3 points (one quarter of all the cases), where the value of circles are smaller than that of the solid line without symbols. This means \( \Delta t \) can be smaller than the standard value when
Figure 3.3: The value of $\Delta t$ required to cross 1 AU. The solid line without symbols is the case $V_g = 0$. The solid line with circles is the case $V_g = 2$ km s$^{-1}$. The dashed line with triangles is the case $V_g = 4$ km s$^{-1}$. The X axis represents the different realizations of the turbulent magnetic field.
the field lines deviate from a simple Parker spiral. The dashed line with triangles represents the case with $V_g = 4 \text{ km s}^{-1}$ and $N = 50$. There are 4 cases which have a smaller $\Delta t$ than the standard value. Compared to the case $V_g = 2 \text{ km s}^{-1}$, there is a higher probability that the arrival time will be less than the standard result.

To show the relationship between the onset time and the magnetic field-line length, I also calculate the magnetic field-line length for different realizations. The results are shown in Figures (3.4) and (3.5), and (3.6). In Figure (3.4), I can see that the random walk of field lines leads to some field lines that are longer than for the case of the Parker’s spiral. But the minimum length is smaller than that of the Parker’s spiral for both cases. In Figure (3.4), I show that the average of the magnetic field line length is longer than that of the Parker’s spiral. In Figure (3.5) and Figure (3.6), I show the histogram of magnetic field-line length. I can see that in Figure (3.5), there are about 526 out of about 2000 magnetic lines which have shorter length than the nominal Parker spiral. In Figure (3.6), about 358 out of about 2000 are shorter.

I can see from Equation (2.22) that the average magnetic field line will be more radial and its length will be shorter when the Sun rotates more slowly. On the other hand, the stochastic mechanism stretches the lines and makes them longer. When the Sun rotates very slowly, I would expect the latter effect to dominate. Figure (3.7) and Figure (3.8) illustrate this for the special case of a slower solar rotation rate which is $\omega = 2.7 \times 10^{-7} \text{ radian s}^{-1}$. This value is 10 times smaller than the normal value which means the nominal Parker spiral field is more radial. For this case, all of the onset time are later than the nominal value, which is 0.859 hour here. Since the lines of force are almost radial, any large-scale fluctuation will result in a longer path length. Thus, there is very little chance of shortcut as in the case of the actual rotation value.
Figure 3.4: The magnetic field line length in different realizations. The solid line without symbols is the case $V_g = 0$. The solid line with circles is the mean field line length when $V_g = 2 \text{ km s}^{-1}$. The dashed line with downward triangles is the mean field line length when $V_g = 4 \text{ km s}^{-1}$. The solid line with squares is the minimum field line length when $V_g = 2 \text{ km s}^{-1}$. The dashed line with upward triangles is the minimum field line length when $V_g = 4 \text{ km s}^{-1}$. The solid line with diamonds is the maximum field line length when $V_g = 2 \text{ km s}^{-1}$. The dashed line with stars is the maximum field line length when $V_g = 4 \text{ km s}^{-1}$. The X axis represents different realizations of the turbulent magnetic field.
Figure 3.5: The histogram of magnetic field line lengths. Here $V_g = 2 \text{ km s}^{-1}$. The nominal Parker spiral is nearly 1 AU in this case.
Figure 3.6: The histogram of magnetic field line lengths. Here $V_g = 4 \text{ km s}^{-1}$. The nominal Parker spiral is nearly 1 AU in this case.
Figure 3.7: The value of $\Delta t$ required to cross 1 AU. In this case, the angular velocity of solar rotation is $2.7 \times 10^{-7}$ radian s$^{-1}$ which is 10 times less than the actual value. The solid line without symbols is the case $V_g = 0$. The solid line with circles is the case $V_g = 2$ km s$^{-1}$. The dashed line with triangles is the case $V_g = 4$ km s$^{-1}$. The X axis represents the different realizations of the turbulent magnetic field.
Figure 3.8: The magnetic field line length in different realizations. In this case, the angular velocity of solar rotation is $2.7 \times 10^{-7}$ radian s$^{-1}$ which is 10 times less than the actual value. The solid line without symbols is the case $V_g = 0$. The solid line with squares is the minimum field line length when $V_g = 2$ km s$^{-1}$. The dashed line with upward triangles is the minimum field line length when $V_g = 4$ km s$^{-1}$. The X axis represents the different realizations of the turbulent magnetic field.
3.4 Conclusions

I have shown that because of the nature of the stochastic heliospheric magnetic field, the path length measured along the line of force can be shorter than that of the nominal Parker spiral. This leads to a time interval required for solar-energetic particles to cross 1 AU that is shorter than expected based on a nominal spiral field. Our results may help explain recent observations (Hilchenbach et al., 2003).

The quantitative model of the heliospheric magnetic field that is used here is similar to that used by Giacalone (2001). Our numerical results show that the onset time of a given SEP event can be about 90–200 seconds sooner than estimated by assuming particles move along the usual Parker spiral (see Fig. (3.3)), which is generally believed to be the earliest time. This can be understood in terms of a random magnetic field in which there are some magnetic field lines that are shorter than the nominal (Parker spiral) ones, although the average of the magnetic field line length is a little bit longer than that of the Parker’s spiral. Of course, the shortest possible field line is 1 AU.
The numerical method discussed in this chapter is different from the one used in Chapter 2 and Chapter 3. In Chapter 2, Newton’s iteration method is used to solve the nonlinear Bernoulli equation. In Chapter 3, Bulirsch-Stoer method is used to integrate the Lorentz equation. SEPs are treated individually as “test” particles in the Lorentz equation, which is a “microscopic” treatment. In this and later chapters, we take a different view to study the SEP behaviors by using Parker’s diffusion equation (1.2). This diffusion equation is the statistical outcome for SEPs and can be viewed as a “macroscopic” treatment for SEPs.

4.1 Governing equations

If we expand Equation (1.2) and drop the source term, the Parker’s equation in spherical coordinates is

\[
\frac{\partial f}{\partial t} = \frac{\kappa_{rr}}{r^2} \frac{\partial^2 f}{\partial r^2} + \frac{\kappa_{\theta\theta}}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{\kappa_{\phi\phi}}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2} \\
+ \frac{\kappa_{r\theta} + \kappa_{r\phi}}{r} \frac{\partial f}{\partial r \partial \theta} + \frac{\kappa_{\theta\phi}}{r} \frac{\partial^2 f}{\partial r \partial \phi} + \frac{\kappa_{\phi\theta}}{r} \frac{\partial^2 f}{\partial \theta \partial \phi} \\
+ \left( \frac{2\kappa_{rr}}{r} + \frac{\cot \theta \kappa_{\theta\theta}}{r} + \frac{1}{r \sin \theta} \frac{\partial \kappa_{\phi\theta}}{\partial \phi} + \frac{1}{r} \frac{\partial \kappa_{\theta\theta}}{\partial \theta} + \frac{1}{r} \frac{\partial \kappa_{r\theta}}{\partial r} \right) \frac{\partial f}{\partial r} \\
+ \left( \frac{\kappa_{r\phi}}{r^2 \sin \theta} + \frac{\cot \theta \kappa_{\theta\phi}}{r^2 \sin^2 \theta} + \frac{1}{r^2 \sin \theta} \frac{\partial \kappa_{\phi\theta}}{\partial \phi} + \frac{1}{r^2} \frac{\partial \kappa_{\theta\phi}}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \kappa_{r\phi}}{\partial r} \right) \frac{\partial f}{\partial \phi} \\
- \left( \frac{u_{\phi}}{r^2 \sin \theta} \frac{\partial f}{\partial \phi} + \frac{u_{\theta}}{r} \frac{\partial f}{\partial \theta} + \frac{u_{r}}{r} \frac{\partial f}{\partial r} \right) \\
+ \frac{1}{3} \left( \frac{2}{r} \frac{u_{\phi}}{r \sin \theta} \frac{\partial f}{\partial \phi} + \frac{1}{r \sin \theta} \frac{\partial u_{\phi}}{\partial \phi} + \frac{1}{r} \frac{\partial u_{\theta}}{\partial \theta} + \frac{\partial u_{r}}{\partial r} \right) \frac{\partial f}{\partial \ln p} 
\] (4.1)
If we only consider the $r$ and the $\theta$ direction and only the diagonal elements of the diffusion coefficient, $u = u(r)$, we have

$$\frac{\partial f}{\partial t} = \kappa_{rr} \frac{\partial^2 f}{\partial r^2} + \frac{\kappa_{\theta\theta} \partial^2 f}{r^2 \partial \theta^2} + \left( \frac{2\kappa_{rr}}{r} + \frac{\partial \kappa_{rr}}{\partial r} \right) \frac{\partial f}{\partial r} + \left( \frac{\cot \theta \kappa_{\theta\theta}}{r^2} + \frac{1}{r^2} \frac{\partial \kappa_{\theta\theta}}{\partial \theta} \right) \frac{\partial f}{\partial \theta} - u_r \frac{\partial f}{\partial r} + \frac{1}{3} \left( \frac{2u_r}{r} + \frac{\partial u_r}{\partial r} \right) \frac{\partial f}{\partial \ln p} \quad (4.2)$$

If all the elements of the diffusion coefficient are considered and assume Parker Spiral Magnetic field, it is

$$\frac{\partial f}{\partial t} = \kappa_{rr} \frac{\partial^2 f}{\partial r^2} + \frac{\kappa_{\theta\theta} \partial^2 f}{r^2 \partial \theta^2} + \left( \frac{2\kappa_{rr}}{r} + \frac{\cot \theta \kappa_{\theta\theta}}{r} + \frac{\partial \kappa_{\theta\theta}}{\partial \theta} + \frac{1}{r} \frac{\partial \kappa_{\theta\theta}}{\partial r} \right) \frac{\partial f}{\partial r} + \left( \frac{\kappa_{\theta\theta}}{r^2} + \frac{\cot \theta \kappa_{\theta\theta}}{r^2} + \frac{1}{r^2} \frac{\partial \kappa_{\theta\theta}}{\partial \theta} + \frac{1}{r} \frac{\partial \kappa_{\theta\theta}}{\partial r} \right) \frac{\partial f}{\partial \theta} - u_r \frac{\partial f}{\partial r} + \frac{1}{3} \left( \frac{2u_r}{r} + \frac{\partial u_r}{\partial r} \right) \frac{\partial f}{\partial \ln p} \quad (4.3)$$

The diffusion coefficient in spherical coordinates according to Equation (1.3) is

$$\begin{align*}
\kappa_{rr} &= \kappa_\perp + \frac{B_r^2}{B^2} (\kappa_\parallel - \kappa_\perp) \\
\kappa_{\theta\theta} &= \kappa_\perp \\
\kappa_{\phi\phi} &= \kappa_\perp + \frac{B_\phi^2}{B^2} (\kappa_\parallel - \kappa_\perp) \\
\kappa_{r\theta} &= \kappa_\perp \frac{B_\phi}{B} \\
\kappa_{r\phi} &= \kappa_\perp \frac{B_r}{B} (\kappa_\parallel - \kappa_\perp) \\
\kappa_{\theta\phi} &= \kappa_\perp \frac{B_\phi}{B} \\
\kappa_{\theta r} &= -\kappa_\perp \frac{B_\phi}{B} \\
\kappa_{\phi r} &= -\kappa_\perp \frac{B_r}{B} (\kappa_\parallel - \kappa_\perp) \\
\kappa_{\phi \theta} &= -\kappa_\perp \frac{B_r}{B} \\
\end{align*} \quad (4.4)
4.2 Transformation of equations

As we know from observations, the energy spectra of SEPs usually follow a power law (Krucker et al., 2007; Wang and Wang, 2006; Mewaldt et al., 2005). The power index is usually between 1 to 10. Based this fact, we can simplify the Parker’s equation by getting rid of the adiabatic cooling term.

For similarity, let \( P = \ln p \).

\[
f = F(x, t)G(P + \nabla \cdot \mathbf{V}_w t/3)
\]  

(4.5)

where \( G \) will be determined by the initial condition.

Thus in spherical coordinates, suppose \( V_w \) is a constant,

\[
G(P, t) = G(P + 2V_w t/3r)
\]  

(4.6)

\[
\frac{\partial f}{\partial r} = F \frac{\partial G}{\partial r} + G \frac{\partial F}{\partial r}
\]  

(4.7)

\[
\frac{\partial^2 f}{\partial r^2} = F \frac{\partial^2 G}{\partial r^2} + G \frac{\partial^2 F}{\partial r^2} + 2 \frac{\partial F}{\partial r} \frac{\partial G}{\partial r}
\]  

(4.8)

\[
\frac{\partial^2 f}{\partial r \partial \phi} = \frac{\partial F}{\partial \phi} \frac{\partial G}{\partial r} + \frac{\partial^2 F}{\partial r \partial \phi}
\]  

(4.9)

If \( f(x, p, 0) = F(x, 0) \exp(-\beta P) \) which means that \( f \) follows power law, then \( G(P) = \exp(-\beta P) \). In this case,

\[
1 \frac{\partial G}{\partial r} = \frac{2\beta V_w t}{3r^2}
\]  

(4.10)

\[
1 \frac{\partial^2 G}{\partial r^2} = \frac{4\beta^2 V_w^2 t^2}{9r^4} - \frac{4\beta V_w t}{3r^3}
\]  

(4.11)

As discussed in 1.3.3, the divergence of the anti-symmetric part of the diffusion tensor is the drift velocity,

\[
(\nabla \cdot \kappa_A)_r = \frac{1}{r} \frac{\partial \kappa_{\theta r}}{\partial \theta} + \frac{\cot \theta}{r} \kappa_{\theta r} = V_{g_r},
\]

\[
(\nabla \cdot \kappa_A)_\theta = \frac{\partial \kappa_{r \theta}}{\partial r} + \frac{\kappa_{r \theta}}{r} + \frac{1}{r \sin \theta} \frac{\partial \kappa_{\phi \theta}}{\partial \phi} = V_{g_\theta},
\]

\[
(\nabla \cdot \kappa_A)_\phi = \frac{1}{r} \frac{\partial \kappa_{\theta \phi}}{\partial \theta} = V_{g_\phi}.
\]  

(4.12)
Note the magnetic field is Parker’s spiral in this calculation.

\[
V_g = \frac{\nu p}{eB^3} \left[ \frac{\sin^2 \alpha}{2} \mathbf{B} \times \nabla B + \frac{\cos^2 \alpha}{B} \mathbf{B} \times (\mathbf{B} \cdot \nabla \mathbf{B}) \right] 
\]

\[
= \frac{2}{3} \frac{\nu p r \Omega_\odot \cos \theta}{V_w} \hat{r} + \left( 2 + \frac{r^2 \Omega_\odot^2 \sin^2 \theta}{V_w^2} \right) \frac{r \Omega_\odot \sin \theta}{V_w} \hat{\theta} + \frac{r^2 \Omega_\odot}{V_w^2} \cos \theta \sin \theta \hat{\phi}
\]

where we take the average among pitch angle (\(\alpha\)). This expression is first given by Jokipii et al. (1977).

By using Equation (4.5) and Equation (4.12), if we assume solar wind velocity is a only constant in radial direction, the original Parker equation (1.2) is transformed to,

\[
\frac{\partial f}{\partial t} = \frac{\kappa_{rr}}{r^2} \frac{\partial^2 f}{\partial r^2} + \frac{\kappa_{\theta\theta}}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{\kappa_{\phi\phi}}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2} + \frac{2 \kappa_{\phi r}}{r} \frac{\partial^2 f}{\partial \phi \partial r} + \left( \frac{2 \kappa_{rr}}{r} + \frac{\partial \kappa_{rr}}{\partial r} - \frac{u_r}{r} + \frac{\nu p r \Omega_\odot \cos \theta}{V_w} \right) \frac{1}{r} \frac{\partial f}{\partial \theta}
\]

\[
+ \left( \frac{\kappa_{r\phi}}{r} + \frac{\partial \kappa_{r\phi}}{\partial r} + V_{g\phi} \right) \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} + \frac{2 u_r}{3 r} \frac{\partial f}{\partial \ln p}
\]

If \(f = F(r, t)G(p, r, t)\),

\[
\frac{\partial F}{\partial t} = \frac{\kappa_{rr}}{r^2} \frac{\partial^2 F}{\partial r^2} + \frac{\kappa_{\theta\theta}}{r^2} \frac{\partial^2 F}{\partial \theta^2} + \frac{\kappa_{\phi\phi}}{r^2 \sin^2 \theta} \frac{\partial^2 F}{\partial \phi^2} + \frac{2 \kappa_{\phi r}}{r} \frac{\partial^2 F}{\partial \phi \partial r} + \left( \frac{2 \kappa_{rr}}{r} + \frac{\partial \kappa_{rr}}{\partial r} - \frac{u_r}{r} + \frac{\nu p r \Omega_\odot \cos \theta}{V_w} \right) \frac{\partial F}{\partial r}
\]

\[
+ \left( \frac{\cot \theta \kappa_{\theta\theta}}{r} + V_{g\theta} \right) \frac{1}{r} \frac{\partial F}{\partial \theta}
\]

\[
+ \left( \frac{\kappa_{r\phi}}{r} + \frac{\partial \kappa_{r\phi}}{\partial r} + \frac{2 \kappa_{r\phi}}{G} \frac{\partial G}{G} + V_{g\phi} \right) \frac{1}{r \sin \theta} \frac{\partial F}{\partial \phi}
\]

\[
+ \left[ \frac{\kappa_{rr}}{G} \frac{\partial^2 G}{\partial r^2} + \frac{1}{r G} \left( \frac{2 \kappa_{rr}}{r} + \frac{\partial \kappa_{rr}}{\partial r} - u_r \right) \right] F
\]

The success of one dimensional solutions of Parker’s equation for solar events and 3D steady solutions of Parker’s equation for galactic cosmic rays suggest that Equation (4.15) with appropriate boundary and initial conditions is a well-posed problem although there is no formal proof has been given.
The complexity of solving Parker’s equation comes from the fact that the time-dependent distribution function, \( f \), depends not only on three spatial variables, \( x_i \), and depends on the magnitude of momentum \( p \) as well. So \( f \) depends on totally eight variables. Here, we simplifies Parker’s equation by getting rid of the \( p \) coordinate based on observations which greatly simplify the numerical method to solve this equation. So \( f \) now only depends on seven variable and the boundary condition for \( p \) is not needed. To author’s knowledge, this transformation of Parker’s equation has never been used before.

4.3 Alternating Direction Implicit methods

4.3.1 2D Alternating Direction Implicit Method (ADI)

The 2D ADI method is unconditionally stable which allows the use of larger time steps as long as the accuracy is good enough.

For simplicity, I use the following equation to illustrate the procedure,

\[
\frac{\partial f}{\partial t} + C_5 \frac{\partial^2 f}{\partial r^2} + C_4 \frac{\partial^2 f}{\partial \theta^2} + C_3 \frac{\partial^2 f}{\partial r \partial \theta} + C_2 \frac{\partial f}{\partial r} + C_1 \frac{\partial f}{\partial \theta} + C_0 f + S(r, \theta, p, t) = 0 \quad (4.16)
\]

Alternating Direction Implicit Method (ADI) for two-dimensional equation uses two half-time steps. The first step is

\[
\frac{f_{i,j}^{n+1/2} - f_{i,j}^n}{\Delta t/2} + C_5 \delta_{rr} f_{i,j}^{n+1/2} + C_4 \delta_{\theta\theta} f_{i,j}^n + C_3 \delta_{r\theta} f_{i,j}^n + C_2 \delta_r f_{i,j}^{n+1/2} + C_1 \delta_\theta f_{i,j}^n + C_0 f_{i,j}^{n+1/2} + S_{n+1/2} = 0 \quad (4.17)
\]

where \( \delta_{rr} \) stands for standard the central difference in \( x \) direction for the second order derivative, \( \delta_r \) for the first order derivative, and \( f_{i,j}^n \) stands for the value of \( f \) at position \((i\Delta r, j\Delta \theta)\) and time \((n\Delta t)\). On this half step, only the quantities in \( r \) direction are updated.

\[
\frac{f_{i,j}^{n+1} - f_{i,j}^{n+1/2}}{\Delta t/2} + C_5 \delta_{rr} f_{i,j}^{n+1/2} + C_4 \delta_{\theta\theta} f_{i,j}^{n+1} + C_3 \delta_{r\theta} f_{i,j}^{n+1} + C_2 \delta_r f_{i,j}^{n+1/2} + C_1 \delta_\theta f_{i,j}^{n+1} + C_0 f_{i,j}^{n+1} + S^{n+1/2} = 0 \quad (4.18)
\]

On the second half step, the quantities in \( \theta \) direction are updated.
4.3.2 3D Alternating Direction Implicit Methods

For 3D calculations, I use Equation (4.15). The ADI method is difficult to extend to three dimensions. I choose a scheme similar to the one developed by Douglas and Gunn (1964). This method includes three time steps. In the first time step, only values in $r$ direction are updated. In the second time step, values in $\theta$ direction are updated. At last, values in $\phi$ direction are updated. The first two steps are the same as in 2D ADI method except there are terms in the $\phi$ direction. Writing down the total discretized equation is tedious but not difficult.

4.4 Boundary conditions

The boundary condition in $r$ is set to be the Dirichlet boundary condition, which means

$$f(r = 0.05\text{AU}, \theta, \phi, p, t) = f(r = 9\text{AU}, \theta, \phi, p) = 0 \quad (4.19)$$

Although other kinds of boundary conditions are possible, these choices do not affect our conclusion as has been discussed in Hamilton (1977); Webb and Quenby (1973).

For the boundary condition in $\phi$, I use the periodic boundary condition which is determined by physics. This is shown in Figure (4.1). In the code, the periodic boundary condition is satisfied by setting

$$f(r, \theta, \phi_0, p, t) = f(r, \theta, \phi_K, p, t)$$
$$f(r, \theta, \phi_{K+1}, p, t) = f(r, \theta, \phi_1, p, t) \quad (4.20)$$

where $K$ the total number of nodes in $\phi$ (the number of nodes start with 1, ends with $K$). That is to say, to calculate the central difference of the node, 1, we use $f(r, \theta, \phi_K, p, t), f(r, \theta, \phi_1, p, t), \text{ and } f(r, \theta, \phi_2, p, t)$. To calculate the central difference of the node, $K$, we use $f(r, \theta, \phi_{K-1}, p, t), f(r, \theta, \phi_K, p, t), \text{ and } f(r, \theta, \phi_1, p, t)$.

For the boundary condition in $\theta$, I use the “periodic” boundary condition too. The actual treatment is shown in Equation (4.21). As we know there are two “singular” points, the poles, where $\sin \theta = 0$ in spherical coordinates. These “singular” points are not physical. Here I choose not to put the calculation node at the pole.
Figure 4.1: Boundary condition in $\phi$. 
to avoid this singular point. The index of the first node in $\theta$ is 1. So when I need to calculate the second order derivative on the first node, I need the node 0. I follow the method provided in Mohseni and Colonius (2000) which is

\[
\begin{align*}
    f(r, \theta_0, \phi_k, p, t) &= f(r, \theta_1, \phi_{k+K/2}, p, t) \\
    f(r, \theta_{M+1}, \phi_k, p, t) &= f(r, \theta_M, \phi_{k+K/2}, p, t).
\end{align*}
\]

(4.21)

4.5 The initial condition

The initial condition is related to injection profile for solar events which usually takes two forms as has been discussed in 1.3.2. Here I choose a compromise of these two, which is a Gaussian,

\[
f(r, \theta, \phi, p, 0) = N_0 \exp \left[ -\frac{(\rho - \rho_0)^2}{\sigma} \right]
\]

(4.22)

where $\|\rho_0 - \rho\|$ is the distance away from the solar event position.

4.6 Non-uniform grid

For non-uniform grid, we use a map function defined by Equation (4.23) which makes the mesh finer close to the Sun and at the transition region in CME simulation. The mesh is shown in Figure (4.2).

\[
q = \arctan \frac{r - r_i}{\epsilon_1} + \arctan \frac{r - r_o}{\epsilon_2}.
\]

(4.23)

Here, $\epsilon_1 = 0.5$ AU and $\epsilon_2 = 0.3$ AU are constants, which control the shape of the mesh to put more grid points near $r = r_i$ and $r = r_o$. Since $r_i$ and $r_o$ change with time, this non-uniform mesh changes with time, too. Thus, an interpolation technique is needed in each time step. To save computational time, we update the mesh every 500 seconds. During this short time, the CME travels a distance much less than the shortest radial step in our mesh.
Figure 4.2: Non-uniform grid by map function (4.23).
The relations between $q$ and $r$ are listed here,

$$ q = \arctan \frac{r - a}{\epsilon_1} + \arctan \frac{r - b}{\epsilon_2} \quad (4.24) $$

$$ \frac{dq}{dr} = \frac{1/\epsilon_1}{1 + (r - a)^2/\epsilon_1^2} + \frac{1/\epsilon_2}{1 + (r - b)^2/\epsilon_2^2} \quad (4.25) $$

$$ \frac{d^2q}{dr^2} = -\frac{2(r - a)/\epsilon_1^3}{(1 + (r - a)^2/\epsilon_1^2)^2} - \frac{2(r - b)/\epsilon_2^3}{(1 + (r - b)^2/\epsilon_2^2)^2} \quad (4.26) $$

$$ \frac{\partial}{\partial r} = \frac{\partial dq}{\partial q} dr \quad (4.27) $$

$$ \frac{\partial^2}{\partial r^2} = \frac{\partial}{\partial r} \left( \frac{dq}{dpr} \frac{\partial}{\partial q} \right) = \left( \frac{dq}{dr} \right)^2 \frac{\partial^2 f}{\partial q^2} + \frac{d^2q}{dr^2} \frac{\partial^2 f}{\partial r^2} \quad (4.28) $$

if $q \neq 0$ we have

$$ r(q) = \frac{a + b}{2} - \frac{\epsilon_1 + \epsilon_2}{2 \tan q} \pm \sqrt{\left( \frac{a + b}{2} - \frac{\epsilon_1 + \epsilon_2}{2 \tan q} \right)^2 - ab + \epsilon_1 \epsilon_2 + \frac{\epsilon_2 a + \epsilon_1 b}{\tan q}} \quad (4.29) $$

Let us take Equation (4.3) as an example. This equation is transformed into

$$ \frac{\partial f}{\partial t} = \kappa_{rr} \left( \frac{dq}{dr} \right)^2 \frac{\partial^2 f}{\partial q^2} + \kappa_{\theta\theta} \frac{\partial^2 f}{r(q)^2 \partial \theta^2} $$

$$ + \left[ \frac{d^2q}{dr^2} \kappa_{rr} + \kappa_{rr} \frac{d^2q}{dr^2} + \left( \frac{dq}{dr} \right)^2 \frac{\partial \kappa_{rr}}{\partial q} \right] \frac{\partial f}{\partial q} $$

$$ + \left( \frac{\cot \theta \kappa_{\theta\theta}}{r(q)^2} \right) \frac{\partial f}{\partial \theta} $$

$$ - \frac{u_r}{dr} \frac{\partial f}{\partial q} + 2u_r \frac{\partial f}{3r(q) \partial \ln p} \quad (4.30) $$
CHAPTER 5

Reservoir phenomenon

During the decay phase of individual gradual solar energetic particle (SEP) events, the intensities measured by different spacecraft can be nearly equal, even if these spacecraft are separated by several AU in radius and by 70 degrees in latitude. This effect is called the “reservoir” phenomenon or “spatial invariance” in the literature. Although there are several qualitative models that explain this by applying adiabatic cooling, interplanetary diffusion, or continued particle acceleration at shocks, there is no consensus to the responsible physical mechanism. In this chapter, I present results from new numerical simulations of energetic particle transport inside a heliosphere distorted by solar disturbances. I compare our numerical solutions with observations from Ulysses, IMP-8, and ACE. Many of the observed features can be reproduced by our model which includes drift, energy loss, and spatial diffusion. This model is based on Parker’s transport equation with the application of a reasonable value of $\kappa_\perp/\kappa_\parallel$ and the effect of a series of coronal mass ejections (CMEs) on the diffusion coefficient. I conclude that anisotropic diffusion may provide an explanation of the “reservoir” phenomenon. This is the first quantitative model for this phenomenon although the treatment of the CME effect is similar to the qualitative analysis of Roelof et al. (1992).

5.1 Introduction

The transport of solar energetic particles (SEPs) from their source through the heliosphere remains one of the most important unsolved problems in heliophysics. Most of SEP measurements have been taken either near the Earth or near the ecliptic plane. Owing to the first in situ measurements at high heliographic latitude made by Ulysses, I now have a better understanding of how SEPs propagate in the inner
heliosphere.

SEP events are traditionally divided into two categories, impulsive events and gradual events, according to their duration and composition (Reames, 1999) (I call it the two-class paradigm in this dissertation). Impulsive solar events usually last a few hours and are characterized by large enhancements of $^3\text{He}^{++}/^4\text{He}^{++}$ relative to coronal abundances. Gradual events last several days and their average abundances often reflect coronal abundances. I also note that recent observations by ACE, Wind, and SOHO show a mixture of these two events indicating that a combination of effects is taking place (Giacalone and Kóta, 2006). So it is possible that particles are accelerated close to the Sun and are experience diffusion for large SEP events.

For many impulsive events and for the early stage of gradual events, the particle mean free path is of the order of 1 AU which means the particles are almost scatter free from the Sun to the Earth. In this case, the transport of the SEPs is governed by the focused transport equation (Roelof, 1969; Ng and Wong, 1979; Ruffolo, 1995). Although it may be possible to incorporate the perpendicular diffusion into these equations, to our knowledge, this has not yet been done.

For many gradual events the particle’s mean free path is of the order of 0.01 AU which means the scattering is strong enough that the distribution function is almost isotropic in the plasma frame. Although Reames (1999) argues that even for gradual events, SEPs are almost scatter-free, High-heliographic-latitude observations from Ulysses show that the onset time of particles were not organized by the particle speed (Zhang et al., 2003), which can be see as a signature of diffusion. This indicates that the scatter-free assumption maybe not applicable during some large solar events. The anisotropy observations by Ulysses also suggest that a large cross-field diffusion is needed to explain high heliographic latitude observations. In this case, Parker’s transport equation (Parker, 1965) is appropriate.

Burlaga (1967) presented a nice analytic analysis on anisotropic diffusion (the parallel and the perpendicular diffusion is different) which is consistent with the observations. However, there are several limitations of this analysis because the magnetic field is a radial field and the interplanetary media is discontinuous at 1 AU.
In this analysis, the solar wind convection, drift, and adiabatic cooling effects were not included. In this chapter I want to investigate the diffusion process numerically including a more reasonable magnetic field and convection, drift, and adiabatic cooling effect.

It was first reported by McKibben (1972) that similar particle intensities were observed by spacecraft separated widely in longitude during the decay phase of large (gradual) events. However, the time profiles observed by different spacecraft are different during the early stage of these large events. This difference maybe implies the anisotropic diffusion which has been shown by Burlaga (1967) (see Fig. 1 of his paper). Hence I believe that this feature is consistent with the idea of perpendicular diffusion. In addition to this longitudinal invariance, Ulysses observations indicate that these similar intensities are also observed at high latitude and at distances > 2 AU (Zhang et al., 2003). This phenomenon is called the “reservoir” phenomenon (Roelof et al., 1992) or “spatial invariance” (Reames, 1999) in the literature. Throughout this chapter, I refer to this as the “reservoir” phenomenon.

Several explanations have been proposed to explain these observations. Because a series of coronal mass ejections (CMEs) can strongly distort the interplanetary magnetic field before the associated flare, Roelof et al. (1992) suggested that the inner heliosphere served as a reservoir for low-energy solar particles during the decay phase of gradual events. In this picture, propagating magnetic disturbances impede the escape of SEPs from the inner heliosphere. Lee (2000) discussed this phenomenon briefly and concluded that the common decay is caused by adiabatic deceleration which dominates the particle transport during the decay phase of solar events (convection and diffusion are neglected in this analysis). His model explains why the decay rates of SEPs are almost identical at different locations, but leaves unexplained why the absolute values of the particle intensity are also nearly invariant with distance. Reames et al. (1997) contended that the SEPs are accelerated by interplanetary shocks, not by solar flares. Thus they interpreted the “reservoir” phenomenon as a result of shock acceleration which can occur far beyond the orbit of the Earth. But for many gradual events no associated shocks are observed (Lario
et al., 2003). And recent observations show that diffusion is important even for impulsive events (Wibberenz and Cane, 2006; Mason et al., 2006). Furthermore, I need a mechanism to explain particle’s migration to high latitude in this theory.

In this chapter, I show that a simple transport model, including perpendicular diffusion, with a region of reduced parallel diffusion to approximate roughly the effect of a series of CMEs, agrees well with the observations. This picture is similar to that proposed by Roelof et al. (1992). Observations are presented in Section 2. Our mathematical model is presented in Section 3. Numerical simulations based on this model are compared with observations from Ulysses, IMP-8, and ACE for the 12 September 2000 event in Section 4.

5.2 Observations

During February - March, 1991, there were 35 large X-ray flares recorded by Solar Geophysical Data (SGD), most of which came from three active regions (AR): 6538, 6545, and 6555. These active regions produced several large X-ray flares. After AR 6538 stopped producing large X-ray flares, AR 6545 began to produce flares two days later. AR 6555 commenced its production just about one day after AR 6545 stopped its activity. This feature is indeed very common in SGD and in reports by Space Environment Center (SEC). During that time Ulysses was about 2.5 AU away from the Sun and well connected to almost the same longitude (W60) as IMP-8. The latitude of Ulysses was around 1.8 degrees (north).

The intensities of 0.3-0.55 MeV/nuc ions measured by Ulysses and by IMP-8 were analyzed by Roelof et al. (1992) for the period of Mar 22 - Apr 20, 1991. The intensities given by these two sets of data were remarkably close to each other during the decay phase of solar events. This implies that the radial gradient of particle intensities during this period was almost zero. Roelof et al. (1992) suggested that this observation is a definitive evidence for the inner heliosphere being a “reservoir” for low-energy particles during the decay phase of gradual events. In this view, CMEs, produced earlier than the associated flare, distorted the interplanetary magnetic
field and kept energetic particles from escaping into the outer heliosphere.

In this chapter, I choose the 12 September 2000 event to test our model, because for this event I have the advantage of three spacecraft observations (Ulysses, ACE, and IMP-8). Moreover, because Ulysses was at high heliographic latitude during this period, I am able to determine the latitudinal gradient as well. All observational data was obtained from NASA’s web site (http://nssdc.gsfc.nasa.gov).

From 1 Sep 2000 to 12 Sep 2000 there were more than 60 X-ray flares recorded by SEC. Most of them were C-class flares. There were 3 M-class flares and no X-class flares recorded during this period. Three active regions, AR 9149, AR 9154, and AR 9158, were the main source for these flares. They behaved similarly as the active regions of March 1991 discussed above. The X-ray flares were produced alternately and in succession by these three regions. Although there is no direct observation of a CME available, it is possible that the interplanetary magnetic field was distorted, which reduces the transport of energetic particles in this region, as Roelof et al (1992) postulated. For this event, Ulysses was about 2.7 AU away from the Sun. Its latitude was about 70 degrees (south). The relative positions of Ulysses and the Earth are shown in Figure 5.1.

5.3 Modeling of the effect

To investigate the diffusion process during the large solar events, I use Parker’s equation (Parker, 1965) (please refer to Equation 1.2 or 4.1). Although many numerical simulations has been done based on this equation, for example, Hamilton (1977); Webb and Quenby (1973), these are all 1D simulations which implies the source of SEPs is a spherical surface, not point source. I try a 2D description in space in this chapter. In a 2D simulation, the SEP source is not point source. Apparently, a 2D simulation makes more sense than a 1D simulation. A full 3D simulation is more desirable, and one is under construction. Since no ones has been working on this, I believe it worth a try.

The calculation is carried out in two spatial dimensions, $r$ and $\theta$. The boundary
Figure 5.1: 3D view of Ulysses trajectory in September 2000.
conditions in radial direction are specified as \( f(0.01\text{AU}, \theta, p, t) = f(5\text{AU}, \theta, p, t) = 0 \). The boundaries in \( \theta \) direction are reflecting boundaries, i.e., \( \partial / \partial \theta = 0 \). I also considered other boundary conditions in the radial direction, and found no significant effect on our conclusions. The initial condition is given by \( f(r, p, t = 0) \propto \delta(r - 0.1\text{AU})\delta(\theta - \pi/2)p^{-5} \).

I consider the transport of energetic particles in the heliospheric magnetic field that consists of an average component (the Parker spiral) with superimposed magnetic irregularities. The average magnetic field is given as

\[
B_r(r, \phi, \theta) = B_0 \left( \frac{r_0}{r} \right)^2, \\
B_\phi(r, \phi, \theta) = -B_0 \frac{\omega_\odot r_0}{V_w} \frac{r_0}{r} \sin \theta \left[ 1 - \left( \frac{r_0}{r} \right)^2 \right], \\
B_\theta(r, \phi, \theta) = 0. 
\]

(5.1)

where \( r, \theta, \) and \( \phi \) are the radial, zenith, and azimuth coordinates in spherical coordinates, respectively. I take \( r_0 \), the Alfvén radius, to be 12 times the solar radius \( r_\odot \) which is taken to be \( 6.995 \times 10^{10} \text{ cm} \). I take the magnetic field on the solar surface \( B_0 = 1.6 \text{ Gauss} \), radial solar wind speed \( V_w = 4 \times 10^7 \text{ cm s}^{-1} \), and rotational rate of the Sun is \( \omega_\odot = 2.7 \times 10^{-6} \text{ radian s}^{-1} \). Note that the heliospheric current sheet is not included in this model. This does not affect our results because the energetic-particle diffusion coefficient does not dependent on the magnetic polarity.

As we know, CMEs are usually associate with magnetic re-connections or shocks happened near the solar surface. So the magnetic field inside a CME can be distorted by these strong nonlinear effects. When CMEs fly away from the Sun, they carry these disturbances.

In order to simulate the effect of the disturbances inside CMEs on the diffusion coefficient, the spatial diffusion tensor is multiplied by a ‘shape’ function \( S(r, \theta, t) \) whose effect is to reduce the diffusion coefficient in the region disturbed by CME. This region is represented by a “structure” in our simulation which is 120 degrees (\( \pi/3 \text{radians} \)) and symmetric about equator. \( r_o \) is the outer boundary and \( r_i \) is the inner boundary of this structure. The initial value of \( r_i \) is set to be 0.5 AU,
\[ r_i(t) = 0.5\text{AU} + V_w t. \] The width of the plasma disturbance, \( W \), is set to either 1 AU or 2 AU, \( r_o(t) = r_i(t) + W. \)

\[ S(r, \theta, t) = \left( \tanh \frac{r - r_o}{L} - \tanh \frac{r - r_i}{L} \right) \left[ \tanh b \left( \theta - \frac{\pi}{3} \right) - \tanh b \left( \theta - \frac{2\pi}{3} \right) \right] + a, \] (5.2)

where \( L \) is the characteristic length of the disturbance and is set to be approximately equal to \( \lambda_\parallel \), which is the mean-free path of a particle with energy of 10 MeV/nuc. \( a \) controls the ratio of the maximum value of the diffusion coefficient to the minimum value. Higher \( b \) corresponds to a sharper transition at \( \theta = \pi/3 \) and \( \theta = 2\pi/3 \).

To illustrate this function, \( \kappa_\parallel \) is shown in Figure 5.2 (b) and (c). The solid line represents \( \kappa_\parallel \propto \beta R^\alpha \), and the dashed line represents \( \kappa_\parallel \propto \beta R^\alpha \sqrt{r} \). These two different choices have no significant effect on our conclusions.

I use a non-uniform computational mesh in the radial direction to reduce the computation time and to focus on the transition region and the region near the Sun, where the diffusion coefficient changes dramatically. The map function from \( r \) to the new variable \( q \) is given by Equation (4.23). To save computational time, I update the mesh every 500 seconds. During this short time, the CME travels a distance much less than the shortest radial step in our mesh (please refer to Section (4.6) for more discussion).

5.4 Results

Figures 5.3 and 5.4 shows the results from our simulation compared with ACE and Ulysses observations. The ion energy range for the ACE (EPAM) data is 1.9-4.75 MeV/nuc. And that for Ulysses (HISCALE) is 1.8-5.0 MeV/nuc.

First, from the observations of Ulysses and ACE (Figure 5.3), I can see that there is a delay in the time of the onset of the event seen at Ulysses compared to ACE. After the onset, Ulysses sees a slower increase than does ACE. The time difference between the maximum measured by Ulysses and that by ACE is more than 3 days. So when the intensity at Ulysses reaches its maximum, the intensity at ACE (at the same time) is already several orders of magnitude smaller than its peak.
Figure 5.2: $\kappa_\parallel$ as a function of position. The solid line and the dashed line are two different cases for $\kappa_\parallel$. 
Figure 5.3: Data from Ulysses (solid line with x-mark) and ACE (solid line) in Sep 2000. The ion channel of ACE is 1.9-4.75 MeV/nuc. The ion channel of Ulysses is 1.8-5.0 MeV/nuc. Numerical simulations are in dashed line and dotted line.
Figure 5.4: Flux ratio between Ulysses and ACE. The solid line is the simulation result.
value. After the peak, the intensity decays at an almost constant rate for several days. Note that there is a spike on day 267 in the intensity time profile seen at ACE which is not recorded by Ulysses. These spikes are usually called “energetic storm particle” (ESP) events in literature. In our simulation, I don’t take the spike into account because most of the “reservoir” events don’t have spikes in their intensity time profile (McKibben et al., 2001). I believe that ESP events are not a major contributor to the “reservoir” phenomenon.

Figure 5.4 shows the ratio in particle intensities seen by the spacecraft, and the numerical simulated results during the same period. In this figure, it is clear that from day 260 to day 266 of year 2000, the intensities measured from Ulysses and ACE are almost equal. Our simulation starts shortly after day 256 of 2000 which is shown by the dash line and the dotted line in Figure 5.3 and by the solid line in Figure 5.4. The simulated particle intensities for Ulysses and ACE are similar to the observations during the rise phase and throughout the event. Thus that our simulations reproduce the “reservoir” phenomenon. Note that the observation data of the two spacecraft overlapped almost perfectly for more than four days during the decay phase. I point out here that the simulations do not follow the observation exactly. Our simulations do not reproduce the oscillations as seen in Figure 5.3 and 5.4 which is possibly because the existing plasma turbulence is not fully incorporated into this model. However, our simulation falls in between the observations, which means this model is valid in principle.

In Figure 5.5 and Figure 5.6 I show the time intensity profile for the same event but for different particle energies and different spacecraft pair (8-19 MeV/nuc for Ulysses and 15 - 25 MeV/nuc for IMP-8). For these ions, there is a spike recorded by Ulysses that was not seen by IMP-8. The ratio between the observed data from Ulysses and IMP-8 is larger than low-energy particles shown in Figure 5.3 and Figure 5.4. This may due to the larger energy range of Ulysses compared to than that for IMP-8. The same feature is also reported by Roelof et al. (1992) Figure 6 panel d. (The IMP-8 raw data include extremely large values which are due to the effect of the magnetosphere. I removed most of these “bad” data.)
Figure 5.5: Measurements made by Ulysses(x-mark) and IMP-8(solid line). The ion channel is 8-19 MeV/nuc for Ulysses and 15-25 MeV/nuc for IMP-8. Numerical simulations are in dashed line and dotted line.
Figure 5.6: The flux ratio of Ulysses over IMP-8. The solid line is the simulation result.
Figure 5.7 shows the comparison between different terms in Equation (1.2) to help identify the important effects. During the decay phase of large SEP events, the convection term is very small compared to other terms. The terms involving diffusion and adiabatic deceleration are comparable. Lee (2000) points out that because of the small gradient of particle intensities, the diffusion term is negligible. Our results indicate that diffusion is not negligible although the cooling term is about 2 times larger than the diffusion term. The sum of these two effects dominates the particle transport.

I also show another event in Nov 2000 in Figure 5.8, observed by Ulysses and IMP-8. The ion energy range is 8-19 MeV/nuc for Ulysses and 15-25 MeV/nuc for IMP-8. It shows that the reservoir phenomenon is common and our model can fit them well.

Although there is no consensus on the dependence of $\kappa_\parallel$ on solar radius, our calculations show that in a wide range of choice that this doesn’t have a noticeable effect on our conclusion. A quantitative indication of the agreement between our model and observations is the correlation coefficient. For example, the correlation coefficient of our simulation and observed data during the decay phase only changes from 0.88 (IMP-8), 0.88(Ulysses) to 0.89 (IMP-8), 0.88 (Ulysses) by changing $\kappa_\parallel$ from $\kappa_\parallel \propto \beta R^\alpha$ to $\kappa_\parallel \propto \beta R^\alpha \sqrt{r}$. Here correlation coefficient is defined as

$$R = \frac{C(x, y)}{\sqrt{C(x, x)C(y, y)}}$$

(5.3)

where $C(x, y)$ is the covariance of $x$ (observed data) and $y$ (numerical data) which is defined as

$$C(x, y) = \frac{\sum_{n=1}^{N} (x_n - \mu_x)(y_n - \mu_y)}{N}$$

(5.4)

where $\mu_x$ is the mean of $x_n$, $\mu_y$ is the mean of $y_n$, and $N$ is the total number of values that I are interested in. If the observed data and numerical simulation are perfectly correlated, $R = 1$.

I have tested different values of the width of the plasma disturbance, $W$, and the reduced $\kappa$ inside the disturbance in our simulations. The numerical simulations
Figure 5.7: The time profile of different terms in equation 1.2. The dotted line shows the convection term. The line with triangle mark shows the diffusion term. The solid line is the adiabatic cooling rate. The dashed line is $df/dt$ term.
Figure 5.8: Ulysses and IMP-8 observations of Nov 2000 event. The ion channel is 8-19 MeV/nuc for Ulysses and 15-25 MeV/nuc for IMP-8. Numerical simulations are in dashed line and dotted line. The solid line is IMP-8 measurement and the line with a star mark is Ulysses measurement.
show that the correlation coefficient changes with these parameters. But the dependence on $W$ and $\kappa$ is very weak. For example, if I change the width of the plasma disturbance from 1 AU to 2 AU, the correlation coefficient of simulation and observation changes from 0.75 (IMP-8), 0.87 (Ulysses) to 0.74 (IMP-8), 0.85 (Ulysses). If I reduce the diffusion coefficient by half and keep the width unchanged, the correlation coefficient of simulation and observation changes from 0.74 (IMP-8), 0.85 (Ulysses) to 0.75 (IMP-8), 0.87 (Ulysses). However, the correlation coefficient of simulation and observation keeps almost unchanged if I change the width from 1 AU to 2 AU and reduce the diffusion coefficient by half at the same time. So the relation between the correlation coefficient, $R$ and $W$ or $\kappa$ cannot be expressed in an analytic form.

5.5 Conclusions

I find that the “reservoir” phenomenon is very common in the heliosphere by examining observation data from Ulysses and other near-earth spacecraft although I only present two solar events in this dissertation. This phenomenon cannot be explained by using spherically symmetric diffusion model (1D model) for two reasons. First, for spherically symmetric diffusion models, the intensity-time profiles measured by near-earth spacecraft should be the same if SEPs are accelerated by solar flare which can be viewed as a point source. Obviously observations do not support this (McKibben, 1972). So the transport of SEPs is not a 1D process in nature. Second, the high latitude measurements by Ulysses suggest that the perpendicular diffusion is important which is consistent with Wind observations (Dwyer et al., 1997; Zhang et al., 2003). But standard spherically symmetric models cannot incorporate the perpendicular diffusion. Thus multi-dimensional diffusion model is the right choice for the transport of SEPs. Because analytic solutions are impossible, I seek a numerical solution for multi-dimensional diffusion. In this chapter, I provide a method to solve 2D diffusion for SEPs. By applying this method, I can quantitatively explain both the reservoir phenomenon and the high latitude observations by Ulysses.
6.1 Introduction

It is well accepted that particles associated with solar flares (high charge states, high abundance of heavy elements, and enrichment of $^3$He) are present in large SEP events which are usually associated with CME-driven showers. But it is still being debated whether flare-related particles are CME-driven-shock accelerated remnants from previous flares or these particles are mainly accelerated by solar flares, especially for $> 10$ MeV/nuc ions and for $> 250$ keV electrons.

After the fall of “flare myth”, it is often assumed that it is the CME-driven shock that accelerates particles for gradual events in Reames’ two-class paradigm (Reames, 1999). This theory has several problems. First, there are no open field lines along which flare-accelerated particles can escape (Reames, 2002), so these particles have no access to the interplanetary space. Second, if the particles are from the solar flares, it is hard to explain why we can observe gradual events over a very wide range of longitude (about 160 degrees), because the particles are scatter-free in this paradigm and the flares are considered point sources or very small source regions (Tylka et al., 2005). The shock acceleration mechanism can explain this by assuming an expanding shock away from the Sun.

However, new high-sensitivity observations during solar Cycle 23 (roughly from 1997 to 2007) show that many large gradual SEP events have high abundance of Fe/O and $^3$He/$^4$He well above the “consensus” values given by Reames (1999) (Readers please refer to Table (1.1) to see these values). Cane et al. (2002) indicated that flare-accelerated electrons do have access to the interplanetary medium based on the type III radio emissions. Laitinen et al. (2000) suggested that CMEs are effective to accelerate low energy protons (around 1 MeV) but not effective to accelerate
electrons above 250 keV. von Rosenvinge et al. (2001) concluded that CME-driven shock model is only consistent with solar events happened east of W15°. For solar events west of W15°, maybe they require flare accelerated particle components.

Cane et al. (2003) divided gradual events into three groups according to the relative abundance of Fe/O. In the first group, they found that, Fe/O can be very high and close to the values measured during impulsive events (19 in 29 events fall in this group). The flare accelerated SEPs dominate in observations above 25 MeV/nuc ions in this first group. For the second and the third group (8 in 29), the shock accelerated SEPs dominate. They also examined the shock speed for these three groups. For events in the first group, the associated shock speeds are around 500 km/sec to 800 km/sec in all but three events. The mean speed of the shocks was 1150 km/sec for group two and 1350 km/sec for group three. As has been pointed out by Cane (1985), such high speed shocks are rare, with less than 10 occurrences per solar cycle.

Cane et al. (2006) extended the study of Cane et al. (2003) to include the analysis of > 25 MeV/nuc Fe and O for 97 events between 1997 and 2005. They found that the abundances are reasonably organized by event profiles at 1 AU such that events with peak intensity shortly after the associated flare are Fe-rich and events peaking at the passage of an interplanetary shock are Fe-poor. These variations in elemental composition suggest that high Fe/O is attributed to the associated impulsive flares and low Fe/O is attributed to shock acceleration.

All these new observations mentioned above indicate that the flare-accelerated particles are dominant for a large portion of gradual events, especially for > 10 MeV/nuc ions and for > 250 keV electrons. New observations also show that for many gradual events, the SEPs are not scatter-free, but diffusive. Figure (6.2) shows the time-intensity profile for Fe and O observed by ACE spacecraft (Mason et al., 2006). On the top, Fe and O profiles are plotted and compared at the same energy. Clearly, these profiles are different. On the bottom, iron’s energy is the same as on the top plot. But oxygen’s energy is doubled. Fe’s profile and O’s profile follow each other, even for small structures. If these particles are scatter-
Figure 6.1: Three-day plots of Fe (filled circles) and O (asterisks) intensities at about 30 MeV/nuc (upper panels), 50 MeV/nuc (middle panels), and 80 MeV/nuc (lower panels) showing examples of the three groups of events at these energies, differentiated by their intensity-time profiles. The vertical dotted line indicates the time of the flares and the dashed lines indicated the times of shock passages. (Cane et al., 2003)
Figure 6.2: Top: Oxygen (solid red line) and iron (dashed blue line) hourly averaged intensities at 273 keV/nuc and $\approx 12$ MeV/nuc. Bottom: same as left panel as Fe intensity; The Oxygen intensity are at approximately twice the kinetic energy per nucleon as the left panel and renormalized as shown to facilitate comparison with Fe intensity. (Mason et al., 2006).
free, then for 546 keV/nuc O would reach 1 AU 2 hours earlier than 273 keV/nuc Fe. Although we can not rule out the possibility that Fe escaped 2 hours earlier from the shock, this seems unlikely to me because this coincidence occurs for other energies. For example, there is about 15 minutes difference for about 10 MeV O and Fe. What’s more, this behavior of Fe and O happens in a large fraction of solar events between 1997 to 2005 (more than 70%). The most likely explanation for this requires interplanetary diffusion.

So we need an quantitative 3D diffusion model to test these theories. In this chapter, I present our 3D solution of Parker’s transport equation with the method described in chapter 4. Our simulations are compared with obervations made by Helios, ISEE 3, and IMP-8.

6.2 Observation

This study is based on electron observations by IMP-8, ISEE 3, Helios 1, and Helios 2. The nominal energy range is 0.3 - 0.8 MeV for the Helios spacecraft, 0.2-2.5 MeV for IMP-8, and 0.2-2 MeV for ISEE 3.

There are 19 impulsive events are selected for the period of December 1977 to March 1980 based on duration time and type III radio bursts and confirmed by low proton to electron ratios.

The connection angle $\Delta$ is defined as the longitudinal distance between the flare longitude and the footpoint of the magnetic field line connected to the spacecraft. The positive value means the footpoint is at the east of the flare. Figure 6.3 shows the maximum intensity (normalized to 1 AU) observed by three spacecraft.

6.3 Model

The interplanetary magnetic field is described as an average field (the Parker spiral) upon which are superimposed irregularities (refer to Equation 5.1).

The evolution of the omni-directional distribution $f(p, x, t)$ satisfies Parker (1965) (please refer to Equation 1.2 or 4.1). As in Chapter 5, we use central differ-
ence in space and choose ADI method to solve this equation.

We take $\kappa_\perp = \kappa_\parallel / 4$ and $\kappa_\perp = \kappa_\parallel / 8$ in our calculation. We assume that $\kappa_\parallel \propto \beta R^\alpha$, where $R$ is the rigidity. $\beta$ is the ratio of particle speed over speed of light. We choose $\alpha$ to be 0.5.

The radial step is 0.04 AU. In $\theta$ direction, the step is $\pi/24 = 0.13089969375$. In $\phi$ direction, the step is $\pi/40 = 0.078539815$. The initial and boundary conditions are described in Chapter 4. The total time is about 5 hours.

6.4 Results

The peak intensities observed by the spacecraft and calculated by my model are shown in Figure 6.3. Peak intensities for the same event are marked by the symbols and colors. The x axis is the connection angle where positive value means the footpoint of the magnetic field line connected to the spacecraft is east of the flare. My simulations are shown in three curves. The dashed blue line is the simulation with $\kappa_\perp = \kappa_\parallel / 4$. Red lines are simulations with $\kappa_\perp = \kappa_\parallel / 8$. These two red lines are the same in shape, but the high one is shifted to match the event with blue x symbols.

Figure 6.4 to 6.6 show the simulations for three different events. The definition of the blue line and red lines is the same as before. We can see that sometime, the blue line fits the observations better, sometimes, the red line fits better. So it seems that for these 13 events, $\kappa_\perp / \kappa_\parallel$ is in the order of 0.125 $\sim$ 0.25.

One important feature of Figures 6.3 to 6.6 is that these events can be observed over a range of angular distance from the flare of more than 160 degrees. The observed peak intensities decrease with connection angle. As mentioned before, these events are all impulsive events which is observable in $\pm$ 20 degree from the flare in Reames’ two-class paradigm. Possible explanations for the discrepancy include coronal transport and interplanetary diffusion.

The coronal transport is a process in which energetic particles are scattered in the corona before they have a chance to follow open field line to escape. The extent
Figure 6.3: Variation of peak intensities with connection angle for 13 events.
Figure 6.4: Numerical simulation vs Observation for a single event.
Figure 6.5: Numerical simulation vs Observation for a single event.
Figure 6.6: Numerical simulation vs Observation for a single event.
of this region in low corona may be limited by the active region’s outer edges, so it is not easy to explain the large angular extents.

The interplanetary model as shown by Figures 6.3 to 6.6 which is our numerical model based on, is promising.

6.5 Discussion

In this chapter I show that a three dimensional solution of Parker’s transport equation can explain the multi-spacecraft observations of peak intensity of SEPs. Clearly, we can understand the transport of SEPs better if we have more spacecraft measuring the same event. Thus, a fully 3D numerical model with the real plasma turbulence incorporated and with the flexibility of using different theories is desirable. With the help of newly launched STEREOs, Ulysses, ACE, Wind and Voyagers, it is possible to validate this numerical model in a wider scenarios.

One important assumption used to simplify the 3D equation is that the injection of particles takes an power-law distribution. Although this is true in principle, there is an uncertainty in determining the injection profile. Without this assumption, this method will need to add one more dimension and one more initial condition in momentum space which required more computation time and storage.
CHAPTER 7

Future Work

For the interplanetary magnetic field, a new model should include gravity, thermal pressure, magnetic force, and time-dependent terms. Although a fully 3D MHD model is desirable, a simplified successful model is more realistic since solving the 3D equations is not an easy task to do.

A detailed discussion of turbulence theory of the plasma is absent in this dissertation because linking the turbulent properties to the diffusion tensor is beyond the scope of this dissertation.

Jokipii (1966) first showed how to calculate the parallel mean free paths from turbulence observations by using quasilinear theory, although the calculated values are about one tenths of the observed values. Subsequent research has been done for parallel diffusion by Jaekel and Schlickeiser (1992); Bieber et al. (1994); Ng and Reames (1995); Shalchi et al. (2004) which took into account of more complicated turbulence models and nonlinear scattering effects. Still, there is no consensus on the governing theories for the parallel diffusion.

Forman et al. (1974) presented analytic solutions for perpendicular diffusion by using quasilinear theory. Subsequent theories include theories based on Taylor-Green-Kubo equation (Bieber and Matthaeus, 1997), “nonlinear guiding center” (NLGC) theory (Matthaeus et al., 2003). Kóta and Jokipii (2000) pointed out that any perpendicular model less than 3D gives smaller perpendicular coefficient because in this case it is sub-diffusive.

Thus, a fully 3D numerical model with the real plasma turbulence incorporated and with the flexibility of using different theories is desirable, in order to test and determine the performance of these theories. Fortunately, we now have many observations by Ulysses, ACE, WIND and Voyagers to validate numerical models.
Another important issue of the SEP transport concerns the 3D shape of CMEs. The LASCO coronagraph on board SOHO spacecraft provided nice 2D pictures of CMEs. Although there are simulations of CMEs based on time-dependent MHD equations, for example, Roussev et al. (2004), the actual 3D shape is not available because we don’t have enough space probes to do the multi-point three-dimensional measurement of CMEs. Hopefully, with the launch of STEREO, we can understand this better.

The acceleration and the transport of SEPs must collaborate with the evolution of CMEs to give a full picture of SEPs for CME events in the inner heliosphere. So this is a more challenging task compared to the 3D transport numerical model.
APPENDIX A

Gyro-radius in the Interplanetary Magnetic Field

Supposing the first adiabatic invariant holds, we have

\[ \frac{W_\perp}{B} = C_0 \]  

(A.1)

where \( W_\perp \) is the momentum perpendicular to the magnetic field, \( B \) is the magnetic field strength, and \( C_0 \) is a constant. Or

\[ \frac{1 - \mu^2}{B} = C_1, \]  

(A.2)

where \( C_1 \) is another constant and \( \mu \) is the pitch angle cosine.

By definition,

\[ r_g = \frac{V_\perp}{\Omega}, \]  

(A.3)

where \( V_\perp \) is the perpendicular speed of the particle, \( \Omega \) is gyro-frequency, and \( r_g \) is the gyro-radius.

\[ \Omega = \frac{eB}{\gamma mc} \]  

(A.4)

where \( m \) is the particle mass, \( e \) is the particle charge, \( c \) is the speed of light, and \( \gamma = 1/\sqrt{1 - V^2/c^2} \).

We have

\[ r_g^2 = \frac{\gamma^2 m^2 c^2 V^2 (1 - \mu^2)}{e^2 B^2} = \frac{\gamma^2 m^2 c^2 V^2 C_1}{e^2 B} \]  

(A.5)

If there is no electric field, \( V \) is a constant. So is \( \gamma \). Hence if \( B \) is decreasing with \( r \), \( r_g \) is increasing with \( r \). So the cartoon shown in Roelof (1969) is wrong (see Figure A.1). Figure A.2 is correct. Obviously, if we define \( R_g \) as \( R_g = V/\Omega \), \( R_g \) still increases as \( B \) decreases. Note that the pitch angle cosine becomes closer and closer to 1 if there is no scattering involved.
Figure A.1: Particle trajectory Roelof (1969) shows the gyro-radius decreasing with $r$. This one is wrong.
Figure A.2: Particle trajectory shows the gyro-radius increasing with $r$. This one is correct.
REFERENCES


