VISCOUS RELAXATION OF CRATERS ON ENCELADUS

by

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ACKNOWLEDGEMENTS

“The fact that we live at the bottom of a deep gravity well, on the surface of a gas covered planet going around a nuclear fireball 90 million miles away and think this to be normal is obviously some indication of how skewed our perspective tends to be.” — Douglas Adams

First, I wish to dedicate this thesis to my dear husband, David Smith, for his loving support, encouragement, guidance, and extensive computer assistance. Without him, none of this would have been achievable.

This research would not have been possible without Dr. Elizabeth Turtle, who first got me started on this project from the time I entered LPL. I also want to thank her for seeing this project through to the end, even though she has been across the country at Johns Hopkins Applied Physics Laboratory the last couple of years.

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DIANA ELIZABETH SMITH
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ABSTRACT

Cassini spacecraft images of Enceladus’ surface have revealed diverse terrains—some heavily cratered, others almost devoid of craters, and even some with ridges and fractures. We have documented crater morphologies in regions for which high-resolution data are available (140 to 360°W and 90°S to 60°N). The south polar region shows a dearth of craters, in sharp contrast to the heavily cratered northern latitudes. Tectonized regions such as Sarandib and Diyar Planitia also have low crater densities. Viscously relaxed craters are found in the apparently young regions of the anti-Saturnian and trailing hemispheres, as well as in the older, upper northern latitudes. By modeling the viscoelastic relaxation of craters on Enceladus using TEKTON, a finite-element code, we predict large geographical variation in heat flow and a complicated thermal history on Enceladus. Our results are consistent with the planitiae being older examples of the South Polar Terrain, supporting a satellite-reorientation hypothesis.
CHAPTER 1

Introduction

1.1 Enceladus

Although Enceladus was discovered in 1789 by William Herschel, until recently, very little was actually known about this tiny satellite of Saturn besides a reflectance spectrum that showed a dominance of pure water ice and a surface that reflects nearly all of the sunlight that hits it (Porco et al., 2006). At a diameter of only about 500 kilometers, making it only the sixth-largest moon of Saturn, one would expect to find an unimpressive, cold icy body with very little interesting geology to be discovered. However, early in the 1980s Voyager 1 and 2 discovered a much more interesting world with a broad range of terrains, from old and heavily cratered to young and tectonically deformed (Smith et al., 1982; Squyres et al., 1983; Kargel and Pozio, 1996). It was also suspected, since Enceladus orbited in the very dense part of Saturn’s E-ring, that there might be a connection between this little satellite and the material making up the E-ring (Pang et al., 1984).

Many of the questions raised by Voyager have now been re-examined with higher-resolution data obtained by the Cassini spacecraft in recent years. Cassini has also opened up many new questions. In 2005, Cassini executed three close flybys
of Saturn’s moon Enceladus, during which the Imaging Science Subsystem (ISS) observed many regions of diverse terrain, some heavily cratered and others almost devoid of craters (Porco et al., 2006).

In addition, Cassini has also discovered an active water-rich plume emanating from Enceladus’ South Polar Terrain (SPT) (Porco et al., 2006; Spencer et al., 2006). Many speculate that this plume may originate from a body of liquid water beneath the surface, which has fueled some discussions of Enceladus’ astrobiological importance. The plume has also brought convincing evidence to the table that the material in the plume is supplying the material in Saturn’s diffuse E-ring.

Enceladus’ icy surface displays evidence of a wide range of diverse terrains and geologic processes including fractures, ridges, active plumes, and variable crater density (e.g., Porco et al., 2006; Brown et al., 2006). Considering Enceladus’ lack of radiogenic heating (with a diameter of only approximately 500 km) and low availability of tidal heating, this amount of geologic activity is somewhat mysterious (Porco et al., 2006).

The study of impact craters provides a method to examine the subsurface structure of a body. Crater morphology is influenced by many factors including gravity and properties of the target material (Melosh, 1989). Viscous relaxation of impact craters presents a means of estimating heat flow, surface age, and rheology. Overall, impact craters can furnish a wealth of information and insight into a planetary body’s physical properties.
Enceladus’ surface displays an interesting dichotomy when it comes to crater density. There are both heavily cratered areas and areas almost devoid of craters, and the craters display a range of morphologies including both relaxed and unre- laxed forms. In some areas both relaxed and unrelaxed craters are present together. Such a heterogeneous surface raises many questions into the nature of Enceladus’ subsurface and rheology. By studying craters that have undergone different degrees of viscous relaxation and the distribution thereof, we can constrain Enceladus’ subsurface rheologic and thermal properties.

Here we present mapping, measurements, and modeling results for viscously relaxed impact craters on Enceladus. Observations of these craters were provided by Cassini Imaging Science Subsystem (ISS) images.
2.1 Introduction

The Imaging Science Subsystem (ISS) aboard the Cassini spacecraft observed the surface of Enceladus across many encounters (e.g., Brown et al., 2006; Porco et al., 2006; Spencer et al., 2006). The images taken revealed diverse terrain, including ridges, fractures, and both relaxed and unrelaxed craters. Such variety of terrain suggests that Enceladus may have a complex internal structure and thermal history. Also, the surface exhibits a significant age dichotomy—the northern hemisphere is heavily cratered, while the southern hemisphere is apparently younger and tectonically resurfaced. A lack of craters south of about 50°S further supports a young age for this region.

An active plume erupting from an obvious heat source at the South Pole observed by Spencer et al. (2006) provides evidence that the South Pole is currently geologically active. This activity has raised questions about Enceladus’ internal thermal structure, since the large amount of geologic activity observed on such a tiny body is surprising. The general lack of craters in the South Polar Terrain (SPT) indicates that the surface is relatively young, and a heat source as significant as that
observed (Spencer et al., 2006) would have a profound effect on the relaxation of geologic features. The lack of craters in the southern latitudes could be due to erasure of craters through relaxation, a young surface, or some other tectonic mechanism. Crater morphology is known to evolve in response to local heat flow. Therefore, study of the impact craters can reveal variations of heat flow based on the change in relaxation state over the surface.

2.2 Terrains

2.2.1 Northern Old Cratered Terrain

The north polar terrain is among the most heavily cratered and oldest on the surface of Enceladus. Figures 2.1 and 2.2 are images of such cratered terrain on the trailing (270°W) hemisphere of Enceladus for latitudes 40 to 60°N. One can see that the surface is peppered with craters, with sizes ranging from greater than 20 km all the way down to craters at the limit of resolution. Both pristine as well as relaxed crater morphologies are seen throughout these images.

2.2.2 Equatorial and Mid-Latitudes

Terrains near the equator and mid-latitudes on the trailing (270°W) side of Enceladus are much less heavily cratered and are characterized by intense zones of fracturing and faulting.

Figure 2.3 is an example of a region that transitions from cratered to tectonically
modified terrain. The westward portion of the image displays a region dominated by sub-parallel grooves and ridges and marks the boundary of Diyar Planitia, an area centered around 0.5°N, 240°W that is relatively uncratered. Craters that border this terrain are degraded and appear shallow. Cross-cutting relations suggest that the majority of these craters predate the fracturing events associated with the periphery of Diyar Planitia. Further to the east in Figure 2.3, the density of surface fracturing decreases and the terrain is instead dominated by craters.

Figure 2.4 shows the region immediately west of that in Figure 2.3, revealing
Figure 2.2: Region of Enceladus from approximately 270 to 360°W in longitude and approximately 40 to 60°N in latitude.

the grooved and ridged terrain to be bounded by cratered terrain to the west. The approximate east-west trending band of relaxed and fractured craters in the transitional terrain seen in Figure 2.3 extends into the section shown in Figure 2.4, continuing to border the highly tectonized region of Diyar Planitia.

2.2.3 Southern Latitudes

Figure 2.5 shows a region of the trailing hemisphere of Enceladus between 40 and 60°S, including a section of the boundary between cratered terrain in the north of the
Figure 2.3: Region of Enceladus from approximately 180 to 270°W in longitude and approximately 0 to 40°N in latitude.

image and the tectonized terrain closer to the South Pole. The region surrounding the South Pole consists of dramatically deformed and fractured terrain that is nearly devoid of impact craters. The South Polar Terrain is the youngest on Enceladus’ surface.

2.3 Crater Data Collection

Global mosaics of Enceladus’ surface assembled from Cassini images were used to identify craters and record their latitude and longitude (using the approximate crater center as reference), relaxation state, and diameter. All images were reprojected to 200 m/pixel resolution in order to reduce any biases due to variations in image resolution across the Cassini coverage of the surface. Crater diameter measurements are, thus, only reliable down to a minimum size, and craters smaller than 4 kilometers
Figure 2.4: Region of Enceladus from approximately 270 to 360°W in longitude and approximately 0 to 40°N in latitude.

in diameter were excluded from our study.

Orthographic projections lead to distortions at images’ edges, so, in order to make accurate measurements of crater diameter, it was necessary to reproject images around the approximate center of each crater to reduce errors. A program named “circlefit” (originally developed by Elizabeth Turtle) was used to select points on the visible crater rim and subsequently calculate a circle through these points with error bars. A very small number (∼5) of severely degraded crater-forms that had no obvious rim could not be reliably identified and fit.

Data were collected and relaxation criteria documented for approximately 250 craters from the aforementioned Cassini images. This chapter focuses on measurements of relaxed craters over the extent of Cassini coverage available. For the purposes of this work, we have classified the craters as “unrelaxed”, “relaxed”, or
Figure 2.5: Region of Enceladus from approximately 180 to 270°W in longitude and approximately 40 to 60°S in latitude.

“uncertain or shallowed due to minor in-fill”.

2.4 Classification Scheme

2.4.1 Fresh Craters

Craters were classified as “fresh” if they appeared to have a well-preserved bowl and relatively steep walls. An example of a fresh crater on Enceladus is shown in Figure 2.6. This particular crater is clearly simple, unfractured, and undegraded, and was measured to be approximately 4 kilometers in diameter.

Figure 2.7 demonstrates of a group of larger unrelaxed craters around 42°N, 344°W. The largest crater in this image is ~10 km in diameter. Many small,
fresh craters dominate much of the surface of Enceladus.

2.4.2 Relaxed Craters

Additionally, many of Enceladus’ craters were classified as “relaxed”. These are craters that have been modified over time through viscous flow of the surrounding ice. An upbowed and/or shallowed crater floor and the presence of a persistent rim are classic characteristics of a crater that has undergone viscous relaxation (Passey and Shoemaker, 1982; Melosh, 1989). Figure 2.8 illustrates an example of an impact crater that has undergone a large degree of relaxation, and Figure 2.9 demonstrates
Figure 2.7: Group of unrelaxed craters in the northern latitudes centered at about 42°N, 344°W.

a circular fit for this same crater as an example.

2.4.3 Uncertain Craters

A crater was classified as “uncertain” if it could not clearly be identified as either relaxed or fresh. These craters generally looked old and shallow; however, in these cases it was unclear as to whether they had been shallowed by infill or by relaxation. Craters that looked anomalously shallow may have been blanketed by plume material. This effect intensified with closer proximity to the South Pole and contributed to a higher number of craters at southern latitudes being classified as “uncertain”.

Other examples of craters classified in this category include those that had
Figure 2.8: Example of a relaxed crater centered at approximately 52°S, 177°W.
Figure 2.9: Example of a relaxed crater centered at approximately 52°S, 177°W showing circular fit. The rim-to-rim diameter of this crater was measured to be approximately 22 km.
Figure 2.10: (a) Example of a crater classified as “uncertain” and centered at approximately 42°N, 298°W. (b) The same crater seen on the left showing the fit of the diameter (measured to be approximately 6 km).

been disturbed in some way after impact, making it difficult to tell whether they were really as shallow as they look. Figure 2.10 shows an example of a crater that exhibits both of these aspects. This specific crater has a shallowed appearance and has fractures crossing it, which makes it difficult to determine whether it has relaxed and, if so, to what degree.

2.5 Observational Results

The locations of relaxed and unrelaxed craters larger than about 4 kilometers in diameter were mapped over regions of Enceladus from about 140° to 360°W and
Figure 2.11: Distribution of sizes for all craters on Enceladus. High-resolution data for longitudes from approximately 0 to 140°W were not available at the time of this survey. Similarly, such data for latitudes above 60°N were also not available.
latitudes 90°S to 60°N. The region between 0° and ~140°W had only been imaged at low resolution (~1 km/pixel) at the time of our analysis, and, therefore, is not included in this survey. This region, or parts of it, may be observed at higher resolution during subsequent Cassini observations.

Figure 2.11 shows crater locations and sizes over Enceladus’ surface. One can see that the northern plains of Enceladus are heavily cratered, as well as a belt that runs roughly north-south on the anti-Saturnian hemisphere of Enceladus (around 180°W). Conversely, Diyar Planitia, a tectonically deformed area approximately 300 kilometers across and located at approximately 0.5°N, 240°W, shows a relative lack of craters. One possible explanation for the lack of craters seen in Diyar Planitia is that it is actually an ancient South Polar Terrain (SPT) (Helfenstein et al., 2006; Bray et al., 2007) that is now in its present location due to reorientation of the satellite (Nimmo and Pappalardo, 2006).

Plotting the fraction of relaxed craters on Enceladus as a function of crater diameter produces some expected results (Figure 2.12). We see a clear trend that bigger craters tend to relax more than small ones (see Section 3.1). Additionally, there are many more small craters available for study, which means that the Poisson error (±√N) is much smaller than the error on the large craters. In fact, the error is unity for the 20 to 25 km bin in this plot due to the fact that there was only one relaxed crater and no unrelaxed craters in this size category.

We see, in general, an increasing trend in the number of relaxed craters as we
Figure 2.12: Fraction of relaxed craters as a function of crater diameter. Error bars are computed using Poisson statistics. The triangles illustrate the effects of including craters classified as “uncertain”. The upper, upside-down triangles indicate the possibility that all the uncertain craters are actually relaxed, whereas the lower triangles indicate the possibility that all uncertain craters are fresh in reality. The two bins for the largest craters do not contain any craters classified as uncertain.
Figure 2.13: Fraction of relaxed craters as a function of latitude for diameters less than 6 km. Error bars are computed using Poisson statistics. The triangles indicate the error when including craters classified as “uncertain”. The upper, upside-down triangles indicate the possibility that all the uncertain craters are actually relaxed, whereas the lower triangles indicate the possibility that all uncertain craters are fresh in reality.
Figure 2.14: Fraction of relaxed craters as a function of latitude for diameters greater than 6 km. Error bars are computed using Poisson statistics. The triangles indicate the error when including craters classified as “uncertain”. The upper, upside-down triangles indicate the possibility that all the uncertain craters are actually relaxed, whereas the lower triangles indicate the possibility that all uncertain craters are fresh in reality.
move toward the southern latitudes (Figures 2.13 and 2.14). Given the rather low number of craters, it was necessary to divide the crater data up into just two main size classes. Roughly equal numbers of craters were distributed between diameter categories of 4 to 6 km versus 6 km or greater. Figure 2.13 reveals that the fraction of “small” craters (4 to 6 km in diameter) that are relaxed increases moving from northern to southern latitudes. When looking at Figure 2.14 of the fraction of “large” craters (6 km and greater in diameter) that are relaxed, we see a very slight increase moving from north to south, as well as a slight increase in the fraction of large craters relaxed around the equator. However, due to the small number statistics, the error on this seeming trend is large. Given both the Poisson error coupled with the error contribution from a number of craters labeled as “uncertain”, it is hard to determine whether or not a significant trend exists in the fraction of “large” relaxed craters with latitude based on our sample.

Changing the cutoff value between these two size classes of craters from 6 km to 10 km makes the statistics for “large” craters even worse. However, the basic trends do not change. The fraction of small craters that are relaxed still increases moving from northern to southern latitudes, and the trend in the fraction of large relaxed craters is even more uncertain (e.g., the southernmost and equatorial latitude bins only contain one large relaxed crater each in this scenario).

It is important to note that we do observe the southern terrains of Enceladus to contain relaxed craters that demonstrate, qualitatively, a greater degree of relaxation
when compared to relaxed craters in the north. Additionally, the most noticeable collections of relaxed craters are either close to the South Polar Terrain or on the boundaries of heavily fractured regions. These areas may have also experienced higher heat flows during their formation.
CHAPTER 3

Analytic Solutions

The flow behavior of ice is non-Newtonian (e.g., Durham et al., 1997; Petrenko and Whitworth, 1999). However, to first order, we can use the Newtonian approximation in order to produce analytical solutions that reveal the basic behavior of crater relaxation on Enceladus.

3.1 Newtonian Viscous Relaxation Equations

In the solution for the case of a uniform viscosity down to very large depth, the relaxation time of craters is (Melosh, 1989):

\[ t_R = \frac{2\eta k}{\rho g}, \]  

(3.1)

where \( \eta \) is dynamic viscosity, \( k \) is wavenumber, \( \rho \) is the density of the material, and \( g \) is the acceleration due to gravity. One can see some basic behavioral characteristics from this: namely, as the viscosity increases, the relaxation time also increases; as the density of the material or gravity increases, the relaxation time decreases; but, most importantly, large-wavelength features (small wavenumber, \( k \)) will tend to relax faster than small-wavelength features.
The solution for the case in which viscosity decreases exponentially with depth is (Brennen, 1974):

\[ \eta(z) = \eta_0 \exp \left( -\frac{z}{d} \right), \quad (3.2) \]

where \( z \) is depth, and \( d \) is the distance over which the viscosity drops by a factor of \( e \).

The relaxation time for this case is:

\[ t_R = \frac{2\eta_0 k}{\rho g} \left\{ \frac{1 - 2\gamma}{4kd\gamma} \left[ 1 + \left( \frac{\gamma}{kd} \right)^2 \left( 1 - 2\gamma \right) \right] \right\}, \quad (3.3) \]

where \( \gamma \) is the function:

\[ \gamma = (2)^{-\frac{3}{2}} \left[ 1 + 4(kd)^2 + \sqrt{16(kd)^4 + 24(kd)^2 + 1} \right]^{1/2} \quad (3.4) \]

This solution is significant in that, for most planetary objects, the temperature increases linearly with depth. For a material of uniform composition, this corresponds to an approximately exponential decrease of viscosity with depth. Hence, the above equation is highly relevant for our system of interest.

In the short-wavelength limit, topographic features relax at the same rate as for a uniform viscosity mantle. However, in the long-wavelength limit, such features
relax faster. Therefore, in observation of craters following this type of behavior, we should see features such as the crater rim (short wavelength) persist longer than the crater bowl (long wavelength).

The analysis of crater relaxation is greatly simplified due to the fact that most craters are axially symmetric, which means crater profiles can be decomposed into $J_0$ Bessel functions (Melosh, 1989). Each function relaxes self-similarly so that the component remains the same while its amplitude decreases (Melosh, 1989). The crater profile can be represented as

$$z(r,t) = \int_0^\infty \zeta(k) \exp(-t/t_R)J_0(kr)k \,dk,$$

where $\zeta(k)$ is the Hankel transform of the initial crater profile, $k$ is wavenumber ($k = 2\pi/\lambda$), $J_0$ is the cylindrical Bessel function, and $t_R$ is relaxation time. An approximately parabolic profile for a sharp-rimmed crater is assumed for our analytic calculations. Using the solution for the relaxation time and Equation 3.5, crater profiles at various stages of relaxation can be derived.

The viscosity of water ice is also strongly temperature dependent (e.g., Ranalli, 1995; Mitri and Showman, 2008):

$$\eta(T) = \eta_0 \exp \left[ A \left( \frac{T_{\text{ref}}}{T} - 1 \right) \right],$$

(3.6)
where $\eta_0$ is the viscosity of ice at a reference temperature, $T_{\text{ref}}$, of 273 K, the constant $A \approx 26$, which corresponds to an activation energy for ice of 60 kJ/mol, and $T$ is temperature in K.

Another way to write this equation is:

$$\eta(T) = \eta_{\text{surf}} \exp \left[ A T_{\text{ref}} \left( \frac{1}{T} - \frac{1}{T_{\text{surf}}} \right) \right],$$  \hspace{1cm} (3.7)

where $\eta_{\text{surf}}$ is the viscosity at the surface and $T_{\text{surf}}$ is the surface temperature.

Combining this with our exponential dependence of viscosity with depth, we can calculate viscosity e-folding depths, $d$, for a variety of temperature gradients, $dT/dz$, by:

$$d = T_{\text{surf}} \frac{dT}{dz} \left( A T_{\text{ref}} \frac{T_{\text{surf}}}{T_{\text{surf}}} - 1 \right)$$  \hspace{1cm} (3.8)

We can also estimate corresponding heat fluxes from

$$F = k \frac{dT}{dz},$$  \hspace{1cm} (3.9)

where $F$ is heat flux, $dT/dz$ is thermal gradient, and $k$ is the thermal conductivity of ice ($k = (567 \text{ W m}^{-1})/T$) (Klinger, 1980).
Table 3.1: Viscosity e-folding depths and heat fluxes for a range of linear temperature gradients.

<table>
<thead>
<tr>
<th>( \frac{dT}{dz} ) (K/km)</th>
<th>d (km)</th>
<th>F (mW m(^{-2}))</th>
</tr>
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<tbody>
<tr>
<td>2.0</td>
<td>≈ 0.55</td>
<td>≈ 11</td>
</tr>
<tr>
<td>10</td>
<td>≈ 0.11</td>
<td>≈ 57</td>
</tr>
<tr>
<td>20</td>
<td>≈ 0.055</td>
<td>≈ 113</td>
</tr>
<tr>
<td>45</td>
<td>≈ 0.024</td>
<td>≈ 255</td>
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</tbody>
</table>

Enceladus has a mean surface temperature of approximately 75 K (e.g., Porco et al., 2006; Brown et al., 2006), and for our analytic calculations we assume a surface viscosity of approximately \(10^{24}\) Pa s based on the results of Passey (1983). However, the exact nature of Enceladus’ ice is not well understood.

Table 3.1 shows values of e-folding depths and corresponding heat fluxes calculated for a range of temperature gradients using the constant \(A = 26\), a reference temperature of 273 K, and a surface temperature of 75 K. The trend is clear—increasing temperature gradients lead to shorter e-folding depths.

3.2 Results of Analytic Modeling

Using a depth-to-diameter ratio of 1/5 and applying the above equations to craters between 5 and 25 km in diameter on Enceladus, we have considered a suite of basic scenarios. For all of our analytic models, we apply a surface viscosity of approximately \(10^{24}\) Pa s based on the results of Passey (1983). We also assume an acceleration due to gravity of 0.11 m s\(^{-2}\) and a density of water ice of \(≈ 950\) kg m\(^{-3}\).

Figure 3.1 demonstrates how a crater profile (in this case a 25-km diameter
crater) relaxes over various classic relaxation times. We model various viscosity profiles for craters of 5-, 15-, and 25-km diameter for both Newtonian uniform viscosities as well as exponentially decaying profiles. We apply different viscous e-folding depths, $d$, of $5D/2$, $D/2$, and $D/8$, where $D$ is the diameter of the crater, to demonstrate the effect on relaxation time (Figures 3.2, 3.3, and 3.4). The $5D/2$ case represents the most broadly decaying viscosity profile and is very close to the uniform case. One can see, however, that as the viscosity gradient becomes much steeper, the effect on the relaxation time becomes more apparent as the times decrease. We also see the role that crater diameter plays in the relaxation time. For our steepest viscosity case ($D/8$), our relaxation time of approximately $6 \times 10^{10}$ years for our 5-km diameter crater drops to about $10^{10}$ years for a crater of 25 km. This is consistent with our previous discussion regarding long-wavelength features relaxing faster.
Crater Relaxation (Analytic): $D = 25$ km, Uniform Newtonian Viscosity $\eta_0 = 10^{24}$ Pa-sec

Figure 3.1: Profile of a 25-km diameter crater relaxed over several classic relaxation times, $t_R$, using a uniform surface viscosity of $10^{24}$ Pa-sec, surface gravity of 0.11 m s$^{-2}$, and density of water ice of $\approx 950$ kg m$^{-3}$. A certain amount of upbowing of the crater floor can be seen as the crater becomes more and more relaxed.
Figure 3.2: Plot of crater center depth as a function of time for a 5-km-diameter crater for a variety of viscosity profiles assuming a surface viscosity of $10^{24}$ Pa-sec, surface gravity of 0.11 m s$^{-2}$, and density of water ice of $\approx 950$ kg m$^{-3}$. For reference, a horizontal line has been drawn at the relaxation fraction corresponding to the classic relaxation time, $t_R$ ($RF \approx 0.632$).
Crater Center Relaxation for Various Viscosity Profiles (Analytic): $D = 15$ km, $\eta_0 = 10^{24}$ Pa-sec

Figure 3.3: Plot of crater center depth as a function of time for a 15-km-diameter crater for a variety of viscosity profiles assuming a surface viscosity of $10^{24}$ Pa-sec, surface gravity of $0.11$ m s$^{-2}$, and density of water ice of $\approx 950$ kg m$^{-3}$. For reference, a horizontal line has been drawn at the relaxation fraction corresponding to the classic relaxation time, $t_R$ ($RF \approx 0.632$).
Figure 3.4: Plot of crater center depth as a function of time for a 25-km-diameter crater for a variety of viscosity profiles assuming a surface viscosity of $10^{24}$ Pa-sec, surface gravity of 0.11 m s$^{-2}$, and density of water ice of $\approx 950$ kg m$^{-3}$. For reference, a horizontal line has been drawn at the relaxation fraction corresponding to the classic relaxation time, $t_R$ ($RF \approx 0.632$).
CHAPTER 4

Finite-Element Modeling

Newtonian viscosity is independent of the state of stress, and its behavior can be represented analytically (Melosh, 1989). We have already shown that analytic solutions to relaxation can be derived in the Newtonian case by decomposition of the crater topography into $J_0$ cylindrical Bessel functions. If these modes are damped exponentially with a characteristic relaxation time, the first-order evolution of crater topography subject to viscous relaxation is reproduced (Smith et al., 2007).

However, ice is non-Newtonian (e.g., Durham et al., 1997; Petrenko and Whitworth, 1999), which means that its viscosity is actually stress dependent. Analytical modeling, although good to first order, fails in this regime (Smith et al., 2007). To more accurately model Enceladus’ icy lithosphere, it is necessary to employ numerical modeling. We thus explore crater relaxation on Enceladus using TEKTON, a finite-element analysis tool (Melosh and Raefsky, 1980).

4.1 Overview

The finite-element (FE) method is very useful for calculating stresses and strains in a structure under some applied stress or strain. In the FE method, a structure is divided up into smaller units called elements, and each element has its own set of
characteristic properties. The displacements and constitutive relation of each element govern its stress-strain behavior. Neighboring elements must satisfy displacement continuity and be in stress equilibrium. Each of these elements is constructed from a set of nodes that define the elements’ boundaries, and the whole system of nodes and elements connected together makes up the entire mesh. This method is useful in the solution of complex geometries for physical problems (Becker, 2004).

Instabilities may result in a system undergoing a large amount of deformation if one individual element suffers too much deformation. To prevent these instabilities from occurring, the mesh must be designed such that the element aspect ratios and gradients across the elements in the material properties are small in regions that undergo a large amount of strain. As long as the strain is distributed over a large number of elements, these instabilities may not occur. Each element in the finite-element mesh is allowed to have its own unique material and physical properties, which allows extremely complex systems to be studied. It is thus very suitable for application to practical geophysical and engineering problems.

The system (or specifically, the geologic structure) is first divided up into elements that are given individual material and physical properties, and the nodes that define the elements are given specific initial locations. Boundary conditions for these nodes are also assigned. The problem can then be described mathematically
by a system of equations:

$$Ku = f$$  \hspace{1cm} (4.1)$$

where $K$ is the stiffness matrix, $u$ is the vector of node displacements, and $f$ is the vector of forces applied to the nodes. This system of equations is then solved simultaneously, and, with each time-step through the simulation, the force vector and stiffness matrix are updated based on the previous time-step’s nodal displacements and stresses in the elements. The system of equations is then solved again. This process is repeated over the number of time-steps specified until the system reaches equilibrium or numerical instabilities develop (i.e., elements become overly deformed).

4.2 About TEKTON

TEKTON is a finite-element model code written by Melosh and Raefsky (1980). The code operates via an input file that contains data about the mesh (nodes and elements and their initial locations), boundary conditions, external forces, and material and rheologic properties. Once the problem is defined, TEKTON calculates the stiffness matrix and produces an elastic solution of displacements and stresses compatible with the applied loads and boundary conditions. On subsequent time-steps, it calculates the viscous solution by iteratively solving the system of equations.
described previously. TEKTON requires user-specification of the time-steps, which makes it very flexible in that both the number and size of each time-step can be defined. This allows materials with very low viscosities to be studied, since the time step can be as small as necessary in order to be shorter than the Maxwell time of the material. Here, we only model viscoelastic crater relaxation, although TEKTON is capable of handling a variety of other material behaviors, such as plastic yielding and flow.

Craters, when formed by projectiles with incidence angles of $> 10^\circ$ retain their circularity as shown in laboratory experiments and observational evidence (Melosh, 1989). Therefore, we have used the axially symmetric version of Tekton to model crater relaxation. This simplifies our problem to a single radial cross-section of the crater mesh.

4.2.1 Benchmarking to Analytic Theory

We have successfully benchmarked TEKTON to our analytic calculations for Newtonian viscosity cases for 5, 15, and 25-kilometer diameter craters (Figure 4.1). A surface viscosity of $10^{24}$ Pa-sec was adopted following the results of Passey (1983). Viscous creep should dominate for times much longer than the Maxwell time ($\tau_M \equiv 2\eta(1 + \nu)/E$), and elastic effects are clearly negligible. Also, topographic amplitudes, not elastic parameters, determine the driving stresses as argued by Dom- bard and McKinnon (2006). Classic relaxation times for these Newtonian viscosity
cases are very long, on the order of $\sim 10^{11}$ years and in excess of the age of the solar system.

4.3  Viscoelastic Crater Relaxation Model

The following sections describe the details of our relaxation model for craters on Enceladus.

4.3.1  Crater Mesh and Boundary Conditions

The axial symmetry of most craters greatly simplifies the geometry of our problem. The mesh generated is 5 crater radii wide by 5 crater radii deep, where the crater radius is considered to be one-half of the crater rim-to-rim diameter. The mesh is much larger than the crater so that the boundaries at the edge do not significantly affect the solution. Following the work of Dombard and McKinnon (2006), elements are generated such that many elements are concentrated near the surface and the crater floor where most of the deformation occurs. This prevents instabilities in the finite-element code from forming in regions of high stress gradients.

An example of a crater mesh used in these simulations can be seen in Figure 4.2. Since our mesh is axially symmetric, the axis of symmetry cannot move in the horizontal direction by definition. Hence, nodes along this line are allowed to move vertically, but not horizontally. The same set of boundary conditions is applied to the outer edge of the mesh, but the large size of the mesh relative to the crater
Figure 4.1: A comparison between analytic Newtonian viscosity cases and those simulated by TEKTON. For reference, a horizontal line has been drawn at the relaxation fraction corresponding to the classic relaxation time, $t_R \ (RF \approx 0.632)$. 

**Crater Center Relaxation vs. Time: Uniform Newtonian Viscosity ($10^{24}$ Pa-sec)**
Figure 4.2: Example crater mesh used in TEKTON.
means that very little deformation is present at this edge anyway. At the very bottom boundary of the mesh, we fix both the vertical and horizontal directions, although, again, no motion is expected to occur here if the mesh is sufficiently deep. The surface, of course, is free of stresses, so nodes there are allowed to move either vertically or horizontally.

4.3.2 Initial Stresses

Relaxation occurs as a result of loading due to topography; i.e., a system with either an excess or deficiency of mass will ultimately settle to an equipotential surface under gravitational forces (Turcotte and Schubert, 2002). Initial hydrostatic stresses in TEKTON can be set using one of the two following methods, and both compute the stresses at the center of each element.

Dombard and McKinnon (2000) propose an initial hydrostatic stress state of

\[ \sigma_{rr} = \sigma_{\theta\theta} = \sigma_{zz} = -\rho g \left[ z - h(r) \exp \left( \frac{z - h(r)}{D/2\pi} \right) \right], \quad (4.2) \]

where \( \rho \) is density, \( g \) is the acceleration due to gravity, \( z \) is the average vertical depth in the mesh, \( D \) is rim-to-rim diameter, and \( h(r) \) is the initial topography. Equation 4.2 contains an exponential that corresponds to the exponential decay of the effect of topography with an \( e \)-folding depth of \( D/2\pi \) (Dombard and McKinnon, 2006). An example of stresses due to planar, periodic topography can be found
in Jaeger and Cook (1979).

Initial hydrostatic stresses can also be set by assuming an even simpler stress state given by:

\[ \sigma_{rr} = \sigma_{\theta\theta} = \sigma_{zz} = -\rho g z, \]  

(4.3)

where, again, \( \rho \) is density, \( g \) is the acceleration due to gravity, and \( z \) is the average vertical depth in the mesh.

Simulations run using both methods were consistent in producing the same results for the relaxation times. This is shown in Figure 4.3.

### 4.3.3 Temperature

The thermal gradient beneath the crater is assumed to be linear (that is, we assume pure conduction in a uniform material) with no internal heat generation:

\[ T = T_s + \left( \frac{dT}{dz} \right) z, \]  

(4.4)

where \( T_s \) is the surface temperature (K), \( dT/dz \) is the temperature gradient (K/km), and \( z \) is depth (km). We consider cases ranging from zero heat flow to the highest probable temperature gradients seen in Enceladus’ South Polar Region. Surface temperatures on Enceladus range from approximately 34 K to 145 K with an average
Figure 4.3: A comparison of different initial stress states set in TEKTON and effects on relaxation time. The two initial stress profiles do not significantly affect the crater evolution. For reference, a horizontal line has been drawn at the relaxation fraction corresponding to the classic relaxation time, $t_R$ ($RF \approx 0.632$).
of about 75 K (Porco et al., 2006).

Temperature cannot increase indefinitely with depth, however, so we apply an adiabatic cutoff temperature of 180 K. This is also necessary to maintain numerical stability as higher temperatures decrease the viscosity to the point where instabilities develop.

4.3.4 Ice I Rheology

All of the current evidence suggests that Enceladus is made up of mostly pure water ice (Porco et al., 2006). Therefore, for these relaxation models, we use elastic constants of water ice determined through Brillouin spectroscopy (Gammon et al., 1983). The two constants used are Young’s modulus, $E$, and Poisson’s ratio, $\nu$. For these simulations we assume $E = 9.33$ GPa and $\nu = 0.325$.

We assume that the viscous relaxation is due only to steady-state secondary creep, and mathematically this rheology is given by a power-law relationship of the form (Goldsby and Kohlstedt, 2001):

$$
\dot{\epsilon} = A(\delta)^{-m} \sigma^n \exp\left(\frac{-Q - PV}{RT}\right),
$$

where $\dot{\epsilon}$ is the strain rate due to a differential stress, $\sigma$, and $\delta$ is grain size. $A$, $m$, and $n$ are the material-specific pre-exponential constant, grain-size index, and power-law index, respectively. $Q$ and $V$ are the activation energy and volume, and
$P$ and $T$ are pressure and temperature. Because the activation energies of creep processes in ice are much higher than the activation volume energies, we ignore the activation volume term in our models (Durham et al., 1997).

The total strain rate is a combination of both elastic and power-law viscous rheologies given by:

\[ \dot{\varepsilon}_{\text{total}} = \dot{\varepsilon}_{\text{elastic}} + \dot{\varepsilon}_{\text{viscous}} \] (4.6)

The combination of three microphysical deformation mechanisms results in a composite flow law for strain in Ice I (Goldsby and Kohlstedt, 2001; Durham and Stern, 2001),

\[ \dot{\varepsilon}_{\text{viscous}} = \dot{\varepsilon}_{\text{diff}} + \dot{\varepsilon}_{\text{GSS}} + \dot{\varepsilon}_{\text{disl}} \] (4.7)

where $\dot{\varepsilon}_{\text{viscous}}$ is the strain rate from volume diffusion (diff), grain-size-sensitive creep (GSS), and dislocation creep (disl). The strain rate of each deformation mechanism depends on temperature, stress, and grain size as given in Equation 4.5. Table 4.1 gives the relevant rheological parameters necessary to describe each regime.

Equation 4.7 describes three separate independent creep mechanisms. In this scenario, the mechanism with the highest strain rate will dominate the overall flow (Durham and Stern, 2001). However, GSS creep is actually a combination
Table 4.1: Rheological Data for Ice Ih

<table>
<thead>
<tr>
<th>Regime</th>
<th>$n$</th>
<th>$m$</th>
<th>$Q, \text{kJ mol}^{-1}$</th>
<th>$A, \text{MPa}^{-n} \text{s}^{-1}$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime A ($&gt;240$ K)</td>
<td>$4 \pm 0.6$</td>
<td>$91 \pm 2$</td>
<td>$10^{11.8\pm0.04}$</td>
<td>Kirby et al. (1987)</td>
<td></td>
</tr>
<tr>
<td>Regime B</td>
<td>$4 \pm 0.1$</td>
<td>$61 \pm 2$</td>
<td>$10^{5.1\pm0.03}$</td>
<td>Kirby et al. (1987)</td>
<td></td>
</tr>
<tr>
<td>Regime C ($&lt;195$ K)</td>
<td>$6 \pm 0.4$</td>
<td>$39 \pm 5$</td>
<td>$10^{-3.8}$</td>
<td>Durham et al. (1997)</td>
<td></td>
</tr>
<tr>
<td>GBS</td>
<td>$1.8$</td>
<td>$1.4$</td>
<td>$49 \pm 1$</td>
<td>$3.9 \times 10^{-3}$</td>
<td>Goldsby and Kohlstedt (2001)</td>
</tr>
<tr>
<td>ES</td>
<td>$2.4$</td>
<td>$60(+2, -5)$</td>
<td>$5.5 \times 10^{7}$</td>
<td>Goldsby and Kohlstedt (2001)</td>
<td></td>
</tr>
</tbody>
</table>
of two dependent creep mechanisms: GBS-accommodated basal slip (GBS) and basal slip-accommodated GBS (basal) (Goldsby and Kohlstedt, 2001). Since these mechanisms operate dependently, the combined GSS creep law is

\[ \dot{\epsilon}_{\text{GSS}} = \left( \frac{1}{\dot{\epsilon}_{\text{basal}}} + \frac{1}{\dot{\epsilon}_{\text{gbs}}} \right)^{-1}, \]

(4.8)

where subscripts refer to basal or easy slip (basal) and grain-boundary sliding (gbs), respectively. In this case, the overall GSS creep rate will be dominated by the slower mechanism (Durham and Stern, 2001; Goldsby and Kohlstedt, 2001).

Since the pre-exponential constants given in Table 4.1 were measured under uniaxial strain or stress experiments, the values must be multiplied by a factor such that (Ranalli, 1995)

\[ A_m = \frac{3^{(n+1)/2}}{2} A. \]

(4.9)

Combining Equations 4.5 and 4.9 leads to a general relation for the viscosity of the material:

\[ \eta = \frac{\delta^m}{3A_m} \sigma^{1-n} \exp \left( \frac{Q}{RT} \right). \]

(4.10)

One can see right away that for Newtonian viscosities, \( n = 1 \), and viscosity is just
a simple function of temperature. For non-Newtonian cases, $n > 1$, so the viscosity is also dependent on stress. The specific value of $n$ will determine the degree of sensitivity. At planetary temperatures and stresses (low), the dominant mechanism is expected to be grain-size-sensitive (GSS) creep (Goldsby and Kohlstedt, 2001; Durham and Stern, 2001), and we assume this for purposes of our simulations. This is also discussed further and justified in Section 5.1.
CHAPTER 5

Modeling Results

We apply our viscoelastic model to craters on Enceladus using a gravitational acceleration of 0.11 m s\(^{-2}\) and assuming a mean surface temperature of 75 K. We also apply material parameters of water ice consistent with the temperature and stress regimes in question. Linear temperature gradients from 2 K/km up to 45 K/km are modeled for a range of crater diameters using a grain size of 1 mm in all simulations (Bland and Showman, 2007; Bland et al., 2007). However, temperature cannot increase with depth without bound, so we impose an adiabatic cutoff temperature of 180 K for all simulations. This was also necessary to maintain numerical stability.

We found it convenient to cast results in terms of a relaxation fraction of the apparent crater depth. The relaxation fraction is defined as

\[
RF = 1 - \frac{d_a(t)}{d_a(0)},
\]

where \(d_a(t)\) is the crater center apparent depth at time \(t\), and \(d_a(0)\) is the initial apparent depth. The classic relaxation time is defined as the time when the crater reaches one e-folding depth, hence, the relaxation fraction at the classic relaxation time is
\[ RF(t_{\text{relax}}) = 1 - \frac{1}{e} \approx 0.632. \]  \hspace{1cm} (5.2)

5.1 Grain-size-sensitive (GSS) Creep Model Results

We discussed the rheology of Ice I in Section 4.3.4. GSS creep accommodated by grain-boundary-sliding creep (GBS) dominates at stresses below a few 0.1 MPa for most temperatures and grain sizes of 1 mm (Figure 5.1). We take this to be the dominant creep mechanism for purposes of our simulations. Figure 5.1 justifies these assumptions in that the majority of our simulation space falls into the GBS regime. The differential stress and temperature is plotted for each element over various timesteps as well as over multiple simulations.

One can also see from Figure 5.1 that many data points fall into what might considered the easy or basal slip regime. In order to truly and accurately model the flow of ice at these temperatures and pressures, it would be necessary to model both the flow through GBS-accommodated basal slip as well as basal slip-accommodated GBS together, as they are dependent rate-limiting mechanisms (i.e., the slower one will determine the flow rate) (Durham and Stern, 2001). At low planetary stresses, GBS will be the controlling mechanism, as basal slip is accommodated by GBS (Goldsby and Kohlstedt, 2001) and will follow an \( n = 1.8 \) regime (see Table 4.1). At even lower stresses, GBS is faster than basal slip, and instead, GBS is accommodated by basal slip in the \( n = 2.4 \) regime. It is possible, therefore, that
including basal slip in the simulations, could increase the relaxation times slightly, as the overall creep rate would be limited further.

The basal slip regime is only a concern at the very lowest differential stresses and lowest temperatures. Therefore, basal slip would be the dominate creep mechanism in areas that are cold and where horizontal and vertical stresses are very close in magnitude. These areas in the mesh correspond to elements far away from the crater where both temperatures and differential stresses are small. This would not be expected to significantly affect the overall crater relaxation, and it is not, therefore, expected that including basal slip would change our results significantly. The effect of basal slip on our simulations is also expected to be much smaller than other uncertain parameters in our simulations (e.g., grain size, ice composition, thermal gradient, etc.).

Figure 5.2 illustrates an example of a deformed mesh from a crater relaxation simulation. This case is a 15-km-diameter crater using a surface temperature of 75 K and a temperature gradient of 45 K/km after approximately 9 million years under GSS creep. The upbowed floor and persistent rim that is characteristic of viscous relaxation can be clearly seen (c.f. Section 2.4.2 and Figure 3.1).

Upper limits on the relaxation times for craters on Enceladus could be determined by modeling cases of zero heat flow. However, simulations were run in an attempt to achieve this, but even the largest diameter case (25 km) using a surface temperature of 75 K did not relax in times less than the age of the solar system.
Figure 5.1: Relevant creep regimes on Enceladus are demonstrated through the use of a differential stress (in MPa) versus Temperature (in K) plot. Data were plotted for each simulation for every element at every time in the mesh. Approximate boundaries of the Ice I deformation map for polycrystalline ice of 1-mm grain size are drawn after Durham and Stern (2001). One mechanism of deformation will dominate the strain rate for a given set of conditions. Three possible creep regimes are shown here: dislocation creep, grain-boundary sliding (GBS, also referred to as grain-size-sensitive or GSS) creep, and basal slip.
Figure 5.2: A 15-km-diameter crater mesh relaxed after $9 \times 10^6$ yr using a surface temperature of 75 K and temperature gradient of 45 K/km.
The upper limit is, therefore, much longer than the age of the solar system, and longer simulations in order to determine the exact timescale were not carried out.

Figure 5.3 shows our model results for relaxation under a warm temperature gradient of 45 K/km for crater diameters ranging from 5 kilometers up to 25 kilometers. Here, a 25-km crater has a relaxation time of roughly $1.5 \times 10^6$ yrs, whereas a 7-km crater takes around $2 \times 10^{10}$ yrs to relax, and we see the large wavelength effect on relaxation time that was discussed in Section 3.1. Even at such a high temperature gradient, a 5-km crater only relaxes by approximately 1% after $\sim 10^{11}$ years. The smallest crater to relax appreciably in a geologically plausible amount of time was the 10-km case at $\approx 3 \times 10^7$ years. A warm surface temperature coupled with a warm temperature gradient as in these models may be representative of conditions encountered in Enceladus’ south polar region. Therefore, it is plausible that 10-km diameter craters and greater could be relaxed away within about 10 million years, if not erased due to resurfacing before then.

Intermediate temperature gradients of 20 K/km (Figure 5.4) and 10 K/km (Figure 5.5) were also modeled. A gradient of 20 K/km produces a relaxation time for a 25-km-diameter crater close to $2 \times 10^6$ years, but a 15-km crater takes $10^{10}$ years under this lower heat flow. Lowering the temperature gradient another factor of two to 10 K/km produces even more drastic results in that the 25-km crater only relaxes by about 10% after roughly $5 \times 10^9$ years (Figure 5.5).

A very low temperature gradient of 2 K/km was also used to model the 25-km
diameter crater case for completeness (Figure 5.6). One can clearly see here that there is no appreciable amount of relaxation even after $10^{10}$ years. The low mean surface temperature of Enceladus (75 K) combined with such a low temperature gradient does not produce plausible conditions for relaxation of even the largest craters observed on Enceladus. We expect to see retention of craters at conditions such as these. Thus, we would also expect areas with lower surface temperatures (i.e., the north polar regions, especially during northern winter) under low temperature gradient conditions to have high crater retention ages.
Figure 5.3: Relaxation fraction as a function of time for a few different crater diameters seen on Enceladus. A surface temperature of 75 K and a grain size of 1 mm are assumed. A linear temperature gradient 45 K/km is applied with an adiabatic cutoff at 180 K. Relaxation times are shortest for larger craters and longer than the age of the solar system for very small craters on Enceladus. For reference, a horizontal line has been drawn at the relaxation fraction corresponding to the classic relaxation time, $t_R$ ($RF \approx 0.632$).
Figure 5.4: Relaxation fraction as a function of time for a few different crater diameters seen on Enceladus. A surface temperature of 75 K and a grain size of 1 mm are assumed. A linear temperature gradient 20 K/km is applied with an adiabatic cutoff at 180 K. Relaxation times are slower than in 45 K/km case, and a 5-km-diameter crater could not be relaxed in an appreciable amount of time. For reference, a horizontal line has been drawn at the relaxation fraction corresponding to the classic relaxation time, $t_R$ ($RF \approx 0.632$).
Figure 5.5: Relaxation fraction as a function of time for a few different crater diameters seen on Enceladus. A surface temperature of 75 K and a grain size of 1 mm are assumed. A linear temperature gradient of 10 K/km is applied with an adiabatic cutoff at 180 K. Relaxation times are so long in this scenario that even the 25-km case would not relax in a geologically relevant timescale. For reference, a horizontal line has been drawn at the relaxation fraction corresponding to the classic relaxation time, $t_R (RF \approx 0.632)$. 
Figure 5.6: Relaxation fraction as a function of time for a 25-km crater on Enceladus. A surface temperature of 75 K and a grain size of 1 mm are assumed. A linear temperature gradient of 2 K/km is applied with an adiabatic cutoff at 180 K. For reference, a horizontal line has been drawn at the relaxation fraction corresponding to the classic relaxation time, $t_R$ ($RF \approx 0.632$).
CHAPTER 6

Discussion and Conclusions

6.1 Summary

We study the long-term relaxation of impact crater topography on Enceladus using a viscoelastic rheological model. Impact craters on Enceladus display a continuum of relaxation states over a wide range of crater diameters, from fresh, unmodified to relaxed craters. Our simulations produce results that either predict the retention of crater topography over long timescales or elimination of topography over relatively short geologic timescales. Therefore, we can reproduce the plethora of relaxation states observed.

High heat flow conditions (thermal gradients of order 45 K/km) for large (approximately > 10 km) diameter craters can effectively eliminate topography within roughly tens of millions of years for our models (see Figure 5.3). High heat flows have been observed in the south polar regions of Enceladus (Spencer et al., 2006) and were, perhaps, present in other regions of Enceladus sometime in its early history. Lowering the heat flow produces drastically different results, and, under low temperature gradients and surface temperature conditions, it is very difficult to relax topography even for large craters.
Recent models by Bland et al. (2007) of Sarandib and Diyar Planitia, regions near Enceladus’ equator, suggest heat flows of 110 to 220 mW m$^{-2}$ during their formation. This implies intense, localized heating in this region during a period of extension. These regions contain extensive ridges and troughs over large parts of the areas between 225 to 330°W and 20°S to 35°N. Our observations of craters in these regions reveal that there is indeed a relative lack of craters. The few craters that we do observe in these areas are distinctly small, fresh, and overlay topography, suggesting recent formation. The fact that these regions probably experienced high thermal gradients in their history ($\sim$ 40 K/km) supports both erasure of craters in this region from viscous relaxation as well as from resurfacing due to the tectonic activity.

Strong, localized heating of these regions is consistent with observations of groups of relaxed craters that border these tectonized regions. These craters are likely to have experienced higher heat flows in the past due to their close proximity to the planitia. This could explain why craters adjacent to these areas exhibit clear signs of relaxation, but crater relaxation drops off as one moves farther away. It has also been suggested that the planitia are older examples of the current south polar terrain (SPT) (Helfenstein et al., 2006) that is in its present location due to satellite reorientation (Nimmo and Pappalardo, 2006).

Observations by Cassini’s Composite Infrared Spectrometer (CIRS) have revealed thermal emissions of 3 to 7 GW in the regions south of 65°S, which cor-
responds to an average heat flux of 250 mW m$^{-2}$ (Spencer et al., 2006). As suggested by Bland et al. (2007), the ancient heat fluxes deduced for the formation of Sarandib and Diyar Planitiae are generally consistent with these measurements for the south polar region. Our data show a distinct lack of craters in the south polar regions. It must again be noted, however, that it is unclear whether craters here have simply been erased due to tectonic activity or by viscous relaxation. In either scenario, a high local heat flux is required. Additionally, the present plume at the South Pole (Porco et al., 2006) also introduces another uncertainty, as plume material falling back onto Enceladus’ surface under gravity may be blanketing and infilling craters at the South Pole. Some plume material may even be blanketing craters at slightly higher southern latitudes, leading to minor in-fill of craters, and further confusing their relaxation state.

An average global surface heat flux for Enceladus of 5 mW m$^{-2}$, corresponding to a thermal gradient of $\sim 1$ K/km, was estimated by Ross and Schubert (1989) from estimates of total power by tidal heating of Enceladus in its current orbital state. Results of our models indicate that such a low heat flow should predict retention of craters over billion year timescales. CIRS also observed Enceladus’ North Pole (currently in northern winter) to have an estimated surface temperature of approximately 33 K (Spencer et al., 2006), and ruled out the presence of a north polar hotspot. These low temperatures hence suggest high crater-retention ages at northern latitudes and the lack of a north polar hot spot similar to the one at the
South Pole. Indeed, these regions appear to contain the most heavily populated areas of craters on Enceladus, given currently available data.

Our observations also reveal puzzling examples of small but nonetheless relaxed craters at northern latitudes. The fact that we see both relaxed (though only a few examples) as well as unrelaxed craters of similar sizes less than 25-km in diameter in the northern latitudes suggests that this area may have experienced a change in thermal gradient over time (i.e., a warmer ancient thermal gradient). Our models suggest that, even for moderate thermal gradients at a mean surface temperature of 75 K, craters of 15-km diameter or smaller do not relax in less than tens of billions of years. Surprisingly, we do see some evidence of relaxation in a few craters of this size in the northern latitude regions. However, we also see evidence of fresh craters of this size in this same area. The presence of these few examples of relaxed craters scattered within a majority of fresh craters of various sizes would seem to indicate that these craters are much older and experienced a higher heat flow at some time in their past.

In summary, our results suggest an overall increase in relaxation and the fraction of relaxed craters moving from northern to southern latitudes, ending with a distinct lack of craters in the entire south polar region. These results are consistent with relaxation under high heat flows and high surface temperatures leading to elimination of topography in this region. Conversely, the north polar region is heavily cratered and has a much colder surface temperature and thermal gradient. These
cold conditions make it very difficult to relax topography on geologically plausible timescales, thus leading to high crater retention ages. The fact that we do observe some relaxed craters in these northern latitudes suggests that these areas most likely experienced higher heat conditions in the past.

Reorientation of the satellite (Nimmo and Pappalardo, 2006) either by an ancient diapir (Helfenstein et al., 2006) or some other mechanism could explain the large range in inferred ancient heat conditions as well as current heat conditions in explaining the diversity of terrains observed on Enceladus. Localized tidal dissipation may explain the high heat flux observed at Enceladus’ South Pole, leading to upwelling of a low-density diapir. This diapir would lead to the large amount of surface deformation that is indeed observed at Enceladus’ south polar region (Nimmo and Pappalardo, 2006). A similar ice diapir at an ancient South Pole may have led to the formation of the Sarandib and Diyar Planitiae, subsequently causing reorientation (Helfenstein et al., 2006; Nimmo and Pappalardo, 2006; Bland et al., 2007). This reorientation may have even been the latest in a series of reorientations throughout Enceladus’ history (Nimmo and Pappalardo, 2006; Bland et al., 2007). Overall, our current observations for the distribution of relaxed craters on the surface of Enceladus coupled with models of crater relaxation are consistent with the planitiae being older examples of the South Polar Terrain (SPT). The fact that our models are consistent with formation models for SPT-type terrains supports the reorientation hypothesis.
6.2 Future Work

This work could be expanded on in several different ways. The most obvious of these is to make further observations of Enceladus' surface from future Cassini observations. Filling in gaps in our data sample would allow for a more thorough analysis of the distribution of crater morphologies over Enceladus' surface. Similarly, previously acquired Voyager images could provide data as a supplement to available Cassini coverage.

Additionally, further simulations in the modeling of these craters could be performed. We have not investigated varying grain sizes, though this may have a significant effect on the relaxation times. It is also possible to incorporate a multiple flow law element library into TEKTON (e.g., Bland et al., 2007). This would allow investigations to simultaneously model the multiple flow mechanisms relevant for water ice.
REFERENCES


